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Optimal Bidding Strategy for Physical Market Participants With Virtual Bidding Capability in Day-Ahead Electricity Markets

HOSSEIN MEHDIPOURPICHA, (Student Member, IEEE), AND RUI BO, (Senior Member, IEEE)
Department of Electrical and Computer Engineering, Missouri University of Science and Technology, Rolla, MO 65409, USA
Corresponding author: Rui Bo (rbo@mst.edu)

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ABSTRACT Virtual bidding provides a mechanism for financial players to participate in wholesale day-ahead (DA) electricity markets. The price difference between DA and real-time (RT) markets creates financial arbitrage opportunities for financial players. Physical market participants (MP), referred to as participants with physical assets in this paper, can also take advantage of virtual bidding but in a different way, which is to further amplify the value of their physical assets. Therefore, this work proposes a model for such physical MPs to maximize the profits. This model employs a bi-level optimization approach, where the upper-level subproblem maximizes the total profit from both physical generations and virtual transactions while the lower-level model mimics the multi-period network-constrained DA market clearing process. In this model, uncertainties associated with other MPs as well as RT market prices are considered. Moreover, the conditional value-at-risk (CVaR) metric is utilized to measure the risk of diverse strategies. The optimal strategy of the strategic physical MP is derived by solving this bi-level optimization model. The proposed bi-level model is transformed to a single level mixed integer linear programming (MILP) model using Karush–Kuhn–Tucker (KKT) optimality conditions and the duality theory. Case studies show the effectiveness of the proposed method and reveal physical MPs may choose to deploy virtual transactions in a very different way than pure financial MPs.

INDEX TERMS Bidding strategy, bi-level optimization, financial products, physical market participants, profit maximization, virtual bidding.

NOMENCULTURE

A. SETS AND INDICES

t	Time periods
i	Generating units of strategic producers
v	Virtual bid
j	Other generating units of the nonstrategic producer
d	Demands
k	Generation/demand offer/bid blocks
n	Buses
l	Transmission lines
s	Scenarios related to other MPs' behaviors
w	Scenarios related to RT market prices
ψ_n	Sets of players located at bus n

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B. PARAMETERS

λ_{tn}^{RT}	Real Time price at bus n at time t
λ_{tik}^{G}	Marginal cost of strategic unit i at time t
λ_{tjk}^{R}	Marginal cost of nonstrategic unit j at time t
λ_{tdk}^{D}	Marginal utility of demand d at time t
V_{tv}^{max}	Upper limit of virtual bid v quantity at time t in DA Market
$\overline{P}_{tik}^{G}$	Upper limit generation of strategic unit i at time t
$\overline{P}_{tjk}^{R}$	Upper limit generation of nonstrategic unit j at time t
$\overline{P}_{tdk}^{D}$	Upper limit of demand d at time t
RU_i	Ramp up limit of generating unit i
RD_i	Ramp down limit of generating unit i

$PTDF_{nl}$	Power Transfer Distribution Factor (sensitivity of power flows through line l to the injection power to bus n)
$proxy_{tv}$	Proxy of virtual transaction v at time t
B	Proxy Budget
$\overline{C}_l$	Power flow Capacity of line l
β	Weighting parameter
α	Confidence level
Π_s	Scenario s probability in DA market
τ_w	Scenario w probability

Note that some of these constants contain index s or w when they refer to scenario s or w.

C. VARIABLES

γ_{tv}	bid price of virtual bid v at time t
γ_{tik}^G	Offer price of generating unit i of the strategic producer at time t
V_{tv}	Virtual bid v power in DA market at time t
P_{tik}^G	Power of generating unit i of the strategic producer in DA market at time t
$\overline{V}_{tv}$	Offer energy of the virtual bid v at time t
P_{tjk}^R	Nonstrategic unit j cleared power at time t
P_{tdk}^D	Cleared power consumed by demand d at time t
LF_{tl}	Line l power flow at time t
λ_{tn}^{DA}	DA LMP at bus n and time t
η_{sw}	Auxiliary variable for CVaR computation
VaR	Value-at-Risk

Note that some of these variables contain index s or w when they refer to scenario s or w.

D. DUAL VARIABLES

λ_{tf}^{DA}	System-wide Generation-load equilibrium at time t
$\underline{\rho}_{tv}$	Minimum offer/bid quantity of virtual bid v at time t
$\overline{\rho}_{tv}$	Maximum offer/bid quantity of virtual bid v at time t
$\underline{\rho}_{tik}^G$	Minimum production of strategic unit i at time t
$\overline{\rho}_{tik}^G$	Capacity of strategic unit i at time t
$\underline{\rho}_{tjk}^R$	Minimum production of nonstrategic unit j at time t
$\overline{\rho}_{tjk}^R$	Capacity of nonstrategic unit j of at time t
$\underline{\rho}_{tdk}^D$	Minimum power of demand d at time t
$\overline{\rho}_{tdk}^D$	Capacity of demand d at time t
$\underline{\vartheta}_{tl}$	Line l capacity in negative direction
$\overline{\vartheta}_{tl}$	Line l capacity in positive direction

Note that some of these dual variables contain index s when they refer to scenario s.

I. INTRODUCTION

From microeconomics market competition perspective, market structure can be classified into two categories: perfect and imperfect competitions. In perfect competition, there are a large number of producers and consumers that compete on a homogeneous product whose price is decided by the demand and supply forces and no firm can influence the market price by changing their strategies. On the other hand, imperfect competition opens up the opportunity for some market participants (MPs) to influence the market prices in favor of their own interests [1], [2]. As a major player in the imperfect electricity market environment, electricity generating companies (GenCos) aim to maximize their profit by exercising offer strategies in one or multiple markets. In recent years, exploring the best offering strategy for the physical GenCos in a single or multiple markets has attracted the attention of many researchers and numerous methods such as optimization-based, game theory-based, and agent-based models have been examined under the deregulated electricity market environment [3]. Antonio *et al.* [4] described how a price-taker thermal GenCo should design effective bidding curves, which are based on the profit maximizing self-schedule and price forecast. A binary expansion approach for the price-maker MP's offer problem has been presented in [5]–[7] presented the bi-level optimization method to derive the optimized offers for physical GenCos to maximize its profit. An optimization-based scheduling for a building energy management system and bidding strategy of small-scale residential prosumers are formulated as a stochastic bi-level optimization problem in [8] to minimize the energy cost and prosumer's inconveniences in the upper-level and lower-level, respectively. A non-cooperative game theory approach to design the best strategy for MPs was reported in [9]. Offering strategy analysis applying agent-based simulation can be found in [10]. Optimal strategy determination of a GenCo in three sequential markets has been studied in [11], in which the GenCo is considered to be the price-taker MP in the day-ahead (DA) and automatic generation control (AGC) markets and the price-setter MP in the balancing market. A multi-stage stochastic model to develop the offering strategy of a generator in the chronological DA and balancing markets has been reported in [12], while [13] employed the same method to design an optimal bidding strategy of an aggregator of prosumers in energy and reserve markets. [14] applied a two-tier matrix game model to optimize the offering strategy of MPs in the FTR auction and DA electricity market. This method reflected the FTR game in the top tier and the energy game in the bottom tier, which was solved in an iterative process. An optimal bidding for a microgrid (MG) incorporated with the flexible ramping product in multiple markets has been presented in [15], which not only increases the MG's revenue, but also improves the dispatch flexibility in the power system.

