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Stabilization of Markovian Jump Systems With Quantized Input and Generally Uncertain Transition Rates

WON IL LEE[1] AND BUM YONG PARK[2]

[1]The Division of Electrical Engineering, Kumoh National Institute of Technology, Gumi-si 39177, Republic of Korea

[2]The Division of Electrical Engineering, Department of IT Convergence Engineering, Kumoh National Institute of Technology, Gumi-si 39177, Republic of Korea

Corresponding author: Bum Yong Park (bumyong.park@kumoh.ac.kr)

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ABSTRACT This study addresses the stabilization condition for Markovian jump systems with quantized input and generally uncertain transition rates. To stabilize Markovian jump systems with nonlinear inputs using only partial knowledge regarding to the transition rates, the proposed controller is designed as two control parts: a linear control part for stabilizing the system and a nonlinear control part for eliminating undesirable effects from the quantized input. Moreover, an appropriate weighting method, using lower and upper bounds of the partial transition rate, is employed to derive a less conservative stability condition. Finally, two numerical examples are provided to illustrate the effectiveness of the proposed controller.

INDEX TERMS Linear matrix inequality, Markovian jump system, quantized input, uncertain transition rate.

I. INTRODUCTION

Recently, Markovian jump systems (MJSs) have been used to investigate random abrupt variations in dynamic systems. Until now, MJSs that are associated with analysis and synthesis [1]–[10], filter design [11]–[13], and robust controller design [14]–[18] have been investigated hitherto. These studies have contributed to the stability and stabilization of various practical systems, such as power [19], economic [20], and manufacturing systems [21]. Random variations in MJSs exist due to transition probabilities, which, in turn, are dependent on transition rates, which are vital to the stability and stabilization of MJSs. In the jumping process of MJSs, the transition rates influence the system operation significantly. Various studies pertaining to the analysis and synthesis of MJSs have assumed known transition rates [22]–[26]. However, in several practical applications, the transition rates are unknown or partially known.

Hence, some stability analysis and synthesis methods for MJSs with uncertain transition rates have been proposed. In [27], the authors introduced a state observer for discrete-time MJSs with time-varying delays and disturbances. The authors employed the variation of the transition probability, which is considered as a polytope with several vertices. In [28], the authors employed exactly known transition rates to design an L_∞ controller for continuous-time MJSs with time-varying delays and disturbances. The authors of [29], [30] introduced a stabilization method for MJSs with bounded transition rates. In their study, the exact values of the transition rates were unknown; however, the upper and lower bounds were known. Nevertheless, in practical applications, it is difficult to obtain the bounds of transition rates. In [31], the authors employed transition rates that contained both known and unknown values. Because partly unknown transition rates require some exactly known transition rates, it poses a limitation for various practical systems. Recently, generally uncertain transition rates with unknown and estimated transition rates were proposed in [32]; these proposed rates have piqued the interest of researchers [33].

Currently, in practical systems, the controller, plant, and measurement devices can be placed in different locations. These are connected over a network and called networked control systems (NCSs). NCSs have several benefits in NCSs, such as flexibility, low cost, easy maintenance, and

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simple installation [34], [35]. Although the NCSs have several advantages, some issues, such as quantization, packet dropout, and network induced-delay, occur owing to limitation in network bandwidth [35], [36]. These issues often result in degrading system performance or even unstable systems [37]. Especially, quantized input is an essential phenomenon in NCSs, because the system input need to be quantized before being transmitted from the controller.

Therefore, quantized inputs for the analysis and synthesis of control systems have been widely investigated [38]–[43], as they can degrade the performance and stability of modern control systems. Recently, some studies have addressed the stabilization problem of MJSs with quantized inputs [44]–[47]. The stabilization problem for discrete-time MJSs with a quantized feedback input was introduced in [44]. In [45], the authors introduced a stabilization condition for MJSs with quantized input and incomplete transition rates, containing exactly known and unknown transition rates. The authors of [46] developed a sliding mode controller for MJSs with a dynamical uniform quantizer and a static logarithm quantizer. Furthermore, an H_∞ controller was proposed for MJSs with quantized input, where known and unknown transition rates were addressed [47]. The above literature assumed that the transition probabilities in MJSs are exactly known or partially known, which is motivativation for this study.

To the best of our knowledge, the stabilization of MJSs with generally uncertain transition rates and quantized input has not fully been investigated. Although the stability analysis and synthesis of MJS with generally uncertain transition rates have been proposed in [32], [33], the authors have only considered an ideal control input without considering quantization. Furthermore, although incomplete transition rates have been employed previously [45], [47], the researchers assumed that some transition rates were known.

Motivated by the studies above, our aim is to develop a state-feedback controller for continuous-time MJSs with quantized input and generally uncertain transition rates. The proposed controller stabilizes MJSs with nonlinear inputs and only employs partial knowledge regarding to the transition rates. The main contributions of this paper are as follows.

