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A Novel Method of Realizing Stochastic Chaotic Secure Communication by Synchrosqueezed Wavelet Transform: The Finite-Time Case

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ABSTRACT This paper is concerned with the finite-time stochastic synchronization of a chaotic system without linear term and its application in secure communication based on synchrosqueezed wavelet transform (SWT). In order to improve the anti-jamming ability of chaotic systems and forecast the synchronization time, proper controllers are designed to achieve finite-time stochastic synchronization for the chaotic system with random perturbation. Based on the proposed chaotic synchronization systems, we present secure communication scheme about secondary encryption to enhance the level of security. Moreover, a novel SWT algorithm is introduced to correctly recover information signals at the end of the scheme. The SWT based scheme can decompose the information signal into a set of intrinsic mode type (IMT) function components. Then, instantaneous frequency and instantaneous amplitude of each IMT component are calculated through Hilbert transform. Simulation results confirm that both the synchronization approach and the security communication scheme are effective. For multi-component signals with adjacent frequency, the accuracy of recovered signals by SWT based on our scheme is higher than that recovered by the other time-frequency denoising methods.

INDEX TERMS Finite-time synchronization, random perturbation, synchrosqueezed wavelet transform, secure communication.

I. INTRODUCTION

Synchronization of chaotic systems has been investigated since it was studied by Pecora and Carroll [1] in 1990. Chaotic systems display unpredictable behavior, and they are sensitive to initial conditions [2]. Therefore, chaotic systems have been used to resolve many engineering problems, especially for secure communication [3].

Many secure communication schemes based on chaotic synchronization have been proposed. Theesar *et al.* [4] apply synchronization of Lur'e systems using sampled-data control to the secure communication problem. Wang *et al.* [5] investigate a communication strategy based on the adaptive synchronization of the coupling Hindmarsh-Rose neuron model. Li *et al.* [6] propose secure communication approach based on chaos shift keying. Wang *et al.* [7] present secure communication approach with the neural networks based on the fixed-time synchronization. Most existing works mainly study the chaotic systems with linear term and consider how to achieve synchronization. Few researches focus on the performance of chaotic secure communication.

A new chaotic system without linear term and its impulsive synchronization is introduced in [8]. Exponential synchronization of this new chaotic system is applied to secure communication in [3]. There are few researches on this new chaotic system, and thus its application in secure communication can reduce the possibility of being decoded. The existing works focus on the infinite time synchronization, which means that the time of recovering the transmitted signal may be infinite. However, in secure communication process, we want to synchronize chaotic systems as quickly as possible. Just as pointed out in [9], if the decoding time does not correspond to the time during which the chaotic oscillators are synchronized, the message may unfortunately not be recovered or sent. Finite-time control technique is an effective method to achieve faster synchronization in control

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systems. However, most of the existing works mainly focus on how to achieve synchronization in finite-time. There are few researches on improving performance in chaotic secure communication. In addition, stochastic effects exist in real systems. A lot of dynamical systems have variable structures subject to stochastic abrupt changes, which may result in abrupt phenomena such as stochastic failures and repairs of components, changes in the interconnections of subsystems, sudden environment changes [10], [11], etc. Therefore, it is necessary to study the stability of stochastic chaotic system in finite-time for secure communication.

How to correctly abstract information signal from noisy situation is important in secure communication. The traditional information signal extraction methods are based on time domain reconstructs phase space [12] or strange attractor characteristic [13]. These methods require a more complex condition and a large quantity of calculations.

In recent years, some scholars apply time-frequency analysis methods to extract the harmonic signal from noisy interference in chaotic secure communication. An *et al.* [14] apply the continuous wavelet transform (CWT) to remove noise from the information signal. Wang *et al.* [15] propose a method of extracting the harmonic signal from chaos interference based on empirical mode decomposition (EMD). Zhou *et al.* [16] use the complete ensemble empirical mode decomposition (CEEMD) to denoise chaotic signals. Although EMD and CEEMD can achieve a better effect than CWT in harmonic extraction, they are very sensitive to noise [17]. Synchrosqueezed wavelet transform (SWT) can accurately separate the mixed signal disturbed by noise, as it has better robustness than EMD and CEEMD [18]. How to apply SWT to improve the accuracy of extraction method as well as enhance the security level in secure communication has been an unsolved problem.

Motivated by the above discussion, in this paper, we propose a novel multistage stochastic chaotic secure communication scheme based on SWT. The main contributions of this paper are as follows. 1) We establish the finite-time stochastic synchronization criterion of the new chaotic systems without linear term. Different from the existing control scheme in [3], [8], the time of synchroniza-tion is finite. In addition, this criterion solves possible problems of robustness and disturbances for chaotic systems applied in secure communication. 2) We design a simple controller and obtain a fixed synchronization time. This fixed synchronization time does not depend on the parameters of the controller, which is different from the conventional finite-time synchronization works. Thus the time for decryption is predictable, and it will greatly improve the reliability of the secure communication. 3) We present a novel time-frequency extraction scheme based on the synchrosqueezed wavelet transform (SWT) in the secure communication. The SWT based scheme can decompose the information signal into a set of intrinsic mode type (IMT) function components. The instantaneous frequency (IF) and the instantaneous amplitude (IA) of each IMT component can be calculated through Hilbert transform. Simulation results verify the effectiveness of both the synchronization approach and security communication scheme. For multi-component signals with adjacent frequency, the accuracy of recovered signals by SWT based on our scheme is higher than that recovered by the other time-frequency denoising methods.

The outline of this paper is organized as follows. In Section II, the main results for achieving finite-time stochastic synchronization of the chaotic system without linear term are presented. In Section III, we introduce the SWT method in order to extract the information signal from the noisy interference. The multistage stochastic chaotic secure communication scheme based on SWT is detailed in Section IV. In Section V, numerical studies are given to verify the proposed scheme. Finally, this paper is wrapped up with some concluding remarks.

Notations: The Euclidean norm is denoted as $\|\cdot\|$, accordingly, $\|x\|^2 = x^T x$, where x^T denotes the transposition of vector x. $E(\cdot)$ stands for the mathematical expectation of a stochastic process. I represents an identity matrix with appropriate dimension. $A > 0$ implies A is a positive definite matrix.