In addition to physical assets-backed transactions in the electricity markets, virtual transactions offer a useful tool for more effective market operation and can help achieve improved convergence between DA and RT market prices, enhanced market power mitigation and market liquidity [16], [17]. Virtual bidders refer to the market participants, either purely financial players or conventional participants, take part in virtual transactions. They place offers/bids into the DA market without having to supply/consume actual power in the RT market. Despite the zero net energy in DA and RT markets, the net payoff is measured in a two-stage settlement procedure according to the DA and RT markets' price spread. For example, the cleared virtual generation (demand) will be paid (charged) at DA market price in the DA market and charged (paid) at the RT market price in the RT market. Virtual transactions play an important role in electricity markets. For instance, designed as decrement bids (DECs) or increment offers (INCs) in Midwest Independent Transmission System Operator (MISO), virtual transactions made up around 6% of all transactions in MISO in 2010 and 2011 [18]. A recent study [19] has explored the effect of virtual transactions in two-settlement markets with uncertain generations using four different market models. A bi-level optimization approach has been used in [20] to find the best strategy of purely financial MP in the DA market.

A. MOTIVATION

In electricity markets, besides financial players, physical market participants can also participate in virtual bidding. However, their optimal strategy can be very different from purely financial players because of the interdependence with the physical assets they own. For instance, when the RT market price is expected to be higher than the DA market price, virtual DEC should provide a positive virtual profit. However, it may increase the DA market price which may cause the reduction of cleared physical generation and therefore the decrease of a physical generation profit. Therefore, the decision making of physical MP with virtual bidding capability is a new and challenging problem that needs to be investigated. To date, there has been little research in this area reported in the literature.

For a physical MP with virtual bidding capability, the decision making needs to consider the following aspects. First, the goal is to maximize the total profit which consists of the physical generation profit and virtual transaction profit, instead of each part individually. Second, virtual transactions (either INC or DEC) may affect DA market prices, which may subsequently alter the profit from the physical generation. Third, physical generation may also affect DA price and therefore the DA/RT price difference, resulting in changes in the virtual transaction profit. Therefore, an optimal bidding strategy should allow the MP to carefully manage its impact on DA price so that the total profit is maximized.

The decision making of a physical MP with virtual bidding capability also faces two major uncertainties: the forecasted RT market price, and DA market price which is impacted by not only the virtual bids/offers of the physical MP itself but also other MPs' bidding strategy in the DA market.

B. CONTRIBUTION

In order to develop an optimal bidding strategy for physical MP with virtual bidding capability and at the same time account for the uncertainties in DA market price and RT market price forecast, this work proposes a risk-controlled bi-level optimization model to maximize the total profit for the appropriate risk level. The upper-level subproblem aims to maximize the payoff of this market player, whose income is measured according to the cleared DA market price obtained at the lower-level subproblem which represents the market clearing procedure. The outputs of lower-level subproblem include cleared energy, cleared virtual transactions and the DA market LMP, are returned to the upper-level subproblem. Besides, this subproblem incorporates scenario-based uncertainties of rivals' strategies and RT market prices. Finally, the conditional value-at-risk (CVaR) is used to empirically estimate the risk of payoff associated with various strategies.

The contribution of this work is summarized as follows:

1) Established mathematical models for a rarely research yet practical problem, namely, the bidding strategy problem for physical market participants with virtual bidding capability.
2) Incorporated uncertainties associated with RT market prices and with rivals' offers/bids in the DA market, as well as conditional value-at-risk to quantity and control the financial risks associated with the strategies.
3) Revealed the physical MPs may have the incentive to exercise the virtual bidding capability in a very different way than purely financial MPs.

C. PAPER ORGANIZATION

The rest of the paper is organized as follows. Section II describes the bidding strategy problem of a physical MP with the virtual bidding capability. Section III introduces a deterministic model and a risk-controlled stochastic model. Section IV presents a case study for both deterministic and stochastic conditions. Section V concludes the paper.

II. PROBLEM DESCRIPTION

A. BIDDING STRATEGY OF PHYSICAL MP WITH VIRTUAL BIDDING CAPABILITY

DA market is cleared on an hourly basis the day prior to the operating day, and the RT market is cleared on a five-minute basis in the operating day; however, its settlement is performed based on the average of twelve five-minute time slots [21]. Practically, there is often a gap between DA and RT prices, hence, a physical MP with virtual bidding capability can arbitrage the price differences using virtual transactions.

In principal, a physical MP with a considerable market share can change the DA market LMP and maximize its physical generation's profits. However, this change may reduce the DA and RT price differences, which shrinks the virtual transaction's payoff opportunity. Thus, in order to optimize its payoff, the market participant needs to make a compromise not only between the cleared power and DA market LMP, but also between the cleared power and its influence on DA/RT price spread, and in the meantime, monitor the probable risk of profit volatility.

B. UNCERTAINTY CHARACTERIZATION

To set its offers/bids, a market participant encounters several uncertainties, including its rivals' strategies, and RT market prices. To deal with these uncertainties, a number of scenarios are defined to reflect the various realizations of the unknown variables along with their corresponding probabilities.

Two separate scenario sets are specified for the problem formulation in this paper:

1) Day-Ahead market scenarios which denote the different strategies of other generators/demands

2) Real-Time market scenarios which denote the different RT market price predictions

C. RISK MODELING

Some MPs may not be willing to choose a bidding strategy that may bring high profit volatility, which results from the aforementioned uncertainty in the problem. Therefore, the conditional value-at-risk (CVaR) metric has been employed, which empowers the market participant to monitor the risk incorporated with its offers/bids. This metric is linear and easy to integrate into the optimization problem [22].

D. MODEL DESCRIPTION

In this paper, the optimal bidding strategy of a physical MP with the virtual bidding capability in the DA market is formulated by means of a stochastic bi-level optimization model. The upper-level subproblem of the proposed model illustrates the MP's payoff maximization problem, and the evaluation of market clearing procedure under various scenarios, is performed in the lower-level subproblems. The upper-level and lower-level subproblems are connected by their respective decision variables. The decision variables of the upper-level subproblem which consist of the MP's offers/bids to sell/purchase physical power or virtual bids in/from the day-ahead electricity market, are transferred to the lower-level subproblem as parameters. The decision variables of the lower-level subproblem include cleared power sold (purchased) by all generating units (demands), and wholesale energy prices, which are passed back to the upper-level problem for MP profit calculation.

