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A Double Auction Framework for Multi-Channel Multi-Winner Heterogeneous Spectrum Allocation in Cognitive Radio Networks

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ABSTRACT Opportunistic availability of licensed frequency bands enables the secondary users (SUs) to avail the radio spectrum dynamically. Cognitive radio (CR) paradigm extends the dynamic spectrum access techniques to sense for free channels (called spectrum holes) which can be efficiently redistributed amongst SUs. Motivated by the adaptive technology in CR, this paper introduces a sealed-bid double auction mechanism which aims to obtain an effective allocation of the unused radio spectrum. The proposed auction model adopts multi-channel allocation where one SU can access more than one available channel, while imposing the constraints for dynamics in spectrum opportunities and varying channel availability time amongst SUs. Previously designed double auctions miss out the CR constraints which can further degrade the network performance. Also, multi-winner allocation is induced in the model which encourages spectrum reuse by allowing a common channel to be assigned to multiple non-interfering SUs. A preference list of channels is maintained at each SU using which SUs offer their bid values for the heterogeneous channels which the primary owners are competing to lease. To organize channel specific groups of non-interfering SUs, a bidder group formation algorithm is developed such that members of a winner group get access to a common channel. The auctioneer formulates a winner determination strategy and a pricing strategy which achieves truthfulness while assigning the idle spectrum. Effectiveness of the proposed model is studied by comparing it with an existing work which shows that channel allocation gets significantly improved on deploying the proposed model.

INDEX TERMS Cognitive radio, dynamic spectrum access, spectrum sharing, double auction, truthfulness, primary owner, secondary user, spectrum opportunities.

I. INTRODUCTION

Traditional static spectrum assignment policy allocates large chunks of spectrum to licensed users (or primary users) on a long term basis. This permits only authorized users to use these frequency bands. On the other hand, extensive deployment of wireless applications creates shortage of radio spectrum. To deal with this discrepancy in spectrum usage, Federal Communications Commission (FCC) [1] studied the licensed spectrum and declared that most of the statically assigned bands remain underutilized by their owners both spatially and temporally. This encourages the unlicensed users (or secondary users) to dynamically utilize the idle spectrum (spectrum holes) by incorporating a new technology called Cognitive Radio (CR) [2], [3]. To efficiently redistribute the radio resource, CR uses dynamic spectrum access (DSA) techniques [4] which initially senses for spectrum holes and then enables the secondary users (SUs) to opportunistically use the spectrum holes causing no interference to primary users (PUs). A key functionality of CR is to fairly allocate the free channels amongst the coexisting SUs [5], [6]. Several spectrum allocation models have been designed [7], [8] to distribute the unused spectrum which can significantly improve the spectrum utilization. Auction-based model [9] provides a different perspective in channel allocation where spectrum holes being the auctioned item are bid by SUs with appropriate valuation. Both single-sided auction and double-sided auction can be deployed for

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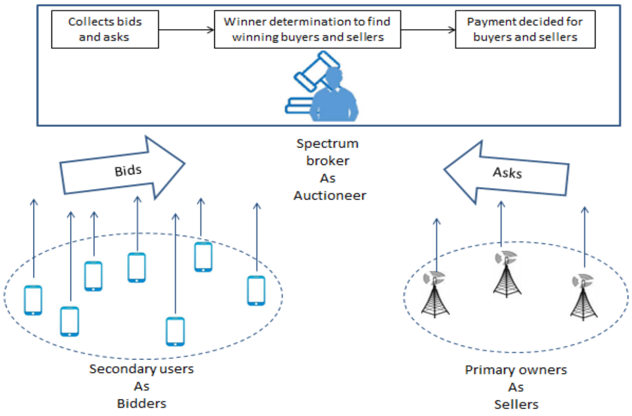


FIGURE 1. **Double-auction in cognitive radio networks.**

spectrum allocation in CR. In single-sided auction, a primary owner (primary base station) of the network performs as auctioneer who auctions the channels from the PUs and also earns a financial benefit. But, in double-sided auction, primary owners (POs) of the network participate as sellers and they compete amongst them to sell the channels left unused by their respective PUs. Spectrum broker (SB) takes the role of auctioneer in a double-sided auction. Fig. 1 shows a double auction model in cognitive radio network (CRN). Although, there can be situations where some channels may remain unassigned in a double-sided auction, but such an auction framework provides a more practical scenario since POs themselves get involve to earn a better monetary profit by submitting ask values for their channels. In today's world of 5G technology [10], double-sided auction anticipates a better flexibility in spectrum usage, where to obtain an increased network capacity, unused licensed channels can be temporarily leased amongst users with the consent of the POs. Also, auction ensures a secured communication since bids/asks from buyers/sellers are announced only to the auctioneer and there needs to be no information exchange amongst the network operators. Therefore, in our proposed model, we develop a double auction mechanism where POs compete amongst them by submitting ask values to the auctioneer to sell their channels, whereas SUs submit bid values to get access to their preferred channels. Applying sealed-bid policy helps to reduce the communication overhead, where both SUs and POs privately submit their values to SB. Moreover, auction-based allocations are widely accepted because every participant gets an equal opportunity to win and SUs with high bid value have a greater chance to get their desired channels.

Auction formulation in CR networks needs to address different network constraints. Dynamics in spectrum opportunities (SOPs) amongst SUs is an important constraint in CRN. Previous research works [12]–[20] on double auction does not tackle this issue due to which all idle channels remain available to every SU during channel allocation. Such an assumption does not exhibit a practical scenario in CRN. Another constraint can be to allow geographically separated SUs to take advantage of spectrum reuse since spectrum auction is much different from the conventional auction which offers an item to only one bidder. Spectrum reusability helps to boost the spectrum utilization as well as increases the profit of sellers. Some of the existing double auction mechanisms consider that a PU contributes only a single channel for the auction [12], [15], [19], [23]. But, if more than one channel is vacant, then it is more likely to constraint a PU to sell all its idle channels at a time. Applying multi-channel allocation can be a good approach in CRN because this enhances the network throughput. Also, multi-channel allocation helps in situations where the PU wants to return back to its channel and the SU using that channel releases it and switches to its other assigned channels avoiding any disturbance during its data transmission. Even so, there can be a situation where an SU gets only one channel and on vacating the channel, the transmission of the SU gets disturbed. To overcome such a situation and to reduce the switching overhead, it is necessary to consider the availability time of the channels kept for lease. None of the existing works on double auction incorporates this network constraint which can thereby degrade the network performance. Homogeneous channel condition in most of the auction models [12]–[19] allows a bidder to submit a uniform bid for all available channels. However, considering the channels to be heterogeneous in their quality proffers a more realistic scenario and this necessitates an SU to submit different bids for different channels. Also, ask values for the channels can differ from a particular PO. To prevent any market manipulation of the bids and asks, the auction mechanism demands a truthful winner determination algorithm such that no bidder/seller can improve its utility with an untruthful bid/ask. Hence, motivated by these observations, we develop a double auction model which includes the discussed network constraints to achieve an effectual spectrum allocation.

In this paper, we propose a multi-channel multi-winner sealed-bid double auction mechanism to model the spectrum allocation problem in CRN. It extends the work carried out in [11] which designs a double auction framework that allows spectrum reuse with single channel allocation of the homogeneous channels. In our proposed model, POs participate in the game to obtain a monetary profit where they report ask values to the SB for leasing their vacant channels. SB acting as the auctioneer collects asks from POs and bids from SUs and consequently decides a clearing price with the winner determination and pricing strategies to achieve an efficient allocation. Both multi-channel allocation and multi-winner allocation for spectrum reuse have been applied in this model. Since we consider that the channels are heterogeneous with respect to their maximum allowable transmission power, so every SU can decide a different bid value for each channel according to the channel availability of the SU. In the seller side, a PO can lease multiple channels which are kept idle. According to the PO's willingness to obtain a monetary amount, it sets the ask value which can also be different for each channel that it wants to auction. Hence, bid values and ask values submitted to SB are channel-specific. Every SU maintains a preference list of its available channels based on which bids are decided by the SU. The bid from an SU is made in terms of the rate of data transfer to be used over the requested channel. Dynamics in SOPs among SUs and differences in availability time of the channels influence the bid submission process. To enable spectrum reuse, a channel-specific bidder group formation algorithm is proposed where for every channel groups of non-interfering SUs are formed. Then, the winning group for each channel is determined using a spectrum allocation algorithm. The developed auction model guarantees truthfulness and individually rationality at the auctioneer. Network simulations are carried out to validate the performance of the proposed model by comparing it with an existing auction model called PreDA [22].

The proposed double auction model makes the following key contributions.

- With the channels being heterogeneous, a preference list of the available channels is maintained at each SU while imposing the network constraints, viz., dynamics in SOPs and differences in channel availability time, in the model. Then based on its preference list, an SU decides the bid values by obtaining the rate of data transfer to be used over the available channels. Monetary form of bidding, as observed in literature, may not allow a good use of the idle radio spectrum.
- In this auction model, every SU in a group is considered during the allocation process. This intends to increase the spectrum utilization, although one or more SUs in a group may obtain zero utility (no profit).
- All winning sellers and auctioneer can earn a monetary benefit from the auction. In the proposed model, we mainly focus on spectrum utilization and on a minimal revenue to be earned by the sellers and the auctioneer. However, we only plan a non-negative profit for both the sellers and the buyers.
- This double auction model dynamically decides on the number of rounds based on situations. In the initial round, all channels are auctioned together. In case some channels remain unassigned, subsequent auction rounds can be used to lease such channels. Also, the leftover availability time of an allocated channel can be auctioned in the next round so as to improve the overall utilization of our resource.

The rest of the paper is organized as follows. In Section 2, we present the literature review. The proposed mechanism is described in Section 3 which contains an illustration of the system model, the auction mechanism and auction properties. Section 4 discusses the performance evaluation of the model and finally we conclude the paper in Section 5.

II. RELATED WORKS

Spectrum allocation can be mentioned as one of the most explored area in the domain of spectrum management. Recent studies on wireless network have come up with different solutions to resolve the spectrum allocation problem. To enable a fair channel allocation in D2D networks, authors in [39] proposed a distributed allocation mechanism with the blind rendezvous method which persuades collision-free concurrent communication. Another such approach has been designed for cognitive Internet of Things (IoT) in [40] which takes upon a bio-inspired algorithm to carry out resource allocation amongst the users. In this paper, we concentrate on operating with ad hoc cognitive radio networks for their efficacy in spectrum utilization. To overcome the spectrum scarcity problem, researchers have applied different allocation models, viz. game theory [25], graph theory [26], evolutionary theory [27], auction theory and so forth, to design spectrum sharing mechanisms for CRN. This paper focuses on auction theoretic models for their effectiveness in channel allocation. Auction models deployed are either single-sided or double-sided. Single-sided auctions include SUs as bidders and the PO as auctioneer who leases the channels of PUs. Authors in [28]–[30] proposed different single-sided auction scenarios. PO auctions the channels and earns a revenue. However, in a more realistic scenario, POs (as sellers) play a part in the auction game together with SUs (as buyers) and SB (as auctioneer). This formulates a double-sided auction which enables the POs to acquire their desired range of profit value from the bidders. One of the primary double auction mechanism is the McAfee auction [31] which leases homogeneous items amongst bidders while satisfying the economic properties for single-unit allocation. Also, it does not allow spectrum reuse. Zhou *et al.* in [12] proposed TRUST as the first truthful double auction which allows spectrum reuse and extends the McAfee auction to decide winning sellers and buyers and their respective payments. TRUST makes every PU sell only a single channel where these channels are homogeneous. Also, an SU can request for only one channel in TRUST. SMALL has been introduced as another strategy-proof double auction in [13] which considers a single seller and initially permits single channel allocation along with spectrum reuse. Interfering SUs are modeled using a conflict graph to form buyer groups. Winner selection and price charged are decided based on group bids computed from the buyer groups. Further, SMALL extends its work for multi-channel allocation of the homogeneous channels. Another double auction mechanism that supports multi-channel request from both buyers and sellers is True-MCSA in [14]. To enable spectrum reuse of the auctioned channels, formation of buyer groups and splitting them into virtual buyer groups (VBG) is carried out. For different design goals, two different bidding policies are generated to decide bid for each VBG. Thereafter, winner determination and pricing strategies use the McAfee auction which guarantees truthfulness in the model. In [15], both buyer side and seller side are decoupled to perform spectrum allocation separately. Buyer side uses the graph partitioning process to create sub graphs for spectrum reuse where pricing is computed in each sub graph. Merging combines the resulting sub graphs. On the seller side, traditional pricing design is used to sell one channel from each seller. The main goal of this approach is to auction the channels such that it achieves

TABLE 1. Existing double-auction mechanisms for CRN.

