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Mixed-Effects Nonhomogeneous Poisson Process Model for Multiple Repairable Systems

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ABSTRACT The nonhomogeneous Poisson process (NHPP) has become a useful approach for modeling failure patterns of recurrent failure data revealed by minimal repairs from an individual repairable system. Sometimes, multiple repairable systems may present system-to-system variability owing to operation environments or working intensities of individual systems. In this paper, we go over the application of generalized mixed-effects models to recurrent failure data from multiple repairable systems based on the NHPP. The generalized mixed-effects models explicitly involve between-system variation through random-effects, along with a common baseline for all the systems through fixed-effects for non-normal data. Details on estimation of the parameters of the mixed-effects NHPP models and construction of their confidence intervals are examined. An applicative example shows prominent proof of the mixed-effects NHPP models for the purpose of reliability analysis.

INDEX TERMS Empirical Bayes, minimal repair, power law process, random-effects model, reliability analysis.

I. INTRODUCTION

Modern systems consist of numerous parts working together, making the maintenance action for the systems more difficult. In general, systems can be classified into repairable and non-repairable systems according to feasibility of maintenance activity. A repairable system is one that can be restored to an operating condition without replacement of the entire system after some repair activity is executed. For the repairable system, the patterns of failure collected after successive repairs are of fundamental importance to establish an effective maintenance policy. For example, increasing time intervals between failures suggest reliability improvement. Conversely, decreasing time intervals imply reliability deterioration. To model the patterns of recurrent failure data, stochastic point processes are commonly employed. Specifically, the nonhomogeneous Poisson process (NHPP) has garnered significant attention in the reliability literature [1], [2].

Occasionally, multiple repairable systems may present system-to-system variability due to changes in operating environments and working intensities of individual systems. In this case, it may be more reasonable to assume a heterogeneity among all the systems. To take the heterogeneity among systems into account, Bayesian methods (both empirical and hierarchical) have been applied to multiple repairable systems due to their flexibility in accounting for parameter uncertainty and allowing the incorporation of a prior knowledge into the process under study (see, e.g., Hamada *et al.* [3]; Reese *et al.* [4]; Arab *et al.* [5]). System heterogeneity may be described via the prior distributions of the model parameters, however, there may also be homogeneity between individual systems. This homogeneity can be explicitly modeled by assuming common parameters in the Bayesian model. If prior distributions are unnecessarily assigned to the common parameters, the prior information employed to the common parameters can make the parameter estimation procedure more complicated. The computational complexity and the difficulty in choosing proper prior distributions have been obstacles for reliability engineers who wish to apply Bayesian methods to such practical reliability problems.

In this article, we will go over the application of mixed-effects models for recurrent failure data from multiple

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repairable systems for the purpose of reliability analysis. The mixed-effects model, which is also called a "random-effects model", allows explicit modeling and analysis of between-individual and within-individual variation, along with a common baseline for all the individuals. In the formation of a mixed-effects model, the probability distributions for multiple observations are each generally assumed to be normal. If a more flexible class of models for non-normal data involving both fixed and random effects is appropriate, a generalized mixed-effects model can be a useful tool for such purposes.

We aim to provide flexible applications of the generalized mixed-effects model for the reliability analysis of multiple repairable systems, based mainly on the NHPP. In biostatistics research, there have been a few studies on the application of the generalized mixed-effects models to Poisson process data. Lawless [6] introduced the random-effects model for mammary tumor data following the Poisson process. Cooil [7] studied Poisson process models with random effects to predict the malpractice claims filed against individual physicians. However, the application of the generalized mixed-effects model seems not to be emphasized in evaluating the reliability of repairable systems. Recently, Tan *et al.* [8] proposed a generalized linear mixed model (GLMM) for the power law process to analyze recurrent failure data from multiple repairable generators. Giorgio *et al.* [9] proposed a regression model for the PLP intensity function of the power train system of a fleet of 33 buses employed in urban or suburban service. They modeled the unobservable heterogeneity among the buses by assuming prior distributions for the model parameters in a Bayesian framework. To our knowledge, applications of nonlinear mixed-effects Poisson process models to multiple repairable systems, however, have been seldom observed in the reliability literature.

The rest of this paper is organized as follows. In Section II, we illustrate several NHPP models with monotonic failure intensity functions in reliability analysis of repairable systems. In Sections III and IV, a generalized mixed-effects model is presented to model recurrent failure data of multiple repairable systems. We also examine issues regarding parameter estimation for the mixed-effects NHPP models and confidence interval construction. Recurrent failure data collected from repairable systems are analyzed in Section V. Analytical results from several mixed-effects NHPP models are compared with corresponding individually fitted NHPP models in terms of parameter estimation and its precision. We conclude in Section VI with a discussion on future research.

II. NONHOMOGENEOUS POISSON PROCESS MODEL

NHPPs represent a broad class of models for failure data generated by repairable systems. The NHPP can provide a *minimal repair* model in which the occurrence of failures and subsequent repairs tend to have a negligible effect on overall system reliability, restoring the system performance to the exact same condition as it was just before the failure. The NHPP is defined by its nonnegative intensity function $\nu(t)$. The expected number of failures in the time interval $(0, t]$ is obtained by $\Lambda(t) = \int_0^t \nu(u)\,du$. The intensity function $\nu(t)$ is equal to the rate of occurrence of failures (ROCOF) associated with the repairable system [2]. When the intensity function is constant, i.e., $\nu(t) \equiv \nu$, the process reduces to a homogeneous Poisson process (HPP). The NHPP has been widely used in modeling failure frequency for repairable systems because of its flexibility and mathematical tractability via its intensity function $\nu(t)$ (Ascher and Feingold [1]; Krivtsov [10]).

