IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received April 13, 2021, accepted April 25, 2021, date of publication May 6, 2021, date of current version May 17, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3078140

Selective Maintenance on a Multi-State Transportation System Considering Maintenance Sequence Arrangement

YAO SUN AND ZHILI SUN
School of Mechanical Engineering and Automation, Northeastern University, Shenyang 110819, China
Corresponding author: Yao Sun (18842501748@163.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 51775097.

ABSTRACT The mission for a type of transportation system is to complete a certain transportation volume within a predetermined period, during which maintenance is also performed. In such a system, any unit selected to be maintained is often connected to the system to participate in the system operation immediately after its maintenance, and its participation in the system operation at different moments has different effects. Based on this consideration, this paper proposes a new selective maintenance model for a multi-state system wherein the maintenance sequence arrangement is considered. In this selective maintenance problem, to maximise the system reliability, a sub-set of all desired maintenance activities is selected, and the sequence of the selected maintenance activities to be performed is arranged, with the transportation volume requirement and the limited budget considered. An ant colony optimisation algorithm is tailored for the resulting optimisation problem. Two example analyses are presented to illustrate the effectiveness of the tailored ant colony optimisation algorithm and the effect of the proposed selective maintenance. The results show that the tailored ant colony optimisation algorithm is effective for the proposed selective maintenance model. This selective maintenance can significantly improve the system reliability when the predetermined period is relatively short, and this maintenance effect of improving the system reliability gradually weakens as the predetermined period becomes longer. Increasing the budget and decreasing the transportation volume requirement can slow this weakening of the maintenance effect.

INDEX TERMS Ant colony optimisation algorithm, maintenance sequence arrangement, multi-state system, selective maintenance.

I. INTRODUCTION

The inevitable deterioration and even failure of engineering systems during their operation make maintenance increasingly important to improve system reliability and prolong system life [1]. Based on this consideration, various maintenance optimisation models have been investigated, aiming at maximising system reliability or minimising maintenance costs [2], [3]. In some industrial environments, not all desired maintenance activities can be completed due to limited resources. A sub-set of the desired maintenance activities is therefore selected to be performed to meet the relevant requirements. This is termed selective maintenance [4].

When selective maintenance was originally proposed, it was applied to a parallel-series system with only the replacement policy being considered [5]. Subsequently, a general framework for selective maintenance was built [6]. Furthermore, selective maintenance was applied to a fleet for improving the mission reliability by combining simulation and optimisation [7] Heuristic methods for selective maintenance problems were investigated and the best methods in various situations were evaluated [8]. In addition, selective maintenance was applied to a manufacturing system, aiming at minimising maintenance cost and production loss within limited maintenance time [9].

The systems investigated in the afore-mentioned works are all binary-state (perfect function or complete failure). In practice, however, engineering systems can work at some intermediate states between perfect function and complete failure, exhibiting a multi-state property [10]. Such systems are called multi-state systems (MSSs). In the last decade, selective maintenance with respect to MSSs has been

The associate editor coordinating the review of this manuscript and approving it for publication was R. K. Saket.

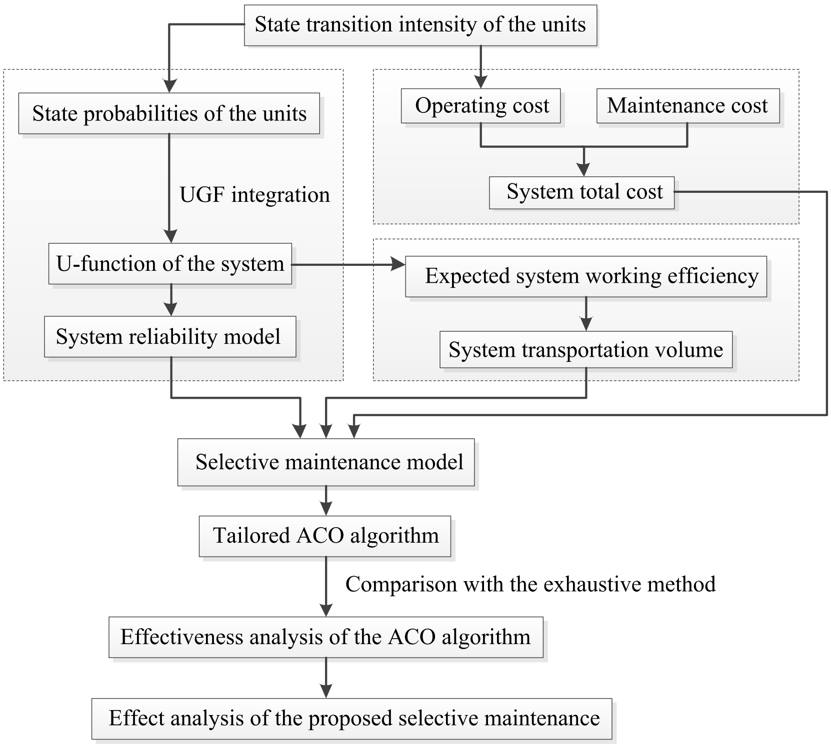

FIGURE 1. Workflow of this study.

studied extensively. The selective maintenance models of MSSs composed of binary-state components and multi-state components were proposed [4], [11]. Three types of dependence were respectively considered in the selective maintenance of MSSs [12]–[14]. The load distribution problem has been considered in selective maintenance modelling. Static loads were optimally distributed for MSSs under a selective maintenance strategy [15]. Based on this, a selective maintenance strategy for MSSs with dynamic load distribution was studied [16].

In most of the previous studies on selective maintenance, missions and breaks alternate, and the maintenance is performed only during breaks and needs to ensure the system operation during missions.

Consider a type of transportation MSS with a mission to complete a certain transportation volume within a predetermined period, while maintenance is also performed during this period. Considering that there is only one maintainer who cannot maintain more than one unit simultaneously, the units selected to be maintained should be maintained in sequence. The transportation volume accumulating as early as possible tends to increase its probability of meeting the requirement. Based on this, all the units not being selected for maintenance are connected to the system to participate in the system operation at the beginning of the predetermined period. The sequenced maintenance of the selected units also starts at the beginning of the predetermined period, and each of them, immediately after maintenance, is connected to the system to participate in the system operation.

Different units participating in the system operation at different moments will result in different effects on the system operation, and the consumable resources are limited. Therefore, in this study, on the premise that the transportation volume meets its requirement, a sub-set of all desired maintenance activities is selected and the sequence of these selected maintenance activities to be performed is arranged, with the aim of maximising the system reliability within the limited resources. This is also a type of selective maintenance, and the decision variable is a maintenance sequence.

The cost of engineering systems considered in this study comprises of the maintenance cost and the operating cost [17]. The operating cost is modelled based on the Markov reward process [18]–[20].

The main contributions of this study are as follows:

1) The selective maintenance model for a type of transportation MSS is first built, in which the maintenance sequence arrangement is considered.

2) To address the computational infeasibility of the exhaustive method when the system scale is large, the ant colony optimisation (ACO) algorithm is tailored for the proposed selective maintenance problem.

3) The operating cost is calculated as a part of the total cost, in contrast to the current studies where only the maintenance cost is considered.

The remainder of this paper is organised as follows: Section 2 describes the studied MSS and its working process. The selective maintenance modelling is detailed in Section 3. Sections 3.1, 3.2, and 3.3, respectively, describe

TABLE 1. Basic parameters of units in the three-unit MSS.

Unit i	λ_{10}^i	λ_{20}^i	λ_{21}^i	g_0^i	g_1^i	g_2^i	$r_i(1,0)$	$r_i(1)$	$r_i(2,0)$	$r_i(2,1)$	$r_i(2)$
1	0.005	0.0015	0.003	0	20	40	0.3	0.5	0.7	0.5	0.7
2	0.008	0.006	0.012	0	15	30	0.4	0.6	0.6	0.4	0.8
3	0.02	0.012	0.016	0	40	70	0.2	0.3	0.4	0.3	0.5

the modelling process of the reliability, total cost and transportation quantity of the system. Section 3.4 describes the resulting optimisation model. Section 4 details the tailored ACO algorithm. Section 5 presents two example analyses to show the effectiveness of the tailored ACO algorithm and the effect of the proposed selective maintenance. Finally, Section 6 gives the conclusions and outlines future work. The workflow is displayed in Fig. 1.

II. PROBLEM STATEMENTS

A. SYSTEM DESCRIPTION

The structure of the MSS investigated in this study is illustrated in Fig. 2. There are m sub-systems connected in series, and sub-system j ($j \in \{1, \ldots, m\}$) has x_j units connected in parallel. The number of all units is denoted as $n = \sum_{j=1}^{m} x_j$, and the units are numbered $1, 2, \ldots, n$ in sequence.

