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System Identification of Lightly Damped Systems Using Recursive Kautz Functions-Case Study With Coriolis Mass Flow Meter

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ABSTRACT This manuscript deals with the system identification of lightly (under) damped processes to find a suitable model structure. To identify such processes, the Kautz model, which is a two-parameter representation of orthogonal basis functions (OBF) in state space format, is used. The process parameters are obtained through the subspace method by sequencing the states obtained from the Kautz model. The obtained states are transformed into convex optimization in which the eigen values poles) to be estimated are confined to lie within user-defined regions in the convex plane. To derive the optimal states, an efficient control scheme based on the state observer design is implemented. The viability of the proposed work has been verified on real-time Coriolis Mass Flow Meter (CMFM) under no flow condition and the corresponding responses are plotted using MATLAB.

INDEX TERMS Kautz function, subspace methods, algebraic Riccati equations, YALMIP toolbox, Coriolis mass flow meter.

I. INTRODUCTION

Modeling of dynamic systems based on the system identification method (black box modeling) is highly an iterative procedure. It requires input-output (I/O) data, model structure (order and type), and selection criteria (rank models based on pre-defined cost function) [1]. The most challenging task in system identification is building the appropriate model structure and the accuracy of the model to mimic the real time system is based on the order of the system, system gain, dominating time constants, etc. If the order of the model is deliberately made high, then the model will perfectly mimic the real-time system irrespective of its nature i.e., linear/nonlinear, time-invariant/time-variant, etc. Increasing the order of the model increases the challenges in validating the real time system and designing an appropriate control algorithm for such systems is cumbersome. These challenges are the serious limitations in Impulse Response (IR) methods, even though, they are highly recommended to approximate any stable linear system as it guarantees stability and neglects truncation errors [2]. Similarly, other black-box models viz., Auto-Regressive (AR), Auto-Regressive Moving Average (ARMA), and Output Error (OE), etc., will suffer while approximating higher order systems and these methods offer less guarantee in terms of stability [1]. The above black-box models can get rid of these limitations (in dealing with a linear system), by choosing the correct basis functions. Basis functions are orthogonal in general, due to the aspect of convergence of model error, its completeness and hence deserves the name Orthogonal Basis Functions (OBF). IR methods have impulse functions as the basis functions to represent linear systems. It has a delay operator in its structure and these delay terms get added if the order of the system is increased thereby making the model more sluggish. The fruitfulness of generalizing the IR structure (Finite Impulse Response) obtained by replacing the delay operator with an all-pass function has been pointed out by Wahlberg [3]. This leads to a simple and efficient OBF that mimics the real-time system. To make the system to be strictly proper, the first delay element will be replaced by the low-pass function and the rest will be replaced by the all-pass function. The first order representation of such systems is called as Laguerre function [4] and the second order representation of such systems is called as Kautz function [5]. Laguerre functions are used to mimic highly damped systems and Kautz functions are used to mimic lightly damped systems. The importance of Laguerre