- To derive a less conservative stability condition, we employ an appropriate weighting method using the lower and upper bounds of partial transition rates. The derived condition is formulated into matrix polynomials.
- The controller comprises two control parts; the first is a linear control part that stabilizes the MJSs, and the second is a nonlinear control part that eliminates the undesirable effects of the quantized input.
- Hitherto, the proposed controller for MJSs with quantized input and generally uncertain transition rates has not been researched.

Therefore, the effectiveness of the proposed controller is illustrated via numerical examples.

This paper is organized as follows. A description of the proposed system and preliminaries are provided in Section 2. The stabilization condition for MJSs with quantized input and generally uncertain transition rates is proposed in Section 3. In Section 4, two numerical examples are provided to demonstrate the effectiveness of the proposed controller. The discussion and conclusion are presented in Section 5 and 6.

Notation: The notations $X \geq Y$ and $X > Y$ indicate that $X - Y$ is positive semidefinite and positive definite, respectively. R^n stands for an n-dimensional Euclidean space. We also use $\|x\|_p$ to indicate the p-norm of x, *i.e.*, $\|x\|_p \triangleq (|x_1|^p + \cdots + |x_n|^p)^{\frac{1}{p}}$, $p \geq 1$. $sgn(\cdot)$ indicates a sign function.

II. SYSTEM DESCRIPTION

First, we consider the following continuous-time MJS with quantized-input.

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)Q(u(t)), \tag{1}$$

where $x(t) \in R^n$ is the system state and $u(t) \in R^m$ is the control input. $Q(\cdot)$ is an uniform quantizer, which is defined by a function $round(\cdot)$ that rounds towards the nearest integer, i.e.,

$$Q(\cdot) \triangleq \epsilon_q round(\cdot/\epsilon_q), \tag{2}$$

where ϵ_q is a positive integer known as the quantizing level. The switching of the system, which is expressed in (1), is governed by the Markov jumping process, $\{r_t, t \geq 0\}$, in a finite integer set, $D = \{1,\ 2,\ \cdots,\ N\}$. The mode transition probability is expressed as follows.

$$P(r_{t+\delta t} = j | r_t = i) = \begin{cases} \pi_{ij}\delta t + o(\delta t) & \text{if } i \neq j \\ 1 + \pi_{ij}\delta t + o(\delta t) & \text{if } i = j \end{cases},$$

where $\delta t > 0$, $\lim_{\delta t \to 0} o(\delta t)/\delta t = 0$, and π_{ij} is the transition rate from mode i to j at time $t + \delta t$. Furthermore, the transition rate matrix Π is in the following set:

$$S_\Pi \triangleq \left\{ [\pi_{ij}]_{i,j \in D} | \pi_{ij} \geq 0 \text{ for } i \neq j, \pi_{ii} = -\sum_{j=1, i \neq j}^{N} \pi_{ij} \right\}. \tag{3}$$

For generally uncertain transition rates that have unknown transition rates or the bounds of the transition rates, the transition rate matrix Π is represented as:

$$\Pi = \begin{bmatrix} [\underline{\pi}_{11}, \bar{\pi}_{11}] & ? & \cdots & [\underline{\pi}_{1N}, \bar{\pi}_{1N}] \\ ? & [\underline{\pi}_{22}, \bar{\pi}_{22}] & \cdots & ? \\ \vdots & \vdots & \ddots & \vdots \\ [\underline{\pi}_{N1}, \bar{\pi}_{N1}] & ? & \cdots & [\underline{\pi}_{NN}, \bar{\pi}_{NN}] \end{bmatrix}, \tag{4}$$

where $\underline{\pi}_{ij}$ and $\bar{\pi}_{ij}$ are the lower and the upper bounds of π_{ij}, respectively, and ? denotes the unknown transition rate. Here, we assume that $\underline{\pi}_{ij}$ and $\bar{\pi}_{ij}$ are known values. By using the definition above of the generally uncertain transition rate, the transition rate is represented as

$$\pi_{ij} = \tilde{\pi}_{ij} + \nabla\pi_{ij}, \tag{5}$$

where $\tilde{\pi}_{ij} = \frac{\underline{\pi}_{ij} + \bar{\pi}_{ij}}{2}$, $\nabla\pi_{ij} \in [-\Delta_{ij},\ \Delta_{ij}]$, and $\Delta_{ij} = \frac{\bar{\pi}_{ij} - \underline{\pi}_{ij}}{2}$.

For convenience, two sets are defined with respect to the measurability of the transition rate for $i, j \in D$, as follows:

$$D_k^i = \{j \mid \underline{\pi}_{ij}, \bar{\pi}_{ij} \text{ are known for } i\},$$
$$D_{uk}^i = \{j \mid \underline{\pi}_{ij}, \bar{\pi}_{ij} \text{ are unknown for } i\}.$$

Furthermore, if $D_k^i \neq \emptyset$, D_k^i is defined as

$$D_k^i = \{\zeta_i^r \mid 1 \leq r \leq q\}, \tag{6}$$

where q is a positive integer with $1 \leq q \leq N$.

