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Knowledge Granularity for Continuous Parameters

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ABSTRACT In the community of Granular Computing, knowledge is interpreted as one classification ability of realistic or abstract objects. Generally, the concept of granularity is used for characterizing such an ability, which has been widely explored in literatures. To calculate the parameterized granularity, a naive approach is to find the granularity in terms of the parameter one by one. Nevertheless, such approach can only generate the single parameter based knowledge granularity, and the difference of knowledge granularities among different parameters may be slight. It follows that the knowledge granularity derived from single parameter may be lack of representativeness. In this paper, the continuous parameters based knowledge granularity is proposed, and the corresponding calculation approach is presented. Inspired by the thinking of definite integral in mathematical problems, the calculation approach is mainly implemented by following steps: firstly, the graph formed by granularity and parameter interval is divided into several small rectangles whose length of interval tends to be 0; secondly, the sum of area values of all the small rectangles is calculated; finally, the obtained area value divided by the whole length of parameter interval can be considered as the continuous parameters based knowledge granularity. This study suggests a new trend of handling problems related to knowledge from the viewpoint of continuity.

INDEX TERMS Continuous parameters, granular computing, granularity, knowledge.

I. INTRODUCTION

Granular Computing [1]–[6] is proposed based on the analyzing of information granularity [7]–[10]. In recent years, the concept of Granular Computing has been widely used in the fields of artificial intelligence, rough set, data mining, knowledge discovery and so on [11]–[18]. Such method simulates the natural pattern of human thinking for problem, and the information granule is taken as the fundamental element for calculating, which aims to process large-scale complex data sets and establish an effective calculation model [19].

In the process of using Granular Computing to deal with problems, information granulation [20]–[23] is a crucial step. The samples in data are combined into a collection of information granules by knowledge [24], [25] and their association degree, which can further form a granular structure [26]–[28]. If the equivalence relation [15] is employed, then information granulation is actually the processes of dividing samples by equivalence relation and obtaining equivalent granular structure; if the neighborhood relation [31], [32] is employed, then information granulation is actually the processes of generating the neighborhood of each sample and obtaining neighborhood granular structure. Among these technologies, it should be noticed that the neighborhood relation has become a practical tool for information granulation in Granular Computing theory, since it enables us to handle the continuous data which is ubiquitous in real-world applications, and neighborhood relation can be directly constructed over the distances between samples.

The parameter radius plays a key role in the constructing of neighborhood relation. This is mainly because if a smaller value of radius is employed to generate neighborhood relation, then the great majority of samples can be distinguished from each other, which indicates a stronger discrimination ability of such parameter based knowledge; if a greater value of radius is used to generate neighborhood relation, then

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most samples cannot be distinguished from each other, which implies a weaker discrimination ability of such parameter based knowledge. According to such property, many knowledge discovery and data mining approaches involving parameter based granularity have been explored. For example, Liu *et al.* [21] designed a framework of multi-granularity feature selection scheme which considers the variation of parameters; Zhu *et al.* [9] learnt an effective distance metric based on neighborhood granule margin maximization from a group of parameters; Zhu and Hu [39] also presented an adaptive selection scheme of neighborhood granularity (i.e., parameters), and provided a solution to margin distribution optimization. Nevertheless, these approaches only exploited the external representations of parameter, instead of describing the reflected granularity, as well as the associated discrimination ability of knowledge. To measure the discrimination ability of knowledge, Miao *et al.* [24] constructively proposed a formal definition of granularity and corresponding calculation approach of granularity; Jing *et al.* [29] utilized such form of knowledge granularity in incremental attribute reduction in which granularity varies by data addition or deletion; Pedrycz and Homenda [1] built a principle of justifiable granularity from the viewpoint of experimental evidence and semantic meaning for the realization of information granule; Li *et al.* [30] investigated the knowledge granularity to offer an exact description for one target granule. In these frameworks, a smaller value of granularity indicates a greater discrimination ability of knowledge, while a greater value of granularity denotes a weaker discrimination ability of knowledge. Note that such granularity is generated based on one and only one parameter, and the corresponding granularity value can only reflect the discrimination ability of knowledge related to single parameter. That means, scant attention has been paid to the representation of knowledge granularity in the form of multiple parameters.

