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Joint Optimization of Test Node Selection and Fiber Thread Connection for Optical Communication Network

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ABSTRACT This paper proposes a method to solve the joint optimization of test node selection and fiber thread connection. The problem is modeled as an integer linear programming problem with two classes of decision variables. The first class of decision variables are composed of the number of parallel fiber threads throughout each potential testing route, the other class of decision variables are imposed just to denote whether a node is decided as a location to place testing equipment. The objective of the joint optimization is to minimize the number of nodes where fiber test equipment based on the optical time domain reflectometer are planned to locate. The objective function is just the sum of the second class of decision variables. Two types of linear constraints are needed. The first type of constraints describes the mutual relation between the two classes of variables, and the other type of constraints are formed to depict the impact of the number of fiber threads mounted within each fiber link on the first class of decision variables. Multiple examples are given to exhibit the advantage of joint optimization over heuristic method.

INDEX TERMS Optical time domain reflectometry, measurement, integer linear programming.

I. INTRODUCTION

Fibers have been widely used in backbones of communication networks for many years throughout the world. As time lapses, faults may occur in some of the fibers within communication networks due to aging, improper usage, or even destruction activities by human beings. The detection and recovery of failed optical communication links thus become a challenging work for technicians in charge of maintaining wide area networks.

Thanks to the profound research efforts in the areas of fiber fault detection and fiber sensors in the past decades, there are several options for us to choose to detect the possible faults within a fiber thread or to do some physical test about the environments along a fiber. Optical time domain reflectometer (OTDR) is a well-known technology proposed in 1970s [1], other early studies on fiber testing can be found in [2], [3]. Subsequent works on OTDR has once become a hot topic and many problems have been solved [4]–[8]. In [4], a method of detecting and locating connection splice faults (events) in fiber optics from noisy OTDR data was proposed, the method combined Gabor series representation of the sampled OTDR data and the application of Rissanen's minimum description length (MDL) criterion to determine the number of events and their location. Afterwards, the authors of [4] developed an algorithm called rank-1 matched subspace detection and estimation to achieve better accuracy in event position estimation [5]. Researches in [6] demonstrated that photon-counting OTDR has the potential to improve the dynamic range by 10dB, as well as the two-point resolution by a factor of 20 against conventional OTDR. Taking the possibility of multiple faults within a single fiber thread into consideration, G. C. Amaral, et al proposed a method to locate multiple faults along a fiber link by using low frequency sub-carrier tone sweep with a signal processing algorithm based on the least absolute shrinkage and selection operator (LASSO) [7]. For those having interests in LASSO, a systematical analysis can be found in [8].

Another important kind of technology of fiber fault monitoring is based on optical frequency domain reflectometer (OFDR). Compared with OTDR, OFDR has the advantage of potential in-service monitoring by arranging a low frequency band for monitoring while the other bands for data transmission. Works on fiber fault location by means of OFDR can be found in [9], [10]. In [9], a method for monitoring fiber links utilizing sub-carrier multiplexing was pro-

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posed and potential application in emerging networks such as relatively short distance analogue mobile front-haul was suggested [11].

In addition to monitoring the fiber itself for fault location, a fiber can be used to monitor some specific physical quantities related to the environments around the fiber. For example, vibration or intrusion detection via distributed fiber sensors was widely adopted for safety monitoring in past decades, here the importance of vibration or intrusion detection for fibers themselves used in communication links should be stressed. Several efforts on improving the reach of vibration or intrusion detection by fiber sensors can be found in [12]–[14].

In order to reduce the number of equipment's required by conducting OTDR test, a major measure adopted by technicians is to connect several relatively short fiber threads to a longer one. In the traditional optical switch, it is impossible to automatically establish optical connection between two arbitrary fiber threads, because the controllable span of the emergent direction of the light originated from one specific incident optical fiber thread is limited owing to the very small adjustable range for the refractive index of the optical device between the incident and emergent fiber threads. Manual welding is inevitable if we expect on-demand connection between fiber threads in traditional optical switch. In recent years, studies on programmable optical device [15]–[18], especially the invention of programmable optical array in optical communication [16], [17], throw light on fully automatic switching or fiber connection controlled by software instead of manually connecting fiber threads.

The previous paragraphs summarized technologies related to fiber testing especially fiber fault location. Nowadays, commercial fiber testing equipment that have maximum reach distance over 150 kilometers based on OTDR bloom worldwide, but they are still expensive. For managers who are in charge of maintaining a wide area network, they want to make decisions to solve the problem of monitoring the status of all the optical links in the network. The most challenging task is to decide the number of fiber testing equipment and which sites are suitable for them. The problem is difficult to get an answer when only idle fiber threads are chosen to be tested. The reason to test only the idle fiber treads lies on three bases: firstly, it has little influence to the communication activity running on working fiber threads; secondly, a fiber usually contains multiple parallel fiber threads close to each other, a fault in one thread may also occur simultaneously in other parallel threads, so that by tests on an idle thread, one can deduce the status of a coaxial working thread; finally, limited by the condition that the maximum length should not be surpassed, idle fiber threads of adjacent fiber links could be concatenated as long as possible, so that the number of testing equipment which are necessary to test all the idle fiber threads of the network can be reduced. The way which fiber threads should be concatenated relies on the decision which sites are chosen to place the testing equipment's, but the latter also relies on the former, this mutual dependence on each other calls for a joint optimization of test node selection and fiber thread connection.

