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Finite-Time Mixed H_∞/Passivity for Neural Networks With Mixed Interval Time-Varying Delays Using the Multiple Integral Lyapunov-Krasovskii Functional

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ABSTRACT In this article, we consider the finite-time mixed H_∞/passivity, finite-time stability, and finite-time boundedness for generalized neural networks with interval distributed and discrete time-varying delays. It is noted that this is the first time for studying in the combination of H_∞, passivity, and finite-time boundedness. To obtain several sufficient criteria achieved in the form of linear matrix inequalities (LMIs), we introduce an appropriate Lyapunov-Krasovskii function (LKF) including single, double, triple, and quadruple integral terms, and estimating the bound of time derivative in LKF with the use of Jensen's integral inequality, an extended single and double Wirtinger's integral inequality, and a new triple integral inequality. These LMIs can be solved by using MATLAB's LMI toolbox. Finally, five numerical simulations are shown to illustrate the effectiveness of the obtained results. The received criteria and published literature are compared.

INDEX TERMS Neural networks, Lyapunov-Krasovskii function, H_∞ and passivity, time-varying delays, finite-time bounded.

I. INTRODUCTION

For a number of years, neural networks have been widely attended in many fields, for instance, model identification, optimization, parallel computation, associative memories design, image processing, and other engineering fields [1]–[4], [6]–[21], [23], [24], [28]. The difficult problems and the increase of system performance have been handled by the powerful efficacy neural networks. At present, due to the fact that there exist communication delays and integration in biological and artificial neural systems, they cause instability, oscillation, or poor performance. In a real system, the occurrences of time-delay phenomena are unavoidable. The existence of time-delay causes networks are unstable and worse dynamic system performance. So, the research of time-delay has attracted much attention in linear systems and neural networks. The sufficient criteria of the neural networks have been presented as delay-dependent and delay-independent. The former is less conservatism than the latter when the size of delay is tiny. A huge number of stability conditions have been presented in the works [1]–[4], [6]–[8], [25].

The stability analysis in neural networks is studied with several inequality techniques and Lyapunov approaches, which are important to less conservatism. So, many inequality techniques have been applied for estimating the upper bound of the time derivative of introduced LKF in the published literature. For Jensen's integral inequality, it was applied to determine the new stability conditions for the neural network in [6]. The free-weighting-based inequality was employed to achieve the conditions with the decline of conservatism [10]. The Wirtinger's integral inequality and reciprocally convex

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optimization are combined in [29]. In addition, to obtain better results, various types of LKF have been presented, for example, activation function based LKF [27], multiple integrals based LKF [26], and so on.

At present, the H_∞ has been played a major key in neural network control, industrial plant, energy management, and other fields. It has been increasing attention to the problem of dynamical systems with H_∞ control such as robust control, associative memories image, processing [30]–[33], and so on. In [34], the authors presented H_∞ control of neural networks with the delay-dependent problem. The problem of H_∞ control for neural networks with interval delay was considered in [35]. However, in recent years, the combination of H_∞ and passivity have been more attraction of attention in the investigation, and the researchers are interested in this problem with the various system presented in [36]. Especially, the authors investigated complex networks with H_∞ and passive synchronization problems in [39].

On the other hand, in the past few decades, there is an indispensable property related to exponential stability: finite-time stability, i.e., the solution of a system reach the equilibrium point in finite-time and more precisely, the time required for solutions to reach the equilibrium concerns engineers. The concepts of finite-time stability were presented by Peter Dorato [47] in 1961. At present, Amato *et al.* [48] extend the finite-time stability to finite-time stability with the external disturbances, which is called the finite-time boundedness. In addition, many types of research have been widely developed in the field of finite-time stability for time-varying delay with neural networks as in [40]–[44]. Also, finite-time boundedness has been extensively studied in [1], [4], [45], [46]. Furthermore, the finite-time passivity and the finite-time H_∞ have been studied for the time-delays system; for example, stochastic systems with finite-time have been discussed in [37]. The authors considered the robust finite-time H_∞ in singular stochastic systems problem in [38]. Finite-time passivity in neural networks has been considered in [3]. However, unfortunately, the finite-time with H_∞/passivity for generalized neural networks has not yet been studied.

With motivation mentioned above, we shall address the above question and study the finite-time stability for mixed H_∞/passivity, finite-time stability, and finite-time boundedness for generalized neural networks with mixed interval time-varying delays problems based on an extended Wirtinger integral inequality, a new triple integral inequality, and Jensen's integral inequality. In the numerical part, we give some examples to present the efficiency of the theorems. The major contributions and highlights of this paper are concluded in the following key points.

- We consider the finite-time mixed H_∞/passivity, finite-time stability, and finite-time boundedness for the generalized neural networks problems with both interval distributed and discrete time-varying delays. It is noted that this work is the first time for studying the combination of H_∞, passivity, and finite-time boundedness.
- We construct the Lyapunov-Krasovskii functionals including single, double, triple, and quadruple integral terms in which more information on the delays ι_1, ι_2, γ_1, γ_2, and a state variable is used. In addition, the LKF consisting of two new triple integral terms $\iota_1^2 \int_{t-\iota_1}^{t} \int_s^t \int_u^t e^{\alpha(t-v)} \dot{z}^T(v) S_1 \dot{z}(v) dv du ds$ and $\iota_2^2 \int_{t-\iota_2}^{t} \int_s^t \int_u^t e^{\alpha(t-v)} \dot{z}^T(v) S_2 \dot{z}(v) dv du ds$, that have not been used in [6], [10]–[13], [15]–[18], [21]. Moreover, we also introduced two new quadruple integral terms $\iota_1^3 \int_{t-\iota_1}^{t} \int_s^t \int_u^t \int_v^t e^{\alpha(t-\lambda)} \dot{z}^T(\lambda) U_1 \dot{z}(\lambda) d\lambda dv du ds$ and $\iota_2^3 \int_{t-\iota_2}^{t} \int_s^t \int_u^t \int_v^t e^{\alpha(t-\lambda)} \dot{z}^T(\lambda) U_2 \dot{z}(\lambda) d\lambda dv du ds$ which have not appeared in [6], [9]–[21].
- We apply tighter inequalities, such as Jensen's integral inequality (Lemma 1), an extended single and double Wirtinger's integral inequalities (Lemma 2, 3), a new triple integral inequality (Lemma 4). Using the above new LKFs and the lemmas leads to less conservatism of obtained results than literature, as demonstrated in numerical examples.
- We obtain finite-time boundedness criterion (Theorem 1), the finite-time stability conditions (Corollary 1) and the finite-time mixed H_∞/passivity criterion (Theorem 2). The proposed conditions are less conservative than the other references as shown in Corollary 2.
- We present the numerical simulations to demonstrate the efficiency and feasibility of the theorems and the corollaries.

The outline of this work is organized in the following form. In section 2, we describe the system model and some preliminary results. In section 3, we discuss some results for neural networks and their proofs. In section 4, we give five numerical examples to present the efficiency of the obtained criteria. Finally, in section 5, we present the conclusions and some suggestions for future directions.

Notations: $\mathbb{R}^n$ denotes the $n-$ dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. For a matrix A, $A > 0$ means that A is a symmetric positive definite matrix, $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the minimum and maximum eigenvalues of A, respectively. The superscript "T" denotes matrix transposition. diag$\{\ldots\}$ denotes the block diagonal matrix. Sym$\{A\} = A + A^T$.

II. PRELIMINARIES

Consider the following both interval distributed and discrete time-varying delayed neural networks:

$$\begin{aligned} \dot{z}(t) &= -Az(t) + B_0 f(Wz(t)) + B_1 g(Wz(t-\iota(t))) \\ &\quad + B_2 \int_{t-\gamma_2(t)}^{t-\gamma_1(t)} h(Wz(s))ds + C\omega(t), \qquad (1) \\ y(t) &= D_1 z(t) + D_2 z(t-\iota(t)) \\ &\quad + D_3 \int_{t-\gamma_2(t)}^{t-\gamma_1(t)} h(Wz(s))ds + D_4\omega(t), \qquad (2) \\ z(t) &= \phi(t), \quad t \in [-\iota_2, 0], \qquad (3) \end{aligned}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector; $y(t)$ is the output vector; $\omega(t) \in \mathbb{R}^n$ is the external disturbance that belongs to the class $\mathcal{L}_2[0, \infty)$; $f(z(t)), g(z(t)), h(z(t)) \in \mathbb{R}^n$ are the neuron activation functions; $A \in \text{diag}\{a_i\} \in \mathbb{R}^{n\times n}$ is a positive diagonal matrix; B_0, B_1, and B_2 are the connection weight matrices; C is the connection disturbance; D_1,D_2, D_3, D_4 are known real constant matrices of suitable dimension; $\phi(t)$ is the initial function defined over $[-\iota_2, 0]$. The variable $\gamma_i(t)(i = 1, 2)$ and $\iota(t)$ represent the interval distributed and discrete time-varying delays satisfying

$$0 \le \iota_1 \le \iota(t) \le \iota_2, \quad \dot{\iota}(t) \le \tau, \tag{4}$$

$$0 \le \gamma_1 \le \gamma_1(t) \le \gamma_2(t) \le \gamma_2. \tag{5}$$

Before moving on, we suggest the following necessary lemmas, assumptions, and definitions for the proofs of our results, and we let

$$\iota_{21} = \iota_2 - \iota_1, \quad \gamma_{21} = \gamma_2 - \gamma_1.$$

Assumption 1 [8]:

(A1) *The activation function $f_i(\cdot)(i = 1, 2, \ldots, n)$ is continuous and bounded satisfying the following inequality*

$$F_i^- \le \frac{f_i(Wu) - f_i(Wv)}{Wu - Wv} \le F_i^+, \tag{6}$$

$u, v \in \mathbb{R}, u \neq v$ where $f_i(0) = 0$, F_i^- and F_i^+ are known real scalars.

(A2) *The activation function $g_i(\cdot)(i = 1, 2, \ldots, n)$ is continuous and bounded satisfying the following inequality*

$$G_i^- \le \frac{g_i(Wu) - g_i(Wv)}{Wu - Wv} \le G_i^+, \tag{7}$$

$u, v \in \mathbb{R}, u \neq v$ where $g_i(0) = 0$, G_i^- and G_i^+ are known real scalars.

(A3) *The activation function $h_i(\cdot)(i = 1, 2, \ldots, n)$ is continuous and bounded satisfying the following inequality*

$$H_i^- \le \frac{h_i(Wu) - h_i(Wv)}{Wu - Wv} \le H_i^+, \tag{8}$$

$u, v \in \mathbb{R}, u \neq v$ where $h_i(0) = 0$, H_i^- and H_i^+ are known real scalars.

Remark 1: The assumption of the neuron activation functions satisfy the condition (6)-(8) may be non-differentiable, non-monotonic, and unbounded of the time-varying delay. The constant F_i^-, F_i^+, G_i^-, G_i^+, H_i^-, and H_i^+ be able to zero, positive, or negative. More especially, in this paper, the assumption are weaker and more general than usual Lipschitz condition $|f(u) - f(v) \le F|u - v|$. Therefore, Our stability criteria with condition (6)-(8) are less conservative than the usual Lipschitz condition.

