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Parallel and Nonparallel Distributed Compensation Controller Design for T-S Fuzzy Discrete Singular Systems With Distinct Difference Item Matrices

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ABSTRACT The issues of admissibility and controller design for a class of extended T-S fuzzy discrete singular systems (FDSSs) with distinct difference item matrices are discussed in this article. A new augmented system which is the equivalent of the original system is introduced to convert these distinct difference item matrices into the easy treatment form. On this basis, using the fuzzy Lyapunov function (FLF), relaxed sufficient condition is given to ensure the admissibility of unforced systems. This condition is described via strict linear matrix inequalities (LMIs), which facilitates to analyze the admissibility. Meanwhile, the design methods of parallel and nonparallel distributed compensation (PDC and Non-PDC) controllers are also proposed. Finally, the advantages of the developed admissibility and controller design approach are illustrated by three examples.

INDEX TERMS PDC and Non-PDC controller, FDSSs, fuzzy Lyapunov function, distinct difference item matrices, augmented system method.

I. INTRODUCTION

Since T-S fuzzy systems [1] are used as a nonlinear function approximation tool, such systems are the wide variety of applications for the control problems of nonlinear systems. Stability is a prior condition of the control system, so stability analysis and control problems of T-S fuzzy continuous/discrete systems have attracted a lot of attention [2]–[5]. Lyapunov stability theory [6]–[8] is the most effective method to solve the stability problems of nonlinear systems. So, using common Lyapunov function (CLF), stability and stabilisation problems of fuzzy systems are solved based on a series of LMIs [9]. But this approach falls into conservatism as the reason that these LMIs need to find common positive definite matrix. Then, fuzzy Lyapunov function (FLF) [10], as a useful tool to reduce the conservatism, is introduced to analyze the stability of these systems. Furthermore, PDC controller and Non-PDC controller which are the two most important type of state feedback controller are widely used for stabilisation problems.

Singular systems has been developed sufficiently in the past several decade and some significant results have been obtained [11]–[13]. They can better describe physical systems than the normal state-space systems and are widely used in many fields such as power system, biological system, economic system, restricted robot and so on. So, T-S fuzzy singular models are defined via extending the normal forms in [15]. And an example is introduced to intuitively show the advantages of fuzzy singular system compared with fuzzy normal system. Currently, the researches of T-S FDSSs can be roughly classified into two categories based on difference item matrices. One is the same difference item matrices in

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fuzzy rules [16]–[21]. In [16], a projection algorithm is given to solve non-strict LMIs in the stability criteria of FDSSs. In [17], the $\mathcal{D}$-stability concept is promoted to fuzzy systems, then the stability criteria of closed-loop FDSSs with delay is proposed via LMIs. The sliding mode control design method of FDSSs with delay and disturbances is given to ensure the admissibility and H_∞ performance in [18]. Using innovative Lyapunov functions and design method, the relaxed sufficient conditions is proposed such that the FDSSs with time delay is admissible in [19]. Based on the novel approximation method and weighting-based FLF, the admissibility and controller design issues of uncertain FDSSs is discussed in [20]. In [21], robust controller design method for FDSSs described by extended models under bounded parameter uncertainties is presented.

The other one is the distinct difference item matrices in fuzzy rules [22]–[25]. In [22], using quadratic/nonquadratic Lyapunov functions, local stabilization conditions of FDSSs with distinct derivative matrices are given. In [23], using delayed Lyapunov functions, the novel method to design the controller for nonlinear discrete systems based on fuzzy system representation. But in [22], [23], the systematic demand is that the compound difference item matrices $\sum_{i=1}^{r} h_i(\xi(k))E_i$ must be invertible. In [24], based on the CLF, the admissibility analysis and control issues for FDSSs with multiple difference matrices E_i at locally models are discussed, which increased the conservatism. An innovative predictor-based control design method for FDSSs with distinct derivative matrices is given based the augmented system method in [25]. However the equivalent of augmented systems and original systems will not be ensured when compound matrices $\sum_{i=1}^{r} h_i(\xi(k))E_i$ is not invertible. At present, the researches of FDSSs with distinct derivative matrices are not plentiful and the key issue is the way to convert these distinct difference item matrices into the easy treatment form.

Through above analysis, the admissibility analysis and controller design for extended FDSSs with distinct derivative matrices is discussed based on FLF. Firstly, based on the new augmented system method, a relaxed sufficient condition is given for the admissibility of FDSSs via strict LMIs. Then, considering the equivalence between the dual system and the original system, the PDC controller and Non-PDC controller design methods are given to ensure the admissibility for closed-loop FDSSs. Finally, it is showed that the proposed method is more effective and feasible by simulation examples. The contributions of this article are summarized below.

I) The admissibility analysis and synthesis issues for extended fuzzy discrete singular models with distinct difference term matrices in the locally singular models are discussed. And this kind of systems can precisely describe a large class of nonlinear discrete singular systems. Different from the existing results, the new augmented system which is admissibility equivalent to the original system is proposed to deal with these difference term matrices.

II) Using fuzzy Lyapunov function and matrix inequality method, a relaxed admissibility criteria for extended FDSSs is given in terms of strict LMIs. The slack matrices are also introduced into this criteria to reduce the conservatism. Then, both PDC controller and Non-PDC controller design methods are further investigated. Using the equivalence between the dual system and the original system and eliminating the coupling relationship between the system matrix and fuzzy Lyapunov matrix, the controller design becomes simple and effective under the proposed approach.

Notations.

$\mathcal{Q}^{\mathrm{T}}$: transpose of $\mathcal{Q}$;

$\mathtt{Det}(\mathcal{Q})$: determinant of the matrix $\mathcal{Q}$

$\mathcal{Q} \succeq 0/\mathcal{Q} > 0$: positive semi-definite/definite matrix;

$\mathcal{Q} \preceq 0/\mathcal{Q} < 0$: negative semi-definite/definite matrix;

$\mathbb{R}^n$: n-dimensional Euclidean space;

$\mathtt{Deg}()$: degree of the polynomial.

$\mathtt{Rank}(\mathcal{Q})$: rank of the matrix $\mathcal{Q}$.

$\mathbb{R}^{m\times n}$: $m \times n$ real matrices set;

II. PRELIMINARIES

Consider T-S fuzzy singular discrete models with **IF-THEN** rules as follows.