The most commonly applied form of NHPP is the power law process (PLP). Crow [11] suggested a PLP model under "find it and fix it" conditions with the intensity function

$$\nu(t) = \frac{\vartheta}{\zeta}\left(\frac{t}{\zeta}\right)^{\vartheta-1}, \quad t > 0, \tag{1}$$

where $\vartheta(> 0)$ and $\zeta(> 0)$ are the shape and scale parameters, respectively. The corresponding mean cumulative number of failures over $(0, t]$ is $\Lambda(t) = (t/\zeta)^{\vartheta}$. As another functional form of NHPP, a log linear process (LLP) has intensity function

$$\nu(t) = \gamma e^{\kappa t}, \quad t > 0 \tag{2}$$

and the corresponding mean cumulative number of failures over $(0, t]$ is $\Lambda(t) = \gamma\kappa^{-1}(e^{\kappa t} - 1)$, for the parameters $\gamma(> 0)$ and κ. The LLP model was first proposed by Cox and Lewis [12] to model air conditioner failures. The PLP and LLP model have been employed to model failure patterns of a repairable system having monotonic intensity, *i*.e., decreasing failure patterns (reliability improvement) with $\vartheta < 1$ ($\kappa < 0$) or increasing failure patterns (reliability deterioration) with $\vartheta > 1$ ($\kappa > 0$). When $\vartheta = 1$ ($\kappa = 0$), the PLP (LLP) reduces to the HPP.

The intensity function of the PLP model tends to infinity as the system age increases, whereas the observed failure process may have a finitely bounded intensity function. Considering NHPPs with a finite and bounded intensity function, Pulcini [13] proposed a *bounded* intensity process (BIP) with intensity function

$$\nu(t) = a[1 - e^{-t/b}]. \quad a, b > 0; \ t > 0 \tag{3}$$

The intensity function is increasing and bounded, approaching an asymptote of a as t tends to infinity. Attardi and Pulcini [14] proposed a modified form of the BIP model (so-called 2-parameter Engelhardt & Bain process (2-EBP) model) to represent a compromise between the PLP and BIP models as

$$\nu(t) = \upsilon \cdot t/(t + \rho), \quad \upsilon, \rho > 0; \ t > 0. \tag{4}$$

The modified BIP model tends to reach at its asymptote more slowly than the BIP intensity (3). See [14] for the characteristics of the 2-EBP model including the physical meaning of its parameters in details.

III. GENERALIZED MIXED-EFFECTS MODEL

Sometimes, inter-individual (or inter-system) variability is substantial enough to warrant inclusion of an individual effect in the model. Lawless [6] refers to such effects as ''unobserved heterogeneity''. In the formulation of mixed-effects model, the unobserved heterogeneity has been explicitly incorporated into the model under study. Mixed-effects models are widely used in medical studies [15], [16] because they can model both between-individual and within-individual variation found in repeated-measurements data.

For many applications dealing with repeated measurements involve non-normal data (e.g., binary data, count data, categorical data), a class of generalized mixed-effects models have been introduced. A generalized mixed-effects model relates the conditional mean for the ith individual to the fixed- and random-effects. Given the vector of fixed-effects $\boldsymbol{\beta}$ and the vector of random-effects $\boldsymbol{b}_i$, which form the ith parameter vector $\boldsymbol{\beta}_i$ as $\boldsymbol{\beta}_i = \boldsymbol{A}_i\boldsymbol{\beta} + \boldsymbol{B}_i\boldsymbol{b}_i$, the response vector $\mathbf{y}_i$ for the ith individual has conditional mean and variance-covariance matrix

$$E(\mathbf{y}_i|\boldsymbol{b}_i) \equiv g^{-1}(\mathbf{x}_i; \boldsymbol{\beta}, \boldsymbol{b}_i) = \boldsymbol{\mu}_i(\boldsymbol{\beta}, \boldsymbol{b}_i),$$

and

$$\mathrm{Var}(\mathbf{y}_i|\boldsymbol{b}_i) = \boldsymbol{V}_i(\boldsymbol{\beta}, \boldsymbol{b}_i),$$

respectively, where g is a monotonic differentiable link function that relates the mean vector $\boldsymbol{\mu}_i$ of response vector $\mathbf{y}_i$ to a covariate vector $\mathbf{x}_i$. In general, the $(n_i \times 1)$ conditional mean vector, $\boldsymbol{\mu}_i(\boldsymbol{\beta}, \boldsymbol{b}_i)$, can be a linear (or nonlinear) function in both $\boldsymbol{\beta}$ and $\boldsymbol{b}_i$. The conditional response $\mathbf{y}_i|\boldsymbol{b}_i$ is assumed to have an exponential family member distribution. The $(k \times 1)$ random-effects $\boldsymbol{b}_i$ are assumed to be normally distributed with mean zero and variance-covariance matrix $\boldsymbol{D}$. The conditional variance-covariance matrix, $\boldsymbol{V}_i(\boldsymbol{\beta}, \boldsymbol{b}_i)$, which may also be dependent on the fixed- and random-effects parameters, is assumed to be positive-definite.

As a kind of generalized mixed-effects model, the generalized linear mixed-effects model (GLMM) is an extension of generalized linear model (GLM) which includes random-effects. The GLM extends the Gaussian-based linear model to the larger class of distributions in the exponential family. Through a linear predictor $\boldsymbol{\eta}_i = \mathbf{x}_i^T\boldsymbol{\beta}$, the GLM can be expressed as: $g(\boldsymbol{\mu}_i) \equiv \boldsymbol{\eta}_i = \mathbf{x}_i^T\boldsymbol{\beta}$. In the GLMM context, the linear predictor is given by $\boldsymbol{\eta}_i = \mathbf{x}_i^T\boldsymbol{\beta} + z_i^T\boldsymbol{b}_i$, where z_i is the $(n_i \times k)$ vector of random-effects associated with the ith individual. The general expression of the conditional mean is given by

$$E(\mathbf{y}_i|\boldsymbol{b}_i) \equiv g^{-1}(\boldsymbol{\eta}_i) = g^{-1}(\mathbf{x}_i^T\boldsymbol{\beta} + z_i^T\boldsymbol{b}_i).$$

When the response and random-effects are normally distributed, the marginal distribution of $\mathbf{y}_i$ is easily obtained by taking expectation and using variance operation (e.g., delta method) through the corresponding model. Due to the potential non-normality in the response, obtaining the marginal distribution of $\mathbf{y}_i$ is a more challenging task for a GLMM. There are three popular approaches for deriving the marginal likelihood in the GLMM settings [17]: (1) linearizing the conditional mean and then repeatedly applying mixed-effects model techniques to the linearized model; (2) using numerical methods to approximate the integrals involved in the marginal likelihood; (3) introducing a Bayesian approach. The linearizing methods include pseudo-likelihood and marginal quasi-likelihood for parameter estimation. Breslow and Lin [18] discussed bias correction problems in the GLMM model and limitations of the pseudo-likelihood and marginal quasi-likelihood methods.

