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Stability of Stochastic Delay Differential Systems With Variable Impulses Due to Logic Choice

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ABSTRACT This paper is concerned with stability problems of stochastic delay differential systems with variable impulses due to logic choice. Firstly, a class of variable impulses due to logic choice is introduced in this paper which is more general than the logic impulses established by (Suo and Sun, 2015) and (Zhang *et al.*, 2018). Then, by establishing a connection between the stochastic delay differential system with logic impulses and a corresponding stochastic delay differential system without logic impulses, some sufficient conditions for stability of the systems are obtained. Finally, the application in a class of linear stochastic delay differential systems with logic impulses is discussed, and several stability criteria together with two numerical examples are given.

INDEX TERMS Stability, stochastic, delay, logic, impulses.

I. INTRODUCTION

It's well known that, impulses described the abrupt changes at certain instants well and a large number of results related to impulsive system have been published both in theoretical research and practical applications in recent years, see [1]–[17] and references therein. In the realistic dynamical systems, the impulsive effects may be influenced by some logic effects. For example, dynamic walking of biped robots is well researched by impulsive system (see [18], [19]), some stable characteristics of biped robots can be achieved by imposing some logic choice on the impulses. Therefore, an interesting recent development of the impulsive system is the logic impulses (see [20]–[22]), that is the impulsive effects are suffered by logic choice which was firstly established in [20].

On the other hand, stochastic effects are commonly encountered in realistic systems and often result in instability, oscillations, divergence and chaos. Many significant results for control problem of stochastic effects have been made recently. For example, adaptive fuzzy control for stochastic high-order nonlinear systems was considered in [23], [24], variable impulsive control for stochastic perturbed multi-agent systems was researched in [25], the control design was proposed from different aspects in [26]–[28]. Stability problems of stochastic systems have also attracted the attention of many researchers. Various stability analysis were made for stochastic delay differential systems (see [29]–[32]), impulsive stochastic delay differential systems (see [33]–[39]), and impulsive stochastic delay differential systems with markovian switching (see [40], [41]), respectively. However, as far as the author knows, there has been no result of stochastic delay system with logic impulses (SDLI), and a SDLI is usually highly complicated to qualitatively analyze due to stochastic effects, delay, logic and impulses exist at the same time. Hence it is challenging to analyze the properties of SDLI.

In this paper, the author aims to construct a class of linear SDLI and study its stability problems. Stability is one of the fundamental concepts which need to be further investigated. However, the traditional methods, such as Itô's formula, to study the stability of stochastic delay differential systems without impulses (SD) cannot be effectively used in SDLI, since it is difficult to integrate the equation on the intervals which contain impulses.

The main results of this paper can be concluded as follows: (i) the author constructs a class of linear stochastic delay differential system with variable impulses due to logic choice, in which the logic impulses generalize the logic impulses established in [20] and [21]; (ii) its stabilities are equivalent respectively to the stabilities of a corresponding SD, if some desired conditions are satisfied. It's worth noting that,

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by establishing the equivalent relation between the solutions of SDLI and SD, it is able to overcome the *Itô*'s formula application difficulty mentioned above that the impulses lie in the integrating intervals; (iii) the application of theorem results in a class of linear stochastic delay differential systems with logic impulses is discussed. Several stability criteria together with two numerical examples are also given.

This paper is organized as follows: in section 2, the author briefly recalls some basic notations and construct a class of linear stochastic delay differential system with variable impulses due to logic choice; in section 3, several sufficient conditions ensuring various stabilities of the SDLI and SD are provided; in section 4, the application of theorem results in a class of linear stochastic delay differential systems with logic impulses is discussed and several stability criteria together with two numerical examples are also given; in section 5, concluding remarks are given.

II. PRELIMINARIES

Let $\{\Omega, F, \{F_t\}_{t\geq 0}, P\}$ be a complete probability space with a filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. right continuous and F_0 containing all p-null sets). Let $PC([-\tau,0],R)$ denote the family of functions which are real-valued absolutely continuous on $[-\tau,0]$, with the norm $\|\phi\| = \sup_{-\tau\leq s\leq 0}|\phi(s)|$, $PC^b_{F_0}([-\tau,0],R) = \{\phi|\phi \in PC([-\tau,0],R)$ and bounded F_0-measurable, satisfying $\sup_{-\tau\leq s\leq 0} E\|\phi\|^p < \infty\}$, $PC^b_{F_0}(\delta) = \{\phi|\phi \in PC^b_{F_0}([-\tau,0],R)$, and $\sup_{-\tau\leq s\leq 0} E\|\phi\|^p \leq \delta\}$, where E denote the expectation of stochastic process. Let $w(t)$ be a one-dimensional Brownian motion defined on the probability space. Let $\Delta_2 = \{\delta_2^i | i = 1,2\}$, where δ_2^i is the ith column of the identity matrix I_2. Moreover, we identify logical values with equivalent vectors as: $T = 1 \sim \delta_2^1, F = 0 \sim \delta_2^2$.