Using the quantization shown in (2), the quantization error $\nabla u(t)$ is defined as

$$\nabla u(t) \triangleq Q(u(t)) - u(t). \tag{7}$$

Based on the definition that is shown in (2) and (7), each component of $\nabla u(t)$ is bounded by one-half of ϵ_q as follows:

$$\| \nabla u(t) \|_\infty \leq \epsilon_q/2. \tag{8}$$

The following lemma is required to prove the main result.

Lemma 1 [48]: The continuous-time MJS with zero input is regarded as mean square stable if symmetric matrices $P_i > 0$ exist for all $i \in D$ such that $A_i^T P_i + P_i A_i + \sum_{j=1}^N \pi_{ij} P_j < 0$.

III. MAIN RESULT

In this section, we discuss the stabilization conditions for the MJS that is expressed in (1) with quantized input and generally uncertain transition rates. Let us propose a controller for the systems shown in (1) as follows:

$$u(t) = K(r_t)x(t) + \bar{u}(r_t, x(t)), \tag{9}$$

where $K(r_t)$ is a linear part to stabilize the proposed system and $\bar{u}(r_t, x(t))$ is a nonlinear part to eliminate the effect of the quantization error. Using the system in (1), the quantization error in (7), and the proposed controller in (9), the closed-loop system is represented as follows, for $r_t = i \in D$.

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i K_i x(t) + B_i\{\nabla u(t) + \bar{u}_i(x(t))\} \\ &= \bar{A}_i x(t) + B_i\{\nabla u(t) + \bar{u}_i(x(t))\}, \end{aligned} \tag{10}$$

where $\bar{A}_i = A_i + B_i K_i$.

Theorem 1: Consider the system expressed in (10) with generally uncertain transition rates. For $i, j \in D$, if positive definite matrices $\bar{P}_i$, $\bar{S}_i$, and $\bar{U}_{ij}$, a symmetric matrix Λ_{ij}, and a matrix $\bar{K}_i$ exist such that

$$\bar{P}_i > 0, \quad \bar{S}_i > 0, \quad i \in D \tag{11}$$
$$\bar{U}_{ij} > 0, \quad i \in D_k^i \tag{12}$$
$$\Lambda_{ij} + \Lambda_{ij}^T > 0, \quad j \in D_k^i \tag{13}$$
$$\bar{P}_j - \bar{S}_i > 0, \quad j \in D_{uk}^i, \ i = j \tag{14}$$
$$\begin{bmatrix} -\bar{S}_i & \bar{P}_i \\ \bar{P}_i & -\bar{P}_j \end{bmatrix} < 0, \quad j \in D_{uk}^i, \ i \neq j \tag{15}$$
$$\begin{bmatrix} -\bar{S}_i - \bar{U}_{ij} & \bar{P}_i \\ \bar{P}_i & -\bar{P}_j \end{bmatrix} < 0, \quad j \in D_k^i \tag{16}$$

$$\begin{bmatrix} \bar{N}_i & N_i^1 & \cdots & N_i^q \\ (*) & -\bar{P}_{\zeta_i^1} & & \\ \vdots & & \ddots & \\ (*) & & & -\bar{P}_{\zeta_i^q} \end{bmatrix} < 0, \tag{17}$$

where

- $i \notin D_k^i$
$$N_i^r = \sqrt{\tilde{\pi}_{i\zeta_i^r} - \Delta_{i\zeta_i^r}}\bar{P}_i,$$
$$\bar{N}_i = M_i - \sum_{j \in D_k^i} (\tilde{\pi}_{ij} - \Delta_{ij})\bar{S}_i,$$
- $i \in D_k^i, i \neq \zeta_i^r$
$$\begin{aligned} N_i^r &= \sqrt{\tilde{\pi}_{i\zeta_i^r} - \Delta_{i\zeta_i^r}}\bar{P}_i, \\ \bar{N}_i &= M_i - \sum_{j \in D_k^i, j \neq i} (\tilde{\pi}_{ij} - \Delta_{ij})\bar{S}_i \\ &\quad + (\tilde{\pi}_{ii} - \Delta_{ii})(\bar{P}_i - \bar{S}_i), \\ M_i &= A_i\bar{P}_i + B_i\bar{K}_i + (A_i\bar{P}_i + B_i\bar{K}_i)^T \\ &\quad + \sum_{j \in D_k^i} 2\Delta_{ij}\bar{U}_{ij} + \sum_{j \in D_k^i} (\bar{\pi}_{ij} - \underline{\pi}_{ij})(\Lambda_{ij} + \Lambda_{ij}^T), \end{aligned}$$

then the closed-loop system that is expressed in (10) is mean square stable. Furthermore, the proposed controller is designed as follows, for $i \in D$.

$$u(t) = K_i x(t) + \bar{u}_i x(t), \tag{18}$$

where $K_i = \bar{K}_i \bar{P}_i^{-1}$ and each component of $\bar{u}_i(x(t))$ is defined as, for $k \in [1, m]$,

$$\bar{u}_{i,k}(x(t)) = -\epsilon_q sgn(\sigma_{i,k}(x(t))), \tag{19}$$

where $\sigma_i(x(t)) \triangleq B_i^T P_i x(t)$ and $P_i = \bar{P}_i^{-1}$.

