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The Evaluation Method for Step-Down-Stress Accelerated Degradation Testing Based on Inverse Gaussian Process

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ABSTRACT In view of the disadvantage that traditional evaluation method of accelerated degradation testing (ADT) complex statistical analysis, in this paper, the randomness and monotonicity of the degradation path for products with high reliability and long lifetime are taken into consideration based on Inverse Gaussian (IG) process. The evaluation method of step-down-stress ADT as well as its reliability function are established based on IG process. At same time, both the simple IG process and IG process with random effects are considered, respectively. The maximum likelihood estimation method and Markov Chain Monte Carlo (MCMC) estimation method are presented to estimate the unknown parameters in the proposed degradation models, respectively. Finally, the proposed evaluation methods are demonstrated by the step-down-stress ADT data of a certain type of missile tank. The results show that the proposed evaluation method is reasonable and valid in this paper. In addition, compared with simple IG process model, IG process with random effects model has many superb properties when dealing with random effects of products in ADT.

INDEX TERMS Evaluation method, Inverse Gaussian process, random effects, reliability, step-down-stress ADT.

I. INTRODUCTION

Nowadays, more and more products with high reliability and long lifetime are used in various fields, such as astronautics, aeronautics, medical treatment, mechanical engineering, etc. Traditional life testing, even accelerated life testing (ALT) have been unable to evaluate the performance of such products efficiently [1]. In such cases, an alternative approach is to collect degradation data at higher-stress level (e.g. higher voltage, temperature, or mechanical load), then through using accelerated degradation equation to predict product's lifetime at normal-stress level, which is called accelerated degradation testing (ADT).

At present, there are two classical ADT methods: constant-stress ADT and step-up-stress ADT. Compared with constant-stress ADT, only smaller sample size is required, as well as the performance degradation of a product deteriorates much faster in the step-up-stress ADT [2]. However, since the primary acceleration stress level is only a little higher than the normal stress level during step-up-stress ADT, which results in the performance degradation rate of the product slower at the initial stage of testing. Based on this situation, Reference [3] proposed a new step-down-stress accelerated testing method based on the assumption that the changing in exerting sequence of stress can improve its efficiency greatly. It is reasonable to assume that this testing method can also be applied to ADT.

For a new ADT method, establishing an efficiently evaluation method for testing data is very important to verify its validity. At present, general degradation path method is the most common analysis method for performance degradation data of products [4]–[6]. It is mostly based on the obtained data to establish a hypothetical model, and then analyze the distribution of pseudo life or performance degradation path. Because of the complex and large amount calculation of this kind evaluation method. Therefore, the evaluation method inconvenient in engineering application. In response to this deficiency, a new evaluation method based on the stochastic process is proposed for ADT data in the reliability fields recently.

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In [7], the authors proposed an optimal evaluation method for step-stress ADT based on the assumption that the underlying degradation path follows a Wiener process. For predicting the lifetime of a certain type of missile electrical connector, an evaluation method of degradation data based on Gamma process was proposed in [8]. Exhibiting a monotone increasing pattern in [9], Gamma process with random effects was used to evaluate the degradation performance of a product subject to constant-stress ADT. In [10] proposed a classic evaluation method for degradation data of products based on Inverse Gaussian (IG) process, which can be easily extend to incorporate random effects and covariates. References [11], [12] investigated the optimal constant-stress ADT planning based on the IG process, as well as, both the IG process and with random effects were considered in this literature, respectively. An evaluation method of performance degradation data based on inverse Normal-Gamma mixture process was proposed in [13].

The above literatures show that proposed evaluation method based on stochastic process can well characterize degradation law of products under constant-stress ADT and step-up-stress ADT conditions. In addition, compared with Wiener process and Gamma process, IG process can better describe performance degradation rule of products with high reliability and long lifetime [10]. However, there are still some deficiencies in the existing literatures, such as these researches are still at the theoretical level, the engineering application is less. In addition, the evaluation method which is more suitable for step-down-stress ADT has not been found. Step-down-stress ADT is likely to become the most commonly used ADT method for the high-requirement products in the future. Therefore, it is urgent to put forward an evaluation method for step-down-stress ADT which is convenient for engineering application.