Unit $i (i \in \{1, \ldots, n\})$ possesses $K_i + 1$ states defined as $0, 1, \ldots, K_i$, which improve from 0 to K_i. States 0 and K_i indicate complete failure and perfect function, respectively. The respective working efficiencies are termed $g_0^i, g_1^i, \ldots, g_{K_i}^i$. The state and working efficiency of unit i are denoted as Y_i and g^i, respectively; $Y_i \in \{0, 1, \ldots, K_i\}$ and $g^i \in \{g_0^i, g_1^i, \ldots, g_{K_i}^i\}$.

For the MSS, with the working efficiency denoted as G, the structure function is expressed as follows:

$$G = \varphi\left(g^1, g^2, \ldots, g^n\right) = \min\left(\sum_{j_1=1}^{x_1} g^{j_1}, \sum_{j_2=x_1+1}^{x_1+x_2} g^{j_2}, \ldots, \sum_{j_m=n-x_m+1}^{n} g^{j_m}\right) \quad (1)$$

Based on the structure function, all possible working efficiencies can be obtained and they are denoted as $G_0, G_1, \ldots, G_Q$, which improve from G_0 to G_Q.

B. WORKING PROCESS

The duration of the predetermined period required to complete the mission is denoted as T, and the time from the beginning of the predetermined period is denoted as t ($t \in [0, T]$). All the units not selected to be maintained are connected to the system at $t = 0$ and start operating immediately. For the selected units, they are maintained in sequence, and the sequential maintenance starts at $t = 0$. For each selected unit, immediately after its maintenance, it is connected to the system and starts operating. The number of selected units is denoted as n', and the maintenance time of the kth ($k \in \{1, \ldots, n'\}$) unit is denoted as T_k; thus, $\sum_{k=1}^{n'} T_k \leq T$. An example of the working process is illustrated in Fig. 3.

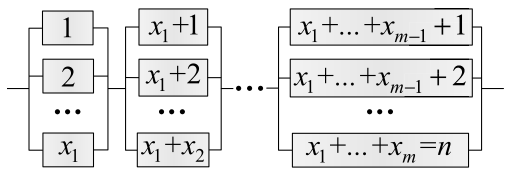

FIGURE 2. System structure block diagram.

TABLE 2. Maintenance parameters of units in the three-unit MSS.

Unit i	$c_i(0,1)$	$c_i(0,2)$	$c_i(1,2)$	$t_i(0,1)$	$t_i(0,2)$	$t_i(1,2)$
1	10	16	12	10	18	14
2	4	10	6	7	15	10
3	6	14	8	12	30	17

As shown in Fig. 3, units 1, 3, 5 and 7 are not selected to be maintained. They are connected to the system at $t = 0$ and participate in the system operation during $[0, T]$. Units 4, 6 and 2 are selected to be maintained. They are maintained in sequence and their maintenance time is T_1, T_2 and T_3, respectively. Therefore, unit 4 is connected to the system at $t = T_1$, and when $t = T_1 + T_2$, unit 6 is connected to the system; whereas for unit 2, it is not connected to the system until $t = T_1 + T_2 + T_3$. Then, they participate in the system operation during $[T_1, T]$, $[T_1 + T_2, T]$ and $[T_1 + T_2 + T_3, T]$, respectively.

Additionally, for unit i ($i \in \{1, \ldots, n\}$), its state at $t = 0$ is denoted as Y_i^{B}, and at the end of its maintenance, its state is denoted as Y_i^{A}. In particular, if unit i is not selected to be maintained, Y_i^{A} is set to be equal to Y_i^{B}. Thus, if unit i is selected to be maintained, $Y_i^{\mathrm{A}} > Y_i^{\mathrm{B}}$; otherwise, $Y_i^{\mathrm{A}} = Y_i^{\mathrm{B}}$.

III. SELECTIVE MAINTENANCE MODELLING

A. SYSTEM RELIABILITY

1) STATE PROBABILITIES OF UNITS

It is assumed that the stochastic degenerative behaviour of each unit follows a homogeneous Markov process with continuous time and discrete states [4], [14], [21]–[23]. The transition intensity matrix of unit i ($i \in \{1, \ldots, n\}$) is expressed as follows:

$$\boldsymbol{E}_i = \begin{bmatrix} \lambda_{00}^i & \lambda_{01}^i & \cdots & \lambda_{0K_i}^i \\ \lambda_{10}^i & \lambda_{11}^i & \cdots & \lambda_{1K_i}^i \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{K_i0}^i & \lambda_{K_i1}^i & \cdots & \lambda_{K_iK_i}^i \end{bmatrix} \quad (2)$$

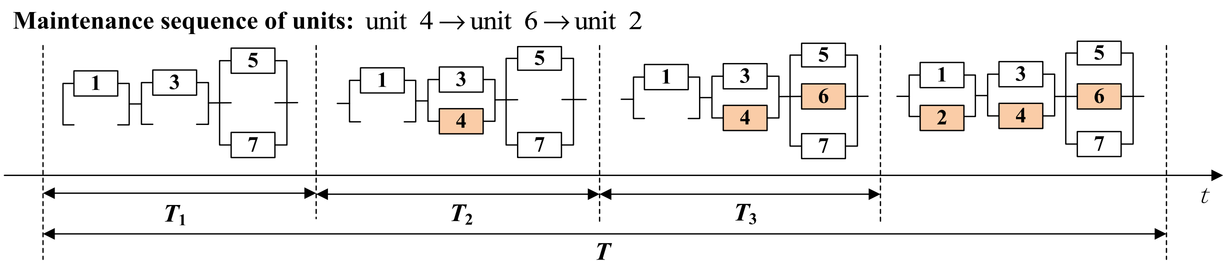

FIGURE 3. **Working process.**

where $\lambda_{ab}^{i}\,(a, b = 0, 1, \ldots, K_i)$ represents the transition rate from state a to b. It is noteworthy that $\lambda_{ab}^{i} = 0\,(a < b)$, for the reason that the state of a unit does not increase during degradation. Additionally, based on the property of homogeneous Markov process, it can be known that $\lambda_{aa}^{i} = -\left(\lambda_{a0}^{i} + \ldots + \lambda_{aa-1}^{i}\right)(a = 1, \ldots, K_i)$.

With the initial state d ($d \in \{0, 1, \ldots, K_i\}$) and operating time t', the state probabilities of unit i are denoted as a vector $\boldsymbol{p}_i^d(t') = \left(p_i^d(t')_0, p_i^d(t')_1, \ldots, p_i^d(t')_{K_i}\right)^{\mathrm{T}}$, which can be obtained by solving (3) [22]:

$$\begin{cases} \dfrac{\mathrm{d}\boldsymbol{p}_i^d(t')}{\mathrm{d}t'} = \boldsymbol{E}_i^{\mathrm{T}} \cdot \boldsymbol{p}_i^d(t') \\ \sum\limits_{u=1}^{K_i} p_i^d(t')_u = 1 \\ p_i^d(0)_d = 1, p_i^d(0)_u = 0\,(u \neq d) \end{cases} \tag{3}$$

where $p_i^d(t')_u\,(u \in \{0, 1, \ldots, K_i\})$ represents the probability of state u with the initial state d and operating time t'.

The state probabilities of unit i ($i \in \{1, \ldots, n\}$) at time t are denoted as a vector $\boldsymbol{P}_i(t) = \left(P_i(t)_0, P_i(t)_1, \ldots, P_i(t)_{K_i}\right)^{\mathrm{T}}$, where $P_i(t)_u\,(u \in \{0, 1, \ldots, K_i\})$ represents the probability of state u at time t. If unit i is not selected to be maintained, it will start operating at $t = 0$ with the initial state Y_i^{B}; otherwise, it will start operating immediately after its maintenance with the initial state Y_i^{A}. Therefore, $\boldsymbol{P}_i(t)$ is as follows:

$$\boldsymbol{P}_i(t) = \begin{cases} \left(p_i^{Y_i^{\mathrm{B}}}(t)_0, p_i^{Y_i^{\mathrm{B}}}(t)_1, \ldots, p_i^{Y_i^{\mathrm{B}}}(t)_{K_i}\right)^{\mathrm{T}}, Y_i^{\mathrm{A}} = Y_i^{\mathrm{B}} \\ (1, 0, \ldots, 0)^{\mathrm{T}}, Y_i^{\mathrm{A}} > Y_i^{\mathrm{B}}, t \leq \sum\limits_{k=1}^{wi} T_k \\ \left(p_i^{Y_i^{\mathrm{A}}}\left(t - \sum\limits_{k=1}^{wi} T_k\right)_0, p_i^{Y_i^{\mathrm{A}}}\left(t - \sum\limits_{k=1}^{wi} T_k\right)_1, \right. \\ \left. \ldots, p_i^{Y_i^{\mathrm{A}}}\left(t - \sum\limits_{k=1}^{wi} T_k\right)_{K_i}\right)^{\mathrm{T}}, Y_i^{\mathrm{A}} > Y_i^{\mathrm{B}}, t > \sum\limits_{k=1}^{wi} T_k \end{cases} \tag{4}$$

where wi represents the position of the maintenance activity of unit i in the maintenance sequence.