Proof: Let us select a Lyapunov function $V(x(t)) = x^T(t)P(r_t)x(t) > 0$, where $P(r_t)$ is a positive definite matrix. Using the weak infinitesimal operator ∇ of the Markov process, $\nabla V(x(t))$ is expressed as

$$\begin{aligned} \nabla V(x(t)) &= 2x^T(t)P_i\dot{x}(t) + x^T(t)\sum_{j \in D} \pi_{ij}P_j x(t) \\ &= x^T(t)\left\{P_i\bar{A}_i + \bar{A}_i^T P_i + \sum_{j \in D} \pi_{ij}P_j\right\}x(t) \\ &\quad + 2\sigma_i^T(x(t))\{\nabla u_i(t) + \bar{u}_i(x(t))\}. \end{aligned}$$

Using the nonlinear control part in (19) and condition (8), the second term of $\nabla V(x(t))$ can be derived as follows:

$$\begin{aligned} &2\sigma_i^T(x(t))\{\nabla u_i(t) + \bar{u}_i(x(t))\} \\ &= 2\sum_{k=1}^m \sigma_{i,k}(x(t))\{\nabla u_{i,k}(t) + \bar{u}_{i,k}(x(t))\} \\ &= 2\sum_{k=1}^m \left\{\sigma_{i,k}(x(t))\nabla u_{i,k}(t) - \epsilon_q|\sigma_{i,k}(x(t))|\right\} \\ &\leq 2\sum_{k=1}^m (|\sigma_{i,k}(x(t))||\nabla u_{i,k}(t)| - \epsilon_q|\sigma_{i,k}(x(t))|) \\ &\leq -\sum_{k=1}^m (\epsilon_q|\sigma_{i,k}(x(t))|) \leq 0. \end{aligned}$$

Then, the upper bound of $\nabla V(x(t))$ is expressed as follows:

$$\nabla V(x(t)) \leq x^T(t) \left\{ P_i\bar{A}_i + \bar{A}_i^T P_i + \sum_{j\in D} \pi_{ij} P_j \right\} x(t). \quad (20)$$

To guarantee the mean square stability of the closed-loop system expressed in (10) from Lemma 1, $\nabla V(x(t)) < 0$ must be guaranteed, thereby resulting in the following condition:

$$P_i\bar{A}_i + \bar{A}_i^T P_i + \sum_{j\in D} \pi_{ij} P_j < 0. \quad (21)$$

From the condition expressed in (3), we can adopt $\sum_{j\in D} \pi_{ij} S_i = 0$ for any $S_i > 0$. The condition above can be converted into the following condition:

$$P_i\bar{A}_i + \bar{A}_i^T P_i + \sum_{j\in D} \pi_{ij} P_j - \sum_{j\in D} \pi_{ij} S_i$$
$$= P_i\bar{A}_i + \bar{A}_i^T P_i + \sum_{j\in D_k^i} \pi_{ij} T_{ij} + \sum_{j\in D_{uk}^i} \pi_{ij} T_{ij} < 0, \quad (22)$$

where $T_{ij} = P_j - S_i$.

If the following two conditions are satisfied for $j \in D_{uk}^i$, i.e.,

$$T_{ij} > 0 \quad \text{if } i = j, \quad (23)$$
$$T_{ij} < 0 \quad \text{if } i \neq j, \quad (24)$$

then based on (3), (22) can be represented as

$$P_i\bar{A}_i + \bar{A}_i^T P_i + \sum_{j\in D_k^i} \pi_{ij} T_{ij} + \sum_{j\in D_{uk}^i} \pi_{ij} T_{ij}$$
$$\leq P_i\bar{A}_i + \bar{A}_i^T P_i + \sum_{j\in D_k^i} \pi_{ij} T_{ij} < 0. \quad (25)$$

Based on (4), substituting (5) into the condition above yields

$$P_i\bar{A}_i + \bar{A}_i^T P_i + \sum_{j\in D_k^i} (\tilde{\pi}_{ij} + \nabla\pi_{ij}) T_{ij}$$
$$= P_i\bar{A}_i + \bar{A}_i^T P_i + \sum_{j\in D_k^i} \tilde{\pi}_{ij} T_{ij} - \sum_{j\in D_k^i} \Delta_{ij} T_{ij}$$
$$+ \sum_{j\in D_k^i} (\nabla\pi_{ij} + \Delta_{ij}) T_{ij} < 0. \quad (26)$$

For $j \in D_k^i$,

$$T_{ij} - U_{ij} < 0, \quad (27)$$

Subsequently,

$$\sum_{j\in D_k^i} (\nabla\pi_{ij} + \Delta_{ij}) T_{ij} \leq \sum_{j\in D_k^i} (\nabla\pi_{ij} + \Delta_{ij}) U_{ij} \leq \sum_{j\in D_k^i} 2\Delta_{ij} U_{ij},$$

where $U_{ij} > 0$.