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polynomials in filtering came into the limelight after an inspirational work done by Lee and Wiener [6]. Kautz [7], obtained a new set of OBF by orthogonalization of the time exponential functions intending to improve the analog filter design called Kautz polynomials. A discrete version of Kautz polynomial has been investigated in [8]. The state space representation of Kautz functions were carried out by Reddy and Saha [9]. Wahlberg has pioneered the application of Laguerre and Kautz functions in system identification. He represented the Laguerre and Kautz functions in the discrete frequency domain (Z-transform). In addition to that, he claimed that no matter whatever may be the value of an unknown parameter, the true system approximated using Laguerre and Kautz functions should be strictly proper and analytic. But choosing any random value of unknown parameters, the system takes more steps to converge. To make the convergence rate faster, the optimal choice for the unknown parameters is the system's dominant poles [10]. Fetching the system's dominant poles using an appropriate system identification method is an uphill task as the system's physical insight is hardly known. The better way to get the system behavior and specifications are by using non-parametric (frequency domain) system identification methods like Spectral estimate [11]. This method is indeed an easy way for modeling as little amount of prior system knowledge will suffice. However, the non-parametric methods suffer from drawbacks as they are not directly used for simulations. On the other hand, in the parametric system identification method, the information of gain, dominant poles should be exploited to decide the order of the model to be used. But the direct information of the gain and poles of the system are seldom or never explicitly utilized. To overcome this issue and to obtain the system poles, the state partition-based subspace method with eigenvalue constraint has been utilized in this paper. The subspace methods (SM) have drawn more attention in the past decades not only because of their numerical stability and simplicity but also for their ease of applicability to MIMO systems [12]. The traditional steps followed in SM are to generate Extended Observability matrix and Toeplitz matrix from measurement data and system parameters are estimated using these two matrices. But this method has limitations i.e., the generated observability matrix obtained from the bulk measurement data leads to uncontrollable and unobservable mode (concern matrices becomes singular). Miller *et al.* [13] proposed a novel method to estimate the system parameters by transforming the entire state space model into convex optimization with constraints in terms of Linear Matrix Inequality (LMI). In this paper, the states obtained from the recursive Kautz functions are used to form a cost function by sequencing it and the constraint in the eigenvalues of the transition matrix obtained from Kautz functions lie in the unit circle to ensure stability. To get the optimal states in order to guarantee noise-free states, the state feedback control law is utilized and the controller gain is calculated using Algebraic Riccati Equation (ARE). Numerical tools like MATLAB R2018A and YALMIP toolbox, a third-party toolbox for MATLAB is used to generate the Kautz model in state space form, to perform convex optimization using SeDuMi solver and to solve ARE to obtain the controller gain [14]. The proposed work is validated with a lightly damped flow-measuring meter called Coriolis Mass Flow Meter (CMFM). The CMFM measures the flow rate of the fluid in terms of mass.

This paper is organized as follows. Following the Introduction,

- Section II deals with the development of the recursive Kautz model in state space form.
- Section III focuses on obtaining system parameters by sequencing the states from the Kautz model. This is done by formulating a cost function (convex optimization) in Frobenius norm with eigenvalue constraint and subsequently, the optimal states are derived by implementing the state feedback control law.
- Section IV shows the simulation results and discussions of the proposed work along with its real-time application (Coriolis Mass Flow Meter at no flow condition).
- Section V discusses the numerical results analysis, which is followed by the conclusion.

II. TWO PARAMETER KAUTZ REPRESENTATION IN STATE SPACE MODEL

The real functions are said to be orthonormal in the interval $[0, \infty)$, if it has the following representation where l and q are merely the number of terms in which the functions are defined.

$$\int_0^\infty m_l(t)m_q(t) = 0, \quad l \neq q \tag{1}$$

The functions that possess orthonormal property are said to be complete. It means the orthogonal functions are analytic as well as convergent for a given predefined space. If the functions are analytic and convergent around a certain point or other function in a predefined space, then the function is said to be complete.

$$\int_0^\infty f(t)m_q(t) = 0 \tag{2}$$

If the function $m_q(t)$ for all $t = 0, 1, 2 \ldots\ldots, n$ is orthonormal and complete in the interval $[0, \infty)$, then the function $f(t)$ has the formal expansion analogous to Fourier series expansion. In the context of approximation, the function has been written as,

$$f(t) = \int_0^\infty p_d m_q(t) \tag{3}$$

where, p_d are the coefficients of the expansion. The paramount feature of OBF for system identification is its simplicity as well as its predominant utilization of coefficients p_d to describe the system properties and functions. The Kautz functions used in this paper is suitable for the system having complex poles and used for resonant ones. The idea behind the complex value poles is that the system shows underdamped behavior. It has initially a transient response and as the time progresses, it reaches a steady state.