$\mathbb{R}_\kappa$: **IF** $\varsigma_1(k)$ is $\mathbb{M}_{1\kappa}$, **AND** $\varsigma_2(k)$ is $\mathbb{M}_{2\kappa}, \ldots,$ **AND** $\varsigma_p(k)$ is $\mathbb{M}_{p\kappa}$, **Then**

$$\begin{cases} \widehat{\mathbb{E}}_\kappa \widehat{x}(k+1) = \widehat{\mathcal{A}}_\kappa \widehat{x}(k) + \widehat{\mathcal{B}}_\kappa u(k) \\ y(k) = \widehat{\mathcal{C}}_\kappa \widehat{x}(k) + \widehat{\mathcal{D}}_\kappa u(k), \kappa = 1, 2, \cdots, n_r \end{cases} \tag{1}$$

where $\mathbb{M}_{\rho\kappa}(\rho = 1, 2, \ldots, p)$ denotes the fuzzy set, n_r is the **IF-THEN** rule number, $\varsigma(k) = [\varsigma_1(k), \varsigma_2(k), \ldots, \varsigma_p(k)]^{\mathrm{T}}$ are premise variables, $\widehat{x}(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ denote the state/input vector, $y(k) \in \mathbb{R}^p$ denotes the measurable output vector; $\widehat{\mathcal{A}}_\kappa$, $\widehat{\mathcal{B}}_\kappa$, $\widehat{\mathcal{C}}_\kappa$, $\widehat{\mathcal{D}}_\kappa$, $\widehat{\mathbb{E}}_\kappa$ are matrices with appropriate dimension and rank$(\widehat{\mathbb{E}}_\kappa) = n_1 \leq n$.

The overall fuzzy discrete models is given as

$$\begin{aligned} \sum_{\kappa=1}^{n_r} h_\kappa(\varsigma(t))\widehat{\mathbb{E}}_\kappa \widehat{x}(k+1) &= \sum_{\kappa=1}^{n_r} h_\kappa(\varsigma(k))[\widehat{\mathcal{A}}_\kappa \widehat{x}(k) + \widehat{\mathcal{B}}_\kappa u(k)] \\ y(k) &= \sum_{\kappa=1}^{n_r} h_\kappa(\varsigma(k))[\widehat{\mathcal{C}}_\kappa \widehat{x}(k) + \widehat{\mathcal{D}}_\kappa u(k)] \end{aligned} \tag{2}$$

The following assumptions is given about difference item matrices $\widehat{\mathbb{E}}_\kappa$ in system (1).

Assumption 1:

$$\widehat{\mathbb{E}}_\kappa = Q_\kappa \bar{\mathbb{E}}, \kappa = 1, 2, \cdots, n_r$$

where Q_κ are invertible constant matrices.

Then, the augmented system can be given as

$$\begin{aligned} \mathbb{E}x(k+1) &= \sum_{\kappa=1}^{n_r} h_\kappa(\varsigma(k))(\mathcal{A}_\kappa x(k) + \mathcal{B}_\kappa u(k)) \\ y(k) &= \sum_{\kappa=1}^{n_r} h_\kappa(\varsigma(k))(\mathcal{C}_\kappa x(k) + \mathcal{D}_\kappa u(k)) \end{aligned} \tag{3}$$

where

$$\mathbb{E}=\begin{bmatrix}\bar{\mathbb{E}} & 0\\ 0 & 0\end{bmatrix},$$
$$\mathcal{A}_\kappa=\begin{bmatrix}0 & I\\ \widehat{\mathcal{A}}_\kappa & -Q_\kappa\end{bmatrix},\quad \mathcal{B}_\kappa=\begin{bmatrix}0\\ \widehat{\mathcal{B}}_\kappa\end{bmatrix},$$
$$\mathcal{C}_\kappa=\begin{bmatrix}\widehat{\mathcal{C}}_\kappa & 0\end{bmatrix},\quad \mathcal{D}_\kappa=\widehat{\mathcal{D}}_\kappa,\ x(k)=\begin{bmatrix}\widehat{x}(k)\\ x_o(k)\end{bmatrix}$$

For convenience, system (3) is rewritten as

$$\begin{cases}\mathbb{E}\dot{x}(k)=\mathcal{A}_\varsigma x(k)+\mathcal{B}_\varsigma u(k)\\ z(k)=\mathcal{C}_\varsigma x(k)+\mathcal{D}_\varsigma u(k)\end{cases}\tag{4}$$

Definition 2:

- System (2) is regular if
$$\mathtt{Det}(z\mathbb{E}_\varsigma-\widehat{\mathcal{A}}_\varsigma)$$
is not identically zero.
- System (2) is causal if
$$\mathtt{Deg}(\mathtt{Det}(z\mathbb{E}_\varsigma-\widehat{\mathcal{A}}_\varsigma))=\mathtt{Rank}(\mathbb{E}_\varsigma)$$
- System (2) is stable if
$$|\lambda(\mathbb{E}_\varsigma,\widehat{\mathcal{A}}_\varsigma)|<1$$
where $\lambda(\mathbb{E},\mathcal{A})=\{z\mid \mathtt{Det}(z\mathbb{E}-\mathcal{A})=0\}$.
- System (2) is admissible if it is regular, causal and stable.

Remark 1: Considering system (2)-(3), we can get

$$\begin{aligned}&\mathtt{Det}(z\mathbb{E}-\mathcal{A}_\varsigma)\\ &=\mathtt{Det}(z\begin{bmatrix}\bar{\mathbb{E}} & 0\\ 0 & 0\end{bmatrix}-\sum_{\kappa=1}^{n_r}h_\kappa(\varsigma(k))\begin{bmatrix}0 & I\\ \widehat{\mathcal{A}}_\kappa & Q_\kappa\end{bmatrix})\\ &=\mathtt{Det}(\begin{bmatrix}z\bar{\mathbb{E}} & -I\\ -\sum_{\kappa=1}^{n_r}h_\kappa(\varsigma(k))\widehat{\mathcal{A}}_\kappa & -\sum_{\kappa=1}^{n_r}h_\kappa(\varsigma(k))Q_\kappa\end{bmatrix})\\ &=\mathtt{Det}(z\widehat{\mathbb{E}}_\varsigma-\widehat{\mathcal{A}}_\varsigma)\end{aligned}$$

So, system (3) is admissibility equivalent to (2). However, as in [25], the following augmented system method is considered.

$$\mathbb{E}x(k+1)=\sum_{\kappa=1}^{n_r}h_\kappa(\varsigma(k))\mathcal{A}_\kappa x(k)\tag{5}$$

where

$$\mathbb{E}=\begin{bmatrix}I & 0\\ 0 & \end{bmatrix},\quad \mathcal{A}_\kappa=\begin{bmatrix}0 & I\\ \widehat{\mathcal{A}}_\kappa & -\widehat{\mathbb{E}}_\kappa\end{bmatrix}$$
$$x(k)=\begin{bmatrix}\widehat{x}(k)\\ \hat{\dot{x}}(k)\end{bmatrix}$$