Consider stochastic delay differential systems with variable impulses due to logic choice as follows:

$$\begin{cases} dy(t) = \sum_{i=1}^{n} p_i(t)y(t-\tau_i(t))dt \\ + \sum_{i=1}^{n} q_i(t)y(t-\tau_i(t))dw(t), \qquad t \neq t_k \\ y(t_k^+) - y(t_k) = \Phi_k(y(t_k)), \qquad t = t_k, k \in N \end{cases} \tag{1}$$

with the initial condition

$$y(t) = \varphi(t), \quad t \in [-\tau, 0] \tag{2}$$

where $0 = t_0 < t_1 < t_2 < \ldots < t_k < \ldots$ are fixed impulsive points with $\lim_{k\to\infty} t_k = \infty$. For $i = 1,2\cdots,n$, $p_i(t)$, $q_i(t) \in C([0,+\infty),R)$, $\tau_i(t) \in C([0,+\infty),[0,\tau_i])$, $\tau_i = \sup_{t\geq 0}\tau_i(t)$, $\tau = \max_{1\leq i\leq n}\tau_i$, $\varphi(t) \in PC^b_{F_0}([-\tau,0],R)$, and $y_i(t_k^+) - y_i(t_k) = \Phi_k(y(t_k))$ are variable impulses due to logic choice in the following form:

$$\Phi_k(y(t_k)) = [I_k(y(t_k)), \tilde{I}_k(y(t_k))]g_1(y(t_k)) + [J_k(y(t_k)), \tilde{J}_k(y(t_k))]g_2(y(t_k))$$

where $I_k, \tilde{I}_k, J_k, \tilde{J}_k \in C(R,R)$ satisfies $I_k(0) = \tilde{I}_k(0) = 0$, $J_k(0) = \tilde{J}_k(0) = 0$. $g_i : R \to \{\delta_2^1, \delta_2^2\}$ is a piecewise logical function as follows:

$$g_i(u) = \begin{cases} \delta_2^2, & |f_i(u)| \geq c_i \\ \delta_2^1, & |f_i(u)| < c_i \end{cases}$$

here, $f_i \in C(R,R)$, $c_i > 0$ is the threshold, for $i = 1,2$.

Obviously, the impulsive effect Φ_k is chosen from the functions $I_k + J_k$, $I_k + \tilde{J}_k$, $\tilde{I}_k + J_k$ and $\tilde{I}_k + \tilde{J}_k$. Furthermore, it can be described as:

$$\Phi_k = [I_k, \tilde{I}_k, J_k, \tilde{J}_k][g_1^T, g_2^T]^T$$

Remark 2.1: The logic impulses introduced in this paper is more general than the logic impulses established in some previous articles. (i) When $f_i(u) = u$, the logical function $g_i(u)$ acts as follows

$$g_i(u) = \begin{cases} \delta_2^2, & |u| \geq c_i \\ \delta_2^1, & |u| < c_i \end{cases}$$

which is established in [20].

(ii) When $f_i(u) = u - \frac{a_i+b_i}{2}$, $c_i = \frac{b_i-a_i}{2}$, the logical function $g_i(u)$ acts as follows

$$g_i(u) = \begin{cases} \delta_2^2, & otherwise \\ \delta_2^1, & u \in \Omega_i \end{cases}$$

where $\Omega_i = (a_i, b_i)$ is a set, which is established in [21].

For any $\varphi(t) \in PC^b_{F_0}([-\tau,0],R)$, we assume that system (1) satisfies necessary conditions for global existence and uniqueness of solution, Φ_k satisfies $\Phi_k(y(t_k)) \neq -y(t_k)$, $\forall k \in N$. Obviously, the system admits an equilibrium solution $y(t) \equiv 0$. In the following, according to [34], Definition 2.1 and 2.2 are given, which are necessary for the discussion.

Definition 2.1: A function $y(t)$ is said to be a solution of system (1) with the initial condition (2) if the following conditions are satisfied:

(i) $y(t)$ is absolutely continuous on each interval $(t_k, t_{k+1}]$, $k \in N$.

(ii) For any t_k, $y(t_k^+) = \lim_{t\to t_k^+} y(t)$ and $y(t_k^-) = \lim_{t\to t_k^-} y(t)$ exist, $y(t_k^-) = y(t)$, $k \in N$.

(iii) $y(t)$ satisfies the system (1) for almost everywhere in $[0,+\infty)\backslash\{t_k\}$ and the impulsive conditions at each $t = t_k$, $k \in N$.

Definition 2.2: The zero solution of system (1) with initial condition (2) is said to be

(i) p-stable, if for any $\varepsilon > 0$, there is a $\delta > 0$ such that the initial function $\varphi(t) \in PC^b_{F_0}(\delta)$ implies $E|y(t)|^p < \varepsilon$ for $t \geq 0$. Especially, when $p = 1$, it is said to be stable.

(ii) asymptotically p-stable, if $y(t)$ is p-stable and there exists a scalar $\delta_0 > 0$, such that the initial function $\varphi(t) \in PC^b_{F_0}(\delta_0)$ implies $\lim_{t\to\infty} E|y(t)| = 0$. when $p = 1$, it is said to be asymptotically stable.

(iii) exponentially p-stable ($p \geq 2$), if there is a pair of positive constants λ and K such that for any initial condition $\varphi(t) \in PC^b_{F_0}([-\tau, 0], R)$,

$$E|y(t)|^p \leq K\|\varphi\|^p e^{-\lambda t}, \quad t \geq 0.$$

Here λ is called the exponential convergence rate. Especially, when $p = 2$, it is said to be exponentially stable in mean square.

III. STABILITY CRITERIA

Motivated by [6], we give the following hypothesis.

Hypothesis 3.1: Assume that the function $\alpha(t)$ satisfies:

(H1) $\alpha(t)$ is continuous differential on $(t_k, t_{k+1}]$, $k = 0, 1, 2 \ldots$

(H2) $\alpha(t_k) = (1 + \frac{\Phi_k(y(t_k))}{y(t_k)})\alpha(t_k^+)$, $k = 1, 2, 3 \ldots$

(H3) $\alpha(t) \neq 0, \forall t > 0$ and $\alpha(t) = 1, \forall t \leq 0$.