The Kautz model is good in approximating the system that has the above-mentioned property. It allows non-identical poles in the construction of resonant systems. The two parameter Kautz model takes two non-identical, but symmetric poles as the free parameters. The generalized Kautz function with complex poles is given as,

$$M_{2n-1}(s) = \sqrt{2a_{n/2}}\frac{s+h_{n/2}}{\left(s+a_{n/2}+jb_{n/2}\right)\left(s+a_{n/2}-jb_{n/2}\right)}\rho_n(s) \quad (4)$$

$$M_{2n}(s) = \sqrt{2a_{n/2}}\frac{s-h_{n/2}}{\left(s+a_{n/2}+jb_{n/2}\right)\left(s+a_{n/2}-jb_{n/2}\right)}\rho_n(s) \quad (5)$$

where,

$$\rho_n(s) = \Pi_{i=1}^{\frac{n}{2}-2}\frac{(s-a_i-jb_i)(s-a_i+jb_i)}{(s+a_i+jb_i)(s+a_i-jb_i)} \quad (6)$$

which represents the all-pass function. Here, $n = \frac{N}{2}$ order of the system, N is the number of terms in the Kautz function. $h_i = \sqrt{a_i + jb_i}$, represents complex poles. a and b denotes real and imaginary terms [15].

The discrete Kautz function in terms of Z-transform is,

$$h_i = \sqrt{a_i + jb_i}\frac{\left(z^{-1}-a\right)^{i-1}\left(\left(z^{-1}-b\right)^{i-1}\right.}{\left(1-az^{-1}\right)^{i}\left(1-bz^{-1}\right)^{i}}$$
$$0 \le a \le 1, 0 \le b \le 1 \quad (7)$$

a, b are the free parameters or scaling factors that the user must define. These scaling factors are called dominant poles in the Kautz function perspective, which are complex conjugate in nature. These functions satisfy the orthogonal property.

$$M(k) = Z^{-1}M(z) \quad (8)$$

However, the inverse of the Kautz function is not easy to construct. The compact form of Kautz function is obtained through state space modeling by recursively solving the Kautz function represented in Equation (7)

$$M_k(z) = M_{k-1}(z)\frac{\left(z^{-1}-a\right)^{1}\left(\left(z^{-1}-b\right)^{1}\right.}{\left(1-az^{-1}\right)^{1}\left(1-bz^{-1}\right)^{1}} \quad (9)$$

where,

$$M_{k-1}(z) \text{ or } M_1(z) = \frac{\sqrt{\left(1-a^2\right)\left(1-b^2\right)}}{\left(1-az^{-1}\right)\left(1-bz^{-1}\right)} \quad (10)$$

By repeating this in similar fashion, the state space format of the Kautz function is obtained.

$$M(k+1) = A1M(k) + A2M(k-1) + BD(k) \quad (11)$$

$A1$ and $A2$ are the matrices of size $n \times n$, D is the input or excitation signal and B is the input vector. The fourth order Kautz function is expressed along with its parameters as,

$$A1 = \begin{pmatrix} v & 0 \\ v(w-1) & v \end{pmatrix} \quad (12)$$

$$A2 = \begin{pmatrix} -w & 0 \\ 1-w^2 & -w \end{pmatrix} \quad (13)$$

$$B = \begin{pmatrix} 1 \\ \gamma v \end{pmatrix} \quad (14)$$

$$\gamma = \sqrt{\left(1-a^2\right)\left(1-b^2\right)} \quad (15)$$

where,

$$v = ab$$
$$w = a + b$$

In contrast to Laguerre function, the Kautz function takes free parameters in complex and conjugate form. To nullify the imaginary term, free parameters in terms of v and w are used to construct matrices instead of a and b.

The transfer function in the discrete domain can be written in terms orthogonal kernel as,

$$T(z) = \sum_{n=1}^{N}\theta_n M_n(z) \quad (16)$$

The input-output relationships in terms of regression phenomena are represented as,

$$Y(k) = \ominus \times \Phi(k) \quad (17)$$

Here, Y is the output of the Kautz function, $\ominus$ is the parameter vector and Φ is the regression in terms of Kautz function as shown below

$$\Phi(k) = \begin{pmatrix} M_1(z) \\ M_1(z) \\ \cdot \\ \cdot \\ M_n(z) \end{pmatrix} \times u(k) \quad (18)$$

The parameter vector takes the form as,

$$\ominus = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \cdot \\ \cdot \\ \theta_n \end{pmatrix}' \quad (19)$$