To solve the joint optimization problem of test node selection and fiber thread connection, the authors propose a mathematical model based on integer linear programming. To check its efficiency, profound comparisons with a heuristic method which was proposed as an early version to find approximate solutions for joint optimization of test node selection and fiber thread connection is presented. These endeavors are the main contributions of this paper.

The remaining part of this paper is organized as follows. Section II gives definitions for the joint optimization problem and symbols used hereafter, section III proposes the mathematical model of the joint optimization problem, section IV provides an example of the joint optimization of test node selection and fiber thread connection and comparisons with a heuristic method, finally, section V concludes the paper.

II. DEFINITION OF SYMBOLS USED IN THE JOINT OPTIMIZATION PROBLEM

A. ABBREVIATIONS AND ACRONYMS

To help readers to understand the joint optimization problem, the following concepts are introduced.

The first concept is ***candidate node***. A candidate node is a node that may be selected to place a fiber testing equipment. Before a solution of the joint optimization problem is found, every node in the network can be regarded as a candidate node. After finding the solution of the joint optimization problem, the set of nodes where to place fiber testing equipment is decided, and any of the nodes that are decided to place a fiber testing equipment is called a ***chosen node***.

Another concept is ***testing route***. A testing route is a route originated from a chosen node to another node and composed of adjacent fiber links. A route originated from a candidate node to another node and composed of adjacent fiber links is called a ***potential testing route***.

The joint optimization problem is a problem related to a specific fiber communication network and has the following features:

The objective is to minimize the number of chosen nodes, but at each chosen node, the number of ports of the testing equipment, or the number of fiber threads that can be tested simultaneously, can be adjusted as required;

Fiber threads of adjacent fiber links can be concatenated so that as many as possible fibers can be tested from a single port, with the proviso that the maximum total length should not be surpassed;

A type of physical constraint exists for the joint optimization problem, for each fiber link, there are a number of specified threads each of which should be tested and used only once by some route passing by.

B. SYMBOLS RELATED

To have a clear description of the topology for an optical communication network, the following concepts and symbols are defined.

N is used to denote the number of nodes in the network;

L is used to denote the number of links in the network;

r_{max} is used to denote the maximum length which a testing equipment allows;

n_j is used to denote the sum of the number of threads need to be tested within fiber links that are directly connected to node j;

m_i is used to denote the number of threads to be tested in fiber link i;

a_{ijk} is used to denote the number of times that the k-th potential testing route originating from node j crosses link i;

K_j is used to denote the number of potential testing routes originated from node j;

X_{jk} is a decision variable used to denote the number of parallel threads allocated for the k-th potential testing route originating from node j;

Y_j is a decision variable used to denote whether the candidate node j is a chosen node.

Z is a column vector composed of the sets of decision variables X_{jk} and Y_j at ascending order by subscripts j, k;

b is a column vector composed of constants, with the same size as Z;

A is a matrix composed of the coefficients of the constraint conditions of the joint optimization problem.

III. MODELING

This section sets up the model of the joint optimization of test node selection and fiber thread connection from four aspects.

A. DECISION VARIABLES

Two kinds of variables are needed to set up the joint optimization problem. The first kind of variables X_{jk}, $k \in \{1, 2, \ldots, K_j\}$, are imposed to describe the number of parallel fiber threads assigned to the k-th potential test route originating from node j.X_{jk} can only take non-negative integer values, its maximum allowable value is the minimum of the number of fiber threads to be tested among the concatenated fiber links across the corresponding route. The second kind of variables $Y_j, j \in \{1, 2, \ldots, N\}$, are imposed to describe which nodes are selected to place testing equipment. Y_j can only take integer values 0 or 1. If node j is eventually a chosen node to place a testing equipment, it takes value 1, else it takes value 0.

In order to have a united expression for the two types of decision variables, a column vector Z is used and it is organized from all the variables X_{jk} and Y_j by the ascending order of the subscripts j, k respectively. Namely,

$$Z = \begin{bmatrix} X_{11}, X_{12}, \ldots, X_{1K_1}, X_{21}, X_{22}, \ldots, X_{2K_2}, \\ \cdots, X_{N1}, X_{N2}, \ldots, X_{NK_N}, Y_1, \ldots, Y_N \end{bmatrix}^T \quad (1)$$

The number of potential testing routes K_j originated from node j depends on many factors such as the network topology, the maximum reachable testing length r_{max} of the equipment and whether loops are permitted in a potential testing route, etc. To have a complete list of all elements X_{jk} in Z, an appropriate route searching algorithm is needed and an example route searching algorithm will be presented in section IV.

B. OBJECTIVE FUNCTION

In the previous subsection, two kinds of decision variables are introduced. The two sets of decision variables are denoted as **X** and **Y** hereafter, and the elements of **X** or **Y** are X_{jk} and Y_j respectively. These symbols have been defined in the previous section.

The motivation of the joint optimization of test node selection and fiber thread connection is to minimize the number of sites selected for equipment placement. This goal of the joint optimization relies on the second kind of decision variables in **Y**, but has no direct relation with those in **X**. Based on this observation, the objective function of the joint optimization problem is defined as follows,

$$F(X, Y) = \sum_{j} Y_j \quad (2)$$

It needs to be stressed that the objective function is a linear function of all the decision variables. The constant coefficients of the second kind of decision variables Y_j appeared in the above expressions are all ones and that of the first kind of decision variables X_{jk} are all zeros.

