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A Linear Parameter Varying Strategy Based Integral Sliding Mode Control Protocol Development and Its Implementation on Ball and Beam Balancer

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ABSTRACT An Integral Sliding Mode (ISM) based robustified Linear Parameter Varying (LPV) control is presented for a laboratory scale ball on a beam balancer system. The inherent input channel nonlinearity and under-actuated coupled dynamics make it a challenging control problem. The ISM control, being famous for reaching phase elimination, operates under the action of a control input which is usually an algebraic sum of a continuous and a discontinuous component. In the design of continuous component, the LPV form is used to linearize the otherwise non linearizable input channel nonlinearity. Hence, an LPV control is designed to cope with the varying dynamics. On the other hand, the discontinuous component diminishes the effect of norm bounded matched disturbances. In addition, the discontinuous control component is made smooth (chattering free) in order to generate a continuous control signal. The stability of the proposed algorithm is presented rigorously in terms of a theorem and is validated experimentally.

INDEX TERMS Ball and beam balancer, integral sliding mode control, linear matrix inequality, linear parameter varying approach.

I. INTRODUCTION

The dynamics of the Ball on a Beam Balancer system offers a variety of challenges to control theoreticians and practitioners. The prominent challenges include the inherent open-loop instability, coupled dynamics and more importantly input channel nonlinearity. Moreover, like many other mechanical systems [1], [2], the dynamics of the ball and beam assemble are also underactuated in nature. Thus a suitable technique to cope with these challenges (see [3], [4]) is of importance. In addition to these challenges, the fact that the ball on a beam balancer system is used to realize/prototype many complex systems, became an other pronounced point of attention. Examples of such systems include the vertical take-off aircraft, control of vertical thrust in rockets and one Degree-of-Freedom (DoF) of a stabilizing platform.

The linear control theory is very mature in its design strategies which include $\mu-$synthesis, loop shaping, LMI, LQR, root locus, Bode and state space etc. These generalized techniques served the demands of control system community by providing constrained performance and robustness. The inherent problems of linear control theory e.g., local performance, local stability and degradation in performance due to the presence of parametric variations, diverted the control

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theoreticians and practitioners towards the development of nonlinear control and Linear Parameter Varying (LPV) based gain scheduling algorithms.

LPV based gain scheduling approach, being famous for its robustness against parametric variations, was proposed via a number of researchers [5]–[7]. In addition, the LPV based controllers, by the virtue of LPV form (in which we do not actually linearize the system but it appears to be linear), facilitated the construction of global linear controllers directly for nonlinear systems [8], [9]. Yet the LPV based gain scheduling controllers were very sensitive to model imperfection, uncertainties and external disturbances [10].

The most popular control algorithm which deals with the nonlinear systems directly, accompanied by its remarkable robustness, is known as the Sliding Mode Control (SMC) [11], [12]. The idea of SMC was to in-force *sliding modes* in a given system's state space via the application of a discontinuous controller. The discontinuous controller undergo excessive actions for bringing the system's dynamics onto a pre-defined sliding manifold. This stage of the algorithm was termed as the reaching phase while the phenomenon of excessive oscillations about the sliding manifold was termed as "*chattering*". A number of SMC variants were proposed to eliminate the chattering and/or to improve the performance. These variants include Higher Order Sliding Mode Control (HOSMC) [13], Dynamic Sliding Mode Control (DSMC) [14] and a class of neural networks based adaptive sliding mode controllers [15], [16]. All of these techniques are extensively reported with their merits (smooth control actions, model-free nature of some the algorithms and accuracy in performance) and demerits (Chattering may appear sooner or later due to the un-modeled fast dynamics, requirement of an extra differentiator, in case of DSMC, can cause noise amplification and obvious computation overhead in case of neural networks based adaptive SMC). The fact that in reaching phase the system dynamics were sensitive to disturbances provoke another variant of SMC. In [17], [18] the Integral Sliding Mode (ISM) control was proposed which eliminated the reaching phase by using an integral manifold as a surface. The elimination of reaching phase improved the performance and robustness but the inherent property of order reduction and hence parameter invariance was sacrificed [14], [19].

A number of control techniques (model-free, model based) have been used for stabilizing the ball and beam system. These include many variants of fuzzy controllers [20], [21], observer based discrete LQR technique [22], adaptive control techniques [23], PID [24] and SMC [25], [26]. Each of these schemes has their certain merits and demerits.