III. MATHEMATICAL FORMULATION

A. MODEL ASSUMPTIONS

The main assumptions of the proposed model are outlined below:

1) Transmission network is modeled using DC power flow to be consistent with contemporary market practices. Power Transfer Distribution Factor (PTDF) has been used to calculate the line flows.
2) Other market participants' offers/bids have been modeled by step-wise curves to be in alignment with the common electricity market practices. It should be pointed out that these unknown parameters can be estimated and forecasted using publicly available market data, which is accessible several months after market clearing [7], [8], [23], [24].
3) The physical MP offers its asset-based physical generation from the location the generators are connected. Its virtual transactions, however, can be offered at other locations.
4) Similar to the assumption taken in [2], [3], [5]–[7], [9], [15], Unit Commitment (UC) is not considered in this work because its nonconvexity makes the problem intractable.

In the following subsections, a deterministic bi-level model of the studied bidding strategy problem is presented first, assuming no model uncertainties. Then, the model is augmented with the modeling of uncertainties and risk, presented as a stochastic bi-level model.

B. DETERMINISTIC BI-LEVEL MODEL

The bidding strategy of a physical MP with virtual bidding capability in the DA market can be formulated using the bi-level optimization model as follows:

1) UPPER-LEVEL

$$\underset{\gamma_{tik}^{G},P_{tik}^{G},\gamma_{tv},\overline{V}_{tv}}{\text{Minimize}} \sum_{tik}\lambda_{tik}^{G}P_{tik}^{G}-\sum_{t(i\in\psi_n)k}\lambda_{tn}^{DA}P_{tik}^{G}-\sum_{t(v\in\psi_n)}\lambda_{tn}^{DA}V_{tv} + \sum_{t(v\in\psi_n)}\lambda_{tn}^{RT}V_{tv} \tag{1a}$$

s.t:

$$\sum_{k}P_{(t+1)ik}^{G}-\sum_{k}P_{tik}^{G}\le R_{i}^{UP},\quad \forall t,\ \forall i \tag{1b}$$

$$\sum_{k}P_{tik}^{G}-\sum_{k}P_{(t+1)ik}^{G}\le R_{i}^{LO},\quad \forall t,\ \forall i \tag{1c}$$

$$\sum_{tv}Proxy_{tv}V_{tv}\le B, \tag{1d}$$

$$-V_{tv}^{max}\le\overline{V}_{tv}\le V_{tv}^{max}\quad \forall t,\ \forall v \tag{1e}$$

2) LOWER-LEVEL

$$V_{tv},P_{tik}^{G}\in arg\left\{\underset{V_{tv},P_{tik}^{G},P_{tjk}^{R},P_{tdk}^{D}}{\text{Minimize}}\sum_{tv}\gamma_{tv}V_{tv}+\sum_{tik}\gamma_{tik}^{G}P_{tik}^{G} + \sum_{tjk}\lambda_{tjk}^{R}P_{tjk}^{R}-\sum_{tdk}\lambda_{tdk}^{D}P_{tdk}^{D}\right. \tag{1f}$$

s.t:

$$\sum_{v}V_{tv}+\sum_{ik}P_{tik}^{G}+\sum_{jk}P_{tjk}^{R}=\sum_{dk}P_{tdk}^{D}:\lambda_{tf}^{DA},\quad \forall t \tag{1g}$$

$$-\overline{V}_{tv}\le V_{tv}\le\overline{V}_{tv}:\ \overline{\rho}_{tv}^{V},\underline{\rho}_{tv}^{V},\quad \forall t,\ \forall v \tag{1h}$$

$$0\le P_{tik}^{G}\le\overline{P}_{tik}^{G}:\ \overline{\rho}_{tik}^{G},\underline{\rho}_{tik}^{G},\quad \forall t,\ \forall i,\ \forall k \tag{1i}$$

$$0\le P_{tjk}^{R}\le\overline{P}_{tjk}^{R}:\ \overline{\rho}_{tjk}^{R},\rho_{tjk}^{R},\quad \forall t,\ \forall j,\ \forall k \tag{1j}$$

$$0\le P_{tdk}^{D}\le\overline{P}_{tdk}^{D}:\ \overline{\rho}_{tdk}^{D},\rho_{tdk}^{D},\quad \forall t,\ \forall d,\ \forall k \tag{1k}$$

$$-\overline{C}_l \leq \sum_n PTDF_{nl}\left(\sum_{(v\in\psi_n)} V_{tv} + \sum_{(i\in\psi_n)k} P^G_{tik} + \sum_{(j\in\psi_n)k} P^R_{tjk} - \sum_{(d\in\psi_n)k} P^D_{tdk}\right) \leq \overline{C}_l : \underline{\vartheta}_{tl}, \overline{\vartheta}_{tl} \quad \forall t,\ \forall l \tag{1l}$$

$$\lambda^{DA}_{tn} = \lambda^{DA}_{tf} - \sum_l PTDF_{nl}\left(\overline{\vartheta}_{tl} - \underline{\vartheta}_{tl}\right) \quad \forall t,\ \forall n \tag{1m}$$

The upper-level subproblem (1a) – (1e) represents the profit maximization of the physical MP with virtual bidding capability, and the lower-level subproblem (1f) – (1m) represents the DA market clearing process. Note that the notations on the right side of the lower-level constraints represent the dual variables of those constraints. The objective function (1a) consists of four terms: the first two terms ($\sum_{tik} \lambda^G_{tik} P^G_{tik} - \sum_{t(i\in\psi_n)k} \lambda^{DA}_{tn} P^G_{tik}$) represent the negative of profits of actual generation in the DA market and the second two terms ($-\sum_{t(v\in\psi_n)} \lambda^{DA}_{tn} V_{tv} + \sum_{t(v\in\psi_n)} \lambda^{RT}_{tn} V_{tv}$) are the negative of profits of virtual transactions which can be obtained by participating in DA and RT markets. Note that this paper deals with DA market bidding strategy, and therefore considers the MP to be a price-maker in the DA market and price taker in the RT market. In other words, the physical generation of the MP in RT is assumed to follow exactly the DA schedule, resulting in zero RT profit based on the two-stage settlement. This is the reason the profit of MP physical generation in the RT market is not presented in (1). Furthermore, RT LMP is modeled for the MP's decision on virtual bidding. Constraints (1b) and (1c) express the ramp-up and ramp-down limits of the physical generating units. Constraint (1d) limits the virtual energy offer/bid according to its virtual proxy which is a financial assurance for submitting virtual transactions. For determining the proxy, the MP needs to open an ISO account for its virtual transactions and deposit some money there to guarantee the capability to pay for the probable loss [25]. Constraint (1e) imposes power limits that this MP can trade as virtual transactions in the DA market.