II. FINITE-TIME STOCHASTIC SYNCHRONIZATION OF THE CHAOTIC SYSTEM WITHOUT LINEAR TERM

A new chaotic system without linear term was found by Xu Y and Wang Y in 2014 [8], which can be described by the following differential equation:

$$\begin{aligned} \dot{x}_1 &= \ln(a + e^{x_2 - x_1}) \\ \dot{x}_2 &= x_1 x_3 \\ \dot{x}_3 &= b - x_1 x_2, \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are state variables and a, b are positive constants. When $a = 0.1, b = 0.25$, it has a chaotic attractor.

Suppose the drive system with stochastic perturbations as follows:

$$\begin{aligned} dx_1 &= \ln(a + e^{x_2 - x_1})dt + H_1(t, x_1)dw(t) \\ dx_2 &= x_1 x_3 dt + H_2(t, x_2)dw(t) \\ dx_3 &= (b - x_1 x_2)dt + H_3(t, x_3)dw(t), \end{aligned} \tag{2}$$

where $w(t)$ is a white noise (i.e. $w(t)$ is a Brownian motion) and satisfies: $E(dw(t)) = 0$, $E(dw(t)^2) = dt$.

$H_i(t, x)$ satisfies the Lipschitz condition, which means there exists a constant $L_i > 0$ such that

$$\|H_i(t, x) - H_i(t, y)\| \leq L_i \|(y - x)\|. \tag{3}$$

We suppose (2) is the master system and define the slave system as follows:

$$\begin{aligned} dy_1 &= [\ln(a + e^{y_2 - y_1}) + u_1]dt + H_1(t, y_1)dw(t) \\ dy_2 &= [y_1 y_3 + u_2]dt + H_2(t, y_2)dw(t) \\ dy_3 &= [b - y_1 y_2 + u_3]dt + H_3(t, y_3)dw(t). \end{aligned} \tag{4}$$

To fulfil the synchronization of slave system (4) and master system (2), we set the error state as:

$$e_1 = y_1 - x_1$$
$$e_2 = y_2 - x_2$$
$$e_3 = y_3 - x_3. \quad (5)$$

From (2), (4), and (5), we obtain the synchronization error system described by

$$\begin{aligned} de_1 &= [\ln(\frac{a + e^{e_2 - e_1} e^{x_2 - x_1}}{a + e^{x_2 - x_1}}) + u_1]dt \\ &\quad + [H_1(t, y_1) - H_1(t, x_1)]dw(t) \\ de_2 &= (e_1 e_3 + e_1 x_3 + e_3 x_1 + u_2)dt \\ &\quad + [H_2(t, y_2) - H_2(t, x_2)]dw(t) \\ de_3 &= -(e_1 e_2 + e_1 x_2 + e_2 x_1 - u_3)dt \\ &\quad + [H_3(t, y_3) - H_3(t, x_3)]dw(t). \end{aligned} \quad (6)$$

Remark 1: In order to eliminate the synchronization error and achieve the complete synchronization, some controllers have been designed in [3], [8]. They are all based on the infinite time synchronization, which is not adapted to secure communication. Therefore, we will design the controller to realize synchronization in finite time, and consider the interference of random noise for practicality.

Now we give the definition of stochastic synchronization in finite time between the master system (2) and the slave system (4) and some lemmas which will be used in Theorem 1.

Definition 1 [19]: The master system (2) and the slave system (4) are said to be stochastic synchronized in finite time if, for a suitable designed feedback controller, there exists a constant $t_1 > 0$ ($t_1 > 0$ depends on the initial state vector value $x(0) = (x_1(0), x_2(0), \cdots, x_n(0))^T$), such that

$$\lim_{t \to t_1} E \|x(t) - y(t)\| = 0,$$

and $\|x(t) - y(t)\| \equiv 0$ for $t > t_1$.

Lemma 1 [19]: Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \le -\eta V^{\theta}(t), \quad \forall t \ge t_0, \ V(t_0) \ge 0,$$

where $\eta > 0$, $0 < \theta < 1$ are constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\theta}(t) \le V^{1-\theta}(t_0) - \eta(1-\theta)(t - t_0), \quad t_0 \le t \le t_1,$$

and $V(t) \equiv 0$, $\forall t \ge t_1$, with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\theta}(t_0)}{\eta(1-\theta)}. \quad (7)$$

Lemma 2 [20] (The One-Dimensional It$\widehat{o}$ Formula): Let $x(t)$ be an It$\widehat{o}$ process on $t \ge 0$ with the stochastic differential

$$dx(t) = f(t)dt + g(t)dw(t).$$

Then $V(x(t), t)$ is also an It$\widehat{o}$ process with the stochastic differential given by

$$\begin{aligned} dV(x(t), t) &= [V_t'(x, t) + V_x'(x, t)f(t) + \frac{1}{2}V_{xx}''(x, t)g^2(t)]dt \\ &\quad + V_x'(x, t)g(t)dw(t). \end{aligned} \quad (8)$$

Lemma 3 [21]: For any $\rho \in R^+$, $X, Y \in R$, the inequality $|X|\,|Y| \le \frac{1}{2}(\rho X^2 + \rho^{-1} Y^2)$ holds.

Now we establish the synchronization criterion between the master system (2) and the slave system (4).