Remember that a column vector Z has been defined to have a unified expression for the two kinds of decision variables X_{jk} and $Y_j (j = 1, 2, \ldots, N;\ k = 1, 2, \ldots, K_j)$ are all elements of **Z**, the objective function can be rewritten as,

$$F(Z) = \sum_{s} c_s Z_s \quad s = 1, 2, \cdots W \quad (3)$$

where the coefficient c_s is zero if Z_s corresponds to a decision variable from **X**, while it is one if Z_s corresponds to a decision variable from **Y**. W stands for the number of rows contained by the column vector Z. It can be observed that $W = (K_1 + K_2 + \cdots + K_N + N)$ by carefully checking the definition of Z in (1).

C. THE CONSTRAINTS

There are two types of constraints on the joint optimization of test node selection and fiber thread connection. The first type of constraints is based on the fact that test node selection and fiber thread connection are closely related to each other. The outcome of a specific fiber thread connecting scheme can be reflected by a potential test route with some number of parallel threads throughout the route. In the previous section, a kind of decision variables X_{jk} have been defined to denote the number of parallel fiber threads allocated to the k-th potential route originated from node j. It is apparent that if node j is finally chosen as a site to place test equipment, the corresponding decision variable Y_j should take value 1 and at least one of the K_j potential test routes can be established as a genuine test route and the number of parallel fiber threads X_{jk} allocated for this route should take a positive integer value; on the contrary, if node j is not chosen as a site to place test equipment, there will not be any test route originated from node j and all the decision variable X_{jk} related to node j will take zero. The above relations between X_{jk} and Y_j when node j is chosen as a site to place testing equipment can be analyzed further by taking into consideration all possible potential test routes originates from j, two cases may occur:

i. Among the end nodes of all the test routes starting from node j, no one is chosen as the site to place test equipment;

ii. Among the end nodes of all the test routes starting from node j, there is at least one node chosen as the site to place test equipment.

For case i, all the fiber threads planned to be tested must be contained by all the test routes originated from the chosen node j, thus the constraint is the following equation,

$$\sum_{k} X_{jk} = n_j Y_j, \quad k = 1, 2, \ldots, K_j \tag{4}$$

where n_j is the sum over the number of threads to be tested of all the fiber links from node j to its direct neighbors. It should be noted that although (4) is derived under the condition $Y_j = 1$, it can readily be verified that it also holds for the case $Y_j = 0$.

For case ii, some fiber threads of the links connecting node j to its direct neighbors may be tested by test routes originated from other chosen nodes, so only a certain part of $n_j Y_j$ need to be tested by the test routes originated from node j, so the following inequality holds for this case,

$$\sum_{k} X_{jk} < n_j Y_j, j = 1, 2, \ldots, N; \quad k = 1, 2, \ldots, K_j \tag{5}$$

To sum up the cases i and ii, the following constraints holds for any node j,

$$\sum_{k} X_{jk} \le n_j Y_j, j = 1, 2, \ldots, N; \quad k = 1, 2, \ldots, K_j \tag{6}$$

Again, although (5) and (6) are derived under the condition $Y_j = 1$, (6) can readily be verified that it also holds for the case $Y_j = 0$.

By moving the right hand side item of (6) to the left, the inequality becomes,

$$\sum_{k} X_{jk} - n_j Y_j \le 0, j = 1, 2, \ldots, N; \quad k = 1, 2, \ldots, K_j \tag{7}$$

The constraint expressed in (7) can be recognized as a linear inequality of the decision variables X_{jk} and Y_j.

The other type of constraints is based on the number of threads to be tested in each fiber link. For each fiber link, the number of threads to be tested should be equal to the sum of the number of threads occupied by each potential test route which passes this fiber link. As each potential test route may be allowed to pass the specific link more than once, the number of threads in link i occupied by the *k*-th potential test route from j can be established as the product of X_{jk} and a_{ijk}. Here a_{ijk} denotes the number of times that link i is traversed by the *k*-th potential test route originated from node j. Thus the following constraints on the joint optimization of test node selection and fiber thread connection should be satisfied,

$$\sum_{j,k} a_{ijk} X_{jk} = m_i, \quad i = 1, 2, \ldots, L \tag{8}$$

As previously declared, m_i is the number of threads to be tested within fiber link i. For each link in the network, a similar constraint like (8) should be imposed to the joint optimization problem, so that L such equations form the second kind of constraints. It is again noticeable that (8) is a linear expression of the decision variables with constant coefficients. These coefficients are constants each of which can be determined by the network topology and the details of the corresponding link and the potential test route.

The constraint expressed in (8) can be rewritten as two inequalities that should be simultaneously satisfied as follows,

$$\sum_{j,k} a_{ijk} X_{jk} \le m_i, \quad i = 1, 2, \ldots, L \tag{9}$$

And

$$\sum_{j,k} -a_{ijk} X_{jk} \le -m_i, \quad i = 1, 2, \ldots, L \tag{10}$$

The inequalities (7), (9) and (10) have united forms and form the complete constraints of the joint optimization of test node selection and fiber threads connection. These similar linear inequalities can be simplified to a single inequality by its matrix representation as follows,

$$AZ \le b \tag{11}$$

where A is the matrix of the coefficients appeared in (7), (9) and (10). A has N + 2 × L rows and $(K_1 + K_2 + \cdots + K_N + N)$ columns. Z is a column vector composed of all the $(K_1 + K_2 + \cdots + K_N + N)$ decision variables, b is a column vector of constants with the same size as that of Z. In accordance with the order of decision variables appeared in (1), the coefficients matrix A can be divided into three parts according to (7), (9) and (10) respectively. The first N rows of A is denoted as A_1, and A_1 can be obtained from the matrix representation of N linear inequalities expressed by (7) as follows, (12), as shown at the bottom of the next page.

