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# Type Synthesis of 2T1R Planar Parallel Mechanisms and Their Moduling Development Applications

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**ABSTRACT** The two translational degrees-of-freedom (DOF) and one rotational DOF (2T1R) planar parallel mechanisms (PMs) have the characteristics of simple structures, simple kinematic and dynamic models, and ease of control and allow for a variety of application prospects. This paper concentrates on the derivations of the 5-DOF hybrid manipulators based on the synthesized mechanisms. To understand the effect of the constraint on the motion of a rigid body clearly, the method of reciprocal product between the wrench screw (constraint) and the twist screw (motion) is used and the relationships between the constraint and the motion of a rigid body are analyzed as well. The results are extended to the relationships between the limb constraint and the axes of limb joints. Then, limbs are divided into planar motion and spatial motion limbs in view of their motion characteristics, and some common limbs with constraint are listed. After that, based on the type synthesis method, a class of 2T1R planar PMs with common limbs is carried out. Meanwhile, some typical planar closed chains with two same limbs are obtained in the process of type synthesis. Finally, a 2T1R kinematically redundancy planar PM and two 2R1T spatial PMs are derived by making simple changes to the obtained mechanisms, and detailed derivations of three 5-DOF hybrid manipulators with the ability of processing spatial complex surface are given. Proved by the test parts, the accomplishment of deriving novel robot configurations with regard to expectant motion are quite valuable and practical.

**INDEX TERMS** Parallel mechanism, type synthesis, degree of freedom, hybrid manipulator.

## I. INTRODUCTION

Industrial robots are most successful in applications like pick and place, motion simulation, and machine processing, etc [1]–[3]. The most common industrial robots are mainly composed of serial robots [4], parallel robots [5], [6], and hybrid serial-parallel robots [7], [8]. It is well known that serial robots have the characteristics of simple structures, large workspace, and high motion flexibility. Compared with serial robots, parallel robots have the advantages of higher stiffness-weight ratio and higher force loading capacity. Hybrid serial-parallel robots with characteristics of high degree of modularity present compromises between advantages and disadvantages of serial and parallel robots and possess significant application prospects for industry.

For a hybrid serial-parallel robot (in the following, we call it hybrid manipulator), in view of the simple structure of its serial part, the core component of the hybrid manipulator should be the parallel part. Thus, parallel mechanisms (PMs) for constructing parallel robots or hybrid manipulators are rising to research hotpot and attract attentions of many scholars. Compared with multi-mobility PMs with five to six degrees-of-freedom (DOF) [9], lower-mobility PMs with two to four DOF are often used as orientation and position adjustment manipulators [10], [11]. For example, the Sprint Z3 tool head [12] can be used as an orientation adjuster and the 2UPR/SPR PM [13] and 3SPR PM [14] can be used as position adjusters

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(R, P, U, and S stand for revolute, prismatic, universal, and spherical joints, respectively).

Among the aforementioned lower-mobility PMs, 3-DOF 2T1R (two translational DOF and one rotational DOF) [15] and 2R1T (two rotational DOF and one translational DOF) [16] PMs especially in the field of constructing 5-DOF hybrid manipulators achieve more and more applications. For example, the parallel part of the five-axis serial-parallel kinematic milling machine SPKM165 [17] is a 3-DOF 2T1R PM, and other well-known five-axis machining centers, such as Ecospeed [18], Tricept [19], Exechon [13], and TriMule [20], are all constructed based on the 3-DOF 2R1T PMs. Up to now, many literatures have studied the valuable 3-DOF 2R1T PMs, but in addition to the 2T1R PM mentioned above, there are few 2T1R PMs available for constructing 5-DOF hybrid manipulators. Therefore, more research on 2T1R PMs is really needed.

According to the closed-loop mechanical chain properties, the 2T1R PMs can be classified into planar and spatial PMs [21], [22]. 3-DOF 2T1R planar PMs with simple structures, simple kinematic models, and ease of control are particularly favored by scholars. For example, Mohan *et al*. [23] developed a 3-DOF 2T1R 2PRP-2PPR planar PM for lower limb rehabilitation. In [3], Wu *et al*. researched the 3-DOF 2T1R PRR-PRR planar PM, which is used as a spray-painting robot. Gosselin *et al*. [24] proposed a 3-DOF 2T1R planar PM with unlimited rotational motion, which is a great fit for the work of pick and place. Yao *et al*. [25] designed a 3-DOF 2T1R planar PM with both actuation and kinematically redundancies. There are relatively few other literatures focused on development applications of 2T1R planar PMs. In a sense, planar closed chains in the planar PMs have potential industrial application value in researching robot configurations.

As a process of determining possible mechanism structures for a given task without consideration of the specific dimensions and shapes of the components [26], type synthesis is the most effective method to obtain PMs with certain required motion of the moving platform (MP). Type synthesis methods such as screw theory [27], $\boldsymbol{G}_F$ set [28], Lie group [29], and Grassmann line geometry [30] are often used in the field of robot configuration analysis. Many significative PMs have been proposed by means of these synthesis methods, which has greatly enriched the diversity of PMs. Above all, using suitable methods to obtain novel robot configurations is important to the design of industrial hybrid manipulators.

The contributions can be summarized as:

- Based on screw theory method, the internal relationships between the constraint and the motion of a rigid body or limb are conducted, which is quite useful for establishing the limb constraint and better understanding the effect of the limb constraint on the motion of the MP.
- Type synthesis method based on the limb constraint and the required motion of the MP is used. A class of 3-DOF 2T1R planar PMs with common limbs is given and some typical planar closed chains with two same limbs are present simultaneously.
- Based on a 2T1R planar PM and two planar closed chains, detailed derivations of three 5-DOF hybrid manipulators with the ability of processing spatial complex surface are given.

The rest of this paper is organized as follows. Section II is devoted to the type synthesis of 2T1R planar PMs. Section III discusses the development applications of the obtained mechanisms. Section IV concludes this paper.

## II. TYPE SYNTHESIS OF 2T1R PLANAR PMS BASED ON CONSTRAINT FORCE/TORQUE

Screw theory as a common mathematical tool has been widely used in the constraint and motion analysis of spatial PMs. Vectors expressed by screws contain the position and orientation information. Therefore, this method has the characteristics of intuition, simplicity, and clear physical meaning.

For a PM, the twist screw (translation and rotation) and wrench screw (force and torque) of the limbs can be easily established by means of screw theory method, and then the constraint and motion types of the MP can be obtained. For a rigid body, its motion is related to its constraint. In other words, constraint can constrain the motion of the rigid body. In this section, we will analyze the relationships between the constraint and the motion of the rigid body and give the relationships between the joint and the constraint of the limb. By combining the limb constraint and space geometry theory, type synthesis of 2T1R planar PMs will be completed.