2) SYSTEM RELIABILITY

Based on (4), the u-function of unit $i (i \in \{1, \ldots, n\})$ is expressed as follows [24]:

$$u_i(t) = \sum_{l=0}^{K_i} P_i(t)_l\, z^{g_l^i} \tag{5}$$

Then, the u-function of the system is calculated as follows:

$$\begin{aligned} u(t) &= \otimes_{\varphi}(u_1(t), u_1(t), \ldots, u_n(t)) \\ &= \sum_{l_1=0}^{K_1}\sum_{l_2=0}^{K_2}\cdots\sum_{l_n=0}^{K_n} P_1(t)_{l_1} P_2(t)_{l_2} \ldots P_n(t)_{l_n} \\ &\quad \times z^{\varphi\left(g_{l_1}^1, g_{l_2}^2, \ldots, g_{l_n}^n\right)} \\ &= \sum_{q=0}^{Q} P^q(t) z^{G_q} \end{aligned} \tag{6}$$

where $\otimes_{\varphi}$ is the combination operator. G_q and $P^q(t)$ represent the qth state and the relevant state probability, respectively.

A system is often expected to perform as many missions as possible throughout its service life; thus, there is a requirement to mitigate system ageing. As for a mission, the mitigation of system ageing can be transformed to a requirement with respect to the probability of $G(T) \geq W$, in which W is the lowest allowable value of G given according to experience. Therefore, the probability of $G(T) \geq W$ is the system reliability for the mission. An indicative function of G is defined as in (7):

$$I(G) = \begin{cases} 1, & G \geq W \\ 0, & G < W \end{cases} \tag{7}$$

Based on (7), the system reliability for the mission is calculated as in (8):

$$R_{\mathrm{S}}(T) = P\{G(T) \geq W\} = \sum_{q=0}^{Q} P^q(T) I(G_q) \tag{8}$$

B. SYSTEM TOTAL COST

The maintenance activity that restores unit i ($i \in \{1, \ldots, n\}$) from state Y_i^{B} to $j\left(j \in \left\{Y_i^{\mathrm{B}} + 1, \ldots, K_i\right\}\right)$ is denoted as $Y_i^{\mathrm{B}} \rightarrow j$, and the position of $Y_i^{\mathrm{B}} \rightarrow j$ in a maintenance sequence can be represented by a binary decision variable that is defined as follows:

$$H_{w,i}\left(Y_i^{\mathrm{B}},j\right)=\begin{cases}1, & \text{the } w\text{th maintenance activity is } Y_i^{\mathrm{B}}\to j\\ 0, & \text{otherwise}\end{cases}\tag{9}$$

The maintenance cost of unit i from state a to $b(0\le a<b\le K_i)$ is denoted as $c_i(a,b)$. Because the dependence of units is not considered in this study, the maintenance cost of the system for the mission is expressed as follows:

$$C_{\mathrm{MS}}=\sum_{w=1}^{n}\sum_{i=1}^{n}\sum_{j=Y_i^{\mathrm{B}}+1}^{K_i}H_{w,i}\left(Y_i^{\mathrm{B}},j\right)\cdot c_i\left(Y_i^{\mathrm{B}},j\right)\tag{10}$$

For unit i, the sojourn at state a $(a\in\{0,1,\ldots,K_i\})$ leads to a cost per unit time to keep it operating, which is defined as $r_i(a)$; and the cost associated with the state transition from state a to b is defined as $r_i(a,b)$ $(a,b\in\{0,1,\ldots,K_i\})$. Based on this, a Markov reward model can be built for unit i to calculate its operating cost [22], with the Markov reward matrix of unit i expressed as follows:

$$\boldsymbol{R}_i=\begin{bmatrix} r_i(0) & r_i(0,1) & \ldots & r_i(0,K_i)\\ r_i(1,0) & r_i(1) & \ldots & r_i(1,K_i)\\ \ldots & \ldots & \ldots & \ldots\\ r_i(K_i,0) & r_i(K_i,1) & \ldots & r_i(K_i)\end{bmatrix}\tag{11}$$

Generally, a completely failed unit does not incur any operational cost; thus, $r_i(0)=0$. Moreover, it is impossible for a unit to reach a higher state during a degenerative process; thus, $r_i(a,b)$ $(a<b)$ will be set to 0, which will not affect the calculation of the operating cost. However, for a unit, the state decrease during its operation can lead to degenerative damage, which cannot be completely eliminated by repairing it to the original state. Therefore, there exists a depreciation cost with respect to the state decrease of a unit; that is, $r_i(a,b)$ $(a>b)$.

With the initial state d $(d\in\{0,1,\ldots,K_i\})$ and operating time t', the expected accumulative operating cost of unit i is denoted as $v_i^d\left(t'\right)$, which can be obtained by solving (12) [22]:

$$\begin{cases}\dfrac{\mathrm{d}\boldsymbol{v}_i\left(t'\right)}{\mathrm{d}t'}=\boldsymbol{u}_i+\boldsymbol{E}_i\cdot\boldsymbol{v}_i\left(t'\right)\\ v_i^d(0)=0\,(d=0,\ldots,K_i)\end{cases}\tag{12}$$

where vector $\boldsymbol{v}_i\left(t'\right)=\left(v_i^0\left(t'\right),v_i^1\left(t'\right),\ldots,v_i^{K_i}\left(t'\right)\right)^{\mathrm{T}}$ and $\boldsymbol{u}_i=\left(u_0^i,u_1^i,\ldots,u_{K_i}^i\right)^{\mathrm{T}}$, in which $u_a^i=r_i(a)+\sum_{b=0,b\neq a}^{K_i}\lambda_{ab}^i r_i(a,b)$ $(a=0,1,\ldots,K_i)$.

The expected accumulative operating cost of unit i up to time t is denoted as $V_L^i(t)$. If unit i is not selected, its operating cost will accumulate from $t=0$ with the initial state Y_i^{B}; otherwise, its operating cost will accumulate after its maintenance with the initial state Y_i^{A}. Thus, $V_L^i(t)$ is as follows:

$$V_L^i(t)=\begin{cases}v_i^{Y_i^{\mathrm{B}}}(t), & Y_i^{\mathrm{A}}=Y_i^{\mathrm{B}}\\ 0,\ Y_i^{\mathrm{A}}>Y_i^{\mathrm{B}}, & t\le\sum_{k=1}^{wi}T_k\\ v_i^{Y_i^{\mathrm{A}}}\left(t-\sum_{k=1}^{wi}T_k\right), & Y_i^{\mathrm{A}}>Y_i^{\mathrm{B}},\ t>\sum_{k=1}^{wi}T_k\end{cases}\tag{13}$$

Given that the dependence of units is not considered in this study, the accumulative operating cost of the system denoted as C_{LS} is the sum of the accumulative operating costs of all units. Thus, the expected accumulative operating cost of the system up to time t is expressed as follows:

$$E\left(C_{\mathrm{LS}}\right)(t)=\sum_{i=1}^{n}V_L^i(t)\tag{14}$$

Therefore, the expected accumulative operating cost of the system for the mission is computed as follows:

$$E\left(C_{\mathrm{LS}}\right)(T)=\sum_{i=1}^{n}V_L^i(T)\tag{15}$$

In this study, the total cost is composed of the maintenance cost and the operating cost. Thus, based on (10) and (15), the expected accumulative total cost of the system for the mission is computed as:

$$\begin{aligned}E\left(C_{\mathrm{S}}\right)(T)&=C_{\mathrm{MS}}+E\left(C_{\mathrm{LS}}\right)(T)\\&=\sum_{w=1}^{n}\sum_{i=1}^{n}\sum_{j=Y_i^{\mathrm{B}}+1}^{K_i}H_{w,j}\left(Y_i^{\mathrm{B}},j\right)\cdot c_i\left(Y_i^{\mathrm{B}},j\right)\\&\quad+\sum_{i=1}^{n}V_{\mathrm{L}}^i(T)\end{aligned}\tag{16}$$

