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The Edge Version of Metric Dimension for the Family of Circulant Graphs $C_n(1, 2)$

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ABSTRACT Graph theory is widely used to analyze the structure models in chemistry, biology, computer science, operations research and sociology. Molecular bonds, species movement between regions, development of computer algorithms, shortest spanning trees in weighted graphs, aircraft scheduling and exploration of diffusion mechanisms are some of these structure models. Let $G = (V_G, E_G)$ be a connected graph, where V_G and E_G represent the set of vertices and the set of edges respectively. The idea of the edge version of metric dimension is based on the distance of edges in a graph. Let R_{E_G} be the smallest set of edges in a connected graph G that forms a basis such that for every pair of edges $e_1, e_2 \in E_G$, there exists an edge $e \in R_{E_G}$ for which $d_{E_G}(e_1, e) \neq d_{E_G}(e_2, e)$ holds. In this paper, we show that the family of circulant graphs $C_n(1, 2)$ is the family of graphs with constant edge version of metric dimension.

INDEX TERMS Line graph, resolving sets, the edge version of metric dimension, circulant graphs.

I. INTRODUCTION

In research areas of sciences where networks constitute the basic and fundamental study blocks, graph theory (graph labeling, graph coloring etc.) is the most intuitive and fundamental approach to apply and study these sciences [1]–[4]. For example: (i) in computer sciences [5], data mining, database designing, image processing, network algorithms, resource allocation, clustering of web documents [6], phone networks(GSM phones) and bi-processor tasks. (ii) in chemistry, study of molecular bonds, molecular descriptors, three dimensional complicated simulated structure of atoms and chemoinformatics are some study blocks. (iii) in biology, protein-protein interaction networks, cell biology structure, population genetics, bioinformatics and sequences of cell-samples are some more examples. (iv) in operations research, travelling salesman problem, optimization using PERT (Project Evaluation Review Technique), minimum sum coloring, job and time table scheduling [7], [8] and game theory.

Let G be an undirected, connected and simple graph consisting of a nonempty finite set V_G of vertices and E_G as a set of edges. For any two vertices $x, y \in V_G$, the distance $d(x, y)$ is the length of a shortest path between x and y. Let $R = \{r_1, r_2, \ldots, r_p\} \subset V_G$ be an ordered set and let $x \in V_G$, then $r(x, R)$ representation of x with respect to R is the p-tuple $\big(d(x, r_1), d(x, r_2), \ldots, d(x, r_p)\big)$. If there exist different representation of different vertices of G with respect to R, then R is said to be a resolving set of G. The resolving set consisting of minimum number of vertices is called a basis for G and the size of the basis is known as the metric dimension of G, written as $dim(G)$. For $R = \{r_1, r_2, \ldots, r_p\} \subset V_G$, the q^{th} component of $r(x, R)$ is 0 if and only if $x = r_q$. Hence, to prove that R is a resolving set it is enough to show that $r(x, R) \neq r(y, R)$ for each pair $x \neq y \in V_G \setminus R$. Slater in [9] and Harary and Melter in [10] represented the idea of resolvability and metric dimension. Applications of metric dimension in different branches are robot [11], network discovery and verification, navigation [11] and chemistry [12]. In [13], it has been proved that computing the metric dimension of a graph is an NP-hard problem. Metric dimension has been deeply investigated for surveys see: [14] and [15]. Also, metric properties of line graphs were studied to a great extent in [16]–[26]. The line graph $L(G)$ of a graph G is defined as, the graph whose vertices are the edges of G, with two adjacent vertices if the corresponding edges share the same vertex in G. The technique of finding vertex distances is helpful in finding edge distances. In [27] the edge version of metric dimension is defined as:

Definition 1:

i) Let $d_{E_G}(f, g)$ be the edge distance between two edges $f, g \in E_G$, which is the smallest length between vertices f and g in the line graph $L(G)$.

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ii) For the edge $e \in E_G$ to resolve two edges f and g of E_G, the condition $d_{E_G}(e,f) \neq d_{E_G}(e,g)$ must hold.

iii) For an ordered set $R_{E_G} = \{f_1, f_2, \ldots, f_k\} \subseteq E_G$ and an edge e in E_G, the k-tuple $\left(d_{E_G}(e,f_1), d_{E_G}(e,f_2), \ldots, d_{E_G}(e,f_k)\right)$ is the edge version of representation $r_{E_G}(e, R_{E_G})$ of e with respect to R_{E_G}.

iv) The set R_{E_G} is said to be the edge version of a resolving set of G, whenever distinct edges of G have distinct edge version of representations with respect to R_{E_G}.

v) The edge version of a resolving set having minimum cardinality basically forms the edge version of metric basis of G. The cardinality of the edge version of metric basis is denoted by $dim_E(G)$, and is called the edge version of metric dimension of G.

It should be noted that the parameter edge version of metric dimension studied here is actually the metric dimension of the line graph of a graph (namely edges uniquely recognizing edges) and is entirely different from the parameter edge metric dimension (namely vertices uniquely recognizing edges) defined in [28].

Recently, the edge version of metric dimension has been investigated for few classes of graphs. The path graphs were studied by G. Chartrand *et al.* in [12] for the edge version of metric dimension. Complete bipartite graphs for the edge version of metric dimension were studied by J. Caceres *et al.* in [31]. In [14], R.F. Bailey and P.J. Cameron worked on the edge version of metric dimension of complete graphs. In [32], L. Eroh determined $dim_E(G)$ of bouquet graphs and wheel graphs. R. Nasir *et al.* investigated the edge version of metric dimension of the n-sunlet graphs and the prism graphs in [27]. J.B. Liu *et al.* considered the family of necklace graphs for the edge version of metric dimension in [29]. Also, in [33] R. Nasir *et al.* discussed the edge version of metric dimension for the families of grid graphs and generalized prism graphs. In metric dimension, the resolving sets are referred as detecting devices in computer network problems while in the edge version of metric dimension we consider the edge version of resolving sets for the same purpose. Due to this reason graphs with constant edge version of metric dimension become more exciting than those graphs with variable edge version of metric dimension. With this motivation, we have studied those families of graphs for which the edge version of metric dimension is constant.

In Section 2, we will study the family of circulant graphs $C_n(1, 2)$ for the edge version of metric dimension and will show that $dim_E(C_n(1, 2)) = 4$ for $n \geq 6$. In the last section we will conclude our findings.