Construct the following non-impulsive stochastic delay differential system:

$$dx(t) = [\frac{\dot{\alpha}(t)}{\alpha(t)}x(t) + \sum_{i=1}^{n} \frac{\alpha(t)p_i(t)}{\alpha(t-\tau_i(t))}x(t-\tau_i(t))]dt + \sum_{i=1}^{n} \frac{\alpha(t)q_i(t)}{\alpha(t-\tau_i(t))}x(t-\tau_i(t))dw(t) \quad (3)$$

for $t > 0$, with the initial condition:

$$x(t) = \varphi(t), \quad t \in [-\tau, 0] \quad (4)$$

An absolutely continuous function $x(t)$ is said to be a solution of system (3) with initial condition (4), if $x(t)$ satisfies (3) almost everywhere and satisfies the initial condition (4) for $t \geq -\tau$.

Next, a fundamental lemma is presented, which established the connection between the SDLI (1) with (2) and a corresponding SD (3) with (4).

Lemma 3.1: Assume that Hypothesis 3.1 holds,

(i) if $x(t)$ is a solution of (3) with (4), then $y(t) = \alpha^{-1}(t)x(t)$ is a solution of (1) with (2) on $[-\tau, +\infty)$.

(ii) if $y(t)$ is a solution of (1) with (2), then $x(t) = \alpha(t)y(t)$ is a solution of (3) with (4) on $[-\tau, +\infty)$.

Proof: First we prove (i). Let $x(t)$ be a possible solution of system (3) with (4), it is easy to see that $y(t) = \alpha^{-1}(t)x(t)$ is absolutely continuous on (t_k, t_{k+1}), $k \in N$. For any $t \neq t_k$,

$$\begin{aligned} dy(t) &= d(\alpha^{-1}(t)x(t)) \\ &= x(t)d(\alpha^{-1}(t)) + \alpha^{-1}(t)d(x(t)) \\ &= -\frac{\dot{\alpha}(t)}{\alpha^2(t)}x(t)dt \\ &\quad + \frac{1}{\alpha(t)}[\frac{\dot{\alpha}(t)}{\alpha(t)}x(t) + \sum_{i=1}^{n} \frac{\alpha(t)p_i(t)}{\alpha(t-\tau_i(t))}x(t-\tau_i(t))]dt \\ &\quad + \frac{1}{\alpha(t)}[\sum_{i=1}^{n} \frac{\alpha(t)q_i(t)}{\alpha(t-\tau_i(t))}x(t-\tau_i(t))]dw(t) \\ &= \sum_{i=1}^{n} \frac{p_i(t)}{\alpha(t-\tau_i(t))}x(t-\tau_i(t))dt \\ &\quad + \sum_{i=1}^{n} \frac{q_i(t)}{\alpha(t-\tau_i(t))}x(t-\tau_i(t))dw(t) \\ &= \sum_{i=1}^{n} p_i(t)y(t-\tau_i(t))dt + \sum_{i=1}^{n} q_i(t)y(t-\tau_i(t))dw(t) \end{aligned}$$

Thus, $y(t) = \alpha^{-1}(t)x(t)$ satisfies system (1) for almost everywhere in $[0, +\infty)\backslash t_k$.

According to (H2), $\forall t_k \in [0, +\infty)$, $k \in N$,

$$\begin{aligned} y(t_k^+) &= \lim_{t\to t_k^+} \frac{x(t)}{\alpha(t)} = \frac{x(t_k^+)}{\alpha(t_k^+)} = \frac{x(t_k)}{\alpha(t_k)}(1 + \frac{\Phi_k(y(t_k))}{y(t_k)}) \\ &= y(t_k)(1 + \frac{\Phi(y(t_k))}{y(t_k)}) = y(t_k) + \Phi_k(y(t_k)) \end{aligned}$$

From (H1), we know that $\alpha(t)$ is left continuous on t_k, namely $\alpha(t_k^-) = \alpha(t_k)$, $k = 1, 2 \ldots$, so

$$y(t_k^-) = \lim_{t\to t_k^-} \frac{x(t)}{\alpha(t)} = \frac{x(t_k^-)}{\alpha(t_k^-)} = \frac{x(t_k)}{\alpha(t_k)} = y(t_k)$$

From (H3), $y(t) = 1 \cdot x(t) = \varphi(t)$ on $[-\tau, 0]$. Therefore, we arrive at a conclusion that $y(t) = \alpha^{-1}(t)x(t)$ is the solution of (1) with initial condition (2).

Next we prove (ii). Since $y(t)$ is a solution of (1), $x(t) = \alpha(t)y(t)$ is absolutely continuous on $[0, +\infty)\backslash t_k$. Moreover, it follows that, for every t_k situated in $[0, +\infty)$,

$$\begin{aligned} x(t_k^+) &= \lim_{t\to t_k^+} \alpha(t)y(t) = \alpha(t_k^+)y(t_k^+) \\ &= \frac{y(t_k)\alpha(t_k)}{y(t_k) + \Phi_k(y(t_k))}y(t_k^+) = \alpha(t_k)y(t_k) = x(t_k) \end{aligned}$$

and

$$\begin{aligned} x(t_k^-) &= \lim_{t\to t_k^-} \alpha(t)y(t) = \alpha(t_k^-)y(t_k^-) \\ &= \alpha(t_k)y(t_k) = x(t_k) \end{aligned}$$

which implies that $x(t) = \alpha(t)y(t)$ is continuous and easy to prove absolutely continuous on $[0, +\infty)$. Furthermore, $x(t) = 1 \cdot y(t) = \varphi(t)$ on $[-\tau, 0]$. Therefore, $x(t) = \alpha(t)y(t)$ is the solution of (3) with the initial condition (4).