In this work, a novel control scheme, making use of LPV form and ISM control, is presented for stabilizing the dynamics of the ball and beam assembly. The hybrid algorithm carries with itself the merits of both (ISM and LPV) the algorithms. The discontinuous part of the ISM control is designed such that it provides a smooth control action and robustness against disturbances of matched type while the continuous control component is designed making use of LPV based gain scheduling state feedback controller, which ensures parameter in-variance while steering the dynamics to an equilibrium. In addition, the LPV form enables the otherwise non-linearizable system to look linear for design purposes (the fact will be discussed later in detail). The technique is validated experimentally on the physical system and simulations are performed for comparison purposes.

The rest of this paper is organized as follow. Section II gives a brief introduction to the problem statement. Also how this problem is handled for a SISO nonlinear/linear system, is given in this section. Section III explores the mathematical setup for stability and design of Smooth ISM Control (SISMC). The design of integral manifold, smooth discontinuous control component and LPV based gain scheduling continuous part is carried out in this section. Also the main results are given in this section in the form of a theorem. Section IV starts with a physical and mathematical description of experimental setup followed by a brief introduction to the problem statement. In addition, a comparison of different control schemes is presented. The section ends with the experimental results and their brief description. The paper is concluded in Section V followed by the acknowledgment and references.

II. PROBLEM FORMULATION

Consider a Single Input Single Output (SISO) dynamical system, in the standard state space form.

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ &\cdot\\ &\cdot\\ \dot{x}_n &= f(\mathbf{x(t)}) + bg(\mathbf{x(t)}) + bu(t),\end{aligned} \tag{1}$$

where $\mathbf{x(t)} \in \Re^n$ is the state vector, $g(\mathbf{x(t)}) : \Re^n \rightarrow \Re$ is the matched disturbance bounded by the non-negative constant η i.e., $|g(\mathbf{x(t)})| < \eta$, $u(t) \in \Re$ is the control input and $b \in \Re$ is the input channel. The function $f(\mathbf{x(t)}) : \Re^n \rightarrow \Re$ is a linear or nonlinear function of states, which may have time or state dependent parameter/s.

Assumption 1: The possible nonlinearity and/or parametric variations in the function $f(\mathbf{x(t)})$ can be formulated as an LPV problem to get $f(\rho, \mathbf{x(t)})$ such that $f(\rho, \mathbf{x(t)})$ is linear in states and depends affinely on scheduling parameter ρ [27].

As mentioned earlier the ISM control being famous for reaching phase elimination, generates a control input $u(t)$ which is the algebraic sum of a continuous part $u_0(t)$ and a discontinuous part $u_1(t)$ i.e., $u(t) = u_0(t) + u_1(t)$. Then based on assumption 1, and the fact that $u_1(t) = 0$ (as we will see later) on the surface, the dynamics during *sliding* can be represented as follows.

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&. \\
&. \\
\dot{x}_n &= f(\rho, \mathbf{x(t)}) + bu_0(t).
\end{aligned} \tag{2}
$$

The function $f(\rho, \mathbf{x(t)})$ is now linear in state variables $(x_1(t), x_2(t), x_3(t) \ldots\ldots x_n(t))$ and depends affinely on the parameter ρ. Moreover, ρ is upper bounded by $\overline{\rho}$ and lower bounded by $\underline{\rho}$ i.e., $\underline{\rho} \leq \rho \leq \overline{\rho}$.

The clear dependence of the dynamics, in (2), on ρ reveals the fact that a simple state feedback linear controller or a robust H_∞ with the pre-defined gains/weights, for steering these sliding mode dynamics to an equilibrium, will not be able to perform well for all the values of ρ [7].

III. CONTROLLER CONSTRUCTION

The ISM control is re-formulated in the following three steps.

A. DESIGN OF INTEGRAL MANIFOLD

The integral manifold S for ISM control, given in [14] is,

$$S = S_0 + Z, \tag{3}$$

where $S_0 = \sum_{i=1}^{n} c_i x_i$ with $c_n = 1$, is the linear combination of states and appears as a Hurwitz and monic polynomial while Z is the integral part which will be defined in the subsequent paragraphs.