II. THE EDGE VERSION OF METRIC DIMENSION FOR THE FAMILY OF CIRCULANT GRAPHS

The family of circulant graphs, denoted by $C_n(1, 2)$ is the family of graphs with the vertex set $V_{C_n(1,2)} = \{v_1, v_2, \ldots, v_n\}$ and the edge set $E_{C_n(1,2)} = H \cup L$ where $H = \{h_i = v_i v_{i+1} : 1 \leq i \leq n\}$ and $L = \{l_i = v_i v_{i+2} : 1 \leq i \leq n\}$ modulo n as shown in Figure. 1. Metric dimension of $C_n(1, 2)$ was studied in [34] and the result is given below:

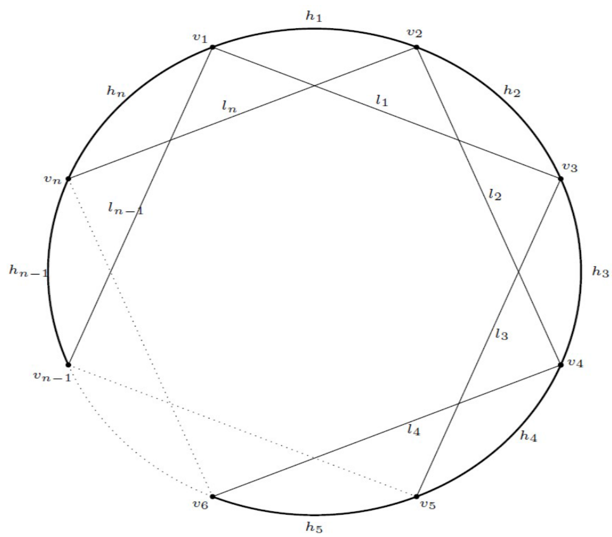

FIGURE 1. The family of circulant graphs $C_n(1, 2)$.

Theorem 1: [34] Let $C_n(1, 2)$ be the family of circulant graphs with $n \geq 5$, then $dim(C_n(1, 2)) = 3$ when $n \equiv 0, 2, 3(mod 4)$ and $dim(C_n(1, 2)) \leq 4$ otherwise.

For the edge version of metric dimension of the family of circulant graphs, we have the following theorem:

Theorem 2: Let G be the family of circulant graphs $C_n(1, 2)$ for $n \geq 6$, then $dim_E(G) = 4$.

Proof: In order to compute the edge version of metric dimension, we have the following cases:

Case 1: For $n \equiv 0(mod 4)$ i.e. $n = 4k$ for $k \geq 2$.

Consider the set of edges $R_{E_G} = \{h_1, h_2, h_{\frac{n}{2}-1}, h_{\frac{n}{2}}\}$, then representation of the edges h_{2i-1} with respect to R_{E_G} is:

$$r_{E_G}(h_{2i-1}, R_{E_G}) = \begin{cases} (0, 1, k, k) & if\ i = 1; \\ (i, i-1, k-i+1, k-i+1) & if\ 2 \leq i \leq k-1; \\ (k, k-1, 0, 1) & if\ i = k; \\ (i, k, 2, 1) & if\ i = k+1; \\ (2k-i+2, 2k-i+2, & \\ i-k+1, i-k) & if\ k+2 \leq i \leq 2k. \end{cases}$$

Representation of the edges h_{2i} with respect to R_{E_G} is:

$$r_{E_G}(h_{2i}, R_{E_G}) = \begin{cases} (1, 0, k-1, k) & if\ i = 1; \\ (i, i, k-i, k-i+1) & if\ 2 \leq i \leq k-1; \\ (k, k, 1, 0) & if\ i = k; \\ (2k-i+1, 2k-i+2, & \\ i-k+1, i-k+1) & if\ k+1 \leq i \leq 2k. \end{cases}$$

Representation of the edges l_{2i-1} with respect to R_{E_G} is:

$$r_{E_G}(l_{2i-1}, R_{E_G}) = \begin{cases} (1, 1, k-1, k) & \text{if } i = 1; \\ (i, i-1, k-i, k-i+1) & \text{if } 2 \le i \le k-1; \\ (k, i-k+4, i-k+1, 1) & \text{if } k \le i \le k+1; \\ (2k-i+1, 2k-i+2, \\ i-k+1, i-k) & \text{if } k+2 \le i \le 2k-1; \\ (1, 2, k, k) & \text{if } i = 2k. \end{cases}$$

Representation of the edges l_{2i} with respect to R_{E_G} is:

$$r_{E_G}(l_{2i}, R_{E_G}) = \begin{cases} (i, i, k-i, k-i) & \text{if } 1 \le i \le k-1; \\ (k, k, i-k+1, i-k+1) & \text{if } k \le i \le k+1; \\ (2k-i+1, 2k-i+1, \\ i-k+1, i-k+1) & \text{if } k+2 \le i \le 2k-1; \\ (1, 1, k, k) & \text{if } i = 2k. \end{cases}$$

Case 2: For $n \equiv 1(mod4)$ i.e. $n = 4k+1$ for $k \ge 2$.

Consider the set of edges $R_{E_G} = \{h_1, h_{\lfloor \frac{n}{2} \rfloor}, h_{\lfloor \frac{n}{2} \rfloor + 1}, h_n\}$, then representation of the edges h_{2i-1} with respect to R_{E_G} is:

$$r_{E_G}(h_{2i-1}, R_{E_G}) = \begin{cases} (0, k, k+1, 1) & \text{if } i = 1; \\ (i, k-i+1, k-i+2, i) & \text{if } 2 \le i \le k; \\ (k+1, 1, 0, k+1) & \text{if } i = k+1; \\ (2k-i+2, i-k, i-k, \\ 2k-i+2) & \text{if } k+2 \le i \le 2k; \\ (1, k+1, k+1, 0) & \text{if } i = 2k+1. \end{cases}$$

Representation of the edges h_{2i} with respect to R_{E_G} is:

$$r_{E_G}(h_{2i}, R_{E_G}) = \begin{cases} (i, k-i+1, k-i+1, i+1) & \text{if } 1 \le i \le k-1; \\ (k, 0, 1, k+1) & \text{if } i = k; \\ (2k-i+2, i-k+1, \\ i-k, 2k-i+1) & \text{if } k+1 \le i \le 2k. \end{cases}$$