C. SYSTEM TRANSPORTATION VOLUME

For the system, the decrease of its working efficiency is stochastic because of the stochastic degeneration of each unit, and the expectation of the working efficiency at time t is expressed as follows:

$$E(G)(t)=\sum_{q=0}^{Q}P^q(t)G_q\tag{17}$$

Regarding the transportation system, the working efficiency is typically defined as the transportation volume per unit time. Therefore, according to the property of mathematical expectation, its expected accumulative transportation quantity up to time t is calculated as follows:

$$E\left(O_{\mathrm{S}}\right)(t)=\int_0^t E(G)(t)\,dt$$

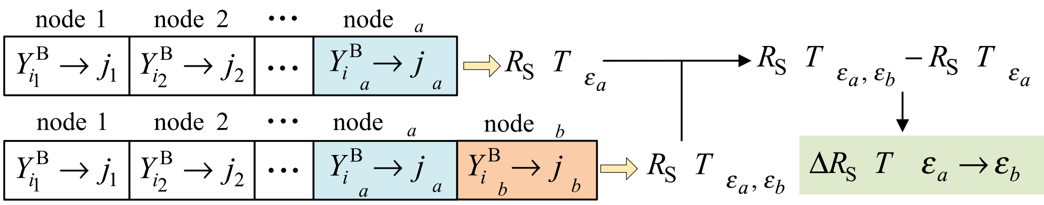

FIGURE 4. Increment of the system reliability corresponding to a maintenance sequence.

$$= \begin{cases} \int_0^t \sum_{q=1}^{Q} P^q(t)\cdot G_q dt, & w' = 0 \\ 0, & w' \geq 1, t \leq \sum_{k=1}^{w'} T_k \\ \int_{\sum_{k=1}^{w'} T_k}^{t} \sum_{q=1}^{Q} P^q(t)\cdot G_q dt, & w' \geq 1, t > \sum_{k=1}^{w'} T_k \end{cases} \tag{18}$$

Based on (18), the expected accumulative transportation quantity of the system for the mission is as follows:

$$\begin{aligned} E(O_\mathrm{S})(T) &= \int_0^T E(G)(t)dt \\ &= \begin{cases} \int_0^T \sum_{q=1}^{Q} P^q(t)\cdot G_q dt, w' = 0 \\ \int_{\sum_{k=1}^{w'} T_k}^{T} \sum_{q=1}^{Q} P^q(t)\cdot G_q dt, w' \geq 1 \end{cases} \end{aligned} \tag{19}$$

D. SELECTIVE MAINTENANCE MODEL

The maintenance time of unit $i\,(i \in \{1,\ldots,n\})$ from state a to b is denoted as $t_i(a,b)$. Then, for a maintenance sequence, the maintenance time of the wth $(w = 1,\ldots,n')$ maintenance activity is expressed as follows:

$$T_w = \sum_{i=1}^{n} \sum_{j=Y_i^\mathrm{B}+1}^{K_i} H_{w,i}\left(Y_i^\mathrm{B}, j\right)\cdot t_i\left(Y_i^\mathrm{B}, j\right) \tag{20}$$

Based on (20), the total maintenance time is calculated as follows:

$$\sum_{w=1}^{n'} T_w = \sum_{w=1}^{n'} \sum_{i=1}^{n} \sum_{j=Y_i^\mathrm{B}+1}^{K_i} H_{w,i}\left(Y_i^\mathrm{B}, j\right)\cdot t_i\left(Y_i^\mathrm{B}, j\right) \tag{21}$$

It is known that the system aging should be mitigated as much as possible. From this perspective, maximising the system reliability should be the objective. In addition, the transportation volume should meet its requirement and the consumable resources are limited, which should be regarded as constraints. Thus, the selective maintenance optimisation model is expressed as in (22) and (23).

$$\mathrm{Max}\ R_\mathrm{S}(T) \tag{22}$$

$$s.t. \begin{cases} E(C_\mathrm{S})(T) \leq C' & \text{(a)} \\ \sum_{w=1}^{n'} T_w \leq T & \text{(b)} \\ E(O_\mathrm{S})(T) \geq O'_\mathrm{S} & \text{(c)} \\ \sum_{w=1}^{n} \sum_{j=Y_i^\mathrm{B}+1}^{K_i} H_{w,i}\left(Y_i^\mathrm{B}, j\right) \leq 1\ (i = 1,\ldots,n) & \text{(d)} \\ \sum_{i=1}^{n} \sum_{j=Y_i^\mathrm{B}+1}^{K_i} H_{w,i}\left(Y_i^\mathrm{B}, j\right) \leq 1\ (w = 1,\ldots,n) & \text{(e)} \\ H_{w,i}\left(Y_i^\mathrm{B}, j\right) = 1 \text{ or } 0\ (w = 1,\ldots,n; i = 1,\ldots,n) & \text{(f)} \end{cases} \tag{23}$$

where C' and O'_S are the budget and the transportation volume requirement, respectively, and the decision variable is a maintenance sequence composed of optional maintenance activities.

IV. TAILORED ACO ALGORITHM

Without considering constraints (23)(a) and (b), the number of solutions in the search space is as follows:

$$1 + \sum_{n'=1}^{n} \left[\sum_{dd=1}^{\frac{n!}{(n-n')!}} \prod_{i \in \mathbf{D}_{dd}} \left(K_i - Y_i^\mathrm{B}\right) \right] \tag{24}$$

where $\frac{n!}{(n-n')!}$ is the n' permutation of n, and $\mathbf{D}_{dd}$ is the set of selected units for the ddth permutation.

It is observed from (24) that the search space grows rapidly with the increase of n and $K_i - Y_i^\mathrm{B}\ (i = 1,\ldots,n)$. Thus, if the system scale is relatively large, it is not computationally feasible to exhaust all possible solutions. However, the maintenance sequence arrangement is very similar to the process in the ACO algorithm where ants visit multiple nodes in sequence. Based on this, the ACO algorithm is tailored for the maintenance optimisation model described in Section III.D, with the constraints and the infeasible solutions addressed by the construction of a tabu list.

The description of the tailored ACO algorithm is as follows:

A. CONSTRUCTION OF THE SOLUTION

Each optional maintenance activity is regarded as an accessible node, and each ant has a tabu list to store its inaccessible nodes.

```
Step 1: Initialization
        ite ← 0 ;
        Correspond optional maintenance activities to nodes (ε_1 ~ ε_BY) ;
        τ(ε_a,ε_b) ← 1(ε_a,ε_b = 1,...,BY; ε_a ≠ ε_b); τ(ε_a,ε_a) ← 0(ε_a = 1,...,BY) ;
Step 2: Put each ant on its initial node.
        ite ← ite + 1 ;
        Δτ(ε_a,ε_b) ← 0(ε_a,ε_b = 1,...,BY; ε_a ≠ ε_b) ;
Step 3: For np=1 to NP do
        | Tabu ← ∅ ;
        | Add the initial node denoted as ε_c1 and the nodes whose units are the same as ε_c1 to Tabu ;
        | C′ ← C′ − c(ε_c1) ; T′ ← T′ − t(ε_c1) ;
        | Add the nodes whose c(ε_•) is greater than C′ or t(ε_•) is greater than T′ to Tabu ;
        | For inn=2 to n do
        |  | If C′ > 0 ⋂ T′ > 0 ⋂ There exist nodes not being added into Tabu. Then
        |  |  | Choose the next node denoted as ε_cnn according to Eq.(25), i.e., P(ε_cnn−1, ε_cnn) ;
        |  |  | Add ε_cnn and the nodes whose units are the same as ε_cnn to Tabu ;
        |  |  | Ant np move from ε_cnn−1 to ε_cnn ;
        |  |  | C′ ← C′ − c(ε_cnn) ; T′ ← T′ − t(ε_c1) ;
        |  |  | Add the nodes whose c(ε_•) is greater than C′ or t(ε_•) is greater than T′ to Tabu ;
        |  | Else
        |  |  | Go to Step 3 ;
        |  | End if
        | End for
      End for
Step 4: For np=1 to NP do
        | Evaluate E(O_S)(T) of the path passed by ant np according to Eq. (19) ;
        | If E(O_S)(T) ≥ O_S′ Then
        |  | Evaluate R_S(T) of the path passed by ant np according to Eq. (8) ;
        | End if
        | Obtain Δτ^np(ε_a,ε_b)_ite (ε_a,ε_b = 1,...,BY; ε_a ≠ ε_b) according to Eq.(27) ;
      End for
      Obtain τ(ε_a,ε_b)_ite+1 (ε_a,ε_b = 1,...,BY; ε_a ≠ ε_b) according to Eq.(26) ;
      R_S(T)_ite ← max(R_S(T)) ;
      R_S(T)_max ← max(R_S(T)_1, R_S(T)_2, ..., R_S(T)_ite) ;
Step 5: If ite ≤ G_m Then
      | Go to Step 2 ;
      End if

Annotations:
c(ε_a) : Total cost of node ε_a
t(ε_a) : Maintenance time of node ε_a
R_S(T)_ite : Maximum R_S(T) of the iteth iteration
G_m : Maximum iteration
```

FIGURE 5. Pseudocode of the tailored ACO algorithm.