Taking the time derivative of (3) along the trajectories of (1) with $f(\mathbf{x(t)})$ formulated as $f(\rho, \mathbf{x(t)})$.

$$\dot{S} = \sum_{i=1}^{n-1} c_i x_{i+1} + \dot{x}_n + \dot{Z}. \tag{4}$$

Evaluating (4) for $\dot{S}$ gives,

$$\dot{Z} = -\sum_{i=1}^{n-1} c_i x_{i+1} - f(\rho, \mathbf{x(t)}) - bu_0(t). \tag{5}$$

This choice of Z makes the integral manifold, (3), dependent upon the parameter ρ.

B. DESIGN OF DISCONTINUOUS PART

The discontinuous part of the controller may be a first order or a second order SMC [28]. Evaluating (4) with $\dot{Z}$ given in (5),

$$\dot{S} = b(g(\mathbf{x(t)}) + u_1(t)). \tag{6}$$

Taking a smooth discontinuous term:

$$u_1(t) = -K|S|^\kappa \operatorname{sign}(b)\operatorname{sign}(S), \tag{7}$$

where $|S|^\kappa$ is the smoothing term and the constant κ is defined as:

$$\kappa = \begin{cases} -1/2, & \text{if } 0 < S < 1 \\ 0, & \text{if } S = 0 \\ 1/2, & \text{if } S < 0 \text{ or } S \geq 1. \end{cases} \tag{8}$$

Then taking the total time derivative of a Lyapunov function $V(t, \mathbf{x(t)}) = \frac{1}{2}S^2$, we have:

$$
\begin{aligned}
\dot{V}(t, \mathbf{x(t)}) &= Sb(g(x(t)) - K|S|^\kappa \operatorname{sign}(b)\operatorname{sign}(S)), \\
&\leq Sb(\eta - K|S|^\kappa \operatorname{sign}(b)\operatorname{sign}(S)), \\
&\leq Sb\eta - K\,|S|^{\kappa+1}\,|b|\,.
\end{aligned}
$$

Since, $|S|^{\kappa+1} \geq S$, $|b| \geq b$ and choosing K to be a positive non-zero constant such that $K > \eta$, makes $\dot{V}(t, x(t)) \leq 0$ (semi-negative definite). This ensures the existence of *sliding modes* in the integral manifold. Moreover, the problem of excessive gains in the vicinity of the manifold is also coped with using the adaptive gain. This gain adaptivity of the discontinuous part ensures chattering elimination and hence smoothed out the control effort.

C. DESIGN OF CONTINUOUS PART

The ISMC begins with established sliding modes (see (2)) and a valid $u_1(t)$ ((7)) keeps the dynamics thereafter but still an appropriate $u_0(t)$ is needed to steer the dynamics in (2) to an equilibrium. In this regard representing (2) in the following state space form.

$$\dot{\mathbf{x}}(\mathbf{t}) = A(\rho)\mathbf{x(t)} + Bu_0(t), \tag{9}$$

where $A(\rho) \in \Re^{n\times n}$ is a parameter dependent system matrix, $\mathbf{x(t)} \in \Re^n$ is the state vector and $B \in \Re^n$ is the input matrix.

Remark 1: The continuous part $(u_0(t))$ of the SISMC is to be designed as a gain scheduled controller in order to stabilize the dynamics in (9) for any value of parameter $(\underline{\rho} \leq \rho \leq \overline{\rho})$.

Assumption 2: The current value of the parameter ρ is available.

The introduction of gain scheduling state feedback controller $u_0(t) = M(\rho)x(t)$, where $M(\rho) \in \Re^n$ is the parameter dependent gain matrix, gives the following closed loop dynamics.

$$
\begin{aligned}
\dot{\mathbf{x}}(\mathbf{t}) &= (A(\rho) + BM(\rho))\mathbf{x(t)}, \\
&= A_{cl}(\rho)\mathbf{x(t)},
\end{aligned} \tag{10}
$$

where $A_{cl}(\rho)$ is the closed loop parameter dependent system matrix. The closed loop dynamics in (10) will be stable if there exist a common, symmetric and positive definite matrix P such that the following inequality is satisfied for all the values of the parameter ρ [29], [30].

$$
\begin{aligned}
&PA_{cl}^T(\rho) + A_{cl}(\rho)P < 0, \\
&\quad PA^T(\rho) + A(\rho)P + PM^T(\rho)B^T + BM(\rho)P < 0.
\end{aligned} \tag{11}
$$

Note that $\rho \in co(\underline{\rho}, \overline{\rho})$ which mean that (10) represent a polytopic system and (11) represent the infinite number of LMIs [31]. So despite solving the infinite LMIs for a common P, it is worthy to solve them only at the vertices of the convex hull.