Representation of the edges l_{2i-1} with respect to R_{E_G} is:

$$r_{E_G}(l_{2i-1}, R_{E_G}) = \begin{cases} (i, k-i+1, k-i+1, i) & \text{if } 1 \le i \le k; \\ (2k-i+2, i-k, \\ i-k, 2k-i+1) & \text{if } k+1 \le i \le 2k; \\ (1, k, k+1, 1) & \text{if } i = 2k+1. \end{cases}$$

Representation of the edges l_{2i} with respect to R_{E_G} is:

$$r_{E_G}(l_{2i}, R_{E_G}) = \begin{cases} (i, k-i, k-i+1, i+1) & \text{if } 1 \le i \le k-1; \\ (k, 1, 1, k+1) & \text{if } i = k; \\ (2k-i+1, i-k+1, \\ i-k, 2k-i+1) & \text{if } k+1 \le i \le 2k. \end{cases}$$

Case 3: For $n \equiv 2(mod4)$ i.e. $n = 4k+2$ for $k \ge 1$.

Consider the set of edges $R_{E_G} = \{h_1, h_{\frac{n}{2}-1}, h_{\frac{n}{2}}, h_n\}$, then representation of the edges h_{2i-1} with respect to R_{E_G} is:

$$r_{E_G}(h_{2i-1}, R_{E_G}) = \begin{cases} (0, k, k+1, 1) & \text{if } i = 1; \\ (i, k-i+1, k-i+2, i) & \text{if } 2 \le i \le k; \\ (k+1, 1, 0, k+1) & \text{if } i = k+1; \\ (2k-i+3, i-k, i-k, \\ 2k-i+2) & \text{if } k+2 \le i \le 2k+1. \end{cases}$$

Representation of the edges h_{2i} with respect to R_{E_G} is:

$$r_{E_G}(h_{2i}, R_{E_G}) = \begin{cases} (i, k-i+1, k-i+1, i+1) & \text{if } 1 \le i \le k-1; \\ (k, 0, 1, k+1) & \text{if } i = k; \\ (2k-i+2, i-k+1, \\ i-k, 2k-i+2) & \text{if } k+1 \le i \le 2k; \\ (1, k+1, k+1, 0) & \text{if } i = 2k+1. \end{cases}$$

Representation of the edges l_{2i-1} with respect to R_{E_G} is:

$$r_E E_G(l_{2i-1}, R_{E_G}) = \begin{cases} (i, k-i+1, k-i+1, i) & \text{if } 1 \le i \le k; \\ (2k-i+2, i-k, i-k, \\ 2k-i+2) & \text{if } k+1 \le i \le 2k+1. \end{cases}$$

Representation of the edges l_{2i} with respect to R_{E_G} is:

$$r_{E_G}(l_{2i}, R_{E_G}) = \begin{cases} (i, k-i, k-i+1, i+1) & \text{if } 1 \le i \le k-1; \\ (k, 1, 1, k+1) & \text{if } i = k; \\ (2k-i+2, i-k+1, i-k, \\ 2k-i+1) & \text{if } k+1 \le i \le 2k; \\ (1, k, k+1, 1) & \text{if } i = 2k+1. \end{cases}$$

Case 4: For $n \equiv 3(mod4)$ i.e. $n = 4k+3$ for $k \ge 1$.

Consider the set of edges $R_{E_G} = \{h_1, h_{n-2}, h_{\lfloor \frac{3n}{2} \rfloor - 1}, h_{\lfloor \frac{3n}{2} \rfloor}\}$, then representation of the edges h_{2i-1} with respect to R_{E_G} is:

$$r_{E_G}(h_{2i-1}, R_{E_G}) = \begin{cases} (0, 2, k, k+1) & \text{if } i = 1; \\ (i, i+1, k-i+1, k-i+2) & \text{if } 2 \le i \le k; \\ (k+1, k+1, 1, 1) & \text{if } i = k+1; \\ (2k-i+3, 2k-i+2, i-k, \\ k-i-1) & \text{if } k+2 \le i \le 2k; \\ (2, 0, k+1, k) & \text{if } i = 2k+1; \\ (1, 2, k+1, k+1) & \text{if } i = \lfloor \frac{n+1}{2} \rfloor. \end{cases}$$

Representation of the edges h_{2i} with respect to R_{E_G} is:

$$r_{E_G}(h_{2i}, R_{E_G}) = \begin{cases} (i, i+2, k-i+1, k-i+1) & \text{if } 1 \le i \le k-1; \\ (i, 2k-i+1, 1, 1) & \text{if } k \le i \le k+1; \\ (2k-i+3, 2k-i+1, \\ i-k, i-k) & \text{if } k+2 \le i \le 2k; \\ (2, 1, k+1, k+1) & \text{if } i = 2k+1. \end{cases}$$

Representation of the edges l_{2i-1} with respect to R_{E_G} is:

$$r_{E_G}(l_{2i-1}, R_{E_G}) = \begin{cases} (i, i+1, k-i+1, k-i+1) & \text{if } 1 \le i \le k-1; \\ (i, 2k-i+1, 2, k-i+1) & \text{if } k \le i \le k+1; \\ (2k-i+3, 2k-i+1, i-k, \\ i-k-1) & \text{if } k+2 \le i \le 2k; \\ (i-2k+1, i-2k, 3k-i+2, \\ i-k-1) & \text{if } 2k+1 \le i \le \lfloor \frac{n+1}{2} \rfloor. \end{cases}$$

Representation of the edges l_{2i} with respect to R_{E_G} is:

$$r_{E_G}(l_{2i}, R_{E_G}) = \begin{cases} (i, i+2, k-i, k-i+1) & \text{if } 1 \le i \le k-1; \\ (i, 2k-i+1, i-k, 2) & \text{if } k \le i \le k+1; \\ (2k-i+2, 2k-i+1, \\ i-k, i-k) & \text{if } k+2 \le i \le 2k; \\ (1, 1, k+1, k+1) & \text{if } i = 2k+1. \end{cases}$$

From the above representations it is clear that no two edges have the same representations, so all the edges have distinct representations which imply that R_{E_G} is the edge version of resolving set and hence $dim_E(G) \le 4$. Next, we have to show that $dim_E(G) \ge 4$. Suppose, to the contrary that $dim_E(G) = 3$, then we have the following possibilities and in all possibilities we will consider indices modulo n along with $h_0 = h_n$ or $l_o = l_n$:

Case 1: For $n \equiv 0(mod 4)$ i.e. $n = 4k$ for $k \ge 2$.