For the convenience of presentation, we denote

$$F_p = \text{diag}\{F_1^- F_1^+, F_2^- F_2^+, \ldots, F_n^- F_n^+\},$$
$$F_m = \text{diag}\left\{\frac{F_1^- + F_1^+}{2}, \frac{F_2^- + F_2^+}{2}, \ldots, \frac{F_n^- + F_n^+}{2}\right\},$$
$$G_p = \text{diag}\{G_1^- G_1^+, G_2^- G_2^+, \ldots, G_n^- G_n^+\},$$
$$G_m = \text{diag}\left\{\frac{G_1^- + G_1^+}{2}, \frac{G_2^- + G_2^+}{2}, \ldots, \frac{G_n^- + G_n^+}{2}\right\},$$
$$H_p = \text{diag}\{H_1^- H_1^+, H_2^- H_2^+, \ldots, H_n^- H_n^+\},$$
$$H_m = \text{diag}\left\{\frac{H_1^- + H_1^+}{2}, \frac{H_2^- + H_2^+}{2}, \ldots, \frac{H_n^- + H_n^+}{2}\right\}.$$

Assumption 2: For any given positive constant ρ and time constant T, the external disturbance satisfies

$$\int_0^T \omega^T(t)\omega(t)dt \le \rho.$$

Definition 1 (Finite-Time Bounded [7]): The system (1) is finite-time bounded with reference to (c_1, c_2, T, V, ρ) with time constant $T > 0$, a matrix $V > 0$, and numbers $c_2 > c_1 > 0$, if the following inequality holds:

$$\sup_{-\iota_2 \le s \le 0} \{z^T(s)Vz(s), \dot{z}^T(s)V\dot{z}(s)\} \le c_1$$
$$\Rightarrow z^T(t)Vz(t) < c_2, \quad \forall t \in [0, T].$$

Definition 2 (Finite-Time Stable [7]): For a given time $T > 0$, numbers $c_2 > c_1 > 0$, and a matrix $V > 0$, the system (1) with $\omega(t) = 0$ is finite-time stable with respect to (c_1, c_2, T, V), if the following inequality holds:

$$\sup_{-\iota_2 \le s \le 0} \{z^T(s)Vz(s), \dot{z}^T(s)V\dot{z}(s)\} \le c_1$$
$$\Rightarrow z^T(t)Vz(t) < c_2, \forall t \in [0, T].$$

Definition 3 (Finite-Time H_∞/Passivity Performance [39]): For given $\sigma \in [0, 1]$, the system (1) is finite-time bounded with a mixed H_∞ and passivity performance δ, if the following two conditions are satisfied:

(1) *the system (1) is finite-time bounded in the sens of Definition 2.*

(2) *under zero initial condition, there exists $\delta > 0$ such that the output $y(t)$ satisfies*

$$\int_0^T [-\sigma y^T(t)y(t) + 2(1-\sigma)\delta y^T(t)\omega(t)]dt$$
$$\ge -\delta^2 \int_0^T \omega^T(t)\omega(t)dt, \tag{9}$$

for any $T \ge 0$ and any non-zero $\omega(t) \in \mathcal{L}_2[0, \infty)$.

Remark 2: The performance index in (9) is mixed H_∞ and passivity index. By substitute the weighting parameter σ, the expression in (9) become to the passivity performance or H_∞ performance index. Furthermore, if $\sigma = 0$, the expression in (9) reduce to the passivity performance index δ, if $\sigma = 1$, the expression in (9) degenerates into the H_∞ performance index δ, and if σ take the value in $(0, 1)$, the expression in (9) represent the mixed H_∞ and passivity performance index δ.

Lemma 1 [6]: For a given matrix $R > 0$ scalar $\alpha_1 < \alpha_2$ and vector $z : [\alpha_1, \alpha_2] \to \mathbb{R}^n$ such that the following integrals are well defined, then the inequality holds:

$$(\alpha_2 - \alpha_1) \int_{\alpha_1}^{\alpha_2} z^T(s)Rx(s)ds \ge \int_{\alpha_1}^{\alpha_2} z^T(s)dsR \int_{\alpha_1}^{\alpha_2} z(s)ds.$$

Lemma 2 [5]: For a positive definite matrix $R \in \mathbb{R}^{n\times n}$, for any continuously differentiable function $z : [\alpha_1, \alpha_2] \to \mathbb{R}^n$, the following inequality holds:

$$\begin{aligned}\int_{\alpha_1}^{\alpha_2} &\dot{z}^T(s)R\dot{z}(s)ds\\ &\geq \frac{1}{\alpha_2-\alpha_1}\chi_1^T R\chi_1 + \frac{3}{\alpha_2-\alpha_1}\chi_2^T R\chi_2\\ &\quad+\frac{5}{\alpha_2-\alpha_1}\chi_3^T R\chi_3 + \frac{7}{\alpha_2-\alpha_1}\chi_4^T R\chi_4,\end{aligned}$$

where

$$\begin{aligned}\chi_1 &= z(\alpha_2) - z(\alpha_1),\\ \chi_2 &= z(\alpha_2) + z(\alpha_1) - \frac{2}{\alpha_2-\alpha_1}\int_{\alpha_1}^{\alpha_2} z(s)ds,\\ \chi_3 &= z(\alpha_2) - z(\alpha_1) + \frac{6}{\alpha_2-\alpha_1}\int_{\alpha_1}^{\alpha_2} z(s)ds\\ &\quad-\frac{12}{(\alpha_2-\alpha_1)^2}\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2} z(u)duds,\\ \chi_4 &= z(\alpha_2) + z(\alpha_1) - \frac{12}{\alpha_2-\alpha_1}\int_{\alpha_1}^{\alpha_2} z(s)ds\\ &\quad+\frac{60}{(\alpha_2-\alpha_1)^2}\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2} z(u)duds\\ &\quad-\frac{120}{(\alpha_2-\alpha_1)^3}\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2}\int_{u}^{\alpha_2} z(v)dvduds.\end{aligned}$$

Lemma 3 [5]: For a positive definite matrix $R \in \mathbb{R}^{n\times n}$, for any continuously differentiable function $z : [\alpha_1, \alpha_2] \to \mathbb{R}^n$, the following inequality holds:

$$\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2} \dot{z}^T(u)R\dot{z}(u)du \geq 2\chi_1^T R\chi_1 + 4\chi_2^T R\chi_2 + 6\chi_6^T R\chi_3,$$

where

$$\begin{aligned}\chi_1 &= z(\alpha_2) - \frac{1}{\alpha_2-\alpha_1}\int_{\alpha_1}^{\alpha_2} z(s)ds,\\ \chi_2 &= z(\alpha_2) + \frac{2}{\alpha_2-\alpha_1}\int_{\alpha_1}^{\alpha_2} z(s)ds\\ &\quad-\frac{6}{(\alpha_2-\alpha_1)^2}\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2} z(u)duds,\\ \chi_3 &= z(\alpha_2) - \frac{3}{\alpha_2-\alpha_1}\int_{\alpha_1}^{\alpha_2} z(s)ds\\ &\quad+\frac{24}{(\alpha_2-\alpha_1)^2}\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2} z(u)duds\\ &\quad-\frac{60}{(\alpha_2-\alpha_1)^3}\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2}\int_{u}^{\alpha_2} z(v)dvduds.\end{aligned}$$

Lemma 4 [2]: For a positive definite matrix $R \in \mathbb{R}^{n\times n}$, for any continuously differentiable function $z : [\alpha_1, \alpha_2] \to \mathbb{R}^n$, the following inequality holds:

$$\begin{aligned}\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2}\int_{u}^{\alpha_2} &\dot{z}^T(v)R\dot{z}(v)dvduds\\ &\geq \frac{6}{(\alpha_2-\alpha_1)^3}\chi_1^T R\chi_1 + \frac{10}{(\alpha_2-\alpha_1)^3}\chi_2^T R\chi_2,\end{aligned}$$

where

$$\begin{aligned}\chi_1 &= \frac{(\alpha_2-\alpha_1)^2}{2}z(\alpha_2)\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2} z(u)duds,\\ \chi_2 &= -\frac{(\alpha_2-\alpha_1)^2}{6}z(\alpha_2)\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2} z(u)duds\\ &\quad+\frac{4}{\alpha_2-\alpha_1}\int_{\alpha_1}^{\alpha_2}\int_{s}^{\alpha_2}\int_{u}^{\alpha_2} z(v)dvduds.\end{aligned}$$

III. MAIN RESULTS

In this section, we will present sufficient conditions of finite-time boundedness, finite-time stability, and finite-time mixed H_∞/passivity for generalized neural networks (1), and we will demonstrate new stability criteria. To simplify the illustration, some notations for vectors and matrices are presented in the form:

$$\begin{aligned}e_i &= \begin{bmatrix}0_{n\times(i-1)n} & I_{n\times n} & 0_{n\times(25-i)n}\end{bmatrix} \in \mathbb{R}^{n\times 25n}\\ &\quad (i = 1, 2, \ldots, 25),\\ \varrho_1 &= \frac{1}{\iota_{21}^3}\int_{t-\iota_2}^{t-\iota_1}\int_{s}^{t-\iota_1}\int_{u}^{t-\iota_1} z^T(v)dvduds,\\ \varrho_2 &= \frac{1}{\iota_2^2}\int_{t-\iota_2}^{t}\int_{s}^{t} z^T(u)duds,\\ \varrho_3 &= \frac{1}{\iota_1^3}\int_{t-\iota_1}^{t}\int_{s}^{t}\int_{u}^{t} z^T(v)dvduds,\\ \varrho_4 &= \frac{1}{\iota_2^3}\int_{t-\iota_2}^{t}\int_{s}^{t}\int_{u}^{t} z^T(v)dvduds,\\ \varpi(t) &= \Bigg[z^T(t),\ \int_{t-\iota_2}^{t-\iota_1} z^T(s)ds,\\ &\qquad \int_{t-\iota_2}^{t-\iota_1}\int_{s}^{t-\iota_1} z^T(u)duds,\ \iota_{21}^3\varrho_1\Bigg]^T,\\ \xi(t) &= \Bigg[z^T(t),\ z^T(t-\iota_1),\ z^T(t-\iota_2),\ z^T(t-\iota(t)),\\ &\qquad \dot{z}^T(t),\ \frac{1}{\iota_1}\int_{t-\iota_1}^{t} z^T(s)ds,\ \frac{1}{\iota_{21}}\int_{t-\iota_2}^{t-\iota_1} z^T(s)ds,\\ &\qquad \frac{1}{\iota_2}\int_{t-\iota_2}^{t} z^T(s)ds,\ \frac{1}{\iota_1^2}\int_{t-\iota_1}^{t}\int_{s}^{t} z^T(u)duds,\\ &\qquad \frac{1}{\iota_{21}^2}\int_{t-\iota_2}^{t-\iota_1}\int_{s}^{t-\iota_1} z^T(u)duds,\ \varrho_2,\ \varrho_3,\ \varrho_1,\\ &\qquad \varrho_4,\ f^T(Wz(t)),\ f^T(Wz(t-\iota_t)),\\ &\qquad f^T(Wz(t-\iota_1)),\ f^T(Wz(t-\iota_2)),\\ &\qquad g^T(Wz(t)),\ g^T(Wz(t-\iota(t))),\\ &\qquad g^T(Wz(t-\iota_1)),\ g^T(Wz(t-\iota_2)),\\ &\qquad h^T(Wz(t)),\ \int_{t-\gamma_2(t)}^{t-\gamma_1(t)} h^T(Wz(s))ds,\ \omega^T(t)\Bigg]^T,\\ \Psi_w &= \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 + \nu_1 + \nu_2\\ &\quad+\nu_3 + \nu_4 + \nu_5 + \nu_6,\\ \Phi_1 &= \mathrm{Sym}\Big\{[e_1^T,\ \iota_{21}e_7^T,\ \iota_{21}^2 e_{10}^T,\ \iota_{21}^3 e_{13}^T]P\\ &\quad\times\Big[e_5^T,\ e_2^T - e_3^T,\ \iota_{21}e_2^T + \iota_1 e_6^T - \iota_2 e_8^T,\end{aligned}$$