Then, the following sufficient condition for (26) is as follows:

$$P_i\bar{A}_i + \bar{A}_i^T P_i + \sum_{j\in D_k^i} \tilde{\pi}_{ij} T_{ij} + \sum_{j\in D_k^i} 2\Delta_{ij} U_{ij} - \sum_{j\in D_k^i} \Delta_{ij} T_{ij} < 0.$$

Multiplying both sides of the condition above with P^{-1} yields

$$\bar{A}_i\bar{P}_i + \bar{P}_i\bar{A}_i^T + \sum_{j\in D_k^i} \tilde{\pi}_{ij} \bar{P}_i T_{ij} \bar{P}_i + \sum_{j\in D_k^i} 2\Delta_{ij} \bar{P}_i U_{ij} \bar{P}_i$$
$$- \sum_{j\in D_k^i} \Delta_{ij} \bar{P}_i T_{ij} \bar{P}_i < 0, \quad (28)$$

where $\bar{P}_i \triangleq P_i^{-1}$.

Moreover, for the appropriate weighting method, the following equation can be derived using (4):

$$\sum_{j\in D_k^i} (\bar{\pi}_{ij} - \underline{\pi}_{ij})(\Lambda_{ij} + \Lambda_{ij}^T) \geq 0, \quad (29)$$

where $\Lambda_{ij} + \Lambda_{ij}^T > 0$. Here, matrix Λ_{ij} is a slack variable to relax the condition shown in (28), resulting in a reduced the conservatism of the condition that is shown in (28).

By the S-procedure, if condition (28) is satisfied whenever condition (29) is ensured, then the following sufficient condition holds:

$$\bar{A}_i\bar{P}_i + \bar{P}_i\bar{A}_i^T + \sum_{j\in D_k^i} (\tilde{\pi}_{ij} - \Delta_{ij})(\bar{P}_i P_j \bar{P}_i - \bar{S}_i)$$
$$+ \sum_{j\in D_k^i} 2\Delta_{ij} \bar{U}_{ij} + \sum_{j\in D_k^i} (\bar{\pi}_{ij} - \underline{\pi}_{ij})(\Lambda_{ij} + \Lambda_{ij}^T) < 0, \quad (30)$$

where $\bar{S}_i = \bar{P}_i S_i \bar{P}_i$ and $\bar{U}_{ij} = \bar{P}_i U_{ij} \bar{P}_i$.

Then, (30) can be represented as follows.

- $i \notin D_k^i$

$$M_i + \sum_{j\in D_k^i} (\tilde{\pi}_{ij} - \Delta_{ij})(\bar{P}_i P_j \bar{P}_i - \bar{S}_i) < 0. \quad (31)$$

- $i \in D_k^i, i \neq \zeta_i^r$

$$M_i + (\tilde{\pi}_{ii} - \Delta_{ii})(\bar{P}_i - \bar{S}_i)$$
$$+ \sum_{j\in D_k^i, j\neq i} (\tilde{\pi}_{ij} - \Delta_{ij})(\bar{P}_i P_j \bar{P}_i - \bar{S}_i) < 0. \quad (32)$$

The two cases above can be converted into the following condition.

- $i \notin D_k^i$

$$M_i - \sum_{j\in D_k^i} (\tilde{\pi}_{ij} - \Delta_{ij})\bar{S}_i - \mathcal{N}_i^T \bar{\mathcal{P}} \mathcal{N}_i < 0, \quad (33)$$

- $i \in D_k^i, i \neq \zeta_i^r$

$$M_i + (\tilde{\pi}_{ii} - \Delta_{ii})(\bar{P}_i - \bar{S}_i) - \sum_{j\in D_k^i, j\neq i} (\tilde{\pi}_{ij} - \Delta_{ij})\bar{S}_i$$
$$- \mathcal{N}_i^T \bar{\mathcal{P}} \mathcal{N}_i < 0, \quad (34)$$

where

$$\mathcal{N}_i = [N_i^1 \; N_i^2 \; \cdots \; N_i^q]^T, \quad \bar{\mathcal{P}} = diag\{-\bar{P}_{\zeta_i^1}, \cdots, -\bar{P}_{\zeta_i^q}\} < 0.$$

If the linear matrix inequality (LMI) condition, which is shown in (17), is satisfied, then the conditions (33) and (34) are guaranteed by the Schur complement. Furthermore, condition (30) holds because of the LMI conditions (11)-(17). Moreover, the LMI condition in (14) can be obtained by multiplying both sides of (23) with $\bar{P}_i$. The LMI conditions in (15) and (16) are induced by multiplying both sides of (24) and (27), respectively, with $\bar{P}_i$ and using the Schur complement. Therefore, the existence of the proposed controller (18) is guaranteed by LMI conditions (11)-(17). □

Remark 1: Based on the condition (29), *we employ information about the MJSs with generally uncertain transition rates and more slack variables than those in the previous studies [32], [33]. This therefore increases the freedom that is required to obtain the solution of the LMI conditions in Theorem 1, making the proposed method less conservative.*

Remark 2: All solutions for the proposed method are obtained using an off-line computational process and the method is implemented via the following steps:

- *Step 1: Obtain the system matrices A_i and B_i, the quantizing level ϵ_q, and observe the transition rates if they are known.*
- *Step 2: Use Theorem 1 to obtain the controller gains K_i and P_i by solving the LMI conditions.*
- *Step 3: Design the proposed controller* (18) *using* (19).
- *Step 4: Use the controller developed in Step* 3 *as the input of the MJSs with generally uncertain transition rates and quantized input.*

Remark 3: In this study, we consider the stabilization problem of the MJSs with generally uncertain transition rates and quantized input, which is a phenomenon that is caused by transmitting control signals through a network in NCSs. To obtain the stabilization condition of the proposed system, $\nabla V(x(t))$ is derived using the weak infinitesimal operator ∇ of the Markov process, which has a nonlinear term due to the quantization error. This makes deriving the stabilization condition difficult. For rejecting the nonlinear term, we introduce the nonlinear control part in (19), *which eliminates the energy in terms of the Lyapunov function that is caused by a quantization error.*

IV. NUMERICAL EXAMPLES

In this section, we illustrate the effectiveness of the proposed controller using two numerical examples.

A. EXAMPLE 1

Let us consider the following MJS with three modes ($N = 3$) [33]. For the initial condition $x(0) = [0.6 \;\; 0.4]^T$,

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}, \; A_3 = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \; B_3 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix},$$
$$\Pi = \begin{bmatrix} [-2, -3] & ? & [1, 1.5] \\ ? & ? & ? \\ [2, 2.5] & [0, 1] & ? \end{bmatrix}, \quad \epsilon_q = 0.1,$$

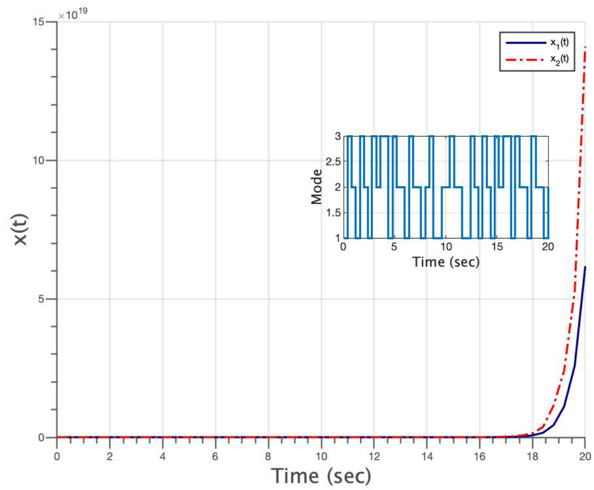

FIGURE 1. State trajectories of open-loop system for Example 1.

where the transition rate matrix Π contains unknown transition rates and their bounds, as shown below.

$$\underline{\pi}_{11} = -2, \quad \bar{\pi}_{11} = -3,$$
$$\underline{\pi}_{13} = 1, \quad \bar{\pi}_{13} = 1.5,$$
$$\underline{\pi}_{31} = 2, \quad \bar{\pi}_{31} = 2.5,$$
$$\underline{\pi}_{13} = 0, \quad \bar{\pi}_{13} = 1.$$

Based on transition rate matrix Π above, the following sets can be obtained:

$$D_k^1 = \{1, 3\}, \quad D_k^2 = \emptyset, \; D_k^3 = \{1, 2\},$$
$$D_{uk}^1 = \{2\}, \quad D_{uk}^2 = \{1, 2, 3\}, \; D_{uk}^3 = \{3\}.$$

The open-loop system for this system with $u(t) \equiv 0$ is not stochastically stable, as shown in Fig 1.

From Theorem 1, we can obtain the proposed controller as expressed in (9), comprising a linear part and a nonlinear part. The control gains of the linear part are as follows.

$$K_1 = \begin{bmatrix} -2.1083 & -2.8696 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -1.7093 & -1.1489 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -2.9196 & -3.8731 \end{bmatrix}.$$

The nonlinear part shown in (19) is constructed using the following gains.

$$P_1 = \begin{bmatrix} 5.0172 \times 10^{-3} & 1.5257 \times 10^{-3} \\ 1.5257 \times 10^{-3} & 5.7631 \times 10^{-3} \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 7.9572 \times 10^{-3} & 2.8029 \times 10^{-3} \\ 2.8029 \times 10^{-3} & 8.1733 \times 10^{-3} \end{bmatrix},$$
$$P_3 = \begin{bmatrix} 6.8209 \times 10^{-3} & 2.1261 \times 10^{-3} \\ 2.1261 \times 10^{-3} & 5.9316 \times 10^{-3} \end{bmatrix}.$$

Fig. 2 shows the state trajectories and mode transition using the proposed controller. Fig. 3 shows the quantized input. It was demonstrated that the proposed controller stabilized the MJS with quantized input and generally uncertain transition rates.