For an ant, when it passes through node $Y_i^{\mathrm{B}} \rightarrow j\left(i \in \{1, \ldots, n\}, j \in \left\{Y_i^{\mathrm{B}}+1, \ldots, K_i\right\}\right)$, all nodes of unit i will be put into its tabu list. Thus, it will not visit the nodes of units that have been maintained; i.e., the infeasible solutions that do not satisfy constraints (23)(d) and (e) can be automatically avoided. To handle constraints (23)(a) and (b), nodes causing $E(C_S)(T)$ to exceed C' or $\sum_{w=1}^{n'} T_w$ to exceed T will also be put into the tabu list when the ant passes through a node. Once all the accessible nodes are placed in its tabu list, the ant stops moving. Therefore, a path passed through by an ant is a feasible solution.

B. MOVEMENT PROBABILITY

An ant selects the next node according to the pheromone and the heuristic information [21]. There are multiple methods to integrate the two elements. In this study, the probability of moving from node ε_a to ε_b is defined as follows [25], [26]:

$$P(\varepsilon_a, \varepsilon_b) = \begin{cases} \dfrac{[\tau(\varepsilon_a, \varepsilon_b)]^{\alpha} [\eta(\varepsilon_a, \varepsilon_b)]^{\beta}}{\sum\limits_{\varepsilon_c \notin \mathbf{Tabu}} [\tau(\varepsilon_a, \varepsilon_c)]^{\alpha} [\eta(\varepsilon_a, \varepsilon_c)]^{\beta}}, & \varepsilon_b \notin \mathbf{Tabu} \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

where **Tabu** is the tabu list. $\tau(\varepsilon_a, \varepsilon_b)$ represents the quantity of pheromone on the directed edge from node ε_a to ε_b denoted by $\varepsilon_a \rightarrow \varepsilon_b$, whereas $\eta(\varepsilon_a, \varepsilon_b)$ represents the heuristic information of moving from node ε_a to ε_b. In addition, α and β reflect the randomness and the certainty during path search, respectively.

Because the objective is to maximise $R_S(T)$, the effectiveness of the next node is quantified by the increment of $R_S(T)$. The $R_S(T)$ corresponding to a maintenance sequence ending with ε_a is denoted as $R_S(T)_{\varepsilon_a}$. If ε_b is added after ε_a, the $R_S(T)$ will be denoted as $R_S(T)_{\varepsilon_a,\varepsilon_b}$. Based on this, the increment of $R_S(T)$ denoted by $\Delta R_S(T)(\varepsilon_a \rightarrow \varepsilon_b)$ is illustrated in Fig. 4.

It is assumed that an ant is currently at node ε_a. If there exists a node ε ($\varepsilon \notin$ **Tabu**) satisfying $\Delta R_S(T)(\varepsilon_a \rightarrow \varepsilon) > 0$, then, $\forall \varepsilon_b \notin$ **Tabu**,

$$\eta(\varepsilon_a, \varepsilon_b) = \begin{cases} \Delta R_S(T)(\varepsilon_a \rightarrow \varepsilon_b), & \Delta R_S(T)(\varepsilon_a \rightarrow \varepsilon_b) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

otherwise, $\eta(\varepsilon_a, \varepsilon_b)$ ($\varepsilon_b \notin$ **Tabu**) will be set as identical constants. This may slow the optimisation process; however, it can be maximally offset by the pheromone.

C. PHEROMONE UPDATING RULES

The number of optional maintenance activities is denoted as $BY = \sum_{i=1}^{n} K_i - Y_i^{\mathrm{B}}.\tau(\varepsilon_a, \varepsilon_b)(\varepsilon_a, \varepsilon_b = 1, \ldots, BY; \varepsilon_a \neq \varepsilon_b)$ will be updated at the end of each iteration, which is expressed as follows:

$$\tau(\varepsilon_a, \varepsilon_b)_{ite+1} = (1-\rho)\,\tau(\varepsilon_a, \varepsilon_b)_{ite} + \sum_{np=1}^{NP} \Delta\tau^{np}(\varepsilon_a, \varepsilon_b)_{ite} \tag{27}$$

where $\tau(\varepsilon_a, \varepsilon_b)_{ite}$ is the quantity of pheromone on $\varepsilon_a \rightarrow \varepsilon_b$ in the iteth iteration, and ρ ($0 < \rho < 1$) is the evaporation rate of the pheromone. NP is the number of ants in each iteration. Further, $\Delta\tau^{np}(\varepsilon_a, \varepsilon_b)_{ite}$ represents the quantity of pheromone left on $\varepsilon_a \rightarrow \varepsilon_b$ by the npth ant in the iteth iteration, which is defined as:

$$\Delta\tau^{np}(\varepsilon_a, \varepsilon_b)_{ite} = \begin{cases} \omega \cdot R_S(T)_{(np,ite)}, & (\varepsilon_a \rightarrow \varepsilon_b) \in \text{path passed by the } np\text{th ant in the } ite\text{th iteration} \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

where ω is a positive constant, and $R_S(T)_{(np,ite)}$ is the $R_S(T)$ corresponding to the maintenance sequence represented by the path passed by the npth ant in the iteth iteration. Additionally, the initial quantity of pheromone on each edge is set as follows:

$$\tau(\varepsilon_a, \varepsilon_b)_0 = \begin{cases} 1, & \varepsilon_a \neq \varepsilon_b \\ 0, & \varepsilon_a = \varepsilon_b \end{cases} \tag{29}$$

Based on Steps 1-3, the search space can be explored throughout as much as possible.

D. PSEUDO CODE OF THE TAILORED ACO ALGORITHM

The pseudocode of the tailored ACO algorithm is shown in Fig. 5.

V. EXAMPLE ANALYSIS

In this section, a simple three-unit system is used to demonstrate the effectiveness of the tailored ACO algorithm. The effect of the selective maintenance proposed in this study is analysed in terms of coping with a larger scale system.

A. EFFECTIVENESS ANALYSIS OF THE TAILORED ACO ALGORITHM

A simple three-unit MSS is used to illustrate the effectiveness of the tailored ACO algorithm, with its structure shown in Fig. 6. Each unit possesses three possible states (0, 1, 2), and the parameters of each unit are listed in Tables 1 and 2. The initial states of the three units are $Y_1^{\mathrm{B}} = 0$, $Y_2^{\mathrm{B}} = 1$, $Y_3^{\mathrm{B}} = 0$, respectively, and the optional maintenance activities are numbered in Table 3.

TABLE 3. Optional maintenance activities of the three-unit MSS.

Unit i	Maintenance activity——number
1	$Y_1^{\mathrm{B}} \rightarrow 1$——1 $Y_1^{\mathrm{B}} \rightarrow 2$——2
2	$Y_2^{\mathrm{B}} \rightarrow 2$——3
3	$Y_3^{\mathrm{B}} \rightarrow 1$——4 $Y_3^{\mathrm{B}} \rightarrow 2$——5

To demonstrate the impact of the sequence in which the maintenance activities are performed, the cases where all the units are replaced sequentially are investigated. The results are shown in Fig. 7.

It is known that the maintenance cost of the six maintenance strategies are equal, and the same is true for the maintenance time. However, as shown in Fig. 7, the sequence in which the maintenance activities are performed has a significant impact on system reliability, operating cost, and transportation volume. For example, a later replacement of unit 3 brings higher reliability, but this also leads to a

TABLE 4. Basic parameters of pipelines in the oil transportation system.

Pipeline i	K_i	g (kilotons per day)				λ					
		g_0^i	g_1^i	g_2^i	g_3^i	λ_{10}^i	λ_{20}^i	λ_{21}^i	λ_{30}^i	λ_{31}^i	λ_{32}^i
1	3	0	30	50	80	0.2	0.08	0.12	0.03	0.1	0.2
2	3	0	30	60	90	0.16	0.08	0.15	0.02	0.1	0.18
3	3	0	20	60	80	0.23	0.15	0.2	0.08	0.1	0.12
4	2	0	80	120	-	0.09	0.04	0.08	-	-	-
5	2	0	70	130	-	0.1	0.08	0.12	-	-	-
6	3	0	50	80	100	0.18	0.12	0.15	0.04	0.12	0.14
7	3	0	30	60	80	0.1	0.06	0.09	0.05	0.08	0.13
8	3	0	20	50	80	0.08	0.05	0.12	0.02	0.06	0.15
9	3	0	30	70	120	0.045	0.03	0.045	0.02	0.04	0.06
10	3	0	50	80	120	0.09	0.06	0.09	0.03	0.075	0.12
11	3	0	10	40	80	0.08	0.05	0.07	0.02	0.035	0.06
12	3	0	20	50	70	0.16	0.12	0.14	0.04	0.08	0.2
13	3	0	10	30	50	0.12	0.07	0.13	0.06	0.1	0.17
14	3	0	10	30	60	0.05	0.035	0.04	0.02	0.03	0.045

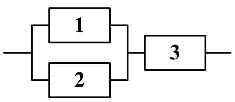

FIGURE 6. Structure block diagram of the three-unit MSS.

higher operating cost. The transportation volumes of the two maintenance strategies in which unit 3 is replaced last are lower than those of the other four maintenance strategies. Therefore, it is necessary to optimise the maintenance sequence. Based on this, effectiveness analysis of the tailored ACO algorithm is requisite.