A) Let $R_{E_G} = \{h_i, h_j, h_k\}$ be the edge version of resolving set from the edge set H.

i) For $j - i \le \frac{n}{2} - 3$ and $i \le k \le \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ even with $\frac{n}{2} + i - 1 \le k \le i - 1$.

ii) For $\frac{n}{2} - 2 \le j - i \le \frac{3n}{4} + 1$:

a) i +j odd: When $i - 1 \le k \le \frac{n}{2} + i - 3$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\frac{n}{2} + i - 2 \le k \le n + i - 4$, we have $r_{E_G}(l_{\frac{n}{2}+i-4}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-3}, R_{E_G})$ and for $n+i-3 \le i-2$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

b) i +j even: When $i \le k \le \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\frac{n}{2} + i - 1 \le k \le n + i - 4$, we have $r_{E_G}(l_{\frac{n}{2}+i-4}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-3}, R_{E_G})$ and for $n+i-3 \le i-1$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

iii) For $\frac{3n}{4} + 2 \le j - i \le n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \le k \le$ j-i $- \frac{n}{2}$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is odd and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is even with $j - i - \frac{n}{2} + 1 \le k \le j - i$, a contradiction.

B) Let $R_{E_G} = \{l_i, l_j, l_k\}$ be the edge version of resolving set from the edge set L.

i) For $j - i \le 4$: When $i \le k \le \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is even then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i+k$ odd with $\frac{n}{2} + i \le k \le n + i - 5$ and $r_{E_G}(l_{j+2}, R_{E_G}) = r_{E_G}(h_{j+2}, R_{E_G})$ with $n + i - 4 \le k \le i - 1$, a contradiction.

ii) For $5 \le j - i \le \frac{n}{2} - 2$: When $i \le k \le \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ and $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ for $i + k$ even with $\frac{n}{2} + i \le k \le n + i - 5$. For odd $i + j$, $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$ and $r_{E_G}(l_{i+3}, R_{E_G}) = r_{E_G}(h_{i+3}, R_{E_G})$ for even $i+j$ with $n+i-4 \le k \le i-1$, a contradiction.

iii) For $\frac{n}{2} - 1 \le j - i \le \frac{3n}{4}$:

a) i +j odd: When $i \le k \le \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\frac{n}{2} + i \le k \le n + i - 3$, we have $r_{E_G}(l_{\frac{n}{2}+i-3}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-2}, R_{E_G})$ and for $n+i-2 \le i-1$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

b) i +j even: When $i - 1 \le k \le \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\frac{n}{2} + i - 1 \le k \le n + i - 3$, we have $r_{E_G}(l_{\frac{n}{2}+i-3}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-2}, R_{E_G})$ and for $n+i-2 \le i-2$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

iv) For $\frac{3n}{4} + 1 \le j - i \le n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \le k \le$ j-i $- \frac{n}{2}$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is even and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j+k$ is odd with $j - i - \frac{n}{2} + 1 \le k \le j - i - 4$. Also for $j - i - 3 \le k \le j - i$, we get $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$, a contradiction.

C) Let $R_{E_G} = \{h_i, h_j, l_k\}$ be the edge version of resolving set with $h_i, h_j \in H$ and $l_k \in L$.

i) For $j - i \le \frac{n}{2} - 3$ and $i \le k \le \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is even then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for odd $i + k$ with $\frac{n}{2} + i \le k \le n + i - 5$. When $n + i - 4 \le k \le i - 1$, we have:

a) i + j odd: $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$.

b) i + j even: $r_{E_G}(l_{i+1}, R_{E_G}) = r_{E_G}(h_{i+1}, R_{E_G})$.

ii) For $\frac{n}{2} - 2 \le j - i \le \frac{3n}{4} + 2$:

a) i +j odd: When $i - 1 \le k \le \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\frac{n}{2} + i - 1 \le k \le n + i - 3$, we have $r_{E_G}(l_{\frac{n}{2}+i-4}, R_{E_G}) =$

$r_{E_G}(h_{\frac{n}{2}+i-3}, R_{E_G})$ and for $n + i - 2 \leq k \leq i - 2$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

b) i +j even: When $i \leq k \leq \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\frac{n}{2} + i \leq k \leq n + i - 3$, we have $r_{E_G}(l_{\frac{n}{2}+i-4}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-3}, R_{E_G})$ and for $n + i - 2 \leq k \leq i - 1$, we have $r_{E_G}(h_{i+1}, R_{E_G}) = r_{E_G}(l_{i+1}, R_{E_G})$, a contradiction.

iii) For $\frac{3n}{4} + 3 \leq j - i \leq n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \leq k \leq$ j-i $- \frac{n}{2}$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is even and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j+k$ is odd with $j - i - \frac{n}{2} + 1 \leq k \leq j - i - 4$. Also for $j - i - 3 \leq k \leq j - i$, we have $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$, a contradiction.

D) Let $R_{E_G} = \{l_i, l_j, h_k\}$ be the edge version of resolving set with $l_i, l_j \in L$ and $h_k \in H$.

i) For $j - i \leq \frac{n}{2} - 2$ and $i \leq k \leq \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ even with $\frac{n}{2} + i - 1 \leq k \leq i - 1$.

ii) For $\frac{n}{2} - 1 \leq j - i \leq \frac{3n}{4}$:

a) i +j even: When $i - 1 \leq k \leq \frac{n}{2} + i - 3$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\frac{n}{2}+i-2 \leq k \leq n + i - 3$, we have $r_{E_G}(l_{\frac{n}{2}+i-3}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-2}, R_{E_G})$ and for $k = \frac{n}{2} + i - 1$, $n + i - 2 \leq k \leq i - 2$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

b) i +j odd: When $i \leq k \leq \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\frac{n}{2} + i - 1 \leq k \leq n + i - 3$, we have $r_{E_G}(l_{\frac{n}{2}+i-3}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-2}, R_{E_G})$ and for $n + i - 2 \leq k \leq i - 1$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

iii) For $\frac{3n}{4} + 1 \leq j - i \leq n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \leq k \leq$ j-i $- \frac{n}{2} - 1$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is odd and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is even with $j - i - \frac{n}{2} \leq k \leq j - i$, a contradiction.