The rest of the paper is structured as follows. Section II describes the step-down-stress ADT and accelerated degradation equation. Section III presents the simple IG process and the IG process with random effects, respectively. Section IV introduces the step-down-stress ADT modeling method based on the simple IG process and the IG process with random effects, respectively. Section V conducts models validation through a real example. Section VI concludes this paper.

II. STEP-DOWN-STRESS ADT DESCRIBE

A. STEP-DOWN-STRESS ACCELERATED TEST

Randomly select n samples from test products. Suppose that there are k accelerated stress levels S_i $(i = 1, 2, \cdots, k)$, and $S_k > S_{k-1} \cdots > S_1 > S_0$, S_0 is normal stress level of product. Usually, k should not less than 3 for the sake of statistical precision. Under stress levels S_i, T_i is duration time. So, $T_1 = t_1$; $T_2 = t_1 + t_2$; $T_k = t_1 + t_2 + \cdots + t_k = \sum_{i=1}^{k} t_i$, where T_1, T_2, $\cdots$, T_k are stress transition moment. m_i are measurement times under stress level S_i. In the step-down-stress ADT, all test units are initially placed at predetermined the highest stress level S_k for a specified length of time t_1. Then, the samples will subject to a lower stress level S_{k-1} to test again for another specified length of time t_2. The stress level of the samples is thus decreased step by step until the final stress level S_1, and the test cannot be stopped until an appropriate termination time t_k is reached. The loading sequence of the test stress in the step-down-stress ADT is shown in Fig. 1.

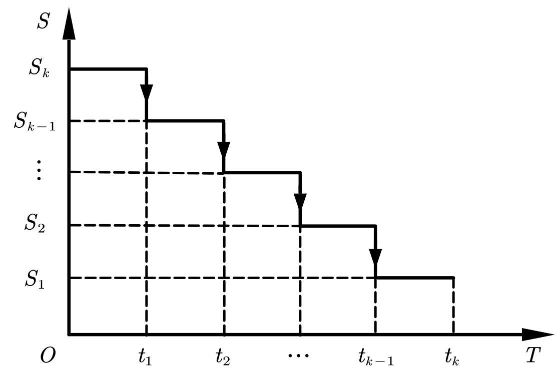

FIGURE 1. Loading sequence of the test stress in the step-down-stress ADT.

B. ACCELERATED DEGRADATION MODEL

1) BASIC ASSUMPTIONS

Without loss of generality, three assumptions are made as follows:

① The degradation paths of testing samples are monotonic increasing over time strictly.

② At each measurement time, the samples are tested simultaneously.

③ The remaining lifetime of the products depend only on the cumulative damage amount and current stress level, but has nothing to do with the cumulative manner [14].

2) ACCELERATED DEGRADATION EQUATION

Accelerated degradation equation determine a relationship between performance degradation measurements and accelerated stress levels. There are several well-known accelerated degradation equations, such as Arrhenius, Power law, and Exponential. The ADT data of products can be converted to degradation data under normal stress level by using acceleration degradation equation. Arrhenius model is used when temperature is accelerated stress. It is defined as:

$$h(S) = \xi_0 \cdot e^{-\omega/S} \tag{1}$$

where ξ_0 is a constant, which depends on the product's geometry, the specimen size and test method, etc.; $\omega = E/K$, E is the activation energy of the reaction; K the Boltzmann's constant, equals to 8.6171×10^{-5}eV/°C; S is accelerated stress level, where it is the absolute temperature in Kelvin.

According to the relationship between accelerated stress level and degradation rate, the accelerated stress levels can be standardized, as follows [15]:

$$x_i = \frac{1/S_0 - 1/S_i}{1/S_0 - 1/S_k} \tag{2}$$

Based on the above standardization, it is seen that $x_0 = 0$, $x_k = 1$, and $0 < x_i < 1 (i = 1, 2, \cdots, k)$ readily. Equation (1) can be rewritten as:

$$h(x) = \exp(\alpha_0 + \alpha_1 x) \tag{3}$$

where $\alpha_0 = \ln \xi_0 - \omega/S_0$, $\alpha_1 = \omega(1/S_0 - 1/S_k)$.