Next, we have

$$\begin{aligned}&\mathtt{Det}(z\mathbb{E}-\mathcal{A}_\varsigma)\\ &=\mathtt{Det}(z\begin{bmatrix}I & 0\\ 0 & 0\end{bmatrix}-\sum_{\kappa=1}^{n_r}h_\kappa(\varsigma(k))\begin{bmatrix}0 & I\\ \widehat{\mathcal{A}}_\kappa & \widehat{\mathbb{E}}_\kappa\end{bmatrix})\\ &=\mathtt{Det}(\begin{bmatrix}zI & -I\\ -\sum_{\kappa=1}^{n_r}h_\kappa(\varsigma(k))\widehat{\mathcal{A}}_\kappa & -\sum_{\kappa=1}^{n_r}h_\kappa(\varsigma(k))\widehat{\mathbb{E}}_\kappa\end{bmatrix})\\ &=\mathtt{Det}(\begin{bmatrix}zI & -I\\ -\widehat{\mathcal{A}}_\varsigma & \widehat{\mathbb{E}}_\varsigma\end{bmatrix})\end{aligned}$$

Then, we can find that

$$\mathtt{Det}\widehat{\mathbb{E}}_\varsigma\mathtt{Det}(z\mathbb{E}-\mathcal{A}_\varsigma)=\mathtt{Det}\widehat{\mathbb{E}}_\varsigma\mathtt{Det}(z\widehat{\mathbb{E}}_\varsigma-\widehat{\mathcal{A}}_\varsigma)$$

So, only when $\widehat{\mathbb{E}}_\varsigma$ is nonsingular, this system can be admissibility equivalent to the original system. At this moment, this system can only described the normal nonlinear system rather than singular system.

Remark 2: As in [15], consider the following fuzzy discrete singular model:

$$\sum_{\rho=1}^{n_e}\mu_\rho(\varsigma(t))\widehat{\mathbb{E}}_\rho\widehat{x}(k+1)=\sum_{\kappa=1}^{n_r}h_\kappa(\varsigma(k))[\widehat{\mathcal{A}}_\kappa\widehat{x}(k)+\widehat{\mathcal{B}}_\kappa u(k)]$$

the proposed method in the article can also used for these systems as

$$\mathbb{E}x(k+1)=\sum_{\kappa=1}^{n_r}\sum_{\rho=1}^{n_e}h_\kappa(\varsigma(k))\mu_\rho(\varsigma(k))(\mathcal{A}_{\kappa\rho}x(k)+\mathcal{B}_\kappa u(k))$$

where

$$\mathbb{E}=\begin{bmatrix}\bar{\mathbb{E}} & 0\\ 0 & 0\end{bmatrix},\quad \mathcal{A}_{\kappa\rho}=\begin{bmatrix}0 & I\\ \widehat{\mathcal{A}}_\kappa & -Q_\rho\end{bmatrix},\quad \mathcal{B}_\kappa=\begin{bmatrix}0\\ \widehat{\mathcal{B}}_\kappa\end{bmatrix}$$

Then, the analysis and control problems of such systems can be solved based on the framework of FDSSs with same differential term matrices.

Lemma 3 [26]: Given matrices $\Phi_{ij}\in\Re^{n\times n}$, the following two statements are equivalent.

a)

$$\sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r}h_\iota(\phi(k))h_\kappa(\phi(k))\Phi_{\iota\kappa}>0\tag{6}$$

b)

$$\Phi_{\iota\iota}>0,\quad \iota=1,2,\cdots,n_r,\tag{7}$$
$$\frac{2}{n_r-1}\Phi_{\iota\iota}+\Phi_{\iota\kappa}+\Phi_{\kappa\iota}>0,\quad 1\le\iota\ne\kappa\le n_r\tag{8}$$

III. MAIN RESULT

A. ADMISSIBILITY ANALYSIS

The admissibility theorem of FDSSs (4) is obtained as follows.

Theorem 4: System (4) is admissible if there exist symmetric matrices $P_\kappa\in\mathbb{R}^{2n\times 2n}$, $P_\kappa>0$, $Q^i_{\kappa\kappa}\in\mathbb{R}^{4n\times 4n}$, $Q_\kappa\in\mathbb{R}^{(2n-n_1)\times(2n-n_1)}$, $W^i_{\kappa\rho}\in\mathbb{R}^{4n\times 4n}$ and matrices $Q^i_{\kappa\rho}\in\mathbb{R}^{4n\times 4n}$, $M_\kappa\in\mathbb{R}^{2n\times 2n}$, $H_\kappa\in\mathbb{R}^{2n\times 2n}$, and such that the following LMIs hold.

$$W^i_{\kappa\kappa}>0,\quad i,\kappa=1,2,\cdots,n_r\tag{9}$$

$$\frac{2}{n_r-1}W^i_{\kappa\kappa}+W^i_{\kappa\rho}+W^i_{\rho\kappa} > 0, i,\rho,\kappa=1,2,\cdots,n_r, \rho\neq\kappa \quad (10)$$

$$\Theta^i_{\kappa\kappa}+W^i_{\kappa\kappa} < Q^i_{\kappa\kappa}, \kappa, i=1,2,\cdots,n_r$$

$$\Theta^i_{\kappa\rho}+\Theta^i_{\rho\kappa}+W^i_{\kappa\rho}+W^i_{\rho\kappa} < Q^i_{\kappa\rho}+(Q^i_{\kappa\rho})^{\mathrm{T}}, \rho>\kappa, \kappa,\rho,i=1,2,\cdots,n_r$$

$$\begin{bmatrix} Q^i_{11} & * & \cdots & * \\ Q^i_{12} & Q^i_{22} & & \vdots \\ \vdots & & \ddots & * \\ Q^i_{1n_r} & \cdots & Q^i_{(n_r-1)n_r} & Q^i_{n_rn_r} \end{bmatrix} < 0, i=1,2,\cdots,n_r \quad (11)$$

where

$$\Theta^i_{\kappa\rho}=\begin{bmatrix} -\mathbb{E}^{\mathrm{T}}P_\kappa\mathbb{E}-M^{\mathrm{T}}_\rho\mathcal{A}_\kappa-\mathcal{A}^{\mathrm{T}}_\kappa M_\rho & * \\ -H^{\mathrm{T}}_\rho\mathcal{A}_\kappa+M_\rho & \mathbb{P}_i+H_\rho+H^{\mathrm{T}}_\rho \end{bmatrix}$$

$$\mathbb{P}_i=P_i-\mathbb{E}^{\perp}Q_i(\mathbb{E}^{\perp})^{\mathrm{T}}$$

and full-column rank matrix $\mathbb{E}^{\perp}$ satisfies $\mathbb{E}^{\mathrm{T}}\mathbb{E}^{\perp}=0$