Theorem 1: With $u_1(t)$ given in (7), if the inequality in (11) is satisfied with a common symmetric and positive definite matrix P at the vertices of the convex hull $co(\underline{\rho}, \overline{\rho})$, then the dynamics in (1) will exhibit convergent sliding modes. Moreover, if the following LMI,

$$L \otimes P + N \otimes A_{cl}(\rho)P + N^T \otimes (A_{cl}(\rho)P)^T < 0, \quad (12)$$

where, $\otimes$ stands for Kronecker product, is satisfied then (2) will be D-stable and will have poles in the D-region defined by the matrices L and N.

Proof: The first part of the theorem has already been proved but is mentioned here for the sake of completeness. Taking a positive definite Lyapunov function

$$V(t, \mathbf{x(t)}) = \frac{1}{2}S^2.$$

Taking the total time derivative of $V(.)$ along the trajectories of the system in (1) and using equations (3), (5), (6) and (7), we have,

$$\begin{aligned}\dot{V} &= Sb\,(g(t) + u_1(t))\,,\\ &\leq Sb\,(\eta + u_1(t))\,,\\ &\leq Sb\left(\eta - K|S|^{\kappa}\text{sign}(b)\text{sign}(S)\right),\\ &\leq Sb\eta - K|S|^{\kappa+1}\,|b|\,.\end{aligned}$$

As $S \leq |S|^{\kappa+1}$, $b \leq |b|$ and $\eta < K$, so

$$\dot{V} \leq -\left|Sb\eta - k|S|^{\kappa+1}\,|b|\right| \leq 0, \quad (13)$$

which confirms the existence of sliding modes.

For the proof of second part, solving the LMI in (12), at the vertices, for P. In this regard let v be the left eigen vector of $A_{cl}(\rho)$ corresponding to eigen value λ, then

$$v^H A_{cl}(\rho) = \lambda v^H,$$

and by congruence transformation

$$\begin{aligned}&(I \otimes v^H)[L \otimes P + N \otimes A_{cl}(\rho)P \ldots\\ &\quad + N^T \otimes (A_{cl}(\rho)P)^T](I \otimes v) < 0,\\ &\quad L \otimes v^H P v + N \otimes v^H A_{cl}(\rho)Pv \ldots\\ &\quad + N^T \otimes v^H P A_{cl}^T(\rho)v < 0,\\ &\quad L + \lambda N + \lambda^* N^T < 0, \quad (14)\end{aligned}$$

where λ^* represent the complex conjugate of eigen value λ. The inequality in (14) defines the sufficient condition for the eigen values of $A_{cl}(\rho)$ to be in the LMI region characterized by L and N.

The algebraic expression for the scheduled gain $M(\rho)$ can be found directly by evaluating (12) at the vertices.

$$\begin{aligned}&L \otimes P + N \otimes (A(\underline{\rho}) + BM(\underline{\rho}))P \ldots\\ &\quad + N^T \otimes ((A(\underline{\rho}) + BM(\underline{\rho}))P)^T < 0, \quad (15)\end{aligned}$$

where, $M(\underline{\rho})$ is the controller gain at vertex characterized by $\underline{\rho}$. The inequality (15) is nonlinear in the variables $M(\underline{\rho})$ and P. In order to make it linear a new variable $\psi(\underline{\rho}) = M(\underline{\rho})P$ is introduced.

$$\begin{aligned}&L \otimes P + N \otimes (A(\underline{\rho})P + B\psi(\underline{\rho}))\ldots\\ &\quad + N^T \otimes ((A(\underline{\rho})P + B\psi(\underline{\rho}))^T < 0. \quad (16)\end{aligned}$$

Solution of (16) for P and $\psi(\underline{\rho})$ gives $M(\underline{\rho}) = \psi(\underline{\rho})P^{-1}$. Similarly the gain $M(\overline{\rho})$ can be found from (16) using the same P.

The final scheduled controller gain $M(\rho)$ can be obtained from the algebraic weighted sum of $M(\underline{\rho})$ and $M(\overline{\rho})$.