Theorem 1: Let the controller be

$$\begin{aligned} u_1 &= -A - k_1 e_1 - \eta sign(e_1)\,|e_1|^{\theta} \\ u_2 &= -k_2 e_2 - \eta sign(e_2)\,|e_2|^{\theta} \\ u_3 &= -k_3 e_3 - \eta sign(e_3)\,|e_3|^{\theta}, \end{aligned} \quad (9)$$

where $\eta > 0$ is a tunable constant, the real number θ satisfies $0 < \theta < 1$. Suppose that constant A and the feedback gains $k_i > 0$ $(i = 1, 2, 3)$ satisfy

$$\begin{aligned} A &> \ln(\frac{a + e^{e_2 - e_1} e^{x_2 - x_1}}{a + e^{x_2 - x_1}}) \\ k_1 &> \frac{1}{2}M(\rho_1 + \rho_2) + L_1^2 \\ k_2 &> \frac{1}{2}M\rho_1^{-1} + L_2^2 \\ k_3 &> \frac{1}{2}M\rho_2^{-1} + L_3^2, \end{aligned} \quad (10)$$

where $M = \max\{|x_1|, |x_2|, |x_3|\}$ and ρ_1, ρ_2 are positive constants. Then the master system (2) and the slave system (4) are stochastic synchronized in a finite time $t_1 = \frac{V^{\frac{1-\theta}{2}}(0)}{\eta(1-\theta)}$, where $V(0) = \frac{1}{2}\sum_{i=1}^{3} e_i^2(0)$, $e_i(0)$ is the initial condition of $e_i(t)$.

Proof: Choose the Lyapunov function as

$$V(t) = e(t)^T Pe(t), \quad (11)$$

where $P = diag(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, $e(t) = (e_1, e_2, e_3)^T$.

Set

$$\tilde{H}_i(t, e_i) = H_i(t, y_i) - H_i(t, x_i).$$

By (8), differentiating both side of (11) we obtain:

$$\begin{aligned} dV(t, e(t)) &= LV(t, e(t))dt + e_1\tilde{H}_1(t, e_1)dw(t) \\ &\quad + e_2\tilde{H}_2(t, e_2)dw(t) + e_3\tilde{H}_3(t, e_3)dw(t), \end{aligned}$$

where

$$\begin{aligned} LV(t, e(t)) &= e_1(\ln(\frac{a + e^{e_2 - e_1} e^{x_2 - x_1}}{a + e^{x_2 - x_1}}) + u_1) \\ &\quad + e_2(e_1 e_3 + e_1 x_3 + e_3 x_1 + u_2) \\ &\quad + e_3(-e_1 e_2 - e_1 x_2 - e_2 x_1 + u_3) \\ &\quad + \tilde{H}_1(t, e_1)^2 + \tilde{H}_2(t, e_2)^2 + \tilde{H}_3(t, e_3)^2. \end{aligned} \quad (12)$$

Substituting (9) and (3) into (12) yields

$$\begin{aligned} &LV(t, e(t)) \\ &\le -(k_1 e_1^2 + k_2 e_2^2 + k_3 e_3^2) + M\,|e_1|\,|e_2| + M\,|e_1|\,|e_3| \\ &\quad + L_1^2 e_1^2 + L_2^2 e_2^2 + L_3^2 e_3^2 - \eta e_1 sign(e_1)\,|e_1|^{\theta} \\ &\quad - \eta e_2 sign(e_2)\,|e_2|^{\theta} - \eta e_3 sign(e_3)\,|e_3|^{\theta}. \end{aligned}$$

By Lemma 3, we have

$$|e_1|\,|e_2| \le \frac{\rho_1}{2}e_1^2 + \frac{\rho_1^{-1}}{2}e_2^2, \quad |e_1|\,|e_3| \le \frac{\rho_2}{2}e_1^2 + \frac{\rho_2^{-1}}{2}e_3^2.$$

Hence

$$\begin{aligned} LV(t,e(t)) \le& -(k_1 - M\frac{\rho_1}{2} - M\frac{\rho_2}{2} - L_1^2)e_1^2 \\ &- (k_2 - \frac{1}{2}M\rho_1^{-1} - L_2^2)e_2^2 \\ &- (k_3 - \frac{1}{2}M\rho_2^{-1} - L_3^2)e_3^2 - \eta e_1 sign(e_1)\,|e_1|^\theta \\ &- \eta e_2 sign(e_2)\,|e_2|^\theta - \eta e_3 sign(e_3)\,|e_3|^\theta, \end{aligned}$$

where $\sum_{i=1}^{3} e_i sign(e_i)\,|e_i|^\theta = \sum_{i=1}^{3} |e_i|^{\theta+1} = [2V(t)]^{\frac{\theta+1}{2}}$.

As $E(\mathrm{d}w(t)) = 0$, if feedback gains $k_i > 0 (i = 1, 2, 3)$ satisfy (10), we can prove

$$\begin{aligned} E(\dot{V}(t,e(t))) &\le E(\sum_{i=1}^{3} -\eta e_i sign(e_i)\,|e_i|^\theta) \\ &\le -2\eta E(V^{\frac{\theta+1}{2}}(t,e(t))). \end{aligned}$$

For any $t_0 > 0$, we have $E(V^{\frac{\theta+1}{2}}(t_0, e(t_0))) = E(V(t_0, e(t_0)))^{\frac{\theta+1}{2}}$, so $E(\dot{V}(t, e(t))) \le -2\eta E(V(t, e(t)))^{\frac{\theta+1}{2}}$.

By Lemma 2, $E(V(t, e(t)))$ converge to zero in the finite time t_1, and t_1 is estimated by (7):

$$t_1 = \frac{V^{1-\frac{1+\theta}{2}}(0)}{2\eta(1-\frac{1+\theta}{2})} = \frac{V^{\frac{1-\theta}{2}}(0)}{\eta(1-\theta)}. \tag{13}$$

According to Definition 1, the slave system (4) is finite-timely stochastic synchronized with the master system (2).

Remark 2: Theorem 1 provides an effective method to design the desired controller by (9). In fact, given scalars η, θ, ρ_1 and ρ_2, one can obtain the desired feedback gains k_i by solving (10).

Remark 3: In secure communication, we can estimate the synchronization time t_1 by (13), to ensure the decryption after the systems achieved the complete synchronization. However, the synchronization time t_1 depends on the parameters θ of the controllers. Once the controllers change, t_1 changes accordingly, which in not convenient for the application of secure communication.

In secure communication process, we want to synchronize chaotic systems as quickly as possible. Therefore, we try to discuss the minimum value of synchronization time t_1. Then, we get the following corollary.