Proof: Based on Lemma 3 and inequalities (9-11), we get

$$\begin{aligned}\Omega &= \begin{bmatrix} -\mathbb{E}^{\mathrm{T}}P_\varsigma\mathbb{E}-M^{\mathrm{T}}_\varsigma\mathcal{A}_\varsigma-\mathcal{A}^{\mathrm{T}}_\varsigma M_\varsigma & * \\ -H^{\mathrm{T}}_\varsigma A^{\mathrm{T}}_\varsigma+M_\varsigma & \mathbb{P}^+_\varsigma+H_\varsigma+H^{\mathrm{T}}_\varsigma \end{bmatrix} \\
&= \sum_{i=1}^{n_r} h^+_i(\varsigma(k))(\sum_{\kappa=1}^{n_r} h^2_\kappa(\varsigma(k))\Theta^i_{\kappa\kappa} \\
&\quad +\sum_{\kappa=1}^{n_r}\sum_{\rho>\kappa}^{n_r} h_\kappa(\varsigma(k))h_\rho(\varsigma(k))(\Theta^i_{\kappa\rho}+\Theta^i_{\rho\kappa})) \\
&< \sum_{i=1}^{n_r} h^+_i(\sum_{\kappa=1}^{n_r} h^2_\kappa(\varsigma(k))\Theta^i_{\kappa\kappa} \\
&\quad +\sum_{\kappa=1}^{n_r}\sum_{\rho>\kappa}^{n_r} h_\kappa(\varsigma(k))h_\rho(\varsigma(k))(\Theta^i_{\kappa\rho}+\Theta^i_{\rho\kappa}) \\
&\quad +\sum_{i=1}^{n_r} h^+_i\sum_{\kappa=1}^{n_r}\sum_{\rho=1}^{n_r} h_\kappa h_\rho W^i_{\kappa\rho} \\
&< \sum_{i=1}^{n_r} h^+_i(\sum_{\kappa=1}^{n_r} h^2_\kappa(\varsigma(k))(\Theta^i_{\kappa\kappa}+W^i_{\kappa\kappa}) \\
&\quad +\sum_{\kappa=1}^{n_r}\sum_{\rho>\kappa}^{n_r} h_\kappa(\varsigma(k))h_\rho(\varsigma(k))(\Theta^i_{\kappa\rho}+\Theta^i_{\rho\kappa}+W^i_{\kappa\rho}+W^i_{\rho\kappa}) \\
&< \sum_{i=1}^{n_r} h^+_i(\sum_{\kappa=1}^{n_r} h^2_\kappa(\varsigma(k))(\varsigma(k))Q^i_{\kappa\kappa} \\
&\quad +\sum_{\kappa=1}^{n_r}\sum_{\rho>\kappa}^{n_r} h_\kappa(\varsigma(k))h_\rho(\varsigma(k))(Q^i_{\kappa\rho}+(Q^i_{\kappa\rho})^{\mathrm{T}})<0\end{aligned}$$

Then pre-multiply and post-multiply Ω with $\Pi=\begin{bmatrix} I & A^{\mathrm{T}}_\varsigma\end{bmatrix}$ and Π^{T} respectively, we can get

$$A^{\mathrm{T}}_\varsigma\mathbb{P}^+_\varsigma A_\varsigma-\mathbb{E}^{\mathrm{T}}P_\varsigma\mathbb{E}<0 \quad (12)$$

Next, two nonsingular matrices $\mathbb{U}$ and $\mathbb{V}$ can be given such that

$$\mathbb{U}E\mathbb{V}=\begin{bmatrix} I_{n_1} & 0 \\ 0 & 0\end{bmatrix}.$$

Accordingly, take

$$\mathbb{V}^{-\mathrm{T}}P_\kappa\mathbb{V}^{\mathrm{T}}=\begin{bmatrix} P_{1\kappa} & P_{2\kappa} \\ P^{\mathrm{T}}_{2\kappa} & P_{4\kappa}\end{bmatrix}, \mathbb{U}\mathcal{A}_\kappa\mathbb{V}=\begin{bmatrix} \mathcal{A}_{1\kappa} & \mathcal{A}_{2\kappa} \\ \mathcal{A}_{3\kappa} & \mathcal{A}_{4\kappa}\end{bmatrix}$$

$$\mathbb{V}^{-\mathrm{T}}\mathbb{E}^+=\begin{bmatrix} 0 \\ I\end{bmatrix}\mathbb{W}$$

where $\mathbb{W}\in\mathbb{R}^{(2n-n_1)\times(2n-n_1)}$ is the nonsingular matrix.

Further, pre-multiplying and post-multiplying (12)with $\mathbb{V}^{\mathcal{T}}$ and $\mathbb{V}$, respectively, it can be obtained by

$$\mathbb{U}\mathcal{A}_\kappa\mathbb{V}=\begin{bmatrix} \star & \star \\ \star & \mathcal{A}^{\mathrm{T}}_{4\varsigma}WQ_\varsigma W^{\mathrm{T}}\mathcal{A}_{4\varsigma}\end{bmatrix}<0$$

Thus $\mathcal{A}_{4\varsigma}$ is non-singular, then system (4) is regular and causal.

Next, choose the FLF as

$$\mathcal{V}(x(k))=x^{\mathrm{T}}(k)\mathbb{E}^{\mathrm{T}}\mathbb{P}_\varsigma\mathbb{E}x(k) \quad (13)$$

By $\mathbb{P}_\kappa=P_\kappa-(\mathbb{E}^{\perp})^{\mathrm{T}}Q_i\mathbb{E}^{\perp}$ and $P_\kappa>0$, we get $\mathbb{E}^{\mathrm{T}}\mathbb{P}_\varsigma\mathbb{E}=\mathbb{E}^{\mathrm{T}}P_\varsigma\mathbb{E}\geq 0$.