Case 2: For $n \equiv 1(mod 4)$ i.e. $n = 4k + 1$ for $k \geq 2$.

A) Let $R_{E_G} = \{h_i, h_j, h_k\}$ be the edge version of resolving set from the edge set H.

i) For $j - i \leq \lfloor\frac{n}{2}\rfloor - 2$ and $i \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is even then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ odd with $\lfloor\frac{n}{2}\rfloor + i \leq k \leq i - 1$.

ii) For $\lfloor\frac{n}{2}\rfloor - 1 \leq j - i \leq \lfloor\frac{3n}{4}\rfloor + 2$:

a) i +j even: When $i - 1 \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\frac{n}{2}+i-1 \leq k \leq n + i - 5$, we have $r_{E_G}(l_{\frac{n}{2}+i-4}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-3}, R_{E_G})$ and for $n+i-4 \leq i-2$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

b) i +j odd: When $i \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\frac{n}{2} + i \leq k \leq n + i - 4$, we have $r_{E_G}(l_{\frac{n}{2}+i-4}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-3}, R_{E_G})$ and for $n + i - 3 \leq k \leq i - 1$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

iii) For $\lfloor\frac{3n}{4}\rfloor + 3 \leq j - i \leq n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \leq k \leq$ j-i $- \lfloor\frac{n}{2}\rfloor$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is even and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is odd with $j - i - \lfloor\frac{n}{2}\rfloor + 1 \leq k \leq j - i$, a contradiction.

B) Let $R_{E_G} = \{l_i, l_j, l_k\}$ be the edge version of resolving set from the edge set L.

i) For $j - i \leq 4$: When $i \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ even with $\lfloor\frac{n}{2}\rfloor + i - 1 \leq k \leq n + i - 5$ and $r_{E_G}(l_{j+2}, R_{E_G}) = r_{E_G}(h_{j+2}, R_{E_G})$ with $n + i - 4 \leq k \leq i - 1$, a contradiction.

ii) For $5 \leq j - i \leq \lfloor\frac{n}{2}\rfloor - 3$. When $i \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ even with $\lfloor\frac{n}{2}\rfloor + i - 1 \leq k \leq n + i - 5$ and $r_{E_G}(l_{j+2}, R_{E_G}) = r_{E_G}(h_{j+2}, R_{E_G})$ with $n + i - 4 \leq k \leq i - 1$. When $i + j$ is odd then, $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$ and $r_{E_G}(l_{i+3}, R_{E_G}) = r_{E_G}(h_{i+3}, R_{E_G})$ for $i + j$ even with $n + i - 4 \leq k \leq i - 1$, a contradiction.

iii) For $\lfloor\frac{n}{2}\rfloor - 2 \leq j - i \leq \lfloor\frac{3n}{4}\rfloor + 1$:

a) i +j even: When $i \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\lfloor\frac{n}{2}\rfloor + i - 1 \leq k \leq n + i - 4$, we have $r_{E_G}(l_{\lfloor\frac{n}{2}\rfloor+i-3}, R_{E_G}) = r_{E_G}(h_{\lfloor\frac{n}{2}\rfloor+i-2}, R_{E_G})$ and for $n + i - 3 \leq k \leq i - 1$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

b) i +j odd: When $i - 1 \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 3$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $k = \lfloor\frac{n}{2}\rfloor + i - 2$, we have $r_{E_G}(l_{\lfloor\frac{n}{2}\rfloor+i-5}, R_{E_G}) = r_{E_G}(h_{\lfloor\frac{n}{2}\rfloor+i-4}, R_{E_G})$ and for $\lfloor\frac{n}{2}\rfloor + i - 1 \leq k \leq n + i - 4$, we have $r_{E_G}(l_{\lfloor\frac{n}{2}\rfloor+i-3}, R_{E_G}) = r_{E_G}(h_{\lfloor\frac{n}{2}\rfloor+i-2}, R_{E_G})$ and for $n + i - 3 \leq k \leq i - 2$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

iv) For $\lfloor\frac{3n}{4}\rfloor + 2 \leq j - i \leq n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \leq k \leq$ j-i $- \lfloor\frac{n}{2}\rfloor - 2$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is even and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is odd with $j - i - \frac{n}{2} - 1 \leq k \leq j - i$, a contradiction.

C) Let $R_{E_G} = \{h_i, h_j, l_k\}$ be the edge version of resolving set with $h_i, h_j \in H$ and $l_k \in L$.

i) For $j - i \leq \lfloor\frac{n}{2}\rfloor - 2$ and $i \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for even $i + k$ with

$\lfloor \frac{n}{2} \rfloor + i - 1 \le k \le n + i - 5$. When $n + i - 4 \le k \le i - 1$, we have:

a) i + j odd: $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$.

b) i + j even: $r_{E_G}(l_{i+1}, R_{E_G}) = r_{E_G}(h_{i+1}, R_{E_G})$.

ii) For $\lfloor \frac{n}{2} \rfloor - 1 \le j - i \le \lfloor \frac{3n}{4} \rfloor + 1$:

a) i +j even: When $i - 1 \le k \le \lfloor \frac{n}{2} \rfloor + i - 3$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\lfloor \frac{n}{2} \rfloor + i - 2 \le k \le n + i - 5$, we have $r_{E_G}(l_{\lfloor \frac{n}{2} \rfloor + i - 4}, R_{E_G}) = r_{E_G}(h_{\lfloor \frac{n}{2} \rfloor + i - 3}, R_{E_G})$ and for $n + i - 4 \le k \le i - 2$, we have $r_{E_G}(h_{i+1}, R_{E_G}) = r_{E_G}(l_{i+1}, R_{E_G})$, a contradiction.

b) i +j odd: When $i \le k \le \lfloor \frac{n}{2} \rfloor + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\lfloor \frac{n}{2} \rfloor + i - 1 \le k \le n + i - 5$, we have $r_{E_G}(l_{\lfloor \frac{n}{2} \rfloor + i - 4}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2} + i - 3}, R_{E_G})$ and for $n + i - 4 \le k \le i - 1$, we have $r_{E_G}(h_{i+1}, R_{E_G}) = r_{E_G}(l_{i+1}, R_{E_G})$, a contradiction.

iii) For $\lfloor \frac{3n}{4} \rfloor + 2 \le j - i \le n - 1$. We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 2 \le k \le$ j-i $- \lfloor \frac{n}{2} \rfloor - 2$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is even and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is odd with $j - i - \frac{n}{2} - 1 \le k \le j - i - 4$. Also for $j - i - 3 \le k \le j - i$, we have $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$, a contradiction.