Corollary 1: If the master system (2) and the slave system (4) are stochastic synchronized in a finite time $t_1 = \frac{V^{\frac{1-\theta}{2}}(0)}{\eta(1-\theta)}$ as mentioned in Theorem 1, when $V(0) > e^2$, there is a minimum value of

$$t_1(\hat{\theta}) = \frac{e\ln V(0)}{2\eta}, \tag{14}$$

where $\hat{\theta} = 1 - \frac{2}{\ln V(0)}$.

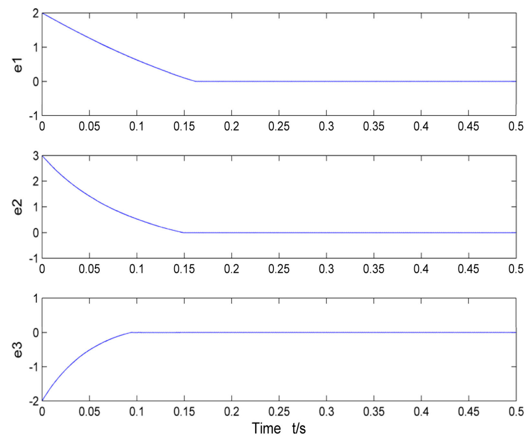

FIGURE 1. Time responses of the error variables under the controller.

Proof: Denote $t_1(\theta) = \frac{V^{\frac{1-\theta}{2}}(0)}{\eta(1-\theta)} > 0$, so $\ln t_1(\theta) = \frac{1-\theta}{2}V(0) - \ln\eta - \ln(1-\theta)$.

Differentiating both side of the former equation, it yields $\frac{t_1'(\theta)}{t_1(\theta)} = -\frac{1}{2}\ln V(0) + \frac{1}{1-\theta}$.

Let $t_1'(\theta) = 0$, we obtain $\hat{\theta} = 1 - \frac{2}{\ln V(0)}$.

As $t_1''(\theta) = t_1(\theta)\{[-\frac{1}{2}\ln V(0) + \frac{1}{1-\theta}]^2 + \frac{1}{(1-\theta)^2}\} > 0$, there is a minimum value of $t_1(\hat{\theta}) = \frac{e\ln V(0)}{2\eta}$.

While $0 < \hat{\theta} = 1 - \frac{2}{\ln V(0)} < 1$, so $V(0) > e^2$.

Remark 4: If η increases, then t_1 decreases. The above analysis provides guidance on how to regulate these parameters to achieve minimum synchronization time. For example, if $V(0) > e^2$, we can take $\hat{\theta} = 1 - 2(\ln V(0))^{-1}$ to obtain the fixed synchronization time $t_1(\hat{\theta})$. It does not depend on the parameters of the controller, which is different from the conventional finite-time synchronization works. Thus the time for decryption is predictable, and it will greatly improve the reliability of the secure commu-nication.

Example 1: To demonstrate and verify the validity of our proposed synchronization criterion, we present and discuss the numerical results. In these numerical simulations, the initial states for the master system and the slave system are given by $(x_1(0), x_2(0), x_3(0)) = (1, 1, 3)$, $(y_1(0), y_2(0), y_3(0)) = (1, 2, 1)$, respectively. Stochastic perturbations are set as $H(t, y-x) = (0.9(\sin y_1 - \sin x_1),\ 0.8(y_2 - x_2),\ 0.7(y_3 - x_3))^T$. Moreover, the states are bounded with $M = 3.3$. By simple computation, we get $V(0) = 8.5 > e^2$, t_1 has a minimum value at $\hat{\theta} = 0.065$. Take $A = 5$ and $\eta = 10$. By computing (14), we get fixed synchronization time $t_1 = 0.2909s$. According to Theorem 1, $k_1 = 10$, $k_2 = 15$, $k_3 = 17$ are chosen for control law. The synchronization errors are plotted in Fig. 1. It can be seen that the controller (9) can synchronize the chaotic system at about $T_s \approx 0.24s$, which is smaller than the fixed synchronization time $t_1 = 0.2909s$.

Remark 5: The result demonstrates that our technique cannot only achieve synchronization within the fixed synchronization time as in [7], but also achieve stochastic synchronization by considering the influence of random

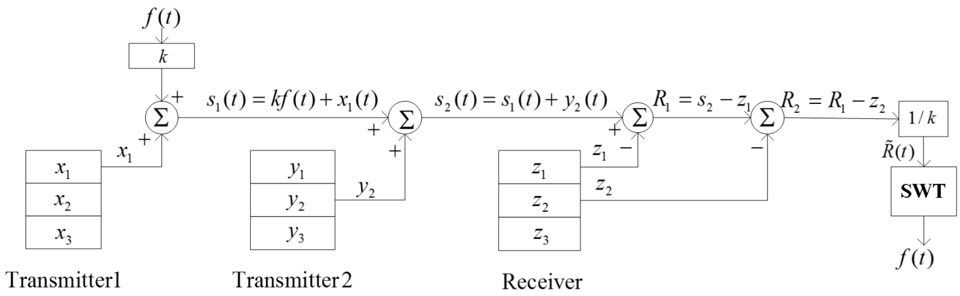

FIGURE 2. Diagram of multistage chaotic secure communication system based on SWT.

perturbation, which improves the anti-jamming ability of the chaotic systems.

III. MULTI-COMPONENT SIGNAL EXTRACTION FROM NOISY INTERFERENCE BY SWT

The SWT proposed by Daubechies *et al.* [22] is a new time-frequency reassigned method based on the CWT. It aims at improving the time-scale representation resulted from CWT and allowing for mode reconstruction. For a given signal $f = \sum_{k=1}^{K} f_k = \sum_{k=1}^{K} A_k(t)e^{i2\pi\phi_k(t)}$, the CWT is defined as:

$$W_f(a,b) = \int f(t)a^{-\frac{1}{2}}\psi^*(\frac{t-b}{a})dt$$

where b is the time shift factor and a is the time scale factor. ψ^* is the complex conjugate of mother wavelet.