However, a group of multiple parameters used for constructing learning model is common in real-world applications. Consequently, the knowledge granularity on single parameter discussed above may be lack of universality. It is desirable to develop an effective solution to characterizing knowledge granularity from continuous parameters. To this end, a calculation approach for continuous parameters based knowledge granularity will be investigated in this paper. It is worth noting that two difficulties will be addressed: 1) existing findings are insufficient to exploit the continuous parameters for knowledge granularity, this study may be the first time to consider such case, so how to present and calculate it; 2) continuous parameters naturally implied in data are required to construct the granularity, so how to derive complete parameters involving global granularities. From the thinking of definite integral in mathematics, the calculation of continuous parameters based knowledge granularity can be realized by the following steps: firstly, the graph formed by knowledge granularity and parameter is considered in one given parameter interval; secondly, the parameter interval in the graph is divided into several small rectangles whose length of interval approaches 0; finally, the area values of all the small rectangles are calculated, and all the values are accumulated, then the obtained whole area value divided by the length of the parameter interval will be regarded as output, i.e., continuous parameters based knowledge granularity. The granularity value can reflect the average discrimination ability of knowledge corresponding to parameters in such interval.

Additionally, the contribution of our study is three-fold:

- We firstly develop a continuous parameters based mechanism to build the formation of granularity, and thereby extend the knowledge representation.
- We present a novel calculation approach to quantifying the knowledge granularity across a group of continuous parameters effectively.
- We discuss and explore the properties of the proposed granularity which can be expected to have a great potential in various applications such as feature selection.

The reminder of this paper is organized as follows. In Section 2, we will introduce the knowledge representation and the calculation of discrete parameters based knowledge granularity. The calculation approach of continuous parameters based knowledge granularity is proposed in Section 3, and the related properties is further discussed. This paper is ended with conclusions and future research in Section 4.

II. DISCRETE PARAMETERS BASED KNOWLEDGE GRANULARITY

A. KNOWLEDGE REPRESENTATION

Formally, let $S = \langle U, AT \rangle$ be a knowledge representation system, in which U is the nonempty finite set of all the samples, i.e., universe, $\forall x_i \in U$, x_i implies a sample in U; AT is the nonempty finite set of all the attributes, $\forall a \in AT$, a indicates one attribute in AT.

Definition 1: Given a knowledge representation system $S = \langle U, AT \rangle$, radius $\delta \geq 0$, $\forall A \subseteq AT$, the neighborhood relation is defined as

$$R_A^{\delta} = \{(x_i, x_j) \in U \times U : dis_{ij}^{A} \leq \delta\}, \tag{1}$$

in which dis_{ij}^{A} indicates the distance between samples x_i and x_j over the attribute subset A.

Following Definition 1, a neighborhood granular structure [33] over U can be derived through granulating the knowledge representation system with neighborhood relation, and such neighborhood granular structure is also one neighborhood system [34] $\{R_A(x_1), R_A(x_2), \ldots, R_A(x_n)\}$, where $R_A(x_i) = \{x_j \in U : (x_i, x_j) \in R_A^{\delta}\}$. Obviously, the knowledge plays a key role in the generating of neighborhood granular structure, and the discrimination ability of knowledge has a direct effect on the size of neighborhood granular structure. A stronger discrimination ability of knowledge indicates fewer samples are considered to be indistinguishable, and a finer neighborhood granular structure will be obtained; while a weaker discrimination ability of knowledge implies more sample are considered to be indistinguishable, and a coarser neighborhood granular structure will be obtained. To characterize the discrimination ability of knowledge quantitatively, Miao *et al.* [24] proposed the concept of granularity.