The above equation with a single quote represents the transpose of $\ominus$. This parameter vector $\ominus$ can be estimated through the least square technique. From (17), it is assumed that the Kautz functions $M(z)$ acts as the system states. Hence, the formal mapping of states with Kautz functions in terms of second order all pass function is,

$$x_{odd,1} = \frac{\sqrt{\left(1-a^2\right)\left(1-b^2\right)}}{\left(1-az^{-1}\right)\left(1-bz^{-1}\right)} \quad (20)$$

$$x_{odd,2} = \frac{\left(z^{-1}-a\right)^{1}\left(z^{-1}-b\right)^{1}}{\left(1-az^{-1}\right)\left(1-bz^{-1}\right)} \quad (21)$$

The generalized N^{th} term is given as,

$$x_{\mathrm{odd}_N} = \frac{\left(z^{-1}-a\right)^{N-1}\left(\left(z^{-1}-b\right)^{N-1}\right.}{\left(1-az^{-1}\right)^{N-1}\left(1-bz^{-1}\right)^{N-1}} \quad (22)$$

Therefore, the generalized state representation for the N-th order system is given as,

$$x_{odd}(k) = \begin{pmatrix} x_1(k) \\ x_3(k) \\ x_5(k) \\ \vdots \\ x_{2n-1}(k) \end{pmatrix} \quad (23)$$

$$x_{\mathrm{even}}(k) = \begin{pmatrix} x_2(k) \\ x_4(k) \\ x_6(k) \\ \vdots \\ x_{2n}(k) \end{pmatrix} = \begin{pmatrix} x_1(k-1) \\ x_3(k-1) \\ x_5(k-1) \\ x_{2n-1}(k-1) \end{pmatrix} \quad (24)$$

The regression function in terms of odd and even is written as,

$$\Phi(k) = \begin{pmatrix} \varphi_{N-1}(k) \\ \varphi_N(k) \end{pmatrix} \quad (25)$$

where,

$$\varphi_N(k) = C1_{1xn}\{x_{odd}(k) - A1_{nxn}x_{\mathrm{even}}(k)] \quad (26)$$
$$\varphi_N(k) = C2_{1xn}\{x_{odd}(k) - A2_{nxn}x_{\mathrm{even}}(k)] \quad (27)$$

Equations (25), (26) and (27), the regression model is partitioned in terms of odd and even ones like the partition of states and this regression model is in the form of time exponential functions stated by Kautz [7].

$$C1 = \sqrt{\frac{\left(1-v^2+2w^2\right)(1-w)}{\left(1+A1^2\right)\left(1+A2^2\right)+2A1v}} \quad (28)$$
$$C2 = \sqrt{\frac{\left(1-v^2+3w^2\right)(1-w)}{\left(1+A2^2\right)\left(1+A2^2\right)+2A1v}} \quad (29)$$

The overall state space equation is written as,

$$X_{\mathrm{odd}}(k+1) = A1X_{\mathrm{odd}}(k) + A2X_{\mathrm{even}}(k) + BD(k) \quad (30)$$

where,

$$X_{\mathrm{even}}(k) = X_{\mathrm{odd}}(k-1) \quad (31)$$
$$Y(k) = C1X_{\mathrm{odd}}(k-1) + C2X_{\mathrm{even}}(k-1) \quad (32)$$

The output matrix is represented as,

$$C = \begin{pmatrix} C1 & C2 \end{pmatrix}$$

$Y(k)$ is estimated through linear regression concept and the least square approximation is shown in the equations (18), (27) and (28). The parameter vector obtained through the least square technique is the output vector and it satisfies the linear dependent property with the output vector C obtained from the Kautz function. It is possible only if careful analysis is done in generating regression function.

The real-time or benchmark output is compared with the Kautz function output and the discrepancies are minimized using Euclidean norm analysis with constraints in terms of eigenvalues of the system matrix $A1$ and $A2$. It is a kind of quadratic optimization with non-linear constraints. Let us assume the real time output to be $Y1$, then

$$\delta = \| Y1 - Y \|_2 \quad (33)$$

subjected to

$$x^2 + y^2 \leq 1 \quad (34)$$

The above constraint pertains to the unity circle. x is the eigenvalue of $A1$ and y is the eigenvalue of $A2$.