Reference	Spectrum reuse	Multi-channel allocation	Multiple channel from a PO	Heterogeneous channel	Dynamics in SOPs	Channel availability time	Truthful
McAfee [31]	×	×	×	×	×	×	✓
TRUST [12]	✓	×	×	×	×	×	✓
SMALL [13]	✓	✓	✓	×	×	×	✓
True-MCSA [14]	✓	✓	✓	×	×	×	✓
[15]	✓	✓	×	×	×	×	✓
DOTA [16]	✓	✓	✓	×	×	×	✓
[17]	✓	✓	✓	×	×	×	✓
District [18]	✓	✓	✓	×	×	×	✓
[19]	✓	✓	×	×	×	×	✓
STRUCTURE [20]	✓	×	✓	✓	×	×	✓
TAMES [21]	✓	✓	✓	✓	✓	×	✓
PreDA [22]	✓	✓	✓	✓	✓	×	✓
TAHES [23]	✓	×	×	✓	✓	×	✓
[24]	✓	×	✓	✓	×	×	✓
Proposed model	✓	✓	✓	✓	✓	✓	✓

truthfulness and budge balance. Wang *et al.* proposed DOTA in [16] as another double auction which takes care of both range request and strict request of SUs. Sellers/buyers can sell/bid multiple channels along with reusing the channels. Similarly, a multi-unit truthful double auction framework with spectrum reuse characteristic is proposed in [17] which aims at improving the user satisfaction degree. However, heterogeneous channel condition and dynamics in SOPs are not reflected in [17]. A locality based spectrum auction is performed in [18] where a seller auctions one channel in each round and designs the allocation with or without prior knowledge about bid distribution. Zhang *et. al.* developed another auction mechanism in [19] which allows PUs along with the SUs to share the auctioned channels. Interference temperature maintains a threshold level to prevent the interference from the SUs to the PUs so that both of them can simultaneously utilize the channel. Every PU leases a single idle channel during the auction process in [19]. Authors in [20] and [21] introduced double auctions with heterogeneous spectrum in terms of channel frequencies. As such, even though all channels are available to the SUs, an SU may be interested in a part of the spectrum. STRUCTURE in [20] applies a bid-dependent buyer group formation algorithm to enable spectrum reuse while allowing a truthful single-channel allocation among SUs. Similarly, in [21], TAMES designs a multi-seller-multi-buyer double auction where the heterogeneous interference graph for spatial reusability is grouped using sequential grouping. Then, the allocation and pricing rules decide the winning sellers and buyers which satisfy all the economic properties. Khairullah *et. al* introduced PreDA in [22] as another double auction mechanism for heterogeneous channels. A preference list is built up based on the SINR values of channels and bids from SUs are submitted considering this preference list. Multi-channel allocation and spectrum reuse are incorporated by forming virtual groups of non-conflicting SUs. Moreover, the CR constraint of dynamics in SOPs has been attained in PreDA. TAHES in [23] is a single-round multi-unit double auction which incorporates the constraint for dynamics in SOPs in the model. TAHES facilitates spectrum reuse of its heterogeneous channels, but it permits only single-channel allocation of SUs and allows each seller to contribute only a single channel. A truthful double auction in [24] aims at optimizing the profit and energy of CRN, where spectrum reusability is modeled using SINR model and each SU can demand only one channel. A relay based double auction is proposed in [32], where to use the homogeneous channels, SUs are grouped according to their relay access and the interference between them. Another relay based auction model that uses both decode-and-forward and amplify-and-forward relay protocols for spectrum sharing is proposed in [33] where Vickrey auction is used to decide the relay selection. Apart from these approaches, two online double auctions are discussed in [34] and [35]. TODA in [34] uses a complete graph to represent reusability of its homogeneous available spectrum. Whereas, LOTUS in [35] uses the concept of interference discount to design the bid submission process. Hence, the state-of-art on channel allocation in CRN unveils several auction mechanisms working under different network scenarios to effectively lease the idle spectrum. Table 1 lists some of the double auction models discussed in literature.

Now, although the existing double auction mechanisms in CRN have managed to resolve the channel allocation problem, but authors in these works have not incorporated one or the other network constraint discussed above. The proposed double auction mechanism encompasses all the explored network constraints to enable a better use of the dynamically available channels. Also, previously designed auction models take the bid value from SUs as some monetary amount since they are more interested in acquiring a good benefit from the market. But, CRN focuses mainly on utilizing the spectrum. With the monetary bidding strategy, a bidder with the highest bid value, on winning the channel cannot assure that he will make the best use of the assigned channel in terms of its channel characteristic, such as, channel capacity, data rate, bandwidth etc. In case we apply monetary bidding in our proposed model (the way it has been applied in other existing works) where the highest paid group wins, then there may arise some SUs in the group who will not abide by the network constraints taken up in this model. This will not allow those SUs to complete their transmission, resulting in a wastage of radio spectrum.

Hence, with these motivations, we plan to develop a multi-channel allocation with spectrum reusability where the bid value from an SU is given in terms of the rate of data transfer which the SU shall use over the auctioned channel. Also, channels from the POs are considered to be heterogeneous which results in channel specific bid/ask values.

III. DOUBLE AUCTION MECHANISM

This section discusses the challenges arising due to CR constraints, the detailed system model and the double auction mechanism deployed to allocate channels in CRN. The economic properties which make the proposed model economically robust are also analyzed in this section.

A. CHALLENGES

1) DYNAMICS IN SPECTRUM OPPORTUNITIES

Due to differences in SUs' capabilities, the whole set of free channels may not be available to every SU. This gives rise to dynamics in spectrum opportunities amongst the SUs. In CR network, if each SU bids for all the available channels, then there may arise situations where an SU wins a channel which is actually unavailable at the SU. As such, the SU cannot transmit. This reduces the utilization of the radio bands as well as hampers the network throughput. To overcome such a constraint on spectrum availability, SUs should identify and maintain the list of free channels available to them using spectrum sensing process. This will eliminate SUs' bidding for the auctioned channels which are not accessible or available to them. Now, when the auction process starts and a channel is being auctioned, an SU can bid for the channel and possibly win the channel only when the channel is found to be available or accessible to the SU.

2) CHANNEL AVAILABILITY TIME

In CRN, a channel assigned to an SU may be reclaimed by the licensed owner of the channel. This necessitates the SU to immediately vacate the channel and switch to another free channel if available. Meanwhile, if the SU was transmitting over the released channel, then the transmission gets interrupted until it gets the next channel. When an SU is being assigned more than one channel, the return of a PU has no adverse effect, except the inclusion of a switching overhead. But, when an SU gets only one channel and it has to leave the channel in the midway of its transmission, then this lowers the network throughput and an efficient utilization of the channel cannot be provided. So, to overcome such a constraint, every SU needs to know the availability time of the channels. This prevents an SU to bid for a channel whose availability time is less than the time for which the SU requires the channel. As such, the SU acquires a channel which it can use to complete its transmission. Also, this reduces the switching overhead in the network.

B. SYSTEM MODEL

In the designed model, we consider a cognitive radio network that coexists with a primary network. Here, spectrum broker acts as auctioneer, N number of SUs, $\mathcal{N} = \{1, 2, 3, \ldots, N\}$, are the bidders requesting for channels and M number of POs, $\mathcal{M} = \{1, 2, 3, \ldots, M\}$ are the sellers who compete to lease their unused channels. When a channel is said to be available from a PU, it implies that the channel is completely free as in interweaved CRN [2], and it is not being shared with the legitimate PU. Each PO q contributes k_q number of channels which are vacant, and along with decides on a maximum transmissible power limit for every channel. Therefore, total number of channels available for the auction are $\mathcal{Y} = (k_1 + k_2 + \ldots + k_q + \ldots + k_M)$. We assume that $N > \mathcal{Y}$. To communicate the bids and asks to the auctioneer, we take a dedicated licensed common control channel (CCC) [36] which can be used with OFDMA access mechanism. Now, since we consider that the auctioned channels are heterogeneous in quality, the set of channels from a PO q is given as $\Lambda_q = \{q_1, q_2, \ldots, q_j, \ldots, q_{k_q}\}$, where q_j is the j^{th} channel of PO q. All the channels are maintained in a vector $\mathcal{K}$ which is given as follows.

$$\mathcal{K} = \{\underbrace{1, \ldots, k_1}_{\Lambda_1}, \underbrace{(k_1+1), \ldots, (k_1+k_2)}_{\Lambda_2}, \ldots, \\ \underbrace{((k_1+\ldots+k_{M-1})+1), \ldots, (k_1+\ldots+k_M)}_{\Lambda_M}\}_{1\times\mathcal{Y}}$$

This model permits geographically separated SUs to share a common channel simultaneously which in turn improves the spectrum utilization and the revenue. To determine the interference relationship between two SUs, we apply the distance-based interference mechanism which has been used in [12], [14], [23]. An interference matrix, $X = \{x_{ik} | x_{ik} \in \{0, 1\}\}_{N\times N}$, maintains the interfering SUs such that, $x_{ik} = 1$ if SUs i and k cannot get the same channel simultaneously, otherwise $x_{ik} = 0$. Before starting the allocation process, every SU senses for its available channels. During the spectrum sensing phase, different hardware constraints may arise resulting in different SU capabilities [37]. Due to such constraints, every SU may not be able to sense all the free channels which gives rise to dynamics in SOPs. Every SU informs the auctioneer about their spectrum availability. This constructs a channel availability matrix at the auctioneer, $C = \{c_{ij} | c_{ij} \in \{0, 1\}\}_{N\times\mathcal{Y}}$. $c_{ij} = 1$ when a channel $j \in \mathcal{K}$ is available at SU i. Otherwise, $c_{ij} = 0$ implying that SU i

will not bid for channel j since SU i cannot sense it. To avoid interruption in data transmission of SU due to PU's arrival, channel availability time should be taken into consideration. For a channel $j \in \mathcal{K}$, $T_{A(j)}$ represents its availability time which is obtained during the sensing process [38]. The OFF period in the ON-OFF model in [38] specifies the duration for which the channel remains available without any interruption from the legitimate owner. During this period, an SU can use the channel to complete its transmission. For all SUs who can sense the channel j, $T_{A(j)}$ gives an approximately similar value. Further, an SU i determines the channel requirement time for a channel j as $T_{R(ij)}$ when $c_{ij} = 1$ and accordingly it bids for the channel only when $T_{A(j)}$ is greater than or equal to $T_{R(ij)}$. This helps to reduce the switching overhead due to PU activity. Also, an SU who is assigned a single channel can carry out its transmission without any disturbance. To obtain the channel requirement time $T_{R(ij)}$ for channel j, SU i takes the transmission time and propagation delay. Message size of SU i and data rate used over channel j gives the transmission time. Here, we consider that the data rate used by SU i to transmit over the channel j forms the bid value offered by SU i for channel j. Then, the propagation speed and distance to receiver gives the propagation delay. Channels auctioned are heterogeneous with respect to their maximum allowable transmission power. This implies that SUs cannot transmit over a channel j with power more than λ_j. Due to heterogeneous channel conditions, bid values from SUs and ask values from POs will be channel-specific. Now, to compute the channel capacity (Shannon's capacity) of channel j, SU i uses Eq. 1.

$$\Psi_{ij} = W \log_2(1 + \lambda_j \cdot \frac{P_{L(i)}}{I_i + \sigma^2}) \tag{1}$$

Channel bandwidth, W, and noise variance, σ^2, are identical for all channels. $P_{L(i)}$ is the path loss factor between SU i's transmitter and receiver and I_i is the interference from primary network. However, Ψ_{ij} is obtained only when $c_{ij} = 1$. Otherwise, if $c_{ij} = 0$, then $\Psi_{ij} = 0$. So, the channel capacities of the available channels for SU i can be given as:

$$\widetilde{\Psi}_i = \{\Psi_{i1} \geq \Psi_{i2} \geq \ldots \geq \Psi_{iy}\}, \quad s.t. \quad 0 \leq y \leq \mathcal{Y}$$

Now, using the channel capacity, Ψ_{ij}, SU i obtains the channel requirement time of channel j. Then, depending on the channel capacity and the difference between availability time and requirement time of the channel, an SU decides on the number and the order of the channels in its preference list. In the list a more preferred channel will be assigned a higher valuation than a less preferred channel. If for an SU i, $T_{R(ij)} > T_{A(j)}$, then there is no bid for channel j and it is excluded from the preference list of SU i. SU i decides its preference list as per the following conditions:

- If $\Psi_{ij} > \Psi_{ik}$, then channel j is preferable over k.
- If $\Psi_{ij} = \Psi_{ik}$ and $(T_{A(j)} - T_{R(ij)}) = (T_{A(k)} - T_{R(ik)}) = 0$, then both channels are equally preferable.
- If $\Psi_{ij} = \Psi_{ik}$ and $(T_{A(j)} - T_{R(ij)}) < (T_{A(k)} - T_{R(ik)})$, then channel j is preferable over k.

And accordingly, the preference list of channels for SU i is:

$$\gamma_i = \{1, 2, 3, \ldots, y\}, \quad s.t. \quad 0 \leq y \leq \mathcal{Y}$$

Now, to compute the valuations of the channels, SU i uses its preference list of channels, such that, for two consecutive channels $\{j, k\} \in \gamma_i$, valuation of channel k is computed before channel j where $k = j+1$. This implies that we will first compute the valuation of the least preferred channel $y \in \gamma_i$. Then the valuations of the channels $\{y-1, y-2, \ldots, 3, 2, 1\} \in \gamma_i$ are computed in order. For a channel $j \in \mathcal{K}$, $v_{ji}^{(b)}$ represents the valuation from SU i. To get the valuation of channel j when $T_{A(j)} \geq T_{R(ij)}$, one of the following condition is applied by SU i.

- When channel j is preferred over channel k (such that $k = j + 1$), then $\Psi_{ij} \geq v_{ji}^{(b)} \geq \Psi_{ik}$.
- When both channels j and k (such that $k = j + 1$) are equally preferable, then $\Psi_{ij} = \Psi_{ik} = v_{ji}^{(b)}$.