B. THE CALCULATION FOR DISCRETE PARAMETERS BASED KNOWLEDGE GRANULARITY

Definition 2 ([24], [35]): Given a knowledge representation system $S = \langle U, AT\rangle$, radius $\delta \geq 0$, $\forall A \subseteq AT$, the knowledge granularity based on radius δ is defined as

$$G_A^\delta = \frac{|R_A^\delta|}{|U|^2}, \tag{2}$$

in which $|X|$ denotes the cardinal number of the set X.

The smaller the granularity value G_A^δ is, the finer the obtained neighborhood granular structure will be, which indicates a stronger discrimination ability of knowledge corresponding to parameter.

Property 1: Given a knowledge representation system $S = \langle U, AT\rangle$, radius $\delta \geq 0$, $\forall A \subseteq AT$, G_A^δ is bounded, and $1/|U| \leq G_A^\delta \leq 1$ holds.

If $R_A^\delta = \omega = \{(x_i, x_i) \in U \times U : \forall x_i \in U\}$, then the granularity will achieve the minimal value $1/|U|$, the finest neighborhood granular structure will be obtained, and the corresponding discrimination ability of knowledge is strongest; if $R_A^\delta = \eta = \{(x_i, x_j) \in U \times U : \forall x_i, x_j \in U\}$, then the granularity will achieve the maximal value 1, the coarsest neighborhood granularity structure will be obtained, and the corresponding discrimination ability of knowledge is weakest.

Property 2: Given a knowledge representation system $S = \langle U, AT\rangle$, radius $\delta_1 \geq 0$, $\delta_2 \geq 0$, and $\delta_1 \leq \delta_2$, $\forall A \subseteq AT$, $G_A^{\delta_1} \leq G_A^{\delta_2}$ holds.

Proof: Following Definition 1, it is easy to know that if $\delta_1 \leq \delta_2$, then $R_A^{\delta_1} \subseteq R_A^{\delta_2}$, it follows that $|R_A^{\delta_1}| \leq |R_A^{\delta_2}|$. By Definition 2, we have $G_A^{\delta_1} = |R_A^{\delta_1}|/|U|^2$, $G_A^{\delta_2} = |R_A^{\delta_2}|/|U|^2$. It follows that $G_A^{\delta_1} \leq G_A^{\delta_2}$ holds. □

Property 2 shows that with the increasing of used radius, the value of granularity will be increased. In others words, the discrimination ability of knowledge will be decreased with the increasing of used radius. Therefore, the variation tendency of granularity can be described by following Fig. 1.

As illustrated in Fig. 1, the horizontal ordinate and vertical ordinate represent the radius and the granularity value, respectively. Additionally, it is not difficult to observe that for one given knowledge representation system, the variation tendency of granularity with parameters can be expressed by a piecewise constant function. This is mainly because only when the parameter and the obtained neighborhood granular structure vary, then the corresponding granularity value will vary; if the parameters varies, the obtained neighborhood granular structure does not change, then the corresponding granularity value will not vary.

Note that if the parameter $\delta_{\max} = \max\{dis_{ij}^A : (x_i, x_j) \in U \times U\}$, then the neighborhood granular structure will be

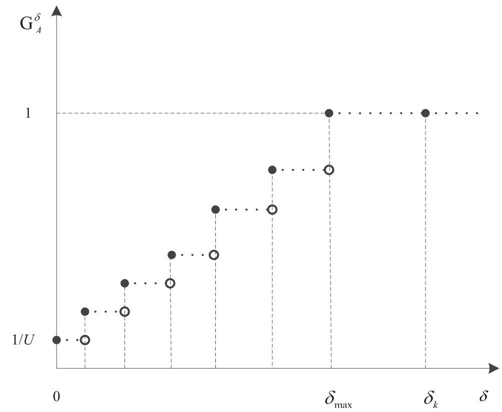

FIGURE 1. The variation of discrete parameters based knowledge granularity.

the coarsest one, and the corresponding granularity value will achieve the maximal value 1; if the value of parameter keeps increasing until $\delta_k \geq \delta_{\max}$, then the neighborhood granular structure induced by such parameter based the knowledge will not change, and the granularity value will also not increase. Therefore, for the parameter $\delta_k > \delta_{\max}$, the variation tendency of corresponding granularity value can be expressed by a constant function with function value of 1.