The cleared power V_{tv} and P^G_{tik} are part of the feasible region specified by the lower-level subproblem (1f) – (1m). The objective function (1f) minimizes the negative of the social welfare. Constraint (1g) represents the generation-load balance for the whole system, and the dual variable of this constraint denotes the system-wide DA market price (λ^{DA}_{tf}). Constraints (1h) – (1j) define the power limits for virtual transaction, physical generation of strategic MP and other nonstrategic generators, respectively. Constraint (1k) represents the demand limits. Transmission line capacity limits are denoted by constraint (1l). Constraint (1m) represents the DA market LMP at bus n and time t. Note that $(i, j, d, v) \in \psi_n$ identifies that these generators/demands are located at bus n and offers/bids from this bus.

C. SOLUTION METHODOLOGY

To convert the bi-level optimization problem described in section III.B into a single level problem, we replaced the lower-level linear optimization problem (represented by (1f) – (1m) by its KKT optimality conditions. The obtained single-level problem, which is known as mathematical problem with equilibrium constraints (MPEC), is illustrated as follows.

$$\underset{\gamma^G_{tik}, P^G_{tik}, \gamma_{tv}, V_{tv}}{\text{Minimize}} \sum_{tik} \lambda^G_{tik} P^G_{tik} - \sum_{t(i\in\psi_n)k} \lambda^{DA}_{tn} P^G_{tik} - \sum_{t(v\in\psi_n)} \lambda^{DA}_{tn} V_{tv} + \sum_{t(v\in\psi_n)} \lambda^{RT}_{tn} V_{tv} \tag{2a}$$

s.t:

$$\text{Constraints (1b)} - \text{(1e)} \tag{2b}$$

$$\gamma^G_{tik} - \lambda^{DA}_{tn} + \overline{\rho}^G_{tik} - \underline{\rho}^G_{tik} = 0, \quad \forall t,\ \forall i \in \psi_n,\ \forall k \tag{2c}$$

$$\gamma_{tv} - \lambda^{DA}_{tn} + \overline{\rho}^V_{tv} - \underline{\rho}^V_{tv} = 0, \quad \forall t,\ \forall v \in \psi_n \tag{2d}$$

$$\lambda^R_{tjk} - \lambda^{DA}_{tn} + \overline{\rho}^R_{tjk} - \underline{\rho}^R_{tjk} = 0, \quad \forall t,\ \forall j \in \psi_n,\ \forall k \tag{2e}$$

$$-\lambda^D_{tdk} + \lambda^{DA}_{tn} + \overline{\rho}^D_{tdk} - \underline{\rho}^D_{tdk} = 0, \quad \forall t,\ \forall d \in \psi_n,\ \forall k \tag{2f}$$

$$\text{Constraints (1g)} - \text{(1m)} \tag{2g}$$

$$0 \leq V_{tv} + \overline{V}_{tv} \perp \underline{\rho}^V_{tv} \geq 0, \quad \forall t,\ \forall v \tag{2h}$$

$$0 \leq \overline{V}_{tv} - V_{tv} \perp \overline{\rho}^V_{tv} \geq 0, \quad \forall t,\ \forall v \tag{2i}$$

$$0 \leq P^G_{tik} \perp \underline{\rho}^G_{tik} \geq 0, \quad \forall t,\ \forall i,\ \forall k \tag{2j}$$

$$0 \leq \overline{P}^G_{tik} - P^G_{tik} \perp \overline{\rho}^G_{tik} \geq 0, \quad \forall t,\ \forall i,\ \forall k \tag{2k}$$

$$0 \leq P^R_{tjk} \perp \underline{\rho}^R_{tjk} \geq 0, \quad \forall t,\ \forall j,\ \forall k \tag{2l}$$

$$0 \leq \overline{P}^R_{tjk} - P^R_{tjk} \perp \overline{\rho}^R_{tjk} \geq 0, \quad \forall t,\ \forall j,\ \forall k \tag{2m}$$

$$0 \leq P^D_{tdk} \perp \underline{\rho}^D_{tdk} \geq 0, \quad \forall t,\ \forall d,\ \forall k \tag{2n}$$

$$0 \leq \overline{P}^D_{tdk} - P^D_{tdk} \perp \overline{\rho}^D_{tdk} \geq 0, \quad \forall t,\ \forall d,\ \forall k \tag{2o}$$

$$0 \leq \overline{C}_l + \sum_n PTDF_{nl}\left(\sum_{(v\in\psi_n)} V_{tv} + \sum_{(i\in\psi_n)k} P^G_{tik} + \sum_{(j\in\psi_n)k} P^R_{tjk} - \sum_{(d\in\psi_n)k} P^D_{tdk}\right) \perp \underline{\vartheta}_{tl} \geq 0 \quad \forall t,\ \forall l \tag{2p}$$

$$0 \leq \overline{C}_l - \sum_n PTDF_{nl}\left(\sum_{(v\in\psi_n)} V_{tv} + \sum_{(i\in\psi_n)k} P^G_{tik} + \sum_{(j\in\psi_n)k} P^R_{tjk} - \sum_{(d\in\psi_n)k} P^D_{tdk}\right) \perp \overline{\vartheta}_{tl} \geq 0 \quad \forall t,\ \forall l \tag{2q}$$

Constraints (2c) – (2f) are the set of partial derivatives of the Lagrangian function of the lower-level subproblem ((1f) – (1l)) regarding to the lower-level decision variables. Constraints (2g) are the primal equality constraints of the lower-level subproblem ((1g) – (1m)), and the remaining constraints are the complementarity constraints [26].

Model (2) is a single-level nonlinear problem, whose nonlinearity comes from three terms: terms $\lambda^{DA}_{tn} P^G_{tik}$ and $\lambda^{DA}_{tn} V_{tv}$ in the objective function (2a) and the complementarity constraints (2h) – (2q). The nonlinear terms in (2a) can be translated to their equivalent linear expressions applying the strong duality theorem (SDT) [6]. Furthermore, the Fortuny-Amat Transformation technique [26] is used to replace the complementarity constraints with their equivalent mixed integer linear terms. Therefore, model (2) is converted to a mixed integer linear programming (MILP) problem, which can be handled using available solvers and accessible commercial software [27].