Under the assumption that the errors in the generalized mixed-effects model are independent of the random-effects, the marginal distribution of the response can be obtained as

$$p(\mathbf{y}_i) = \int p(\mathbf{y}_i|\boldsymbol{b}_i)p(\boldsymbol{b}_i)\, d\boldsymbol{b}_i,$$

where $p(\mathbf{y}_i|\boldsymbol{b}_i)$ is the conditional distribution of $\mathbf{y}_i$ given the random-effects $\boldsymbol{b}_i$, and $p(\boldsymbol{b}_i)$ is the probability density of $\boldsymbol{b}_i$. In general, this integral does not have a closed-form expression and the linearization methods do not work with the marginal distribution directly. Instead, approximation methods such as Laplace approximation [19] and adaptive Gaussian quadrature [20] are used to numerically approximate the integral involved in the marginal likelihood.

IV. MIXED-EFFECTS NHPP MODEL

Suppose that there are m independent systems; the system i is observed over the time interval $(0, \tau_i)$ and n_i failures are observed to occur, at times $t_{i1} < \cdots < t_{in_i}$. For the parameters $\boldsymbol{\theta}$ of the NHPP, the likelihood function is

$$\mathcal{L}(\boldsymbol{\theta}) = \prod_{i=1}^{m}\left\{\prod_{j=1}^{n_i}\nu(t_{ij}; \boldsymbol{\theta})\right\}\exp\{-\Lambda(\tau_i; \boldsymbol{\theta})\}, \tag{5}$$

with failure intensity $\nu(\cdot)$ and its cumulative mean function $\Lambda(\cdot)$. By incorporating the inter-individual variation into the random effects $\boldsymbol{b}_i$ for ith system, along with fixed effects $\boldsymbol{\beta}$ (identical to all the systems), the conditional mean for a failure process of the ith system $\boldsymbol{t}_i = (t_{i1}, \ldots, t_{in_i})^T$ is $E[\boldsymbol{t}_i|\boldsymbol{b}_i] \equiv \boldsymbol{\mu}_i = \Lambda(\boldsymbol{t}_i|\boldsymbol{b}_i)$. The distribution of random effects $\boldsymbol{b}_i$ is the same over all the systems. The contribution to the likelihood function (5) having observed failures n_i at times t_{ij} for individual system i is

$$\mathcal{L}_i(\boldsymbol{\beta}) = \int_{\boldsymbol{b}_i}\left\{\prod_{j=1}^{n_i}\nu(t_{ij}|\boldsymbol{b}_i)\right\}\exp\{-\Lambda(\tau_i|\boldsymbol{b}_i)\}p(\boldsymbol{b}_i)\,\mathrm{d}\boldsymbol{b}_i.$$

The likelihood function with parameters $\boldsymbol{\beta}$ and $\boldsymbol{b}_i$ from the sample of m systems has the form

$$\mathcal{L}(\boldsymbol{\beta}) = \prod_{i=1}^{m}\int_{\boldsymbol{b}_i}\left\{\prod_{j=1}^{n_i}\nu(t_{ij}|\boldsymbol{b}_i)\right\}\exp\{-\Lambda(\tau_i|\boldsymbol{b}_i)\}p(\boldsymbol{b}_i)\,\mathrm{d}\boldsymbol{b}_i, \tag{6}$$

and maximizing the likelihood function (6) yields the maximum likelihood estimate (MLE) of $\boldsymbol{\beta}$, denoted by $\hat{\boldsymbol{\beta}}$.

Suppose that the failure process of each system follows the PLP, then the recurrent failure-times have the mean $\Lambda(t_{ij}) = (t_{ij}/\zeta_i)^{\vartheta_i}$, where ζ_i and ϑ_i are the scale and shape parameters for system i, respectively. With a functional form of η that relates the mean Λ of the failure times in the GLMM conxext, the mean function of PLP model can be expressed

$$\eta_{ij} \equiv \ln(\Lambda(t_{ij})) = \vartheta_i \left\{\ln(t_{ij}) - \ln(\zeta_i)\right\}, \tag{7}$$

and by re-parameterizing the log-linear function (7) as

$$\beta_{i1} = -\vartheta_i \ln(\zeta_i), \quad \beta_{i2} = \vartheta_i, \quad \text{and} \quad x_{ij} = \ln(t_{ij}),$$

we have $\eta_{ij} = \mathbf{x}_i^T \boldsymbol{\beta}_i$, for $\mathbf{x}_i = (1, x_{ij})^T$ and $\boldsymbol{\beta}_i = (\beta_{i1}, \beta_{i2})^T$. Reflecting the system-to-system variation into the parameters $\boldsymbol{\beta}_i$ as $\beta_{i1} = \beta_1 + b_{i1}$ and $\beta_{i2} = \beta_2 + b_{i2}$, where (b_{i1}, b_{i2}) are assumed to be bivariate normally distributed with mean $\mathbf{0}$ and variance $\boldsymbol{\Sigma}$, the GLMM for multiple repairable systems under the PLP is represented as

$$\eta_{ij} = \mathbf{x}_i^T \boldsymbol{\beta} + z_i^T \boldsymbol{b}_i, \quad \boldsymbol{b}_i \sim \mathcal{N}_2(\mathbf{0}, \boldsymbol{\Sigma}), \tag{8}$$

for $\boldsymbol{\beta} = (\beta_1, \beta_2)^T$ and $\boldsymbol{b}_i = (b_{i1}, b_{i2})^T$. The likelihood function for multivariate normally distributed $\boldsymbol{b}_i$ is

$$\mathcal{L}(\boldsymbol{\beta}) \approx \prod_{i=1}^{m} \int_{\boldsymbol{b}_i} \left\{ \prod_{j=1}^{n_i} \nu(t_{ij}|\boldsymbol{b}_i) \right\} \exp\{-\Lambda(\tau_i|\boldsymbol{b}_i)\} \times |\boldsymbol{\Sigma}|^{-1/2} \exp\left\{ -\frac{1}{2} \boldsymbol{b}_i^T \boldsymbol{\Sigma}^{-1} \boldsymbol{b}_i \right\} \, d\boldsymbol{b}_i.$$