Property 3: Given a knowledge representation system $S = \langle U, AT\rangle$, radius $\delta \geq 0$, $\forall A, B \subseteq AT$, and $A \subseteq B$, $G_B^\delta \leq G_A^\delta$ holds.

Property 3 indicates that the granularity value will be decreased with the increasing number of used attributes. That is to say, the discrimination ability of knowledge will be boosted if more attributed are employed to characterize the target samples.

It should be noticed that the granularity shown in Definition 2 can only characterize the single discrete parameter based knowledge granularity. However, the knowledge granularities corresponding to different parameters may be entirely different. Hence, we can say that the knowledge granularity corresponding to single discrete parameter may be lack of representativeness. In view of this, a calculation approach for continuous parameters based knowledge granularity will be proposed in the next section.

III. CONTINUOUS PARAMETERS BASED KNOWLEDGE GRANULARITY

A. CONTINUOUS PARAMETERS BASED KNOWLEDGE GRANULARITY AND ITS CALCULATION

Definition 3: Given a knowledge representation system $S = \langle U, AT\rangle$, radius interval $[\delta_m, \delta_n]$, $\forall A \subseteq AT$, then continuous radius interval $[\delta_m, \delta_n]$ based knowledge granularity is referred to as the continuous parameters based knowledge granularity, which is denoted as $G_A^{[\delta_m,\delta_n]}$.

The granularity shown in Definition 3 is the one with universality over the continuous parameters $[\delta_m, \delta_n]$. In particular, if $\delta_m = \delta_n$, then such granularity will degenerate into the single parameter based knowledge granularity.

To calculate the continuous parameters based granularity shown in Definition 3, a naive approach is to employe the thinking of average value. Precisely, firstly, compute the knowledge granularities corresponding to all the parameters; secondly, calculate the average value of these granularities, and such average value may be regarded as the continuous parameters based knowledge granularity. Nevertheless, the continuous parameter interval contains innumerable number of parameter, it is difficult to calculate knowledge granularity corresponding to each parameter in such parameter interval, and it is also impractical. Therefore, how to calculate the continuous parameters based knowledge granularity becomes an open problem.

According to the thinking of definite integral in mathematics, the graph formed by granularity and parameter interval can be divided into several small rectangles whose length of interval approaches 0, as shown in Fig. 2. Calculate the area values of all the small rectangles, and the sum of these area values divided by the whole length of parameter interval can be considered as continuous parameters based knowledge granularity. The detailed calculation process is shown as follows.

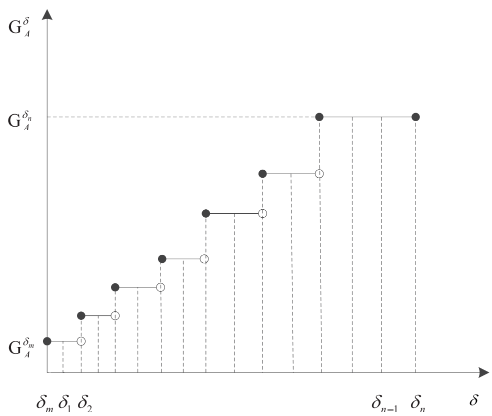

FIGURE 2. The decomposition figure for computing continuous parameters based knowledge granularity.