### A. CONSTRAINT ANALYSIS OF A RIGD BODY

For the motion of a rigid body, it is determined by the constraint. For example, spatial constraint force and torque can constrain the translation DOF and rotation DOF of the rigid body, respectively. Based on screw theory method, Li *et al*. [31] discussed the geometrical condition for the axis of a feasible rotation of a rigid body constrained by a pure force. In the following, the effect of the constraint on the motion of a rigid body will be analyzed.

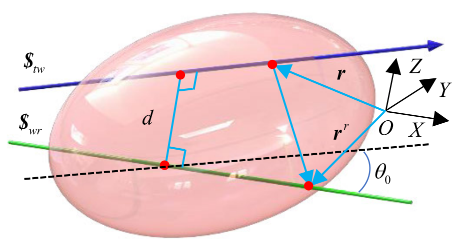



**FIGURE 1.** Schematic diagram of a rigid body with wrench and twist screws.

As shown in Fig. 1, the schematic diagram of a rigid body with wrench screw $\$_{wr}$ (constraint) and twist screw $\$_{tw}$ (motion) is given. Wrench screw $\$_{wr}$ is composed of constraint force $\$_{cf}$ and constraint torque $\$_{ct}$. Twist screw $\$_{tw}$ is composed of pure rotational motion $\$_{rm}$ and pure translational motion $\$_{tm}$. According to screw theory, the reciprocal product between the wrench screw $\$_{wr}$ and the twist screw

$\$_{tw}$ can be expressed as:

$$\lambda = \$_{wr} \circ \$_{tw} \tag{1}$$

where "∘" represents reciprocal product, λ represents instantaneous power.

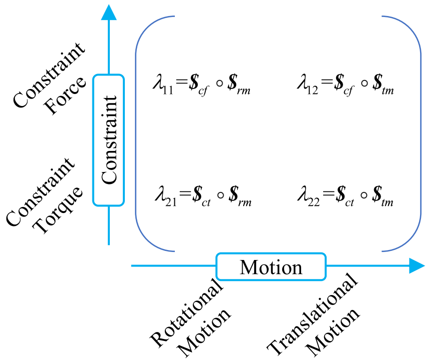



**FIGURE 2.** **Reciprocal product matrix of the constraint and motion.**

As shown in Fig. 2, a reciprocal product matrix of the constraint and motion is given. The components of the matrix are composed of four different forms of reciprocal product $\lambda_{ij}$ $(i = 1, 2; j = 1, 2)$; and $\lambda_{ij}$ represents the instantaneous power done by constraint force or constraint torque on rotational motion or translational motion, respectively.

Constraint force $\$_{cf}$, constraint torque $\$_{ct}$, rotational motion $\$_{rm}$, and translational motion $\$_{tm}$ can be expressed as:

$$\begin{cases} \$_{cf} = f(\boldsymbol{s}^r; \boldsymbol{r}^r \times \boldsymbol{s}^r) \\ \$_{ct} = t(\boldsymbol{0}; \boldsymbol{s}^r) \\ \$_{rm} = \omega(\boldsymbol{s}; \boldsymbol{r} \times \boldsymbol{s}) \\ \$_{tm} = v(\boldsymbol{0}; \boldsymbol{s}) \end{cases} \tag{2}$$

where $\boldsymbol{s}$ and $\boldsymbol{s}^r$ represent unit vectors along the axes of twist screw and wrench screw, respectively; $\boldsymbol{r}$ and $\boldsymbol{r}^r$ represent the radius vectors from original point to any one point on the axes of twist screw and wrench screw, respectively; $f$, $t$, $w$, and $v$ represent amplitude value of constraint force, constraint torque, angular velocity, and linear velocity, respectively; $\boldsymbol{0}$ represents a zero vector.

According to Fig. 2 and (2), the instantaneous power done by constraint force $\$_{cf}$ to rotational motion $\$_{rm}$ can be expressed as:

$$\begin{aligned} \lambda_{11} &= \$_{cf} \circ \$_{rm} = f\omega[\boldsymbol{s}^r \cdot (\boldsymbol{r} \times \boldsymbol{s}) + (\boldsymbol{r}^r \times \boldsymbol{s}^r) \cdot \boldsymbol{s}] \\ &= f\omega d \ sin\theta_0 \end{aligned} \tag{3}$$

where $d$ represents the length of the common perpendicular between the axes of constraint force $\$_{cf}$ and the rotational motion $\$_{rm}$, $\theta_0$ represents the angle between the axes of constraint force $\$_{cf}$ and the rotational motion $\$_{rm}$.

Similarly, the instantaneous power done by constraint force $\$_{cf}$ to translational motion $\$_{tm}$ can be expressed as:

$$\lambda_{12} = \$_{cf} \circ \$_{tm} = fv[\boldsymbol{s}^r \cdot \boldsymbol{s} + (\boldsymbol{r}^r \times \boldsymbol{s}^r) \cdot \boldsymbol{0}] = fv \cos\theta_0. \tag{4}$$

The instantaneous power done by constraint torque $\$_{ct}$ to rotational motion $\$_{rm}$ can be expressed as:

$$\lambda_{21} = \$_{ct} \circ \$_{rm} = tw[\boldsymbol{0} \cdot (\boldsymbol{r} \times \boldsymbol{s}) + \boldsymbol{s}^r \cdot \boldsymbol{s}] = tw \cos\theta_0. \tag{5}$$

The instantaneous power done by constraint torque $\$_{ct}$ to translational motion $\$_{tm}$ can be expressed as:

$$\lambda_{22} = \$_{ct} \circ \$_{tm} = tv(\boldsymbol{0} \cdot \boldsymbol{s} + \boldsymbol{s}^r \cdot \boldsymbol{0}) = 0. \tag{6}$$

By means of the virtual work principle, $\lambda_{ij}$ should be zero. Therefore, for (3), only $d = 0$ or $\theta_0 = k\pi$, $(k = 0, 1, 2, \ldots)$, $\lambda_{12} = 0$. That is to say, the axes of constraint force and rotational motion are coplanar. For (4) and (5), only $\theta_0 = (0.5 + k)\pi$, $(k = 0, 1, 2, \ldots)$, $\lambda_{12} = 0$, and $\lambda_{21} = 0$. That is to say, the axes of constraint force and translational motion are orthogonal, and the axes of constraint torque and rotational motion are orthogonal. For (6), $\lambda_{22} = 0$ is always satisfied. Thus, the axis of constraint force is independent of the axis of translational motion.