In the nonlinear mixed-effects model (NLMM) context, the functional form of η relating the mean $\Lambda(t)$ of the failure-times may not exist. The NLMM is a further generalization of the GLMM. The NLMM permits more flexible variance-covariance structure for the parameters of the NLMM, as well as random errors. The likelihood function with the LLP failure intensity for multivariate normally distributed $\boldsymbol{b}_i \equiv (b_{i1}, b_{i2})^T$, for example, is

$$\mathcal{L}(\boldsymbol{\beta}) \approx \prod_{i=1}^{m} \int_{b_{i1}} \int_{b_{i2}} \left\{ \prod_{j=1}^{n_i} (\beta_1 + b_{i1}) e^{(\beta_2 + b_{i2}) t_{ij}} \right\} \times \exp\left\{ -\left(\frac{\beta_1 + b_{i1}}{\beta_2 + b_{i2}} \right) \left(e^{(\beta_2 + b_{i2}) t_{ij}} - 1 \right) \right\} |\boldsymbol{\Sigma}|^{-1/2} \times \exp\left(-\frac{1}{2} (b_{i1}, b_{i2})^T \boldsymbol{\Sigma}^{-1} (b_{i1}, b_{i2}) \right) db_{i1}\, db_{i2}.$$

A. ESTIMATION OF PARAMETERS IN MIXED-EFFECTS NHPP MODEL

When the mixed-effects model contains a single random-effect, it is relatively easy to evaluate the integral in the likelihood function. The likelihood function can be maximized numerically to find the ML estimates. In general, the integral calculations in the likelihood function (6) involve high-dimensional integration, and do not produce closed-form expressions, requiring numerical integration techniques to estimate the likelihood function. Bae and Kvam [21] introduced various approximation methods to numerically optimize the likelihood function from repeated-measured degradation data of vacuum fluorescent displays (VFDs) when the distribution of $\boldsymbol{b}_i$ is multivariate normal. SAS® NLMIXED procedure provides several approximation methods including adaptive Gaussian quadrature [20] and first-order method [22] for the mixed-effects model.

In the mixed-effects NHPP model, ML estimates of $\boldsymbol{\beta}$ are obtained by maximizing the likelihood function (6) numerically or using approximation methods (if necessary). The random-effects in the mixed-effects NHPP model are assumed to have normal distributions with zero means. Their specific values for a given individual are just realizations from the normal distributions. These random effects can be efficiently estimated using empirical Bayes methods [23]. Empirical Bayes methods are concerned first with estimating distributions from which random-effects have been generated. Once a distribution of random-effects has been estimated, this distribution is used to estimate the realized values of random-effects using Bayes' theorem. For the failure process of the ith system $\boldsymbol{t}_i$, empirical Bayes estimates of $\boldsymbol{b}_i$ (denoted by $\hat{\boldsymbol{b}}_i$) is given by the posterior mean of $\boldsymbol{b}_i$ as

$$\hat{\boldsymbol{b}}_i = E(\boldsymbol{b}_i|\boldsymbol{t}_i) = \frac{\int_{\boldsymbol{b}_i} \boldsymbol{b}_i\, p(\boldsymbol{t}_i|\boldsymbol{b}_i)\, p(\boldsymbol{b}_i)\, d\boldsymbol{b}_i}{\int_{\boldsymbol{b}_i} p(\boldsymbol{t}_i|\boldsymbol{b}_i)\, p(\boldsymbol{b}_i)\, d\boldsymbol{b}_i},$$

for the conditional probability function of $\boldsymbol{t}_i$ given $\boldsymbol{b}_i$, $p(\boldsymbol{t}_i|\boldsymbol{b}_i)$. If parametric assumptions on the distribution of random-effects are made, e.g., normal, then empirical Bayes methods are equivalent to best linear unbiased prediction (BLUP) methods [24].

B. CONSTRUCTING CONFIDENCE INTERVALS

Confidence intervals can be constructed for the parameters of the mixed-effects model or their functions based on standard errors derived from the (observed) Fisher information matrix. In generalized mixed-effects NHPP model without covariates, a large-sample approximation of standard errors of the ML estimators is given through the estimated variance-covariance matrix $\hat{\boldsymbol{\Xi}}_{\hat{\boldsymbol{\beta}}}$, which is computed as the inverse of the observed Fisher information matrix. That is, $\hat{\boldsymbol{\Xi}}_{\hat{\boldsymbol{\beta}}} \equiv \mathcal{I}(\hat{\boldsymbol{\beta}})^{-1}$ for $\mathcal{I}(\hat{\boldsymbol{\beta}}) = -\partial^2 l/\partial \boldsymbol{\beta}^2$ evaluated at $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$, where $l = \log \mathcal{L}(\boldsymbol{\beta})$.

Based on the ML estimates of $\boldsymbol{\beta} = (\beta_1, \beta_2)^T$, the point estimates of the shape and scale parameters in the PLP, for example, can be obtained by $\hat{\vartheta} = \hat{\beta}_2$, and $\hat{\zeta} = \exp(-\hat{\beta}_1/\hat{\beta}_2)$, respectively. The estimated standard errors of (or functions of) $\hat{\vartheta}$ and $\hat{\zeta}$ are computed using the delta method; that is, $\widehat{\text{s.e.}}(\hat{\beta}_1) = \sqrt{\hat{\boldsymbol{\Xi}}_{\hat{\boldsymbol{\beta}}}(1,1)}$ and $\widehat{\text{s.e.}}(\hat{\beta}_2) = \sqrt{\hat{\boldsymbol{\Xi}}_{\hat{\boldsymbol{\beta}}}(2,2)}$. Their Wald-type confidence intervals are also computed based on the estimated standard errors. Approximate $100(1-\alpha)\%$ point-wise confidence intervals for the fixed-effects parameters are, respectively

$$\hat{\beta}_1 \pm t_{\alpha/2}(\psi) \cdot \widehat{\text{s.e.}}(\hat{\beta}_1) \quad \text{and} \quad \hat{\beta}_2 \pm t_{\alpha/2}(\psi) \cdot \widehat{\text{s.e.}}(\hat{\beta}_2)$$

where $t_{\alpha/2}(\psi)$ is the $100(1-\alpha/2)$ quantile of the t-distribution with ψ degrees of freedom. The degrees of freedom are approximated by simply using the minimum number of degrees of freedom contributed by random-effects that affact the term being tested or using the Satterthwaite's method [25]. The Wald t-statistic accounts for the uncertainty in the estimates of overdispersion for the GLMMs. Unlike classical balanced ANOVA algorithms, because computing the test statistics for fixed effects do not generate correct values of degree of freedom automatically, the Wald z-statistic can be used instead of the Wald t-statistic for the GLMMs without overdispersion. Then $100(1-\alpha)\%$ point-wise confidence intervals for the fixed-effects parameters are approximated as: $\hat{\beta}_1 \pm z_{\alpha/2} \cdot \widehat{\text{s.e.}}(\hat{\beta}_1)$, and $\hat{\beta}_2 \pm z_{\alpha/2} \cdot \widehat{\text{s.e.}}(\hat{\beta}_2)$, respectively, where $z_{\alpha/2}$ is the $(1-\alpha/2)$ quantile of the standard normal distribution.