Define

$$\triangle\mathcal{V}(x(k))=\mathcal{V}(x(k+1))-\mathcal{V}(x(k))$$

Then, we have

$$\begin{aligned}\triangle\mathcal{V}(x(k)) &= x^{\mathrm{T}}(k+1)\mathbb{E}^{\mathrm{T}}\mathbb{P}^+_\varsigma\mathbb{E}x(k+1) \\ &\quad - x^{\mathrm{T}}(k)\mathbb{E}^{\mathrm{T}}\mathbb{P}_\varsigma\mathbb{E}x(k) \\ &= x^{\mathrm{T}}(k)[\mathcal{A}^{\mathrm{T}}_\varsigma\mathbb{P}^+_\varsigma\mathcal{A}_\varsigma-\mathbb{E}^{\mathrm{T}}\mathbb{P}_\varsigma\mathbb{E}]x(k) \\ &= x^{\mathrm{T}}(k)[\mathcal{A}^{\mathrm{T}}_\varsigma\mathbb{P}^+_\varsigma\mathcal{A}_\varsigma-\mathbb{E}^{\mathrm{T}}P_\varsigma\mathbb{E}]x(k)<0\end{aligned}$$

Therefore, system (4) is stable. Together with the result that system (4) is regular and causal, it follows that the system (4) is admissible. ■

Remark 3: By the proposed augmented system method, the time-varying problem of global system matrix $\sum_{\kappa=1}^{n_r} h_\kappa(\varsigma(t))\widehat{\mathbb{E}}_\kappa$ caused by distinct differential term matrices is solved. Then, combining with fuzzy Lyapunov function, the admissibility theorem of this kind of system is given via strict Lmis. By introducing the new slack matrices $W^i_{\kappa\rho}$, the conservatism of the admissibility theorem is reduced. Meanwhile, the proposed theorem can also be directly applied to the stability analysis of FDSSs with same differential term matrices.

B. FUZZY CONTROLLER DESIGN

In this section, PDC controller design issue for nonlinear system (2) is solved. Based on the PDC method, the state-feedback controller is obtained by

$$\begin{aligned} u(k) &= \sum_{\kappa=1}^{n_r} h_\kappa(\varsigma(k))\widehat{K}_\kappa \hat{x}(k) \\ &= \sum_{\kappa=1}^{n_r} h_\kappa(\varsigma(k))K_\kappa x(k) = K_\varsigma x(k) \end{aligned} \tag{14}$$

where $K_\kappa = [\widehat{K}_\kappa, 0]$.

Then, the closed-loop model is

$$\mathbb{E}x(k+1) = (\mathcal{A}_\varsigma + \mathcal{B}_\varsigma K_\varsigma)x(k) \tag{15}$$

Since

$$\mathtt{Det}(z\mathbb{E}^{\mathrm{T}} - (\mathcal{A}_\varsigma + \mathcal{B}_\varsigma K_\varsigma)^{\mathrm{T}}) = \mathtt{Det}(z\mathbb{E} - (\mathcal{A}_\varsigma + \mathcal{B}_\varsigma K_\varsigma))$$

the dual system is equivalent to the original system under the concern of admissibility.

Then, replacing $(\mathbb{E}, \mathcal{A}_\varsigma)$ by $(\mathbb{E}^{\mathrm{T}}, (\mathcal{A}_\varsigma + \mathcal{B}_\varsigma K_\varsigma)^{\mathrm{T}})$ and selecting $M_\kappa = \begin{bmatrix} Y & \alpha Y \\ M_{21} & M_{12} \end{bmatrix}$, $H_\kappa = \begin{bmatrix} \beta Y & \gamma Y \\ H_{21} & H_{22} \end{bmatrix}$ in Theorem 5, the following theorem is obtained directly.

Theorem 5: System (15) is admissible if there exist scalars $\alpha > 0$, $\beta > 0$, $\gamma > 0$, symmetric matrices $P_\kappa \in \mathbb{R}^{2n\times 2n}$, $P_\kappa > 0$, $Q^i_{\kappa\kappa} \in \mathbb{R}^{4n\times 4n}$, $Q_\kappa \in \mathbb{R}^{(2n-n_1)\times(2n-n_1)}$, $W^i_{\kappa\rho} \in \mathbb{R}^{4n\times 4n}$, and matrices $Q^i_{\kappa\rho} \in \mathbb{R}^{4n\times 4n}$, $M_\kappa = \begin{bmatrix} Y & \alpha Y \\ M_{21} & M_{12} \end{bmatrix} \in \mathrm{R}^{2n\times 2n}$, $H_\kappa = \begin{bmatrix} \beta Y & \gamma Y \\ H_{21} & H_{22} \end{bmatrix} \in \mathrm{R}^{2n\times 2n}$, such that the following LMIs hold.

$$W^i_{\kappa\kappa} > 0, \quad i, \kappa = 1, 2, \cdots, n_r \tag{16}$$

$$\frac{2}{n_r - 1} W^i_{\kappa\kappa} + W^i_{\kappa\rho} + W^i_{\rho\kappa} > 0, i, \rho, \kappa = 1, 2, \cdots, n_r, \rho \neq \kappa \tag{17}$$

$$\begin{aligned} &\Theta^i_{\kappa\kappa} + W^i_{\kappa\kappa} < Q^i_{\kappa\kappa}, \kappa, i = 1, 2, \cdots, n_r \\ &\Theta^i_{\kappa\rho} + \Theta^i_{\rho\kappa} + + W^i_{\kappa\rho} + W^i_{\rho\kappa} < Q^i_{\kappa\rho} + (Q^i_{\kappa\rho})^{\mathrm{T}}, \rho > \kappa, \kappa, \rho, i = 1, 2, \cdots, n_r \\ &\begin{bmatrix} Q^i_{11} & * & \cdots & * \\ Q^i_{12} & Q^i_{22} & & \vdots \\ \vdots & & \ddots & * \\ Q^i_{1n_r} & \cdots & Q^i_{(n_r-1)n_r} & Q^i_{n_r n_r} \end{bmatrix} < 0, i = 1, 2, \cdots, n_r \end{aligned} \tag{18}$$

where

$$\begin{aligned} \Theta^i_{\kappa\rho} &= \begin{bmatrix} -\mathbb{E}P_\kappa\mathbb{E}^{\mathrm{T}} - \Phi - \Phi^{\mathrm{T}} & * \\ -H^{\mathrm{T}}\mathcal{A}^{\mathrm{T}}_\kappa + \bar{I}^{\mathrm{T}}S^T_\rho\mathcal{B}^{\mathrm{T}}_\kappa + M & \mathbb{P}_i + H + H^{\mathrm{T}} \end{bmatrix} \\ \mathbb{P}_i &= P_i - \mathbb{E}^\dagger Q_i(\mathbb{E}^\dagger)^{\mathrm{T}} \\ \Phi &= M^{\mathrm{T}}\mathcal{A}^{\mathrm{T}}_\kappa + \hat{I}^{\mathrm{T}}S^T_\rho\mathcal{B}^{\mathrm{T}}_\kappa \\ \bar{I} &= [I, \alpha I], \hat{I} = [\beta I, \gamma I] \end{aligned}$$

and full-column rank matrix $\mathbb{E}^\dagger$ satisfies $\mathbb{E}\mathbb{E}^\dagger = 0$.