Under three gradually strict constraint conditions, the tailored ACO algorithm is compared with the exhaustive method. The results are shown in Fig. 8.

Owing to the simple system structure, the number of ants and iterations are set to be relatively small; that is, $NP = 3$ and $G_{\rm m} = 10$, which can better reflect the ability of the tailored ACO algorithm. As shown in Fig. 8, under each of the three constraint conditions, with the iteration increases, $R_{\rm S}(T)$ initially increases, until it reaches the theoretical maximum $R_{\rm S}(T)$ obtained by the exhaustive method; then it remains unchanged at this theoretical maximum $R_{\rm S}(T)$. The increase of $R_{\rm S}(T)$ when the iteration is small demonstrates that the probabilistic search based on the pheromone quantity and the heuristic information are effective. Moreover, $R_{\rm S}(T)$ can reach the theoretical maximum value, indicating that the search space has the opportunity to be explored thoroughly.

Therefore, with the reasonable selection of the number of ants NP and the maximum iteration $G_{\rm m}$, the tailored ACO algorithm is effective for the proposed selective maintenance model.

B. EFFECT ANALYSIS OF THE PROPOSED SELECTIVE MAINTENANCE

An oil transportation system is used as an example to investigate the effect of the selective maintenance. Its structure is shown in Fig. 9. It is designed to transport oil from an oil tanker to some carriages.

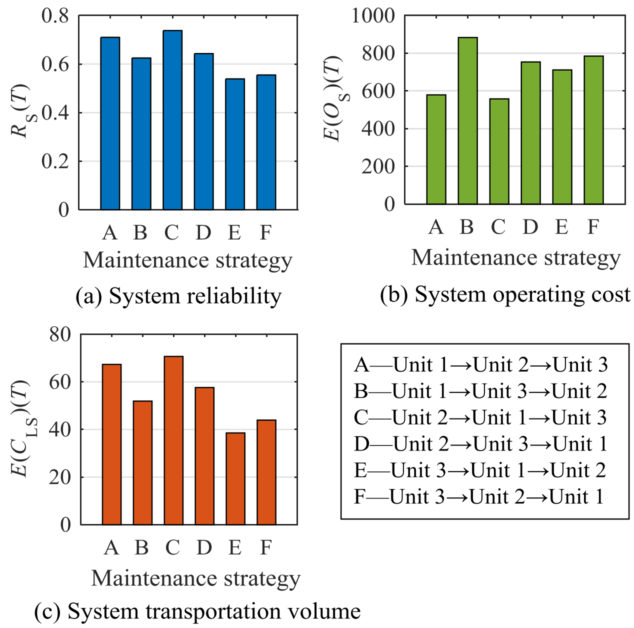

FIGURE 7. Impact of the sequence in which the maintenance activities are performed (W = 35, T = 70).

The oil tanker is connected to the carriages through five pipeline groups connected in series. Each pipeline group consists of several pipelines of the same type in parallel. Additionally, each pipeline is equipped with a pump to drive the oil flow according to the current state of the pipeline. Thus, a certain amount of electricity should be supplied per unit of time. This is the operating cost per unit time of the pipeline in the current state. The parameters of the pipelines are listed in Tables 4 and 5.

The mission of the oil transportation system is to transport a certain volume of oil within a predetermined period.

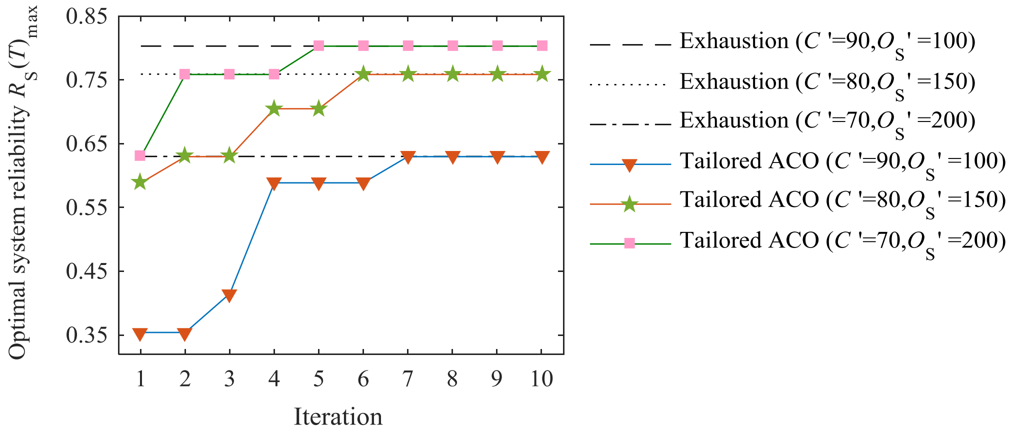

FIGURE 8. Iterative processes of the tailored ACO algorithm ($W = 30$, $T = 55$).

TABLE 5. Parameters of pipelines related with consumable resources in the oil transportation system.

Pipeline i		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Maintenance cost (USD)	c_i (0,1)	25	30	35	20	25	30	25	30	25	20	30	20	30	20
	c_i (0,2)	45	40	50	35	45	40	40	40	40	35	45	30	40	35
	c_i (0,3)	50	55	60	-	-	55	50	45	50	45	60	40	50	55
	c_i (1,2)	35	35	45	35	45	35	20	35	35	25	35	25	35	30
	c_i (1,3)	50	55	60	-	-	55	50	45	50	45	60	40	50	55
	c_i (2,3)	50	55	60	-	-	55	50	45	50	45	60	40	50	55
Maintenance time (day)	t_i (0,1)	0.3	0.4	0.3	0.2	0.3	0.4	0.3	0.5	0.4	0.3	0.2	0.5	0.4	0.2
	t_i (0,2)	0.5	0.6	0.6	1.1	0.6	0.5	0.5	2	0.6	0.5	0.4	1	1.8	0.4
	t_i (0,3)	2.5	2.6	0.7	-	-	0.6	0.7	2.1	0.7	0.6	0.9	2.8	2	0.8
	t_i (1,2)	0.4	0.3	0.4	1.1	0.6	0.3	0.2	1.8	0.3	0.3	0.3	0.8	1.6	0.3
	t_i (1,3)	2.5	2.6	0.7	-	-	0.6	0.7	2.1	0.7	0.6	0.9	2.8	2	0.8
	t_i (2,3)	2.5	2.6	0.7	-	-	0.6	0.7	2.1	0.7	0.6	0.9	2.8	2	0.8
Operating cost (USD)	r_i (1,0)	6	6	5	2	1	9	2	7	1	1.5	2	4	5	2
	r_i (1)	5	8	4	2.5	1.2	6	2	4	1.5	2	4	7	4	4
	r_i (2,0)	8	8	7	2.5	1.4	12	3	10	2	4	4	7	8	4
	r_i (2,1)	4	5	4	1.5	0.8	10	1	6	1.5	2.5	3	4	6	3
	r_i (2)	7	14	8	3.5	2.4	8	3	8	2	6	5.5	10	7	6
	r_i (3,0)	9	12	10	-	-	15	4	14	2.5	6	7.5	13	15	8
	r_i (3,1)	6	9	8	-	-	12	3.5	12	2	4	6.5	10	12	6.5
	r_i (3,2)	3	4	6	-	-	7	2	9	1	3	4	8	7	5
	r_i (3)	10	16	11	-	-	11	4	14	2.5	7	7	15	18	8

TABLE 6. Initial states of pipelines and optional maintenance activities of the oil transportation system.