C. GOODNESS-OF-FIT TEST & MODEL CHECKING

After fitting mixed-effects NHPP model to failure-time data from multiple repairable systems, we need to assess the significance of the terms in the model. The significance test can be done through a likelihood ratio statistic. Denote $\mathcal{L}_F$ as the likelihood for the full model, and $\mathcal{L}_R$ as the likelihood for the reduced model. Then under the null hypothesis that the reduced model is adequate, the likelihood ratio test (LRT) statistic

$$2\log(\mathcal{L}_F/\mathcal{L}_R) = 2(\log\mathcal{L}_F - \log\mathcal{L}_R)$$

will approximately follow a χ^2 distribution with $(\varphi_F - \psi_R)$ degrees of freedom, where ψ_F and ψ_R are the number of parameters to be estimated in the full and reduced model, respectively. However, the LRT statistic is not recommended for testing or constructing confidence intervals for fixed-effects in the generalized mixed-effects model because it is unreliable for small to moderate sample sizes [26]. In general, the likelihood ratio approach is more appropriate for inference on the random-effects compared to the Wald-type statistic, which requires stronger assumptions on the parameters to be estimated [27].

Even though the LRT can assess the significance of particular terms, model selection procedure via such pairwise comparisons has been criticized owing to an overuse of hypothesis testing. By contrast, an information-based model selection procedure allows comparison of multiple candidate models. Two widely used information criteria for assessing model fit are Akaike's information criterion (AIC) [28] and the Bayesian information criterion (BIC) [29]. For the log-likelihood of a model, l, the AIC and BIC are, respectively

$$\text{AIC} = -2\,l + 2p^*, \quad \text{and} \quad \text{BIC} = -2\,l + p^*\log N,$$

where p^* denotes the total number of parameters in the model, and N denotes the total number of observations in the data set; that is, $N = \sum_{i=1}^{m} n_i$ for the mixed-effects NHPP model. If we use the AIC to compare several models for the same data, we prefer the model with the lowest AIC value.

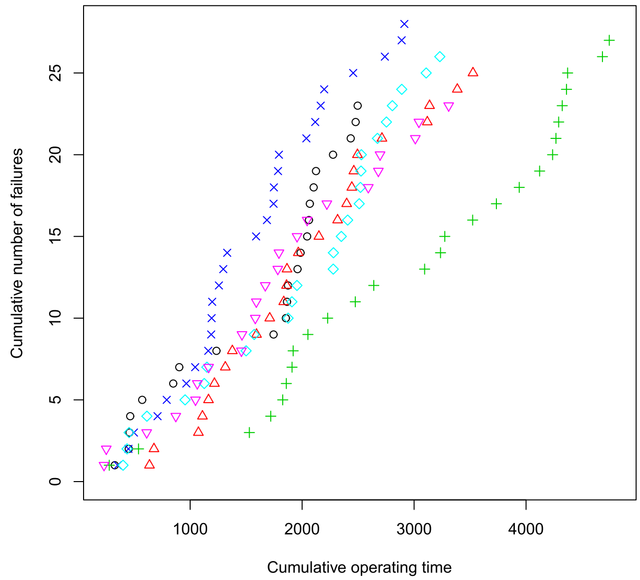


FIGURE 1. Plot of the cumulative number of failures against cumulative operating time for six LHD machines. (∘: LHD1, △: LHD3, +: LHD9, ×: LHD11, ⋄: LHD17, ▽: LHD20).

Similarly, when using BIC we prefer the model with the lowest BIC value.

Residuals can be set up to provide checks on the assumed model. Under the NHPP model, the quantities $\Lambda(t_{ij}) - \Lambda(t_{i,j-1}), j = 1, \ldots, n_i$, are independent standard exponential random variables only when the failure process is failure truncated, that is, when $\tau_i = t_{i,n_i}$. Therefore, residuals $e_{ij} = \hat{\Lambda}(t_{ij}) - \hat{\Lambda}(t_{i,j-1})$ should look like standard exponential random variables if the NHPP model under assumptions is correct. The deviation from the model assumptions can be checked by plotting $(e_{ij}, e_{i,j-1})$ to detect serial correlation with respect to j in the e_{ij}'s. See Lawless [6] for more details on the properties of residuals and formal model assessment using the residuals.

V. HYDRAULIC SYSTEMS DATA

Kumar and Klefsjö [30] analyzed the times between successive failures of the hydraulic system of some load-haul-dump (LHD) machines at Kiruna mine for a period of two years. The data consists of failures from three different machine groups; old(LHD1 and LHD3), medium old(LHD9 and LHD11), and new machines (LHD17 and LHD20). Cumulative failure plots for six LHD machines are given in Figure 1. After fitting a power law process model to the failure intensities of the six LHD machines, they set up maintenance policies for such repairable items. Later, Attardi and Pulcini [14] provided a reliability analysis for recurrent failure data of the LHD machines with the 2-EBP model to represent a compromise between the PLP and BIP models.

Figure 1 suggests that recurrent failure data of the LHD machines have monotonic failure intensities with variability among individual machines, which implies random-effects model would be most appropriate for modeling machine effects in the failure model. In such a case, fixed-effects models may lead to biased parameter estimates by incorporating all failure data from each of the LHD machines to estimate the parameters of the model under consideration, which ignores the variability among individual items. First, we individually fitted the PLP, LLP, and 2-EBP models to the failure intensities of six LHD machines. The MLEs of the PLP model, for example, are given by $\hat{\vartheta} = n_i / \sum_{j=1}^{n_i-1} \ln(t_{n_i}/t_j)$, and $\hat{\zeta} = t_{n_i}/n_i^{1/\hat{\vartheta}}$. The MLEs of the parameters for the three NHPP models are summarized (with corresponding standard errors in parentheses) in Table 1.