Following Fig. 1 in Section 2, it is easy to know that single parameter based granularity G_A^δ is bounded in one given parameter interval $[\delta_m, \delta_n]$. Some parameter points are arbitrarily inserted into the parameter interval $[\delta_m, \delta_n]$ such that:

$$\delta_m < \delta_1 < \delta_2 < \ldots < \delta_{n-1} < \delta_n, \tag{3}$$

then the parameter interval $[\delta_m, \delta_n]$ will be divided into n small intervals:

$$[\delta_m, \delta_1], [\delta_1, \delta_2], \ldots, [\delta_{n-1}, \delta_n], \tag{4}$$

it follows that the length of small interval is:

$$\Delta\delta_1 = \delta_1 - \delta_m, \Delta\delta_2 = \delta_2 - \delta_1, \ldots, \Delta\delta_n = \delta_n - \delta_{n-1}. \tag{5}$$

Let $\delta_t (s-1 \leq t \leq s)$ be an arbitrary point in each small interval $[\delta_{s-1}, \delta_s]$, calculate the product of granularity $G_A^{\delta_t}$ and length of small interval $\Delta\delta_s$, and then compute the sum of them such that:

$$S = \sum_{t=1}^{n} G_A^{\delta_t} \Delta\delta_s. \tag{6}$$

Let $\lambda = \max\{\Delta s_1, \Delta s_2, \ldots, \Delta s_n\}$. If λ tends to be 0, then there is the limit of Equation (6), and the limit has nothing to do with the division of parameter interval $[\delta_m, \delta_n]$ as well as the selection of point δ_t. Consequently, the area value of graph formed by granularity and radius can be formulated as:

$$\lim_{\lambda \to 0} \sum_{t=1}^{n} G_A^{\delta_t} \Delta\delta_s = \int_{\delta_m}^{\delta_n} G_A^\delta d\delta. \tag{7}$$

In general, if $\delta_m < \delta_n$, then the continuous parameters based knowledge granularity is:

$$G_A^{[\delta_m, \delta_n]} = \frac{\int_{\delta_m}^{\delta_n} G_A^\delta d\delta}{\delta_n - \delta_m}, \tag{8}$$

In particular, if $\delta_m = \delta_n$, then the continuous parameters based knowledge granularity will degenerate into single parameter based knowledge granularity. In such case, the continuous parameters $[\delta_m, \delta_n]$ based knowledge granularity is:

$$G_A^{[\delta_m, \delta_n]} = G_A^{\delta_m} = G_A^{\delta_n}. \tag{9}$$

To sum up, the continuous parameters $[\delta_m, \delta_n]$ based knowledge granularity can be formulated as:

$$G_A^{[\delta_m, \delta_n]} = \begin{cases} \dfrac{\int_{\delta_m}^{\delta_n} G_A^\delta d\delta}{\delta_n - \delta_m}, & \delta_m < \delta_n; \\ G_A^{\delta_m}, & \delta_m = \delta_n. \end{cases} \tag{10}$$

Consequently, the calculation approach for continuous parameters based knowledge granularity can be defined as follows.

Definition 4: Given a knowledge representation system $S = \langle U, AT \rangle$, radius interval $[\delta_m, \delta_n]$, $0 \leq \delta_m \leq \delta \leq \delta_n$, $\forall A \subseteq AT$, continuous radii $[\delta_m, \delta_n]$ based knowledge granularity is:

$$G_A^{[\delta_m, \delta_n]} = \begin{cases} \dfrac{\int_{\delta_m}^{\delta_n} G_A^\delta d\delta}{\delta_n - \delta_m}, & \delta_m < \delta_n; \\ G_A^{\delta_m}, & \delta_m = \delta_n. \end{cases} \tag{11}$$

Remark 1: The smaller the value of continuous parameters based knowledge granularity is, the stronger the average discrimination ability of the knowledge corresponding to such group of parameters will be. It should be noticed that if δ_m is equal to δ_n, then the calculation approach for continuous parameters based knowledge granularity shown in Definition 4 will degenerate into the single discrete parameter based one.

Example 1: Given the knowledge representation system $S = \langle U, AT \rangle$ shown in Tab. 1, in which $U = \{x_1, x_2, x_3, x_4, x_5\}$ and $AT = \{a_1, a_2, a_3, a_4\}$.