$$
\begin{aligned}
&\qquad \frac{\iota_{21}^2}{2} e_2^T - \iota_{21}^2 e_{10}^T \Big]^T \Big\},\\
\Phi_2 &= e^{-\alpha\iota_1} e_2^T Q_1 e_2 - (1-\tau) e^{\alpha\iota_2} e_4^T Q_1 e_4 + e_1^T Q_2 e_1\\
&\quad - e^{\alpha\iota_1} e_1^T Q_2 e_2 + e_1^T Q_3 e_1 - e^{\alpha\iota_2} e_3^T Q_3 e_3,\\
\Phi_3 &= \iota_1^2 e_5^T R_1 e_5 + \iota_{21}^2 e_5^T R_2 e_5\\
\Phi_3 &= \iota_1^2 e_5^T R_1 e_5 + \iota_{21}^2 e_5^T R_2 e_5\\
&\quad - [e_1^T - e_2^T] R_1 [e_1^T - e_2^T]^T\\
&\quad - 3[e_1^T + e_2^T - 2e_6^T] R_1 [e_1^T + e_2^T - 2e_6^T]^T\\
&\quad - 5[e_1^T - e_2^T + 6e_6^T - 12 e_9^T] R_1\\
&\quad \times [e_1^T - e_2^T + 6e_6^T - 12 e_9^T]^T\\
&\quad - 7[e_1^T + e_2^T - 12 e_6^T + 60 e_9^T - 120 e_{12}^T] R_1\\
&\quad \times [e_1^T + e_2^T - 12 e_6^T + 60 e_9^T - 120 e_{12}^T]^T\\
&\quad - [e_2^T - e_3^T] R_2 [e_2^T - e_3^T]^T\\
&\quad - 3[e_2^T + e_3^T - 2e_7^T] R_2 [e_2^T + e_3^T - 2e_7^T]^T\\
&\quad - 5[e_2^T - e_3^T + 6e_7^T - 12 e_{10}^T] R_2\\
&\quad \times [e_2^T - e_3^T + 6e_7^T - 12 e_{10}^T]^T\\
&\quad - 7[e_2^T + e_3^T - 12 e_7^T + 60 e_{10}^T - 120 e_{13}^T] R_2\\
&\quad \times [e_2^T + e_3^T - 12 e_7^T + 60 e_{10}^T - 120 e_{13}^T]^T,\\
\Phi_4 &= \frac{\iota_4}{2} e_5^T S_1 e_5 + \frac{\iota_4}{2} e_5^T S_2 e_5\\
&\quad - 2\iota_1^2 [e_1^T - e_6^T] S_1 [e_1^T - e_6^T]^T\\
&\quad - 4\iota_1^2 [e_1^T + 2e_6^T - 6e_9^T] S_1\\
&\quad \times [e_1^T + 2e_6^T - 6e_9^T]^T\\
&\quad - 6\iota_1^2 [e_1^T - 3e_6^T + 24 e_9^T - 60 e_{12}^T] S_1\\
&\quad \times [e_1^T - 3e_6^T + 24 e_9^T - 60 e_{12}^T]^T\\
&\quad - 2\iota_2^2 [e_1^T - e_8^T] S_2 [e_1^T - e_8^T]^T\\
&\quad - 4\iota_2^2 [e_1^T + 2e_8^T - 6e_{11}^T] S_2 [e_1^T + 2e_8^T - 6e_{11}^T]^T\\
&\quad - 6\iota_2^2 [e_1^T - 3e_8^T + 24 e_{11}^T - 60 e_{14}^T] S_2\\
&\quad \times [e_1^T - 3e_8^T + 24 e_{11}^T - 60 e_{14}^T]^T,\\
\Phi_5 &= \frac{\iota_1^6}{6} e_5^T U_1 e_5 + \frac{\iota_1^6}{6} e_5^T U_2 e_5\\
&\quad - 6\left[\frac{\iota_1^2}{2} e_1^T - \iota_1^2 e_9^T\right] U_1 \left[\frac{\iota_1^2}{2} e_1^T - \iota_1^2 e_9^T\right]^T\\
&\quad - 10\left[-\frac{\iota_1^2}{6} e_1^T - \iota_1^2 e_9^T + 4\iota_1^2 e_{12}^T\right] U_1\\
&\quad \times \left[-\frac{\iota_1^2}{6} e_1^T - \iota_1^2 e_9^T + 4\iota_1^2 e_{12}^T\right]^T\\
&\quad - 6\left[\frac{\iota_2^2}{2} e_1^T - \iota_2^2 e_{11}^T\right] U_2 \left[\frac{\iota_2^2}{2} e_1^T - \iota_2^2 e_{11}^T\right]^T\\
&\quad - 10\left[-\frac{\iota_2^2}{6} e_1^T - \iota_2^2 e_{11}^T + 4\iota_2^2 e_{14}^T\right] U_2\\
&\quad \times \left[-\frac{\iota_2^2}{6} e_1^T - \iota_2^2 e_{11}^T + 4\iota_2^2 e_{14}^T\right]^T,\\
\Phi_6 &= \sigma_{21}^2 e_{23}^T Y e_{23} - e_{24}^T Y e_{24},
\end{aligned}
$$

$$
\begin{aligned}
\nu_1 &= [(F_p W(e_1^T - e_2^T)) - e_{15}^T + e_{17}^T] L_{f1}\\
&\quad \times [e_{15} - e_{17} - (F_m W(e_1 - e_2))]\\
&\quad + [(F_p W(e_1^T - e_3^T)) - e_{15}^T + e_{18}^T] L_{f2}\\
&\quad \times [e_{15} - e_{18} - (F_m W(e_1 - e_3))]\\
&\quad + [(F_p W(e_1^T - e_4^T)) - e_{15}^T + e_{16}^T] L_{f3}\\
&\quad \times [e_{15} - e_{16} - (F_m W(e_1 - e_4))]\\
&\quad + [(F_p W(e_4^T - e_2^T)) - e_{16}^T + e_{17}^T] L_{f4}\\
&\quad \times [e_{16} - e_{17} - (F_m W(e_4 - e_2))]\\
&\quad + [(F_p W(e_4^T - e_3^T)) - e_{16}^T + e_{18}^T] L_{f5}\\
&\quad \times [e_{16} - e_{18} - (F_m W(e_4 - e_3))]\\
&\quad + [(F_p W(e_2^T - e_3^T)) - e_{17}^T + e_{18}^T] L_{f6}\\
&\quad \times [e_{17} - e_{18} - (F_m W(e_2 - e_3))],\\
\nu_2 &= [F_p W e_1^T - e_{15}^T] V_{f1} [e_{15} - F_m W e_1]\\
&\quad + [F_p W e_4^T - e_{16}^T] V_{f2} [e_{16} - F_m W e_4]\\
&\quad + [F_p W e_2^T - e_{17}^T] V_{f3} [e_{17} - F_m W e_2]\\
&\quad + [F_p W e_3^T - e_{18}^T] V_{f4} [e_{18} - F_m W e_3],\\
\nu_3 &= [(G_p W(e_1^T - e_2^T)) - e_{19}^T + e_{21}^T] L_{g1}\\
&\quad \times [e_{19} - e_{21} - (G_m W(e_1 - e_2))]\\
&\quad [(G_p W(e_1^T - e_3^T)) - e_{19}^T + e_{22}^T] L_{g2}\\
&\quad \times [e_{19} - e_{22} - (G_m W(e_1 - e_3))]\\
&\quad [(G_p W(e_1^T - e_4^T)) - e_{19}^T + e_{20}^T] L_{g3}\\
&\quad \times [e_{19} - e_{20} - (G_m W(e_1 - e_4))]\\
&\quad [(G_p W(e_4^T - e_2^T)) - e_{20}^T + e_{21}^T] L_{g4}\\
&\quad \times [e_{20} - e_{21} - (G_m W(e_4 - e_2))]\\
&\quad [(G_p W(e_4^T - e_3^T)) - e_{20}^T + e_{22}^T] L_{g5}\\
&\quad \times [e_{21} - e_{22} - (G_m W(e_2 - e_3))]\\
&\quad [(G_p W(e_2^T - e_2^T)) - e_{21}^T + e_{21}^T] L_{g6}\\
&\quad \times [e_{20} - e_{21} - (G_m W(e_4 - e_2))],\\
\nu_4 &= [G_p W e_1^T - e_{19}^T] V_{g1} [e_{19} - G_m W e_1]\\
&\quad + [G_p W e_4^T - e_{20}^T] V_{g2} [e_{20} - G_m W e_4]\\
&\quad + [G_p W e_2^T - e_{21}^T] V_{g3} [e_{21} - G_m W e_2]\\
&\quad + [G_p W e_3^T - e_{22}^T] V_{g4} [e_{22} - G_m W e_3],\\
\nu_5 &= [H_p W e_1^T - e_{23}^T] V_{h1} [e_{23} - H_m W e_1],\\
\nu_6 &= \mathrm{Sym}\{[e_1^T X_1 + e_5^T X_2][-e_5 - A e_1 + B_0 e_{15}\\
&\quad + B_1 e_{20} + B_2 e_{24} + C e_{25}]\},\\
\Omega_1 &= \frac{e^{\alpha\iota_2} - e^{\alpha\iota_1}}{\alpha},\ \Omega_2 = \frac{e^{\alpha\iota_1} - 1}{\alpha},\ \Omega_3 = \frac{e^{\alpha\iota_2} - 1}{\alpha},\\
\Omega_4 &= \frac{e^{\alpha\iota_1} - \alpha\iota_1 - 1}{\alpha^2},\ \Omega_5 = \frac{e^{\alpha\iota_2} - e^{\alpha\iota_1} - \alpha\iota_{21}}{\alpha^2},\\
\Omega_6 &= \frac{-2\iota_1^2 - 2\alpha\iota_1^3 - \alpha^2\iota_1^4 + 2\iota_1^2 e^{\alpha\iota_1}}{2\alpha^3},\\
\Omega_7 &= \frac{-2\iota_2^2 - 2\alpha\iota_2^3 - \alpha^2\iota_2^4 + 2\iota_2^2 e^{\alpha\iota_2}}{2\alpha^3},\\
\Omega_8 &= \frac{-6\iota_1^3 - 6\alpha\iota_1^4 - 3\alpha^2\iota_1^5 - \alpha^3\iota_1^6 + 6e^{\alpha\iota_1}}{6\alpha^4},
\end{aligned}
$$

$$\Omega_9 = \frac{-6\iota_2^3 - 6\alpha\iota_2^4 - 3\alpha^2\iota_2^5 - \alpha^3\iota_2^6 + 6e^{\alpha\iota_2}}{6\alpha^4},$$
$$\Omega_{10} = \frac{e^{\alpha\gamma_2} - e^{\alpha\gamma_1} - \alpha\sigma_{21}}{\alpha^2},$$
$$\zeta_1 = \lambda_{\min}(\hat{P}),\ \zeta_2 = \lambda_{\max}(\hat{P}),\ \zeta_3 = \lambda_{\max}(\hat{Q}_1),$$
$$\zeta_4 = \lambda_{\max}(\hat{Q}_2),\ \zeta_5 = \lambda_{\max}(\hat{Q}_3),\ \zeta_6 = \lambda_{\max}(\hat{R}_1),$$
$$\zeta_7 = \lambda_{\max}(\hat{R}_2),\ \zeta_8 = \lambda_{\max}(\hat{S}_1)\ ,\zeta_9 = \lambda_{\max}(\hat{S}_2),$$
$$\zeta_{10} = \lambda_{\max}(\hat{U}_1),\ \zeta_{11} = \lambda_{\max}(\hat{U}_2),\ \zeta_{12} = \lambda_{\max}(\hat{Y}),$$
$$\zeta_{13} = \lambda_{\max}(\hat{M}).$$

A. FINITE-TIME BOUNDEDNESS

In this subsection, we study finite-time boundedness for the generalized neural networks in the following form:

$$\dot{z}(t) = -Az(t) + B_0 f(Wz(t)) + B_1 g(Wz(t-\iota(t))) + B_2\int_{t-\gamma_2(t)}^{t-\gamma_1(t)} h(Wz(s))ds + C\omega(t), \tag{10}$$
$$z(t) = \phi(t), \quad t\in[-\iota_2, 0]. \tag{11}$$

Theorem 1: For given positive scalars ι_2, τ and α, the system (10) is finite-time bounded if there exist symmetric positive definition matrices P, $Q_i (i = 1, 2, 3)$, R_j, S_j, $U_j (j = 1, 2)$, Y, M, any matrices X_1, X_2 such that the following LMIs hold:

$$\Psi_w - \alpha e_{25}^T M e_{25} < 0, \tag{12}$$
$$\begin{aligned}
&\zeta_1 I \le \hat{P} \le \zeta_2 I,\ 0 \le \hat{Q}_1 \le \zeta_3 I,\\
&0 \le \hat{Q}_2 \le \zeta_4 I,\ 0 \le \hat{Q}_3 \le \zeta_5 I,\\
&0 \le \hat{R}_1 \le \zeta_6 I,\ 0 \le \hat{R}_2, \le \zeta_7 I\\
&0 \le \hat{S}_1 \le \zeta_8 I,\ 0 \le \hat{S}_2 \le \zeta_9 I,\\
&0 \le \hat{U}_1 \le \zeta_{10} I,\ 0 \le \hat{U}_2 \le \zeta_{11} I,\\
&0 \le \hat{Y} \le \zeta_{12} I,\ 0 \le \hat{M} \le \zeta_{13} I,
\end{aligned} \tag{13}$$
$$e^{\alpha T}\left[\Pi c_1 + \omega\zeta_{13}(1 - e^{-\alpha T})\right] < \zeta_1 c_2. \tag{14}$$