III. ESTIMATING THE SYSTEM POLES USING SUBSPACE METHOD

The states obtained from the Kautz model is sequenced in two segments in a recursive fashion. The Kalman gain is estimated by solving ARE and its co-efficient matrix P [16]. To solve the Riccati equations, we must know the system parameters, Q and R matrices. To estimate the system parameters via state estimation, the spectral radius concept has been implemented [17].

The identified system states from the Kautz model are given as,

$$\begin{aligned} \dot{X} &= AC * X + BC * D \\ Y &= \ominus * X \end{aligned} \quad (35)$$

where,

$$AC = \begin{pmatrix} A1_{nxn} & A2_{nxn} \\ 1_{nxn} & 0_{nxn} \end{pmatrix} \quad (36)$$
$$BC = \begin{pmatrix} B_{nxn} \\ 0_{nxn} \end{pmatrix} \quad (37)$$

The states obtained from the Kautz model as,

$$X = \left[X_{(\mathrm{odd}\,,nx1)}; X_{(\mathrm{even}\,,nx1)}\right] \quad (38)$$
$$D = [D(0)D(1)\ldots D(n)] \quad (39)$$

The states can be sequenced as follows,

$$X_{n-1} = X_{(:,1:n-1)}X_n = X_{(:,2:n)} \quad (40)$$

The input sequence is given by,

$$D = [D(0)D(1)\ldots D(n-1)] \quad (41)$$

From Miller and Raymond [13]

$$X_n = [AC * BC] * \begin{bmatrix} X_{n-1} \\ U \end{bmatrix} \quad (42)$$

Then, we estimate the parameters $\widehat{AC}\widehat{BC}$ by minimizing the cost function as,

$$J(AC, BC) = \left\| [AC * BC] \begin{bmatrix} \widehat{X}_{n-1} \\ U \end{bmatrix} - \widehat{X} \right\|_F \quad (43)$$

In the unconstrained case with the least square approximation,

$$[\widehat{AC}\widehat{BC}] = \widehat{X}_n * \begin{pmatrix} X_{n-1} \\ U \end{pmatrix}^+ \quad (44)$$

$^+$ is the Pseudo-Inverse. The constraint takes the form as the spectral radius which is defined in terms of the eigenvalues of the matrix AC.

$$Q = \{z \in \mathbb{C}, Z \leq 1 - \text{eig}(\widehat{AC}), -1 \leq \text{eig}(\widehat{AC}) \leq 1\} \quad (45)$$

The main intention of state observer design is to get the optimal control gain matrix K by estimating the system states and using the state feedback control law, and then one shall obtain the necessary control signals.

$$U = -K * X \quad (46)$$

The basic concept of the state feedback using Luenberger gain is mentioned in the following equation

$$\hat{X}(k+1) = AC^*\hat{X}(k) + BC^*u(k) + L\left(Y - \Theta^*\hat{X}(k)\right) \quad (47)$$

where, $u(k)$ is the control signal, L is the Luenberger gain matrix.

The purpose of finding the Luenberger gain is to find how much the output Y is penalized in order to get the desired output $\hat{Y}$.

The closed loop system is represented as,

$$\hat{X}(k+1) = (AC - LC)\hat{X}(k) + L\left(Y - \Theta^*\hat{X}(k)\right) \quad (48)$$

This optimal controller gain K_{opt} is calculated using ARE,

$$P * \widehat{AC} + \widehat{AC}' * P - P * \widehat{BC} * R^{-1} * \widehat{BC}' * P + Q = 0$$

$$U_{opt} = -K_{opt} * X_{opt}$$

$$K_{opt} = P * \widehat{BC} * R^{-1} * \widehat{BC}' * P \quad (49)$$

To obtain Riccati coefficient P, the YALMIP toolbox with SeDuMi solver is used. Similarly, the state weighting matrix Q and control weighting matrix R can be calculated using the YALMIP toolbox.