The continuous parameters [0.4, 0.6] based knowledge granularity can be calculated as follows:

TABLE 1. A toy example of knowledge representation system.

	a_1	a_2	a_3	a_4
x_1	0.28	0.31	0.77	0.65
x_2	0.05	0.95	0.80	0.71
x_3	0.10	0.03	0.19	0.75
x_4	0.82	0.44	0.49	0.28
x_5	0.69	0.38	0.45	0.68

1) *Compute the distances between samples, and then the distances between samples are 0.43, 0.53, 0.68, 0.68, 0.72, 0.74, 0.93, 1, 1.06, 1.11, respectively.*
2) *From the distances between samples, neighborhood relation in Definition 1 and discrete parameters based granularity knowledge, it is not difficult to observe the following: i)* $\forall \delta_t \in [0.4, 0.43)$ *and based on Definition 2,* $G_{AT}^{\delta_t}$ *can be obtained such that* $G_{AT}^{\delta_t} = 1/5$*; ii)* $\forall \delta_t \in [0.43, 0.53)$ *and based on Definition 2,* $G_{AT}^{\delta_t}$ *can be obtained such that* $G_{AT}^{\delta_t} = 7/25$*; iii)* $\forall \delta_t \in [0.53, 0.6]$ *and based on Definition 2,* $G_{AT}^{\delta_t}$ *can be obtained such that* $G_{AT}^{\delta_t} = 9/25$.
3) *Following Definition 4 and Equation (7), as shown at the bottom of the page.*

Finally, the continuous radii [0.4, 0.6] base knowledge granularity can be calculated as 101/250.

B. THE PROPERTIES OF CONTINUOUS PARAMETERS BASED KNOWLEDGE GRANULARITY

Property 4: Given a knowledge representation system $S = \langle U, AT\rangle$*, radius interval* $[\delta_m, \delta_n]$*,* $0 \le \delta_m \le \delta_n$*,* $\forall A \subseteq AT$*, then* $1/|U| \le G_A^{[\delta_m,\delta_n]} \le 1$ *holds.*

Proof: For the first case, if $\delta_m < \delta_n$, then we have:

$$G_A^{[\delta_m,\delta_n]} = \frac{\int_{\delta_m}^{\delta_n} G_A^{\delta} d\delta}{\delta_n - \delta_m},$$

following Equation (7), we have:

$$G_A^{[\delta_m,\delta_n]} = \frac{\int_{\delta_m}^{\delta_n} G_A^{\delta} d\delta}{\delta_n - \delta_m} = \frac{\lim_{\lambda\to 0}\sum_{t=1}^{n} G_A^{\delta_t}\Delta\delta_s}{\delta_n - \delta_m}.$$

Consequently, if $\forall t \in [1, n]$ and $G_A^{\delta_t}$ is the minimal value $1/|U|$, then $G_A^{[\delta_m,\delta_n]}$ can achieve the minimal value, i.e., $G_A^{[\delta_m,\delta_n]} = (\delta_n - \delta_m)G_A^{\delta_t}/(\delta_n - \delta_m) = 1/|U|$; if $\forall t \in [1, n]$ and $G_A^{\delta_t}$ is the maximal value 1, then $G_A^{[\delta_m,\delta_n]}$ can achieve the maximal value, i.e., $G_A^{[\delta_m,\delta_n]} = (\delta_n - \delta_m)G_A^{\delta_t}/(\delta_n - \delta_m) = 1$.

For the second case, if $\delta_m = \delta_n$, then continuous parameters $[\delta_m, \delta_n]$ based knowledge granularity will degenerate into single parameter based knowledge granularity, and by Property 1, $1/|U| \le G_A^{\delta_m} \le 1$ holds.

To sum up, Property 4 holds. □

Property 4 shows that if the interval $[\delta_m, \delta_n]$ is small, and both the knowledge granularities corresponding to parameters δ_m and δ_n are $1/|U|$, then the knowledge granularity corresponding to each parameter in interval $[\delta_m, \delta_n]$ is $1/|U|$, accordingly, the continuous parameters $[\delta_m, \delta_n]$ based knowledge granularity achieves the minimal value $1/|U|$, which also indicates the corresponding average discrimination ability is the strongest; if the interval $[\delta_m, \delta_n]$ is large, and both the knowledge granularities corresponding to parameters δ_m and δ_n are 1, then the knowledge granularity based on each parameter in interval $[\delta_m, \delta_n]$ are 1, accordingly, the continuous parameters $[\delta_m, \delta_n]$ based knowledge granularity achieves the maximal value 1, which also implies the corresponding average discrimination ability is the weakest.