D. STOCHASTIC BI-LEVEL MODEL WITH UNCERTAINTY AND RISK MODELING

Bidding strategy of the intended MP is affected by the uncertainties of other MPs' offers/bids and the RT market prices. These uncertainties can be incorporated into the main problem (1) by employing a sets of scenarios, each of which represents the realization of different uncertain parameters. In this modeling, the probability distribution functions (PDF) of all uncertain parameters are assumed to be known or estimated based on historical information. Adding the conditional value-at-risk (CVaR) measure to control the profit risk, the resulted formulation will be as follows:

1) UPPER-LEVEL

$$\underset{\Delta}{\text{Maximize}}\,(1-\beta)\sum_{s}\Pi_s\Bigg(\sum_{t(i\in\psi_n)k}\lambda^{DA}_{tns}P^{G}_{tiks}+\sum_{t(v\in\psi_n)}\lambda^{DA}_{tns}V_{tvs}-\sum_{tik}\lambda^{G}_{tik}P^{G}_{tiks}-\sum_{t(i\in\psi_n)w}\tau_w\lambda^{RT}_{tnw}V_{tvs}\Bigg)+\beta\Big(VaR-\frac{1}{1-\alpha}\sum_{sw-}\Pi_s\tau_w\eta_{sw}\Big) \tag{3a}$$

s.t:

$$\sum_k P^{G}_{(t+1)iks}-\sum_k P^{G}_{tiks}\le R^{UP}_i,\quad \forall t,\ \forall i,\ \forall s \tag{3b}$$

$$\sum_k P^{G}_{tiks}-\sum_k P^{G}_{(t+1)iks}\le R^{LO}_i,\quad \forall t,\ \forall i,\ \forall s \tag{3c}$$

$$\sum_{tv} Proxy_{tv}V_{tvs}\le B,\quad \forall s \tag{3d}$$

$$-V^{max}_{tv}\le \overline{V}_{tv}\le V^{max}_{tv}\quad \forall t,\ \forall v,\ \forall s \tag{3e}$$

$$VaR-\Bigg(\sum_{t(i\in\psi_n)k}\lambda^{DA}_{tns}P^{G}_{tiks}+\sum_{t(v\in\psi_n)}\lambda^{DA}_{tns}V_{tvs}-\sum_{tik}\lambda^{G}_{tik}P^{G}_{tiks}-\sum_{t(v\in\psi_n)}\lambda^{RT}_{tnw}V_{tvs}\Bigg)\le\eta_{sw}\quad\forall s,\ \forall w \tag{3f}$$

$$\eta_{sw}\ge 0,\quad \forall s,\ \forall w \tag{3g}$$

2) LOWER-LEVEL

$$(V_{tvs},P^{G}_{tiks})\in arg\Bigg\{\underset{P^{G}_{tiks},V_{tvs},P^{R}_{tjks},P^{D}_{tdks}}{\text{Minimize}}\sum_{tv}\gamma_{tv}V_{tvs}+\sum_{tik}\gamma^{G}_{tik}P^{G}_{tiks}+\sum_{tjk}\lambda^{R}_{tjks}P^{R}_{tjks}-\sum_{tdk}\lambda^{D}_{tdks}P^{D}_{tdks} \tag{3h}$$

s.t:

$$\sum_v V_{tvs}+\sum_{ik}P^{G}_{tiks}+\sum_{jk}P^{R}_{tjks}=\sum_{dk}P^{D}_{tdks}:\lambda^{DA}_{tfs},\quad\forall t,\ \forall s \tag{3i}$$

$$-\overline{V}_{tv}\le V_{tvs}\le\overline{V}_{tv}:\overline{\rho}^{V}_{tvs},\underline{\rho}^{V}_{tvs},\quad\forall t,\ \forall v,\ \forall s \tag{3j}$$

$$0\le P^{G}_{tiks}\le\overline{P}^{G}_{tik}:\overline{\rho}^{G}_{tiks},\underline{\rho}^{G}_{tiks},\quad\forall t,\ \forall i,\ \forall k,\ \forall s \tag{3k}$$

$$0\le P^{R}_{tjks}\le\overline{P}^{R}_{tjk}:\overline{\rho}^{R}_{tjks},\rho^{R}_{tjks},\quad\forall t,\ \forall j,\ \forall k,\ \forall s \tag{3l}$$

$$0\le P^{D}_{tdks}\le\overline{P}^{D}_{tdk}:\overline{\rho}^{D}_{tdks},\rho^{D}_{tdks},\quad\forall t,\ \forall d,\ \forall k,\ \forall s \tag{3m}$$

$$-\overline{C}_l\le\sum_n PTDF_{nl}\Bigg(\sum_{(v\in\psi_n)}V_{tvs}+\sum_{(i\in\psi_n)k}P^{G}_{tiks}+\sum_{(j\in\psi_n)k}P^{R}_{tjks}-\sum_{(d\in\psi_n)k}P^{D}_{tdks}\Bigg)\le\overline{C}_l:\underline{\vartheta}_{tls},\overline{\vartheta}_{tls}\quad\forall t,\ \forall l,\ \forall s \tag{3n}$$

$$\lambda^{DA}_{tns}=\lambda^{DA}_{tfs}-\sum_l PTDF_{nl}\left(\overline{\vartheta}_{tls}-\underline{\vartheta}_{tls}\right)\quad\forall t,\ \forall n,\ \forall s \tag{3o}$$

In this formulation all variables are $\Delta = \{\gamma_{tv}, V_{tvs}, \overline{V}_{tv}, VaR, \eta_{sw}, \gamma^{G}_{tik}, P^{G}_{tiks}, \lambda^{DA}_{tfs}, :\overline{\rho}^{V}_{tvs}, \underline{\rho}^{V}_{tvs}, \overline{\rho}^{G}_{tiks}, \underline{\rho}^{G}_{tiks}, \overline{\rho}^{R}_{tjks}, \rho^{R}_{tjks}, \overline{\rho}^{R}_{tjks}, \rho^{R}_{tjks}, \overline{\rho}^{D}_{tdks}, \rho^{D}_{tdks}, \underline{\vartheta}_{tls}, \overline{\vartheta}_{tls}\ and\ \lambda^{DA}_{tns}\}$ and the binary variables created during applying Fortuny-Amat transformation. The objective function (3a) is the negative of the expected profit and Π_s represents the probability associated with scenario s. RT market price uncertainty has been represented in the fourth term of the objective function, in which τ_w is the probability of scenarios associated with RT market price scenarios. The last term of the objective function (3a) is conditional value-at-risk (CVaR). Weighting parameter β is used to compromise between the expected profit and CVaR. The lower β is, the more risk-taker the MP is. However, risk-averse MP accepts the higher value of β. It means if β is large enough (close to 1), the MP neglects its expected profit but guarantees the minimum profit for a given confidence level α. Constraints (3f) and (3g) are used to compute the CVaR [22]. All other constraints are similar to the deterministic model, while the lower-level subproblem is solved for each scenario s. The procedure of constructing the MPEC and MILP for problem (3) is completely similar to the deterministic model.

IV. CASE STUDIES

The proposed models have been tested on systems of different sizes, and for different conditions (including uncongested and congested conditions). For demonstration purpose, two IEEE standard systems (IEEE 14-bus test system and IEEE 39-bus test system) have been studied in an uncongested condition. Detailed data and results are illustrated as follows.

A. DATA

Systems' data used in this paper such as generation capacities, maximum load quantities, transmission line capacities, and etc. have been taken from [28], [29]. Moreover, forecasted offers/bids prices of generators/loads are obtained from [6], and have been slightly modified to match the assumptions made in this paper.