Eq. 2 computes the valuation $v_{ji}^{(b)}$ of channel j for SU i. For the least preferred channel of an SU, we directly apply Eq. 2 to get its valuation. For a channel which is not included in the preference list of SU, valuation is equal to 0 since such a channel shall satisfy one of the first two conditions of Eq. 2.

$$v_{ji}^{(b)} = \begin{cases} 0 & \text{if } c_{ij} = 0 \\ 0 & \text{if } (T_{R(ij)} > T_{A(j)}) \\ \Psi_{ij} & \text{if } (T_{R(ij)} = T_{A(j)}) \\ < \Psi_{ij} & \text{if } T_{R(ij)} < T_{A(j)}, \text{ but } v_{ji}^{(b)} \text{ is chosen} \\ & \text{s.t. } T_{A(j)} \geq T_{R(ij)} \text{ continues to hold} \end{cases} \tag{2}$$

$c_{ij} = 0$ implies that channel j is not available at the SU, so SU i has no valuation for channel j. The computation of channel requirement time for the channel is carried out only when $c_{ij} = 1$, so that valuation for the channel can be determined. When channel requirement time computed using the channel capacity becomes greater than the availability time, SU i does not submit any bid for the channel. This helps to avoid interruption in the transmission process of the SU. But, when both requirement time and availability time are same, then channel capacity becomes the valuation. This is because, if the SU chooses a data rate (less than channel capacity), then the requirement time increases and the constraint for channel availability time gets violated. And lastly, when the requirement time is less than the availability time, then we choose a data rate as the valuation. But, here again we have to check whether the chosen value for data rate (to be used for computing the requirement time) satisfies the availability time constraint. This accordingly proceeds to give us a data rate which assures the channel availability time constraint to yield a productive network performance. Now, with truthful bidding strategy, the bid value decided by SU i for a channel j, $b_{ji}^{(b)}$, is equal to the valuation $v_{ji}^{(b)}$. So, on receiving bids from the SUs, a bid vector is formed for each channel j, which is given as, $B_j^{(b)} = \{b_{j1}^{(b)}, b_{j2}^{(b)}, \ldots, b_{ji}^{(b)}, \ldots, b_{jN}^{(b)}\}$. Considering all channels, we get a bid matrix $\mathcal{B} = \{B_1^{(b)}; B_2^{(b)}; \ldots; B_{\mathcal{Y}}^{(b)}\}$.

$$\mathcal{B} = \begin{bmatrix} b_{11}^{(b)} & b_{12}^{(b)} & \ldots & b_{1i}^{(b)} & \ldots & b_{1N}^{(b)} \\ \ldots & \ldots & \ldots & \ldots & & \\ b_{j1}^{(b)} & b_{j2}^{(b)} & \ldots & b_{ji}^{(b)} & \ldots & b_{jN}^{(b)} \\ \ldots & \ldots & \ldots & \ldots & & \\ b_{\mathcal{Y}1}^{(b)} & b_{\mathcal{Y}2}^{(b)} & \ldots & b_{\mathcal{Y}i}^{(b)} & \ldots & b_{\mathcal{Y}N}^{(b)} \end{bmatrix}$$

To represent the channel allocation among SUs, a channel allocation matrix is used, $\mathcal{A} = \{a_{ij} | a_{ij} \in \{0, 1\}\}_{N \times \mathcal{Y}}$, where $a_{ij} = 1$ if channel j has been assigned to SU i, otherwise $a_{ij} = 0$. On winning a channel, the SU needs to make a payment to auctioneer. Since, multi-channel allocation enables an SU to acquire more than one channel, so a bidder payment matrix, $P^{(b)}$, is created such that it holds the payment paid by every SU for the channels which it won. $P^{(b)} = \{p_{ij}^{(b)}\}_{N \times \mathcal{Y}}$ where $p_{ij}^{(b)}$ represents the payment from SU i when $a_{ij} = 1$ for channel $j \in \mathcal{K}$. $p_{ij}^{(b)} = 0$ when channel j is not assigned to SU i. Total payment paid by SU i is given by $\rho_i^{(b)} = \sum_{j=1}^{\mathcal{Y}} p_{ij}^{(b)}$, $\forall i \in \mathcal{N}$. Utility of an SU i, $u_i^{(b)}$, on winning a channel is the difference between true valuation of SU for the channel and payment paid by SU for the channel. So, $\widetilde{u_i^{(b)}}$ represents the total utility of SU i on winning its desired channels which can be obtained using Eq. 3.

$$\widetilde{u_i^{(b)}} = \begin{cases} \sum_{j=1}^{\mathcal{Y}} a_{ij} v_{ji}^{(b)} - \rho_i^{(b)} & \text{if } \sum_{j=1}^{\mathcal{Y}} a_{ij} \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

If an SU i remains deprived from channel allocation, then both $\rho_i^{(b)} = \widetilde{u_i^{(b)}} = 0$. Now, with an allocation $\mathcal{A} = \{a_{ij}\}_{N \times \mathcal{Y}}$ obtained for the auctioned channels in different auction rounds, spectrum utilization, S_u, can be defined as the sum total of winning bid values of SUs for the assigned channels and is expressed in Eq. 4 [11], [29]. Spectrum utilization gives a measure of the total data rate that has been allowed over the used channels.

$$S_u = \sum_{j=1}^{\mathcal{Y}} \sum_{i=1}^{N} b_{ji}^{(b)} a_{ij} \tag{4}$$

In the seller side, POs compete amongst them and submit ask values to the auctioneer for their idle channels. This allows them to obtain certain financial gain as sellers. Every PO q decides a valuation, $v_{qj}^{(s)}$, for its channel j such that $q_j \in \Lambda_q$. Since the channels are heterogeneous, the valuation for every channel in Λ_q can be different. Considering truthfulness in the model, the ask value of PO q for its j^{th} channel, $b_{qj}^{(s)}$, is equal to the valuation $v_{qj}^{(s)}$. So, by collecting the ask values from all POs, we get an ask vector $\mathcal{H}$ which is given as follows.

$$\mathcal{H} = \{\underbrace{b_{11}^{(s)}, \ldots, b_{1\,k_1}^{(s)}}_{\text{Ask values for } \Lambda_1}, \ldots, \underbrace{b_{q1}^{(s)}, \ldots, b_{qk_q}^{(s)}}_{\text{Ask values for } \Lambda_q}, \ldots, \times \underbrace{b_{M1}^{(s)}, \ldots, b_{Mk_M}^{(s)}}_{\text{Ask values for } \Lambda_M}\}_{1 \times \mathcal{Y}}$$

A winner vector, W_q, in formed for each PO q such that $W_q = \{w_{q1}, \ldots, w_{qj}, \ldots, w_{qk_q}\}$, where w_{qj} represents the number of non-interfering SUs who are assigned channel j of PO q. Also, the payment earned by a PO q on selling its channel j is given as $p_{qj}^{(s)}$. As such, we get a seller payment vector, $P_q^{(s)}$, for each PO q where, $P_q^{(s)} = \{p_{q1}^{(s)}, \ldots, p_{qj}^{(s)}, \ldots, p_{qk_q}^{(s)}\}$. If $w_{qj} = m$, $(0 \leq m \leq N)$, then $p_{qj}^{(s)}$ is the payment earned from m number of SUs who won the channel j of PO q. But, if $w_{qj} = p_{qj}^{(s)} = 0$, then this implies that channel j could not be assigned to any SU in the network. So, total payment earned by a PO q is given as $\rho_q^{(s)} = \sum_{j=1}^{k_q} p_{qj}^{(s)}$, $\forall q \in \mathcal{M}$. Again, to find the utility of a PO q on selling a channel, $u_q^{(s)}$, difference between the payment earned by the PO for the channel and its valuation for the channel is taken. So, total utility of PO q, on selling all its channel, $\widetilde{u_q^{(s)}}$, can be computed using Eq. 5.

$$\widetilde{u_q^{(s)}} = \begin{cases} \rho_q^{(s)} - \sum_{\substack{j=1 \\ w_{qj} \neq 0}}^{k_q} v_{qj}^{(s)} & \text{if } \sum_{j=1}^{k_q} w_{qj} \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Hence in this paper, we deploy a sealed-bid double auction mechanism where heterogeneous channels are auctioned by POs so that SUs in the CRN can fulfill their spectrum requirement while abiding by the constraints which become apparent in CRN.

C. AUCTION MECHANISM

In the proposed model, an auction is being designed where the bidders and the sellers submit their bids and asks respectively to the auctioneer. At the start, POs submit ask values along with the maximum allowable transmission power for their respective channels to the auctioneer (SB). SB announces the maximum allowable transmission power of all auctioned channels, and the SUs, while abiding by the power limits, offer their bid values to SB. And then, based on these values, the auctioneer decides the winners. Initially, one round is used with the aim to auction all the channels together by the auctioneer. But, if some channels remain unassigned after deciding the winner per channel, the auction proceeds to the next round by allowing the SUs and POs to resubmit their bids and ask values respectively for such channels. This is primarily done to allow an increase in the overall spectrum utilization and to allow more allocation chance to the SUs. Also, another round of auction can be carried out for those assigned channels which have some leftover availability time. In this case, the SUs can utilize the leftover availability time of those channels while abiding by the channel availability time constraint. Fig 2 gives a diagrammatic representation of the proposed double auction model. Following steps are carried out by the auctioneer for the proposed model.

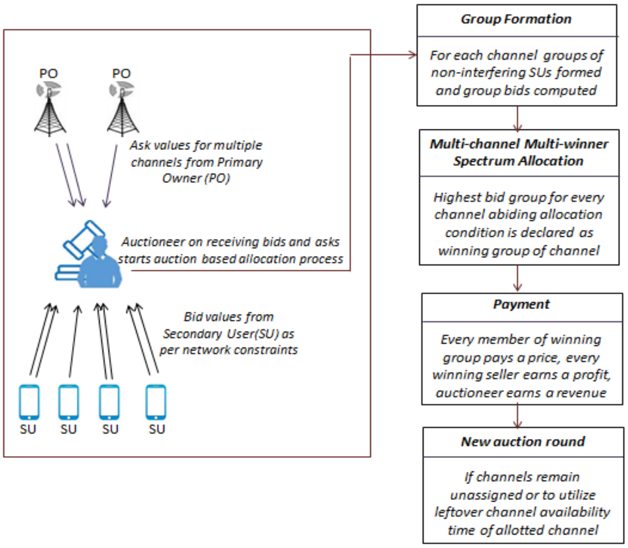


FIGURE 2. **Diagrammatic representation of proposed double auction model.**

Algorithm 1 Group Formation Algorithm

```
Input: Matrices $X$, $\mathcal{B}$ and channel $j \in [1, \mathcal{Y}]$
Output: Group vector $G_j = (g_1^j, g_2^j, \ldots, g_z^j, \ldots, g_{|G_j|}^j)$
$G_j = \phi$;
for $i \leftarrow 1$ to $N$ do
    if $b_{ji}^{(b)} \neq 0$ then
        for $k \leftarrow 1$ to $N$ do
            $SUList = \phi$ ;
            for $q \leftarrow k$ to $(N + k - 1)$ do
                Calculate $m = q \ \% \ (N+1)$;
                if $m < q$ then
                    $m = m+1$;
                end
                if $m = i$ then
                    $SUList = SUList \cup \{m\}$
                else
                    if $b_{jm}^{(b)} \neq 0$ then
                        if $x_{im} \neq 1$ then
                            if $SUList = \phi$ then
                                $SUList = SUList \cup \{m\}$;
                            else
                                for $l \leftarrow 1$ to $length(SUList)$ do
                                    $f = SUList(l)$;
                                    if $x_{mf} = 1$ then
                                        $flag = 0$;
                                        $break$;
                                    else
                                        $flag = 1$;
                                    end
                                end
                                if $flag = 1$ then
                                    $SUList = SUList \cup \{m\}$;
                                end
                            end
                        end
                    end
                end
            end
            if $\nexists g_y \in G_j$ s.t. $g_y = SUList$ then
                $G_j = G_j \cup \{SUList\}$;
            else
                $SUList$ not included in $G_j$
            end
        end
    end
end
```

1) WINNER DETERMINATION

SB carries out the winner determination strategy to obtain the winning SUs who can acquire their preferred channels and winning POs who can temporarily leased their unused channels. Two algorithms are developed to determine the winners for each channel. In the first algorithm (group formation algorithm), we carry out the group formation process where groups of non-interfering SUs are formed for each channel. In this process, for each channel, groups are formed by taking those SUs together who have a bid for the given channel and who can transmit over the given channel without causing any interference to each other. A number of such groups of non-interfering SUs for each channel are formed separately due to the constraints specified in the model. Thereafter, for each group (formed for a channel), a group bid is computed based on the bid values of the group members. Finally, the spectrum allocation algorithm is applied to allocate the given channel, which takes the group bid values as its input.