The next step is to find the optimal gain K through the matrices R, P and $\ominus$.

The parameter vector $\ominus$ is obtained in (19)

$$K_{opt} = P * R^{-1} \quad (50)$$

The optimal control law is calculated by,

$$U_{opt} = -K_{opt} * X_{opt} \quad (51)$$

where,

$$K_{opt} = P * \widehat{BC} * R^{-1} * \widehat{BC}' * P \quad (52)$$

Finally, the optimal states are calculated as,

$$X_{opt} = \widehat{AC} * X + \widehat{BC} * D + K_{opt}(Y - \widehat{Y}) \quad (53)$$

where,

$$Y = \ominus * X_{\text{opt}} \quad (54)$$

Using the optimal control law, the closed loop dynamics can be obtained as,

$$\hat{X} = \left(\widehat{AC} - \widehat{BC} * K_{opt}\right) X + K_{opt}\left(Y - \odot X_{opt}\right) \quad (55)$$

where $\widehat{X}$ is the desired states. The stability is guaranteed through eigenvalues of the closed loop matrix $\left(\widehat{AC} - \widehat{BC} * K_{\text{opt}}\right)$ that lies within the unity circle by making ARE equation as spectral radius in terms of Linear Matrix Inequalities (LMI) mentioned in (46).

The above equation is solved iteratively for the length of the I/O sequence and the optimal states are obtained. Once the optimal states are obtained, move to (30) and find AC and BC matrices as per (36) and (37). Thus, the stability is guaranteed through the eigenvalues of the matrix AC in terms of unity circle constraint i.e., the eigenvalues should not go beyond the limits [-1;1].

A. ALGORITHM FOR GETTING THE OPTIMAL STATES AND PARAMETERS

Step 1: Assign values for a and b in the Kautz function with some numerical values that should be within the unity circle to guarantee stability.

Step 2: Get the Kautz parameters in a recursive fashion.

Step 3: Use the Kautz states and parameters, formulate the objective function in Frobenius norm by splitting the states as mentioned in (40) and estimate the parameters $\widehat{AC}$ and $\widehat{BC}$ using (36) and (37).

Step 4: Calculate the optimal Riccati coefficient, P, by solving ARE using optimization solver SeDuMi.

Step 5: Find the optimal control gain, K_{opt} from using(52).

Step 6: Using K_{opt}, find the optimal states X_{opt}

Step 7: Repeat steps 3-6 until the Frobenius norm is minimized or reaches a predefined tolerance value as mentioned by the user.

IV. RESULTS AND DISCUSSION

A. REAL-TIME EXAMPLE: KAUTZ APPROXIMATION FOR CORIOLIS MASS FLOW METER WITH PIEZO SENSORS

Coriolis mass flow meter is a type of flow measuring instrument that measures the flow in terms of mass. CMFM relies on the vibratory interaction of the fluid and its conveying tube that induces the Coriolis acceleration on the fluid and senses its effect on the tube. It consists of twin vibrating tubes with a transmitter that maintains the flow tube vibrations for measurement [18]. The experimental setup consists of two sensors attached to the bending end of the tube and one more is fixed at the center of the tube, which acts as the exciting element. Since, the piezo sensor is also an inverse sensor, it is used for dual purposes i.e., actuation and sensing. The essential measurements using CMFM are amplitude and frequency of tube at no flow condition (analyzed in the proposed work) and tube frequency at flow condition, which is the function of the phase difference between two sensor measurements. The working principle of CMFM is when the fluid travels through the pipe it generates vibrations based on the Coriolis effect [19]. The Coriolis force $\vec{F}_c$ depends on the moving mass of the fluid flow Δm, angular velocity $\vec{w}$ and radial velocity $\vec{v}$ of the oscillating system.

$$\vec{F}_c = \Delta m(\vec{w} * \vec{v}) \tag{56}$$

CMFM measures the flow rate in terms of mass and not in terms of volume. It measures mass in terms of kilograms per second. In volumetric flow measurement, the flow rate is measured in terms of the mass of the fluid by density (good for constant density). If the density varies, then the relationship becomes complicated. The density depends on temperature and pressure. At the time of flow, the pressure exerted on the tube changes and it changes the density. If density varies, mass varies. CMFM acts typically on the resonance condition. i.e., when the tube frequency is equal to the excitation frequency it generates sustained oscillations. If there is any fluid flow, the mass of the fluid varies due to the density variation in terms of pressure and the frequency of the tube also varies. It is necessary to tune the excitation frequency equal to the tube frequency to keep CMFM in the resonant condition. The mass has a relationship with the angular frequency in terms of ratio of spring constant/mass.