The middle L rows of A is denoted as A_2 and it is obtained from the matrix representation of L linear inequalities expressed by (9). A_2 is organized as follows, (13), as shown at the bottom of the next page.

The bottom L lines of matrix A can easily be established as-A_2 according to (10), this means that each element in the bottom block of A is just the inverse number of that in A_2 with the equal in-block row indices and column indices, respectively. In summary, matrix A can be established as a block-wise matrix as follows,

$$A = \begin{bmatrix} A_1 \\ A_2 \\ -A_2 \end{bmatrix} \tag{14}$$

Accordingly, the constant vector b can also be divided to three row blocks with number of rows N, L and L for each block. After careful speculations, the block-wise expression can be obtained as follows,

$$\mathrm{b} = \begin{bmatrix} 0 \\ \mathrm{m} \\ -\mathrm{m} \end{bmatrix} \tag{15}$$

where **0** is a column vector of N zeros, and **m** is a column vector composed of L numbers $\mathrm{m_i}, \mathrm{i} = 1, 2, \ldots, \mathrm{L}$. As defined in section II-B, $\mathrm{m_i}$ denotes the number of fiber threads to be tested in the fiber link i.

Now, with the objective function defined in (3) and the constraints expressed in (11), the entire model of the joint optimization of test node selection and fiber thread connection has been set up. By checking the objective function and the constraints of the joint optimization problem, some important features can be found as follows:

i. the objective function is a linear function of its decision variables;

ii. all the constraints can be expressed as linear equalities or inequalities;

iii. all the decision variables can only take integer values.

Fortunately, these features of the joint optimization problem indicate that the joint optimization of test node selection and fiber thread connection is an integer linear programming problem (ILP).

D. SOLUTION OF THE ILP

The task of joint optimization of test node selection and fiber thread connection has an objective function as (3) and constraints as (11). In summary, it can be modeled as an ILP problem as follows,

$$\begin{aligned} &\text{objectives: } \min \sum_{\mathrm{j}} \mathrm{Y_j} \\ &\text{with constraints: } \begin{cases} \sum_{\mathrm{k}} \mathrm{X_{jk}} - \mathrm{n_j Y_j} \le 0, \\ \quad \mathrm{j} = 1, 2, \ldots, \mathrm{N};\ \mathrm{k} = 1, 2, \ldots, \mathrm{K_j} \\ \sum_{\mathrm{j,k}} \mathrm{a_{ijk} X_{jk}} = \mathrm{m_i}, \\ \quad \mathrm{i} = 1, 2, \ldots, \mathrm{L}. \end{cases} \end{aligned} \tag{16}$$

Or in its equivalent matrix form as,

$$\begin{aligned} &\text{objectives: } \min_{Z} \mathrm{F}(\mathrm{Z}) \\ &\text{with constraints: } \mathrm{AZ} \le \mathrm{b} \end{aligned} \tag{17}$$

where $\mathrm{F}(\cdot)$ is the linear function defined by (3) and Z is a column matrix each element of which can only take non-negative integer value. In general, linear programming without the constriction of integer values for the decision variables can be efficiently solved by the simplex method [19]. The goal of finding integer solutions to an ILP problem needs further efforts. There are two fundamental methods to get integer solutions to an ILP problem based on the simplex method. The first one is the cutting plane method [19]. The other method is the branch and bound method [20]. Nowadays these two methods have been integrated into comprehensive algorithms by software developers to solve ILP problems. For example, the ILP model expressed by (16) may be solved by modern software tools such as the function intlinprog() in MATLAB 2016 or higher versions [21], [22].

IV. ALGORITHMS AND NUMERICAL EXAMPLES

In this section, the theoretic model of joint optimization of test node selection and fiber thread connection will be applied to a real optical communication network set up for power grid management in a southern province in China.

A. DETAILS OF THE OPTICAL NETWORK

Fig. 1 gives the topology of the example network. The distance and the number of threads to be tested within each link are also provided in this figure.

The network has 27 nodes and 27 fiber links. The nodes are labeled with numbers and the links are unidirectional. The length and the number of threads to be tested for each link are also provided in the figure. For example, the pair of numbers 30.08 and 10 labeled in brackets in the vicinity of the link between node 1 and node 2 claims that the length of this link is 30.08 km while the number of fiber threads to be tested in this fiber link is 10. Hereafter, the fiber testing technology adopted is assumed to be traditional OTDR and the maximum total length that equipment can reach is 150 kilometers. Concatenation of multiple fiber threads can be tested if only