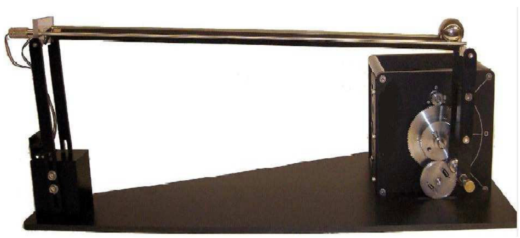
$$M(\rho) = r_1 M(\overline{\rho}) + r_2 M(\underline{\rho}),$$

where $r_1 + r_2 = 1$, are the weighting constants and has the following mathematical representation.

$$\begin{aligned}r_1 &= \frac{\overline{\rho} - \rho}{\overline{\rho} - \underline{\rho}},\\ r_2 &= \frac{\rho - \underline{\rho}}{\overline{\rho} - \underline{\rho}}.\end{aligned}$$

In the next section the proposed algorithm is applied to laboratory test bench, Ball on a Beam Balancer (BBB). The subject test bench is under actuated in nature with coupled dynamics and an inherent nonlinearity in control input channel. The conventional linear approximations of this system fail due to practically invalid assumption of *input being small enough*. The proposed algorithm does not actually linearize the system during sliding rather makes it *look linear* for controller design purposes.

IV. BALL ON A BEAM BALANCER



The *Ball on a Beam Balancer* is considered as an important test bench in the field of control engineering because of the wider spectrum in the form of nonlinearity and inherent open loop instability. The major task of this test bench is to control the position of a stainless steel ball over a metallic beam.

A. PHYSICAL DESCRIPTION

The left most end of the beam is fixed while the right most end can be stimulated for up and down motion by means of a DC servo motor and gear assembly. The ball position is measured from voltage variations, created by the ball movement, at the beam while the angular position of the motor is measured by an absolute potentiometer. The control signal generated by an interfaced computer is given to DC servo motor via a power amplifier.

TABLE 1. Physical specifications.

Entity	Notation/Value/Unit
Beam Length	L, 41 cm
Lever Arm Offset	r, 2.5 cm
Servo Gear Angle	θ_l, ± 60 deg
Moment of Inertia	J_{eq}, 2.084×10^{-3} $kg.m^2$
Back EMF Constant	C_1, 7.68×10^{-3} $V.Sec/rad$
Damping co-efficient	B_{eq}, 8.40×10^{-2}
Torque per unit voltage	γ, 0.13 $N.m/volt$
Control Voltage	V, $Volts$
Gravitation Constant	g, 9.81 m/sec^2
Ball Position	x, -0.2 to 0.2 m

B. MATHEMATICAL DESCRIPTION

The simplified mathematical model of the subject test bench is reported as two coupled systems (see [32] and www.quanser.com) i.e., the dynamics of servo DC motor

$$\ddot{\theta}_l(t) = -\frac{B_{eq}}{J_{eq}}\dot{\theta}_l(t) + \frac{\gamma}{J_{eq}}V(t), \tag{17}$$

and the dynamics of ball moving over the beam.

$$\ddot{x}(t) = \frac{5rg}{7L}\sin(\theta_l). \tag{18}$$

The variables and parameters are given in Table 1. Note that the negative x values are used to differentiate between the right and left side of the beam center. Moreover, the mathematical model shows that the ball is stimulated by the shaft angle of the DC servo motor (θ_l) while θ_l is the result of the voltage applied (V) to the DC servo motor.

Remark 2: The only actual input in this system is V, which makes it an under actuated system.

C. PROBLEM DESCRIPTION

The control objective is to operate the DC servo motor in such a way that the ball is kept robustly at the beam center, despite all nonlinearities and the under actuated nature of the systems' dynamics.

The main difficulty in designing a linear controller for the ball dynamics (see (18)) is that $\sin(\theta_l)$ cannot be approximated equivalent to θ_l because the variation in θ_l is not small (see Table 1). This is shown in Fig. 1, where the response of the linearized system, to an arbitrary sinusoidal input, becomes unstable while the nonlinear and LPV (to be presented next) models, of the same system, follow each other.