The instantaneous frequency is obtained:

$$\omega_f(a,b) = -iW_f^{-1}(a,b)\frac{\partial W_f(a,b)}{\partial b}.$$

After obtaining the instantaneous frequency $\omega_f(a,b)$, the synchrosqueezing $W_f(a,b)$ is defined by

$$S_{f,\varepsilon}^{\delta}(b,\omega) = \int_{A_{\varepsilon,f(b)}} W_f(a,b)\frac{1}{\delta}h\left(\frac{\omega-\omega_f(a,b)}{\delta}\right)\frac{da}{a^{3/2}},$$

where $A_{\varepsilon,f(b)}(b) = \left\{a \in R_+;\ \left|W_f(a,b)\right| > \varepsilon\right\}$.

If ε is small enough, f_k can be completely reconstructed by

$$\tilde{f}_k(b) = \lim_{\delta\to 0}\left(R_\psi^{-1}\int_{|\omega-\varphi_k'(b)|<\varepsilon} S_{f,\varepsilon}^{\delta}(b,\omega)\,d\omega\right).$$

At present, most of the research on the secure communication is based on complete synchronization [3], [8]. In this case, the extraction of the information signal is relatively simple. However, the channel is often mixed with noise. The noise can be regarded as an error term e. Consider the signal $g = f + e$, where e satisfies $\|e\| \le \tilde{\varepsilon}$. The SWT can extract the signal f from error term by the following equation [23]:

$$\tilde{f}_k(b) = \lim_{\delta\to 0}\left(R_\psi^{-1}\int_{|\omega-\varphi_k'(b)|<\varepsilon} S_{g,\tilde{\varepsilon}}^{\delta}(b,\omega)\,d\omega\right).$$

IV. MULTISTAGE CHAOTIC SECURE COMMUNICATION SYSTEM BASED ON SWT

The synchronization criterion that we proposed in Section II plays an important role in chaotic communication. The secure communication theory is that the information signal is mixed with the chaos signal to be the pretend transmitted signal, which is transmitted to the receiver. The information signal is extracted by means of a synchronous regime between the master and slave systems. In addition, the level of security mostly depends on the complexity level of dynamics for chaotic systems. Therefore, we propose a secure communication scheme about secondary encryption based on this chaotic synchronized system to enhance the level of security. The secure performance of our secure communication scheme will be displayed in Section V.

This paper proposes a scheme about secondary encryption based on multistage chaos synchronized system for secure communication. A sketch design for the communication scheme is shown in Fig.2. Based on the sensitive of chaotic system to the initial value, we apply the new chaotic system (1) with three different initial values to Transmitter 1, Transmitter 2, and Receiver respectively. By multiplying the compression ratio constant k, the original message $f(t)$ is added to the first stage chaotic system variable x_1. Then we get the first stage pretend chaos signal $s_1(t) = kf(t) + x_1$, which is transmitted to Transmitter 2. Then, $s_1(t)$ is added to the second stage chaotic system variable y_2 to be the second stage pretend chaos signal $s_2(t) = s_1(t) + y_2$, which is transmitted to Receiver. Suppose $R_1 = s_2 - z_1$ is the first recovery of information signal and $R_2 = R_1 - z_2$ be the second one, where z_1, z_2 are the third stage chaotic system variables. By employing the mentioned controllers from Theorem 1 in Section II, the synchronization will be achieved among the three chaos systems in the fixed synchronization time $t_1(\hat{\theta})$, then $z_2 - y_2 \to 0$ and $z_1 - x_1 \to 0$. Therefore, we can recover the original signal by $\tilde{R}(t) = k^{-1}R_2 = k^{-1}(kf(t) + x_1 - z_1 + y_2 - z_2) = f(t)$ at the rece-iver end.

However, the recovered signal by traditional scheme is $\tilde{R}(t) = f(t) + n(t)$, as the channel is often mixed with noise. Fortunately, SWT has been found to be a useful tool for analyzing and decomposing multi-component. So, we apply SWT at the receiving terminal to accurately recover the multi-component signal $f = \sum_{k=1}^{K} f_k = \sum_{k=1}^{K} A_k(t) e^{i2\pi\phi_k(t)}$, and separate each component f_k.

The steps of extracting the information signal and each component from mixed signal $\tilde{R}(t)$ based on SWT are briefly described as:

1) Calculate the continuous wavelet coefficients of signal. Choose the mother wavelet as $\psi(t) = \frac{1}{\sqrt{2\pi}} e^{i2\pi t} e^{-t^2/2}$. Calculate the continuous wavelet coefficients $W_{\tilde{R}}(a, b)$ of the output signal $W_{\tilde{R}}(a, b)$ at the receiving terminal, and discretize it as $W_{\tilde{R}}(a, b)$. Calculate the instantaneous frequency as $\tilde{\omega}_{\tilde{R}}(a_j, t_n) = \frac{\partial_t(\tilde{W}_{\tilde{R}}(a_j, t_n))}{2\pi i \cdot \tilde{W}_{\tilde{R}}(a_j, t_n)}$.
2) Divide the frequency interval. The length of mixed signal $\tilde{R}(t)$ is $N = 2^{L+1}$, and the sampling time inter-val is Δt. Set $n_v = 32,\ n_a = Ln_v,\ \Delta\omega = \frac{1}{n_a - 1}\log_2(\frac{N}{2})$, and $\omega_0 = \frac{1}{N\Delta t}$. Fix $\omega_l = 2^{l\Delta\omega}\omega_0, l = 0, 1, \cdots, n_a - 1$, and then the whole frequency will be divided into different intervals as $W_l = [\frac{\omega_{l-1} + \omega_l}{2}, \frac{\omega_l + \omega_{l+1}}{2}]$.
3) Calculate the synchrosqueezed value in the time-frequency plane:
$$\tilde{T}_{\tilde{R}}(\omega_l, b) = \sum_{0 \le j \le n_a - 1, \{j: |\tilde{\omega}(a_j, t_n| \in W_l\}} \tilde{W}_{\tilde{R}}(a_j, t_n) \frac{\log 2}{n_v} a_j^{-\frac{1}{2}}.$$
4) Extract each component of the mixed signal. Calculate the curve c_q^* according to 18]. Set the interval $L_k(t_n) = [c_q^* - nv/2, c_q^* + nv/2]$, then calculate each component by $\tilde{f}_k(t_n) = \frac{2}{R_\psi} \mathrm{Re}(\sum_{l \in L_k(t_n)} \tilde{T}_{\tilde{R}}(\omega_l, t_n))$.
5) Calculate modal parameters of each component. Calculate the Hilbert transform of $y_k(t) =$ by $y_k(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{f_k(t)}{t - \tau} d\tau$. Calculate the IA of each component by $\tilde{A}_k(t) = \sqrt{f_k^2(t) + y_k^2(t)}$, and calculate the IF by $\tilde{\omega}_k(t) = \frac{1}{2\pi} \frac{d \arctan(y_k(t)/f_k(t))}{dt}$. Then use the least squares method to fit the $\tilde{A}_k$ and $\tilde{\omega}_k$.