Next, Non-PDC controller is consider as

$$\begin{aligned} u(k) &= \sum_{\kappa=1}^{n_r} h_\kappa(\varsigma)\widehat{K}_\kappa M^{-1}_{\varsigma 11}\hat{x}(k) \\ &= \sum_{\kappa=1}^{n_r} h_\kappa(\varsigma)K_\kappa \begin{bmatrix} M_{\varsigma 11} & 0 \\ M_{\varsigma 21} & M_{\varsigma 22} \end{bmatrix}^{-1} x(k) \\ &= K_\varsigma M^{-1}_\varsigma x(k) \end{aligned} \tag{19}$$

where $K_\kappa = [\widehat{K}_\kappa, 0]$.

Then, the closed-loop model is

$$\mathbb{E}x(k+1) = (\mathcal{A}_\varsigma + \mathcal{B}_\varsigma K_\varsigma M^{-1}_\varsigma)x(k) \tag{20}$$

Next, replacing $(\mathbb{E}, \mathcal{A}_\varsigma)$ by $(\mathbb{E}^{\mathrm{T}}, (\mathcal{A}_\varsigma + \mathcal{B}_\varsigma K_\varsigma M^{-1}_\varsigma)^{\mathrm{T}})$ and selecting $H_\kappa = \alpha M_\kappa$ and $M_\kappa = \begin{bmatrix} M_{\kappa 11} & 0 \\ M_{\kappa 21} & M_{\kappa 22} \end{bmatrix}$ in Theorem 4, the following result can be obtained directly.

Theorem 9: System (20) is admissible if there exist a scalar $\alpha > 0$, symmetric matrices $P_\kappa \in \mathbb{R}^{2n\times 2n}$, $P_\kappa > 0$, $Q^i_{\kappa\kappa} \in \mathbb{R}^{4n\times 4n}$, $Q^i_{\kappa\kappa} < 0$, $Q_\kappa \in \mathbb{R}^{(2n-n_1)\times(2n-n_1)}$, $W^i_{\kappa\rho} \in \mathbb{R}^{4n\times 4n}$ and matrices $Q^i_{\kappa\rho} \in \mathbb{R}^{4n\times 4n}$, $M_\kappa = \begin{bmatrix} M_{\kappa 11} & 0 \\ M_{\kappa 21} & M_{\kappa 22} \end{bmatrix} \in \mathrm{R}^{2n\times 2n}$, $H_\kappa = \alpha M_\kappa \in \mathrm{R}^{2n\times 2n}$, such that the following LMIs hold.

$$W^i_{\kappa\kappa} > 0, \quad i, \rho = 1, 2, \cdots, n_r \tag{21}$$

$$\frac{2}{n_r - 1} W^i_{\kappa\kappa} + W^i_{\kappa\rho} + W^i_{\rho\kappa} > 0, i, \rho, \kappa = 1, 2, \cdots, n_r, \rho \neq \kappa \tag{22}$$

$$\begin{aligned} &\Theta^i_{\kappa\kappa} + W^i_{\kappa\kappa} < Q^i_{\kappa\kappa}, \kappa, i = 1, 2, \cdots, n_r \\ &\Theta^i_{\kappa\rho} + \Theta^i_{\rho\kappa} + W^i_{\kappa\rho} + W^i_{\rho\kappa} < Q^i_{\kappa\rho} + (Q^i_{\kappa\rho})^{\mathrm{T}}, \rho > \kappa, \kappa, \rho, i = 1, 2, \cdots, n_r \\ &\begin{bmatrix} Q^i_{11} & * & \cdots & * \\ Q^i_{12} & Q^i_{22} & & \vdots \\ \vdots & & \ddots & * \\ Q^i_{1n_r} & \cdots & Q^i_{(n_r-1)n_r} & Q^i_{n_r n_r} \end{bmatrix} < 0, i = 1, 2, \cdots, n_r \end{aligned} \tag{23}$$

where

$$\begin{aligned} \Theta^i_{\kappa\rho} &= \begin{bmatrix} -\mathbb{E}P_\kappa\mathbb{E}^{\mathrm{T}} - \Phi - \Phi^{\mathrm{T}} & * \\ -\alpha\Phi + M_\rho & \mathbb{P}_i + \alpha M_\rho + \alpha M^{\mathrm{T}}_\rho \end{bmatrix} \\ \mathbb{P}_i &= P_i - \mathbb{E}^\dagger Q_i(\mathbb{E}^\dagger)^{\mathrm{T}} \\ \Phi &= M^{\mathrm{T}}_\rho\mathcal{A}^{\mathrm{T}}_\kappa + K^T_\rho\mathcal{B}^{\mathrm{T}}_\kappa \\ K_\kappa &= [\widehat{K}_\kappa, 0] \end{aligned}$$

and full-column rank matrix $\mathbb{E}^\dagger$ satisfies $\mathbb{E}\mathbb{E}^\dagger = 0$.

IV. ILLUSTRATIVE EXAMPLES

Example 6: Consider the following 2-rules fuzzy singular model:

$$\sum_{\kappa=1}^{2} h_\kappa(\varsigma(t))\widehat{\mathbb{E}}_\kappa \widehat{x}(k+1)=\sum_{\kappa=1}^{2} h_\kappa(\varsigma(k))\widehat{\mathcal{A}}_\kappa \widehat{x}(k) \tag{24}$$

where

$$\widehat{\mathbb{E}}_1=\begin{bmatrix}2&1&0\\0&1&0\\0&0&0\end{bmatrix},\quad \widehat{\mathbb{E}}_2=\begin{bmatrix}1&0&0\\1&1&0\\0&0&0\end{bmatrix}$$

$$\widehat{\mathcal{A}}_1=\begin{bmatrix}-0.5&-0.9&-0.8\\1&0.4&\frac{1}{3}(a-1)\\0.4&0&1\end{bmatrix}$$

$$\widehat{\mathcal{A}}_2=\begin{bmatrix}\frac{1}{4}(b-1)&-0.6&0.5\\0.4&0.2&0.4\\0.4&0.5&1\end{bmatrix}$$

Choosing a series of values (a, b) with $a \in [-6, -3]$ and $b \in [-2, 4]$ and comparing the admissibility conditions of [21], [24] and Theorem 4, the feasible area is shown in Fig.1. It can be intuitively seen that the feasible region of theorem 4 is much larger than the existing results in [21] and [24].