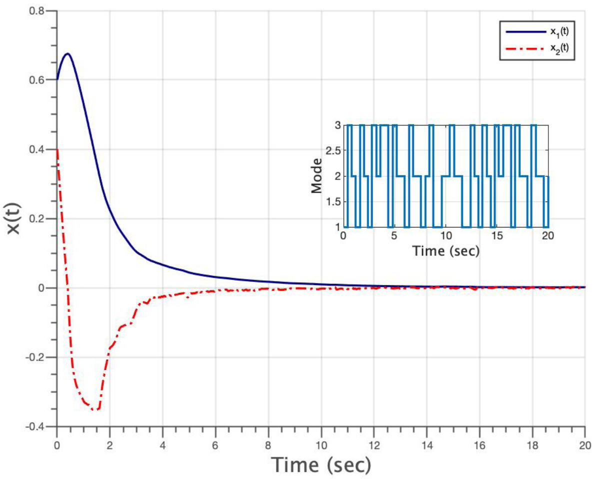

FIGURE 2. State trajectories for Example 1.

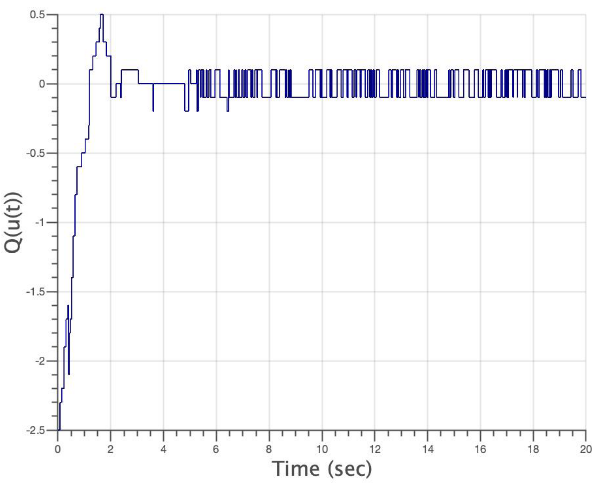

FIGURE 3. Quantized input for Example 1.

B. EXAMPLE 2

Let us consider the following MJS with two modes ($N = 2$). For the initial condition $x(0) = [0.8\ \ 0.6\ \ 0.3]^T$,

$$A_1 = \begin{bmatrix} -2.85 & 0 & 0 \\ 2.85 & -3.6 & 0 \\ 0 & 0 & -1.36 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 2.85 & 0 & 0 \\ 0 & -3.6 & 0 \\ 2.85 & 0 & -1.36 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} [-2, -0.5] & ? \\ ? & ? \end{bmatrix}, \quad \epsilon_q = 0.05,$$

where the transition rate matrix Π contains unknown transition rates and their bounds, as shown below.

$$\underline{\pi}_{11} = -2, \quad \bar{\pi}_{11} = -0.5.$$

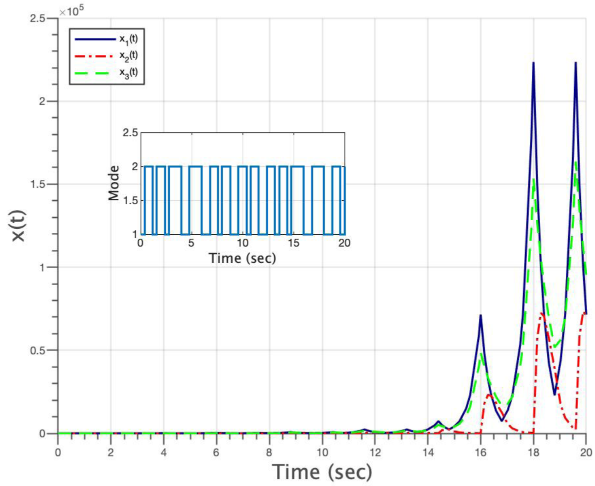

FIGURE 4. State trajectories of open-loop system for Example 2.

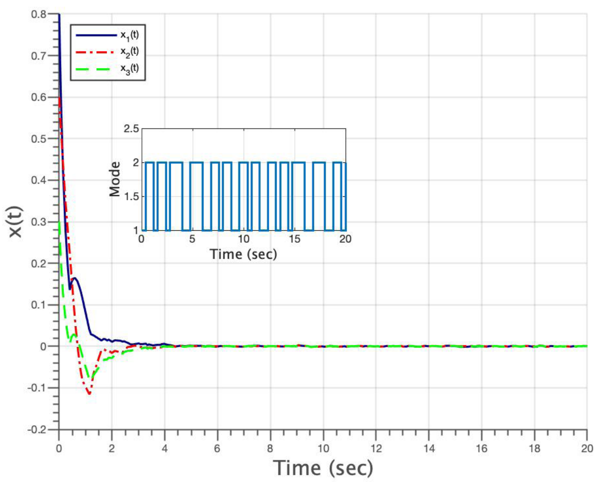

FIGURE 5. State trajectories for Example 2.

Based on the transition rate matrix Π above, the following sets can be obtained:

$$D_k^1 = \{1\}, \quad D_k^2 = \emptyset,$$
$$D_{uk}^1 = \{2\}, \quad D_{uk}^2 = \{1, 2\}.$$

The open-loop system for this system with $u(t) \equiv 0$ is not stochastically stable, as shown in Fig 4.