TABLE 1. Generators data.

	G1	G2	G3	G4	G5
Bus #	1	2	3	6	8
Capacity (MW)	182.4	130	100	100	100
$\overline{P}_1$ (MW)	150.2	99	55	50	55
$\overline{P}_2$ (MW)	32.2	41	45	50	45
λ_1 (\$/MWh)	10.37	10.08	11.32	11.71	19.32
λ_2 (\$/MWh)	11.41	10.97	13.19	14.93	22.19
RU/RD (MW/h)	150	120	100	80	90

B. 14-BUS TEST SYSTEM

The IEEE 14-bus test system has 14 buses, 5 generators, 11 loads, and 20 transmission lines [28]. Modifications and additional parameters have been made to the system for better illustration. The generators' data is summarized in Table 1. It is assumed that generators submit two-block offer curves for each hour. The two-block offer generations are shown by $\overline{P}_1$ and $\overline{P}_2$, and the corresponding marginal costs are depicted by λ_1 and λ_2. RU/RD represents the generator ramp up and ramp down rate. We consider that a strategic MP has two generators G1 and G3 located at buses 1 and 3 with installed capacities of 182.4 MW and 100 MW, respectively.

TABLE 2. Demand power (MW).

Load #	Block 1	Block 2	Load #	Block 1	Block 2
1	21.7	20.5	7	9	7
2	94.2	88.4	8	3.5	6
3	47.8	32.5	9	6.1	12.1
4	7.6	10.1	10	13.5	15
5	11.2	14.3	11	14.9	21.2
6	29.5	30			

TABLE 3. Demand bid price (\$/MWh).

Hour	Block 1	Block 2	Hour	Block 1	Block 2
1	17.43	16.79	13	25.00	20.61
2	17.25	16.38	14	24.97	20.38
3	17.22	16.32	15	20.38	18.93
4	17.22	16.32	16	20.38	18.93
5	16.89	16.13	17	20.88	19.53
6	16.89	16.13	18	25.00	20.61
7	17.25	16.38	19	25.00	20.61
8	17.94	17.22	20	25.00	20.61
9	19.23	18.15	21	25.00	20.61
10	20.38	18.93	22	24.97	20.38
11	24.97	20.38	23	19.53	18.34
12	25.00	20.61	24	17.94	17.22

Table 2 and Table 3 display the demand bids and the corresponding bid prices of the two blocks, respectively. For the sake of simplicity, the similar 24-hour bid price profile is employed for all loads.

Power Transfer distribution Factors (PTDFs) of the IEEE 14-bus test system is obtained from MATPOWER [29]. Forecasted real-time market prices are obtained through simulation and shown in Table 4. It is worth mentioning that the price forecast is for an uncongested system and the forecast will change for congested systems.

TABLE 4. Predicted real time market price (\$/MWh).

Hour	Price	Hour	Price
1	15.79	13	19.61
2	15.38	14	19.38
3	15.32	15	19.43
4	15.32	16	19.43
5	15.13	17	20.03
6	15.13	18	21.11
7	15.38	19	21.11
8	16.22	20	21.11
9	17.15	21	21.11
10	17.93	22	20.88
11	19.38	23	18.84
12	19.61	24	17.72

Three different case studies have been designed to test the Deterministic Model (2) and Stochastic Model (3). Different conditions, including uncongested and congested systems, have been tested, and the results for uncongested system are presented for illustration:

- *Case 1:* All MPs offer their marginal costs and the strategic MP does not have virtual bidding capability.
- *Case 2:* Strategic MP offers strategically without virtual bidding capability while other MPs offer their marginal costs.
- *Case 3:* Strategic MP offers strategically with virtual bidding capability while other MPs offer their marginal costs.

C. RESULTS FOR DETERMINISTIC CONDITION (MODEL (2))

The total cleared power and total profits of the strategic MP for the three cases are depicted in Fig. 1. It shows that, in comparison to Case 1, where all MPs, including the strategic MP put in their marginal costs as offer prices, this strategic MP makes significantly higher profits in Case 2. In Case 2, the MP offers a strategically determined higher price so that the market clearing price is increased, as shown in Fig. 2(a). Although the cleared power is reduced, the profit increases. In Case 3, the presence of virtual transactions makes the decision making process more complicated for the strategic MP since virtual transactions may change the DA market prices and subsequently alter the DA/RT price which affects the virtual transaction profit, and the changed DA price has a direct impact on physical generation profit. Therefore, the strategic MP needs to make a compromise between the physical generation profit and the virtual transaction profit through

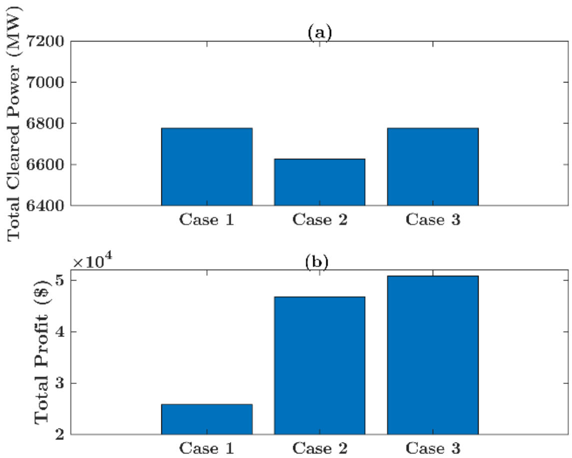

FIGURE 1. Results of the strategic MP in the IEEE 14-bus system. a) Total cleared power and b) Total profits of MP.

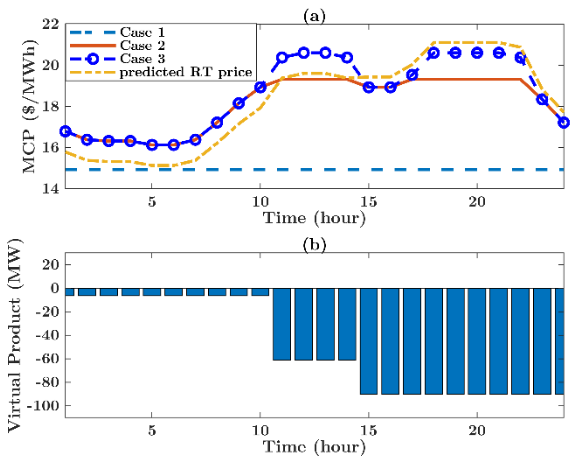

FIGURE 2. Effect of virtual transactions on market price in the IEEE 14-bus system. a) DA market prices for the three cases and the predicted RT price; b) Virtual transaction in Case 3.