Property 5: Given a knowledge system $S = \langle U, AT\rangle$*, radius intervals* $[\delta_m, \delta_n]$ *and* $[\delta_p, \delta_q]$*,* $0 \le \delta_m \le \delta_p$*,* $0 \le \delta_n \le \delta_q$*, and* $\delta_n - \delta_m = \delta_q - \delta_p$*,* $\forall A \subseteq AT$*, then* $G_A^{[\delta_m,\delta_n]} \le G_A^{[\delta_p,\delta_q]}$ *holds for continuous radii* $[\delta_m, \delta_n]$ *and* $[\delta_p, \delta_q]$.

Proof: For the first case, if $\delta_m < \delta_n$ and $\delta_p < \delta_q$, we have:

$$G_A^{[\delta_m,\delta_n]} = \frac{\int_{\delta_m}^{\delta_n} G_A^{\delta} d\delta}{\delta_n - \delta_m};\ G_A^{[\delta_p,\delta_q]} = \frac{\int_{\delta_p}^{\delta_q} G_A^{\delta} d\delta}{\delta_q - \delta_p}.$$

By Equation (7), we have:

$$G_A^{[\delta_m,\delta_n]} = \frac{\int_{\delta_m}^{\delta_n} G_A^{\delta} d\delta}{\delta_n - \delta_m} = \frac{\lim_{\lambda\to 0}\sum_{t=1}^{n} G_A^{\delta_t}\Delta\delta_s}{\delta_n - \delta_m};$$

$$G_A^{[\delta_p,\delta_q]} = \frac{\int_{\delta_p}^{\delta_q} G_A^{\delta} d\delta}{\delta_q - \delta_p} = \frac{\lim_{\lambda\to 0}\sum_{t=1}^{q} G_A^{\delta_t}\Delta\delta_s}{\delta_q - \delta_p}.$$

Considering that $0 \le \delta_m \le \delta_p$ and $0 \le \delta_n \le \delta_q$, hence,

$$\sum_{t=1}^{n} G_A^{\delta_t} \le \sum_{t=1}^{q} G_A^{\delta_t}.$$

It follows that

$$\int_{\delta_m}^{\delta_n} G_A^{\delta} d\delta \le \int_{\delta_p}^{\delta_q} G_A^{\delta} d\delta,$$

$$\begin{aligned} G_{AT}^{[0.4,0.6]} &= \frac{\int_{0.4}^{0.6} G_{AT}^{\delta} d\delta}{0.6 - 0.4} = \frac{\lim_{\lambda\to 0}\sum_{t=1}^{n} G_{AT}^{\delta_t}\Delta\delta_s}{0.6 - 0.4} \\ &= \frac{1/5 \cdot (0.43 - 0.4) + 7/25 \cdot (0.53 - 0.43) + 9/25 \cdot (0.6 - 0.53)}{0.6 - 0.4} \\ &= \frac{101}{250}. \end{aligned}$$

furthermore, considering that $\delta_n - \delta_m = \delta_q - \delta_p$. Consequently, $G_A^{[\delta_m,\delta_n]} \leq G_A^{[\delta_p,\delta_q]}$ holds.

For the second case, if $\delta_m = \delta_n$ and $\delta_p = \delta_q$, then both the knowledge granularities corresponding to continuous parameters $[\delta_m, \delta_n]$ and $[\delta_p, \delta_q]$ will degenerate into single parameter based knowledge granularity. By $0 \leq \delta_m \leq \delta_p$ and Property 2, $G_A^{\delta_m} \leq G_A^{\delta_p}$ holds.