Proof: We construct the LKF as follows:

$$V(t, x_t) = \sum_{j=1}^{6} V_j(t, x_t) \tag{15}$$

where

$$V_1(t,x_t) = \varpi^T(t)P\varpi(t),$$
$$V_2(t,x_t) = \int_{t-\iota(t)}^{t-\iota_1} e^{\alpha(t-s)} z^T(s)Q_1 z(s)ds + \int_{t-\iota_1}^{t} e^{\alpha(t-s)} z^T(s)Q_2 z(s)ds + \int_{t-\iota_2}^{t} e^{\alpha(t-s)} z^T(s)Q_3 z(s)ds,$$
$$V_3(t,x_t) = \iota_1\int_{t-\iota_1}^{t}\int_s^t e^{\alpha(t-u)}\dot{z}^T(u)R_1\dot{z}(u)duds + \iota_{21}\int_{t-\iota_2}^{t-\iota_1}\int_s^t e^{\alpha(t-u)}\dot{z}^T(u)R_2\dot{z}(u)duds,$$
$$V_4(t,x_t) = \iota_1^2\int_{t-\iota_1}^{t}\int_s^t\int_u^t e^{\alpha(t-v)}\dot{z}^T(v)S_1\dot{z}(v)dvduds + \iota_2^2\int_{t-\iota_2}^{t}\int_s^t\int_u^t e^{\alpha(t-v)}\dot{z}^T(v)S_2\dot{z}(v)dvduds,$$
$$V_5(t,x_t) = \iota_1^3\int_{t-\iota_1}^{t}\int_s^t\int_u^t\int_v^t e^{\alpha(t-\lambda)}\dot{z}^T(\lambda)U_1\dot{z}(\lambda)d\lambda dvduds + \iota_2^3\int_{t-\iota_2}^{t}\int_s^t\int_u^t\int_v^t e^{\alpha(t-\lambda)}\dot{z}^T(\lambda) \times U_2\dot{z}(\lambda)d\lambda dvduds,$$
$$V_6(t,x_t) = \gamma_{21}\int_{t-\gamma_2}^{t-\gamma_1}\int_s^t e^{\alpha(t-u)}h^T(Wz(u))Yh(Wz(u))duds.$$

Then, the time derivative of (15) are computed as follows:

$$\begin{aligned}
\dot{V}_1(t,x_t) &= 2\varpi^T(t)P\dot{\varpi}(t) - \alpha\varpi^T(t)P\varpi(t) + \alpha V_1\\
&= \xi^T(t)\Phi_1\xi(t) + \alpha V_1,
\end{aligned}$$
$$\begin{aligned}
\dot{V}_2(t,x_t) &= e^{\alpha\iota_1}z^T(t-\iota_1)Q_1 z(t-\iota_1)\\
&\quad -(1-\dot{\iota}(t))e^{\alpha\iota_2}z^T(t-\iota(t))Q_1 z(t-\iota(t))\\
&\quad + z^T(t)Q_2 z(t) - e^{\alpha\iota_1}z^T(t-\iota_1)Q_2 z(t-\iota_1)\\
&\quad + z^T(t)Q_3 z(t) - e^{\alpha\iota_2}z^T(t-\iota_2)Q_3 z(t-\iota_2)\\
&\quad + \alpha V_2\\
&\le e^{\alpha\iota_1}z^T(t-\iota_1)Q_1 z(t-\iota_1)\\
&\quad -(1-\tau)e^{\alpha\iota_2}z^T(t-\iota(t))Q_1 z(t-\iota(t))\\
&\quad + z^T(t)Q_2 z(t) - e^{\alpha\iota_1}z^T(t-\iota_1)Q_2 z(t-\iota_1)\\
&\quad + z^T(t)Q_3 z(t) - e^{\alpha\iota_2}z^T(t-\iota_2)Q_3 z(t-\iota_2)\\
&\quad + \alpha V_2\\
&= \xi^T(t)\Phi_2\xi(t) + \alpha V_2,
\end{aligned} \tag{16, 17}$$
$$\begin{aligned}
\dot{V}_3(t,x_t) &= \iota_1^2\dot{z}^T(t)R_1\dot{z}(t) + \iota_{21}^2\dot{z}^T(t)R_2\dot{z}(t)\\
&\quad -\iota_1\int_{t-\iota_1}^{t}\dot{z}^T(s)R_1 e^{\alpha(t-s)}\dot{z}(s)ds\\
&\quad -\iota_{21}\int_{t-\iota_2}^{t-\iota_1} e^{\alpha(t-s)}\dot{z}^T(s)R_2\dot{z}(s)ds + \alpha V_3\\
&\le \iota_1^2\dot{z}^T(t)R_1\dot{z}(t) + \iota_{21}^2\dot{z}^T(t)R_2\dot{z}(t)\\
&\quad -\iota_1\int_{t-\iota_1}^{t}\dot{z}^T(s)R_1\dot{z}(s)ds\\
&\quad -\iota_{21}\int_{t-\iota_2}^{t-\iota_1}\dot{z}^T(s)R_2\dot{z}(s)ds + \alpha V_3\\
&= \xi^T(t)\Phi_3\xi(t) + \alpha V_3.
\end{aligned} \tag{18}$$

Using Lemma2, it gives

$$\begin{aligned}
&-\iota_1\int_{t-\iota_1}^{t}\dot{z}^T(s)R_2\dot{z}(s)ds\\
&\le -[z(t) - z(t-\iota_1)]^T R_1[z(t) - z(t-\iota_1)]\\
&\quad -3\left[z(t) + z(t-\iota_1) - \frac{2}{\iota_1}\int_{t-\iota_1}^{t}z(s)ds\right]^T R_1\\
&\quad \times\left[z(t) + z(t-\iota_1) - \frac{2}{\iota_1}\int_{t-\iota_1}^{t}z(s)ds\right]\\
&\quad -5\left[z(t) - z(t-\iota_1) + \frac{6}{\iota_1}\int_{t-\iota_1}^{t}z(s)ds\right.
\end{aligned}$$

$$
\begin{aligned}
&-\frac{12}{\iota_1^2}\int_{t\iota_1}^{t}\int_s^t z(u)duds\Bigg]^T R_1\\
&\times\Bigg[z(t)-z(t-\iota_1)+\frac{6}{\iota_1}\int_{t-\iota_1}^t z(s)ds\\
&-\frac{12}{\iota_1^2}\int_{t\iota_1}^t\int_s^t z(u)duds\Bigg]\\
&-7\Bigg[z(t)+z(t-\iota_1)-\frac{12}{\iota_1}\int_{t-\iota_1}^t z(s)ds\\
&+\frac{60}{\iota_1^2}\int_{t\iota_1}^t\int_s^t z(u)duds-120\varrho_3\Bigg]^T R_1\\
&\times\Bigg[z(t)+z(t-\iota_1)-\frac{12}{\iota_1}\int_{t-\iota_1}^t z(s)ds\\
&+\frac{60}{\iota_1^2}\int_{t\iota_1}^t\int_s^t z(u)duds-120\varrho_3\Bigg],\\
&-\iota_{21}\int_{t-\iota_2}^{t-\iota_1}\dot z^T(s)R_2\dot z(s)ds\\
\le&-[z(t-\iota_1)-z(t-\iota_2)]^T R_2[z(t-\iota_1)-z(t-\iota_2)]\\
&-3\Bigg[z(t-\iota_1)+z(t-\iota_2)-\frac{2}{\iota_{21}}\int_{t-\iota_2}^{t-\iota_1}z(s)ds\Bigg]^T R_2\\
&\times\Bigg[z(t-\iota_1)+z(t-\iota_2)-\frac{2}{\iota_{21}}\int_{t-\iota_2}^{t-\iota_1}z(s)ds\Bigg]\\
&-5\Bigg[[z(t-\iota_1)-z(t-\iota_2)+\frac{6}{\iota_{21}}\int_{t-\iota_2}^{t-\iota_1}z(s)ds\\
&-\frac{12}{\iota_{21}^2}\int_{t-\iota_2}^{t-\iota_1}\int_s^{t-\iota_1}z(u)duds\Bigg]^T\\
&\times R_2\Bigg[[z(t-\iota_1)-z(t-\iota_2)+\frac{6}{\iota_{21}}\int_{t-\iota_2}^{t-\iota_1}z(s)ds\\
&-\frac{12}{\iota_{21}^2}\int_{t-\iota_2}^{t-\iota_1}\int_s^{t-\iota_1}z(u)duds\Bigg]\\
&-7\Bigg[z(t-\iota_1)+z(t-\iota_2)-\frac{12}{\iota_{21}}\int_{t-\iota_2}^{t-\iota_1}z(s)ds\\
&+\frac{60}{\iota_{21}^2}\int_{t-\iota_2}^{t-\iota_1}\int_s^{t-\iota_1}z(u)duds-120\varrho_1\Bigg]^T R_2\\
&\times\Bigg[z(t-\iota_1)+z(t-\iota_2)-\frac{12}{\iota_{21}}\int_{t-\iota_2}^{t-\iota_1}z(s)ds\\
&+\frac{60}{\iota_{21}^2}\int_{t-\iota_2}^{t-\iota_1}\int_s^{t-\iota_1}z(u)duds-120\varrho_1\Bigg].
\end{aligned}
$$

$$
\begin{aligned}
\dot V_4(t,x_t)=&\frac{\iota_1^4}{2}\dot z^T(t)S_1\dot z(t)+\frac{\iota_2^4}{2}\dot z^T(t)S_2\dot z(t)\\
&-\iota_1^2\int_{t-\iota_1}^t\int_s^t e^{\alpha(t-u)}\dot z^T(u)S_1\dot z(u)duds\\
&-\iota_2^2\int_{t-\iota_2}^t\int_s^t e^{\alpha(t-u)}\dot z^T(u)S_2\dot z(u)duds+\alpha V_4\\
\le&\frac{\iota_1^4}{2}\dot z^T(t)S_1\dot z(t)+\frac{\iota_2^4}{2}\dot z^T(t)S_2\dot z(t)\\
&-\iota_1^2\int_{t-\iota_1}^t\int_s^t \dot z^T(u)S_1\dot z(u)duds\\
&-\iota_2^2\int_{t-\iota_2}^t\int_s^t \dot z^T(u)S_2\dot z(u)duds+\alpha V_4\\
=&\xi^T(t)\Phi_4\xi(t)+\alpha V_4. \qquad (19)
\end{aligned}
$$