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$$\mathrm{A_1} = \begin{bmatrix} 1 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{n_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 1 & 0 & 0 & 0 & 0 & -\mathrm{n_2} & 0 & 0 \\ \vdots & & & & \cdots & & & & & & 0 & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 1 & 0 & 0 & 0 & -\mathrm{n_N} \end{bmatrix} \tag{12}$$

$$\mathrm{A_2} = \begin{bmatrix} \mathrm{a_{111}} & \cdots & \mathrm{a_{11K_1}} & \mathrm{a_{121}} & \cdots & \mathrm{a_{12K_2}} & \mathrm{a_{1N1}} & \cdots & \mathrm{a_{1NK_N}} & 0 & 0 & \cdots & 0 \\ \mathrm{a_{211}} & \cdots & \mathrm{a_{21K_1}} & \mathrm{a_{221}} & \cdots & \mathrm{a_{22K_2}} & \mathrm{a_{2N1}} & \cdots & \mathrm{a_{2NK_N}} & 0 & 0 & \cdots & 0 \\ \vdots & \cdots & & \vdots & \cdots & & \vdots & \cdots & & & & \vdots & \\ \mathrm{a_{L11}} & \cdots & \mathrm{a_{L1K_1}} & \mathrm{a_{L21}} & \cdots & \mathrm{a_{L2K_2}} & \mathrm{a_{LN1}} & \cdots & \mathrm{a_{LNK_N}} & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{13}$$

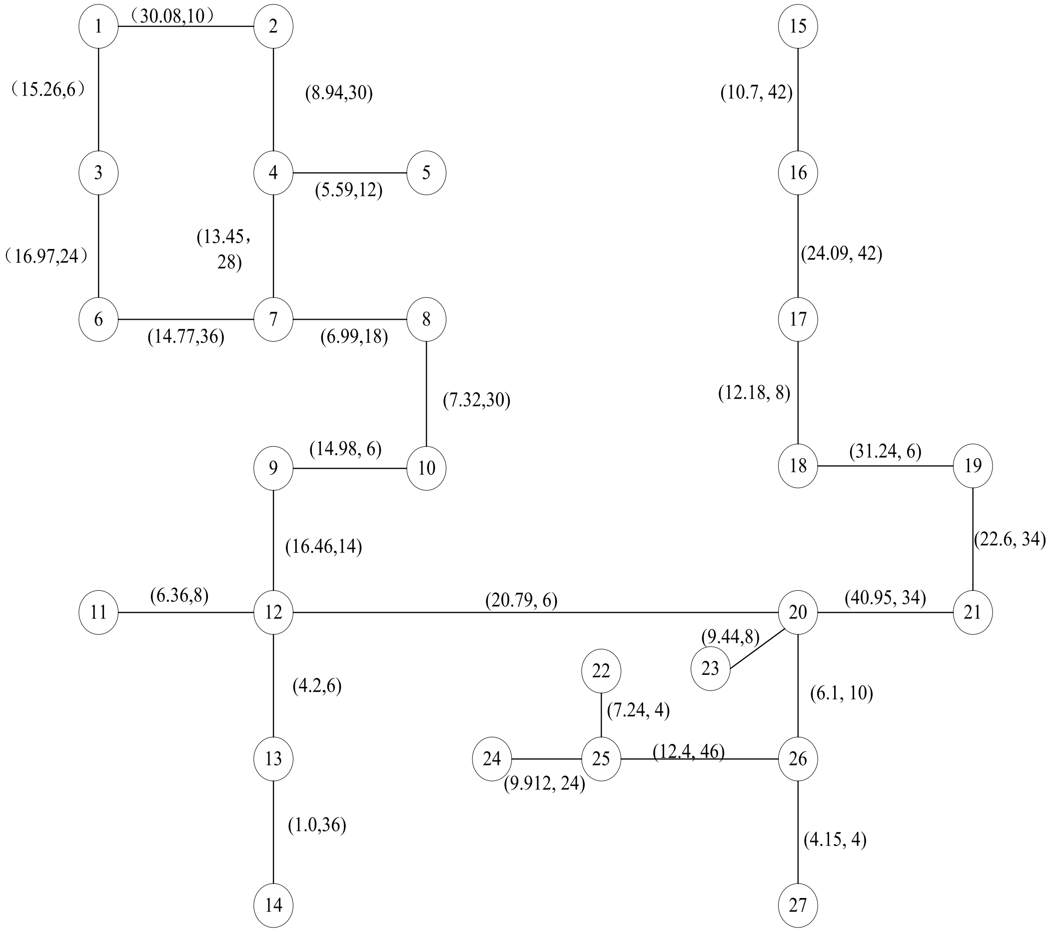

FIGURE 1. Topology of the network, each side of the graph has a pair of numbers, and the first number means the distance, another number means the number of threads to be tested.

the total length of the threads does not exceed the limit of 150 km.

B. ENUMERATION OF ROUTES

As mentioned previously in section II, enumeration of all the potential test routes is a necessary work in setting up the ILP model for the joint optimization of test node selection and fiber thread connection. Fig. 2 gives the flowchart of finding all the potential test routes for a pair of specified nodes ns and ne. The subroutine calls itself iteratively and includes 7 input parameters. N is the number of nodes contained in the network, r_{max} is the maximum allowed total length of any potential testing route, ns is the specified starting node, ne is the specified ending node, cn is the current node just added to the current potential test route, **cp** records the probed route from ns to cn which is part of the current potential test route, **mr** is a table which contains the distances between any starting node to its direct neighbors. The output of the subroutine is given by the two quantities nr and **rp**, nr stores the number of potential test routes found out between ns and ne, **rp** is a table which stores all the potential test routes, one route per row. Owing to the iterative nature of the subroutine, the outputs of the subroutine are declared as global variables for simplicity.