To sum up, Property 5 holds. □

Property 5 shows that if the interval length of continuous parameters is the same, then the continuous parameters based knowledge granularity will be decreased with the decreasing of the value of interval endpoints. In other words, the average discrimination ability of continuous parameters based knowledge will be increased with the decreasing of the value of interval endpoints.

Property 6: Given a knowledge representation system $S = \langle U, AT \rangle$*, radius interval* $[\delta_m, \delta_n]$, $0 \leq \delta_m \leq \delta_n$, $\forall A, B \subseteq AT$*, and* $A \subseteq B$*, then* $G_B^{[\delta_m,\delta_n]} \leq G_A^{[\delta_m,\delta_n]}$ *holds.*

Proof: For the first case, if $\delta_m < \delta_n$, then we have:

$$G_A^{[\delta_m,\delta_n]} = \frac{\int_{\delta_m}^{\delta_n} G_A^{\delta} d\delta}{\delta_n - \delta_m}; \quad G_B^{[\delta_m,\delta_n]} = \frac{\int_{\delta_m}^{\delta_n} G_B^{\delta} d\delta}{\delta_n - \delta_m}.$$

By Equation (7), we have:

$$G_A^{[\delta_m,\delta_n]} = \frac{\int_{\delta_m}^{\delta_n} G_A^{\delta} d\delta}{\delta_n - \delta_m} = \frac{\lim\limits_{\lambda \to 0} \sum\limits_{t=1}^{n} G_A^{\delta_t} \Delta\delta_s}{\delta_n - \delta_m};$$

$$G_B^{[\delta_m,\delta_n]} = \frac{\int_{\delta_m}^{\delta_n} G_B^{\delta} d\delta}{\delta_n - \delta_m} = \frac{\lim\limits_{\lambda \to 0} \sum\limits_{t=1}^{n} G_B^{\delta_t} \Delta\delta_s}{\delta_n - \delta_m}.$$

Considering that $A \subseteq B$, and by Property 3, we have:

$$\sum_{t=1}^{n} G_B^{\delta_t} \leq \sum_{t=1}^{n} G_A^{\delta_t}.$$

It follows that

$$\int_{\delta_m}^{\delta_n} G_B^{\delta} d\delta \leq \int_{\delta_m}^{\delta_n} G_A^{\delta} d\delta.$$

For the second case, if $\delta_m = \delta_n$, then continuous parameters $[\delta_m, \delta_n]$ based knowledge granularity will degenerate into single parameter based knowledge granularity. Correspondingly, by Property 3, if $A \subseteq B$, then $G_B^{\delta_m} \leq G_A^{\delta_m}$ holds.

To sum up, Property 6 holds. □

Property 6 reveals that the value of continuous parameters based knowledge granularity will be decreased with the growing of number of used attributes. That is, the average discrimination ability of continuous parameters based knowledge will be increased with the growing of number of used attributes.

IV. CONCLUSION AND FUTURE PERSPECTIVES

In the calculating of parameter based granularity, the conventional calculation approach can only find the knowledge granularity corresponding to single parameter. However, there may be little difference in knowledge granularities among different parameters, and the single parameter based knowledge granularity may be lack of representativeness. In view of this, inspired by the thinking of definite integral in mathematical problems, a calculation approach for continuous parameters based knowledge granularity is proposed, and the related properties are further discussed. The following topics deserve our further researches.

1) Only the calculation approach for continuous parameters based knowledge granularity is presented in this paper. Following such approach, the computing approach of various measure criteria based on continuous parameters will be further explored and investigated, which may be helpful for providing new research thinking for attribute selection problem.
2) Only the calculation approach for continuous parameters based knowledge granularity is presented in this paper. The source and selection of continuous parameters have not been discussed from the viewpoint of data-driven, which will be the main research direction of our future works.
3) Only the theoretical framework of granularity is discussed in this paper. The quantification and quantization of uncertainty in data will be explored, and further exploited in potential applications such as feature extraction, selection and fusion for intelligent monitoring. [36]–[38]

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