To carry out the group formation process (Algorithm 1), every channel $j \in \mathcal{K}$ is taken one-by-one to form a group vector $G_j = (g_1^j, g_2^j, \ldots, g_z^j, \ldots, g_{|G_j|}^j)$, where g_z^j represents the z^{th} group in G_j such that all members in g_z^j can be given channel j which in turn facilitates spectrum reuse. To construct the group vector for channel j, we take every SU i one after another and check the bid matrix. If bid from SU i for channel j is 0, we move to the next SU since SU i cannot be included in any group. Otherwise, we get all possible combinations of SUs who can form groups with SU i while considering the interference amongst the SUs. Moreover, on applying the algorithm to form G_j, if a group is formed which is similar to a group that already exists in G_j, then the newly formed group is discarded so as to avoid duplication of groups in the group vector. For instance, there are 4 SUs (SU 1, SU 2, SU 3, SU 4) and one channel C1. SU 2 has no bid for the channel and there is interference between SU 1 and SU 3, SU 2 and SU 3. According to the algorithm, we take every SU i one-by-one. Since, SU 1 ($i = 1$) bids for the channel C1, so we start with SU 1. Now we look for the groups where SU 1 will be one of the members. For this, we take every SU $k \in \{1, 2, 3, \ldots, N\}$

to get an *SUList* for each k. When $k = 1$, SU i = SU k, so SU 1 gets included in the *SUList*, i.e., $SUList = \{1\}$. Now to check for the other SUs which can be included in the *SUList* for $k = 1$, we take the SUs sequentially from SU 2 to SU 4. SU 2 has no bid for C1, so it cannot be in the list. SU 3 interferes with SU 1, so it cannot be in the list. However, SU 4 gets included, so $SUList = \{1, 4\}$. Once all the SUs are checked for $k = 1$, we obtain the complete *SUList* for $k = 1$. Now, since G_1 is empty, so $G_1 = \{\{1, 4\}\}$. Next for $k = 2$, $k = 3$ and $k = 4$, the same $SUList = \{1, 4\}$ is obtained and this will not be included in G_1. Then, we take SU $i = 2$ and there is no bid from SU 2. So, we go for SU $i = 3$. Here, $SUList = \{3, 4\}$ gets formed for $k = 1$, $k = 2$, $k = 3$ and $k = 4$. Since, this is not present in G_1, so $G_1 = \{\{1, 4\}, \{3, 4\}\}$. Then we move to SU $i = 4$ where for $k = 1$, $SUList = \{1, 4\}$, for $k = 2$, $SUList = \{3, 4\}$, for $k = 3$, $SUList = \{3, 4\}$, for $k = 4$, $SUList = \{1, 4\}$. This keeps G_1 unchanged, and finally the group vector obtained for the channel is $G_1 = \{\{1, 4\}, \{3, 4\}\}$.

Once the group vector G_j is obtained for the channel j, a group bid has to be computed for each group in G_j. For a group $g^j_z \in G_j$, group bid, μ^j_z, is taken from one of the members in g^j_z. If SU $h \in g^j_z$ has the lowest bid value amongst all the SUs in g^j_z, then μ^j_z can be obtained from the bid value submitted by SU h, i.e., $b^{(b)}_{jh}$. So, group bid for group g^j_z can be computed as given in Eq. 6.

$$\mu^j_z = \min\{b^{(b)}_{jh} | h \in g^j_z\}.|g^j_z| \tag{6}$$

However, if there appears two or more minimum bid SU in the group, then the bid from any one of the SU is randomly picked to compute the group bid. In a few double auction models discussed in literature [12]–[14], [21], [22], it has been found that when a group bid is decided, the member with minimum bid value in the group is excluded from the group. This reduces the number of SUs who can be assigned the channel, which in turn reduces the spectrum reuse and spectrum utilization. Also, when a group has only one member, such a group cannot participate in auction since the lone member gets excluded in this case. No such member exclusion is performed in our model while computing the group bids. This improves the use of the radio spectrum and the possibility of allocating a channel gets higher due to greater group bid values. For computational complexity of Algorithm 1, we find that for every i^{th} iteration of outermost loop, the first inner loop runs N times and for every k^{th} iteration of first inner loop, the second inner loop again runs N times. Then, for every q^{th} iteration of the second inner loop, the third inner loop runs $(q - 1)$ times. So, we get the following.

$$\begin{aligned} T(N) &= N^2[(1 + 2 + \ldots + (N - 1)) + (N + y)] \\ &= N^2\left[\left(\frac{N(N+1)}{2} - N\right) + (N + y)\right] \\ &= (N^4 + N^2 y) \\ &= O(N^4) \end{aligned}$$

Algorithm 2 Spectrum Allocation Algorithm

```
Input: Group vector $G_j = (g^j_1, g^j_2, \ldots, g^j_z, \ldots, g^j_{|G_j|})$ for
        channel $j \in [1, \mathcal{Y}]$
Output: Allocation matrix $\mathcal{A}$
Get $\theta_j = (\mu^j_1, \mu^j_2, \ldots, \mu^j_z, \ldots, \mu^j_{|G_j|})$;
Let channel $j$ be $d^{th}$ channel of PO $q$, so get $b^{(s)}_{qd}$;
if $b^{(s)}_{qd} \le \max(\theta_j)$ then
    if count($\max(\theta_j)$)= 1 then
        Channel $j$ allocated to $g^j_z$ when $\mu^j_z = \max(\theta_j)$;
    else
        Get every group $g^j_f$ with group bid $\max(\theta_j)$;
        $\forall g^j_f$, compute sumValue $= \sum_{\forall m \in g^j_f} b^{(b)}_{jm}$ ;
        Let $g^j_z$ be the group with highest sumValue,
        allocate channel $j$ to $g^j_z$;
    end
    $\forall n \in g^j_z$, $a_{nj} = 1$ and $w_{qd} = |g^j_z|$ ;
else
    Channel $j$ remains unallocated
end
```

To carry out the spectrum allocation process for a channel j (Algorithm 2), SB takes the group vector G_j and the corresponding group bid vector $\theta_j = (\mu^j_1, \mu^j_2, \ldots, \mu^j_z, \ldots, \mu^j_{|G_j|})$, where μ^j_z is the group bid of g^j_z. Now, if channel $j \in \mathcal{K}$ is the d^{th} channel of PO q, then we take the ask value $b^{(s)}_{qd}$ for the channel from PO q and compare it with the highest value in θ_j. If the group g^j_z has the highest bid amongst all groups in G_j, and μ^j_z is greater than or equal to $b^{(s)}_{qd}$, then the channel gets assigned to all members of g^j_z. Otherwise, the channel remains unassigned. However, if there appears two or more groups having the same highest group bid value, then in such case, we get the total bid value of each group by computing the summation of bids of every member in a group. Amongst them, the group which gives the highest total bid is considered as the winning group. But again, if more than one group shows a similar total bid value, then we go for the random selection of any one group (having the highest total bid) to be declared as winner. So, this process repeats for every channel present in $\mathcal{K}$ to get the allocation pattern. For the computational complexity of Algorithm 2, we find that there can be $(N + y)$ groups (y is some constant number) with maximum group bid and for every f^{th} group, there can be (N-1) SUs to compute the sum. Also, N number of SUs can win a particular channel. So, we get the following.

$$\begin{aligned} T(N) &= [((N + y)(N - 1)) + N] \\ &= O(N^2) \end{aligned}$$

2) PAYMENT

Now, once the channels get assigned to their respective winner SUs, every winning SU has to pay a price to the auctioneer. On allocating a channel $j \in \mathcal{K}$ to the members of a group g_z^j, the auctioneer earns the group valuation δ_z^j as its profit. According to the truthful bidding strategy, valuation of SU i for channel j, $v_{ji}^{(b)}$, is equal to the bid $b_{ji}^{(b)}$. Consequently, SB computes δ_z^j using Eq. 7.

$$\delta_z^j = \min\{v_{jh}^{(b)} | h \in g_z^j\}.|g_z^j| \tag{7}$$

The group valuation computed is equal to the group bid. Now, according to Eq. 7, every winning SU i in g_z^j pays a uniform price. That is, $p_{ij}^{(b)} = \min\{v_{jh}^{(b)} | h \in g_z^j\}$, which is the minimum valuation offered by some SU h in g_z^j. As such, payment from SU h is its own valuation which results in zero utility for SU h. Also, any other SU having a valuation equal to SU h will have a zero utility. An SU i who cannot win channel j, does not make any payment for the channel, i.e., $p_{ij}^{(b)} = 0$. Subsequently, a winning PO q on selling its channel j to the SUs of a group g_z^j earns a payment from the auctioneer. Auctioneer obtains the valuation from PO q for the channel which is equal to the ask value. This valuation is then given as the payment to the winning PO. That is, $p_{qj}^{(s)} = v_{qj}^{(s)}$ and this gives a zero utility to the PO. So, every winning PO obtains a zero utility. Also, when PO q cannot sell the channel, it receives zero payment. Such a payment strategy guarantees truthfulness in the model, where all SUs in a group are considered for winner determination. Also, it helps the auctioneer to earn a monetary benefit as its revenue. In previously discussed auction models [21], auctioneer does not earn a revenue. In the proposed model, we allow every winning seller and the auctioneer to earn a minimal monetary benefit, which showcases a more practical scenario in auction. However, we only concentrate on obtaining a non-negative utility amongst the buyers and the sellers since our main focus is to improve the spectrum utilization. And finally, auctioneer computes its revenue by taking the difference between the payment earned from the winning SUs and the payment paid to the winning POs. Revenue r_{qj} obtained on selling channel j from PO q to a group g_z^j is given using Eq. 8. But, if the channel remains unsold, then $r_{qj} = 0$.

$$r_{qj} = \delta_z^j - v_{qj}^{(s)} \tag{8}$$

3) NEW AUCTION ROUND

After completing one round of auction where all the channels are auctioned together, we may find one or more unassigned channels. Also, for a channel that has been assigned to a group of non-interfering SUs, the leftover availability time (in excess to SU's requirement) of the channel gets wasted. So, to utilize these resources, subsequent auction rounds can be carried out. For a channel j which remains unassigned, the new round of auction starts by allowing the SUs and the PO to resubmit their bids and ask values respectively for the channel. To increase the chance of channel allocation in the new round, the PO decreases the channel's ask value with respect to the previous round. However, every PO sets a reserve value below which it cannot decide its ask price to sell the channel. Another channel k, which has been allocated to a group of SUs, can also participate in the subsequent auction round. The leftover availability time of the channel can be utilized by the SUs who could not win that channel in any of the previous rounds. For channel k, $T_{A(k)}$ in the new round is equal to the difference between $T_{A(k)}$ of previous round and $T_{R(ik)}$, where SU i is a winner of channel k in previous round whose channel requirement time is maximum among all winners of the channel. The PO resubmits the ask value and the maximum allowable power limit for the channel to the auctioneer. Then, an SU l bids for channel k in the new round only if it has not been assigned channel k in any of the previous rounds and along with it adheres to the network constraints. The preference list of the participating SUs decides their bid values. Then, the winner determination and payment steps are executed in the new round and this accordingly updates the allocation matrix $\mathcal{A}$. The auction stops when for the auctioned channels there is no group vector for any of the channel. This implies that either there are no participating SUs or the participating SUs do not satisfy the network constraints for the auctioned channels.

D. ILLUSTRATIVE EXAMPLE

Fig. 3 discusses the proposed double auction model using an example. In this figure, 7 SUs are acting as buyers and 2 POs are acting as sellers. All total there are 5 channels out of which 3 channels are auctioned by PO 1 and remaining 2 channels by PO 2. Due to the network constraints, all channels may not be available to every SU. As for SU 1, only three channels (C1, C2 and C4) out of the 5 channels are available for which SU 1 submits its bids. Similarly, the SOPs of all other SUs are shown along with each SU in the figure. Also, the figure shows the interfering SUs using the interference matrix. The shaded region in the first row second column of the matrix implies that SU 1 and SU 2 cannot be assigned a common channel simultaneously due to their interference. Ask values from the POs for their respective channels are given in Table 2. Table 3 shows the preference list of each SU and accordingly the bids from the SUs based on their preference list are given in Table 4. And, on collecting all the bids and asks, the auctioneer first of all obtains the channel-specific groups for each channel and their respective group bids. Then, the winning bidders and sellers and their respective payments are determined as shown in Table 5. Channel 1 gets assigned to SUs S1, S4 and S7 since they are non-interfering amongst them and their group bid computes to be the highest amongst other groups. Similarly, we get the winners for channels 3, 4 and 5. But channel 2 remains unassigned because the highest group bid value is less than the ask value of channel 2. Now, we start the 2nd round of auction as there are SUs participating for the unassigned channel 2 and also for the channel 3 where the channel is auctioned for the remaining available time. For the channels 1, 4 and 5, even though their available time may not be fully utilized, but there are no SUs who look for these channels. So, Table 6 shows the ask values, Table 7 and Table 8 gives the preference list and bids of the SUs respectively for the two channels. For channel 3, the SUs

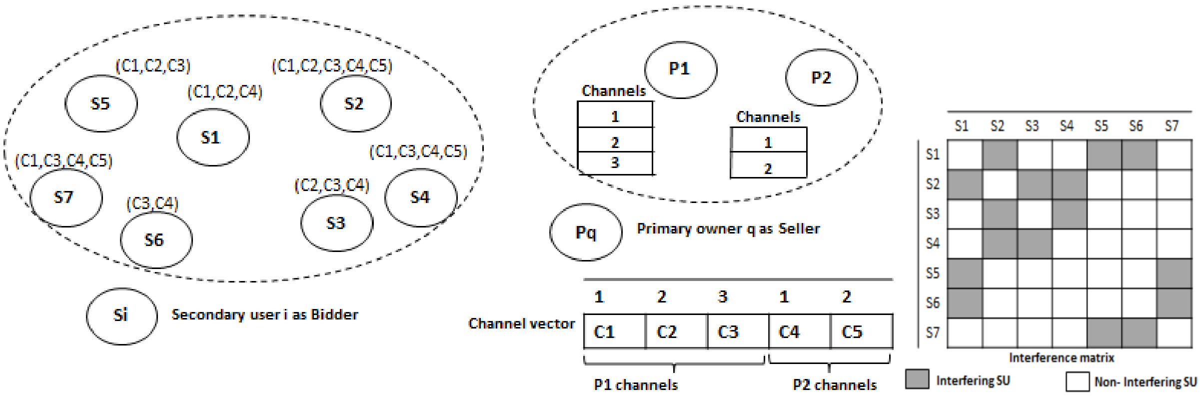


FIGURE 3. Illustrative example of the double auction model.