At the first time the subroutine is called, cn has an initial value which is equal to ns, and **cp** contains only one node which is just equal to ns. At the current node cn, a neighboring node of cn is checked and judged whether it is suitable to add this neighbor to the current potential test route **cp**. Only if the total length after adding a new node to the current route does not exceed the maximum r_{max}, the new node is permitted to the route. If the new node is just the specified ending node ne, then the current potential test route is accomplished and can be stored in **rp**, in addition, nr, the number of routes found, should be increased by one. Eventually, the number of potential test routes is counted by nr and each route from node ns to ne can be enumerated by reading out the corresponding row of the table stored by **rp**. By similar calls to the subroutine exhrout(·) with other specified node pairs (ns,ne), all potential test routes can be enumerated. It needs to be stressed that during the iterative process of calling exhrout(·), a direct neighbor of the node cn is added to the current route and then cn is replaced with the neighboring node just added to the route. No restriction is made on selecting a direct neighbor

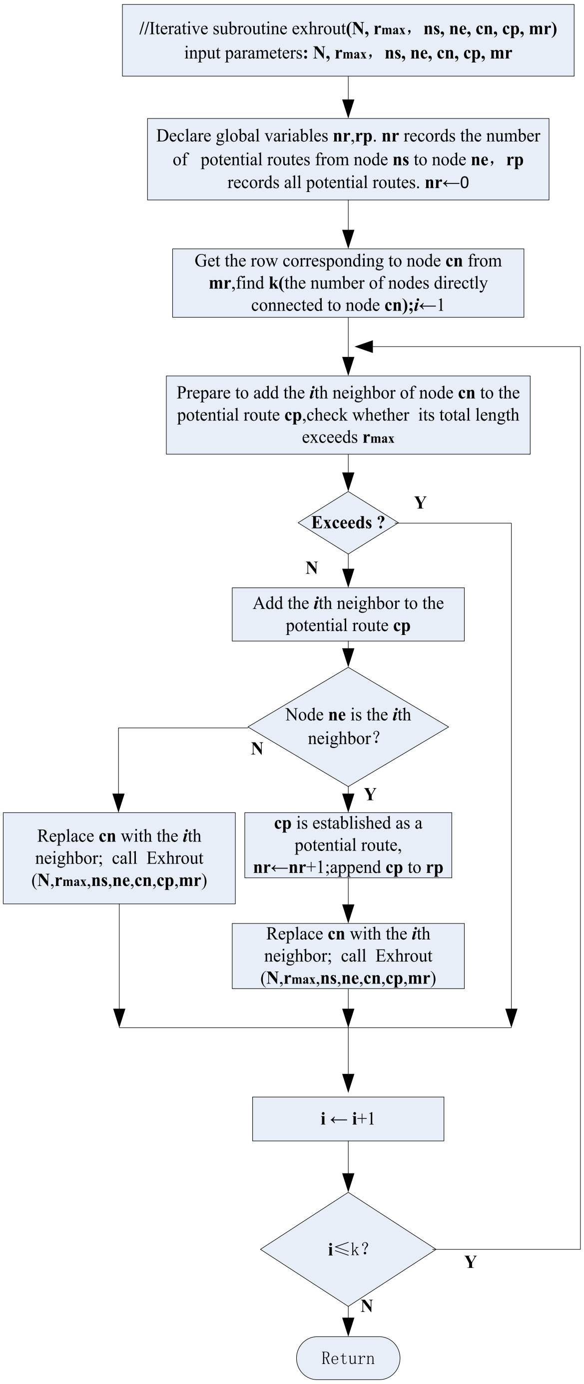

FIGURE 2. **Flowchart of the iterative subroutine of enumeration of potential test routes.**

for the node cn, so rings may occur and no upper limit for rings has been set in this algorithm.

C. HEURISTIC SEARCHING

Before the method of joint optimization of test node selection and fiber thread connection is found, the problem of test node selection had to be solved approximately by some heuristic searching methods. By such heuristic algorithm, one node is selected as the site to place test equipment according to the election standard that the node with maximum residual number of threads to be tested within a predefined hops starting from the candidate node is the winner. The number of hops is also called the depth of searching. On selecting a node to place test equipment, a subsequent fiber thread connection process is planned to maximize the number of threads that can be tested starting from the equipment at the just selected node, and then the residual fiber threads to be tested are re-calculated for further processing. The above mentioned ''node selection and connection'' process repeats until all the fiber threads have been tested. Such heuristic method can be listed as follows.

Step 1. *Initializing,* input the topology of the network including the matrix of distances between each pair of direct neighbors, numbers of threads to be tested within each fiber link; residual numbers of threads to be tested within each fiber link; input **D**, the depth of searching for selecting a node to place a testing equipment; input the maximum of the total length of any route r_{max}.

Step 2. *Judging*, if there is no link with positive remaining fiber threads to be tested, the task has completed and finishes; otherwise continue the following step.

Step 3. *Selection and connection*, calculate the sum of local residual numbers of fiber threads that can be tested within **D** hops for each candidate node, choose the one with the largest sum as the winner, then set up an local ILP model to maximize the number of residual fiber threads that can be connected and tested from the equipment placed at the winner; on solving the ILP problem, the parts of fiber threads connected and tested need to be wiped out from the residual number of threads of each corresponding fiber link.

Step 4. *Loop*, Turn to step 2.