a delicate balance between the amount of physical/virtual transactions and its impact on DA prices. In comparison to Case 2, more physical generation power of the strategic MP is cleared in Case 3, leading to higher profit for the following reasons:

a) From hour 1 to hour 10, the predicted RT price is lower than the DA price in Case 2, as illustrated in Fig. 2(a). For virtual transaction without physical generation, which is often studied in the literature, the virtual transaction would always choose to act as a virtual generation (INC) in order to make virtual transaction profit. For virtual transactions with physical generation, which is the focus of the work, the MP may choose a different strategy because virtual generation can cause a negative impact on the physical generation profit through its impact on DA prices. In this case study, the strategic MP chooses to bid in as a virtual demand (of 6.3 MW) instead of virtual generation and manages to keep the DA/RT price difference unchanged. Although the virtual demand leads to a negative virtual transaction profit, the strategic MP's generation increases as a result of the virtual demand, and the physical generation profit increases more than the loss in virtual transaction profit, resulting in an increase of the net profit.

b) From hour 11 to hour 14, the predicted RT prices are slightly higher than the DA prices in Case 2 (as seen in Fig. 2(a)), leaving small room to make virtual transaction profit alone. The strategic MP decides to bid virtual demand in large quantity which substantially increases the DA prices. As the DA prices become higher than the RT prices, it incurs significant loss to the virtual transaction profit. However, the negative virtual transaction profit is offset by the much-increased physical generation profit (as seen in Fig. 3 (a)) that benefits from increased DA prices. As a result, the total profit has increased.

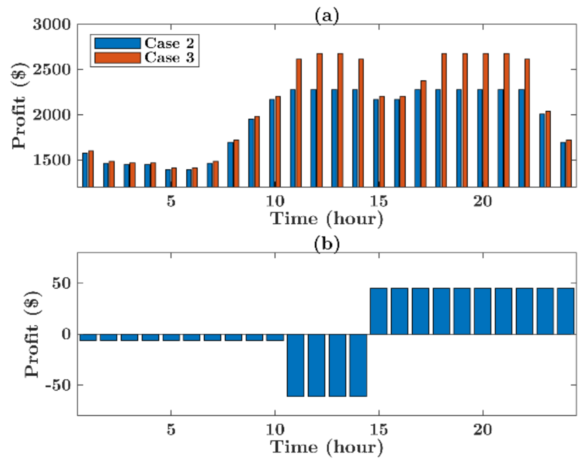

FIGURE 3. Profit comparison. a) Hourly physical generation profit for Case 2 and Case 3. b) Hourly virtual transaction profit in Case 3.

c) From hour 15 to hour 24, the predicted RT price is considerably higher than the DA price (as seen in Fig. 2(a)), and strategic MP bids in a virtual demand which is expected to bring virtual transaction profit as long as the resulting DA price is maintained to be lower than the predicted RT price. In addition, the virtual demand increases the DA price and therefore brings higher physical generation profit.

For the above reasons, the strategic MP with virtual bidding capability (Case 3) achieves a higher total profit than Case 2, as shown in Fig. 1 (b).

D. RESULTS FOR STOCHASTIC CONDITION (MODEL 3)

Scenario generation methods applied for power system applications can be classified into three general categories: sampling-based, forecasting-based and optimization-based approaches [30]; and different works apply various methods to generate an appropriate number of scenarios [31], [32]. Since the number of generated scenarios is normally huge in these methods, scenario reduction methods are applied to reduce the number of generated scenarios [33]–[35]. For the sake of illustration in this paper, we generate 15 scenarios to model the other MPs' behaviors and RT market prices, however, the proposed method can be applied to a larger number of scenarios.

In this section, it is assumed that the amount of power offered/bid by other MPs are known parameters by the strategic MP. Moreover, their unknown offer/bid prices are modeled by multiplying the marginal costs (Table 1) and bid prices (Table 3), respectively, with an uncertainty factor vector [1, 1.1, 1.3, 0.9, 0.75]. Therefore, five independent scenarios of rivals' strategies are designated, with the predefined probabilities of [0.7, 0.05, 0.1, 0.1, 0.05].

Moreover, to model the RT market price uncertainty, three scenarios (A, B and C) are generated by multiplying the RT market predicted prices (Table 4) with the uncertainty factor vector of [1, 1.25, 0.8]. The probabilities of these scenarios are assumed to be 0.8, 0.1, and 0.1, respectively. Due to the simplicity of illustration in this case study, transmission constraints were overlooked so that all buses have the same RT price.

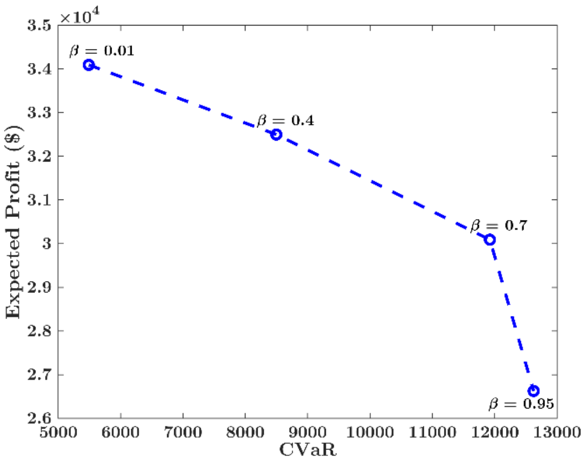

FIGURE 4. Efficient frontier of profit vs risk for the IEEE 14-bus test system.

Considering the confidence level α to be 0.95, the single-level model (3) is solved for multiple value of β. Fig. 4 depicts the efficient frontier and indicates the reduction in the expected profit as the weighting factor β increases. It means that the strategic MP expects a higher profit when it takes the risk-taker position. However, it may experience money losses in certain situations, such as RT scenario B in conjunction with DA scenario 5, as shown in Fig. 5(a). On the other hand, when the strategic MP adopts the risk-averse position, its expected profit declines, while its tailored optimal strategy assures positive profits in all situations, as illustrated in Fig. 5(b). In other words, the strategic MP decreases its profit volatility and its expected profit.

E. 39-BUS TEST SYSTEM

To show the consistency of the results even for the bigger system with more buses, lines and market participants, the proposed model has been implemented for the 39-bus test system which data can be found in [29]. In this system,

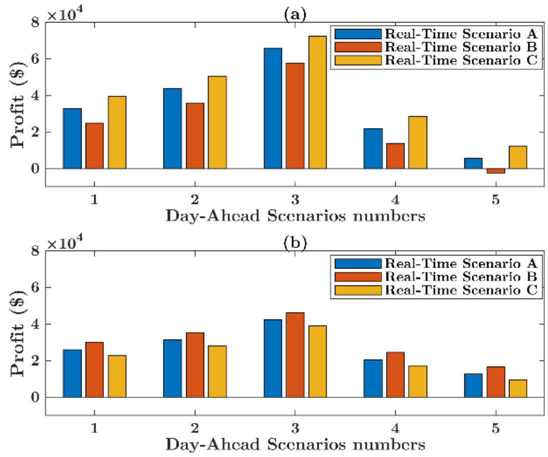

FIGURE 5. Profits of risk-taker MP versus risk-averse MP in different scenarios. a) risk-taker MP; b) risk-averse MP.

we select a strategic MP that owns 3 physical units and is able to bid virtual transactions in 4 different locations. The three generators are located at buses 34, 36 and 39 respectively, while the virtual transactions bid from buses 7, 12, 18 and 23 respectively. The same three cases (namely, Case 1, Case 2 and Case 3) defined in section IV-A are studied here. Fig. 6 illustrates that, by applying the Deterministic Model (2), the strategic MP with the virtual bidding capability can gain more profit than the other two cases. Changes of market prices in the three cases are shown in Fig. 7, which reinforces the observation from the previous section that the price influence of virtual transactions plays an important role in the profit maximization of the strategic MP.