The signals received from the sensors are given to the data acquisition system (DAQ). The actuating signal is given to the piezo sensor which is placed at the center of the tube using a function generator. These actuating and sensing signals are taken as input-output to design the Kautz function. At no flow, the tube is at resonance condition. i.e., having sustained oscillations with no phase shift or negligible phase shift. But at flow condition, the Coriolis effect creates the twisting force and subsequently, the Coriolis acceleration makes the tube wobble and alters the tube's vibrational mode. This forces the tube to vibrate at some other frequency and creates the phase difference between the two sensor signals. The difference in the phase is proportional to the mass of the fluid and hence this flow meter is called CMFM.

In addition to the meter under test (CMFM), the real-time experimental setup also includes a multivariable flowmeter designed by Endress + Hauser (with specification Proline Promass 80F). This flow meter can act as a reference CMFM meter and the twin tube under test attached parallel to the reference CMFM is used as a working tube where the Piezo transmitter and sensors are attached.

The illustrative results are provided to validate the proposed model. Fig. 1 displays the pseudo random signal tracked by the Kautz model and user defined second order underdamped system. The pseudo random signal is fed to the user defined second order underdamped system obtained using the Kautz model. Similarly, the Kautz model is validated with the same underdamped system using Chirp signal (variable frequency signal) as excitation signal shown in Fig. 2.

$$G(z) = \frac{0.9615z - 0.5002}{z^2 + 0.2822z + 0.1795}$$

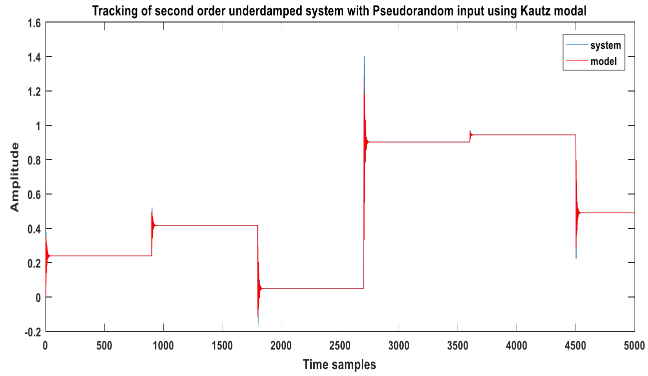

FIGURE 1. Responses for Pseudo random signal excitation.

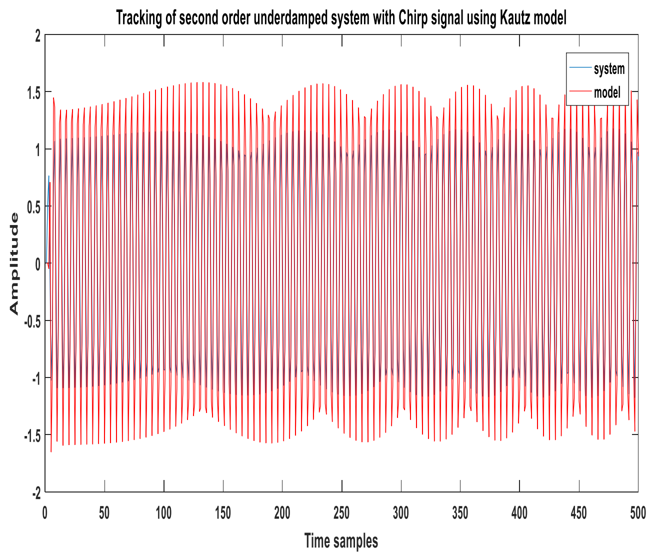

FIGURE 2. Responses for Chirp Signal Excitation.