The local ILP model appeared in step 3 has decision variables of only the first type, namely X_{jk}. Here j is the selected winner. Because the local ILP model is rebuilt in each round of the above algorithm, the set of variables has to be redefined, so that the first subscript j is implicitly specified and can be abbreviated in the local ILP model. The objective function of the local ILP is the sum of the numbers of fiber threads tested and can be expressed as the sum over each related fiber links, this is merely $F_L(X) = \Sigma_{i,k} a_{ijk} X_{jk}$. The constraints of the local ILP contain only one class. The class of constraints emerge because of the residual number of fiber threads to be tested within each fiber, the expression of this kind of constraints is similar to (8) without the summation over the subscript j.

D. ADVANTAGES OF JOINT OPTIMIZATION

For the optical fiber network presented in Fig. 1, both the heuristic searching and joint optimization methods are applied to get solutions to the testing node selection and fiber thread connection problem respectively. The joint optimization of test node selection and fiber thread connection is compared with the heuristic searching algorithm in this subsection. It has been assumed for both methods that the

TABLE 1. Comparison of node selection and number of testing routes.

	Heuristic Searching	Joint Optimization
Number of nodes	6	4
Nodes selected	7,9,14,17,21,25	7,13,16,26
Number of testing routes	25	30

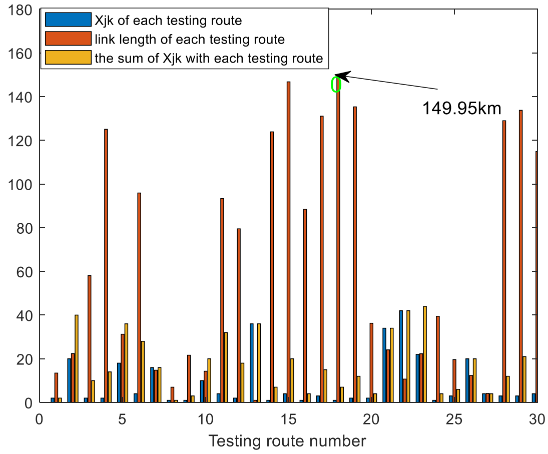

FIGURE 3. data related to each testing route when the size of network is 27.

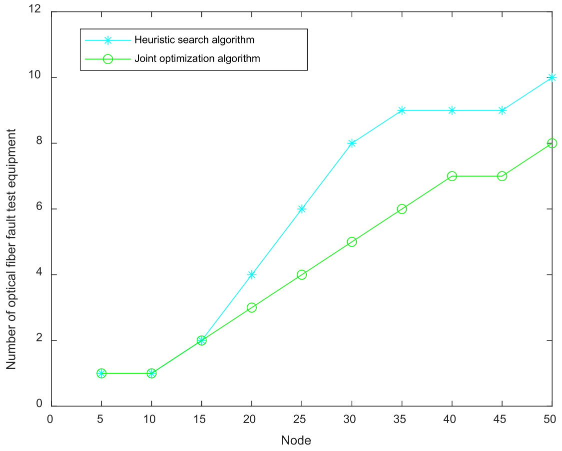

FIGURE 4. The number of necessary test nodes for different network sizes.

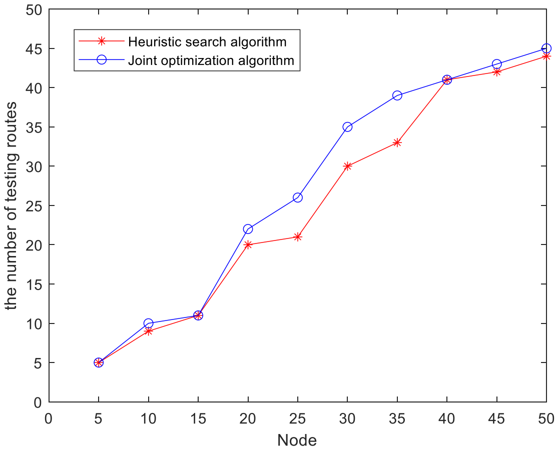

FIGURE 5. The number of testing routes for different network sizes.

error in the total length of each testing route owing to fiber thread connection is neglected. The maximum of the total length of each test route is 150 km. For the heuristic searching algorithm, the depth or the number of hops considered in choosing a winner from the candidates is two. Increase in the depth may help finding out some correct nodes to place equipment and change the whole set of nodes selected to place equipment but it can hardly cut down the number of nodes needed for placing equipment.

Table 1 gives comparison on the number of nodes selected, the set of nodes selected for placing test equipment and the number of test routes between the two methods.

From the results given in Table 1, it is apparent that the joint optimization method needs less number of nodes to place test equipment. Another fact may be noticed that the number of test routes given by heuristic search is less than that given by the joint optimization method, this implies that the objective to maximize the number of fiber threads to be tested from the equipment located at the selected node leads to some test routes composed of too many links. As each thread can only be tested or used for once, excessive use of threads in bottleneck links which are frequently traversed by other potential test routes counts against subsequent searching for a minimum number of nodes to place test equipment.

In order to verify the correctness of the joint optimization algorithm, the 30 testing routes searched by the algorithm have been listed by ascending order of originating node code in Table 2. It can be seen from Fig. 3, link length of each testing route is not more than 150 kilometers, and the number of parallel threads X_{jk} and total fiber threads of each test route are also shown in Fig. 3. The sum of the fiber threads of 30 testing routes is equal to 532. This is consistent with the total number of fibers to be monitored in Fig. 1. In summary, it is verified that the joint optimization algorithm can achieve the goal of minimizing the number of nodes to place fiber fault detection apparatus while meeting the requirement that all fibers must be fully monitored and the total length of each routing link should not exceed 150 km.