Moreover, the robustness of the proposed LPV based SISMC, in the presence of a matched disturbance, is compared with a First Order Sliding Mode (FOSM) controller and a standalone LPV based gain scheduling controller, shown in Fig. 2. All the controllers perform well under steady state. However, with the introduction of the disturbance, at 18^{th} *Sec* of the simulation, the stand alone LPV is proved to be the least robust while the proposed LPV based SISMC outperforms the other controllers. In addition, an inspection of the control efforts generated by all the three controllers is also presented

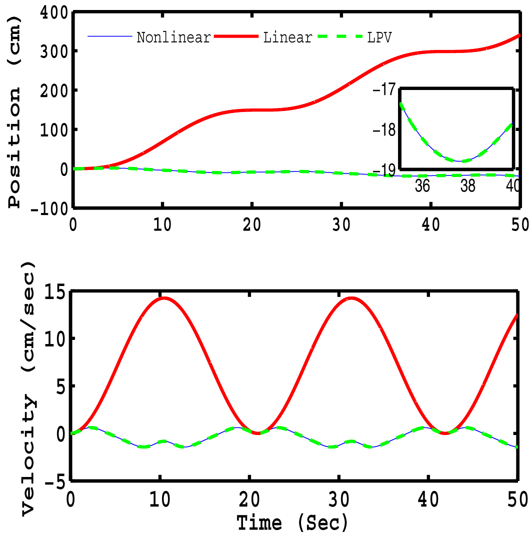


FIGURE 1. Comparison of linear, Nonlinear and LPV models.

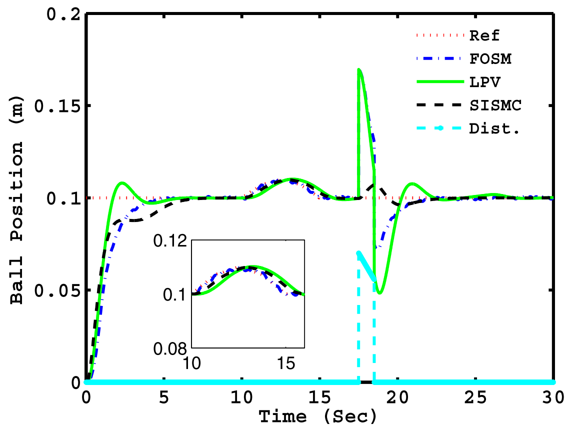


FIGURE 2. Robustness comparison of FOSM, LPV and SISMC.

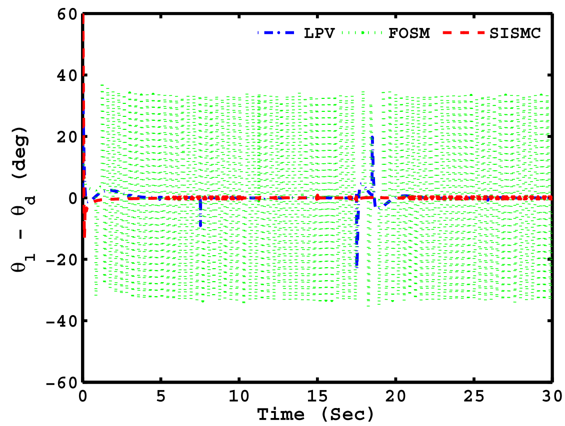


FIGURE 3. Comparison of control signals.

in Fig. 3. This clearly reveals the smoothening capability of the proposed algorithm.

Remark 3: The existence of the discontinuous part in the hybrid SISMC ensures robustness while the variable gain ensures the chattering elimination.

In the next section the proposed algorithm is validated experimentally.

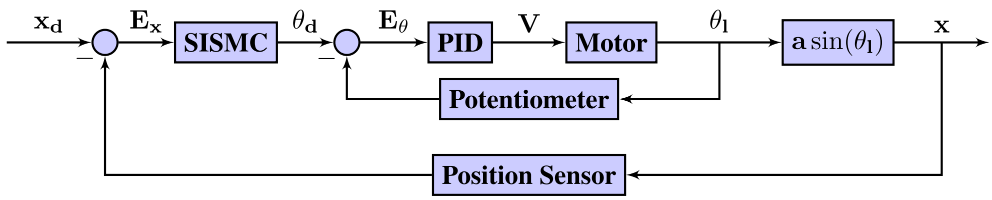


FIGURE 4. Control configuration for ball on a beam balancer.