Pipeline i	Y_i^B	Maintenance activity——number	Pipeline i	Y_i^B	Maintenance activity——number
1	1	$Y_1^B \to 2$ ——1 $Y_1^B \to 3$ ——2	8	1	$Y_8^B \to 2$ ——9 $Y_8^B \to 3$ ——10
2	2	$Y_2^B \to 3$ ——3	9	1	$Y_9^B \to 2$ ——11 $Y_9^B \to 3$ ——12
3	0	$Y_3^B \to 1$ ——4 $Y_3^B \to 2$ ——5 $Y_3^B \to 3$ ——6	10	2	-
4	1	$Y_4^B \to 2$ ——7	11	1	$Y_{11}^B \to 2$ ——13 $Y_{11}^B \to 3$ ——14
5	1	$Y_5^B \to 2$ ——8	12	2	$Y_{12}^B \to 3$ ——15
6	3	-	13	1	$Y_{13}^B \to 2$ ——16 $Y_{13}^B \to 3$ ——17
7	3	-	14	2	$Y_{14}^B \to 3$ ——18

The initial states of the pipelines and the optional maintenance activities are shown in Table. 6.

Based on the system design, the pipelines can be connected to the oil transportation system during the system operation.

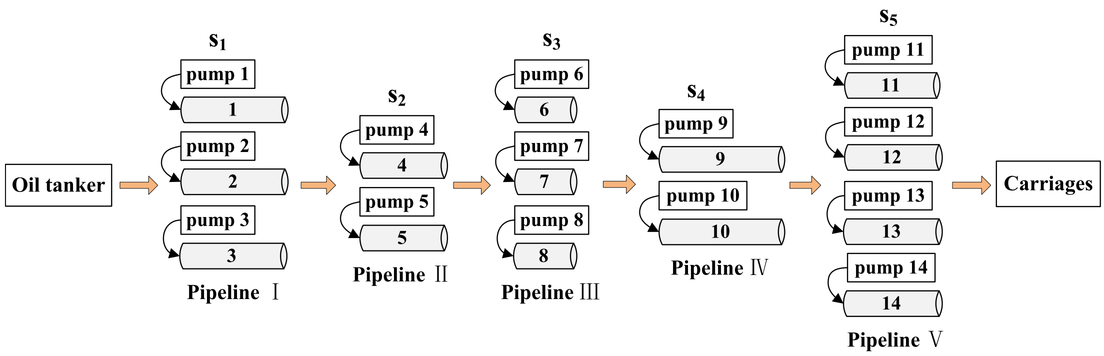

FIGURE 9. Structure of the oil transportation system.

Thus, the oil transportation system is consistent with the proposed selective maintenance model.

The demand W is set to 20 kilotons per day, and the parameters of the tailored ACO algorithm are listed in Table 7.

The sensitivity of the optimisation result to the constraints is shown in Fig. 10, which is used to illustrate the effect of the variation of the proposed selective maintenance.

As shown in Fig. 10, $R_S(T)_{max}$ in cases A and B is visibly higher than that of case A, indicating that the proposed selective maintenance can significantly improve the system reliability.

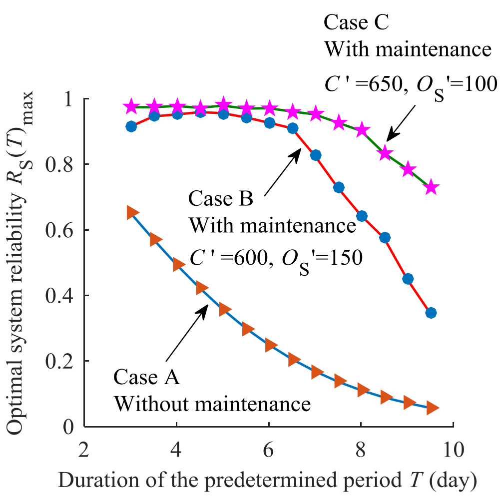

FIGURE 10. Sensitivity of the optimisation result to the constraints.

Fig. 10 also shows that in case B, as T increases, $R_S(T)_{max}$ initially increases until $T = 4.5$, after which it decreases and decreases more significantly after $T = 6.5$. This reflects the dual impacts of T on $R_S(T)_{max}$, which is explained as follows. First, the increase in T allows more pipelines to be maintained, thus increasing $R_S(T)_{max}$ (impact 1). Second, the increase in T also prolongs the operating time of the system, which counteracts the effect of maintenance and thus decreases $R_S(T)_{max}$ (impact 2). When T is relatively small, its increase improves $R_S(T)_{max}$, indicating that impact 1 is dominant when T is relatively small. When T is relatively large, its increase leads to the decrease in $R_S(T)_{max}$, indicating that impact 2 transforms to be dominant. Regarding case C, a similar scenario is observed, except that when T is relatively small, $R_S(T)_{max}$ is high and remains almost unchanged with the increase in T owing to the relaxation of the constraints. In other words, impacts 1 and 2 of T are almost equivalent when T is relatively small, and when T is relatively large, impact 2 transforms to be dominant.

TABLE 7. Parameters of the tailored ACO algorithm.

α	β	ω	NP	G_m
3	3	100	50	100

In summary, as the predetermined period gradually becomes longer, its impact 1 weakens and its impact 2 strengthens. Thus, the proposed selective maintenance can significantly improve the system reliability if the predetermined period is relatively short. However, this effect of the proposed selective maintenance gradually weakens as the duration of the predetermined period increases owing to the increasing dominance of impact 2.

As shown in Fig. 10, for a fixed T, $R_S(T)_{max}$ in case C is higher than that in case B. This indicates that the increase in budget and the reduction in transportation volume requirement (i.e. the relaxation of the constraints) can slow the weakening of the effect of the proposed selective maintenance.

The optimisation processes of several randomly selected constraints are shown in Fig. 11.

As shown in Fig. 11, $R_S(T)_{max}$ increases with the iterative process until it reaches a certain value, after which it remains unchanged. $R_S(T)_{max}$ reaches that certain value before the 30*th* iteration in the cases in Figs. 11(b), (c), (e) and (f). For each of the cases in Figs. 11 (a) and (d), although $R_S(T)_{max}$ reaches the specified certain value at approximately the 50*th* iteration, the increment of last increase in $R_S(T)_{max}$ is small. Therefore, the iteration processes in Fig. 11 also reflect the effectiveness of the ACO algorithm. Thus, the maximum value $R_S(T)_{max}$ reaches can be regarded as the maximum reliability that the system can achieve.

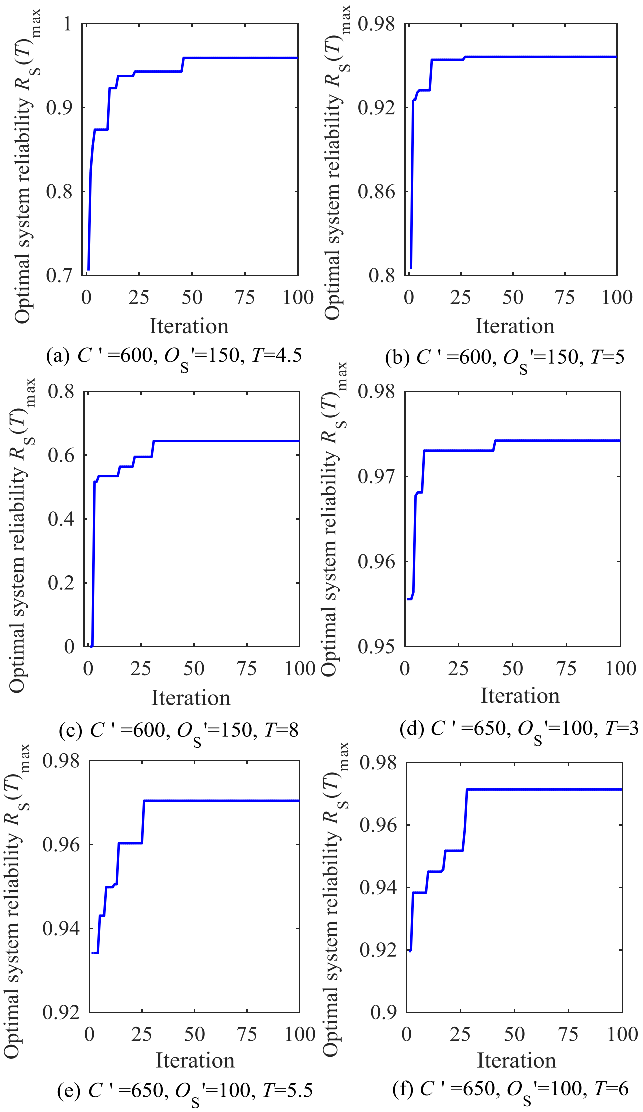

FIGURE 11. Optimisation process.