By applying Lemma3, we can deduce

$$
\begin{aligned}
&-\iota_1^2\int_{t-\iota_1}^t\int_s^t \dot z^T(u)S_1\dot z(u)duds\\
\le&-2\iota_1^2\Bigg[z(t)-\frac{1}{\iota_1}\int_{t-\iota_1}^t z(s)ds\Bigg]^T S_1\\
&\times\Bigg[z(t)-\frac{1}{\iota_1}\int_{t-\iota_1}^t z(s)ds\Bigg]\\
&-4\iota_1^2\Bigg[z(t)+\frac{2}{\iota_1}\int_{t-\iota_1}^t z(s)ds\\
&-\frac{6}{\iota_1^2}\int_{t-\iota_1}^t\int_s^t z(u)duds\Bigg]^T S_1\\
&\Bigg[z(t)+\frac{2}{\iota_1}\int_{t-\iota_1}^t z(s)ds-\frac{6}{\iota_1^2}\int_{t-\iota_1}^t\int_s^t z(u)duds\Bigg]\\
&-6\iota_1^2\Bigg[z(t)-\frac{3}{\iota_1}\int_{t-\iota_1}^t z(s)ds\\
&+\frac{24}{\iota_1^2}\int_{t-\iota_1}^t\int_s^t z(u)duds-60\varrho_3\Bigg]^T S_1\\
&\times\Bigg[z(t)-\frac{3}{\iota_1}\int_{t-\iota_1}^t z(s)ds\\
&+\frac{24}{\iota_1^2}\int_{t-\iota_1}^t\int_s^t z(u)duds-60\varrho_3\Bigg],\\
&-\iota_2^2\int_{t-\iota_2}^{t-\iota_1}\int_s^t \dot z^T(u)S_2\dot z(u)duds\\
\le&-2\iota_2^2\Bigg[z(t)-\frac{1}{\iota_2}\int_{t-\iota_2}^t z(s)ds\Bigg]^T S_2\\
&\times\Bigg[z(t)-\frac{1}{\iota_2}\int_{t-\iota_2}^t z(s)ds\Bigg]-4\iota_2^2\Bigg[z(t)\\
&+\frac{2}{\iota_2}\int_{t-\iota_2}^t z(s)ds-\frac{6}{\iota_2^2}\int_{t-\iota_2}^t\int_s^t z(u)duds\Bigg]^T S_2\\
&\times\Bigg[z(t)+\frac{2}{\iota_2}\int_{t-\iota_2}^t z(s)ds-\frac{6}{\iota_2^2}\int_{t-\iota_2}^t\int_s^t z(u)duds\Bigg]\\
&-6\iota_2^2\Bigg[z(t)-\frac{3}{\iota_2}\int_{t-\iota_2}^t z(s)ds\\
&+\frac{24}{\iota_2^2}\int_{t-\iota_2}^t\int_s^t z(u)duds-60\varrho_4\Bigg]^T S_2\\
&\Bigg[z(t)-\frac{3}{\iota_2}\int_{t-\iota_2}^t z(s)ds\\
&+\frac{24}{\iota_2^2}\int_{t-\iota_2}^t\int_s^t z(u)duds-60\varrho_4\Bigg].
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_5(t,x_t) &= \frac{\iota_1^6}{6}\dot{z}^T(t)U_1\dot{z}(t)+\frac{\iota_2^6}{6}\dot{z}^T(t)U_2\dot{z}(t) \\
&\quad -\iota_1^3\int_{t-\iota_1}^{t}\int_{s}^{t}\int_{u}^{t}e^{\alpha(t-v)}\dot{z}^T(v)U_1\dot{z}(v)dvduds \\
&\quad -\iota_2^3\int_{t-\iota_2}^{t}\int_{s}^{t}\int_{u}^{t}e^{\alpha(t-v)}\dot{z}^T(v)U_2\dot{z}(v)dvduds \\
&\quad +\alpha V_5 \\
&\le \frac{\iota_1^6}{6}\dot{z}^T(t)U_1\dot{z}(t)+\frac{\iota_2^6}{6}\dot{z}^T(t)U_2\dot{z}(t) \\
&\quad -\iota_1^3\int_{t-\iota_1}^{t}\int_{s}^{t}\int_{u}^{t}\dot{z}^T(v)U_1\dot{z}(v)dvduds \\
&\quad -\iota_2^3\int_{t-\iota_2}^{t}\int_{s}^{t}\int_{u}^{t}\dot{z}^T(v)U_2\dot{z}(v)dvduds+\alpha V_5 \\
&= \xi^T(t)\Phi_5\xi(t)+\alpha V_5. \qquad (20)
\end{aligned}
$$

By utilizing Lemma4, we have

$$
\begin{aligned}
&-\iota_1^3\int_{t-\iota_1}^{t}\int_{s}^{t}\int_{u}^{t}\dot{z}^T(v)U_1\dot{z}(v)dvduds \\
&\le -6\Big[\frac{\iota_1^2}{2}z(t)-\int_{t-\iota_1}^{t}\int_{s}^{t}z(u)duds\Big]^T U_1 \\
&\quad \Big[\frac{\iota_1^2}{2}z(t)-\int_{t-\iota_1}^{t}\int_{s}^{t}z(u)duds\Big] \\
&\quad -10\Big[\frac{\iota_1^2}{6}z(t)-\int_{t-\iota_1}^{t}\int_{s}^{t}z(u)duds \\
&\quad +\frac{4}{\iota_1}\int_{t-\iota_1}^{t}\int_{s}^{t}\int_{u}^{s}z(v)dvduds\Big]^T U_1 \\
&\quad \times\Big[\frac{\iota_1^2}{6}z(t)-\int_{t-\iota_1}^{t}\int_{s}^{t}z(u)duds \\
&\quad +\frac{4}{\iota_1}\int_{t-\iota_1}^{t}\int_{s}^{t}\int_{u}^{s}z(v)dvduds\Big], \\
&\quad -\iota_2^3\int_{t-\iota_2}^{t}\int_{s}^{t}\int_{u}^{t}\dot{z}^T(v)U_2\dot{z}(v)dvduds \\
&\le -6\Big[\frac{\iota_2^2}{2}z(t)-\int_{t-\iota_2}^{t}\int_{s}^{t}z(u)duds\Big]^T U_2 \\
&\quad \times\Big[\frac{\iota_2^2}{2}z(t)-\int_{t-\iota_2}^{t}\int_{s}^{t}z(u)duds\Big] \\
&\quad -10\Big[\frac{\iota_2^2}{6}z(t)-\int_{t-\iota_2}^{t}\int_{s}^{t}z(u)duds \\
&\quad +\frac{4}{\iota_2}\int_{t-\iota_2}^{t}\int_{s}^{t}\int_{u}^{s}z(v)dvduds\Big]^T U_2 \\
&\quad \times\Big[\frac{\iota_2^2}{6}z(t)-\int_{t-\iota_2}^{t}\int_{s}^{t}z(u)duds \\
&\quad +\frac{4}{\iota_2}\int_{t-\iota_2}^{t}\int_{s}^{t}\int_{u}^{s}z(v)dvduds\Big].
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_6(t,x_t) &= \sigma_{21}^2h^T(Wz(t))Yh(Wz(t)) \\
&\quad -\sigma_{21}\int_{t-\gamma_2}^{t-\gamma_1}e^{\alpha(t-u)}h^T(Wz(s))Yh(Wz(s))ds \\
&\le \sigma_{21}^2h^T(Wz(t))Yh(Wz(t))-(\gamma_2(t)-\gamma_1(t)) \\
&\quad \times e^{\alpha\iota_2}\int_{t-\gamma_2(t)}^{t-\gamma_1(t)}h^T(Wz(s))Yh(Wz(s))ds \\
&= \xi^T(t)\Phi_6\xi(t)+\alpha V_6. \qquad (21)
\end{aligned}
$$

By Lemma1, we obtain

$$
\begin{aligned}
&-(\gamma_2(t)-\gamma_1(t))\int_{t-\gamma_2(t)}^{t-\gamma_1(t)}h^T(Wz(s))Yh(Wz(s))ds \\
&\quad \le -\int_{t-\gamma_2(t)}^{t-\gamma_1(t)}h^T(Wz(s))dsY\int_{t-\gamma_2(t)}^{t-\gamma_1(t)}h(Wz(s))ds.
\end{aligned}
$$

By utilizing Assumption 1, we get that

$$
\begin{aligned}
l_{fi}(v_1,v_2) &:= 2[f(z(v_1))-f(z(v_2)) \\
&\quad -F_mW(z(v_1)-z(v_2))]^TL_{fi} \\
&\quad \times[F_pW(z(v_1)-z(v_2)) \\
&\quad -f(z(v_1))+f(z(v_2))]\ge 0, \\
v_{fi}(v) &:= 2[f(z(v))-F_mWz(v)]^TV_{fi} \\
&\quad [F_pWz(v)-f(z(v))]\ge 0 \\
l_{gi}(v_1,v_2) &:= 2[g(z(v_1))-g(z(v_2)) \\
&\quad -G_mW(z(v_1)-z(v_2))]^TL_{gi} \\
&\quad \times[G_pW(z(v_1)-z(v_2)) \\
&\quad -g(z(v_1))+g(z(v_2))]\ge 0, \\
v_{gi}(v) &:= 2[g(z(v))-G_mWz(v)]^TV_{gi} \\
&\quad \times[G_pWz(v)-g(z(v))]\ge 0 \\
v_h(v) &:= 2[h(z(v))-H_mWz(v)]^TV_h \\
&\quad \times[H_pWz(v)-h(z(v))]\ge 0
\end{aligned}
$$

where

$$
\begin{aligned}
L_{fi} &= \mathrm{diag}\{l_{1fi},l_{2fi},\dots,l_{nfi}\}, \\
V_{fj} &= \mathrm{diag}\{v_{1fj},v_{2fj},\dots,v_{nfj}\}, \\
L_{gi} &= \mathrm{diag}\{l_{1gi},l_{2gi},\dots,l_{ngi}\}, \\
V_{gj} &= \mathrm{diag}\{v_{1gj},v_{2gj},\dots,v_{ngj}\}, \\
L_{hi} &= \mathrm{diag}\{l_{1hi},l_{2hi},\dots,l_{nhi}\}, \\
V_h &= \mathrm{diag}\{v_{1h},v_{2h},\dots,v_{nh}\}. \\
i &= 1,2,\dots,6, j=1,2,3,
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
&l_{f1}(t,t-\iota_1)+l_{f2}(t,t-\iota_2)+l_{f3}(t,t-\iota(t)) \\
&\quad +l_{f4}(t-\iota(t),t-\iota_1)+l_{f5}(t-\iota(t),t-\iota_2) \\
&\quad +l_{f6}(t-\iota_1,t-\iota_2)=\xi^{(}t)\nu_1\xi(t)\ge 0, \qquad (22) \\
&v_{f1}(t)+v_{f2}(t-\iota(t)) \\
&\quad +v_{f3}(t-\iota_1)+v_{f4}(t-\iota_2)=\xi^{(}t)\nu_2\xi(t)\ge 0, \qquad (23) \\
&l_{g1}(t,t-\iota_1)+l_{g2}(t,t-\iota_2)+l_{g3}(t,t-\iota(t))) \\
&\quad +l_{g4}(t-\iota(t),t-\iota_1)+l_{g5}(t-\iota(t),t-\iota_2) \\
&\quad +l_{g6}(t-\iota_1,t-\iota_2)=\xi^{(}t)\nu_1\xi(t)=\xi^{(}t)\nu_3\xi(t)\ge 0, \\
&\qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
&v_{g1}(t)+v_{g2}(t-\iota(t))+v_{g3}(t-\iota_1) \\
&\quad +v_{g4}(t-\iota_2)=\xi^{(}t)\nu_4\xi(t)\ge 0, \qquad (25) \\
&v_h(t)=\xi^{(}t)\nu_5\xi(t)\ge 0. \qquad (26)
\end{aligned}
$$

Moreover, the following equality holds for any $n \times n$ matrices X_1, X_2:

$$\begin{aligned}0 &= 2\left[z^T(t)X_1 + \dot{z}^T(t)X_2\right] \\ &\quad \times [-\dot{z}(t) - Az(t) + B_0 f(Wz(t)) \\ &\quad + B_1 g(Wz(t-\iota(t))) + B_2 \int_{t-\sigma_2(t)}^{t-\sigma_1(t)} h(Wz(s))ds\Bigg] \\ &= \xi^T(t)\nu_6\xi(t). \end{aligned} \tag{27}$$

Combining (16)-(27), it can be inferred that

$$\dot{V}(t,x_t) - \alpha V(t,x_t) - \alpha\omega^T(t)M\omega(t) \leq \xi^T(t)\{\Psi_w - \alpha e_{25}^T M e_{25}\}\xi(t). \tag{28}$$

It follows from (12) and (28), we have

$$\dot{V}(t,x_t) - \alpha V(t,x_t) - \alpha\omega^T(t)M\omega(t) < 0. \tag{29}$$

By multiplying of (29) with $e^{-\alpha t}$, then (29) becomes

$$\frac{d}{dt}\left(e^{-\alpha t}V(t,x_t)\right) < \alpha e^{-\alpha t}\omega^T(t)M\omega(t). \tag{30}$$

By integrating (30) on $[0,t]$ where $t \in [0,T]$ and Assumption 2, we obtain

$$\begin{aligned} V(t,x_t) &< e^{\alpha T}\left[V(0,x_0) + \alpha\int_0^T e^{-\alpha s}\omega^(s)M\omega(s)ds\right] \\ &< e^{\alpha T}\left[V(0,x_0) + \alpha\lambda_{\max}(M)\omega\int_0^T e^{-\alpha s}ds\right] \\ &< e^{\alpha T}\left[V(0,x_0) + \omega\zeta_{13}(1-e^{-\alpha T})\right]. \end{aligned} \tag{31}$$