III. THE IG PROCESS AND RANDOM EFFECTS

A. THE SIMPLE IG PROCESS

Consider a product whose performance degradation is measurable. Let $\{Y(t), t \geq 0\}$ is the degradation path of a randomly selected unit. We assume that the degradation path of products follows an IG process. IG process has the following properties [10].

(1) $Y(0) = 0$;

(2) $Y(t)$ has independent increments, i.e., $Y(t_4) - Y(t_3)$ and $Y(t_2) - Y(t_1)$ are independent for $0 < t_1 < t_2 < t_3 < t_4$;

(3) The degradation increment follows an IG distribution, that is, $Y(t) - Y(s) \sim \mathrm{IG}(\mu\Delta\Lambda, \lambda(\Delta\Lambda)^2)$ for $0 \leq s < t$, where $\Delta\Lambda = \Lambda(t) - \Lambda(s)$, $\Lambda(\cdot)$ is a given, monotone increasing function of time t with $\Lambda(0) = 0$. If $y \sim IG(a, b)$, $(a, b > 0)$ with mean a and variance a^3/b, then, the probability density function(PDF) can be defined as:

$$f(y; a, b) = \sqrt{\frac{b}{2\pi y^3}} \exp\left[-\frac{b(y-a)^2}{2a^2 y}\right], \quad (y > 0) \tag{4}$$

and the cumulative distribution function (CDF) can be defined as:

$$F(y; a, b) = \Phi\left[\sqrt{\frac{b}{y}}\left(\frac{y}{a} - 1\right)\right] + \exp\left(\frac{2b}{a}\right) \times \Phi\left[-\sqrt{\frac{b}{y}}\left(\frac{y}{a}\right) + 1\right] \tag{5}$$

where $\Phi(\cdot)$ is the standard normal CDF.

The above model assumes that the samples are homogeneous, hence, it is called simple IG process.

B. THE IG PROCESS WITH RANDOM EFFECTS

The simple IG assume that all units share the same process of performance degradation. However, in many real applications, there is substantial unit-to-unit variability among the degradation processes of different individuals due to some unobserved factors, such as variations in the raw materials, manufacturing process, measurement error, etc.. Random effects degradation models are believed to be more realistic in performance degradation analysis [16]. The unit-specific random effects can be incorporated into the simple IG process to represent such heterogeneity in the degradation paths. Hence, when degradation data by using an IG process, one encounters situations where one or both of the parameters μ and λ vary from unit-to-unit.

Considering the heterogeneity in both the μ and λ parameters based on the simple IG process. Let $\lambda \sim \mathrm{Gamma}\left(\delta, \gamma^{-1}\right)$, $(\delta, \gamma > 0)$ with PDF as:

$$g(\lambda; \delta, \gamma) = \frac{\gamma^\delta \lambda^{\delta-1}}{\Gamma(\delta)} \exp(-\gamma\lambda), \quad \lambda > 0 \tag{6}$$

where $\Gamma(\delta)$ is the Gamma function, $\delta > 0$, $\gamma > 0$.

According to the above description, the unconditional distribution of $Y(t)$ can be written as:

$$f_{Y(t)}(y) = \frac{\Gamma(\delta + 1/2)}{\Gamma(\delta)} \gamma^\delta \sqrt{\frac{\Lambda^2(t)}{2\pi y^3}} \times \left[\gamma + \frac{(y - \mu\Lambda(t))^2}{2\mu^2 y}\right]^{-\delta-1/2} \tag{7}$$

Here, the mean of $Y(t)$ is still $\mu\Lambda(t)$, while the variance is given by $\gamma\mu^3\Lambda(t)/(\delta - 1)$ when $\delta > 1$.

IV. EVALUATION METHOD OF STEP-DOWN-STRESS ADT BASED ON THE IG PROCESS

A. THE EVALUATION METHOD OF STEP-DOWN-STRESS ADT BASED ON SIMPLE IG PROCESS

We assume that the random variable $Y(t)$ represent the degradation process of products, and $Y(t) \sim \mathrm{IG}\left(\mu\Lambda(t), \lambda\Lambda^2(t)\right)$ with mean $\mu\Lambda(t)$ and variance $\mu^3\Lambda(t)/\lambda$. The parameter μ represent the degradation rate, and the parameter λ has no direct physical meaning. Let $\mu = h(S)$, where $h(S)$ is a link function which reflect the effects of the accelerated stress level S_i to the degradation process of products. This is a legitimate stress-degradation acceleration relation, as both the mean of the degradation path $\mu\Lambda(t)$ and the variation of the degradation path $\mu^3\Lambda(t)/\lambda$ are increasing in S_i, λ is constant at accelerated stress levels S_i. The PDF of $Y(t)$ is obtained based on simple IG process, as follow:

$$f_{Y(t)}(y) = \sqrt{\frac{\lambda\Lambda^2(t)}{2\pi y(t)^3}} \exp\left[-\frac{\lambda(y(t) - h(x)\Lambda(t))^2}{2h(x)^2 y(t)}\right] \tag{8}$$

Assumed that n samples are tested in the step-down-down ADT. Let $Y_{ji}(t_l)$, $(j = 1, 2, \cdots, n;\ i = 1, 2, \cdots, k;\ l = 1, 2, \cdots, m_i)$ denote the degradation path of the j test unit at times t_l under accelerated stress levels S_i. Let $\Delta y_{ij,l} = Y_{ij,l} - Y_{ij,l-1}$ be the degradation increments and $\Lambda_{ij,l}(t) = \Lambda(t_{ij,l}) - \Lambda(t_{ij,l-1})$. The likelihood function of the simple IG process model is obtained as:

$$L(\boldsymbol{\theta}) = \prod_{j=1}^{n}\prod_{i=1}^{k}\prod_{l=1}^{m_i} f(\Delta y_{ij,l}; \boldsymbol{\theta}) = \prod_{j=1}^{n}\prod_{i=1}^{k}\prod_{l=1}^{m_i}\left(\sqrt{\frac{\lambda\left(\Lambda_{ij,l}\right)^2}{2\pi\,\Delta y_{ij,l}^3}} \times \exp\left[-\frac{\lambda\left(y_{ij,l} - e^{\alpha_0+\alpha_1 x_i}\left(\Lambda_{ij,l}\right)\right)^2}{2e^{2(\alpha_0+\alpha_1 x_i)}\Delta y_{ij,l}}\right]\right) \tag{9}$$

where $\boldsymbol{\theta}$ denote the unknown parameters and need to be estimated.

The function $\Lambda(\cdot)$ can be selected based on the personal experience of reliability engineers, knowledge of the physics of failure, handbooks, and other sources [17]. Here, we let $\Lambda(t) = t$. Hence, the log-likelihood function of the step-down-stress accelerated degradation model based on simple IG process can be written as:

$$\ln L(\boldsymbol{\theta}) = \sum_{j=1}^{n}\sum_{i=1}^{k}\sum_{l=1}^{m_i}\left[\frac{\ln\lambda}{2} + \ln t_{ij,l} - \frac{\lambda(y_{ij,l}e^{-\alpha_0-\alpha_1 x_i} - t_{ij,l})^2}{2y_{ij,l}}\right] \quad (10)$$

where $\boldsymbol{\theta} = (\alpha_0, \alpha_1, \lambda)'$ are unknown parameters. We will use the maximum likelihood estimate method to estimate them.

Let D_f denote the failure threshold for the degradation path of products. The product's failure lifetime T_D can be defines as the first passage time when the degradation path crosses D_f, that is as:

$$T_D = \inf\left\{t : Y(t) \geq D_f\right\} \quad (11)$$

The CDF of the lifetime distribution T_D can be obtained as:

$$\begin{aligned} F_{IG}(t) &= \Pr(T_D \leq t) \\ &= 1 - \Phi\left[\sqrt{\frac{\hat{\lambda}}{D_f}}\left(\frac{D_f}{e^{(\hat{\alpha}_0+\hat{\alpha}_1 x)}} - t\right)\right] \\ &\quad - \exp\left(\frac{2\hat{\lambda}t}{e^{(\hat{\alpha}_0+\hat{\alpha}_1 x)}}\right)\Phi\left[-\sqrt{\frac{\hat{\lambda}}{D_f}}\left(\frac{D_f}{e^{(\hat{\alpha}_0+\hat{\alpha}_1 x)}} + t\right)\right] \end{aligned} \quad (12)$$