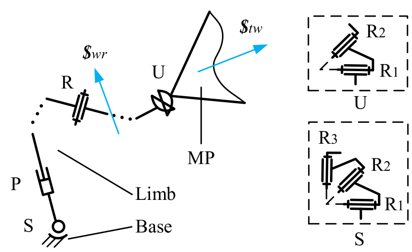



**FIGURE 3.** **Schematic diagram of a general spatial PM.**

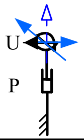
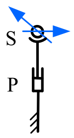
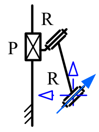
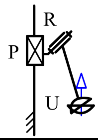
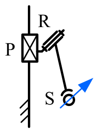
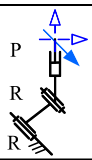
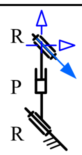
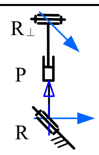
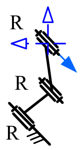
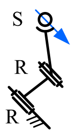
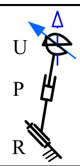
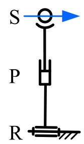
For lower-mobility PM, the output DOF of the MP is determined by limb constraint. Therefore, analyzing the constraint supplied by different limbs can help to obtain the DOF and motion type of the MP. For example, Fig. 3 shows a general spatial PM. The MP is equivalent to the above rigid body. One general limb is composed of different joints, among which, S joint is equivalent to three R joints with ordered orthogonal axes. U joint is equivalent to two R joints with orthogonal axes. According to the obtained relationships between the constraint and the motion of a rigid body, two judgment rules between the limb constraint and the axes of the limb joints can be organized as follow:

(1) Rule 1: The axis of constraint force $\$_{cf}$ must be in the same plane as the axes of R joints and perpendicular to the axes of P joints.
(2) Rule 2: The axis of constraint torque $\$_{ct}$ must be perpendicular to the axes of R joints.

## B. CONSTRAINT ANALYSIS OF SOME COMMON LIMBS

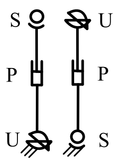
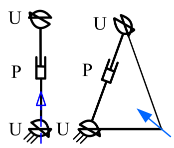
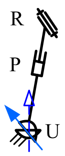
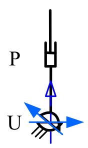
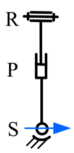
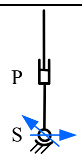
In view of the motion characteristics of four commonly used R, P, U, and S joints, we can divide limbs into two different types: planar motion limbs and spatial motion limbs. If the motion of the limb is restricted in a plane, the limb will belong to a planar motion limb, which mainly takes R or P joint as the first joint. Otherwise, it belongs to spatial motion

**TABLE 1.** Constraint force/torque of planar motion limbs.

| Limb | PU | PS | PRR | PRU |
|---|---|---|---|---|
| Diagram |  |  |  |  |
| **Limb** | **PRS** | **RRP** | **RPR** | **RPR⊥** |
| Diagram |  |  |  |  |
| **Limb** | **RRR** | **RRS** | **RPU** | **RPS** |
| Diagram |  |  |  |  |

Note, ⟶ represents constraint force, - - ▷ represents constraint torque. In above RPU or the following UPR limb, the axis of U joint connecting to P joint is parallel to the axis of R joint. As shown in Table 2, some commonly used spatial motion limbs with constraint are listed.

**TABLE 2.** Constraint force/torque of spatial motion limbs.

| Limb | UPS/SPU | UPU | UPR |
|---|---|---|---|
| Diagram |  |  |  |
| **Limb** | **UP** | **SPR** | **SP** |
| Diagram |  |  |  |

limb, which mainly takes U or S joint as the first joint. Then, according to the above two judgment rules, we can get the constraint provided by limb to the MP. Table 1 lists some planar motion limbs with constraint, which are quite common in lower-mobility PMs.

Note, ⟶ represents constraint force, - - ▷ represents constraint torque. In above RPU or the following UPR limb, the axis of U joint connecting to P joint is parallel to the axis of R joint. As shown in Table 2, some commonly used spatial motion limbs with constraint are listed.

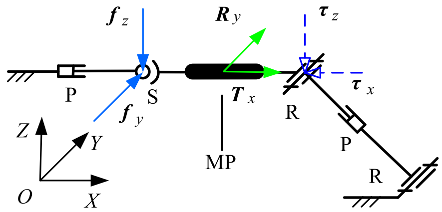



**FIGURE 4.** Structure of the PS/RPR PM.

**TABLE 3.** Limbs for constructing 2T1R planar PM.

| Symbol | Limb |
|---|---|
| A | UPS, SPU |
| B | PRS, RRS, RPS, SPR |
| C | PRU, RPU, UPR |
| D | PRR, RRP, RPR, RRR |

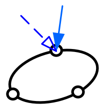
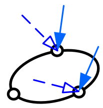
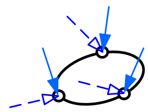
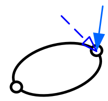
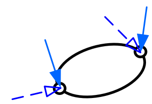
To better understand the effect of limb constraint on the motion of the MP (the bold line), the PS/RPR PM is shown in Fig. 4. Form Table 1, we know that limb PS can provide constraint forces $\$_{cf1}$ and $\$_{cf2}$, which constrain the translational DOF in plane *YOZ*. For limb RPR, constraint torques $\$_{ct1}$ and $\$_{ct2}$ can constrain the rotational DOF around any axis in plane *YOZ*. Constraint force $\$_{cf3}$ also can constrain the translational DOF along *Y* axis. In summary, PS/RPR PM is a planar PM with translational DOF $\boldsymbol{T}_x$ along *X* axis and rotational DOF $\boldsymbol{R}_y$ around *Y* axis.

Considering the motion characteristics of 2T1R planar PM, we need select the limbs which are suitable for constructing 2T1R planar PM. The limbs providing no constraint are UPS and SPU (marked 'A'); the limbs providing one constraint force are PRS, RRS, RPS, and SPR (marked 'B'); the limbs providing one constraint force and one constraint torque are PRU, RPU, and UPR (marked 'C'); the limbs providing one constraint force and two constraint torques are PRR, RRP, RPR, and RRR (marked 'D'). As the constraint type of UPU limb varies with its configuration, assumptions are required for type synthesis of the 2T1R planar PM. To reduce the difficulty of the following analysis, the UPU limb will not be taken into account in the process of type synthesis. Other limbs are reorganized according to the number and types of constraint. The results are shown in Table 3.