Next, we consider $V(0,x_0)$ by Assumption 1, we get

$$\begin{aligned} &V(0,x_0) \\ &= \varpi^T(0)P\varpi(0) + \int_{-\iota(0)}^{-\iota_1} e^{-\alpha s}z^T(s)Q_1 z(s)ds \\ &\quad + \int_{-\iota_1}^{0} e^{-\alpha s}z^T(s)Q_2 z(s)ds \\ &\quad + \int_{-\iota_2}^{0} e^{-\alpha s}z^T(s)Q_3 z(s)ds \\ &\quad + \iota_1\int_{-\iota_1}^{0}\int_s^0 e^{-\alpha u}\dot{z}^T(u)R_1\dot{z}(u)duds \\ &\quad + \iota_{21}\int_{-\iota_2}^{-\iota_1}\int_s^0 e^{-\alpha u}\dot{z}^T(u)R_2\dot{z}(u)duds \\ &\quad + \iota_1^2\int_{-\iota_1}^{0}\int_s^0\int_u^0 e^{-\alpha v}\dot{z}^T(v)S_1\dot{z}(v)dvduds \\ &\quad + \iota_2^2\int_{-\iota_2}^{0}\int_s^0\int_u^0 e^{-\alpha v}\dot{z}^T(v)S_2\dot{z}(v)dvduds \\ &\quad + \iota_1^3\int_{-\iota_1}^{0}\int_s^0\int_u^0\int_v^0 e^{-\alpha\lambda}\dot{z}^T(\lambda)U_1\dot{z}(\lambda)d\lambda dvduds \\ &\quad + \iota_2^3\int_{-\iota_2}^{0}\int_s^0\int_u^0\int_v^0 e^{-\alpha\lambda}\dot{z}^T(\lambda)U_2\dot{z}(\lambda)d\lambda dvduds \\ &\quad + \sigma_{21}\int_{-\gamma_2}^{-\gamma_1}\int_s^0 e^{-\alpha u}h^T(Wz(u))Yh(Wz(u))duds \\ &\leq \varpi^T(0)P\varpi(0) + \int_{-\iota(0)}^{-\iota_1} e^{-\alpha s}z^T(s)Q_1 z(s)ds \\ &\quad + \int_{-\iota_1}^{0} e^{-\alpha s}z^T(s)Q_2 z(s)ds \\ &\quad + \int_{-\iota_2}^{0} e^{-\alpha s}z^T(s)Q_3 z(s)ds \\ &\quad + + \iota_1\int_{-\iota_1}^{0}\int_s^0 e^{-\alpha u}\dot{z}^T(u)R_1\dot{z}(u)duds \\ &\quad + \iota_{21}\int_{-\iota_2}^{-\iota_1}\int_s^0 e^{-\alpha u}\dot{z}^T(u)R_2\dot{z}(u)duds \\ &\quad + \iota_1^2\int_{-\iota_1}^{0}\int_s^0\int_u^0 e^{-\alpha v}\dot{z}^T(v)S_1\dot{z}(v)dvduds \\ &\quad + \iota_2^2\int_{-\iota_2}^{0}\int_s^0\int_u^0 e^{-\alpha v}\dot{z}^T(v)S_2\dot{z}(v)dvduds \\ &\quad + \iota_1^3\int_{-\iota_1}^{0}\int_s^0\int_u^0\int_v^0 e^{-\alpha\lambda}\dot{z}^T(\lambda)U_1\dot{z}(\lambda)d\lambda dvduds \\ &\quad + \iota_2^3\int_{-\iota_2}^{0}\int_s^0\int_u^0\int_v^0 e^{-\alpha\lambda}\dot{z}^T(\lambda)U_2\dot{z}(\lambda)d\lambda dvduds \\ &\quad + \sigma_{21}\int_{-\gamma_2}^{-\gamma_1}\int_s^0 e^{-\alpha u}z^T(u)\hat{H}^T Y\hat{H}z(u)duds \end{aligned}$$

where $\hat{H} = \mathrm{diag}\{H_1^+, \ldots, H_n^+\}$.

Furthermore, we let $\hat{P} = V^{\frac{-1}{2}}PV^{\frac{-1}{2}}$, $\hat{Q}_i = V^{\frac{-1}{2}}Q_iV^{\frac{-1}{2}}$, $i = 1,2,3$, $\hat{R}_j = V^{\frac{-1}{2}}R_jV^{\frac{-1}{2}}$, $\hat{S}_j = V^{\frac{-1}{2}}S_jV^{\frac{-1}{2}}$, $\hat{U}_j = V^{\frac{-1}{2}}U_jV^{\frac{-1}{2}}$, $j = 1,2$, $\hat{Y} = V^{\frac{-1}{2}}\hat{H}^T Y\hat{H}V^{\frac{-1}{2}}$, we can derive that

$$\begin{aligned} &V(0,x_0) \\ &\leq \varpi^T(0)V^{\frac{1}{2}}\hat{P}V^{\frac{1}{2}}\varpi(0) \\ &\quad + \int_{-\iota(0)}^{-\iota_1} e^{-\alpha s}z^T(s)V^{\frac{1}{2}}\hat{Q}_1V^{\frac{1}{2}}z(s)ds \\ &\quad + \int_{-\iota_1}^{0} e^{-\alpha s}z^T(s)V^{\frac{1}{2}}\hat{Q}_2V^{\frac{1}{2}}z(s)ds \\ &\quad + \int_{-\iota_2}^{0} e^{-\alpha s}z^T(s)V^{\frac{1}{2}}\hat{Q}_3V^{\frac{1}{2}}z(s)ds \\ &\quad + \iota_1\int_{-\iota_1}^{0}\int_s^0 e^{-\alpha u}\dot{z}^T(u)V^{\frac{1}{2}}\hat{R}_1V^{\frac{1}{2}}\dot{z}(u)duds \\ &\quad + \iota_{21}\int_{-\iota_2}^{-\iota_1}\int_s^0 e^{-\alpha u}\dot{z}^T(u)V^{\frac{1}{2}}\hat{R}_2V^{\frac{1}{2}}\dot{z}(u)duds \\ &\quad + \iota_1^2\int_{-\iota_1}^{0}\int_s^0\int_u^0 e^{-\alpha v}\dot{z}^T(v)V^{\frac{1}{2}}\hat{S}_1V^{\frac{1}{2}}\dot{z}(v)dvduds \\ &\quad + \iota_2^2\int_{-\iota_2}^{0}\int_s^0\int_u^0 e^{-\alpha v}\dot{z}^T(v)V^{\frac{1}{2}}\hat{S}_2V^{\frac{1}{2}}\dot{z}(v)dvduds \\ &\quad + \iota_1^3\int_{-\iota_1}^{0}\int_s^0\int_u^0\int_v^0 e^{-\alpha\lambda}\dot{z}^T(\lambda)V^{\frac{1}{2}}\hat{U}_1V^{\frac{1}{2}}\dot{z}(\lambda)d\lambda dvduds \\ &\quad + \iota_2^3\int_{-\iota_2}^{0}\int_s^0\int_u^0\int_v^0 e^{-\alpha\lambda}\dot{z}^T(\lambda)V^{\frac{1}{2}}\hat{U}_2V^{\frac{1}{2}}\dot{z}(\lambda)d\lambda dvduds \\ &\quad + \sigma_{21}\int_{-\gamma_2}^{-\gamma_1}\int_s^0 e^{-\alpha u}z^T(u)V^{\frac{1}{2}}YV^{\frac{1}{2}}z(u)duds \end{aligned}$$

$$
\begin{aligned}
&\leq \{\lambda_{\max}(\hat{P}) + \Omega_1\lambda_{\max}(\hat{Q}_1) + \Omega_2\lambda_{\max}(\hat{Q}_2) \\
&\quad +\Omega_3\lambda_{\max}(\hat{Q}_3) + \Omega_4\lambda_{\max}(\hat{R}_1) + \Omega_5\lambda_{\max}(\hat{R}_2) \\
&\quad +\Omega_6\lambda_{\max}(\hat{S}_1) + \Omega_7\lambda_{\max}(\hat{S}_2) + \Omega_8\lambda_{\max}(\hat{U}_1) \\
&\quad +\Omega_9\lambda_{\max}(\hat{U}_2) + \Omega_{10}\lambda_{\max}(\hat{Y})\} \\
&\quad \times \sup_{\iota_2\leq s\leq 0} \{z^T(s)Vz(s), \dot{z}^T(s)V\dot{z}(s)\} \leq \Gamma c_1
\end{aligned}
$$

where

$$
\begin{aligned}
\Gamma &= \zeta_2 + \Omega_1\zeta_3 + \Omega_2\zeta_4 + \Omega_3\zeta_5 + \Omega_4\zeta_6 + \Omega_5\zeta_7 \\
&\quad +\Omega_6\zeta_8 + \Omega_7\zeta_9 + \Omega_8\zeta_{10} + \Omega_9\zeta_{11} + \Omega_{10}\zeta_{12} \quad (32)
\end{aligned}
$$

In addition, it follows from (15) that

$$
\begin{aligned}
V(t, x_t) &\geq z^T(t)Pz(t) \\
&\geq \lambda_{\min}(\hat{P})z^T(t)Vz(t) = \zeta_1 z^T(t)Vz(t). \quad (33)
\end{aligned}
$$

Then, from the inequalities (31)-(33) and the condition (14), we obtain

$$
\begin{aligned}
z^T(t)Vz(t) &\leq \frac{e^{\alpha T}}{\zeta_1}\left[\Gamma c_1 + \rho\zeta_{13}(1 - e^{-\alpha T})\right] \\
&< c_2.
\end{aligned}
$$

By definition (2), the system (10) is finite-time bounded. The proof is complete. □

Remark 3: Choosing (v_1, v_2) *as* $(t, t-\iota_1)$, $(t, t-\iota_2)$, $(t, t-\iota(t))$, $(t-\iota(t), t-\iota_1)$, $(t-\iota(t), t-\iota_2)$, *and* $(t-\iota_1, t-\iota_2)$ *in* $(6)-(8)$. *Therefore, we used more information on cross terms among the terms* t, $t-\iota_1$, $t-\iota_2$, *and* $\iota(t)$. *So, our method lead to less conservative stability criteria.*

Remark 4: In this paper, the LKFs including single, double, triple, and quadruple integral terms in which more information on the delays ι_1, ι_2, γ_1, γ_2, *and a state variable are used. In addition, the LKFs consisting of two new triple integral terms* $\iota_1^2\int_{t-\iota_1}^{t}\int_{s}^{t}\int_{u}^{t} e^{\alpha(t-v)}\dot{z}^T(v)S_1\dot{z}(v)dvduds$ *and* $\iota_2^2\int_{t-\iota_2}^{t}\int_{s}^{t}\int_{u}^{t} e^{\alpha(t-v)}\dot{z}^T(v)S_2\dot{z}(v)dvduds$ *are applied, that have not been used in [6], [10]–[13], [15]–[18], [21]. Moreover, we also introduce two new quadruple integral terms* $\iota_1^3\int_{t-\iota_1}^{t}\int_{s}^{t}\int_{u}^{t}\int_{v}^{t} e^{\alpha(t-\lambda)}\dot{z}^T(\lambda)U_1\dot{z}(\lambda)d\lambda dvduds$ *and* $\iota_2^3\int_{t-\iota_2}^{t}\int_{s}^{t}\int_{u}^{t}\int_{v}^{t} e^{\alpha(t-\lambda)}\dot{z}^T(\lambda)U_2\dot{z}(\lambda)d\lambda dvduds$ *which have not appeared in [6], [9]–[21]. Furthermore, the stability and performance analysis have applied more information on activation functions, that is, in the proof we have* $F_i^- \leq \frac{f_i(Wu)-f_i(Wv)}{Wu-Wv} \leq F_i^+$, $G_i^- \leq \frac{g_i(Wu)-g_i(Wv)}{Wu-Wv} \leq G_i^+$, *and* $H_i^- \leq \frac{h_i(Wu)-h_i(Wv)}{Wu-Wv} \leq H_i^+$. *Therefore, the major keys to improving the results of our work are constructing new LKFs and using new techniques for estimating the time derivatives, which lead to less conservatism. The bound of the time derivative of LKFs is tighter bound than the inequalities [49], [50] when we use the Jensen's integral inequality [6], the new triple integral inequality [2], and extended single and double Wirtinger's integral inequality [5] in the proof. All of these lead to a reduction of the conservatism of obtained results compared with previous works as shown in numerical examples.*