By applying Lemma 3.1, we can study the stability problems of SDLI (1) by the property of SD (3), and some stability criteria of the systems can be provided.

Theorem 3.1: Assume that there exists a positive constant M such that for any $t > 0$,

$$|\alpha^{-1}(t)| \leq M \quad (5)$$

(i) If the zero solution of (3) is p-stable, then the zero solution of (1) is also p-stable.

(ii) If the zero solution of (3) is asymptotically p-stable, then the zero solution of (1) is also asymptotically p-stable.

(iii) If the zero solution of (3) is exponentially p-stable ($p \geq 2$), then the zero solution of (1) is also exponentially p-stable ($p \geq 2$). What's more, they have the same exponential convergence rate.

Proof: We prove (i) only, (ii) and (iii) can be proved similarly and proofs will be omitted here.

Let $x(t)$ and $y(t)$ be the possible solutions of system (3) and (1) corresponding to the initial condition (4) and (2).

From the hypotheses, the zero solution of (3) is p-stable, for any $\varepsilon > 0$, there is a $\delta > 0$ such that the initial condition $\varphi(t) \in PC^b_{F_0}(\delta)$ implies

$$E|x(t)|^p < \frac{\varepsilon}{M^p}.$$

In view of Lemma 3.1, $y(t) = \alpha^{-1}(t)x(t)$ is the unique solution of (1) on $[-\tau, +\infty)$. Furthermore, it is easy to see that,

$$\begin{aligned} E|y(t)|^p &= E|\alpha^{-1}(t)x(t)|^p \leq E(|\alpha^{-1}(t)|^p |x(t)|^p) \\ &\leq E(M^p|x(t)|^p) \leq M^p E(|x(t)|^p) \\ &< M^p \frac{\varepsilon}{M^p} = \varepsilon. \end{aligned}$$

which implies that the zero solution of (1) is also p-stable.

Similarly, the following stability analysis of SD (3) can be deserved.

Theorem 3.2: Assume that there exists a positive constant N such that for any $t > 0$,

$$|\alpha(t)| \leq N \tag{6}$$

(i) If the zero solution of (1) is stable, then the zero solution of (3) is also stable.

(ii) If the zero solution of (1) is asymptotically p-stable, then the zero solution of (3) is also asymptotically p-stable.

(iii) If the zero solution of (1) is exponentially p-stable ($p \geq 2$), then the zero solution of (3) is also exponentially p-stable ($p \geq 2$). What's more, they have the same exponential convergence rate.

By combining Theorem 3.1 and Theorem 3.2, the following interesting result can be easily provided.

Theorem 3.3: Assume that inequalities (5) and (6) are satisfied, then the zero solution of (1) is p-stable, asymptotically p-stable, exponentially p-stable ($p \geq 2$) if and only if the zero solution of (3) is p-stable, asymptotically p-stable, exponentially p-stable ($p \geq 2$) respectively.

IV. APPLICATION TO A CLASS OF LINEAR SDLI

In [4], G. Zhang *et al.* have studied the exponential stability of the following linear impulsive delay differential equation:

$$\begin{cases} \dot{x}(t) = py(t) + qy(t-\tau), & t \geq 0,\ t \neq k\tau \\ x(t_k) - x(t_k^-) = rx(t_k^-), & t = k\tau \\ x(t) = \varphi(t), & t \in [-\tau,\ 0) \end{cases} \tag{7}$$

where $k = 0, 1, 2, \ldots$, $r \neq -1$, $\tau > 0$, p and q are real constants, $\varphi(t) \in C([-\tau, 0), R)$.

In [6], X.Liu *et al.* have shown the analytic and numerical stability results of a more general linear impulsive delay differential equation as follow:

$$\begin{cases} \dot{x}(t) = ay(t) + by(t-\tau), & t > 0,\ t \neq t_k \\ x(t_k^+) - x(t_k^-) = r_k x(t_k), & t = t_k \\ x(t) = \varphi(t), & t \in [t_0 - \tau,\ t_0] \end{cases} \tag{8}$$

where $a, b, r_k \in R$, $\tau > 0$, $t_{k+1} - t_k = \tau$ are fixed points with $\lim_{k\to\infty} t_k = \infty$ and $\varphi(t)$ is a smooth function on $[t_0 - \tau,\ t_0]$.

Note that, the logic effects and stochastic effects were not taken into the above literature and there are few results of this kind of research. Based on such consideration, by applying stability results (Theorems 3.1-3.3) obtained in section 3, we discuss the stability problem of the following linear stochastic delay differential system with variable logic impulses:

$$\begin{cases} dy(t) = (ay(t) + by(t-\tau))dt \\ +(a_1 y(t) + b_1\, y(t-\tau))dw(t), & t \neq t_k \\ y(t_k^+) - y(t_k) = \phi_k y(t_k), & t = t_k \\ y(t) = \varphi(t), & t \in [-\tau,\ 0] \end{cases} \tag{9}$$

where $a, b, a_1, b_1 \in R$, $\tau > 0$, $\phi_k > -1$, $t_{k+1} - t_k = \tau$ are fixed points, $k = 0, 1, 2, \ldots$, and

$$\begin{aligned} \phi_k &= [r_k, \tilde{r}_k] g_1(y(t_k)) + [\lambda_k, \tilde{\lambda}_k] g_2(y(t_k)) \\ &= [r_k, \tilde{r}_k, \lambda_k, \tilde{\lambda}_k][g_1^T, g_2^T]^T \end{aligned}$$

where $r_k, \tilde{r}_k, \lambda_k, \tilde{\lambda}_k$ are real numbers, $g_i : R \to \{\delta_2^1, \delta_2^2\}$ is a piecewise logical function as follows:

$$g_i(u) = \begin{cases} \delta_2^2, & |f_i(u)| \geq c_i \\ \delta_2^1, & |f_i(u)| < c_i \end{cases}$$

here, $f_i \in C(R, R)$, $c_i > 0$ is the threshold, for $i = 1, 2$. Obviously, ϕ_k is chosen from the real numbers $r_k + \lambda_k$, $r_k + \tilde{\lambda}_k$, $\tilde{r}_k + \lambda_k$ and $\tilde{r}_k + \tilde{\lambda}_k$.