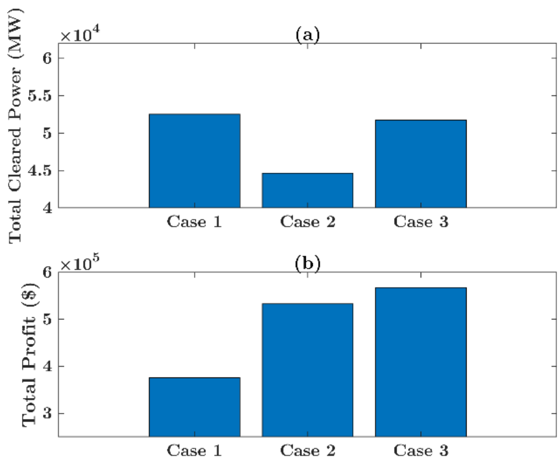

FIGURE 6. Results of the strategic MP in the IEEE 39-bus system. a) Total cleared power and b) Total profits of MP.

To consider the uncertainty in other MPs' offers/bids and RT market prices, 7 different offer/bids of other MPs with a probability vector of [0.6 0.025 0.075 0.1 0.05 0.05 0.1] and 4 different scenarios of RT market price with a probability vector of [0.8 0.075 0.075 0.05] are taken into account to construct 28 scenarios in this case study. The Stochastic

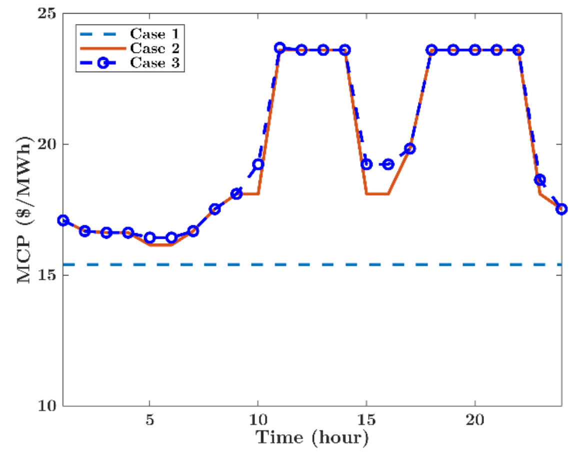

FIGURE 7. Market prices of the IEEE 39-bus system in different cases.

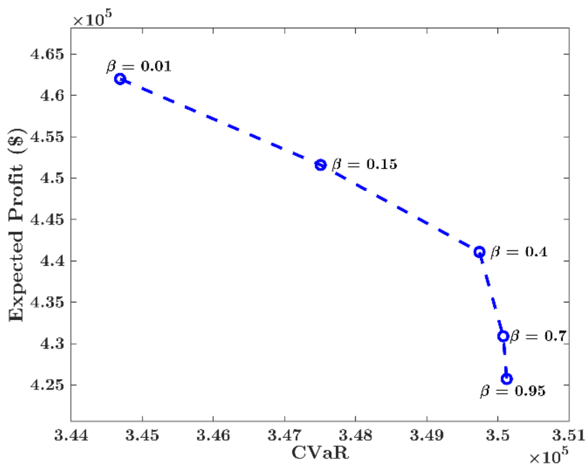

FIGURE 8. Efficient frontier of profit vs risk for the IEEE 39-bus test system.

model (3) is solved for several values of β and $\alpha = 0.95$. The efficient frontier which displays the expected profits of the risk-taker and risk-averse strategic MP is depicted in Fig. 8. Similar to the observations seen in the IEEE 14-bus test system, Fig. 8 shows, as the risk aversion level increases, the strategic MP will have reduced profit and at the same time reduced profit volatility.

V. CONCLUSION

This paper proposes a bi-level model and solution process that enables physical market participants with the virtual bidding capability to maximize their total profit in the participation of both physical assets and virtual bidding. Bi-level optimization programming approach has been utilized, in which LMPs are endogenous generated. Uncertainties of other MPs' offer/bids and RT market prices have been taken into account via scenarios-based modeling. Moreover, the CVaR measure has been applied to quantify the MP's different decision's risk. Duality theorem, KKT optimality conditions, SDT and Fortuny-Amat Transformation are employed to translate the bi-level problem into a MILP problem to be solved. Simulation results illustrate the ability of the proposed model to derive the optimal decisions of the strategic MP. Employing the proposed models, the strategic MP can optimally determine the amount of physical/virtual transactions and manage its impact on the DA price, in order to achieve a balance between the physical generation profit and the virtual transaction profit. A case study on a deterministic condition illustrates a few optimal strategies that utilize virtual transaction to influence DA price in a way that benefits the physical generation profit. Case studies for a stochastic condition demonstrate the proposed method allows the strategic MP to select a risk level which makes the compromise between the expected profit across all scenarios and the profit volatility in those scenarios.

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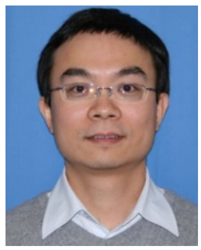
HOSSEIN MEHDIPOURPICHA (Student Member, IEEE) received the B.Sc. degree in electrical engineering from the K. N. Toosi University of Technology (KNTU), in 2010, and the M.Sc. degree in electrical engineering from the Iran University of Science and Technology (IUST), Tehran, Iran, in 2013. He is currently pursuing the Ph.D. degree in power system engineering with the Missouri University of Science and Technology.

His research interests include power system optimization and economics and electricity market simulation.



RUI BO (Senior Member, IEEE) received the B.S.E.E. and M.S.E.E. degrees in electric power engineering from Southeast University, China, in 2000 and 2003, respectively, and the Ph.D. degree from the University of Tennessee, Knoxville (UTK), in 2009. He worked as a Principal Engineer and a Project Manager at Mid-Continent Independent System Operator (MISO), from 2009 to 2017. He is currently an Assistant Professor with the Electrical and Computer Engineering Department, Missouri University of Science and Technology (formerly University of Missouri-Rolla). His research interests include computation, optimization and economics in power system operation and planning, high performance computing, and electricity market simulation, evaluation, and design.