TABLE 2. Seller ask values in 1st round.

Channel	Ask value
C1	5
C2	7
C3	4
C4	2
C5	3

TABLE 3. Preference list of SUs in 1st round.

SU	Preference list
S1	{C1, C4, C2}
S2	{C4, C3, C2, C5, C1}
S3	{C2, C4, C3}
S4	{C1, C3, C4, C5}
S5	{C3, C1, C2}
S6	{C3, C4}
S7	{C5, C1, C3, C4}

who did not get this channel previously can only participate. And finally, groups, group bids, winners and payments for channel 2 and 3 are given in Table 9. Further, there are no more auction rounds since there are no SUs who bids for the channels. In a practical scenario, the base stations (BS) of a cellular network operate as sellers and the SUs in the ad hoc CRN will be the bidders. Then, the channels which are kept unused by the mobile stations (MS) are brought for lease by the respective BS which can earn a revenue to the BS and along with provide the SUs their requisite spectrum.

E. AUCTION PROPERTIES

In this section, we prove the economic properties for the proposed double auction.

Definition 1: Individual rationality: A double auction is said to be individually rational if no winning bidder pays a price which is greater than its valuation and no winning seller earns a price which is less than its valuation. That is, $\forall i \in \mathcal{N}$, $p_{ij}^{(b)} \leq v_{ji}^{(b)}$ *when* $a_{ij} = 1$ *and* $\forall q \in \mathcal{M}$, $p_{qj}^{(s)} \geq v_{qj}^{(s)}$ *when* $w_{qj} \neq 0$.

TABLE 4. Bid values in 1st round.

	C1	C2	C3	C4	C5
S1	5	3	0	4	0
S2	2	3	5	6	3
S3	0	6	2	4	0
S4	7	0	2	2	1
S5	3	1	4	0	0
S6	0	0	7	3	0
S7	4	0	4	3	6

Definition 2: Truthful: A double auction is said to be truthful if there is no bidder or seller in the game who can improve its utility by submitting an untruthful bid or ask value.

Definition 3: Ex-post budget balance: A double auction is said to be ex-post budget balance if the auctioneer earns a non-negative revenue.

Theorem 1: The proposed double auction is individually rational in the buyer side.

Proof: For an auction to be individually rational in the buyer side, the utility obtained by every buyer should be non-negative. That is, for an SU i, $u_i^{(b)} \geq 0$.

In the buyer side, every SU i, who is a member of a winning group g_z^j, pays a price which is equal to the minimum valuation offered by some SU h belonging to g_z^j. This implies that all members of g_z^j including SU h makes a payment which is less than or equal to their respective valuation for the channel j. That is, $\forall i \in g_z^j$, $v_{ji}^{(b)} \geq v_{jh}^{(b)}$ where $v_{jh}^{(b)}$ is the payment. Hence, we get a non-negative utility for the SUs, that is, $u_i^{(b)} \geq 0$. ■

Theorem 2: The proposed double auction is individually rational in the seller side.

Proof: For an auction to be individually rational in the seller side, the utility obtained by every seller should be non-negative. That is, for a PO q, $u_q^{(s)} \geq 0$.

In the seller side, a PO q on selling its channel j $(1 \leq j \leq k_q)$ to the members of a group g_z^j earns a payment $p_{qj}^{(s)}$ from the auctioneer where $p_{qj}^{(s)}$ is equal to the valuation of PO q for the channel j. And since utility is the difference between the payment and the valuation, so utility obtained by

TABLE 5. Winner determination and payment in 1st round.

Channel	Groups	Group bids	Winning SUs	Buyer's pay	Seller's pay	Revenue
C1	(S1, S4, S7), (S2, S5), (S2, S7), (S4, S5)	{12, 4, 4, 6}	S1, S4, S7	12	5	7
C2	(S1, S3), (S2, S5), (S3, S5)	{6, 2, 2}	Unallocated	0	0	0
C3	(S2, S5, S6), (S3, S5, S6), (S4, S5, S6), (S2, S7), (S3, S7)	{12, 6, 6, 8, 4}	S2, S5, S6	12	4	8
C4	(S1, S3, S7), (S1, S4, S7), (S2, S6), (S3, S6), (S4, S6), (S2, S7)	{9, 6, 6, 6, 4, 6}	S1, S3, S7	9	2	7
C5	(S2, S7), (S4, S7)	{6, 2}	S2, S7	6	3	3

TABLE 6. Seller ask values in 2nd round.

Channel	Ask value
C2	5
C3	1

TABLE 7. Preference list of SUs in 1st round.

SU	Preference list
S1	{C2}
S2	{C2}
S3	{C2, C3}
S4	{C3}
S5	{C2}
S7	{C3}

TABLE 8. Bid values in 2nd round.

	C1	C2	C3	C4	C5
S1	0	4	0	0	0
S2	0	3	0	0	0
S3	0	6	4	0	0
S4	0	0	3	0	0
S5	0	2	0	0	0
S6	0	0	0	0	0
S7	0	0	5	0	0

PO q is $u_q^{(s)} = 0$. Hence, no matter what the ask value is for the channel, the utility remains to be zero. This results in a non-negative utility for the POs. ■

Lemma 1: When an SU i wins a channel j by submitting a bid value $b_{ji}^{(b)}$, then SU i also wins the channel with a bid $b_{ji}'^{(b)} > b_{ji}^{(b)}$. (provided that all other bids and asks and network conditions remain same)

Proof: On auctioning a channel j from a PO q, if a group g_z^j wins the channel, then it depends on the lowest bid value submitted by some SU h of the group. As such, every other SU in g_z^j has a bid value greater than or equal to the bid of SU h. Now, for any SU $i \in g_z^j$ such that $i \neq h$, if SU i submits a bid $b_{ji}'^{(b)} > b_{ji}^{(b)}$, SU i still wins the channel j since the group bid remains unchanged. And also, if SU h submits a bid $b_{jh}'^{(b)} > b_{jh}^{(b)}$, the group still wins the channel since the group bid increases or stays same, no matter whether SU h remains the lowest bidder in the group or not. ■

Lemma 2: When a PO q sells a channel j $(1 \leq j \leq k_q)$ by submitting an ask value $b_{qj}^{(s)}$, then PO q can also sell the channel with a value $b_{qj}'^{(s)} < b_{qj}^{(s)}$. (provided that all other bids and asks and network conditions remain same)

Proof: To sell a channel j from PO q to the members of a group g_z^j, the group bid μ_z^j has to be greater than the ask value $b_{qj}^{(s)}$. So, when g_z^j wins channel j, this implies that $b_{qj}^{(s)} \leq \mu_z^j$. As such, even with an ask value $b_{qj}'^{(s)} < b_{qj}^{(s)}$, PO q can sell its channel to the group. ■

Theorem 3: The proposed double auction is truthful in the buyer side.

Proof: A double auction is truthful in the buyer side if on submitting an untruthful bid value $b_{ji}^{(b)} \neq v_{ji}^{(b)}$ by an SU i, its utility cannot be improved. That is, if $u_i^{(b)}$ and $u_i'^{(b)}$ are the utilities for $v_{ji}^{(b)}$ and $b_{ji}^{(b)}$ respectively, then $u_i^{(b)} \geq u_i'^{(b)}$.

Case I: $b_{ji}^{(b)} > v_{ji}^{(b)}$

1) When SU i cannot win the auction by bidding either $v_{ji}^{(b)}$ or $b_{ji}^{(b)}$, then $u_i^{(b)} = u_i'^{(b)} = 0$.

2) When SU i loses by bidding $v_{ji}^{(b)}$, but wins on bidding $b_{ji}^{(b)}$, then we get $u_i^{(b)} = 0$. If an SU initially loses but then wins the auction by increasing its own bid value, this implies that the SU is the one who initially gives the lowest bid $v_{ji}^{(b)}$ in the group which determines the group bid. When SU i submits $b_{ji}^{(b)} > v_{ji}^{(b)}$ and it wins the channel, then there can be two cases. In one case, SU i remains the lowest bid SU in the winning group g_z^j. And in the other case, there can be some SU k in g_z^j such that $b_{jk}^{(b)} \geq v_{ji}^{(b)}$, but $b_{jk}^{(b)} < b_{ji}^{(b)}$. So, SU k becomes the lowest bidder in g_z^j when SU i bids $b_{ji}^{(b)}$. But, since SU i wins, it pays its own valuation in both cases because it is the bidder with lowest valuation $v_{ji}^{(b)}$ in the group, i.e., $p_{ij}'^{(b)} = v_{ji}^{(b)}$ which results in $u_i'^{(b)} = 0$.

3) According to Lemma 1, it cannot be true that SU i wins by bidding $v_{ji}^{(b)}$ and loses on bidding $b_{ji}^{(b)}$.

4) When SU i wins by bidding both $v_{ji}^{(b)}$ and $b_{ji}^{(b)}$, then let $p_{ij}^{(b)}$ and $p_{ij}'^{(b)}$ be the payments respectively. With $v_{ji}^{(b)}$, if SU i is the lowest valuated bidder in the group g_z^j, then with $b_{ji}^{(b)} > v_{ji}^{(b)}$, SU i may or may not be the lowest bidder but it submits the lowest valuation. So in both conditions, SU i on winning channel j pays its own valuation as the payment, i.e., $p_{ij}^{(b)} = p_{ij}'^{(b)}$. Otherwise, with $v_{ji}^{(b)}$, if SU i is not the lowest valuated bidder in g_z^j, then its payment is independent of its valuation and it depends on only the valuation of some SU k who has the lowest value in g_z^j. So, with $b_{ji}^{(b)} > v_{ji}^{(b)}$, we get $p_{ij}^{(b)} = p_{ij}'^{(b)}$ which gives $u_i^{(b)} = u_i'^{(b)}$.

Case II: $b_{ji}^{(b)} < v_{ji}^{(b)}$

1) When SU i cannot win the auction by bidding either $v_{ji}^{(b)}$ or $b_{ji}^{(b)}$, then $u_i^{(b)} = u_i'^{(b)} = 0$.

TABLE 9. Winner determination and payment in 2nd round.

Channel	Groups	Group bids	Winning SUs	Buyer's pay	Seller's pay	Revenue
C2	(S1, S3), (S2, S5), (S3, S5)	{8, 4, 4}	S1, S3	8	5	3
C3	(S3, S7), (S4, S7)	{8, 6}	S3, S7	8	1	7

2) According to Lemma 1, it cannot be true that SU i wins by bidding $b_{ji}^{(b)}$ and loses on bidding $v_{ji}^{(b)}$.

3) When SU i loses by bidding $b_{ji}^{(b)}$, but wins on bidding $v_{ji}^{(b)}$, then we get $u_i^{\prime(b)} = 0$. Now, according to Theorem 1, the utility obtained by a winning SU is non-negative. So, with bid $v_{ji}^{(b)}$, $u_i^{(b)} \geq 0$.

4) When SU i wins by bidding both $v_{ji}^{(b)}$ and $b_{ji}^{(b)}$, we get $u_i^{(b)} = u_i^{\prime(b)}$ due to similar reasons as explained in Case I. ■

Theorem 4: The proposed double auction is truthful in the seller side.

Proof: A double auction is truthful in the seller side if on submitting an untruthful ask value $b_{qj}^{(s)} \neq v_{qj}^{(s)}$ by a PO q for its channel j, the utility of PO q cannot be improved. That is, if $u_q^{(s)}$ and $u_q^{\prime(s)}$ are the utilities for $v_{qj}^{(s)}$ and $b_{qj}^{(s)}$ respectively, then $u_q^{(s)} \geq u_q^{\prime(s)}$. Now, the payment of a winning PO is its valuation for the leased channel. And according to Theorem 2, if PO q wins the channel j with $b_{qj}^{(s)}$ when $b_{qj}^{(s)} < v_{qj}^{(s)}$ or with $v_{qj}^{(s)}$ when $b_{qj}^{(s)} > v_{qj}^{(s)}$ or with both $b_{qj}^{(s)}$ and $v_{qj}^{(s)}$ when either $b_{qj}^{(s)} > v_{qj}^{(s)}$ or $b_{qj}^{(s)} < v_{qj}^{(s)}$, then we get $u_q^{(s)} = u_q^{\prime(s)} = 0$. Also, according to Lemma 2, it cannot be true that PO q loses with an ask value of $v_{qj}^{(s)}$ and wins on submitting the ask value $b_{qj}^{(s)}$ when $b_{qj}^{(s)} > v_{qj}^{(s)}$ or it loses with an ask value of $b_{qj}^{(s)}$ and wins on submitting the ask value $v_{qj}^{(s)}$ when $b_{qj}^{(s)} < v_{qj}^{(s)}$. Hence, the utility of the PO cannot be improved if the ask value differs from its valuation. ■

Theorem 5: The proposed double auction is ex-post budget balance.