TABLE 1. The MLEs and their standard errors for the parameters in the PLP, LLP, and 2-EBP models.

	PLP		LLP		2-EBP	
ID	$\hat{\zeta}$	$\hat{\vartheta}$	$\hat{\gamma}\ (\times 10^{-2})$	$\hat{\kappa}\ (\times 10^{-4})$	$\hat{\upsilon}$	$\hat{\rho}$
LHD 1	363.81	1.6281	0.34	6.967	0.0223	1485.05
	(153.34)	(0.3395)	(0.18)	(3.110)	(0.0251)	(3183.56)
LHD 3	408.00	1.4925	0.48	2.133	0.0125	1011.85
	(184.28)	(0.2985)	(0.21)	(1.990)	(0.0060)	(1225.74)
LHD 9	646.56	1.6539	0.22	3.543	0.0171	4140.93
	(259.12)	(0.3183)	(0.11)	(1.500)	(0.0200)	(7978.64)
LHD 11	231.71	1.3163	0.79	1.272	0.0135	391.04
	(115.74)	(0.2488)	(0.31)	(2.260)	(0.0046)	(471.48)
LHD 17	383.20	1.5284	0.39	4.009	0.0147	1024.49
	(167.58)	(0.2997)	(0.19)	(2.190)	(0.0091)	(1565.88)
LHD 20	253.14	1.2198	0.59	9.941	0.0084	198.25
	(142.41)	(0.2544)	(0.26)	(2.190)	(0.0027)	(346.62)

Meanwhile, because the individual variability can be modeled most effectively using random-effects, we considered a mixed-effects model with random-effects as well as fixed-effects to model recurrent failures of six LHD machines. The likelihood ratio test (LRT) was sequentially executed to compare the mixed-effects NHPP model fit by the ML method to decide which of the terms in the model require random-effects to account for between-individual variation. The model-building strategy is to start with the model which includes random-effects for all NHPP model parameters, and then examine the fitted model to decide which of the random-effects can be eliminated. The random-effects PLP for comparison has a mean failure intensity of the PLP with two random-effects

$$\Lambda_{ij}(t) = \left(\frac{t_{ij}}{\zeta + b_{i1}}\right)^{\vartheta + b_{i2}}, \quad i = 1, \ldots, m, \ \ j = 1, \ldots, n_i$$

where the random-effects (b_{i1}, b_{i2}) have a general covariance structure. Similarly, the general models for the LLP and 2-EBP are

$$\Lambda_{ij}(t) = \left(\frac{\gamma + b_{i1}}{\kappa + b_{i2}}\right)(e^{(\kappa + b_{i2})t_{ij}} - 1),$$

and

$$\Lambda_{ij}(t) = (\upsilon + b_{i1})\left[t_{ij} - (\rho + b_{i2})\ln(1 + t_{ij}/(\rho + b_{i2}))\right],$$

respectively. The model-building structures for the three random-effects NHPP models are summarized in Table 2. We additionally computed the AIC and BIC to ensure parsimony.

For the mixed-effects PLP model, we compared the full model (Model 1) with the reduced models (Model 2, 3, and 4), and the small p-values for the LRT statistic (which follows asymptotically χ^2 distribution with 1 degree of freedom (Model 2) or 2 degrees of freedom

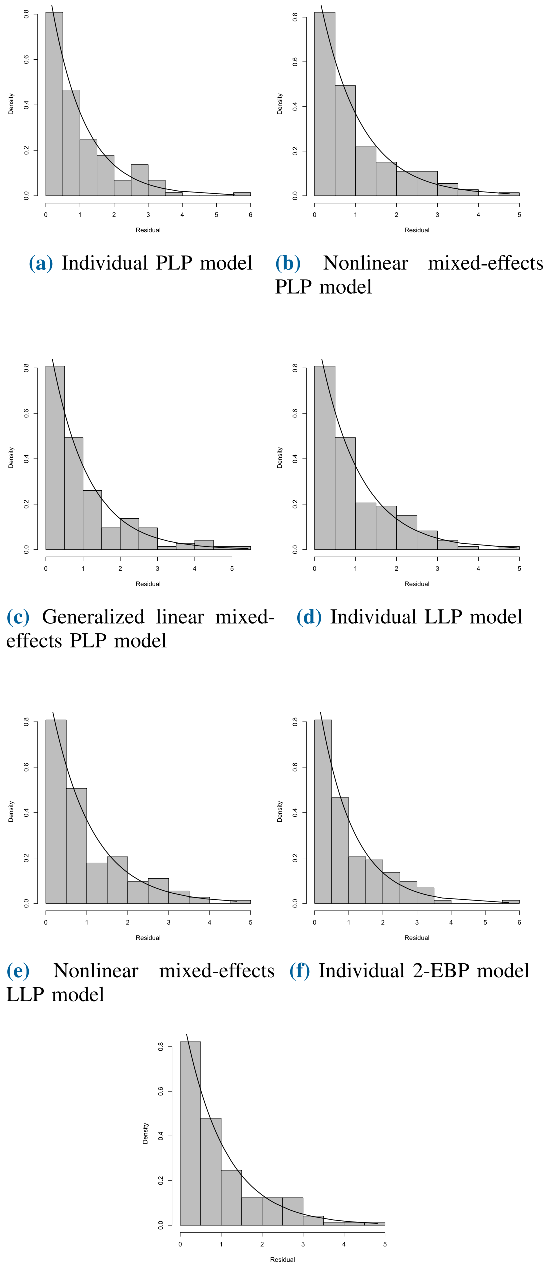
TABLE 2. Likelihood ratio tests comparing different mixed-effects models for the hydraulic systems data. (G: General, D: Diagonal).