Remark 6: The overall computational complexity is $O(n_v N \log_2^2 N)$.

V. NUMERICAL RESULTS

In this section, we provide several numerical examples to illustrate the ideas in Section IV. Moreover we show how SWT compares to other time-frequency denoising algorithms, such as EMD [15], CEEMD [16], WT [24]. All the experiments are carried out in the Matlab (R2015b) environment running on a PC with Intel(R) Core(TM) i7-3770 CPU 3.40GHz.

In these numerical simulations, the initial states of chaotic systems are given by $x(0) = (1, 1, 3)$, $y(0) = (1, 2, 1)$ and $z(0) = (-1, 2, -1)$. Compression ratio constant is set as $k = 0.01$. The abstraction process of the information signal is after the finite time $t_1 = 0.2909s$ under controller $k_1 = 10, k_2 = 15, k_3 = 17$. We consider the signal: $f(t) + n(t) = f_1(t) + f_2(t) + f_3(t) + n(t) = A_1\cos(2\pi\omega_1 t) + A_2\cos(2\pi\omega_2 t) + A_3\cos(2\pi\omega_3 t)$. $\omega_1 = 20,\ \omega_2 = 25,\ \omega_3 = 30$ are their IFs of each component and $A_1 = 1, A_2 = 2, A_3 = 0.5$ are their IAs. $n(t)$ is the Gaussian white noise with different noise level. The

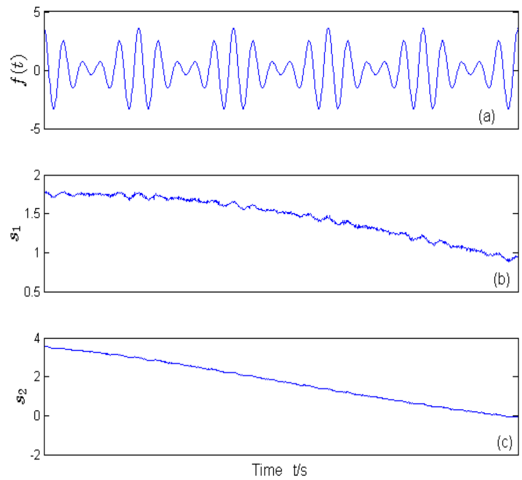

FIGURE 3. The secure performance display in time domain: (a) the information signal $f(t)$; (b) the first encrypted signal $s_1(t)$; (c) the second encrypted signal $s_2(t)$.

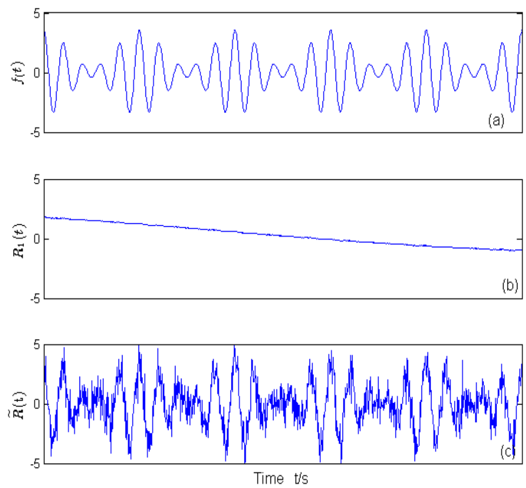

FIGURE 4. The extraction performance by traditional method under $NL = 60\%$: (a) the signal $f(t)$; (b) the first recovered signal $R_1(t)$; (c) the traditional recovered signal $\tilde{R}(t)$.

noise level (NL) is defined as $NL = \sigma_n / \sigma_f \times 100\%$, where σ_n is the standard deviation of noise, and σ_f is the standard deviation of signal.

In Experiment 1, we first show the secure performance of the multistage chaotic secure communication system. As mentioned in Section IV, after the information signal $f(t)$ masked by the states of chaotic systems, $s_1(t) = kf(t) + x_1(t)$ is the first transmitted signal and $s_2(t) = s_1(t) + y_2(t)$ is the second transmitted signal. Fig. 3 depicts the numerical results of encryption process for the multi-component signal $f(x)$. Obviously, the information signal is masked after a short transmission and the secondary encryption method can enhance the security of communication.

In Experiment 2, we show the inaccuracy of the traditional extraction method in noisy situation with $NL = 60\%$. As can be seen from Fig. 4 (a) and (c), with the Gaussian

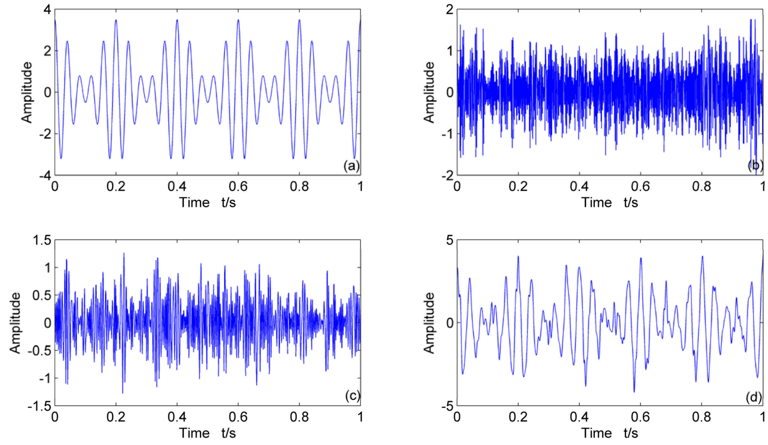

FIGURE 5. The EMD extraction results of the information signal under $NL = 60\%$: (a) The information signal $f(t)$; (b) The imf_1 by EMD; (c) The imf_2 by EMD; (d) The imf_3 by EMD.