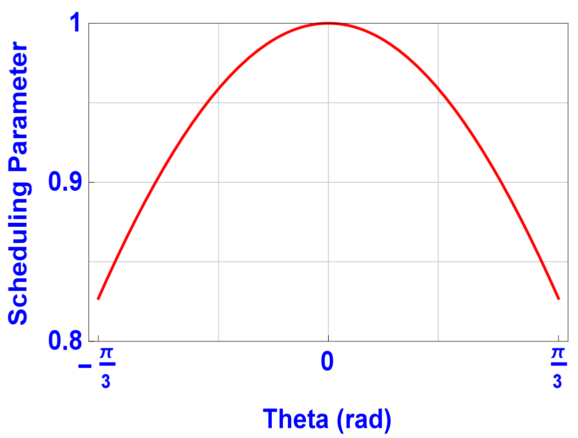


FIGURE 5. Scheduling parameter.

TABLE 2. Controller parameters and software specifications.

Notation	Value/Range
S	$S_0 + Z$
S_0	$\dot{e} + Ce$
Z	$-\int(a\rho u_0 + C\dot{e})dt$
e	$x_d - x$
$\overline{\rho}$	1
$\underline{\rho}$	0.78
$M(\overline{\rho})$	$\begin{bmatrix}20 & 10\end{bmatrix}$
$M(\underline{\rho})$	$\begin{bmatrix}-20 & -10\end{bmatrix}$
u_0	$M(\rho)\begin{bmatrix}e & \dot{e}\end{bmatrix}^T$
u_1	$-K\,\lvert S\rvert^{\kappa}\,Sign(S)Sign(\rho)$
θ_d	$u_0 + u_1$
K	0.4
C	0.5
MATLAB/Simulink	7.10
Visual Studio	2008
Quanser Real Time Control	QuaRC
Mathematica	10
Step Time	0.001 Sec

D. EXPERIMENTAL RESULTS

A multi loop control strategy, to cope with the under actuation, is devised. A virtual controller takes into account the current ball position and the reference position to produce corrective actions for an inner loop PID controller. This is shown in Fig. 4.

The outer loop is equipped with the proposed SISMC algorithm for controlling the ball position by generating a reference angle for the inner loop. A PID controller in the inner loop takes reference angle from SISMC and provides a controlled voltage to the DC motor.

In this work the nonlinearity in control channel is handled such that the system appears linear in states and control input. Let the scheduling parameter is $\rho = \sin(\theta_l)/(\theta_l + \epsilon)$, where $\epsilon \rightarrow 0$. Fig. 5 shows a plot of ρ for range of θ_l given in Tab. 1.

Remark 4: The value of K should be chosen such that it ensures the stability while C is chosen to ensure transient performance of the sliding modes.

Hence, the dynamics of the ball moving over the metallic beam can be represented in LPV form,

$$\ddot{x}(t) = a\rho\theta_l,$$

where $a = 5rg/7L$ and the corresponding state space is,

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= a\rho\theta_l.\end{aligned}$$

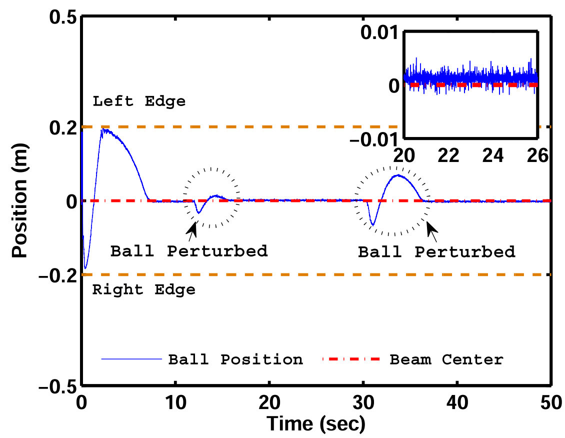


FIGURE 6. Ball position.

These state equations are used to formulate the proposed LPV based SISMC. Details of the algorithms' parameters, used for experimentation, are given in Table 2.

In Fig. 6 it is shown that the ball is balanced, at the beam center, by the SISMC algorithm in less than 10 seconds. Also notice that the ball has been disturbed (perturbed from the center) at around 11^{th} second and 30^{th} second of the experiment. In both the cases the ball effectively comes back to the center of the beam. The zoomed portion in Fig. 6 shows the accuracy of the SISMC algorithm. The negative values on position axis are just for differentiating the left and right side of the beam with respect to its center.