### C. TYPE SYNTHESIS OF 2T1R PLANAR PM

For a general 2T1R PM, according to the limb constraint and space geometry theory, we know that the axis of rotational DOF is perpendicular to the axes of translational DOF. Thus, the constraint forces should be perpendicular to the motion planar of the MP, and the constraint torques should be parallel to the motion planar of the MP. At least one constraint force should be provided by the limbs of the PM, and when there are multiple constraint forces, the axes of constraint forces should be collinear or parallel to each other. The number of

**TABLE 4. Five cases of the MP with different structural constraint.**

| | Case I | Case II | Case III | Case IV | Case V |
|---|---|---|---|---|---|
| Diagram |  |  |  |  |  |
| Constraint description | Constraint on one joint | Constraint on two joints | Constraint on three joints | Constraint on unilateral joint | Constraint on bilateral joints |

constraint torques is related to the constraint forces on the MP. To facilitate the configuration analysis of 2T1R planar PM with three limbs, five cases of the MP with different structural constraint are shown in Table 4.

For each of cases I-III, three joints connecting to the MP are arranged in circular symmetry. For either case IV or V, two of three limbs share the same joint connecting to the MP. Specific analysis of the five cases in Table 4 will be carried out in the following.

### 1) CASE I: CONSTRAINT ON ONE JOINT

For case I in Table 4, there is only one limb providing structural constraint. This limb needs to provide not only one constraint force that constrains the translational DOF of the MP, but also two constraint torques that constrain two rotational DOF of the MP. Also, the axis of constraint force should be perpendicular to the axes of constraint torques. The other two limbs do not need to provide constraint to the MP. By means of Tables 1-3, we can easily obtain 12 A/A/D-type PMs. In Fig. 5, (a), (b), (c), and (d) show four typical 3-DOF 2T1R planar PMs, namely 2UPS/PRR, 2UPS/RRP, 2UPS/RPR, and 2UPS/RRR, respectively.

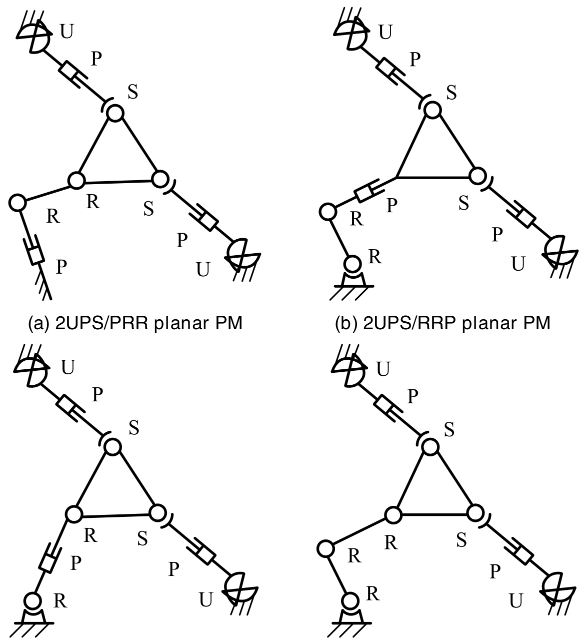
As shown in Fig. 5, all of limbs of each planar PM are in the same plane, and the initial pose of each limb between the base and the MP is general (the same below).

### 2) CASE II: CONSTRAINT ON TWO JOINTS

For case II in Table 4, there are two limbs providing structural constraint. From the preceding analysis, we know that the axes of constraint forces are parallel to each other. As two non-linear constraint forces can only constrain one rotational DOF, at least one limb also needs to provide one or more constraint torques. Furthermore, the axis of constraint torque cannot be perpendicular to the plane of constraint forces. Therefore, we can divide the corresponding 2T1R planar PMs into two types:

(1) Type 1: One limb only provides constraint force; the other one provides constraint force and torque simultaneously. Then, we can obtain 24 A/B/C-type and 32 A/B/D-type PMs.

(2) Type 2: Each of two limbs provides constraint force and torque simultaneously. Then, we can obtain 12 A/C/C-type, 20 A/D/D-type, and 24 A/C/D-type PMs.



(a) 2UPS/PRR planar PM (b) 2UPS/RRP planar PM
(c) 2UPS/RPR planar PM (d) 2UPS/RRR planar PM

**FIGURE 5. Typical 3-DOF 2T1R planar PMs conforming to case I.**

As shown in Fig. 6, four typical 3-DOF 2T1R planar PMs, A/B/C-type UPS/RPS/RPU, A/B/D-type UPS/RPS/RRR, A/C/C-type 2UPR/UPS, and A/D/D-type 2RRR/UPS, are given.

Taking Fig. 6 (c) as an example, we know that all of the limbs are spatial motion limbs. However, the motion of this 3-DOF 2UPR/UPS PM is constrained in a plane. The main reason is caused by the arrangement of the directions and positions of the constraint. It is worth noting that the axes of constraint torques in each PM cannot be parallel. Otherwise, the motion of the MP will not be fully constrained. In Fig. 6 (c), when the axes of two constraint torques provided by two UPR limbs are parallel, the motion of the MP will lose control. In other words, the pose of the MP cannot be completely determined by its limbs.

### 3) CASE III: CONSTRAINT ON THREE JOINTS

For case III in Table 4, all of three limbs provide structural constraint. The axes of constraint forces should be parallel