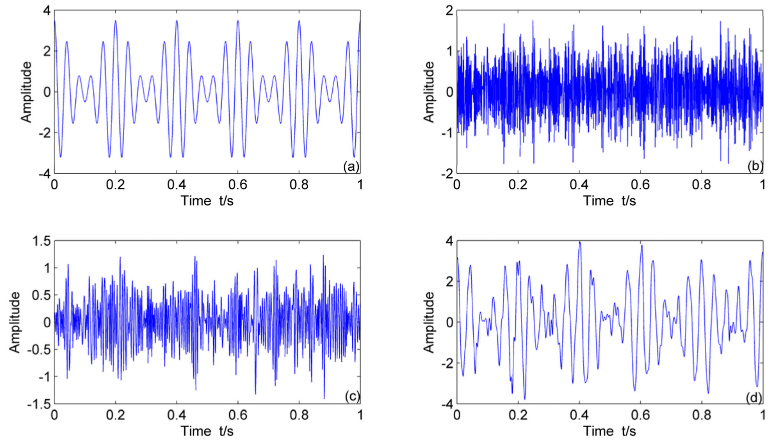

FIGURE 6. The CEEMD extraction results of the information signal under $NL = 60\%$: (a) The information signal $f(t)$; (b) The imf_1 by CEEMD; (c) The imf_2 by CEEMD; (d) The imf_3 by CEEMD.

noise perturbing, the curve is not as smooth as before. The correlation coefficient (CC) between $\tilde{R}(t)$ and $f(t)$ here is $0.693 < 1$. The closer the CC is to 1, the better the accuracy of the recovered signal should be. Thus we cannot use $\tilde{R}(t)$ as a recovery of information signal as shown in [3]. So, we apply SWT at the end of the receiving terminal, in order to eliminate the influence by noise so as to abstract the information signal accurately.

In Experiment 3, we discuss the extraction performance of SWT over EMD and CEEMD under $NL = 60\%$. Fig. 5-Fig. 6 are the extraction results by EMD and CEEMD respectively. The mixed signal $\tilde{R}(t)$ is decomposed into 3 intrinsic mode-type functions imf_1, imf_2 and imf_3. As can be seen from Fig. 5, imf_1 and imf_2 are composite signal of high-frequency noise, and the imf_3 is the approximation of the information signal. The CCs here are respectively 0.4198, 0.8038 and 0.2076, which means EMD cannot separate the three components. As can be seen from Fig. 6, although CEEMD has better denoising effect than EMD, it still cannot separate the three components. The CCs here are respectively 0.2558, 0.8231 and 0.3924. Obviously, neither EMD nor CEEMD can separate the three components of the information signal.

Fig. 7- Fig. 9 show the extraction performances of extracting the information signal $f(x)$ from the mixed signal $\tilde{R}(t)$ by our SWT based scheme mentioned in Section *IV*. Fig. 7 (b)-(d) are the comparison of the three original components $f_1(t), f_2(t), f_3(t)$ and their extraction results. The CCs here are respectively 0.9977, 0.9995, and 0.9879. That means SWT based scheme can separate the three components from the information signal. The least squares fitting curves of IA and IF extracted by SWT based scheme are shown in Fig. 8- Fig. 9. The fitting values of IA are respectively 1.0083, 2.0085, 0.4989, which is closed to the real values 1, 2, 0.5. The fitting values of IF are respectively 19.996, 24.995, 29.994, which is closed to the real values 20, 25, 30. As can be seen, the SWT based scheme can separate the three components from the information signal, and the frequency and

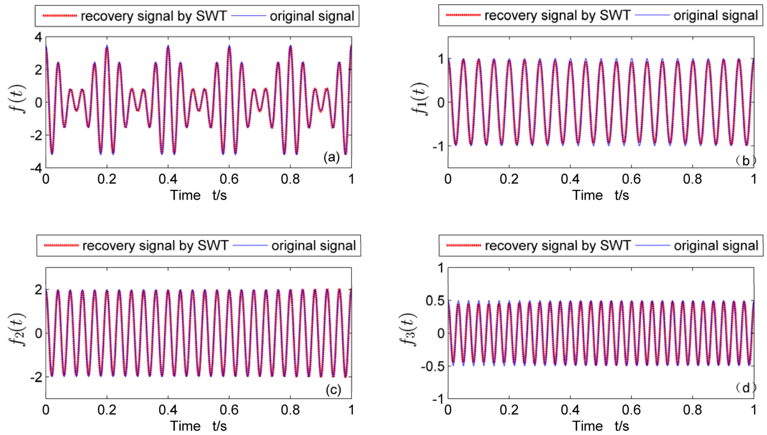

FIGURE 7. The SWT extraction results of the information signal under $NL = 60\%$: (a) The extraction results of $f(t)$; (b) The extraction results of $f_1(t)$; (c) The extraction results of $f_2(t)$; (d) The extraction results of $f_3(t)$.

TABLE 1. The detection results of amplitude for each component with different noise intensity.