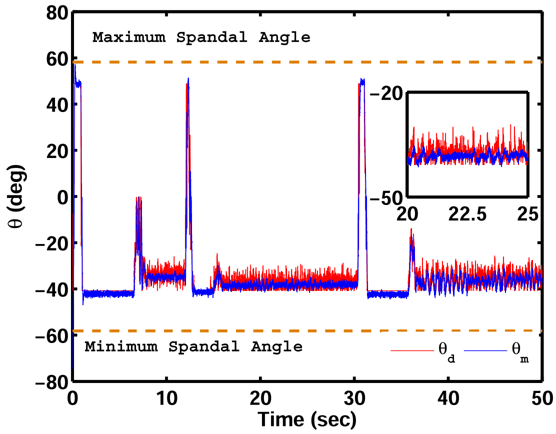


FIGURE 7. Desired and measured angles for servo motor.

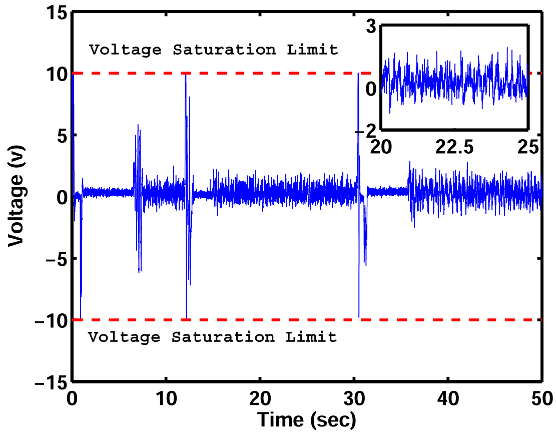


FIGURE 8. Voltage applied to motor.

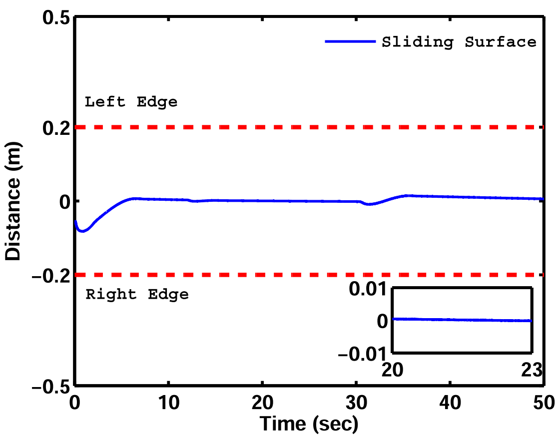


FIGURE 9. Sliding surface.

In Figure 7 the control effort (θ_d) produced by SISMC and the corresponding motor angle (θ_l) are shown. The controller is effectively responding to any change in ball position, keeping the ball at the beam center and importantly without causing saturation to the motor.

Remark 5: Higher values for K and/or C may cause the motor to run into saturation limits.

Remark 6: The smooth nature of the SISMC produces θ_d smooth enough that it does not disturb the performance of the inner loop PID control.

In figure 8 the control effort ($V(t)$) produced by the PID controller such that θ_l tracks θ_d, is shown. It is important to observe that due to the smoothness of the SISMC the performance of the inner loop PID is not affected.

Figure 9 shows the sliding surface (S). $S = 0$ is maintained effectively with sliding accuracy of 10^{-4}.

V. CONCLUSION

The laboratory test bench, ball on a beam balancer, is a challenging control problem due its under actuated nature and the input channel nonlinearity. Since the linearization of the dynamics is practically impossible so the LPV form is preferred which makes the system appear linear in state variables and affine in control input. However, the LPV based gain scheduling controllers are not robust against external disturbances. Hence, a hybrid of Integral Sliding Mode Control (ISMC) and LPV based gain scheduling controller is proposed.

The reaching phase elimination property of ISMC, good for robustness and performance, subtracts a useful advantage of the order reduction from this algorithm. As a result the performance of the sliding dynamics is sensitive to parametric variations. This problem is addressed via modification to the integral manifold and an LPV based gain scheduling controller as the continuous part of the controller. The continuous controller, robustified LPV based gain scheduling, addresses the problem of parametric variations, operates in combination with a discontinuous controller. The discontinuous part of the controller will provide robustness against bounded matched disturbances and will keep the dynamics on the integral manifold. In addition, the discontinuous part of the ISMC algorithm is made smooth in order to produce a chattering free environment. The experimental results obtained from the laboratory test bench, ball on a beam balancer, show the effectiveness of the proposed strategy.

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