D) Let $R_{E_G} = \{l_i, l_j, h_k\}$ be the edge version of resolving set with $l_i, l_j \in L$ and $h_k \in H$.

i) For $j - i \le \lfloor \frac{n}{2} \rfloor - 3$ and $i \le k \le \lfloor \frac{n}{2} \rfloor + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ even with $\lfloor \frac{n}{2} \rfloor + i \le k \le i - 1$.

ii) For $\lfloor \frac{n}{2} \rfloor - 2 \le j - i \le \lfloor \frac{3n}{4} \rfloor$:

a) i +j odd: When $i - 1 \le k \le \lfloor \frac{n}{2} \rfloor + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\lfloor \frac{n}{2} \rfloor + i \le k \le n + i - 5$, we have $r_{E_G}(l_{\lfloor \frac{n}{2} \rfloor + i - 5}, R_{E_G}) = r_{E_G}(h_{\lfloor \frac{n}{2} \rfloor + i - 4}, R_{E_G})$ and for $k = \lfloor \frac{n}{2} \rfloor + i - 1$, $n + i - 4 \le k \le i - 2$ we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

b) i +j even: When $i \le k \le \lfloor \frac{n}{2} \rfloor + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\lfloor \frac{n}{2} \rfloor + i \le k \le n + i - 5$, we have $r_{E_G}(l_{\lfloor \frac{n}{2} \rfloor + i - 5}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2} + i - 4}, R_{E_G})$ and for $n + i - 4 \le k \le i - 1$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

iii) For $\frac{3n}{4} + 1 \le j - i \le n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \le k \le$ j-i $- \frac{n}{2} - 1$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is odd and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is even with $j - i - \frac{n}{2} \le k \le j - i$, a contradiction.

Case 3: For $n \equiv 2(mod4)$ i.e. $n = 4k + 2$ for $k \ge 1$.

A) Let $R_{E_G} = \{h_i, h_j, h_k\}$ be the edge version of resolving set from the edge set H.

i) For $j - i \le \frac{n}{2} - 2$ and $i \le k \le \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ even with $\frac{n}{2} + i \le k \le i - 1$.

ii) For $\frac{n}{2} - 1 \le j - i \le \lfloor \frac{3n}{4} \rfloor + 2$:

a) i +j odd: When $i - 1 \le k \le \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\frac{n}{2} + i - 1 \le k \le n + i - 2$, we have $r_{E_G}(l_{\frac{n}{2} + i - 3}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2} + i - 2}, R_{E_G})$ and for $n + i - 1 \le i - 2$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

b) i +j even: When $i \le k \le \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\frac{n}{2} + i \le k \le n + i - 2$, we have $r_{E_G}(l_{\frac{n}{2} + i - 3}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2} + i - 2}, R_{E_G})$ and for $n + i - 1 \le k \le i - 1$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

iii) For $\lfloor \frac{3n}{4} \rfloor + 3 \le j - i \le n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \le k \le$ j-i $- \frac{n}{2}$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is odd and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is even with $j - i - \frac{n}{2} + 1 \le k \le j - i$, a contradiction.

B) Let $R_{E_G} = \{l_i, l_j, l_k\}$ be the edge version of resolving set from the edge set L.

i) For $j - i \le 4$: When $i \le k \le \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is even then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ odd with $\frac{n}{2} + i - 1 \le k \le n + i - 5$ and $r_{E_G}(l_{j+2}, R_{E_G}) = r_{E_G}(h_{j+2}, R_{E_G})$ with $n + i - 4 \le k \le i - 1$, a contradiction.

ii) For $5 \le j - i \le \frac{n}{2} - 3$: When $i \le k \le \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ and $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ for $i + k$ even with $\frac{n}{2} + i - 1 \le k \le n + i - 5$. For odd $i + j$, $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$ and $r_{E_G}(l_{i+3}, R_{E_G}) = r_{E_G}(h_{i+3}, R_{E_G})$ for even $i + j$ with $n + i - 4 \le k \le i - 1$, a contradiction.

iii) For $\frac{n}{2} - 1 \le j - i \le \lfloor \frac{3n}{4} \rfloor + 1$:

a) i +j odd: When $i \le k \le \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\frac{n}{2} + i - 1 \le k \le n + i - 4$, we have $r_{E_G}(l_{\frac{n}{2} + i - 4}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2} + i - 3}, R_{E_G})$ and for $n + i - 3 \le i - 1$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

b) i +j even: When $i - 1 \le k \le \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\frac{n}{2} + i - 2 \le k \le n + i - 4$, we have $r_{E_G}(l_{\frac{n}{2} + i - 4}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2} + i - 3}, R_{E_G})$ and for $n + i - 3 \le i - 2$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

iv) For $\lfloor \frac{3n}{4} \rfloor + 2 \le j - i \le n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \le k \le$ j-i $- \frac{n}{2}$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is odd and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is even with $j - i - \frac{n}{2} + 1 \le k \le j - i - 4$. Also for $j - i - 3 \le k \le j - i$, we get $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$, a contradiction.

C) Let $R_{E_G} = \{h_i, h_j, l_k\}$ be the edge version of resolving set with $h_i, h_j \in H$ and $l_k \in L$.

i) For $j - i \leq \frac{n}{2} - 2$ and $i \leq k \leq \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for even $i + k$ with $\frac{n}{2} + i \leq k \leq n + i - 5$. When $n + i - 4 \leq k \leq i - 1$, we have:

a) i + j odd: $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$.
b) i + j even: $r_{E_G}(l_{i+1}, R_{E_G}) = r_{E_G}(h_{i+1}, R_{E_G})$.

ii) For $\frac{n}{2} - 1 \leq j - i \leq \lceil\frac{3n}{4}\rceil + 2$:

a) i +j odd: When $i - 1 \leq k \leq \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\frac{n}{2} + i - 1 \leq k \leq n + i - 3$, we have $r_{E_G}(l_{\frac{n}{2}+i-3}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-2}, R_{E_G})$ and for $n + i - 2 \leq k \leq i - 2$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

b) i +j even: When $i \leq k \leq \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\frac{n}{2} + i \leq k \leq n + i - 3$, we have $r_{E_G}(l_{\frac{n}{2}+i-3}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-2}, R_{E_G})$ and for $n + i - 2 \leq k \leq i - 1$, we have $r_{E_G}(h_{i+1}, R_{E_G}) = r_{E_G}(l_{i+1}, R_{E_G})$, a contradiction.

iii) For $\lceil\frac{3n}{4}\rceil + 3 \leq j - i \leq n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \leq k \leq$ j-i $- \frac{n}{2} - 1$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j+k$ is even and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j+k$ is odd with $j-i-\frac{n}{2} \leq k \leq j-i-4$. Also for $j-i-3 \leq k \leq j-i$, we have $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$, a contradiction.