Remark 4.1: When $a_1 = b_1 = 0$, $\phi_k \equiv r$ or $\phi_k \equiv r_k$, system (9) can be degraded into systems (7) or (8), then we can draw a conclusion that system (9) is more general than the systems in [4] and [6].

In the following discussion, we need to introduce a few more notations (see [29]). Let $C^{2,1}(R^n \times R_+, R_+)$ denote the family of all continuous non-negative funcitons $V(x, t)$ which are continuously twice differentiable in x and once in t. Then, for a stochastic delay differential system:

$$dx(t) = f(x(t), x(t-\tau), t)dt + g(x(t), x(t-\tau), t)dw(t)$$

on $t \geq 0$, we define $\mathcal{L}V : R^n \times R^n \times R_+ \to R$ by:

$$\begin{aligned} \mathcal{L}V(x, y, t) = {} & V_t(x, t) + V_x(x, t)f(x, y, t) \\ & + \frac{1}{2} trace\left(g^T(x, y, t)V_{xx}(x, t)g(x, y, t)\right) \end{aligned}$$

where

$$\begin{aligned} V_t(x, t) &= \frac{\partial V(x, t)}{\partial t}, \\ V_x(x, t) &= \left(\frac{\partial V(x, t)}{\partial x_1}, \ldots, \frac{\partial V(x, t)}{\partial x_n}\right), \\ V_{xx}(x, t) &= \left(\frac{\partial^2 V(x, t)}{\partial x_i \partial x_j}\right)_{n\times n} \end{aligned}$$

A. CASE I

For the linear stochastic delay differential system with logic impulses (9), let $\alpha(t)$ as follows,

$$\alpha(t)=\begin{cases}\prod_{0\le t_j<t}\frac{1}{1+\phi_j} & t>\tau\\ 1 & t\le\tau\end{cases}$$

Note that, $\alpha(t)$ is a piecewise constant function, one can easily check that it satisfies (H1) and (H3).

$$\begin{aligned}\alpha(t_k^+)&=\lim_{t\to t_k^+}\prod_{0\le t_j<t}\frac{1}{1+\phi_j}=\prod_{0\le t_j\le t_k}\frac{1}{1+\phi_j}\\&=\frac{1}{1+\phi_k}\prod_{0\le t_j<t_k}\frac{1}{1+\phi_j}=\frac{1}{1+\phi_k}\alpha(t_k)\end{aligned}$$

which implies that $\alpha(t)$ satisfies (H2). So, $\alpha(t)$ satisfies Hypothesis 3.1, that's clear.

Moreover, the corresponding non-impulsive stochastic delay differential system is in the following form,

$$\begin{aligned}dx(t)=(ax(t)&+\frac{b}{1+\phi_k}x(t-\tau))dt\\&+(a_1x(t)+\frac{b_1}{1+\phi_k}x(t-\tau))dw(t)\end{aligned}\quad(10)$$

where $t\in(t_k,t_{k+1}]$, with the initial condition:

$$x(t)=\varphi(t),\qquad t\in[-\tau,\ 0]$$

Theorem 4.1: Assume that there exists a positive constant M such that

(1) $\prod_{k\in N}(1+\omega_k)\le M$,

(2) $-\frac{a_1^2}{2}-a>\frac{|b+a_1b_1|}{1+\ominus}+\frac{1}{2}(\frac{b_1}{1+\ominus})^2$

then the linear SDLI (9) is exponentially stable in mean square. Where $\omega_k=\max\{r_k+\lambda_k,\ r_k+\tilde{\lambda}_k,\ \tilde{r}_k+\lambda_k,\ \tilde{r}_k+\tilde{\lambda}_k\}$, $\ominus=\min_{k\in N}\{r_k+\lambda_k,\ r_k+\tilde{\lambda}_k,\ \tilde{r}_k+\lambda_k,\ \tilde{r}_k+\tilde{\lambda}_k\}$.

Proof: For brevity, let $x(t)=x,\ x(t-\tau)=x_\tau$. Let $x(t)$ be a possible solution of system (10), we choose $V(x,t)=x^2$, then

$$\begin{aligned}\mathcal{L}V&=2x(ax+\frac{b}{1+\phi_k}x_\tau)+(a_1x+\frac{b_1}{1+\phi_k}x_\tau)^2\\&=(2a+a_1^2)x^2+\frac{2(b+a_1b_1)}{1+\phi_k}xx_\tau+(\frac{b_1}{1+\phi_k})^2x_\tau^2\end{aligned}$$

Note that $\phi_k>-1$, we have $1+\omega_k,\ 1+\ominus>0$, then

$$\begin{aligned}\mathcal{L}V&\le(2a+a_1^2)x^2+\frac{|b+a_1b_1|}{1+\phi_k}(x^2+x_\tau^2)+(\frac{b_1}{1+\phi_k})^2x_\tau^2\\&=(2a+a_1^2+\frac{|b+a_1b_1|}{1+\phi_k})x^2+(\frac{|b+a_1b_1|}{1+\phi_k}+(\frac{b_1}{1+\phi_k})^2)x_\tau^2\\&\le(2a+a_1^2+\frac{|b+a_1b_1|}{1+\ominus})x^2+(\frac{|b+a_1b_1|}{1+\ominus}+(\frac{b_1}{1+\ominus})^2)x_\tau^2\end{aligned}$$