Proof: A double auction is said to be ex-post budget balance if the difference between the payment collected from bidders and the payment paid to sellers is non-negative. That is, if auctioneer earns a non-negative revenue. According to the payment strategy, payment earned from a winning group is its group valuation and payment paid to a winning seller is its valuation for the sold channel. Since, the auction model restraints to a truthful bidding strategy and a channel can be assigned to a group of SUs only when the group bid is greater than or equal to the ask value of the channel, so this implies that the profit earned by auctioneer will be non-negative. ■

IV. PERFORMANCE EVALUATION

In this section, we study the MATLAB based simulation of the proposed auction model. In the simulation setup, we consider a network of size 800m×800m where the SUs are randomly distributed. Interference amongst the SUs is modeled by applying the distance-based interference mechanism [12], [14], [23] for the given network size. We assume the distance between two SUs as less than or equal to 200m for them to interfere with each other. Auctioneer announces the maximum allowable transmission power of the channels which is in the range of [0.01,1] Watts. Bandwidth and noise variance are common for all SUs and is taken as 1 KHz and 10^{-5} respectively. The values for the path loss factor and interference from primary network ranges between [2,4] dB and [0.001,0.0001] Watts respectively. For performance analysis, we first compare our model with an existing double-auction model called PreDA [22]. From Table 1, it can be noted that PreDA shows similarities with the proposed model in terms of the deployed network characteristics. PreDA designs the model in such a way that every idle channel may not be available to each user (dynamics in SOPs) and along with considers that the channels auctioned are heterogeneous in quality. But, PreDA excludes the channel availability time constraint in its model, which can thereby have an adverse effect on its network performance. We simulate PreDA by applying the network constraints taken up in our model, and this accordingly influences the bid collection in PreDA. A bid-independent group formation algorithm (as discussed in PreDA) is executed to get non-interfering groups on SUs. Preference list of each SU in PreDA is decided in the similar way as in the proposed method. And then, channels are leased by forming virtual groups for each channel. Further, PreDA designs the winner determination strategy in such a way that on computing the group bid for each buyer group, the SU with minimum bid is eliminated from the group. This decreases the SU count who can acquire the free channels in PreDA. For both the models, the number of channels that a PO can lease is set to be 2. We deploy two different network scenarios for performance evaluation, where in one scenario, the number of SUs are varied from 50 to 90 keeping number of POs fixed at 10, and in the other scenario, number of POs are varied from 5 to 30 keeping number of SUs fixed at 80. Secondly, performance of the proposed model is analyzed using three different sets of SUs while varying the number of POs from 5 to 30. Here again, the number of channels from each PO is taken to be 2. Third, we evaluate the effect of varying number of auctioned channels from the sellers in our proposed model. Then, to show how the network constraints can influence the system performance, we compare our model with PreDA, where in PreDA, the constraint related to channel availability time in excluded as in its actual model. However, we proceed with dynamics in SOPs of SUs since this constraint is also incorporated in the actual model of PreDA. Lastly, the interference range is varied from 100m to 600m to observe its impact on spectrum utilization and spectrum reuse for a single auction round with varying number of SUs as well as channels. All the results are averaged over 500 rounds in every scenario.

The performance metrics used to evaluate the results are given as follows.

- Total allocated bands ($\mathcal{T}_b$): This is the total number of free channels allocated amongst SUs during the auction process as given in Eq. 9.

$$\mathcal{T}_b = \mathcal{Y} - \sum_{j=1}^{\mathcal{Y}} \prod_{i=1}^{N} (1 - a_{ij}) \tag{9}$$

- Allocated units per seller (A_s): This is the ratio of number of allocated channels to the number of participating sellers (POs) as given in Eq. 10.

$$A_s = \frac{\mathcal{Y} - \sum_{j=1}^{\mathcal{Y}} \prod_{i=1}^{N} (1 - a_{ij})}{M} \tag{10}$$

- Spectrum utilization (S_u): This is the total of winning bid values from the SUs as expressed in Eq. 4 which includes the winning bids from all auction rounds.
- Seller's Payment ($\widetilde{S_p}$): This is the total price values earned by the winning sellers for their assigned channels in all the auction rounds. Eq. 11 shows the payment earned by the winning sellers in one auction round (S_p).

$$S_p = \sum_{q=1}^{M} \rho_q^{(s)} \tag{11}$$

- Revenue ($\widetilde{\mathcal{R}}$): This is the total revenue earned by the auctioneer in all auction rounds. Eq. 12 shows the revenue of the auctioneer in one auction round $\mathcal{R}$.

$$\mathcal{R} = \sum_{q=1}^{M} \sum_{j=1}^{k_q} r_{qj} \tag{12}$$

- Channel allocation ratio (C_a): This is the ratio of number of channels which got assigned during the auction process to the total number of free channels which are auctioned as given in Eq. 13.

$$C_a = \frac{\mathcal{Y} - \sum_{j=1}^{\mathcal{Y}} \prod_{i=1}^{N} (1 - a_{ij})}{\mathcal{Y}} \tag{13}$$

- Spectrum reuse (S_r): This is the ratio of number of times the channels are reused amongst the SUs to the number of assigned channels as given in Eq. 14. S_r is obtained for the initial auction round.

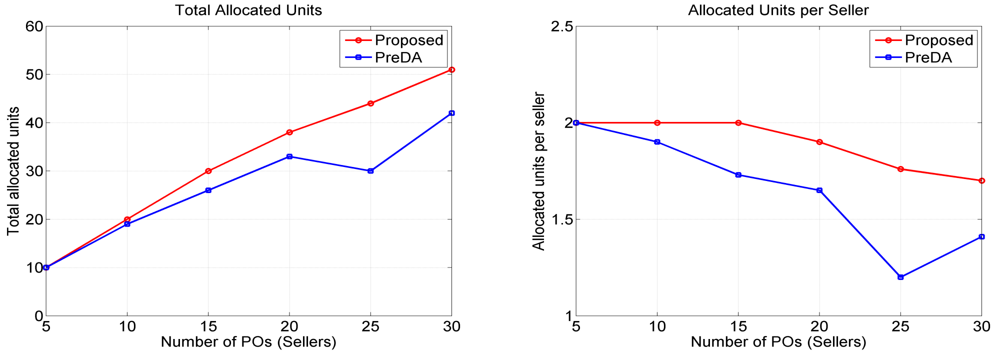
$$S_r = \frac{\sum_{q=1}^{M} \sum_{j=1}^{k_q} w_{qj}}{\mathcal{Y} - \sum_{j=1}^{\mathcal{Y}} \prod_{i=1}^{N} (1 - a_{ij})} \tag{14}$$

- Successful user ratio (U_r): Using this metric, we show how the channel availability time can adversely affect the network performance when it is discarded while designing the auction model. U_r is defined as the ratio of number of winning SUs who abide by the availability time constraint and therefore can complete their transmission to the total number of winning SUs in the network as given in Eq. 15. Therefore, this metric is used while we compare our designed model (in presence of channel availability time constraint) with PreDA (in absence of channel availability time constraint).

$$U_r = \frac{\sum_{i=1}^{N} \sum_{j=1}^{\mathcal{Y}}{}_{T_{A(j)} > T_{R(ij)}} \; a_{ij}}{\sum_{j=1}^{\mathcal{Y}} \sum_{i=1}^{N} a_{ij}} \tag{15}$$

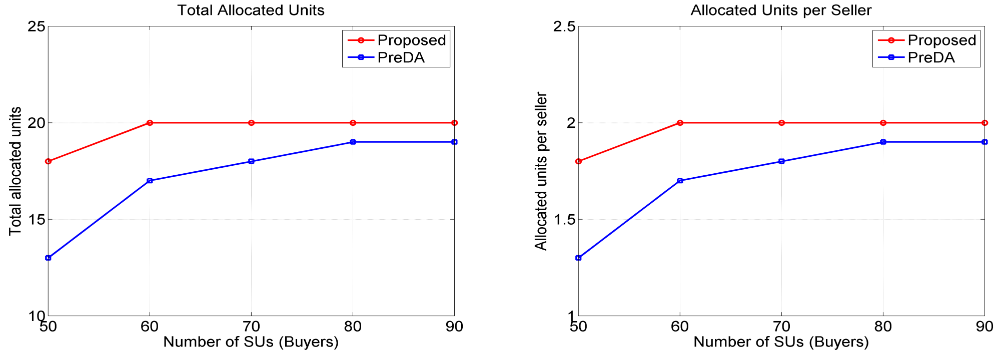
In Fig. 4(a), the count for total allocated bands is obtained for the proposed model and PreDA. From the figure it can be observed that with increase in number of POs (sellers), total allocated bands increases because more number of free channels are available which can be acquired by the SUs. The proposed model makes a good use of the vacant radio spectrum as compared to PreDA. This is because, virtual group formation strategy applied in PreDA creates groups for a channel whose group bids may not be high enough to exceed the ask value of the channel. Also in PreDA, a buyer group having more than one member eliminates the minimum bid SU from the group. This decreases the group bid value as well as winning SU count who can be allocated the channels in PreDA. In our proposed model, channel-specific group formation is carried out for each channel. We assign a channel to every member of the winning group. The group bids calculated in the proposed model give a greater value as compared to the group bids in PreDA due to which there is a greater chance for a channel to get assigned in our model. When number of channels are less, marginally similar results are obtained for both the models since there are sufficient SUs who can get the channels. But when number of channels increases, the possibility of getting unassigned channels also increases. Fig. 4(b) shows the number of allocated channels (units) per seller where the proposed model performs better as compared to PreDA due to the group formation and allocation methods deployed in our model. Increase in number of sellers increases the competition amongst them to sell their free channels. This leaves behind some unassigned channels. So, on increasing the number of sellers the count of allocated channels per seller decreases. PreDA uses signal to interference and noise ratio (SINR) as a preference indicator of channels which is an important criteria for channel selection. Using SINR can provide a different perspective in the quality of available channels. We plan to use SINR in our designed model as a future initiative so that we can allow finer radio bands to be utilized in an efficient way.

Fig 5(a) shows the total allocated bands for both the models with increasing number of SUs (buyers). When number of SU increases for a fixed set of available channels, more



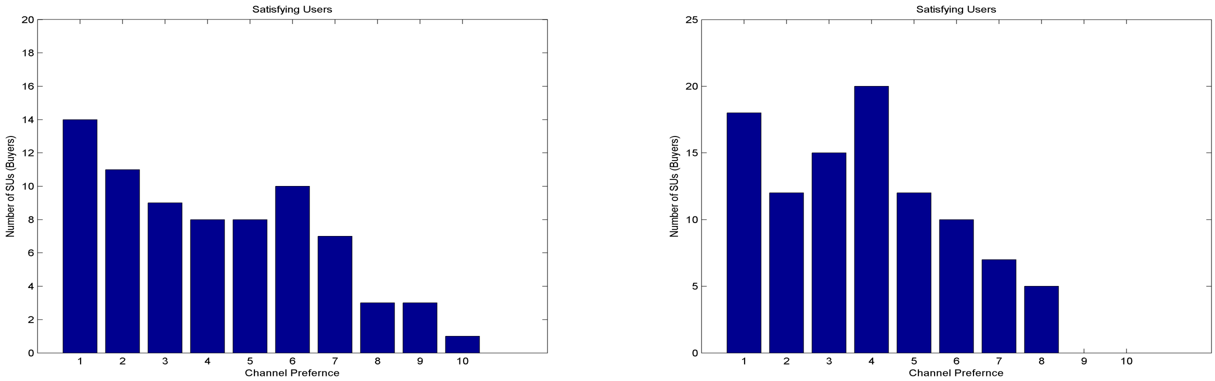
(a) (b)

FIGURE 4. a, b shows total allocated units and allocated units per seller of proposed model and PreDA respectively with respect to number of POs.



(a) (b)

FIGURE 5. a, b shows total allocated units and allocated units per seller of proposed model and PreDA respectively with respect to number of SUs.



(a) (b)

FIGURE 6. a, b shows number of satisfying SUs with respect to channel preference for 50 SUs 5 POs and 70 SUs 5 POs respectively for the proposed model.

SUs are willing to utilize the vacant spectrum. This also increases the SU count who can reuse a particular channel. From the figure we can observe that when SUs are more in number, almost all auctioned channels sell out in our proposed model. But the grouping strategy applied for channel allocation in PreDA reduces the number of allocated bands. In Fig 5(b), allocated channels per seller shows a growth with increasing number of SUs because number of allocated channel increases when more SUs are eager to get the channels.