From Theorem 1, we can obtain the proposed controller as expressed in (9), comprising a linear part and a nonlinear part. The control gains of linear part are as follows.

$$K_1 = \begin{bmatrix} -4.5642 \times 10^{-1} & -4.2015 \times 10^{-1} & -1.3975 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 8.2736 \times 10^{-3} & 8.4794 \times 10^{-1} & -5.4234 \times 10^{-1} \end{bmatrix}.$$

The nonlinear part shown in (19) is constructed using the following gains.

$$P_1 = \begin{bmatrix} 1.8068 \times 10^{-2} & -2.2793 \times 10^{-3} & -1.3027 \times 10^{-3} \\ -2.2793 \times 10^{-3} & 1.6384 \times 10^{-2} & 2.3703 \times 10^{-3} \\ -1.3027 \times 10^{-3} & 2.3703 \times 10^{-3} & 1.8883 \times 10^{-2} \end{bmatrix},$$

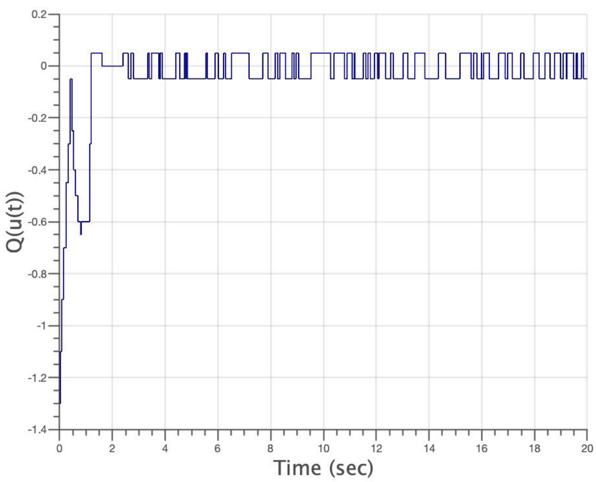

FIGURE 6. Quantized input for Example 2.

$$P_2 = \begin{bmatrix} 2.4901\times10^{-2} & -4.1032\times10^{-3} & -3.3977\times10^{-4} \\ -4.1032\times10^{-3} & 2.3480\times10^{-2} & 3.9442\times10^{-3} \\ -3.3977\times10^{-4} & 3.9442\times10^{-3} & 2.3540\times10^{-2} \end{bmatrix}.$$

By using the aforementioned control gains, the state trajectories and mode evolution were obtained, as shown in Fig. 5. Fig. 6 shows the quantized input, where the proposed controller stabilized the MJS with quantized input and generally uncertain transition rates.

Remark 4: In Examples 1 and 2, the controller gains, K_i and P_i, are obtained by solving the LMI conditions in Theorem 1. To solve the conditions, we used the Robust Control Toolbox in Matlab R2020b. Furthermore, the state and input trajectories are simulated using Simulink in Matlab R2020b. The proposed controller, which is composed of the parameters, stabilizes MJSs with generally uncertain transition rates and quantized input.

V. DISCUSSION

The proposed method has some feasible solutions, because the stabilization condition in Theorem 1 is derived as LMIs. The proposed condition, then, is the sufficient condition of inequality (21) that guarantees the mean square stability for the MJSs. Therefore, although it may not consider all feasible solutions for the proposed MJSs (10), the obtained solution from the proposed method guarantees sufficient stabilization of the MJS with partially or unknown transition rates and quantized input. Furthermore, the proposed method can be extended to an H_2 or H_∞ control of the proposed MJSs with external disturbances, which we will consider in a future study.

VI. CONCLUSION

In this study, we proposed a stabilization condition for MJSs with quantized-input and generally uncertain transition rates. To derive a less conservative stabilization, an appropriate weighting method was employed. The proposed controller comprised two control parts: a linear control part that stabilizes the MJSs and a nonlinear control part that eliminates the undesirable effects of the quantized input. The simulation results validated the effectiveness of the proposed controller.

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WON IL LEE received the B.S. degree from the School of Electrical Engineering and Computer Science, Kyungpook National University, Daegu, South Korea, in 2010, and the Ph.D. degree from the Department of Electrical Engineering, Pohang University of Science and Technology, Pohang, South Korea, in 2017. From 2017 to 2020, he was a Senior Engineer with Samsung Display Company Ltd., Yongin, South Korea. Since 2020, he has been with the School of Electronic Engineering, Kumoh National Institute of Technology, Gumi, South Korea, where he is currently an Assistant Professor. His current research interests include delayed systems, artificial intelligence, and robust control theory.

BUM YONG PARK received the M.S. and Ph.D. degrees in electrical and electronic engineering from the Pohang University of Science and Technology (POSTECH), Pohang, Republic of Korea, in 2011 and 2015, respectively. He has joined the Kumoh National Institute of Technology (KIT), Gumi, Republic of Korea, in 2017, where he is currently an Assistant Professor with the School of Electronic Engineering. His research interests include robust control, signal processing for embedded control systems, and robot manipulator systems.