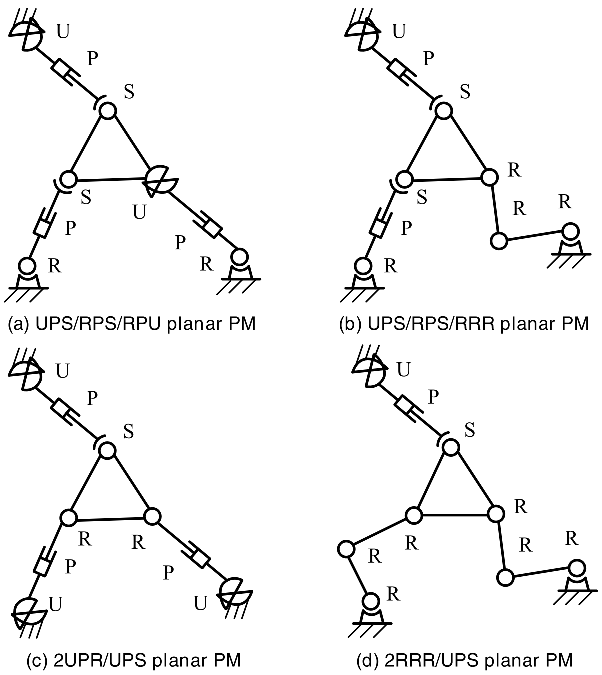



**FIGURE 6.** Typical 3-DOF 2T1R planar PMs conforming to case II.

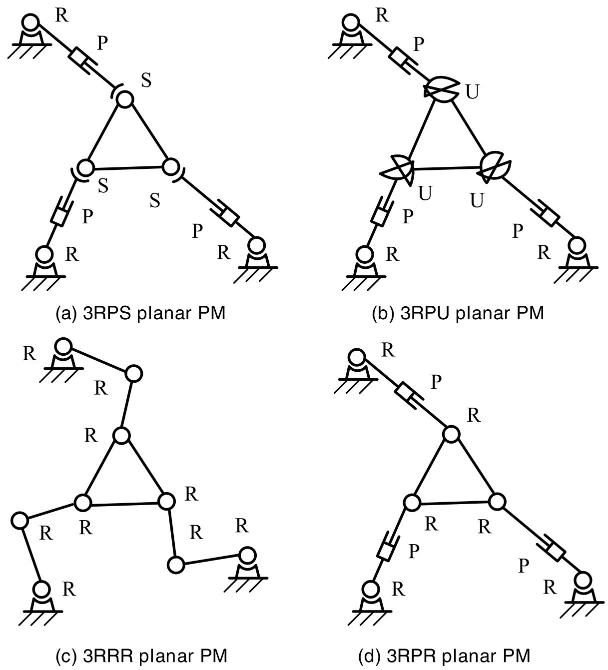



**FIGURE 7.** Typical 3-DOF 2T1R planar PMs conforming to case III.

to each other. Same as the above analysis, we can divide the corresponding 2T1R planar PMs into three types:

(1) Type 1: All of three limbs provide constraint forces only. Then, we can obtain 20 B/B/B-type PMs.
(2) Type 2: Each limb provides constraint force and torque simultaneously. Then, we can obtain 10 C/C/C-type, 20 D/D/D-type, 24 C/C/D-type, and 30 C/D/D-type PMs.
(3) Type 3: Some limbs provide constraint forces only and others provide constraint force and torque together. Then, we can obtain 64 B/C/D-type, 24 B/C/C-type, 40 B/D/D-type, 30 B/B/C-type, and 40 B/B/D-type PMs.

As shown in Fig. 7, four typical 3-DOF 2T1R planar PMs, B/B/B-type 3RPS, C/C/C-type 3RPU, D/D/D-type 3RRR, and D/D/D-type 3RPR, are given.

### 4) CASE IV: CONSTRAINT ON UNILATERAL JOINT

For case IV in Table 4, three are only two joints on the MP. And among which, two limbs share the same joint. To reduce the following analysis difficulty, it is assumed that the structures of these two limbs with shared joint are the same. Thus, at least one limb provides one constraint force and two constraint torques to the MP, and at least one limb provides no constraint to the MP. We can divide the corresponding 2T1R planar PMs into two types:

(1) Type 1: Two limbs with shared joint provide the above constraint forces and torques. Then, we can obtain 8 $D^2$/A-type PMs ($D^2$ means the limbs with shared joint, the same below).
(2) Type 2: The single limb provides one constraint force and two constraint torques. Then, we can obtain 8 $A^2$/D-type PMs.

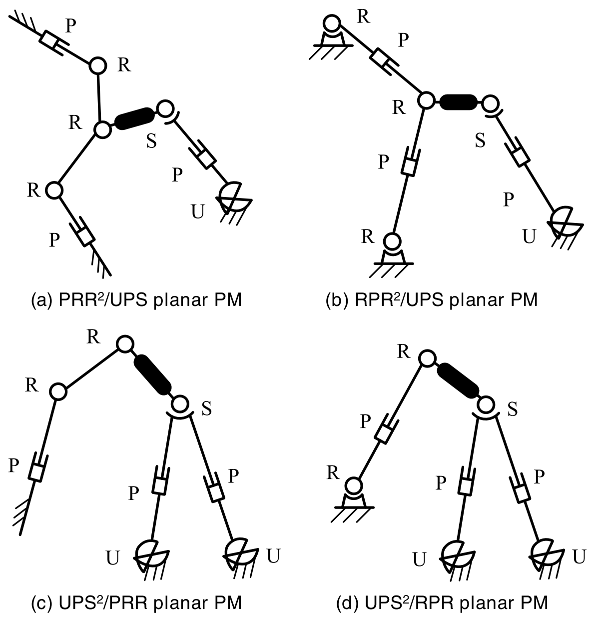


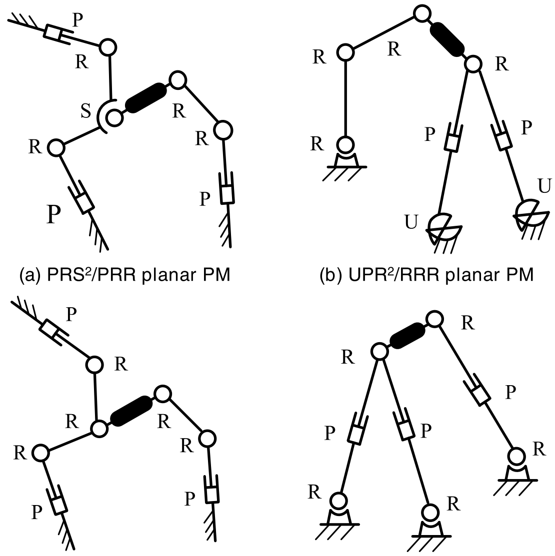

**FIGURE 8.** Typical 3-DOF 2T1R planar PMs conforming to case IV.

As shown in Fig. 8, four typical 3-DOF 2T1R planar PMs, $D^2$/A-type PRR$^2$/UPS, $D^2$/A-type RPR$^2$/UPS, $A^2$/D-type UPS$^2$/PRR, and $A^2$/D-type UPS$^2$ /RPR, are given.

### 5) CASE V: CONSTRAINT ON BILATERAL JOINTS

For case V in Table 4, all of the limbs provide structural constraint to the MP. We can divide the corresponding 2T1R planar PMs into three types:

(3) Type 1: Limb with shared joint provides constraint force only. Then, we can obtain 12 $B^2$/C-type and 16 $B^2$/D-type PMs.
(4) Type 2: Limb with shared joint provides one constraint force and one constraint torque. Then, we can obtain 12 $C^2$/B-type, 9 $C^2$/C-type, and 12 $C^2$/D-type PMs.
(5) Type 3: Limb with shared joint provides one constraint force and two constraint torques. Then, we can obtain 16 $D^2$/B-type, 12 $D^2$/C-type, and 16 $D^2$/D-type PMs.