Fig 6(a) shows the number of satisfying SUs with respect to their channel preference when number of PO is 5 and number of SU is 50. In our proposed model, a new preference list is built up by an SU in each auction round. To obtain the satisfying buyers in Fig 6(a) we consider the initial auction round and the respective preference lists of the SUs in this

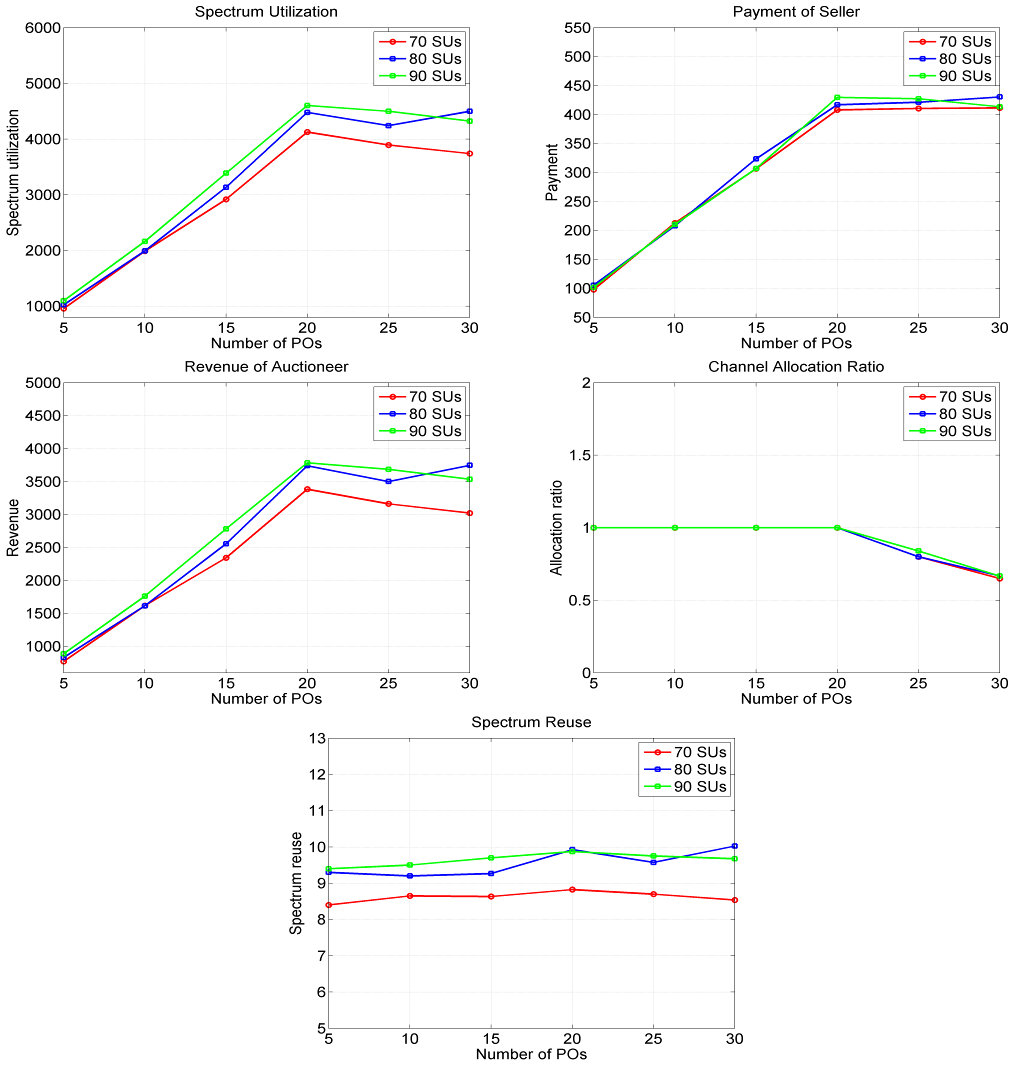


FIGURE 7. a, b, c, d, e shows spectrum utilization, seller payment, revenue, channel allocation ratio and spectrum reuse respectively of proposed model for different sets of SUs on varying the number of POs.

round. In our model, an SU bids for all its available channels according to its preference list. Group formation and channel allocation algorithms do not include channel preference. So from Fig 6(a) we can observe that the number of satisfying SUs varies for different channel preferences. Fig 6(b) shows the number of satisfying SUs with respect to their channel preference when number of PO is 5 and number of SU is 70. Here again, we take the initial auction round to compute the number of satisfying buyers.

In Fig. 7(a), spectrum utilization is obtained for the proposed model for three different sets of SUs on increasing the number of POs from 5 to 30. When number of channels are more, SUs have a greater chance to get access to the spectrum. This increases the spectrum utilization with increase in number of POs. The model also allows the leftover availability time of the assigned channels to be used in successive auction rounds which further boosts the overall spectrum utilization. In some cases spectrum utilization can get reduced even when channels are more in number because some channels may remain unassigned during the auction process or the leftover availability time of the assigned channels may not get used. Again, when number of SU increases for a fixed set of POs, we get an improved performance for spectrum utilization. However, due to interfering SUs and the network constraints taken up in this model, a group with more SU count may achieve a reduced utilization as compared to a group with less number of SUs. Fig. 7(b) shows the payment earned by the winning sellers. On increasing the number of channels, sellers are likely to earn a higher income since the number of auctioned items are more. In the proposed model, group bids computed are high and this enables our model to lease more number of channels which further increases the seller's payment. When SUs are more in number, then depending on the groups formed for the channels, the payment marginally varies among the sets of SUs. Similarly from Fig. 7(c), we can observe that on

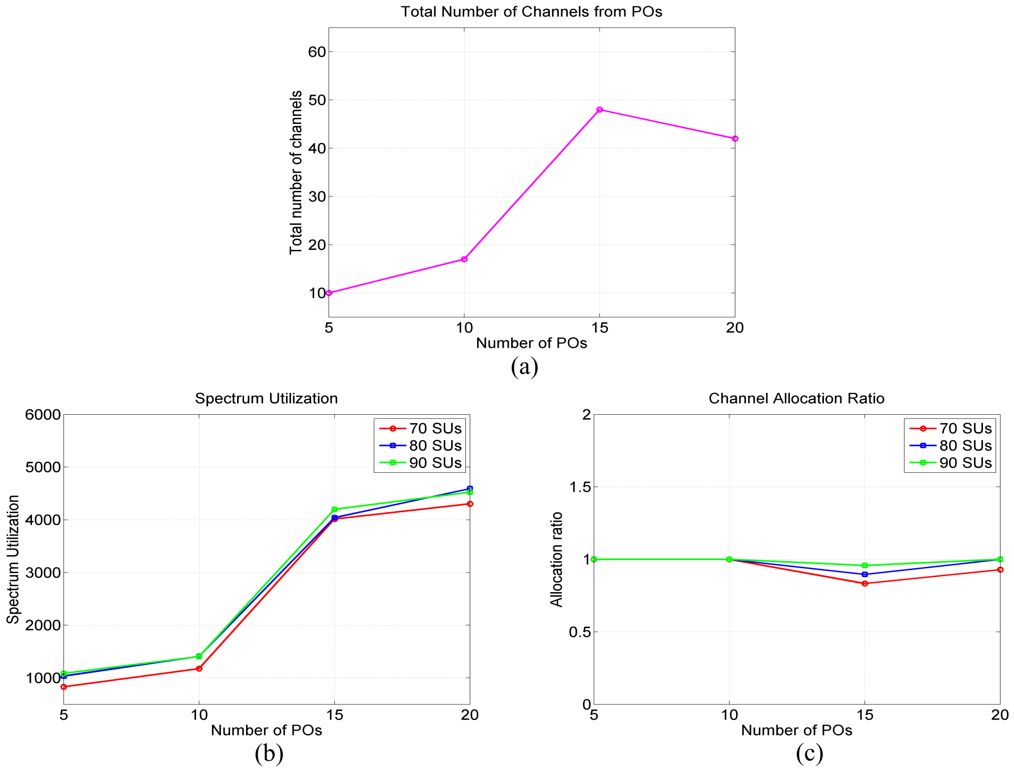


FIGURE 8. a, b, c shows total number of auctioned channels from POs and spectrum utilization and channel allocation ratio for three different sets of SUs when the number of channels at POs are varied.

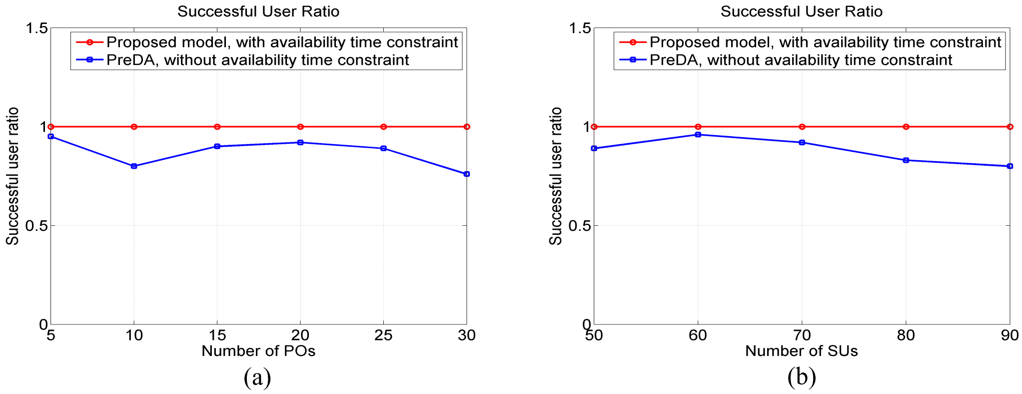


FIGURE 9. a, b shows successful user ratio of proposed model and PreDA with respect to number of SUs and POs in consideration with availability time constraint.

increasing the number of channels the auctioneer gets to earn a higher revenue if all channels get assigned during the auction. Fig. 7(d) shows the channel allocation ratio. When number of channels are less all channels get assigned amongst the SUs. But on increasing the number of channels, it is more likely that some channels remain unassigned which thereby decreases the allocation ratio. Lastly Fig. 7(e) gives the spectrum reusability values where with increasing number of channels spectrum reuse improves subject to the number of assigned channels. But with different sets of SUs, interference amongst the SUs may somehow effect the spectrum reusability.

In Fig. 8(b) and 8(c), we are varying the number of channels which a PO can auction. The number of channels that are available for auction from each PO is randomly chosen between 1 and 3. Fig 8(a) shows the total number of channels that are kept for lease by the POs when the number of POs are varying from 5 to 20. In Fig. 8(b) we can observe the variation in spectrum utilization in three different sets of SUs when the number of POs are increased. With more number of channels, it is expected that some of those channels will remain unassigned which reduces the utilization of the radio bands. Again in some cases, even with increasing the number of POs, spectrum utilization may get reduced because the number

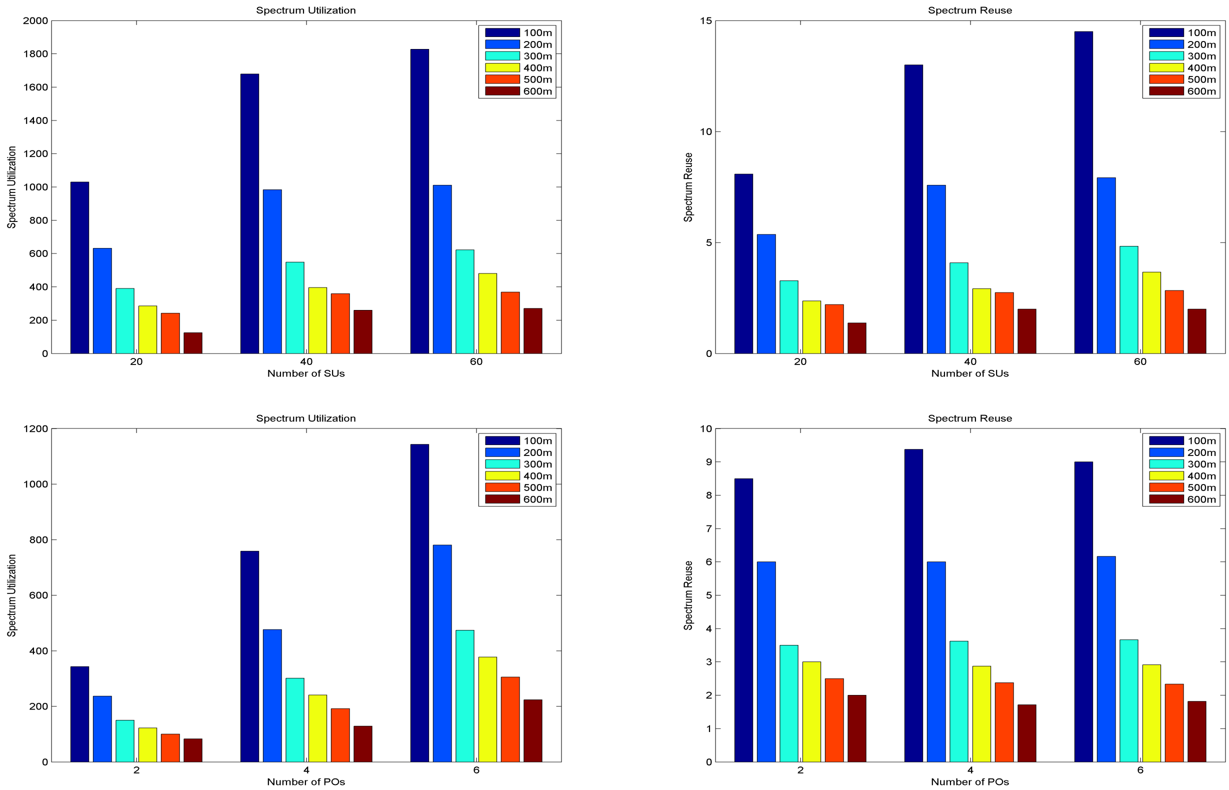


FIGURE 10. a, b, c, d shows spectrum utilization and spectrum reuse for varying interference ranges with respect to number of SUs and number of POs.

of channels available from the POs is less. This reduces the spectrum availability amongst the SUs. Therefore, when the number of channels from POs is different, then an increase in the number of POs cannot indicate an increase in the number of channels or an increase in spectrum utilization. On taking different sets of SUs, we can observe that spectrum utilization increases with increasing SU count because the number of non-interfering SUs who can be simultaneously assigned a channel increases. But, there can be a situation where some available channel remains unassigned because the group bids cannot satisfy the allocation condition and this consequently degrades the spectrum utilization obtained by the set comprising of more number of SUs as compared to the set having lesser number of SUs. In Fig. 8(c), channel allocation ratio has been displayed for different SU sets. For all the sets we get a good performance in channel allocation. This is because in our model, if a channel does not get assigned in an auction round, successive auction rounds can be carried out to lease such channel until a stopping criteria is met. This increases the chances of channel allocation. However, on increasing the number of POs (which can increase the number of channels), there may arise some unassigned channels which can reduce the channel allocation ratio.