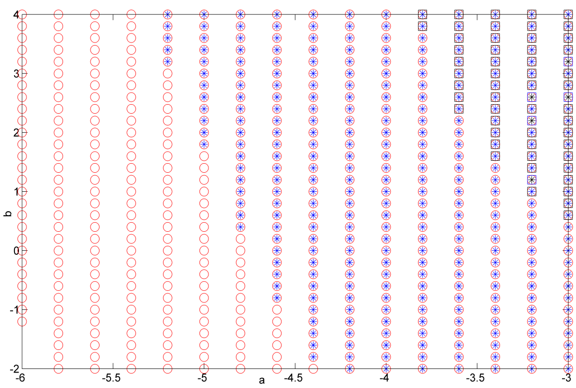

FIGURE 1. Compare feasible area with Theorem 4 (∘), [21](∗) and [24] (□).

Example 7: The fuzzy discrete singular model with two fuzzy rules is given by

$$\sum_{\kappa=1}^{2} h_\kappa(\varsigma(t))\widehat{\mathbb{E}}_\kappa \widehat{x}(k+1)=\sum_{\kappa=1}^{2} h_\kappa(\varsigma(k))[\widehat{\mathcal{A}}_\kappa \widehat{x}(k)+\widehat{\mathcal{B}}_\kappa u(k)] \tag{25}$$

where

$$\widehat{\mathbb{E}}_1=\begin{bmatrix}1&1&0\\0&1&0\\0&0&0\end{bmatrix},\quad \widehat{\mathbb{E}}_2=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},$$

$$\widehat{\mathcal{A}}_1=\begin{bmatrix}-0.1&0.2&0.6\\0&-0.2&0.3\\0.1&0.2&1\end{bmatrix}$$

$$\widehat{\mathcal{A}}_2=\begin{bmatrix}a&0.1&0.2\\0.2&-0.3&0.2\\0.1&0&1\end{bmatrix}$$

$$\widehat{\mathcal{B}}_1=\begin{bmatrix}0.1\\0.1\\0.2\end{bmatrix},\quad \widehat{\mathcal{B}}_2=\begin{bmatrix}b\\0.2\\0.1\end{bmatrix}$$

Comparing PDC controller design conditions of [24] and Theorem 5 with $a \in [-1, 2]$ and $b \in [-1, 1]$, the feasible area is shown in Fig.2. From Figure 2, it can be concluded that theorem 5 is less conservative than the result in [24].

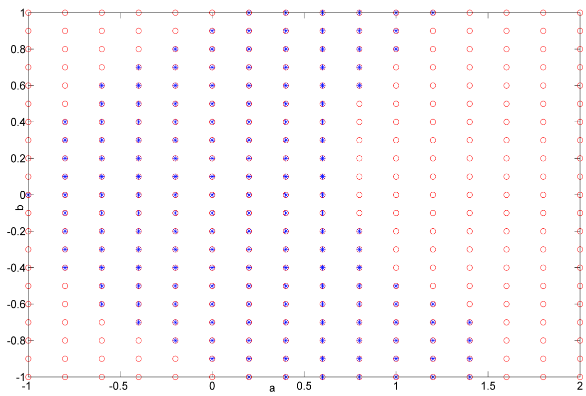

FIGURE 2. Compare feasible area with Theorem 5 (∘) and [24] (∗).

Example 8 [27]: The inverted pendulum is controlled by DC motor through gear train in Fig.3-4. Meanwhile, the system parameters are given by

$$g=9.8\mathrm{m/s}^2,\ K_m=0.1\mathrm{Nm/A},\ M=1\mathrm{kg}$$
$$L=1\mathrm{m},\ R_a=1\Omega,\ N=10,\ K_b=0.1\mathrm{Vs/rad}$$

Then, choosing state variables

$$x_1=\theta_p(t),\ x_2=\dot{\theta}_p(t),\ x_3(t)=I_a(t)$$

this physical system can be given by the following model:

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=9.8\sin x_1+x_3\\ \dot{x}_3&=-2x_2-2x_3+2u(t)\end{aligned}$$

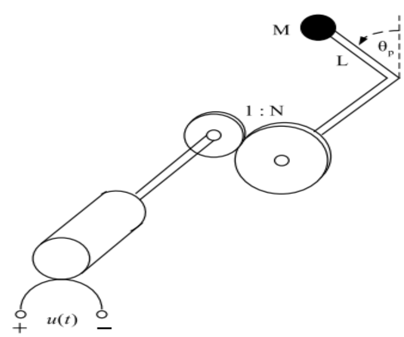

FIGURE 3. Controlled inverted pendulum.

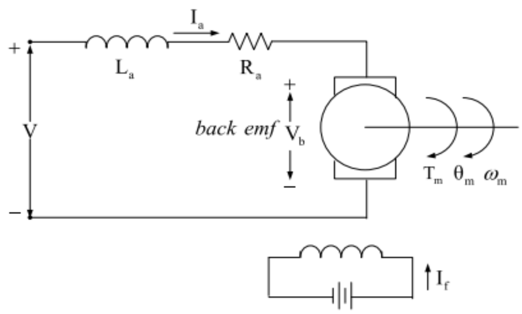

FIGURE 4. Armature-controlled DC motor.