(a) PRS²/PRR planar PM (b) UPR²/RRR planar PM
(c) PRR²/PRR planar PM (d) RPR²/RPR planar PM

**FIGURE 9. Typical 3-DOF 2T1R planar PMs conforming to case V.**

As shown in Fig. 9, four typical 3-DOF 2T1R planar PMs, $B^2$/D-type PRS$^2$/PRR, $C^2$/D-type UPR$^2$/RRR, $D^2$/D-type PRR$^2$/PRR, and $D^2$/D-type RPR$^2$ /RPR, are given.

From the point of structure constraint, some U or S joints in the obtained 3-DOF 2T1R planar PMs can be replaced by R joints. For example, the 3-DOF 2UPS/PRR planar PM, as shown in Fig. 5 (a), can be changed into the 3-DOF 2RPS/PRR over-constraint planar PM, which has been contained in type 3 of case III. Therefore, there are really certain connections among different types of 3-DOF 2T1R planar PMs.

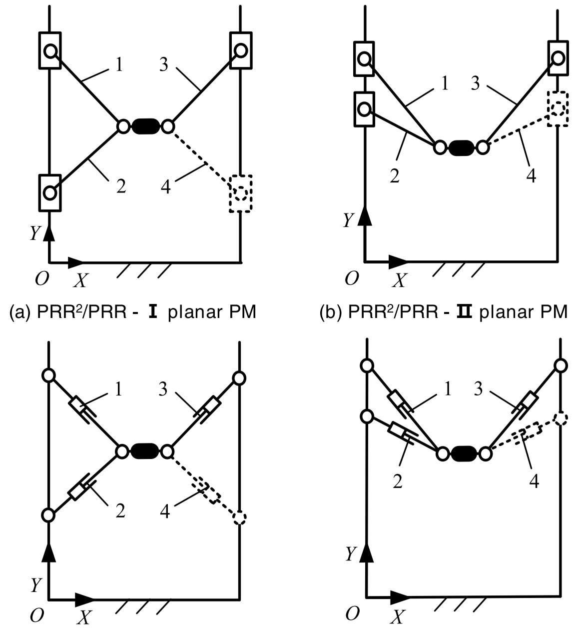
In summary, from the preceding analysis, we know that the type synthesis method based on the limb constraint and space geometry theory is quite intuitive and convenient. A class of 2T1R planar PMs with three limbs is obtained. Except that, in many 2T1R planar PMs, the limbs, the bases, and the MPs can constitute different planar closed chains, which are quite helpful to construct the spatial PMs.

## III. DEVELOPMENT APPLICATIONS OF THE OBTAINED MECHANISMS

Based on the characteristics of large stiffness and high load capacity of PMs, robots expanded by PMs have significant application prospects in industry. In this section, based on some obtained typical planar PMs and planar closed chains, detailed derivations of the industrial robots will be given.

### A. DEVELOPMENT APPLICATIONS OF THE PLANAR PMS

Reference [24] presents two 2T1R planar PMs: 3PRR and 3RPR. The joints connecting to the base are placed in a straight line, which makes these two PMs more suitable for picking and placing. As shown in Fig. 9 (c) and (d), PRR$^2$/PRR and RPR$^2$/RPR PMs with shared joints and the least number of single-DOF joints are composed of P and R joints only, which makes them have the characteristic of simple structures.



(a) PRR²/PRR - Ⅰ planar PM (b) PRR²/PRR - Ⅱ planar PM
(c) RPR²/RPR - Ⅰ planar PM (d) RPR²/RPR - Ⅱ planar PM

**FIGURE 10. 3-DOF 2T1R planar PMs: PRR$^2$/PRR and RPR$^2$/RPR.**

Rearranging the joints (in Fig. 9 (c) and (d)) connecting to the base, we can obtain two novel 2T1R planar PRR$^2$/PRR and RPR$^2$/RPR PMs. As shown in Fig. 10, limbs 1 and 2 are located on one side of the MP and limb 3 on the other side of the MP. In Fig. 10 (a) and (c), the length of each limb is the same, and limb 2 is below of the MP. In Fig. 10 (b) and (d), the length of limbs 1 and 3 is the same, the length of limb 2 is shorter than others. All of the limbs are above the MP.

From Fig. 10, we know that the dexterity of the MP of this type PM is very important. Apart from the advantages of the planar PMs, it is obvious that many planar PMs generally suffer from the drawback of singularities with negative influences on workspaces and the dexterity of the MPs. To alleviate the above drawback, actuation and kinematically redundancies are often used in the planar PMs [24], [25]. Thus, as shown in Fig. 10, we can improve dexterity of the MP by adding actuation redundancy limb 4 (dotted line). Except that, both stiffness and workspace of the PM can be effectively improved by actuation redundancy. However, this method generates internal forces on joints and increases the difficulty of robot control. By making simple changes to the actuation redundancy PMs, we can obtain kinematically redundancy PMs. For example, as shown in Fig. 11 (a), limb 5 is added between the MP and limb 3 (4), which ensures the flexibility of the MP and eliminates the internal force of joints. Then, actuation redundancy PMs become to kinematically redundancy PMs. As shown in Fig. 11 (b), the MP and limb 5 can be transformed into a mechanical gripper. Then, a novel robot configuration with ability of carrying and grabbing can be constructed. As shown in Fig. 11 (c), (d), and (e), three different 2-DOF serial parts are present.

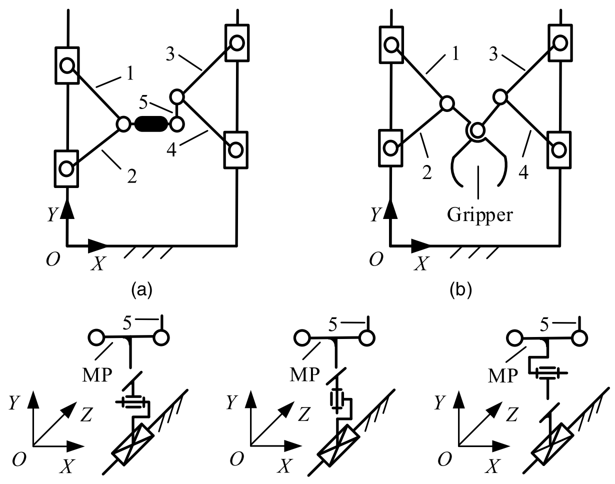
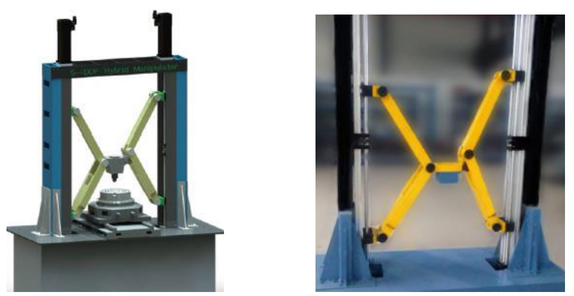