Because the path of the IG process is strictly increasing, the reliability function of the accelerated degradation model based on simple IG process can be derived as:

$$R(t) = \Pr(T_D > t) = \Pr\left(Y(t) < D_f\right) = 1 - F_{IG}(t) \quad (13)$$

Hence, the reliability function of step-down-stress accelerated degradation model based on simple IG process can be written as:

$$\begin{aligned} R(t) = \Phi\left[\sqrt{\frac{\hat{\lambda}}{D_f}}\left(\frac{D_f}{e^{\hat{\alpha}_0+\hat{\alpha}_1 x}} - t\right)\right] + \exp\left(\frac{2\hat{\lambda}t}{e^{\hat{\alpha}_0+\hat{\alpha}_1 x}}\right) \\ \times\Phi\left[-\sqrt{\frac{\hat{\lambda}}{D_f}}\left(\frac{D_f}{e^{\hat{\alpha}_0+\hat{\alpha}_1 x}} + t\right)\right] \end{aligned} \quad (14)$$

where $\Phi(\cdot)$ is the standard normal CDF.

B. THE EVALUATION METHOD OF STEP-DOWN-STRESS ADT BASED ON IG PROCESS WITH RANDOM EFFECTS

In order to capture the heterogeneity of samples, here let $\lambda \sim \text{Gamma}\left(\delta, \gamma^{-1}\right)(\delta, \gamma > 0)$ [10]. It refer to (4) and (6), the unconditional distribution of $Y(t)$ based on IG process with random effects can be written as:

$$f_{Y(t)}(y) = \frac{\Gamma(\delta+1/2)}{\Gamma(\delta)}\gamma^{\delta}\sqrt{\frac{t^2}{2\pi y^3}} \times\left[\gamma + \frac{(y-h(x)t)^2}{2h(x)^2 y}\right]^{-\delta-1/2} \quad (15)$$

Other assumptions about the ADT settings in section IV(A) carry over to this section. The likelihood function is obtained as:

$$\begin{aligned} L(\boldsymbol{\theta}) &= \prod_{j=1}^{n}\prod_{i=1}^{k}\prod_{l=1}^{m_i} f(\Delta y_{ij,l};\boldsymbol{\theta}) \\ &= \prod_{j=1}^{n}\prod_{i=1}^{k}\prod_{l=}^{m_i}\frac{\Gamma(\delta+1/2)}{\Gamma(\delta)}\gamma^{\delta}\sqrt{\frac{t_{ij,l}^2}{2\pi y_{ij,l}^3}} \\ &\quad\times\left[\gamma + \frac{(y_{ij,l}-h(x_i)t_{ij,l})^2}{2h(x_i)^2 y_{ij,l}}\right]^{-\delta-1/2} \end{aligned} \quad (16)$$

where $\boldsymbol{\theta}$ denote the unknown parameters and need to be estimated.

The log-likelihood function of the step-down-stress accelerated degradation model based on IG process with random effects is obtained as:

$$\begin{aligned} \ln L(\boldsymbol{\theta}) = \sum_{j=1}^{n}\sum_{i=1}^{k}\sum_{l=1}^{m_i}\Bigg\{&\ln\Gamma(\delta+m/2) - \ln\Gamma(\delta) + \\ &\times\delta\ln\gamma + (\ln t_{ij,l} - \frac{3}{2}\ln y_{ij,l}) - (\delta + \frac{m_i}{2}) \\ &\times\ \ln\left[\gamma + \frac{\left(y_{ijl} - e^{\alpha_0+\alpha_1 x_i}t_{ij,l}\right)^2}{2e^{2\alpha_0+2\alpha_1 x_i}y_{ij,l}}\right]\Bigg\} \end{aligned} \quad (17)$$

where $\boldsymbol{\theta} = (\alpha_0, \alpha_1, \delta, \gamma)'$ are unknown parameters.

Obviously, above proposed evaluation models are very complicated, the traditional parameter estimation method is difficult to meet the requirement. Bayesian approach is a feasible method to integrate all available information and infer unknown parameters. Markov Chain Monte Carlo (MCMC) is a simulation approximation method that is widely used in Bayesian analysis [17]. A special case of MCMC is the Gibbs sampler, which involves simulating from the conditional posterior distribution of each parameter given the data and all of the other parameters. It is effective in handling high dimensional problems and has been applied on various reliability engineering applications [1]. Moreover, a well-developed software package open-BUGS, is used to carry out implementation of the MCMC [18]. Hence, we will use the MCMC method to estimate the unknown parameters.