D) Let $R_{E_G} = \{l_i, l_j, h_k\}$ be the edge version of resolving set with $l_i, l_j \in L$ and $h_k \in H$.

i) For $j - i \leq \frac{n}{2} - 3$ and $i \leq k \leq \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ even with $\frac{n}{2} + i \leq k \leq i - 1$.

ii) For $\frac{n}{2} - 2 \leq j - i \leq \lceil\frac{3n}{4}\rceil + 1$:

a) i +j even: When $i - 1 \leq k \leq \frac{n}{2} + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\frac{n}{2} + i \leq k \leq n + i - 4$, we have $r_{E_G}(l_{\frac{n}{2}+i-4}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-3}, R_{E_G})$ and for $k = \frac{n}{2} + i - 1$, $n + i - 3 \leq k \leq i - 2$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

b) i +j odd: When $i \leq k \leq \frac{n}{2} + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\frac{n}{2} + i \leq k \leq n + i - 4$, we have $r_{E_G}(l_{\frac{n}{2}+i-4}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2}+i-3}, R_{E_G})$ and for $n + i - 3 \leq k \leq i - 1$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

iii) For $\lceil\frac{3n}{4}\rceil + 2 \leq j - i \leq n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \leq k \leq$ j-i $- \frac{n}{2}$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is odd and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is even with $j - i - \frac{n}{2} + 1 \leq k \leq j - i$, a contradiction.

Case 4: For $n \equiv 3(mod4)$ i.e. $n = 4k + 3$ for $k \geq 1$.

A) Let $R_{E_G} = \{h_i, h_j, h_k\}$ be the edge version of resolving set from the edge set H.

i) For $j - i \leq \lfloor\frac{n}{2}\rfloor - 1$ and $i \leq k \leq \lceil\frac{n}{2}\rceil + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is even then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ odd with $\lceil\frac{n}{2}\rceil + i \leq k \leq i - 1$.

ii) For $\lfloor\frac{n}{2}\rfloor \leq j - i \leq \lfloor\frac{3n}{4}\rfloor + 2$:

a) i +j even: When $i - 1 \leq k \leq \lceil\frac{n}{2}\rceil + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\lceil\frac{n}{2}\rceil + i - 1 \leq k \leq n + i - 3$, we have $r_{E_G}(l_{\lfloor\frac{n}{2}\rfloor+i-3}, R_{E_G}) = r_{E_G}(h_{\lfloor\frac{n}{2}\rfloor+i-2}, R_{E_G})$ and for $n + i - 2 \leq k \leq i - 2$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

b) i +j odd: When $i \leq k \leq \lceil\frac{n}{2}\rceil + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\lceil\frac{n}{2}\rceil + i \leq k \leq n + i - 3$, we have $r_{E_G}(l_{\lfloor\frac{n}{2}\rfloor+i-3}, R_{E_G}) = r_{E_G}(h_{\lfloor\frac{n}{2}\rfloor+i-2}, R_{E_G})$ and for $n + i - 2 \leq k \leq i - 1$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

iii) For $\lfloor\frac{3n}{4}\rfloor + 3 \leq j - i \leq n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \leq k \leq$ j-i $- \lfloor\frac{n}{2}\rfloor$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is even and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is odd with $j - i - \lfloor\frac{n}{2}\rfloor + 1 \leq k \leq j - i$, a contradiction.

B) Let $R_{E_G} = \{l_i, l_j, l_k\}$ be the edge version of resolving set from the edge set L.

i) When $j - i \leq 4$. For $i \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ even with $\lfloor\frac{n}{2}\rfloor + i \leq k \leq n + i - 5$ and $r_{E_G}(l_{j+2}, R_{E_G}) = r_{E_G}(h_{j+2}, R_{E_G})$ with $n + i - 4 \leq k \leq i - 1$, a contradiction.

ii) For $5 \leq j - i \leq \lfloor\frac{n}{2}\rfloor - 1$: When $i \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ even with $\lfloor\frac{n}{2}\rfloor + i \leq k \leq n + i - 5$. When $i + j$ is odd then, $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$ and $r_{E_G}(l_{i+3}, R_{E_G}) = r_{E_G}(h_{i+3}, R_{E_G})$ for $i+j$ even with $n+i-4 \leq k \leq i-1$, a contradiction.

iii) For $\lfloor\frac{n}{2}\rfloor \leq j - i \leq \lfloor\frac{3n}{4}\rfloor + 1$:

a) i +j even: When $i \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\lfloor\frac{n}{2}\rfloor + i \leq k \leq n + i - 4$, we have $r_{E_G}(l_{\lfloor\frac{n}{2}\rfloor+i-3}, R_{E_G}) = r_{E_G}(h_{\lfloor\frac{n}{2}\rfloor+i-2}, R_{E_G})$ and for $n + i - 3 \leq k \leq i - 1$ we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

b) i +j odd: When $i - 1 \leq k \leq \lfloor\frac{n}{2}\rfloor + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\lfloor\frac{n}{2}\rfloor + i - 1 \leq k \leq n + i - 4$, we have $r_{E_G}(l_{\lfloor\frac{n}{2}\rfloor+i-3}, R_{E_G}) = r_{E_G}(h_{\lfloor\frac{n}{2}\rfloor+i-2}, R_{E_G})$ and for $n + i - 3 \leq k \leq i -$

2, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

iv) For $\lfloor \frac{3n}{4} \rfloor + 2 \leq j - i \leq n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \leq k \leq$ j-i $- \lfloor \frac{n}{2} \rfloor - 1$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is even and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is odd with $j - i - \lfloor \frac{n}{2} \rfloor \leq k \leq j - i - 4$. $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$, for $j - i - 3 \leq k \leq j - i$, a contradiction.