**FIGURE 11.** 3-DOF 2T1R planar kinematically redundancy PM.



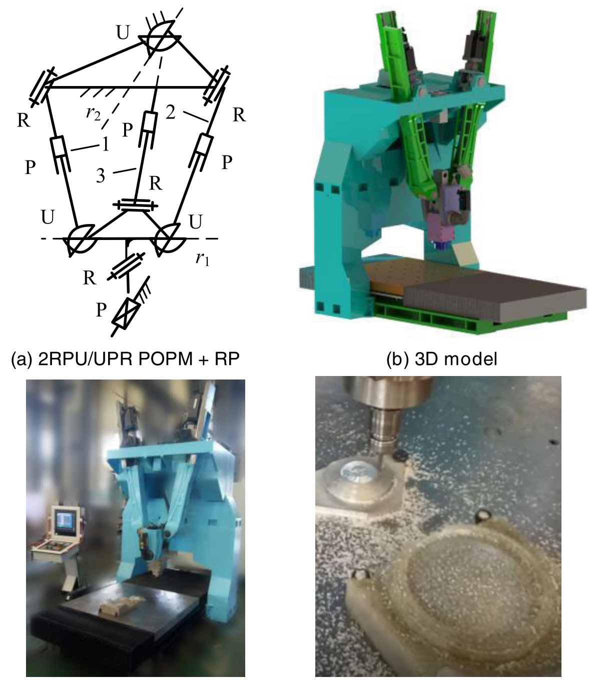
(a) 3D model (b) Prototype of the parallel part

**FIGURE 12.** A 5-DOF hybrid manipulator based on the PRR$^2$/PRR planar PM.

By combining the general 2T1R planar PMs with three limbs in Fig. 10, or actuation redundancy 2T1R planar PMs in Fig. 10, or kinematically redundancy 2T1R planar PMs in Fig. 11 with the 2-DOF serial parts in Fig. 11, we can obtain 36 5-DOF hybrid manipulators. As shown in Fig. 12 (a), the 3D model of a 5-DOF hybrid manipulator [22] (constructed by Fig. 11 (a) and (c)) with the ability of processing spatial complex surface is shown, the prototype of the parallel part of the industrial robot is shown in Fig. 12 (b).

### *B. DEVELOPMENT APPLICATIONS OF THE PLANAR CLOSED CHAINS*

Planar closed chains obtained from the above type synthesis are helpful to construct spatial 2R1T PMs. Compared with planar PMs, spatial PMs with better motion flexibility are more suitable for constructing industrial robots. According to the motion characteristics of two rotational DOF of the MP, spatial PMs can be divided into position-orientation PMs (POPMs) and position-position PMs (PPPMs). For a POPM, one axis of two rotational DOF is close to the MP, and the other one is close to the base. On the contrary, both axes of two rotational DOF of a PPPM are close to the base.



(a) 2RPU/UPR POPM + RP (b) 3D model
(c) Prototype (d) Spherical crown part

**FIGURE 13.** A 5-DOF hybrid manipulator based on the 2RPU/UPR POPM.

#### 1) DEVELOPMENT APPLICATIONS OF THE POPMS

For any two RPU limbs in Fig. 7 (b), axes of two R joints are parallel to each other. Two RPU limbs, the MP line, and the base line are restricted in a same plane, and they can be seen as a planar closed chain (2RPU). This type planar closed chain 2RPU is easy to construct a 3-DOF 2R1T POPM.

As shown in Fig. 13 (a), when the axes of two U joints connecting to the MP are collinear (marked as $r_1$), the rotational DOF of the MP can be determined by the constraint in limb 3. According to the relationships between the constraint and the motion, at least one constraint force is needed. And the axis of constraint force provided by limb 3 should be parallel to the $r_1$ axis. Except for constraint force, if there is one constraint torque provided by limb 3, the axis of constraint torque should be perpendicular to the $r_1$ axis. Thus, the third limb can be UPR limb with one constraint force and one constraint torque or SPR limb with one constraint force.

As shown in Fig. 13 (a), a 3-DOF 2R1T 2RPU/UPR PM with 9 single-DOF joints is given. The axis of R joint in UPR limb is parallel to the $r_1$ axis, and the axis of U joint connecting to the base is parallel to the axis of R joint in RPU limb. For $r_2$ axis, it is collinear with the axis of U joint connecting to the base. Thus, $r_1$ ($r_2$) axis is coplanar with each axis of the constraint forces and perpendicular to each axis of the constraint torques. In summary, the rotational DOF around $r_1$ and $r_2$ axes make the MP have the ability of orientation and position adjustment, respectively. By adding a single-DOF head and a translational table to the 3-DOF 2R1T 2RPU/UPR POPM, a 5-DOF hybrid manipulator configuration [8], as shown in Fig. 13 (a), can be obtained. The 3D model of the 5-DOF hybrid manipulator is shown in Fig. 13 (b). The prototype is shown in Fig. 13 (c), and a test part processed by this manipulator is shown in Fig. 13 (d).

### 2) DEVELOPMENT APPLICATIONS OF THE PPPMS

From Fig. 13 (a), we know that the axes of two rotational DOF of the MP are close to the base and the MP, respectively. Therefore, to obtain the PPPMs with position adjustment ability, the axes of two rotational DOF of the PPPMs must be close to the base. According to relationships between two axes of the rotational DOF of the MP and the constraint generated on the MP, constraint forces in the planar closed chain should be close to the base. As shown in Fig. 6 (c), constraint forces in two UPR limbs are close to the base. When the axes of two U joints connecting to the base are collinear, the planar closed chain (2UPR), composed of two UPR limbs, the MP line, and the base line, is easy to construct a 3-DOF 2R1T PPPM.