To estimate the product's failure lifetime T_D, we use the normal distribution to approximate $[Y(t)|\lambda]$, and then marginalize over λ [14]. Therefore, the CDF of T_D can be approximated as:

$$F_{T_D}(t) = F_{t_{2\delta}}\left(\frac{\delta^{1/2}(e^{\alpha_0+\alpha_1 x}t - D_f)}{e^{\alpha_0+\alpha_1 x}\sqrt{e^{\alpha_0+\alpha_1 x}t\gamma}}\right) \quad (18)$$

where $t_{2\delta}$ is the student t-distribution with 2δ degrees of freedom.

Similarly, refer to (13), the reliability function of step-down-stress accelerated degradation model based on IG process with random effects is written as:

$$R(t) = 1 - F_{t_{2\delta}}\left(\frac{\delta^{1/2}(e^{\alpha_0+\alpha_1 x}t - D_f)}{e^{\alpha_0+\alpha_1 x}\sqrt{e^{\alpha_0+\alpha_1 x}t\gamma}}\right) \tag{19}$$

V. EXAMPLE VERIFICATION

Tank is a missile seeker cooling device which contains a certain amount of freon. This product enjoys long storage lifetime and high reliability, as well as other design and manufacturing advantages. During the storage time, the freon will continue leaking. When the leakage reaches the threshold ($D_f = 1.2$g), the tank cooling effect does not meet the seekers' requirements, then the tank is determined to fail. In [5] the step-down-stress ADT of a certain type of missile tank was conducted, in which temperature is accelerate stress. Average storage lifetime of the tank was predicted based on the equivalent method of accumulative damage theory in [5]. Now, we use the same test data in [5] to validate the proposed evaluation method in this paper.

A. TESTING CONDITIONS

The sample size $n = 6$, the number of accelerated stress levels $k = 3$, the accelerated stress levels are $S_3 = 60$ °C, $S_2 = 50$ °C, $S_1 = 40$ °C, respectively. $S_0 = 25$°C is the normal storage stress. The inspected times of each stress $m_i = (15, 15, 23)'$ for $i = 1, 2, 3$. The stress transition moment are $T_1 = 1258$h, $T_2 = 2672$h, $T_3 = 4802$h, respectively. The inspection interval is about 72h. The failure threshold D_f, as a critical value, equals to 1.2g. The design average storage lifetime of the missile tank under the normal stress level is 5 years, that is 43800h. The degradation path curves of the missile tank in the step-down-stress ADT are shown in Fig.2.

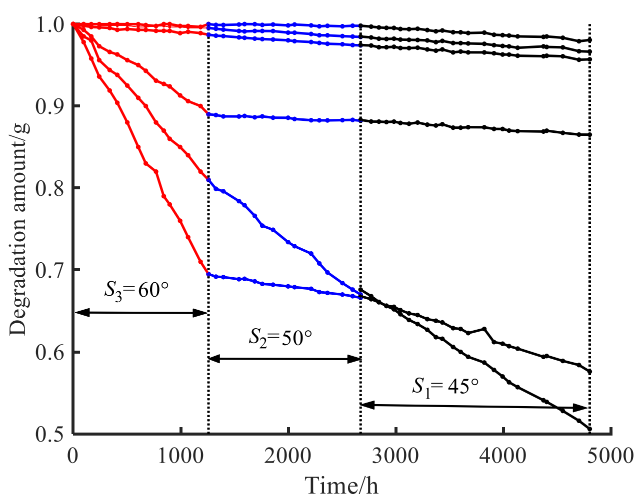

FIGURE 2. Degradation path curves of the test samples.

B. PARAMETER ESTIMATION

1) PARAMETER ESTIMATION OF THE SIMPLE IG PROCESS MODEL

According to the degradation testing data of the missile tank in the step-down-stress ADT, the parameters of step-down-stress accelerated degradation model based on simple IG process (refer to (10)) are estimated through the maximum likelihood estimation. The estimated values of parameters are $\hat{\alpha}_0 = -10.4705$, $\hat{\alpha}_1 = -1.2701$, $\hat{\lambda} = 1.9431 \times 10^{-8}$, respectively.

