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A New Sufficient Criterion for the Stability of 2-D Discrete Systems

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ABSTRACT During the past few decades, two and higher dimensional systems have been extensively applied in many areas of research. The representation of the 2-D systems in the frequency domain is usually given by its transfer function. The bounded-input bounded-output (BIBO) stability of the two dimensional discrete systems depends on the zeros of the characteristic polynomial which is the denominator of this transfer function. In this paper, a new sufficient criterion for the stability of two-dimensional linear shift-invariant discrete systems is presented. The new criterion is based on the sufficient condition for stable polynomials with complex coefficients and the stability criterion for 2-D discrete systems proposed by Murray and Delsarte *et al.*. The new criterion is non-conservative for the stability testing of 2-D discrete systems. It is shown that the proposed sufficient criterion is simple enough to be applied for the stability checking of the 2-D discrete systems. The utility of the proposed criterion is demonstrated by examples.

INDEX TERMS 2-D discrete systems, transfer function, polynomials, stability, sufficient condition.

I. INTRODUCTION

During the past few decades, two and higher dimensional systems have been extensively applied in many areas of the study of broadband beamforming, digital filtering, image processing, multipass processes, gas filtration, thermal processes, geophysics, medical electronics, video and lightfield processing, sensor networks, 2-D discrete control systems, and so on [1]–[11]. In these applications, the signals are functions of two or more variables. The designer of systems designed for such applications is interested in the checking the stability of them. Criteria which provide sufficient and necessary conditions for the stability of 2-D discrete systems have been developed.

The stability analysis of 2-D discrete systems was studied in the frequency domain leading to many well known criteria and tests [12], [13]–[29]. It is a heavy computational task to test whether a 2-D polynomial is devoid of zeros in the unit bidisc. Hence, it is useful to have at hand simple sufficient conditions for the checking the stability. Such contributions for the stability of 2-D discrete systems have been presented in [30], [31]. Simple relations between the coefficients of the 2-D polynomial have been presented in these contributions.

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By using the sufficient conditions in [30], [31], the stability checking of the 2-D discrete systems can be carried out rapidly. In this paper a new sufficient criterion for linear 2-D discrete systems will be proposed. This is based on the sufficient condition for stable polynomials with complex coefficients presented in [34], [35] and the stability criterion proposed by Murray [32] and Delsarte *et al.* [33]. Simple relations between the coefficients of the 2-D polynomial will be presented in the proposed criterion. By using the proposed criterion the stability checking of the 2-D discrete systems can be carried out rapidly.

Notation: The closed unit bidisc is denoted by $\overline{U}^2$

$$\overline{U}^2 = \{(z_1, z_2) | |z_1| \leq 1, |z_2| \leq 1\}$$

the open unit bidisc is denoted by U^2

$$U^2 = \{(z_1, z_2) | |z_1| < 1, |z_2| < 1\}$$

and the boundary of the unit bidisc is denoted by T^2

$$T^2 = \{(z_1, z_2) | |z_1| = 1, |z_2| = 1\}$$

Similar denotes are given for the one-variable regions $\overline{U}$, U and T. **R** and **C** denote the set of real numbers and complex numbers respectively. The region in which a stable polynomial is allowed to have its roots is defined as the stability region of the polynomial.

II. PRELIMINARIES

A single input-single output linear shift-invariant 2-D discrete system is represented in frequency domain by its transfer function $H(z_1, z_2)$ which is a rational function

$$H(z_1, z_2) = \frac{B(z_1, z_2)}{A(z_1, z_2)} \tag{1}$$

where $B(z_1, z_2)$, $A(z_1, z_2)$ are two variable polynomials of the variables z_1, z_2, where $z_1, z_2 \in \mathbf{C}$. Systems (1) are assumed to have no non-essential singularities of the second kind on the unit bidisc T^2. The case of having non-essential singularities has been considered in [36]. The 2-D characteristic polynomial $A(z_1, z_2)$ is given by:

$$A(z_1, z_2) = \sum_{i=0}^{n} \sum_{l=0}^{m} \alpha_{il} z_1^i z_2^l \tag{2}$$

where $\alpha_{il} \in \mathbf{R}$ are the constant real coefficients of the polynomial.

Criteria which provide sufficient and necessary conditions for the bounded-input bounded-output (BIBO) stability of 2-D discrete systems have been developed in [12]. They require that the characteristic polynomial is devoid of zeros in the unit bidisc. The condition for BIBO stability of 2-D discrete systems is given below.

A. STABILITY CONDITION FOR 2-D DISCRETE SYSTEMS [12]

The linear 2-D discrete system (1) is BIBO stable if and only if

$$A(z_1, z_2) \neq 0 \tag{3}$$

in the closed unit bidisc $\overline{U}^2$.