		Random-coefficients included	Covar. structure	d.f.	AIC	BIC	Test	LRT	p-value
PLP	1	(b_1, b_2)	G	6	606.040	624.184			
	2	(b_1, b_2)	D	5	610.270	625.389	1 vs. 2	6.229	0.0126
	3	(b_1)	G	4	612.688	624.783	1 vs. 3	10.647	0.0049
	4	(b_2)	G	4	643.217	655.312	1 vs. 4	41.176	<0.0001
LLP	5	(b_1, b_2)	G	6	611.047	629.190			
	6	(b_1, b_2)	D	5	611.882	627.002	5 vs. 6	2.836	0.0922
	7	(b_1)	G	4	617.288	629.384	6 vs. 7	7.406	0.0065
	8	(b_2)	G	4	686.611	698.706	6 vs. 8	76.728	<0.0001
2-EBP	9	(b_1, b_2)	G	6	602.282	620.426			
	10	(b_1, b_2)	D	5	600.316	615.436	9 vs. 10	0.034	0.8542
	11	(b_1)	G	4	612.494	624.590	10 vs. 11	14.178	0.0002
	12	(b_2)	G	4	603.280	615.375	10 vs. 12	4.963	0.0259

(Models 3 and 4)), along with the smaller value for the AIC and BIC suggest that the full model (including two random-effects which have general covariance structure) is satisfactory for the hydraulic systems data. The final parameter estimates of the mixed-effects PLP model are: $\hat{\zeta} = 294.9230$(hours), $\hat{\vartheta} = 1.3174$, and $(b_{i1}, b_{i2})^T \sim \mathcal{N}\left((0,0)^T, (1.3010 \times 10^4, 12.2149; 12.2149, 0.0157)\right)$. Additionally, we estimated the parameters of the generalized linear mixed PLP model by transforming the mean failure intensity as described in (7), and final parameter estimates of the model are obtained as: $\hat{\beta}_1 = -7.6470$, $\hat{\beta}_2 = 1.3454$, with $(b_{i1}, b_{i2})^T \sim \mathcal{N}\left((0,0)^T, (2.5945, -0.3435; -0.3435, 0.0471)\right)$.

Next, for the mixed-effects LLP model, we compared the full model (Model 5) with the model based on two random-effects having diagonal covariance structure (Model 6), with the outcome being a p-value $>$ 0.05 for the LRT statistic, and the smaller values for the AIC and BIC, which corroborates Model 6. To search for a simpler model, we consider the LLP model with a random-effect. The models including a random-effect (Model 7, and 8) have smaller p-values, and larger AIC and BIC values, showing that Model 6 is the best mixed-effects model in terms of LLP for the hydraulic systems data. The final parameter estimates of the mixed-effects LLP model are: $\hat{\gamma} = 5.1536 \times 10^{-3}$, $\hat{\kappa} = 2.6750 \times 10^{-4}$(hours^{-1}), and $(b_{i1}, b_{i2})^T \sim \mathcal{N}\left((0,0)^T, (2.9467 \times 10^{-6}, 0; 0, 1.6788 \times 10^{-8})\right)$. Similarly, the 2-EBP model including two random-effects which have diagonal covariance structure (Model 10) is selected for the data. The final parameter estimates of the mixed-effects 2-EBP model are: $\hat{\upsilon} = 0.0108$(hours^{-1}), $\hat{\rho} = 489.6058$(hours), and $(b_{i1}, b_{i2})^T \sim \mathcal{N}((0,0)^T, (3.2113 \times 10^{-6}, 0; 0, 1.1977 \times 10^5))$. In choosing the best model among all the mixed-effects models, the minimization of AIC and BIC led to the selection of the mixed-effects 2-EBP model over the mixed-effects PLP and LLP models. All the model parameters were estimated using R NLME library.

As a diagnostics illustration for the fitted models, Figure 2 features the histograms for the residuals derived from each of the NHPP models, $e_{ij} = \hat{\Lambda}(t_{ij}) - \hat{\Lambda}(t_{i,j-1})$, along with probability density lines of a standard exponential distribution. If assumed NHPP models are correct, they should look like standard exponential random variables. The histograms in Figure 2 show that the residuals from all NHPP models appear to follow standard exponential distribution, justifying the assumptions of the NHPP models.



(a) Individual PLP model (b) Nonlinear mixed-effects PLP model

(c) Generalized linear mixed-effects PLP model (d) Individual LLP model

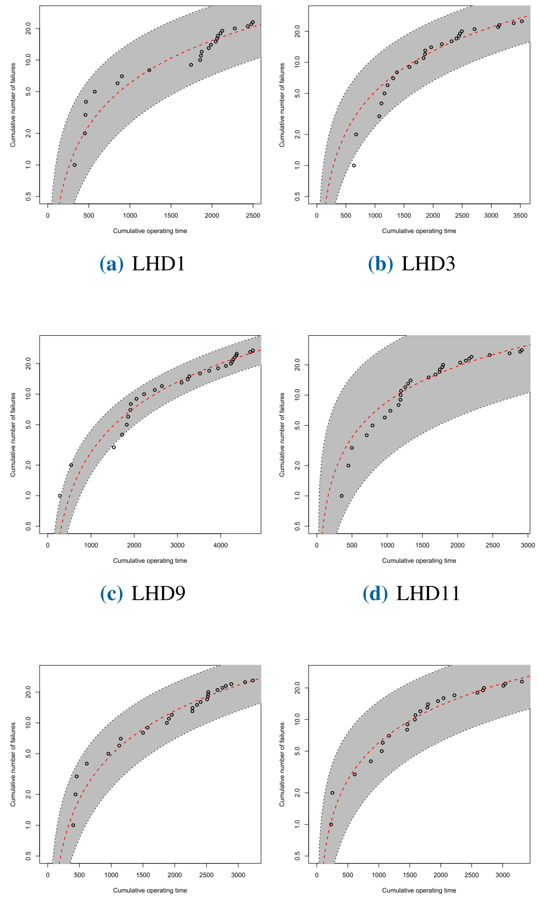
(e) Nonlinear mixed-effects LLP model (f) Individual 2-EBP model

(g) Nonlinear mixed-effects 2-EBP model

FIGURE 2. Histograms of the residuals from each of the NHPP models for six LHD machines.

TABLE 3. Mean squared errors between observed and estimated number of failures from each of NHPP models for individual LHD machines (*NLMM denotes nonlinear mixed-effects model).