Refer to (2), $x_0 = 0$ for $i = 0$. So, $h(S_0)$ can be expressed by $h(x_0) = e^{\alpha_0}$. Substituting the parameter estimated values of the degradation model into (14), the storage reliability function of the missile tank under the normal stress level is obtained as:

$$\begin{aligned} R(t) = \Phi &\left[\sqrt{\frac{1.943 \times 10^{-8}}{1.2}}\left(\frac{1.2}{e^{-10.47054}} - t\right)\right] \\ &+ \exp\left(\frac{2 \times 1.943 \times 10^{-8} t}{e^{-10.47054}}\right) \\ &\times \Phi\left[-\sqrt{\frac{1.943 \times 10^{-8}}{1.2}}\left(\frac{1.2}{e^{-10.47054}} + t\right)\right] \end{aligned} \tag{20}$$

Generally, when the reliability value of the product is lower than 0.9, it is considered that the product has failed in the field of reliability. Refer to (20), when reliability value of the missile tank equals to 0.9 under the normal stress level, cumulative testing time of the sample is 43028h. Hence, the predicted average storage lifetime of the missile tank is 43028h based on the simple IG model.

2) PARAMETER ESTIMATION OF THE IG PROCESS WITH RANDOM EFFECTS MODEL

In the same way, the parameters of step-down-stress accelerated degradation model based on IG process with random effects are estimated through the MCMC method.

For the implementation of MCMC through openBUGS, it takes a certain number of iterations for samples generated from simulation runs are representative of a certain distribution. In our numerical analysis, we have used the practice of not using the first 5000 samples generated through simulation. Instead, the subsequent 10,000 samples are used. The parameter estimated values (refer to (17)) are $\hat{\alpha}_0 = -4.7091$, $\hat{\alpha}_1 = 0.3864$, $\hat{\delta} = 8.2703 \times 10^{-6}$, $\hat{\gamma} = 3.7615 \times 10^{-7}$, respectively.

The corresponding storage reliability function of the model based on IG process with random effects can be obtained by substituting $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\delta}$, $\hat{\gamma}$ and into (19). The average storage lifetime of the step-down-stress accelerated degradation model based on the IG process with random effects is 43359h.

3) SOME MORE CONSIDERATION

In addition, references [19]–[21] proposed the evaluation methods of step-down-stress ADT based on Wiener process and Gamma process, respectively. In practice, however, many degradation processes of products with high reliability and long lifetime are positive and increasing strictly, the Wiener process is not appropriate in such a case. Therefore, in this paper, we only briefly introduce the evaluation method of step-down-stress ADT based on Gamma process in [21].

We assume that the degradation path of products can be described by Gamma process with parameters α, β. The shape parameter α is related to the accelerated stress levels S_i, that is $\alpha = h(S_i)$; β is constant over S_i, the PDF is as:

$$f(x;\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} I_{(0,\infty)}(x), \quad x > 0 \quad (21)$$

where, $I_{(0,\infty)}(x) = \begin{cases} 1, x \in (0,\infty) \\ 0, x \notin (0,\infty) \end{cases}$ is an indicator function; $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$ is the Gamma function.
and corresponding likelihood function can be written as:

$$\begin{aligned} L(\boldsymbol{\theta}) &= \prod_{j=1}^{n}\prod_{i=1}^{k}\prod_{l=m_1}^{m_i} f(\Delta y_{ij,l};\boldsymbol{\theta}) \\ &= \prod_{j=1}^{n}\prod_{i=1}^{k}\prod_{l=m_1}^{m_i} \frac{1}{\Gamma(e^{\alpha_0+\alpha_1 x}\Delta t_{il})\beta^{e^{\alpha_0+\alpha_1 x}\Delta t_{il}}} \\ &\quad \times \Delta y_j(t_{il};x_i)^{e^{\alpha_0+\alpha_1 x}\Delta t_{il}} \exp(-\frac{\Delta y_j(t_{il};x_i)}{\beta}) \end{aligned} \quad (22)$$

where $\boldsymbol{\theta} = (\alpha_0, \alpha_1, \beta)'$ are unknown parameters.