B. STABILITY CRITERION FOR 2-D DISCRETE SYSTEMS [32], [33]

The linear 2-D discrete system (1) is BIBO stable if and only if

$$A(z, ze^{j\omega}) \neq 0, \quad |z| \leq 1, \quad \omega \in [0, 2\pi] \tag{4}$$

In the above criterion, the 2-D stability problem has been reduced in a 1-D stability problem with a characteristic polynomial whose coefficients are functions of $e^{j\omega}$ for $\omega \in [0, 2\pi]$.

It should be mentioned that using the arguments in [32], [33] numerous other stability criteria can be derived from (4) by replacing one or both variables by a finite Blaschke product since this is precisely the class of functions analytic in U, continuous on U and having unit modulus on T. An introduction to finite Blaschke products is presented in [37].

The following lemma is important for the derivation of the new sufficient condition.

Lemma 1 [34], [35]: The polynomial $A(z) = z^n + \alpha_n z^{n-1} + \cdots + \alpha_2 z + \alpha_1$ *with complex coefficients has all its roots inside the unit disc if*

$$\sum_{i=1}^{n} |\alpha_i|^2 < \frac{1}{n} \tag{5}$$

where $|\alpha_i|$ is the absolute value of the coefficient α_i. The above lemma implies that the polynomial $A(z)$ is stable.

III. THE NEW SUFFICIENT CRITERION FOR THE STABILITY OF 2-D DISCRETE SYSTEMS

In the below theorem the new sufficient criterion for the stability of linear 2-D discrete systems will be presented. For the derivation of this new stability criterion, the criterion presented in [32], [33] will be combined with the results in [34], [35].

Theorem: The linear 2-D discrete system (1) is BIBO stable if

$$\sum_{k=1}^{n+m} b_k^2 < \frac{|\alpha_{00}|^2}{n+m} \tag{6}$$

with

$$\left.\begin{aligned}
&b_1 = (|\alpha_{10}| + |\alpha_{01}|) \\
&b_2 = (|\alpha_{20}| + |\alpha_{02}| + |\alpha_{11}|) \\
&b_3 = (|\alpha_{30}| + |\alpha_{03}| + |\alpha_{12}| + |\alpha_{21}|) \\
&b_4 = (|\alpha_{40}| + |\alpha_{04}| + |\alpha_{13}| + |\alpha_{31}| + |\alpha_{22}|) \\
&\qquad\vdots \\
&b_{n+m-3} = \\
&(|\alpha_{n-3,m}| + |\alpha_{n-2,m-1}| + |\alpha_{n-1,m-2}| + |\alpha_{n,m-3}|) \\
&b_{n+m-2} = (|\alpha_{n-2,m}| + |\alpha_{n,m-2}| + |\alpha_{n-1,m-1}|) \\
&b_{n+m-1} = (|\alpha_{n,m-1}| + |\alpha_{n-1,m}|) \\
&b_{n+m} = |\alpha_{nm}|
\end{aligned}\right\} \tag{7}$$

Proof: It follows from (2) and (4) that the polynomial $A(z, ze^{j\omega})$ can be written as

$$A(z, ze^{j\omega}) = \sum_{i=0}^{n} \sum_{l=0}^{m} \alpha_{il} e^{jl\omega} z^{i+l} \tag{8}$$

The reciprocal polynomial $A_1(z)$ of the polynomial $A(z, ze^{j\omega})$ can be written as

$$A_1(z) = z^{n+m} A(z^{-1}, z^{-1} e^{j\omega}) \tag{9}$$

and we define the polynomial $A_2(z)$ as $A_2(z) = A_1(z)/\alpha_{00}$. Then the condition $A(z, ze^{j\omega}) \neq 0$ in $|z| \leq 1$ holds if and only if the condition $A_2(z) \neq 0$ in $|z| \geq 1$ holds. The polynomial $A_2(z)$ is given by:

$$A_2(z) = z^{n+m} + c_{n+m} z^{n+m-1} + \cdots + c_3 z^2 + c_2 z + c_1$$

with

$$\begin{aligned}
c_{n+m} &= (\alpha_{10} + \alpha_{01} e^{j\omega})/\alpha_{00} \\
c_{n+m-1} &= (\alpha_{20} + \alpha_{02} e^{j2\omega} + \alpha_{11} e^{j\omega})/\alpha_{00} \\
c_{n+m-2} &= (\alpha_{30} + \alpha_{03} e^{j3\omega} + \alpha_{12} e^{j2\omega} + \alpha_{21} e^{j\omega})/\alpha_{00} \\
c_{n+m-3} &= (\alpha_{40} + \alpha_{04} e^{j4\omega} + \alpha_{13} e^{j3\omega} + \alpha_{31} e^{j\omega} + \alpha_{22} e^{j2\omega})/\alpha_{00} \\
&\ \ \vdots
\end{aligned}$$

$$c_3 = (\alpha_{n-2,m}e^{jm\omega} + \alpha_{n,m-2}e^{j(m-2)\omega} + \alpha_{n-1,m-1}e^{j(m-1)\omega})/\alpha_{00}$$
$$c_2 = (\alpha_{n,m-1}e^{j(m-1)\omega} + \alpha_{n-1,m}e^{jm\omega})/\alpha_{00}$$
$$c_1 = \alpha_{nm}e^{jm\omega}/\alpha_{00}$$