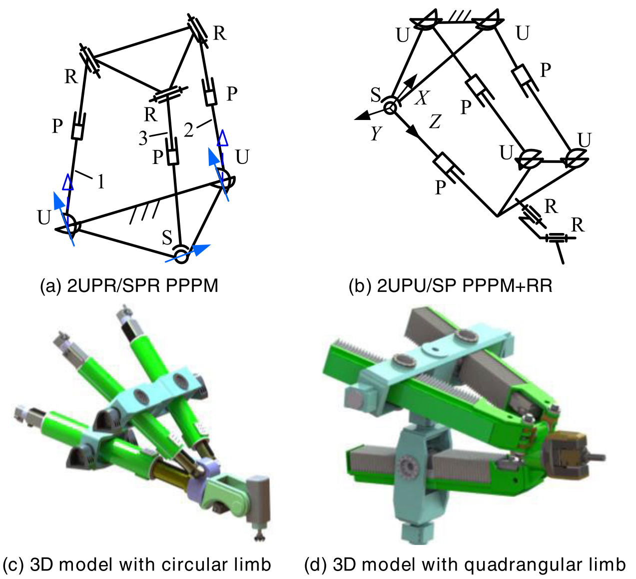



**FIGURE 14. A 5-DOF hybrid manipulator based on the 2UPU/SP PPPM.**

From the point of the constraint provided by the above planar closed chain 2UPR, limb 3 need to provide constraint force as well. And the constraint force provided by limb 3 should be close to the base. As shown in Fig. 14 (a), to satisfy the motion of the MP, the SPR limb with one constraint force is an appropriate choice. The axis of constraint force provided by the SPR limb is parallel to the MP and perpendicular to the axes of two constraint forces provided by two UPR limbs. Then, the 3-DOF 2UPR/SPR PPPM can be obtained, and it is the parallel part of the famous Exechon robot [13], which is a 5-axis machining center. We can change the position of the R joint in SPR limb further. When the axis of the R joint in SPR limb passes through the centers of two R joints in two UPR limbs, the 2UPR/SPR PPPM is changed into the 2UPU/SP PPPM. Then, by adding a 2-DOF tilting head to the end of the SP limb, a 5-DOF hybrid manipulator configuration [11], as shown in Fig. 14 (b), can be obtained. Two 3D models of the 5-DOF hybrid manipulator with two types of limbs are shown in Fig. 14 (c) and (d), respectively.

The 5-DOF hybrid manipulator based on the PPPM has good expansibility. For example, based on the 3D model in Fig. 14 (d), we can obtain different configurations by redesigning the base. Vertical, horizontal, and catercorner configurations of the 5-DOF hybrid manipulators are shown in Fig. 15 (a), (b), and (c), respectively. The addition of vertical and horizontal translational DOF of the 5-DOF hybrid manipulator module can greatly increase the motion area and enhance the processing capacity of the end effector of the 5-DOF hybrid manipulator, as shown in Fig. 15 (d).

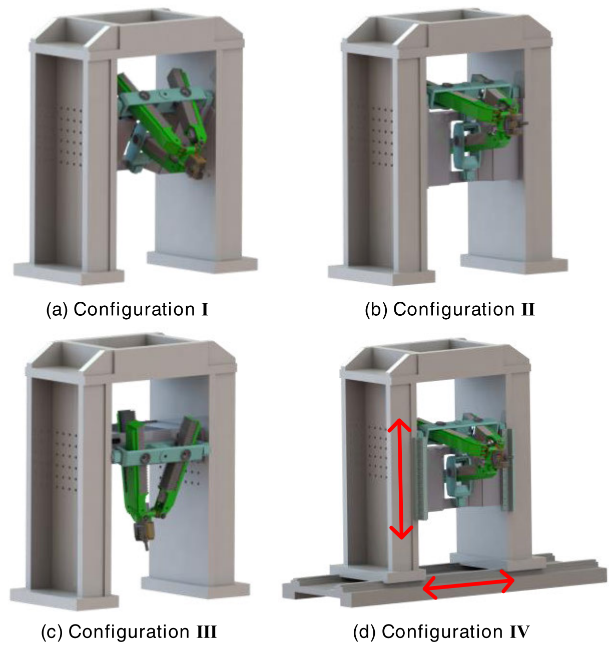
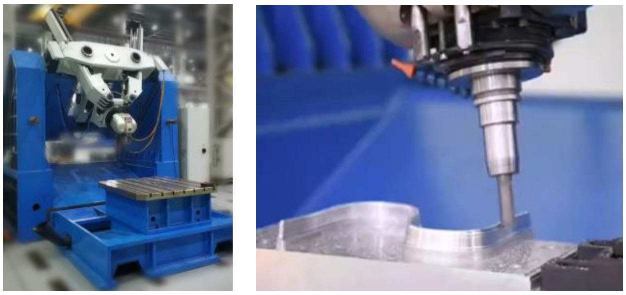



**FIGURE 15. Modular configurations of the 5-DOF hybrid manipulator.**

The prototype of the 5-DOF hybrid manipulator based on the catercorner configuration is shown in Fig. 16 (a), and a test part processed by this 5-DOF hybrid manipulator is shown in Fig. 16 (b) [3].

The test parts processed by the obtained 5-DOF hybrid manipulators, as shown in Fig. 13 (d) and Fig. 16 (b), can



(a) Prototype (b) S-type part

**FIGURE 16. Prototype of the 5-DOF 2UPU/SP PPPM + RR hybrid manipulator.**

indirectly show the ability of the robots and prove the superiority of synthesized mechanisms.

## IV. CONCLUSION

In this paper, the reciprocal product method based on screw theory is used to analyze the effect of the limb constraint on the motion of the MP. Also, the relationships between the limb constraint and the axes of limb joints are easily obtained. To construct 2T1R planar PMs, some common limbs with constraint are classified into planar motion limbs and spatial motion limbs. Thereafter, based on the limb constraint and space geometry theory, type synthesis of 2T1R planar PMs is carried out. Then, a class of 2T1R planar PMs with three limbs is obtained. Meanwhile, some planar closed chains are easily extracted from the synthesized 2T1R planar PMs. Finally, the 3-DOF 2T1R PRR$^2$/PRR planar PM is evolved into two industrial robots, one with the ability of carrying and grabbing and the other one with the ability of processing spatial complex surface. Besides, other two 5-DOF hybrid manipulators based on two different planar closed chains are also derived.

In this study, theoretical analysis of the constraint and motion allows a more thorough understanding of the PMs. Type synthesis and development applications of the obtained mechanisms can enrich the diversity of PMs. In addition, the main contribution of this paper lies in the derivation processes from robot configurations obtained by type synthesis method to robot structures with certain required motion. The theoretical applications from type synthesis method to hybrid manipulators are more helpful for us to obtain novel industrial robots with the characteristics of simple structures and high degree of modularities.

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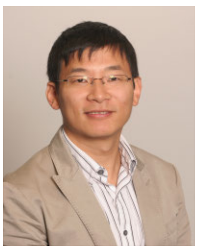

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