Considering sampling time $T = 0.1$ and $x_4(k) = 9.8\,T\sin x_1(k)$, the following nonlinear discrete singular system is obtained.

$$\begin{aligned} x_1(k+1) &= x_1(k) + Tx_2(k) \\ x_2(k+1) &= x_2(k) + Tx_3(k) + x_4(k) \\ x_3(k+1) &= (1-2T)x_3(k) + T(-2x_2(k) + 2u(k)) \\ 0 &= T9.8\sin x_1(k) - x_4(k) \end{aligned}$$

Next, this system can be translated into the following FDSS with $x_1(k) \in [-\pi, \pi]$.

$$\mathbb{E}x(k+1) = \sum_{\kappa=1}^{2} h_\kappa(\varsigma(k))[\mathcal{A}_\kappa x(k) + \mathcal{B}_\kappa u(k)]$$

$$\mathbb{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{A}_1 = \begin{bmatrix} 1 & 0.1 & 0 & 0 \\ 0 & 1 & 0.1 & 1 \\ 0 & -0.2 & 0.8 & 0 \\ 0.98 & 0 & 0 & -1 \end{bmatrix}$$

$$\mathcal{A}_1 = \begin{bmatrix} 1 & 0.1 & 0 & 0 \\ 0 & 1 & 0.1 & 1 \\ 0 & -0.2 & 0.8 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\mathcal{B}_1 = \mathcal{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 0.2 \\ 0 \end{bmatrix} \tag{26}$$

and two fuzzy membership functions are shown in Fig.5. To check PDC controller design condition, the following controller parameter is given by solving the LMIs of Theorem 5.

$$K_1 = \begin{bmatrix} -60.2613 & -26.6432 & -5.7393 & -12.8987 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -58.4214 & -26.0505 & -5.6461 & -12.6253 \end{bmatrix}$$

Next, based on Theorem 9, the parameters of Non-PDC controller can be obtained as

$$K_1 = \begin{bmatrix} 0.1379 \; 0.8969 \; 6.9924 \; -1.6801 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -0.4490 & 2.1484 & 7.3582 & -1.0013 \end{bmatrix}$$

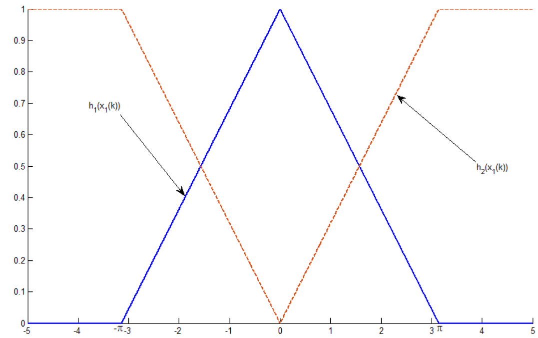

FIGURE 5. Fuzzy membership functions.

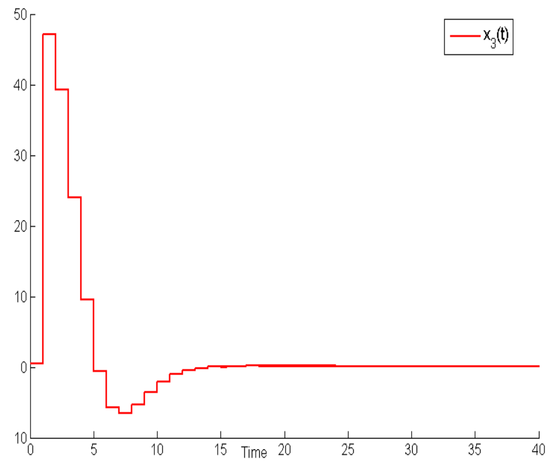

FIGURE 6. State $x_3(t)$: under PDC controller.

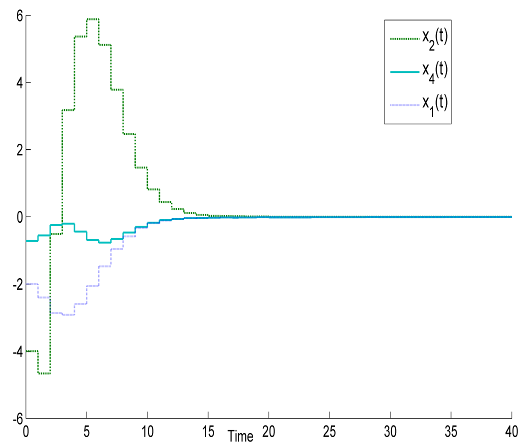

FIGURE 7. State $x_1(t), x_2(t), x_4(t)$: under PDC controller.

Considering initial state $x_1(0) = 2$, $x_2(0) = 4$ and $x_3(0) = 0.5$, the state response of the closed-loop system under PDC controller is given in Fig. 6-7 and under Non-PDC controller is given in Fig. 8-9.

Then, the closed-loop system under PDC controller and under Non-PDC controller are stable.

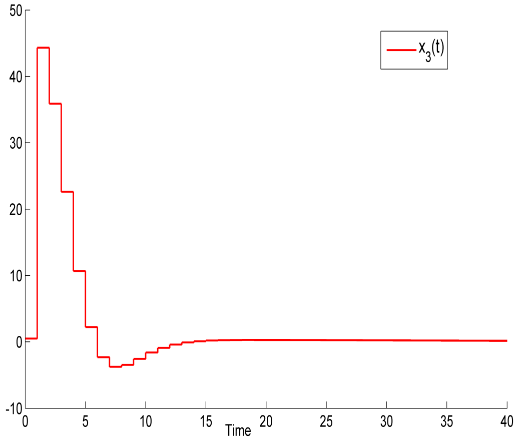

FIGURE 8. State $x_3(t)$: under Non-PDC controller.

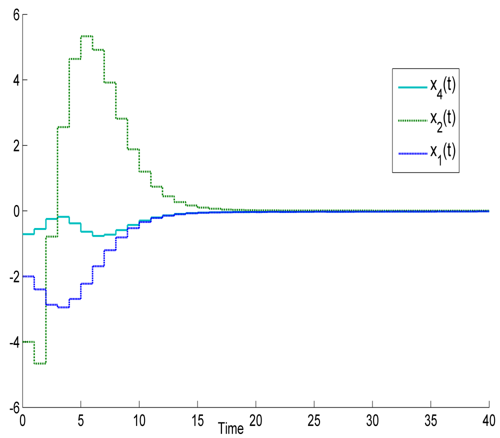

FIGURE 9. State $x_1(t), x_2(t), x_4(t)$: under Non-PDC controller.

V. CONCLUSION

In this article, the admissibility analysis and synthesis issues for a class of FDSSs with distinct difference item matrices has been studied. Based on the augmented system method, this kind of systems has been transformed into the fuzzy singular model with the same difference item matrices, which has been adapted for further admissibility analysis and controller design. The obvious characteristic is that this augmented system is equivalent to the original system under the concern of admissibility. Then, combining with FLF and LMIs technology, a novel and relaxed sufficient condition has been given to ensure the admissibility of such systems. On this basis, the design method of PDC an Non-PDC controller has been proposed via strict LMIs. Compared with the existing results, the advantages of the proposed method have been verified through three examples. It should be noted that the proposed method can also effectively solve the related control issues for fuzzy discrete singular systems with distinct difference item matrices, such as robust control, passive control, output feedback controller design and so on. If the difference item matrices are time varying, the corresponding control issues become more complicated challenging ones. These topics leave for future study.

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