and the coefficients $c_1, c_2, \cdots, c_{n+m}$ are functions of the complex variable $e^{jl\omega}, l = 1, 2, \cdots, m$. The application of the well known Triangle Inequality with complex numbers yields

$$|(\alpha_{10} + \alpha_{01}e^{j\omega})|^2 \leq (|\alpha_{10}| + |\alpha_{01}|)^2$$
$$|(\alpha_{20} + \alpha_{02}e^{j2\omega} + \alpha_{11}e^{j\omega})|^2 \leq (|\alpha_{20}| + |\alpha_{02}| + |\alpha_{11}|)^2$$
$$|(\alpha_{30} + \alpha_{03}e^{j3\omega} + \alpha_{12}e^{j2\omega} + \alpha_{21}e^{j\omega})|^2 \leq (|\alpha_{30}| + |\alpha_{03}| + |\alpha_{12}| + |\alpha_{21}|)^2$$
$$(|(\alpha_{40} + \alpha_{04}e^{j4\omega} + \alpha_{13}e^{j3\omega} + \alpha_{31}e^{j\omega} + \alpha_{22}e^{j2\omega})|)^2 \leq (|\alpha_{40}| + |\alpha_{04}| + |\alpha_{13}| + |\alpha_{31}| + |\alpha_{22}|)^2$$
$$\vdots$$
$$|(\alpha_{n-2,m}e^{jm\omega} + \alpha_{n,m-2}e^{j(m-2)\omega} + \alpha_{n-1,m-1}e^{j(m-1)\omega})|^2 \leq (|\alpha_{n-2,m}| + |\alpha_{n,m-2}| + |\alpha_{n-1,m-1}|)^2$$
$$|(\alpha_{n,m-1}e^{j(m-1)\omega} + \alpha_{n-1,m}e^{jm\omega})|^2 \leq (|\alpha_{n,m-1}| + |\alpha_{n-1,m}|)^2$$
$$|\alpha_{nm}e^{jm\omega}|^2 = |\alpha_{nm}|^2$$

We apply the sufficient condition in Lemma 1 on the polynomial $A_2(z)$ and the conditions (6) - (7) are derived.

Simple relations between the coefficients of the 2-D discrete polynomial have been presented in the proposed criterion. Other sufficient conditions for the stability of multidimensional discrete systems have been presented in the literature. The sufficient criterion proposed in this paper offers a computational efficient alternative to the existing conditions in [30], [31]. Many examples of 2-D discrete polynomials have been studied and in all cases the proposed criterion determined the stability sufficiency and rapidly as the conditions in [30], [31]. The stability checking of the 2-D discrete systems using the proposed sufficient criterion is illustrated in the following examples.

IV. EXAMPLES

Example 1:

Consider the 2-D discrete system with order (1, 2) and the characteristic polynomial is given by

$$A(z_1, z_2) = 2z_1 + z_2 + z_1z_2 + 3z_2^2 + 2z_1z_2^2 + 10$$

with

$\alpha_{01} = 1, \alpha_{10} = 2, \alpha_{02} = 3, \alpha_{11} = 1, \alpha_{12} = 2, \alpha_{00} = 10$

The inequality

$$(|\alpha_{01}| + |\alpha_{10}|)^2 + (|\alpha_{02}| + |\alpha_{11}|)^2 + (|\alpha_{12}|)^2 < \frac{|\alpha_{00}|^2}{3}$$

is satisfied, hence the 2-D system is BIBO stable.

Example 2:

Consider the 2-D discrete system with order (2, 2) and the characteristic polynomial is given by

$$A(z_1, z_2) = 2z_1 + z_2 + z_1^2 + z_1z_2 + z_2^2 + z_1^2z_2 + 2z_1z_2^2 + 3z_1^2z_2^2 + 13$$

with

$\alpha_{01} = 1, \alpha_{10} = 2, \alpha_{02} = 1, \alpha_{11} = 1, \alpha_{20} = 1, \alpha_{12} = 2, \alpha_{21} = 1, \alpha_{22} = 3, \alpha_{00} = 13$

The inequality

$$(|\alpha_{01}| + |\alpha_{10}|)^2 + (|\alpha_{02}| + |\alpha_{11}| + |\alpha_{20}|)^2 + (|\alpha_{12}| + |\alpha_{21}|)^2 + (|\alpha_{22}|)^2 < \frac{|\alpha_{00}|^2}{4}$$

is satisfied, hence the 2-D system is BIBO stable.

V. CONCLUSION

A new sufficient stability criterion for linear shift invariant 2-D discrete systems has been presented. This criterion offers a computational efficient alternative to the existing sufficient conditions. Using the proposed sufficient criterion, the stability checking of the 2-D discrete systems can be carried out rapidly. This has been illustrated by two detailed examples.

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