B. FINITE-TIME STABLE

We define

$$
\begin{aligned}
e_i &= \left[0_{n\times(i-1)n}I_{n\times n}0_{n\times(24-i)n}\right] \in \mathbb{R}^{n\times 24n} \\
&\quad (i = 1, 2, \ldots, 24), \\
\xi(t) &= \Bigg[z^T(t),\ z^T(t-\iota_1),\ z^T(t-\iota_2),\ z^T(t-\iota(t)), \\
&\quad \dot{z}^T(t),\ \frac{1}{\iota_1}\int_{t-\iota_1}^{t} z^T(s)ds,\ \frac{1}{\iota_{21}}\int_{t-\iota_2}^{t-\iota_1} z^T(s)ds, \\
&\quad \frac{1}{\iota_2}\int_{t-\iota_2}^{t} z^T(s)ds,\ \frac{1}{\iota_1^2}\int_{t-\iota_1}^{t}\int_{s}^{t} z^T(u)duds, \\
&\quad \frac{1}{\iota_{21}^2}\int_{t-\iota_2}^{t-\iota_1}\int_{s}^{t-\iota_1} z^T(u)duds,\ \varrho_2,\ \varrho_3,\ \varrho_1, \\
&\quad \varrho_4,\ f^T(Wz(t)),\ f^T(Wz(t-\iota_t)), \\
&\quad f^T(Wz(t-\iota_1)),\ f^T(Wz(t-\iota_2)), \\
&\quad g^T(Wz(t)),\ g^T(Wz(t-\iota(t))), \\
&\quad g^T(Wz(t-\iota_1)),\ g^T(Wz(t-\iota_2)), \\
&\quad h^T(Wz(t)),\ \int_{t-\gamma_2(t)}^{t-\gamma_1(t)} h^T(Wz(s))ds\Bigg]^T, \\
\Psi_{nw} &= \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 + \nu_1 + \nu_2 + \nu_3 \\
&\quad +\nu_4 + \nu_5 + \nu_{c6}, \\
\nu_{c6} &= \mathrm{Sym}\{[e_1^TX_1 + e_5^TX_2][-e_5 - Ae_1 + B_0e_{15} \\
&\quad +B_1e_{20} + B_2e_{24}]\}.
\end{aligned}
$$

Remark 5: The generalized neural networks (1)-(2) without output vector and external disturbance ($y(t) = 0$ *and* $\omega = 0$) *satisfying (4)-(5) becomes*

$$
\begin{aligned}
\dot{z}(t) &= -Az(t) + B_0f(Wz(t)) + B_1g(Wz(t-\iota(t))) \\
&\quad +B_2\int_{t-\gamma_2(t)}^{t-\gamma_1(t)} h(Wz(s))ds, \quad (34) \\
z(t) &= \phi(t), t \in [-\iota_2, 0]. \quad (35)
\end{aligned}
$$

Corollary 1: For given positive scalars ι_2, τ *and* α, *the system (34) is finite-time stable,* P, $Q_i (i = 1, 2, 3)$, R_j, S_j, $U_j (j = 1, 2)$, Y, M, *any matrices* X_1, X_2 *such that the following LMIs hold:*

$$
\begin{aligned}
&\Psi_{nw} < 0, \quad (36) \\
&\zeta_1 I \leq \hat{P} \leq \zeta_2 I,\ 0 \leq \hat{Q}_1 \leq \zeta_3 I,\ 0 \leq \hat{Q}_2 \leq \zeta_4 I, \\
&\quad 0 \leq \hat{Q}_3 \leq \zeta_5 I,\ 0 \leq \hat{R}_1 \leq \zeta_6 I,\ 0 \leq \hat{R}_2 \leq \zeta_7 I, \\
&\quad 0 \leq \hat{S}_1 \leq \zeta_8 I,\ 0 \leq \hat{S}_2 \leq \zeta_9 I,\ 0 \leq \hat{U}_1 \leq \zeta_{10} I, \\
&\quad 0 \leq \hat{U}_2 \leq \zeta_{11} I,\ 0 \leq \hat{Y} \leq \zeta_{12} I, \quad (37) \\
&e^{\alpha T}\Pi c_1 < \zeta_1 c_2. \quad (38)
\end{aligned}
$$

Proof: Similarly to the proof of Theorem 1, therefore, it is omitted here. □

Remark 6: The generalized neural networks (34)-(35) without distributed delay and W *is identity matrix* ($B_2 = 0$ *and* $W = I$) *can be written as follows:*

$$
\begin{aligned}
\dot{z}(t) &= -Az(t) + B_0f(z(t)) + B_1g(z(t-\iota(t))) \quad (39) \\
z(t) &= \phi(t), t \in [-\iota_2, 0]. \quad (40)
\end{aligned}
$$

satisfying $0 \leq \iota(t) \leq \iota_2$ *and* $\dot{\iota}(t) \leq \tau$, *which mean that the system (39)-(40) becomes a special case of the system (34)-(35)*

Corollary 2: For given positive scalars ι_2, τ *and* α, *the system (39) is asymptotically stable, if there exist symmetric positive definition matrices* $Q_i(i = 1, 2, 3)$, R_j, S_j, $U_j(j = 1, 2)$, *any matrices* P, X_1, X_2 *such that the following LMI holds*:

$$\Psi_{nw} < 0. \tag{41}$$

Proof: The proof of Corollary 2 is similar to the proof of Theorem 1, hence it is omitted here. □

Remark 7: As shown in the above, from Corollary 2, we can give stability criterion for a neural network with time-varying delay, even any delay rate τ. *Our results are more effective, which is illustrated in the numerical example part.*

C. FINITE-TIME MIXED H_∞/PASSIVITY ANALYSIS

In this subsection, we consider the finite-time mixed H_∞/passivity for the generalized neural networks as follows:

$$\begin{aligned}\dot{z}(t) &= -Az(t) + B_0 f(Wz(t)) + B_1 g(Wz(t - \iota(t)))\\ &\quad + B_2 \int_{t-\gamma_2(t)}^{t-\gamma_1(t)} h(Wz(s))ds + C\omega(t), \qquad (42)\\ y(t) &= D_1 z(t) + D_2 z(t - \iota(t))\\ &\quad + D_3 \int_{t-\gamma_2(t)}^{t-\gamma_1(t)} h(Wz(s))ds + D_4\omega(t), \qquad (43)\\ z(t) &= \phi(t), t \in [-\iota_2, 0]. \qquad (44)\end{aligned}$$

We define

$$\begin{aligned}\nu_7 &= -\gamma_1[e_1 + D_2e_4 + D_3e_{24} + D_4e_{25}]^T\\ &\quad \times[e_1 + D_2e_4 + D_3e_{24} + D_4e_{25}]\\ &\quad +2(1-\sigma)\delta[e_1 + D_2e_4 + D_3e_{24} + D_4e_{25}]e_{25}\\ &\quad +\delta^2 e_{25}^T e_{25},\\ \Psi_{mp} &= \Psi_w + \nu_7.\end{aligned}$$

Theorem 2: For given positive scalars ι_2, τ *and* α, *the system (42)-(43) is finite-time bounded with a mixed* H_∞ */passivity performance index* δ *if there exist symmetric positive definition matrices* P, $Q_i(i = 1, 2, 3)$, R_j, S_j, $U_j(j = 1, 2)$, Y, M, *any matrices* X_1, X_2 *such that the following LMIs hold*:

$$\Psi_{mp} - \alpha e_{25}^T M e_{25} < 0, \tag{45}$$

$$\begin{aligned}\zeta_1 I &\leq \hat{P} \leq \zeta_2 I,\ 0 \leq \hat{Q}_1 \leq \zeta_3 I,\\ 0 &\leq \hat{Q}_2 \leq \zeta_4 I,\ 0 \leq \hat{Q}_3 \leq \zeta_5 I,\\ 0 &\leq \hat{R}_1 \leq \zeta_6 I,\ 0 \leq \hat{R}_2, \leq \zeta_7 I\\ 0 &\leq \hat{S}_1 \leq \zeta_8 I,\ 0 \leq \hat{S}_2 \leq \zeta_9 I,\\ 0 &\leq \hat{U}_1 \leq \zeta_{10} I,\ 0 \leq \hat{U}_2 \leq \zeta_{11} I,\\ 0 &\leq \hat{Y} \leq \zeta_{12} I,\ 0 \leq \hat{M} \leq \zeta_{13} I,\end{aligned} \tag{46}$$

$$e^{\alpha T}\left[\Pi c_1 + \omega\zeta_{13}(1 - e^{-\alpha T})\right] < \zeta_1 c_2. \tag{47}$$

Proof of Theorem 2: By using LKF and the proof of Theorem 1, we have

$$\begin{aligned}&\dot{V}(t, x_t) - \alpha V(t, x_t) - J(t)\\ &\qquad \leq \xi^T(t)\left(\Psi_w - \alpha e_{25}^T M e_{25}\right)\xi(t)\end{aligned} \tag{48}$$

where

$$J(t) = -\sigma y^T(t)y(t) + 2(1-\sigma)\delta y^T(t)w(t) + \sigma^2\omega^T(t)\omega(t).$$

From (45), we obtain

$$\dot{V}(t, x_t) - \alpha V(t, x_t) - J(t) \leq 0. \tag{49}$$

Multiplying inequality (49) by $e^{-\alpha t}$ *and integration from* 0 *to* T, *we obtain*

$$V(x_t, t) < e^{\alpha T}V(x_0, 0) + e^{\alpha T}\int_0^T e^{-\alpha s}J(s)ds.$$

Then, under the zero original condition $V(x_0, 0) = 0$, *we get*

$$\int_0^T J(s)ds > e^{\alpha T}V(x_t, t).$$

Meanwhile, $V(x_t, t) > 0$. *Thus,*

$$\int_0^T \left[-\sigma y^T(t)y(t) + 2(1-\sigma)\delta y^T(t)w(t) + \sigma^2\omega^T(t)\omega(t)\right]ds > 0.$$

which indicates that

$$\begin{aligned}&\int_0^T \sigma y^T(t)y(t) - 2(1-\sigma)\delta y^(t)\omega(t)\\ &\qquad\qquad \leq \delta^2\int_0^T \omega^T(t)\omega(t)dt.\end{aligned}$$

By definition (3), the system (42) is finite-time bounded with a mixed H_∞*/passivity. This completes the proof.*

IV. NUMERICAL EXAMPLES

Next, we show numerical examples to demonstrate the efficiency of the present results.

Example 1: Consider the generalized neural networks described in (10) with the following matrix parameters:

$$\begin{aligned}A &= \mathrm{diag}\{1, 1\},\ F_m = G_m = H_m = \mathrm{diag}\{0, 0\}\\ F_p &= \mathrm{diag}\{-0.04, 0.04\},\ G_P = \mathrm{diag}\{-0.16, 0.16\},\\ H_P &= \mathrm{diag}\{-1, -1\},\ W = \mathrm{diag}\{1, 1\},\\ B_0 &= \begin{bmatrix} 1.188 & 0.09 \\ 0.09 & 1.188 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.09 & q0.14 \\ 0.05 & 0.09 \end{bmatrix},\\ B_2 &= \begin{bmatrix} 0.44 & -0.21 \\ 0.29 & 0.41 \end{bmatrix},\ C = \begin{bmatrix} 0.2 & -0.6 \\ 0.3 & 0.2 \end{bmatrix}.\end{aligned}$$

Let the interval discrete time-varying is $\iota(t) = 0.8 + 0.5\sin(t)$, *the interval distributed time-varying delays are* $\gamma_1(t) = 0.3 + 0.2\sin(t)$ *and* $\gamma_2(t) = 0.5 + 0.3\sin(t)$, *the neuron activation functions are taken as* $f(z(t)) = [0.2\tanh(z_1(t)),\ 0.2\tanh(z_2(t))]^T$, $g(z(t)) = [0.2(|z_1(t) + 1| - |z_1(t) - 1|),\ 0.2(|z_2(t) + 1| - |z_2(t) - 1|)]^T$ *and* $h(z(t)) = [\tanh(z_1(t)),\ \tanh(z_2(t))]^T$. *For given scalars* $\tau = 0.5$, $\rho = 0.1$, $c_1 = 1.12$, $T = 30$, $\alpha = 0.1$ *and* V

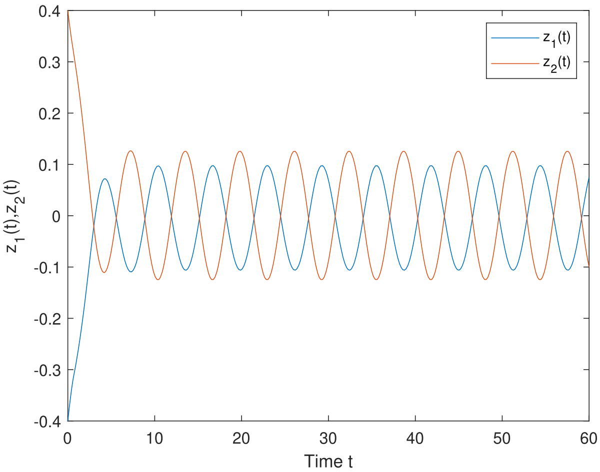

FIGURE 1. The trajectories of $z_1(t)$ and $z_2(t)$ of system (10) in Example 1.