Noise level/%	EMD			CEEMD			WT			SWT		
	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$
40	0.4143	2.1157	0.3278	1.1487	1.9250	0.5283	1.0835	1.7164	0.3706	1.0069	2.0021	0.4980
60	0.5908	2.1156	0.6653	0.9533	1.8179	0.6668	1.0591	1.7414	0.4092	1.0083	2.0085	0.4989
80	0.8404	2.2614	0.9594	1.3109	1.7342	0.6971	1.0906	1.7637	0.4304	1.0089	2.0097	0.4827
100	0.9316	2.3762	1.2984	1.5608	1.6874	1.0018	1.0351	1.7348	0.3893	1.0103	2.0110	0.4766
120	1.1544	2.3668	1.6168	1.7972	1.6312	1.1045	1.1185	1.7177	0.3640	1.0113	2.0215	0.4691

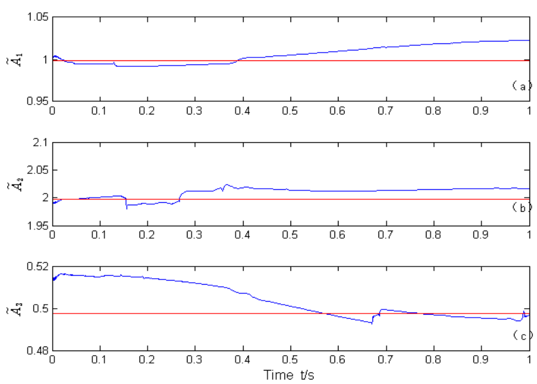

FIGURE 8. The least squares fitting curve of instantaneous amplitude extracted by SWT under $NL = 60\%$: (a) fitting curve of $\tilde{A}_1$ for $f_1(t)$; (b) fitting curve of $\tilde{A}_2$ for $f_2(t)$; (c) fitting curve of $\tilde{A}_3$ for $f_3(t)$.

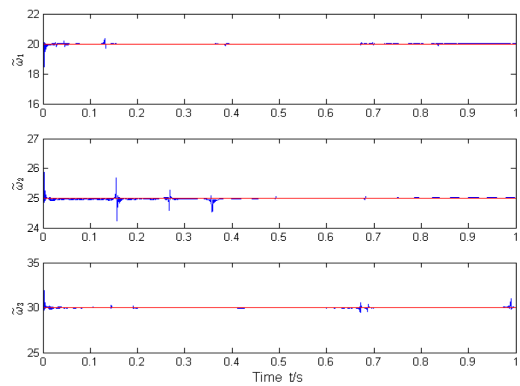

FIGURE 9. The least squares fitting curve of instantaneous frequency extracted by SWT under $NL = 60\%$: (a) fitting curve of $\tilde{\omega}_1$ for $f_1(t)$; (b) fitting curve of $\tilde{\omega}_2$ for $f_2(t)$; (c) fitting curve of $\tilde{\omega}_3$ for $f_3(t)$.

amplitude of the extracted signal are almost the same as those of the original signal. Therefore, when the noise level NL = 60%, the extraction precision of SWT based scheme is higher than that of EMD and CEEMD.

In Experiment 4, we discuss the extraction effects of SWT, EMD, CEEMD and WT under different noise intensities. The values of the noise level (NL) are respectively set to 40%-120%. Table 1 and Table 2 show the detection results of amplitude and frequency of each component respectively by using the four time-frequency denoising algorithms. When $NL = 40\%$, the detection results of amplitude of each component by EMD are respectively 0.4143, 0.3278, 2.1157, which are too far from the real values 1, 2, 0.5. This is the same case for the frequency extraction by EMD. Obviously, EMD cannot abstract amplitude and frequency of each component, not to mention the high noise level. When $NL = 40\%$, the mean square errors (MSE) of amplitude abstracted by CEEMD, WT and SWT are respectively about 0.0095, 0.0347, 0.0002. When $NL = 120\%$, the MSE of amplitude abstracted by CEEMD, WT and SWT are respectively about 0.3790, 0.0374, 0.0005. This is the same case for the frequency extraction by them. Therefore, SWT can achieve

TABLE 2. The detection results of frequency for each component with different noise intensity.

Noise level/%	EMD			CEEMD			WT			SWT		
	$\tilde{\omega}_1$	$\tilde{\omega}_2$	$\tilde{\omega}_3$	$\tilde{\omega}_1$	$\tilde{\omega}_2$	$\tilde{\omega}_3$	$\tilde{\omega}_1$	$\tilde{\omega}_2$	$\tilde{\omega}_3$	$\tilde{\omega}_1$	$\tilde{\omega}_2$	$\tilde{\omega}_3$
40	18.034	27.879	26.597	16.312	21.845	28.539	19.996	24.995	29.993	19.996	24.995	29.999
60	17.676	28.070	28.396	16.500	25.907	27.980	19.996	24.994	29.993	19.996	24.995	29.994
80	18.457	27.117	30.596	16.083	26.878	28.201	19.876	24.654	29.895	19.996	24.995	29.992
100	18.569	28.394	32.590	16.893	27.890	29.783	19.823	24.657	29.689	19.995	24.995	29.991
120	17.157	28.058	34.394	16.470	27.882	27.278	19.789	24.597	29.691	19.994	24.995	29.991

better extraction effect than the other methods, even if the noise level is higher.

Based on the results achieved from the above experiments, the secure communication scheme about secondary encryption can enhance the level of security, and the SWT based scheme has a better extraction effect than EMD, CEEMD and WT, with noise interference.

VI. CONCLUSION

In this paper, the finite-time stochastic synchronization of a chaotic system without linear term under vector-form stochastic perturbations has been solved. Most of existing work only studied the exponential or impulsive synchroniz-ation of this new chaotic system, which means that the time of recovering the information signal may be infinite. The finite-time stochastic synchronization theorem proposed in this paper can solve problems of robustness and disturbances for chaotic systems. In addition, we obtain a fixed synchronization time that does not depend on the parameters of the controller. The time for decryption is predictable. Therefore, the results are helpful to the application in secure communication. Moreover, we proposed a secure communication scheme about secondary encryption based on multistage chaos synchronized system. In this method, the information signal is decoded twice. Considering chaos signal depending on initial values, obviously, this method has greater security than original chaos masking mode. Finally, SWT is used at the receiver end in order to improve the accuracy of the recovered signal. Numerical simulations and the results have showed the effectiveness and feasibility of the proposed synchronization and secure communication scheme. For multi-component sinusoidal signal with adjacent frequency, the accuracy of signals recovered by SWT based scheme is higher than that recovered by EMD, CEEMD and WT.

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