Table 2 gives the list of testing routes and the corresponding numbers of parallel fiber threads for each testing route allocated by both methods. The testing routes are essentially fiber thread connection schemes which contain part of the answer to the joint optimization problem.

There is only one node that is chosen by both the heuristic searching and the joint optimization. The common node is node 7, but the testing routes starting from this node are quite different. By heuristic searching, 15 testing routes originating from node 7 are established, while by joint optimization, 12 testing routes are chosen.

The example shows that the number of nodes selected for placing testing equipment by joint optimization is less than that by heuristic searching, this can lead to reduction in

TABLE 2. List of testing routes and numbers of parallel fiber threads.

ROUTE NUMBER	HEURISTIC SEARCHING		ROUTE NUMBER	JOINT OPTIMIZATION	
	X_{JK}	TESTING ROUTE		X_{JK}	TESTING ROUTE
1	8	7,4	1	2	7,4
2	2	7,4,2,4,5,4,7,4,2,1	2	20	7,4,2
3	1	7,4,2,1	3	2	7,4,5,4,2,1
4	1	7,8,10,8,10,8,10,8,10,8,7,4,2,1	4	2	7,6,3,1,2,4,2,1
5	1	7,4,5,4,2,1,3	5	18	7,6,3
6	10	7,6,3	6	4	7,4,5,4,2,1,3,6
7	3	7,8,10,8,10,8,7,4,2,4,5, 4,2,4,7,6,3	7	16	7,6
8	2	7,4,2,4,2 ,1,3,6	8	1	7,8
9	20	7,6	9	1	7,8,10,8
10	3	7,6,3,6,3,1,2,4,7,8,10	10	10	7,810
11	1	7,8,10	11	4	7,8,10,8,10,9,12,9,12
12	1	7,8,10,9,12,11,12,20,26,27,26,27,26,20,26,20,26,25,24	12	2	7,8,10,8,10,9,12,11,12,11
13	3	7,8,10,9,12,13,14,13,14,13,14,13,14,13,12,20,26,25,24,25,24,25,22	13	36	13,14
14	1	7,8,10,9,12,20,23,20,23,20,26 ,25,24,25,22	14	1	13,12,11,12,20,21,19,21
15	1	7,8,10,9,12,11,12,11,12,11,12,20,23,20,23,20,26	15	4	13,12,20,21,20,21
16	8	9,12	16	1	13,12,20,21,19
17	12	14,13	17	3	16,17,18,19,21,20
18	6	17,18,19,21,20	18	1	16,17,18,19,21,20,23,20
19	2	17,18	19	2	16,17,18,19,21,19,21
20	42	17,16,15	20	24	16,17,18
21	14	21,20,21,19	21	34	16,17
22	14	21,19	22	42	16,15
23	1	25,24	23	22	26,25,24
24	4	25,24,25,26	24	1	26,25,24,25,22
25	37	25,26	25	3	26,25,22
			26	20	26,25
			27	4	26,27
			28	3	26,20,21,20,21
			29	3	26,20,23,20,21,19,21,19
			30	4	26,20,21,19,21,19

TABLE 3. List of execution time for different sizes.

Size of The Network	Runtime of Heuristic Searching (s)	Runtime of Joint Optimization(s)
5	0.094	1.138
10	2.024	2.827
15	78.945	97.10
20	5.675e+03	6.7065e+03
25	9.3231e+03	1.0023e+04
30	2.0023e+04	2.7008e+04
35	3.998e+04	4.7363e+04
40	6.456e+04	7.03e+04
45	8.5122e+04	9.3082e+04
50	1.3289e+05	1.5e+05

construction investment and subsequent maintaining efforts of the fiber testing system.

In order to further verify the performance superiority of the joint optimization algorithm, the size of fiber communication network is changed to compare the optimal number of optical fiber fault testing equipment solved by the two algorithms. As shown in Fig. 4, when the network size is small enough, the obtained numbers of devices of the two algorithms are the same. As the size of the network increases, it is obvious that the difference between the solution of the two algorithms also increases. The larger the number of necessary fiber test apparatus is, the more cost of purchasing equipment and human maintenance endeavor will be needed. So that the joint optimization of test node selection and fiber thread connection takes advantages over the heuristic method. Fig. 5 is the number of testing routes gathered by both heuristic search algorithm and joint optimization algorithm.

For different network sizes, the time needed to complete the joint optimization algorithm and that of heuristic search vary accordingly. Table 3 gives the list of program execution time for each network by both methods. All the computations are executed by the same computer. The CPU is NE2640, with 32GB DDR3 RAM, the operating system is Win10 and the programs are developed in MATLAB2020b.

Although the execution time of the heuristic searching is slightly less than that of the joint optimization, the optimal solution of the joint optimization is apparently better than that of heuristic searching. With the development of computing power of future computers, it is worthy to obtain the optimal solution at the sacrifice of a little more computation time.

V. CONCLUSION

To minimize the number of nodes in an optical communication network where fiber test equipment is planned to place, an integer linear programming model is proposed. To overcome the difficulty that test node selection and fiber thread connection are mutually dependent on each other, two types of decision variables are defined and the key to this ILP model is the perception of the linear relation between the two classes of decision variables. Compared with heuristic searching, the proposed method gives better solutions to the joint test node selection and fiber thread connection problem.

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