From condition (2), we have

$$\begin{aligned}-\frac{a_1^2}{2}-a&>\frac{|b+a_1b_1|}{1+\ominus}+\frac{1}{2}(\frac{b_1}{1+\ominus})^2\\&\ge\frac{|b+a_1b_1|}{1+\phi_k}+\frac{1}{2}(\frac{b_1}{1+\phi_k})^2\end{aligned}$$

Then,

$$-(2a+a_1^2+\frac{|b+a_1b_1|}{1+\phi_k})>\frac{|b+a_1b_1|}{1+\phi_k}+(\frac{b_1}{1+\phi_k})^2>0$$

For Theorem 3.4 in [29], we can give a conclusion that, the non-impulsive stochastic delay differential system (10) is exponentially stable in mean square.

In the other hand, from condition (1)

$$|\alpha^{-1}(t)|\le\prod_{k\in N}(1+\phi_k)\le\prod_{k\in N}(1+\omega_k)\le M$$

According to Theorem 3.1, the linear stochastic delay differential system with logic impulses (9) is exponentially stable in mean square.

Example 4.1: Consider the following linear SDLI:

$$\begin{cases}dy(t)=(ay(t)+by(t-0.1))dt\\+(a_1y(t)+b_1\,y(t-0.1))dw(t), & t\ne0.1k\\ y(t_k^+)-y(t_k)=\phi_ky(t_k), & t=0.1k\end{cases}\quad(11)$$

with initial condition $y(t)=2,t\in[-0.1,0]$, and the logic impulses act as follows:

$$\begin{aligned}y(t_k^+)-y(t_k)&=\phi_ky(t_k)\\&=y(t_k)[\frac{1}{2^k},-\frac{1}{2^k}]g_1(y(t_k))+y(t_k)[\frac{1}{3^k},-\frac{1}{3^k}]g_2(y(t_k))\\&=y(t_k)[\frac{1}{2^k},-\frac{1}{2^k},\frac{1}{3^k},-\frac{1}{3^k}][g_1^T(y(t_k)),g_2^T(y(t_k))]^T\end{aligned}$$

where $g_i:R\to\{\delta_2^1,\delta_2^2\}$, $i=1,2$, is a piecewise logical function as follows:

$$g_1(u)=\begin{cases}\delta_2^2, & |u|\ge0.5,\\ \delta_2^1, & |u|<0.5\end{cases}$$

$$g_2(u)=\begin{cases}\delta_2^2, & |u|\ge0.25,\\ \delta_2^1, & |u|<0.25\end{cases}$$

here, $f_i(u)=u$, $c_1=0.5$ and $c_2=0.25$. Then, ϕ_k is chosen from the real numbers $\frac{1}{2^k}+\frac{1}{3^k},-\frac{1}{2^k}-\frac{1}{3^k},\frac{1}{2^k}-\frac{1}{3^k}$. Furthermore, $\omega_k=\frac{1}{2^k}+\frac{1}{3^k}$, $\ominus=\min_{k\in N}\{-\frac{1}{2^k}-\frac{1}{3^k}\}=-\frac{5}{6}$. Note that, there exists a positive constant M, such that $\prod_{k\in N}(1+\omega_k)\le M$. Therefore, by Theorem 4.1, system (11) is exponentially stable in mean square, if $-\frac{a_1^2}{2}-a>6\,|b+a_1b_1|+18b_1^2$ is hold, for example, $a=-3$, $a_1=2$, $b=\frac{1}{3}$, $b_1=-\frac{1}{6}$, showed in Figure 1.

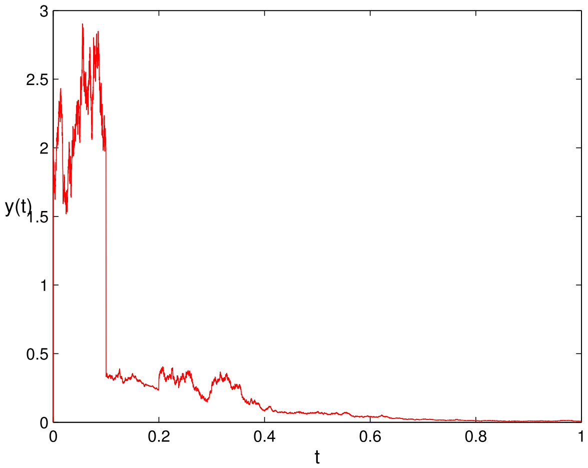

FIGURE 1. $a = -3, a_1 = 2, b = \frac{1}{3}, b_1 = -\frac{1}{6}$.

B. CASE II

For the linear stochastic delay differential system with logic impulses (9), let $\alpha(t)$ as follows,

$$\alpha(t) = (1+\phi_k)^{\frac{t-t_{k+1}}{\tau}}, \quad t \in (t_k, t_{k+1}]$$

It's easily check that $\alpha(t)$ satisfies (H1) and (H3). Furthermore,

$$\begin{aligned}\alpha(t_k) &= (1+\phi_{k-1})^{\frac{t_k-t_k}{\tau}} = 1, \quad t_k \in (t_{k-1}, t_k] \\ \alpha(t_k^+) &= \lim_{t\to t_k^+}(1+\phi_k)^{\frac{t-t_{k+1}}{\tau}} = (1+\phi_k)^{\frac{t_k-t_{k+1}}{\tau}} \\ &= \frac{1}{1+\phi_k} = \frac{1}{1+\phi_k}\alpha(t_k)\end{aligned}$$

which implies that $\alpha(t)$ satisfies (H2). So, $\alpha(t)$ satisfies Hypothesis 3.1, that's clear.