VI. CONCLUSION AND FUTURE WORKS

A selective maintenance model for a type of transportation MSS is studied in which the maintenance sequence arrangement is considered. Thereafter, the ACO algorithm is tailored for this selective maintenance problem. Using two example analyses, the effectiveness of the tailored ACO algorithm and the effect of the proposed selective maintenance are illustrated. The following conclusions can be drawn:

(1) The comparison with the exhaustive method and the analysis of the iterative process demonstrate that the probabilistic search based on the pheromone quantity and the heuristic information is effective and the search space can be explored thoroughly. Thus, the tailored ACO algorithm is effective for the proposed selective maintenance problem.

(2) The predetermined period required to complete the mission has dual impacts on the maximum system reliability the system can achieve. First, the increase in T can provide more opportunities for pipeline maintenance, which can increase $R_S(T)_{max}$ (impact 1). Second, the increase in T also leads to a longer operating time of the system, which counteracts the effect of maintenance and thus decreases $R_S(T)_{max}$ (impact 2).

(3) As the predetermined period gradually becomes longer, its impact 1 weakens and its impact 2 strengthens. Thus, the proposed selective maintenance can significantly improve the system reliability when the predetermined period is relatively short. However, this effect of the proposed selective maintenance gradually weakens as the predetermined period becomes longer owing to the increasing dominance of impact 2. The increase in budget and the reduction in transportation volume requirement can slow the weakening of the effect of the proposed selective maintenance.

Nevertheless, it is assumed that there is only one maintainer in this study. If there are multiple maintainers, maintenance activities can be performed in a parallel mode. In the future, this situation will be considered in this selective maintenance model. The proposed model only applies to one MSS. Meanwhile, a type of MSS combination is quite practical and is characterized by a redundancy [23]. Therefore, this selective maintenance model will be developed for this type of MSS combination.

REFERENCES

[1] M. C. A. Olde Keizer, S. D. P. Flapper, and R. H. Teunter, "Condition-based maintenance policies for systems with multiple dependent components: A review," *Eur. J. Oper. Res.*, vol. 261, no. 2, pp. 405–420, Sep. 2017.

[2] S. Alaswad and Y. Xiang, "A review on condition-based maintenance optimization models for stochastically deteriorating system," *Rel. Eng. Syst. Saf.*, vol. 157, pp. 54–63, Jan. 2017.

[3] H. Wang, "A survey of maintenance policies of deteriorating systems," *Eur. J. Oper. Res.*, vol. 139, no. 3, pp. 469–489, Jun. 2002.

[4] M. Pandey, M. J. Zuo, and R. Moghaddass, "Selective maintenance modeling for a multistate system with multistate components under imperfect maintenance," *IIE Trans.*, vol. 45, no. 11, pp. 1221–1234, Nov. 2013.

[5] W. F. Rice, C. R. Cassady, and J. A. Nachlas, "Optimal maintenance plans under limited maintenance time," in *Proc. 7th Ind. Eng. Res. Conf. (IeRC)*, Banff, BC, Canada, 1998, pp. 1–3.

[6] C. R. Cassady, E. A. Pohl, and W. P. Murdock, "Selective maintenance modeling for industrial systems," *J. Qual. Maintenance Eng.*, vol. 7, no. 2, pp. 104–117, Jun. 2001.

[7] K. Schneider and C. R. Cassady, "Fleet performance under selective maintenance," in *Proc. Annu. Symp. Rel. Maintainability (RAMS)*, Los Angeles, CA, USA, 2004, pp. 571–576.

[8] T. Lust, O. Roux, and F. Riane, "Exact and heuristic methods for the selective maintenance problem," *Eur. J. Oper. Res.*, vol. 197, no. 3, pp. 1166–1177, Sep. 2009.

[9] H. Zhu, F. Liu, X. Shao, Q. Liu, and Y. Deng, "A cost-based selective maintenance decision-making method for machining line," *Qual. Rel. Eng. Int.*, vol. 27, no. 2, pp. 191–201, Mar. 2011.

[10] G. Levitin, "Introduction to multi-state systems," in *The Universal Generating Function in Reliability Analysis and Optimization*, 1st ed. London, U.K.: Springer-Verlag, 2005, pp. 67–72.

[11] Y. Liu and H.-Z. Huang, "Optimal selective maintenance strategy for multi-state systems under imperfect maintenance," *IEEE Trans. Rel.*, vol. 59, no. 2, pp. 356–367, Jun. 2010.

[12] C. D. Dao, M. J. Zuo, and M. Pandey, "Selective maintenance for multi-state series–parallel systems under economic dependence," *Rel. Eng. Syst. Saf.*, vol. 121, pp. 240–249, Jan. 2014.

[13] C. D. Dao and M. J. Zuo, "Selective maintenance of multi-state systems with structural dependence," *Rel. Eng. Syst. Saf.*, vol. 159, pp. 184–195, Mar. 2017.

[14] C. D. Dao and M. J. Zuo, "Selective maintenance for multistate series systems with S-dependent components," *IEEE Trans. Rel.*, vol. 65, no. 2, pp. 525–539, Jun. 2016.
[15] C. Chen, Y. Liu, and H.-Z. Huang, "Optimal load distribution for multi-state systems under selective maintenance strategy," in *Proc. Int. Conf. Qual., Rel., Risk, Maintenance, Saf. Eng. (ICQR MSE)*, Chengdu, China, Jun. 2012, pp. 436–442.
[16] C. D. Dao and M. J. Zuo, "Optimal selective maintenance for multi-state systems in variable loading conditions," *Rel. Eng. Syst. Saf.*, vol. 166, pp. 171–180, Oct. 2017.
[17] A. Lisnianski, I. Frenkel, L. Khvatskin, and Y. Ding, "Maintenance contract assessment for aging systems," *Qual. Rel. Eng. Int.*, vol. 24, no. 5, pp. 519–531, Aug. 2008.
[18] A. Lisnianski, I. Frenkel, L. Khvatskin, and Y. Ding, "Multistate system reliability assessment by using the Markov reward model," in *Statistical Models and Methods for Biomedical and Technical Systems, Statistics for Industry and Technology*. Boston, MA, USA: Birkhäuser, 2008, pp. 153–168.
[19] A. Lisnianski, "The Markov reward model for a multi-state system reliability assessment with variable demand," *Qual. Technol. Quant. Manage.*, vol. 4, no. 2, pp. 265–278, Jan. 2007.
[20] S. Toledano, I. Gartsman, G. Avitan, I. Frenkel, and L. Khvatskin, "On Markov reward approach to failure criticality importance assessment for aging multi-state system," in *Proc. 2nd Int. Symp. Stochastic Models Rel. Eng., Life Sci. Oper. Manage. (SMRLO)*, Beer Sheva, Israel, 2016, pp. 375–379.
[21] Y. Liu, Y. Chen, and T. Jiang, "On sequence planning for selective maintenance of multi-state systems under stochastic maintenance durations," *Eur. J. Oper. Res.*, vol. 268, no. 1, pp. 113–127, Jul. 2018.
[22] Y. F. Li and R. Peng, "Availability modeling and optimization of dynamic multi-state series–parallel systems with random reconfiguration," *Rel. Eng. Syst. Saf.*, vol. 127, pp. 47–57, Jul. 2014.
[23] A. Lisnianski and Y. Ding, "Redundancy analysis for repairable multi-state system by using combined stochastic processes methods and universal generating function technique," *Rel. Eng. Syst. Saf.*, vol. 94, no. 11, pp. 1788–1795, Nov. 2009.
[24] Y. Liu and H.-Z. Huang, "Optimal replacement policy for multi-state system under imperfect maintenance," *IEEE Trans. Rel.*, vol. 59, no. 3, pp. 483–495, Sep. 2010.
[25] Y.-C. Liang and A. E. Smith, "An ant colony optimization algorithm for the redundancy allocation problem (RAP)," *IEEE Trans. Rel.*, vol. 53, no. 3, pp. 417–423, Sep. 2004.
[26] D. Merkle, M. Middendorf, and H. Schmeck, "Ant colony optimization for resource-constrained project scheduling," *IEEE Trans. Evol. Comput.*, vol. 6, no. 4, pp. 333–346, Aug. 2002.

YAO SUN was born in Liaoning, China, in 1993. She received the M.S. degree in mechanical engineering from Northeastern University, Shenyang, China, in 2018, where she is currently pursuing the Ph.D. degree with the School of Mechanical Engineering and Automation.

Her research interests include motion reliability analysis, system reliability evaluation, and maintenance decision.

ZHILI SUN was born in Shandong, China, in 1957. He received the B.S., M.S., and Ph.D. degrees from the School of Mechanical Engineering and Automation, Northeastern University, Shenyang, China, in 1982, 1985, and 1997, respectively.

Since 1985, he has been a Teacher with Northeastern University, where he became a Professor, in 1998. His research interests include mechanical reliability engineering, tribology design, structural design, and fault diagnosis.