The function is not easy for engineering applications, the B-S distribution to approximation regularly [21]. We also use the MCMC method to estimate parameters $\hat{\alpha}_0$, $\hat{\alpha}_1$, and $\hat{\beta}$ refer to (22). The parameter estimated values are $\hat{\alpha}_0 = -6.7776$, $\hat{\alpha}_1 = 1.4966$, and $\hat{\beta} = 0.0281$, respectively. The average storage lifetime of the step-down-stress accelerated degradation model based on Gamma process is 42046h.

C. RESULTS ANALYSIS

From the above reliability formulas, we can obtain the average storage lifetime of missile tank for each model in the condition of 25°C, as shown in Table 1. In addition, in order to facilitate comparison, the prediction result in [5] is also listed in this table.

TABLE 1. Average storage lifetime of missile tank for each model.

Models	Estimation Lifetime/h	Design Lifetime/h
Simple IG process	43028	43800
IG process with random effects	43359	
Gamma process in [21]	42046	
SNADM in [5]	42291	

Table 1 shows that although the estimated storage lifetimes are all close to the design average storage lifetime of the missile tank, proposed the evaluation method based on IG process with random effects is closest to the design lifetime in this paper. Hence, we may safely draw a conclusion that the proposed models and parameter estimation methods are reasonable and valid in this paper.

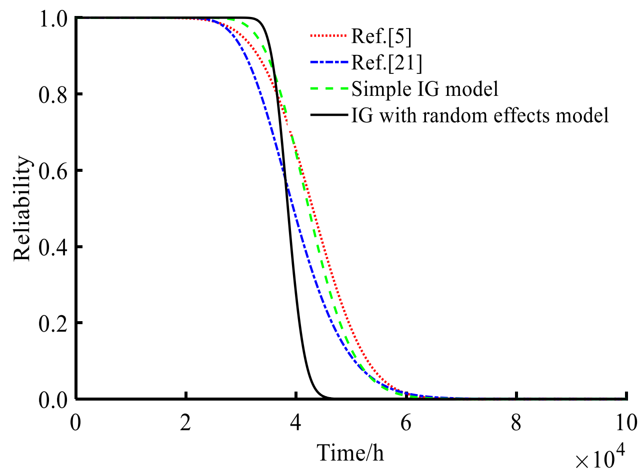

FIGURE 3. Reliability curves for these models.

According to the above reliability models, the reliability curves for each model can be drawn, which are shown in Fig.3.

As can be seen from Fig.3, the reliability curves of these models are without obvious difference except the IG process with random effect model. To quantitatively select the model, the most straightforward way is to use the Akaike Information Criterion (AIC) [22]. The AIC is defined as:

$$\mathrm{AIC} = -2 \times L(\hat{\boldsymbol{\theta}}) + 2\varpi \quad (23)$$

where ϖ is the number of parameters, and $L(\hat{\boldsymbol{\theta}})$ is estimation value of log-likelihood function.

Based on this criterion, the model with the minimum AIC value can be selected as a good fitting model. We obtained the AIC of these models, as shown in Table 2.

TABLE 2. Aic of these models.

Models	Simple IG	IG with Random Effects	In [21] Gamma	In [5] SNADM
AIC	-32.23	-60.89	-24.36	-10.15

Table 2 indicates that the model based on IG process with random effects has the smallest AIC. At the same time, as shown in Fig.2, the unit-to-unit heterogeneity is obvious within the six test samples in step-down-stress ADT of the missile tank. Therefore, the random effects model is meaningful, and result is reasonable. In addition, Tables 1 and 2 show that both the simple IG model and IG process with effect random are super to the SNADM model in [5] and the Gamma model in [21]. So to speak, the proposed evaluation methods for step-down-stress ADT based on the IG process are more accurate and reasonable.

VI. CONCLUSION

This paper provides a new evaluation method of step-down-stress ADT for products with high reliability and long lifetime based on IG process. The method overcomes some disadvantages that traditional analysis methods heavily relies on priori information and complex statistical analysis effectively. Compared with existing evaluation methods, the proposed evaluation method is more efficient in this paper. In addition, both simple IG process and IG process with random effects are considered, respectively, and IG process with random effects model is much more reasonable due to incorporated the unit-to-unit heterogeneity of test samples than the simple IG process model.

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