C) Let $R_{E_G} = \{h_i, h_j, l_k\}$ be the edge version of resolving set with $h_i, h_j \in H$ and $l_k \in L$.

i) For $j - i \leq \lfloor \frac{n}{2} \rfloor - 1$ and $i \leq k \leq \lfloor \frac{n}{2} \rfloor + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is odd then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for even $i + k$ with $\lfloor \frac{n}{2} \rfloor + i \leq k \leq n + i - 5$. When $n + i - 4 \leq k \leq i - 1$, we have:

a) i + j odd: $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$.
b) i + j even: $r_{E_G}(l_{i+1}, R_{E_G}) = r_{E_G}(h_{i+1}, R_{E_G})$.

ii) For $\lfloor \frac{n}{2} \rfloor \leq j - i \leq \lceil \frac{3n}{4} \rceil + 2$:

a) i + j even: When $i - 1 \leq k \leq \lfloor \frac{n}{2} \rfloor + i - 2$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\lfloor \frac{n}{2} \rfloor + i - 1 \leq k \leq n + i - 3$, we have $r_{E_G}(l_{\lfloor \frac{n}{2} \rfloor + i - 3}, R_{E_G}) = r_{E_G}(h_{\lfloor \frac{n}{2} \rfloor + i - 2}, R_{E_G})$ and for $n + i - 2 \leq k \leq i - 2$, we have $r_{E_G}(h_{i+1}, R_{E_G}) = r_{E_G}(l_{i+1}, R_{E_G})$, a contradiction.

b) i + j odd: When $i \leq k \leq \lfloor \frac{n}{2} \rfloor + i - 1$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\lfloor \frac{n}{2} \rfloor + i \leq k \leq n + i - 3$, we have $r_{E_G}(l_{\lfloor \frac{n}{2} \rfloor + i - 3}, R_{E_G}) = r_{E_G}(h_{\frac{n}{2} + i - 2}, R_{E_G})$ and for $n + i - 2 \leq k \leq i - 1$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

iii) For $\lceil \frac{3n}{4} \rceil + 3 \leq j - i \leq n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \leq k \leq$ j-i $- \lfloor \frac{n}{2} \rfloor - 1$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j+k$ is even and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j+k$ is odd with $j - i - \frac{n}{2} \leq k \leq j - i - 4$. Also for $j - i - 3 \leq k \leq j - i$, we have $r_{E_G}(l_{i+2}, R_{E_G}) = r_{E_G}(h_{i+2}, R_{E_G})$, a contradiction.

D) Let $R_{E_G} = \{l_i, l_j, h_k\}$ be the edge version of resolving set with $l_i, l_j \in L$ and $h_k \in H$.

i) For $j - i \leq \lfloor \frac{n}{2} \rfloor - 2$ and $i \leq k \leq \lfloor \frac{n}{2} \rfloor + i$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$. When $i + k$ is even then, $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$ and $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$ for $i + k$ odd with $\lfloor \frac{n}{2} \rfloor + i + 1 \leq k \leq i - 1$.

ii) For $\lfloor \frac{n}{2} \rfloor - 1 \leq j - i \leq \lfloor \frac{3n}{4} \rfloor + 1$:

a) i + j odd: When $i - 1 \leq k \leq \lfloor \frac{n}{2} \rfloor + i - 1$, we have $r_{E_G}(l_{i-3}, R_{E_G}) = r_{E_G}(h_{i-2}, R_{E_G})$, for $\lfloor \frac{n}{2} \rfloor + i \leq k \leq n + i - 4$, we have $r_{E_G}(l_{\lfloor \frac{n}{2} \rfloor + i - 4}, R_{E_G}) = r_{E_G}(h_{\lfloor \frac{n}{2} \rfloor + i - 3}, R_{E_G})$ and for $\lfloor \frac{n}{2} \rfloor + i \leq k \leq +i - 4$, $n + i - 3 \leq k \leq i - 2$, we have $r_{E_G}(h_{i+2}, R_{E_G}) = r_{E_G}(l_{i+2}, R_{E_G})$, a contradiction.

b) i + j even: When $i \leq k \leq \lfloor \frac{n}{2} \rfloor + i$, we have $r_{E_G}(l_{i-2}, R_{E_G}) = r_{E_G}(h_{i-1}, R_{E_G})$, for $\lfloor \frac{n}{2} \rfloor + i + 1 \leq k \leq n + i - 4$, we have $r_{E_G}(l_{\lfloor \frac{n}{2} \rfloor + i - 4}, R_{E_G}) = r_{E_G}(h_{\lfloor \frac{n}{2} \rfloor + i - 3}, R_{E_G})$ and for $n + i - 3 \leq k \leq i - 1$, we have $r_{E_G}(h_{i+3}, R_{E_G}) = r_{E_G}(l_{i+3}, R_{E_G})$, a contradiction.

iii) For $\lfloor \frac{3n}{4} \rfloor + 2 \leq j - i \leq n - 1$: We have $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$ with $j - i + 1 \leq k \leq$ j-i $- \lfloor \frac{n}{2} \rfloor - 1$, $r_{E_G}(l_{j-3}, R_{E_G}) = r_{E_G}(h_{j-2}, R_{E_G})$, when $j + k$ is even and $r_{E_G}(l_{j-2}, R_{E_G}) = r_{E_G}(h_{j-1}, R_{E_G})$, when $j + k$ is odd with $j - i - \frac{n}{2} \leq k \leq j - i$, a contradiction.

All the above possibilities lead to contradiction. Hence, there is no edge version of resolving set of order 3 in the edge set E_G, which implies that $dim_E(G) = 4$. ■

III. CONCLUSION

In this paper, we have investigated the notion of the edge version of metric dimension of the family of circulant graphs $C_n(1, 2)$ which is the least cardinality over all the edge version of resolving sets of $C_n(1, 2)$. It is interesting to consider the family of the circulant graphs because its edge version of metric dimension is independent of parity of n. Finally, we get $dim_E(C_n(1, 2)) = 4$ for $n \geq 6$.

Open Problem 1: Find the edge version of metric dimension of the family of circulant graphs $C_n(1, 3)$.

Open Problem 2: Find the edge version of metric dimension of the family of circulant graphs $C_n(1, 2, 3)$.

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