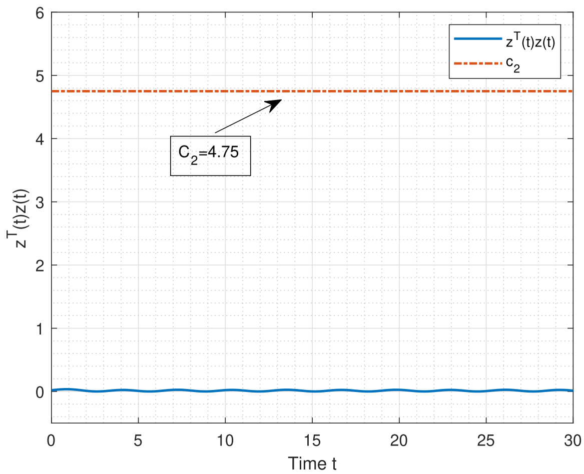

FIGURE 2. Time history of $x^T(t)x(t)$ for closed-loop system in Example 1.

is identity matrix. Solving LMIs (12) in Theorem 1, we obtain $c_2 = 4.75$.

For an initial condition $\phi(t) = [-0.40.4]^T$, *figure 1 demonstrates the trajectories of solution* $z_1(t)$ *and* $z_2(t)$ *of generalized neural networks (10) with various activation functions and mixed time-varying. Figure 2 illustrates the time history of* $z^T(t)z(t)$ *for the delay generalized neural network system (10). In conclusion, system (10) is finite-time boundedness with respect to (0.5, 4.75, 30, I, 0.1). Thus, this proves the effectiveness of our obtained results in Theorem 1.*

Example 2: Consider the generalized neural networks described in (34) with the following matrix parameters:

$$
\begin{aligned}
A &= \mathrm{diag}\{1, 1\},\ W = \mathrm{diag}\{1, 1\},\\
F_m &= G_m = H_m = \mathrm{diag}\{0, 0\},\\
F_p &= \mathrm{diag}\{-0.04, 0.04\},\ G_P = \mathrm{diag}\{-0.16, 0.16\},\\
H_P &= \mathrm{diag}\{-1, -1\},\ B_0 = \begin{bmatrix} 1.188 & 0.09 \\ 0.09 & 1.188 \end{bmatrix},\\
B_1 &= \begin{bmatrix} 0.09 & 0.14 \\ 0.05 & 0.09 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.44 & -0.21 \\ 0.29 & 0.41 \end{bmatrix}.
\end{aligned}
$$

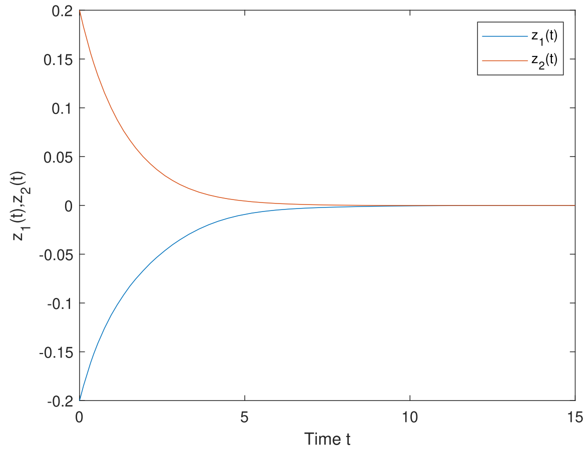

FIGURE 3. The trajectories of $z_1(t)$ and $z_2(t)$ of system (34) in Example 2.

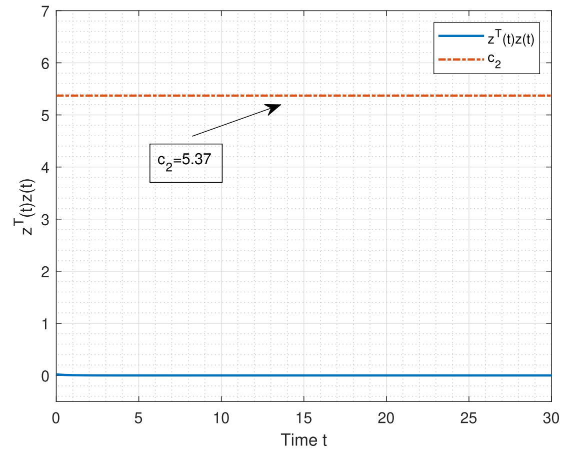

FIGURE 4. Time history of $x^T(t)x(t)$ for closed-loop system in Example 2.

Let the interval discrete time-varying is $\iota(t) = 0.8 + 0.5\sin(t)$, *the interval distributed time-varying delays are* $\gamma_1(t) = 0.3 + 0.2\sin(t)$ *and* $\gamma_2(t) = 0.5 + 0.3\sin(t)$, *the neuron activation functions are taken as* $f(z(t)) = [0.2\tanh(z_1(t)),\ 0.2\tanh(z_2(t))]^T$, $g(z(t)) = [0.2(|z_1(t)+1| - |z_1(t) - 1|),\ 0.2(|z_2(t) + 1| - |z_2(t) - 1|)]^T$ *and* $h(z(t)) = [\tanh(z_1(t)),\ \tanh(z_2(t))]^T$. *For given scalars* $\tau = 0.5$, $c_1 = 1.12$, $\rho = 0.1$, $T = 30$, $\alpha = 0.1$ *and V is identity matrix. Solving LMIs (36) in Corollary 1, we obtain* $c_2 = 5.37$.

For an initial condition $\phi(t) = [-0.20.2]^T$, *figure 3 demonstrates the trajectories of solution* $z_1(t)$ *and* $z_2(t)$ *of generalized neural networks (34) with various activation functions and mixed time-varying. Figure 4 illustrates the time history of* $z^T(t)z(t)$ *for the delay generalized neural network system (34). In conclusion, system (34) is finite-time stable with respect to (0.5, 5.37, 30, I, 0.1). Thus, this proves the effectiveness of our obtained results in Corollary 1.*

Example 3: Consider the neural networks described in (39) with the matrix parameters as follows:

$$
\begin{aligned}
A &= \mathrm{diag}\{2, 2\},\ W = \mathrm{diag}\{1, 1\},\\
F_p &= \mathrm{diag}\{0.4, 0.8\},\ F_m = \mathrm{diag}\{0, 0\},\\
W_0 &= \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix},\ W_1 = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix}.
\end{aligned}
$$

TABLE 1. Delay bounds ι_2 with different τ.

τ	0.80	0.90
[9]	4.5940	3.4671
[10]	4.8167	3.4245
[11]	5.4428	3.6482
[12]	5.6384	3.7718
[6]	6.7186	3.9623
Corollary 2	8.5200	4.0979

TABLE 2. Delay bounds ι_2 with different τ.

τ	0.40	0.45	0.50	0.55
[13]	4.6569	3.7268	3.4076	3.2841
[14]	4.5543	3.8364	3.5583	3.4110
[15]	7.6697	6.7287	6.4126	3.2569
[16]	8.3498	7.3817	7.0219	6.8156
[17]	10.1095	8.6732	8.1733	7.8993
[12]	10.5730	9.3566	8.8467	8.5176
[18]	16.8020	11.6745	9.9098	9.0062
[6]	17.2697	12.0698	10.2903	9.3879
Corollary 2	19.5194	12.2110	12.4201	10.3990

Let the time-varying $\iota(t) = 0.8\sin(t) + 4.8384$, the neuron activation function is taken as $f(z(t)) = [0.4\tanh(z_1(t)),\ 0.8\tanh(z_2(t))]^T$. The proposed criteria, the maximum delay bounds with τ estimated by the Corollary 2 are shown in Table 1. In addition, we compare the obtained results with the published work. The results guarantee that the stability conditions demonstrated in this article are more efficient than the existing literature.

Example 4: Consider the neural networks described in (39) with the matrix parameters in the following:

$$A = \mathrm{diag}\{1.5, 0.7\},\ W = \mathrm{diag}\{1, 1\},$$
$$F_p = \mathrm{diag}\{0.3, 0.8\},\ F_m = \mathrm{diag}\{0, 0\},$$
$$W_0 = \begin{bmatrix} 0.0503 & 0.0454 \\ 0.0987 & 0.2075 \end{bmatrix},\ W_1 = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix}.$$

Let the time-varying $\iota(t) = 0.4\sin(t) + 10.173$, the neuron activation function is taken as $f(z(t)) = [0.3\tanh(z_1(t)),\ 0.8\tanh(z_2(t))]^T$.The proposed conditions, the maximum delay bounds with τ computed by the Corollary 2 are shown in Table 2. In addition, we compare the obtained results with the published work. The results guarantee that the stability conditions demonstrated in this article are more efficient than the existing literature.

Example 5: Consider the neural networks described in (39) with the following matrix parameters:

$$A = \mathrm{diag}\{7.3458, 6.9987, 5.5949\},\ W_0 = \mathrm{diag}\{0, 0, 0\},$$
$$W_1 = \mathrm{diag}\{1, 1, 1\},\ F_m = \mathrm{diag}\{0, 0, 0\}$$
$$F_p = \mathrm{diag}\{0.3680, 0.1795, 0.2876\},$$
$$W = \begin{bmatrix} 13.6014 & -2.9616 & -0.6936 \\ 7.4736 & 21.6810 & 3.2100 \\ 0.7290 & -2.6334 & -20.1300 \end{bmatrix}.$$

Let $f(z(t)) = [0.3680\tanh(z_1(t)),\ 0.1795\tanh(z_2(t)),\ 0.2876\tanh(z_3(t))]^T$. The proposed criteria, the maximum delay bounds with τ estimated by the Corollary 2 are listed in Table 3. Additionally, we compare the obtained results with the published work. The results guarantee that the stability conditions demonstrated in this article are more efficient than the existing literature.

TABLE 3. Delay bounds ι_2 with different τ.

τ	0.00	0.10	0.50
[19]	1.5575	0.9430	0.4417
[20]	1.6409	0.9962	0.4470
[10]	1.7250	1.0408	0.4480
[16]	1.7302	1.0453	0.4486
[21]	1.8898	1.1114	0.4514
[18]	1.8899	1.1194	0.4599
[6]	1.9349	1.1365	0.4678
Corollary 2	3.1150	1.4410	1.0299

Remark 8: From table 1-3, it is noticed that our results presented larger bounds of time-delay than the existing literature by using the multiple integral terms Lyapunov-Krasovskii function combined with inequalities.

Remark 9: The main advantage of this paper is to apply Jensen's integral inequality (Lemma 1), an extended single and double Wirtinger's integral inequalities (Lemma 2, 3), a new triple integral inequality (Lemma 4) with new LKFs to the proof. As a result, our maximum delay is greater than that of [9]–[21].

V. CONCLUSION

In this article, the problem of finite-time mixed H_∞/passivity, finite-time stability, and finite-time boundedness for generalized neural networks with interval distributed and discrete time-varying delays has been studied. It is the first time for the combination of H_∞, passivity, and finite-time boundedness. By constructing a new multiple integral LKF and applying an extended single and double Wirtinger integral inequality, Jensen's integral inequality, and a new triple integral inequality, new stability conditions are established to derive the finite-time boundedness, finite-time stability, and finite-time mixed H_∞/passivity neural networks. Numerical examples are presented to verify the efficiency of presented results and are better than the existing results. We can see that the presented technique can be extended to Takagi–Sugeno fuzzy non-homogeneous Markovian jump systems [22]; in neural networks system, synchronization of coupled reaction-diffusion [23] and synchronization of coupled memristive [24].

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