The corresponding non-impulsive stochastic delay differential system is in the following form,

$$\begin{aligned}&dx(t) \\ &= [(\frac{\ln(1+\phi_k)}{\tau}+a)x(t)+b\left(\frac{1+\phi_k}{1+\phi_{k-1}}\right)^{\frac{t-t_{k+1}}{\tau}}x(t-\tau)]dt \\ &\quad +[a_1x(t)+b_1\left(\frac{1+\phi_k}{1+\phi_{k-1}}\right)^{\frac{t-t_{k+1}}{\tau}}x(t-\tau)]dw(t) \qquad (12)\end{aligned}$$

where $t \in (t_k, t_{k+1}]$, with the initial condition:

$$x(t) = \varphi(t), \quad t \in [-\tau, 0]$$

Theorem 4.2: The zero solution of linear stochastic delay differential system with logic impulses (9) is p-stable, asymptotically p-stable, and exponentially p-stable if and only if the zero solution of the non-impulsive stochastic delay differential system (12) is p-stable, asymptotically p-stable, and exponentially p-stable respectively.

Proof: In linear stochastic delay differential system with logic impulses (9), let $\omega = \max_{k\in N}\{r_k+\lambda_k,\ r_k+\tilde{\lambda}_k,\ \tilde{r}_k+\lambda_k,\ \tilde{r}_k+\tilde{\lambda}_k\}$, $\ominus = \min_{k\in N}\{r_k+\lambda_k,\ r_k+\tilde{\lambda}_k,\ \tilde{r}_k+\lambda_k,\ \tilde{r}_k+\tilde{\lambda}_k\}$.

For $t \in (t_k, t_{k+1}]$, denote $t = t_{k+1} - \theta\tau$ $(0 \le \theta < 1)$, then

$$|\alpha(t)| = (1+\phi_k)^{\frac{t-t_{k+1}}{\tau}} = (1+\phi_k)^{-\theta} \le \max\left\{\frac{1}{1+\ominus}, 1\right\}$$

and

$$|\alpha(t)^{-1}| = (1+\phi_k)^{\frac{-t+t_{k+1}}{\tau}} = (1+\phi_k)^{\theta} \le \max\{1+\omega, 1\}$$

According to Theorem 3.3, we can get a conclusion that the zero solution of linear stochastic delay differential system with logic impulses (9) is p-stable, asymptotically p-stable, and exponentially p-stable if and only if the zero solution of the non-impulsive stochastic delay differential system (12) is p-stable, asymptotically p-stable, and exponentially p-stable, respectively.

Theorem 4.3: Assume that

$$-\frac{a_1^2}{2} - a > \frac{\ln(1+\omega)}{\tau} + \frac{b_1^2}{2}\left(\frac{1+\omega}{1+\ominus}\right)^2 + |b+a_1b_1|\frac{1+\omega}{1+\ominus}$$

then the SDLI (9) is exponentially stable in mean square. Where $\omega = \max_{k\in N}\{r_k+\lambda_k,\ r_k+\tilde{\lambda}_k,\ \tilde{r}_k+\lambda_k,\ \tilde{r}_k+\tilde{\lambda}_k\}$, $\ominus = \min_{k\in N}\{r_k+\lambda_k,\ r_k+\tilde{\lambda}_k,\ \tilde{r}_k+\lambda_k,\ \tilde{r}_k+\tilde{\lambda}_k\}$.

Proof: For brevity, denote $x = x(t)$, $x_\tau = x(t-\tau)$, and

$$\mathcal{K} = \frac{\alpha(t)}{\alpha(t-\tau)} = \left(\frac{1+\phi_k}{1+\phi_{k-1}}\right)^{\frac{t-t_{k+1}}{\tau}}, \quad t \in (t_k, t_{k+1}]$$

It's easily to check that,

$$\frac{1+\ominus}{1+\omega} \le \mathcal{K} \le \frac{1+\omega}{1+\ominus}$$

Let $x(t)$ be a possible solution of system (12), we choose $V(x,t) = x^2$, then

$$\begin{aligned}\mathcal{L}V &= 2x[(\frac{\ln(1+\phi_k)}{\tau}+a)x+b\left(\frac{1+\phi_k}{1+\phi_{k-1}}\right)^{\frac{t-t_{k+1}}{\tau}}x_\tau] \\ &\quad +[a_1x+b_1\left(\frac{1+\phi_k}{1+\phi_{k-1}}\right)^{\frac{t-t_{k+1}}{\tau}}x_\tau]^2 \\ &= (2\frac{\ln(1+\phi_k)}{\tau}+2a+a_1^2)x^2+2(b+a_1b_1)\mathcal{K}xx_\tau \\ &\quad +b_1^2\mathcal{K}^2x_\tau^2 \\ &\le (2\frac{\ln(1+\phi_k)}{\tau}+2a+a_1^2)x^2+|b+a_1b_1|\mathcal{K}(x^2+x_\tau^2) \\ &\quad +b_1^2\mathcal{K}^2x_\tau^2 \\ &= (2\frac{\ln(1+\phi_k)}{\tau}+2a+a_1^2+|b+a_1b_1|\mathcal{K})x^2 \\ &\quad +(b_1^2\mathcal{K}^2+|b+a_1b_1|\mathcal{K})x_\tau^2\end{aligned}$$