ID	PLP	PLP-NLMM*	PLP-GLMM	LLP	LLP-NLMM*	2-EBP	2-EBP-NLMM*
LHD1	4.1244	3.5629	3.968	2.6763	2.8155	4.8004	3.9412
LHD3	5.9701	1.9741	5.3466	3.9958	2.6602	4.5271	1.6121
LHD9	1.824	1.5157	3.1913	1.8075	1.3938	2.0153	1.6806
LHD11	6.8924	2.7614	6.7788	4.2242	3.2632	5.9497	2.3928
LHD17	1.6045	1.5893	2.2047	1.7261	1.2264	1.7504	1.9241
LHD20	2.5777	1.1966	1.1755	1.8546	1.3164	2.0163	1.103
Average	3.8641	2.0946	3.8504	2.7373	2.1212	3.5295	2.0969



(a) LHD1 (b) LHD3

(c) LHD9 (d) LHD11

(e) LHD17 (f) LHD20

FIGURE 3. $\hat{\Lambda}(t)$ and its 95% pointwise confidence intervals, along with observed points $(t_i, N(t_i))$, under the mixed-effects PLP model for six sets of hydraulic systems data (The vertical axis is log-scaled for better representation of the confidence intervals).

To select the most suitable NHPP model for the hydraulic systems data, we calculated mean squared errors between observed number of failures, $\Lambda(t_{ij})$, and estimated number of failures from each of NHPP models, $\hat{\Lambda}(t_{ij})$, for individuals as $\text{MSE}_i = n_i^{-1} \sum_{j=1}^{n_i} (\hat{\Lambda}(t_{ij}) - \Lambda(t_{ij}))^2$. In Table 3, we observe that each of mixed-effects NHPP models has smaller MSE than individually fitted NHPP models, demonstrating that prediction from the mixed-effects NHPP models are substantially more reliable by employing flexible structure in approximating the integrals in the likelihood function (6) such as adaptive Gaussian quadrature method. We chose the nonlinear mixed-effects PLP model which has the smallest average MSE with respect to hydraulic systems data for further analytical study, although the average MSE of the nonlinear mixed-effects PLP model is only slightly smaller than that from the mixed-effects 2-EBP model. Note that the mixed-effects 2-EBP model was chosen in terms of the AIC and the BIC. Table 4 compares parameter estimates of individually fitted PLP model and those of nonlinear mixed-effects PLP model and their 95% pointwise confidence intervals for the six LHD machines.

The parameter estimates of nonlinear mixed-effects PLP model consist of ML estimates of fixed-effects and BLUP estimates of random-effects. Because the parameters are

TABLE 4. Parameter estimates of both individually fitted PLP model and nonlinear mixed-effects PLP model, along with their approximate 95% confidence intervals under the lognormal approximation in parentheses (Unit of measure in $\hat{\zeta}$: hours).

ID	PLP		PLP-NLMM	
	$\hat{\zeta}$	$\hat{\vartheta}$	$\hat{\zeta}$	$\hat{\vartheta}$
LHD 1	363.8135	1.6281	264.2611	1.3491
	(159.2611, 831.0900)	(1.0819, 2.4501)	(113.4034, 615.8012)	(1.1244, 1.6186)
LHD 3	408.0019	1.4925	285.8752	1.3138
	(168.3466, 988.8264)	(1.0085, 2.2088)	(130.7819, 624.8925)	(1.0896, 1.5840)
LHD 9	646.5576	1.6539	508.9101	1.4540
	(294.7609, 1418.2232)	(1.1342, 2.4117)	(327.988, 789.631)	(1.2279, 1.7218)
LHD 11	231.7135	1.3163	160.4545	1.1749
	(87.0558, 616.7441)	(0.9089, 1.9065)	(39.8336, 646.3305)	(0.9531, 1.4483)
LHD 17	383.2002	1.5284	330.2566	1.4281
	(162.6264, 902.9434)	(1.0407, 2.2448)	(167.8277, 649.8894)	(1.2023, 1.6963)
LHD 20	253.1447	1.2198	219.7807	1.1844
	(84.044, 762.4849)	(0.8106, 1.8356)	(79.4739, 607.7916)	(0.9624, 1.4575)

constrained to be nonnegative, we used the lognormal approximation for the parameters to construct confidence intervals as: $\hat{\zeta}\exp\left\{\pm z_{\alpha/2}\cdot \widehat{\text{s.e.}}(\hat{\zeta})/\hat{\zeta}\right\}$ and $\hat{\vartheta}\exp\left\{\pm z_{\alpha/2}\cdot \widehat{\text{s.e.}}(\hat{\vartheta})/\hat{\vartheta}\right\}$. The parameter estimates of the nonlinear mixed-effects PLP model are consistently smaller than those of individually fitted PLP models. Furthermore, their confidence intervals are consistently shorter than those of individually fitted PLP models. It was observed that other mixed-effects NHPP models have shorter confidence intervals than individually fitted NHPP models. Under the mixed-effects PLP model ignoring machine groups, the estimate of cumulative number of failures, $\widehat{\Lambda}(t)$, and 95% (pointwise) confidence intervals for $\widehat{\Lambda}(t)$ are plotted for six individual sets of hydraulic systems data in Figure 3.

VI. CONCLUSION

We introduced generalized mixed-effects models to recurrent failure data from multiple repairable systems, based on the NHPP. The generalized mixed-effects models explicitly involve between-system variation through random-effects, along with a common baseline for all the systems through fixed-effects for non-normal data. The primary focus for the research is on repairable systems that exhibit system-to-system variability. Our featured analysis in Section V shows key advantages to the mixed-effects modeling for a motivating example (hydraulic systems data), with model checking based on both the AIC and BIC. Both model assessment criteria balance model complexity by compromising variance versus bias, and are in general agreement in the example. The model with the smallest BIC has the added convenience, in the Bayesian framework, of being equivalent to selecting one with highest posterior probability [31].

The assumption of normally distributed parameters may produce non-zero probability of being negative when the parameters of the NLMM are restricted to be positive (as a referee pointed out). Alternatively, the random effects in the mixed model can be efficiently estimated using empirical Bayes methods. Conveniently, then empirical Bayes methods are equivalent to best linear unbiased prediction (BLUP) methods in the case the random effects can be assumed normal. Covariates can be introduced to the mixed-effects PLP, for example, to incorporate needed system information. Future applications in which prior system knowledge lends itself to more sophisticated Bayesian models could lead to further improvements in model fitting and model assessment (based on the BIC).

It is possible to investigate the effects of the preventive (or overhaul) maintenance on the reliability of mixed-Effects NHPP models. The optimality of preventive maintenance policy can be defined as the minimization of the expected cost per unit of time, however, the minimization procedure requires the calculation of high-dimension integrals because several random-effects are also involved. Other numerical or mathematical methods (e.g., stochastic optimizations, Monte Carlo simulations) can be introduced to the optimization procedure in future research.

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