Fig. 9(a) and 9(b) shows how the channel availability time constraint can have an impact on the use of radio spectrum. We evaluate the performance using two scenarios. In one scenario, number of POs is varied from 5 to 30 when number of SU is 80, and in other scenario number of SUs is varied from 50 to 90 when number of PO is 10. On simulating PreDA, we have considered that PreDA does not include the availability time constraint in its model as according to [22] and its allocation proceeds as per its designed algorithm. Now when winning SUs are obtained in PreDA, then there we may find one or more winning SUs who are unable to complete their transmission. This is because, such as SU gets a channel which has to be given way to the licensed user while the SU is carrying out its transmission. As such, the successful user ratio declines in PreDA and this results in a wastage of the radio spectrum. But in our proposed model, every wining SU can complete its transmission over the assigned channel due to the constraints imposed in the model. This overall provides a much better successful user ratio and improves the radio spectrum usage.

In Fig. 10(a) and 10(b), spectrum utilization and spectrum reuse values are obtained when the interference range is varied as {100m, 200m, 300m, 400m, 500m, 600m} for three different sets of SUs, {20, 40, 60}, with the number of PO kept fixed at 6. From Fig 10(a), we can observe that on increasing the interference distance for each SU, the spectrum utilization gets reduced since the numbers of members in the groups will get reduced. When the interference distance considered is large enough, then even some channels may remain unassigned resulting in a shrink in spectrum utilization. Similarly, spectrum reuse degrades in its performance with increase in the interference distance as shown in Fig 10(b). Again, Fig. 10(c) and 10(d) shows the spectrum utilization and spectrum reuse values under the similar interference ranges, but in a different scenario where the number of POs are varied as $\{2, 4, 6\}$ while keeping the number of SU fixed at 20. From both the figures, we can observe that there is a decrease in the values of the two parameters which occurs because the group sizes get smaller on increasing the interference distance for each SU. However, with increase in the number of channels, a growth in the performance of both spectrum utilization and spectrum reuse can be observed.

Hence, from the simulation results we can conclude that the proposed double auction mechanism deployed for spectrum allocation in CRN significantly helps in enhancing the spectrum utilization across the network.

V. CONCLUSION

In this paper, we proposed a double auction mechanism for multi-channel multi-winner allocation with heterogeneous channel condition. This urges the sellers and buyers to submit channel-specific asks and bids respectively to the auctioneer. Dynamics in spectrum opportunities and variation in availability time of channels are incorporated in the model which essentially improves the network performance. To allow spectrum reuse, groups are configured specifically for each channel and a group bid from each group decides the channel allocation pattern amongst the SUs. Along with achieving an improved spectrum utilization, we have also proved that the proposed auction is individually rational, truthful and budget balance. Simulations have been carried out to show that the proposed auction outshines in terms of spectrum utilization which significantly resolves the spectrum scarcity problem amongst the wireless devices. As a future initiative, we plan to improve the utility of the buyers and the sellers by considering the network conditions proposed in this model.

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REFERENCES

[1] FCC, *Notice of Proposed Rule Making and Order*, document ET Docket 03–322, 2003.
[2] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, ''NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey,'' *Comput. Netw.*, vol. 50, no. 13, pp. 2127–2159, Sep. 2006.
[3] X. Xing, T. Jing, W. Cheng, Y. Huo, and X. Cheng, ''Spectrum prediction in cognitive radio networks,'' *IEEE Wireless Commun.*, vol. 20, no. 2, pp. 90–96, Apr. 2013.
[4] Q. Zhao and B. M. Sadler, ''A survey of dynamic spectrum access,'' *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.
[5] R. Fantacci and D. Marabissi, ''Cognitive spectrum sharing: An enabling wireless communication technology for a wide use of smart systems,'' *Future Internet*, vol. 8, pp. 1–17, Jun. 2016.
[6] S. Pandit and G. Singh, ''An overview of spectrum sharing techniques in cognitive radio communication system,'' *Wireless Netw.*, vol. 23, no. 2, pp. 497–518, Feb. 2017.
[7] E. Ahmed, A. Gani, S. Abolfazli, L. J. Yao, and S. U. Khan, ''Channel assignment algorithms in cognitive radio networks: Taxonomy, open issues, and challenges,'' *IEEE Commun. Surveys Tuts.*, vol. 18, no. 1, pp. 795–823, 1st Quart., 2016.
[8] E. Z. Tragos, S. Zeadally, A. G. Fragkiadakis, and V. A. Siris, ''Spectrum assignment in cognitive radio networks: A comprehensive survey,'' *IEEE Commun. Surveys Tuts.*, vol. 15, no. 3, pp. 1108–1135, 3rd Quart., 2013.
[9] S. Parsons, J. A. Rodriguez-Aguilar, and M. Klein, ''Auctions and bidding: A guide for computer scientists,'' *ACM Comput. Surveys*, vol. 43, pp. 1–66, Feb. 2011.
[10] F. Hu, B. Chen, and K. Zhu, ''Full spectrum sharing in cognitive radio networks toward 5G: A survey,'' *IEEE Access*, vol. 6, pp. 15754–15776, 2018.
[11] M. Devi, N. Sarma, and S. K. Deka, ''DAMW: Double auction multi-winner framework for spectrum allocation in cognitive radio networks,'' in *Proc. 20th Int. Conf. Innov. Community Services (I CS)*, 2020, pp. 266–281.
[12] X. Zhou and H. Zheng, ''TRUST: A general framework for truthful double spectrum auctions,'' in *Proc. IEEE 28th Conf. Comput. Commun. (INFOCOM)*, Apr. 2009, pp. 999–1007.
[13] F. Wu and N. Vaidya, ''SMALL: A strategy-proof mechanism for radio spectrum allocation,'' in *Proc. IEEE INFOCOM*, Apr. 2011, pp. 81–85.
[14] Z. Chen, H. Huang, Y.-E. Sun, and L. Huang, ''True-MCSA: A framework for truthful double multi-channel spectrum auctions,'' *IEEE Trans. Wireless Commun.*, vol. 12, no. 8, pp. 3838–3850, Aug. 2013.
[15] W. Dong, S. Rallapalli, L. Qiu, K. K. Ramakrishnan, and Y. Zhang, ''Double auctions for dynamic spectrum allocation,'' *IEEE/ACM Trans. Netw.*, vol. 24, no. 4, pp. 2485–2497, Aug. 2016.
[16] Q. Wang, B. Ye, T. Xu, S. Lu, and S. Guo, ''DOTA: A double truthful auction for spectrum allocation in dynamic spectrum access,'' in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Apr. 2012, pp. 1490–1495.
[17] T. Jing, C. Zhao, X. Xing, Y. Huo, W. Li, and X. Cheng, ''A multi-unit truthful double auction framework for secondary market,'' in *Proc. IEEE Int. Conf. Commun. (ICC)*, Jun. 2013, pp. 2817–2822.
[18] W. Wang, B. Li, and B. Liang, ''District: Embracing local markets in truthful spectrum double auctions,'' in *Proc. 8th Annu. IEEE Commun. Soc. Conf. Sensor, Mesh Ad Hoc Commun. Netw. (SECON)*, Jun. 2011, pp. 521–529.
[19] X. Zhang, D. Yang, G. Xue, R. Yu, and J. Tang, ''Transmitting and sharing: A truthful double auction for cognitive radio networks,'' in *Proc. IEEE Int. Conf. Commun. (ICC)*, May 2018, pp. 1–6.
[20] Y.-E. Sun, H. Huang, M. Tian, Z. Sun, W. Yang, H. Guo, and L. Huang, ''STRUCTURE: A strategyproof double auction for heterogeneous secondary spectrum markets,'' in *Proc. Int. Conf. Algorithms Architectures Parallel Process. (ICA PP)*, 2015, pp. 427–441.
[21] Y. Chen, J. Zhang, K. Wu, and Q. Zhang, ''TAMES: A truthful double auction for multi-demand heterogeneous spectrums,'' *IEEE Trans. Parallel Distrib. Syst.*, vol. 25, no. 11, pp. 3012–3024, Nov. 2014.
[22] E. F. Khairullah and M. Chatterjee, ''PreDA: Preference-based double auction for spectrum allocation in heterogeneous DSA networks,'' *Comput. Commun.*, vol. 133, pp. 41–50, Jan. 2019.
[23] X. Feng, Y. Chen, J. Zhang, and Q. Zhang, ''TAHES: A truthful double auction mechanism for heterogeneous spectrums,'' *IEEE Trans. Wireless Commun.*, vol. 11, no. 11, pp. 4038–4047, Sep. 2012.
[24] X. Zhai, T. Zhou, C. Zhu, B. Chen, W. Fang, and K. Zhu, ''Truthful double auction for joint Internet of energy and profit optimization in cognitive radio networks,'' *IEEE Access*, vol. 6, pp. 23180–23190, 2018.
[25] D. Niyato and E. Hossain, ''Competitive pricing for spectrum sharing in cognitive radio networks: Dynamic game, inefficiency of nash equilibrium, and collusion,'' *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 779–797, Jan. 2006.
[26] H. Zheng and C. Peng, ''Collaboration and fairness in opportunistic spectrum access,'' in *Proc. IEEE Int. Conf. Commun. (ICC)*, May 2005, pp. 3132–3136.
[27] Z. Zhao, Z. Peng, S. Zheng, and J. Shang, ''Cognitive radio spectrum allocation using evolutionary algorithms,'' *IEEE Trans. Wireless Commun.*, vol. 8, no. 9, pp. 4421–4425, Sep. 2009.
[28] Z. Shi and G. Luo, ''Multi-band spectrum allocation algorithm based on first-price sealed auction,'' *Cybern. Inf. Technol.*, vol. 17, no. 1, pp. 104–112, Mar. 2017.

[29] M. Devi, N. Sarma, and S. K. Deka, "Multi-winner heterogeneous spectrum auction mechanism for channel allocation in cognitive radio networks," in *Proc. Int. Conf. Distrib. Comput. Internet Technol. (ICDCIT)*, 2020, pp. 251–265.
[30] I. A. Kash, R. Murty, and D. C. Parkes, "Enabling spectrum sharing in secondary market auctions," *IEEE Trans. Mobile Comput.*, vol. 13, no. 3, pp. 556–568, Mar. 2014.
[31] R. P. McAfee, "A dominant strategy double auction," *J. Econ. Theory*, vol. 56, no. 2, pp. 434–450, Apr. 1992.
[32] G. Quaresima, F. Benedetto, and L. Mastroeni, "On the properties of double auction-based models for spectrum management in cognitive radio networks," in *Proc. 42nd Int. Conf. Telecommun. Signal Process. (TSP)*, Jul. 2019, pp. 500–504.
[33] X. Zhang, B. Zhang, K. An, Z. Chen, and D. Guo, "Auction-based secondary relay selection on overlay spectrum sharing in hybrid satellite–terrestrial sensor networks," *Sensors*, vol. 19, no. 22, p. 5039, Nov. 2019.
[34] S. Wang, P. Xu, X. Xu, S. Tang, X. Li, and X. Liu, "TODA: Truthful online double auction for spectrum allocation in wireless networks," in *Proc. IEEE Symp. New Frontiers Dyn. Spectr. (DySPAN)*, Apr. 2010, pp. 1–10.
[35] Y. Chen, P. Lin, and Q. Zhang, "LOTUS: Location-aware online truthful double auction for dynamic spectrum access," *IEEE Trans. Wireless Commun.*, vol. 14, no. 2, pp. 1092–1099, Feb. 2015.
[36] B. F. Lo, "A survey of common control channel design in cognitive radio networks," *Phys. Commun.*, vol. 4, no. 1, pp. 26–39, Mar. 2011.
[37] J. Xiang, Y. Zhang, and T. Skeie, "Medium access control protocols in cognitive radio networks," *Wireless Commun. Mobile Comput.*, vol. 10, no. 1, pp. 1–18, 2009.
[38] H. Kim and K. G. Shin, "Efficient discovery of spectrum opportunities with MAC-layer sensing in cognitive radio networks," *IEEE Trans. Mobile Comput.*, vol. 7, no. 5, pp. 533–545, May 2008.
[39] H. Zhao, K. Ding, N. I. Sarkar, J. Wei, and J. Xiong, "A simple distributed channel allocation algorithm for D2D communication pairs," *IEEE Trans. Veh. Technol.*, vol. 67, no. 11, pp. 10960–10969, Nov. 2018.
[40] J. Li, H. Zhao, A. S. Hafid, J. Wei, H. Yin, and B. Ren, "A bio-inspired solution to cluster-based distributed spectrum allocation in high-density cognitive Internet of Things," *IEEE Internet Things J.*, vol. 6, no. 6, pp. 9294–9307, Dec. 2019.

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