And

$$\begin{aligned}-\frac{a_1^2}{2}-a &> \frac{\ln(1+\omega)}{\tau}+\frac{b_1^2}{2}\left(\frac{1+\omega}{1+\ominus}\right)^2+|b+a_1b_1|\frac{1+\omega}{1+\ominus} \\ &\ge \frac{\ln(1+\phi_k)}{\tau}+\frac{b_1^2}{2}\mathcal{K}^2+|b+a_1b_1|\mathcal{K}\end{aligned}$$

Then,

$$\begin{aligned}&-(2\frac{\ln(1+\phi_k)}{\tau}+2a+a_1^2+|b+a_1b_1|\mathcal{K}) \\ &\qquad > b_1^2\mathcal{K}^2+|b+a_1b_1|\mathcal{K}\end{aligned}$$

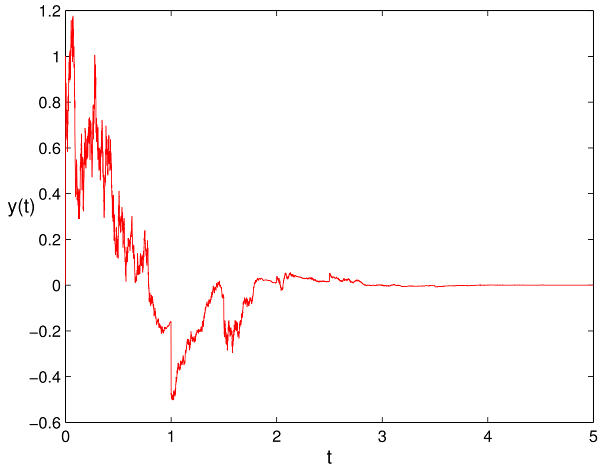

FIGURE 2. $a = -4, a_1 = 1, b = -\frac{1}{3}, b_1 = \frac{2}{3}$.

According to Theorem 3.4 in [29], we can give a conclusion that, the non-impulsive stochastic delay differential system (12) is exponentially stable in mean square. Take Theorem 4.2 into account, the linear stochastic delay differential system with logic impulses (9) is exponentially stable in mean square.

Remark 4.2: Theorem 4.2 in [4] can be drawn from the Theorem 4.3 above, if $r_k = \tilde{r}_k = \lambda_k = \tilde{\lambda}_k = r > -1$ and $a_1 = b_1 = 0$.

Example 4.2: Consider the following linear SDLI:

$$\begin{cases} dy(t) = (ay(t) + by(t-0.5))dt \\ +(a_1 y(t) + b_1\, y(t-0.5))dw(t), & t \neq 0.5k \\ y(t_k^+) - y(t_k) = \phi_k y(t_k), & t = 0.5k \end{cases} \tag{13}$$

with initial condition $y(t) = 1, t \in [-0.5, 0]$, and the logic impulses act as follows:

$$\begin{aligned} y(t_k^+) - y(t_k) &= \phi_k y(t_k) \\ &= y(t_k)[-\frac{1}{4}, -\frac{1}{2}]g_1(y(t_k)) \\ &\quad + y(t_k)[\frac{9}{4}, \frac{3}{2}]g_2(y(t_k)) \\ &= y(t_k)[-\frac{1}{4}, -\frac{1}{2}, \frac{9}{4}, \frac{3}{2}][g_1^T(y(t_k)), g_2^T(y(t_k))]^T \end{aligned}$$

where $g_i : R \to \{\delta_2^1, \delta_2^2\}$, $i = 1, 2$, is a piecewise logical function as follows:

$$g_1(u) = \begin{cases} \delta_2^2, & |u - 0.05| \geq 0.15, \\ \delta_2^1, & |u - 0.05| < 0.15 \end{cases}$$

and

$$g_2(u) = \begin{cases} \delta_2^2, & |u + 0.1| \geq 0.2, \\ \delta_2^1, & |u + 0.1| < 0.2 \end{cases}$$

Obviously, $f_1(u) = u - 0.05, f_2(u) = u + 0.1, c_1 = 0.15$, $c_2 = 0.2$, $g_1(u)$ and $g_2(u)$ can be expressed as

$$g_1(u) = \begin{cases} \delta_2^2, & otherwise, \\ \delta_2^1, & u \in (-0.1, 0.2) \end{cases}$$

and

$$g_2(u) = \begin{cases} \delta_2^2, & otherwise, \\ \delta_2^1, & u \in (-0.3, 0.1) \end{cases}$$

It is easy to obtain ϕ_k, which is chosen from the real numbers $1, \frac{5}{4}, \frac{7}{4}$ and 2. Moreover, $\omega = 2, \ominus = 1$, that's clear. Then by Theorem 4.3, system (13) is exponentially stable in mean squear if $-\frac{a_1^2}{2} - a > 2\ln 3 + \frac{9}{8}b_1^2 + \frac{3}{2}|b + a_1 b_1|$ is hold, for example, $a = -4$, $a_1 = 1$, $b = -\frac{1}{3}$, $b_1 = \frac{2}{3}$, showed in Figure 2.

V. CONCLUSION

In this paper, the stability of stochastic delay differential systems with variable impulses due to logic choice has been investigated. Firstly, the author has introduced a class of linear stochastic delay differential system with variable impulses due to logic choice, in which the logic impulses generalize the logic impulses established in [20] and [21]. Then, by establishing the equivalent relation between the solutions of SDLI and SD, some stability criteria for SDLI and SD have been proposed. Finally, the author has discussed the application in a class of linear stochastic delay differential systems with logic impulses, including several stability criteria and two numerical examples.

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