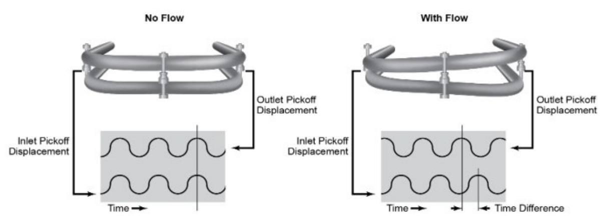

FIGURE 3. Coriolis mass flow meter tube movement.

is the second order underdamped transfer function obtained using Kautz function.

Fig. 3 shows the tube movement of tubes under no flow as well as flow condition. The flow tube of CMFM is vibrated at the frequency called resonant frequency or drive or working frequency. When there is no flow, the tube vibrates at its natural frequency generating signals of same phase in both the inlet and outlet portion of the tube. The flow induces, the Coriolis force which is generated due to fluid-tube interaction and it slightly changes the drive mode signal shape. The change in signal shape is visualized through the time difference of two sensor signals placed at the curved end of the tube. It affects the drive frequency and predominantly changes to a new mode of frequency called Coriolis frequency. Using an arbitrary function generator, a sinusoidal waveform of amplitude one is fed to the Piezo sensor attached at the center of tube. It makes the tube vibrate at the excitation or drive frequency. To find out the tube's natural frequency, a spectral estimation test is performed. DAQ is used as an interfacing unit to load the signals from sensors to the computer. The (DAQ) software module called Tracer DAQ views the signal in Strip chart recorder and oscilloscope. Then these signals are loaded in the MATLAB platform using DAQ toolbox to develop the Kautz function, and parameters are estimated using convex optimization and finally the optimal states are obtained. The components used, the real-time experimental setup of CMFM and the overall block diagram of the proposed approach are shown in Figs 4a, 4b and 4c. The reference meter in Fig. 4a will display the mass flow rate, density, frequency etc., of the fluid that passes through the reference tube shielded near the reference meter. Fig. 4a also depicts the piezoelectric sensors and DAQ.

Fig.4b shows the overall experimental design. The Piezo-actuator, which is housed at the center of the tube, is energized with sinusoidal waveform using an arbitrary function generator. The same signal is given as the input to the Kautz model through the DAQ card (analog input pin1). The tube is vibrated and the two sensors which are placed at the two ends of the tube, senses these vibrations. These sensor signals are fed into PC through DAQ output pins. The discrepancies in the output of model and sensor output have been treated as the minimization function, which can be solved using an optimization procedure with eigenvalue constraints. After this, the state space partitioning, estimation of parameters and state feedback controller design are carried out in PC.

Fig. 4c explicates the block diagram of the proposed approach. The natural frequency of the tube was identified by providing an excitation signal to the tube through a piezo actuator with some range of frequencies in trial-and-error basis shown in Fig.5 The range of frequencies was estimated by the sound generated by the piezo actuator (buzzer) based on its internal properties. The tube frequency estimated in this work is 4.772 KHz as shown and verified in Fig. 6 using Welch Power Spectral Density (WPSD) method. Now, the tube is excited with this estimated sinusoidal frequency through an arbitrary function generator to achieve the resonant condition. At no flow condition, both sensor readings show the vibrating signals with no phase shift as shown in Fig.7. At flow condition, these signals show phase variation as well as amplitude variation. The phase difference in both sensor signals serves as an indication of mass measurement directly. Amplitude variation shows the damping factor induced in the tube.

The same estimated excitation signal is used for Kautz model which is fed to PC through DAQ as indicated in Fig. 4c and the modeling of tube is done using the algorithm explained in Section III. Fig. 8 depicts the tracking of noisy sensor signal by Kautz model and Fig. 9 depicts the tracking of noise free sensor signal by Kautz model. Since, the Kautz model identifies the system dynamics through the information of dominant poles, the initial guess for Kautz function is provided with some arbitrary values. Their numbers depend on the order of the system. The proper selection of the initial

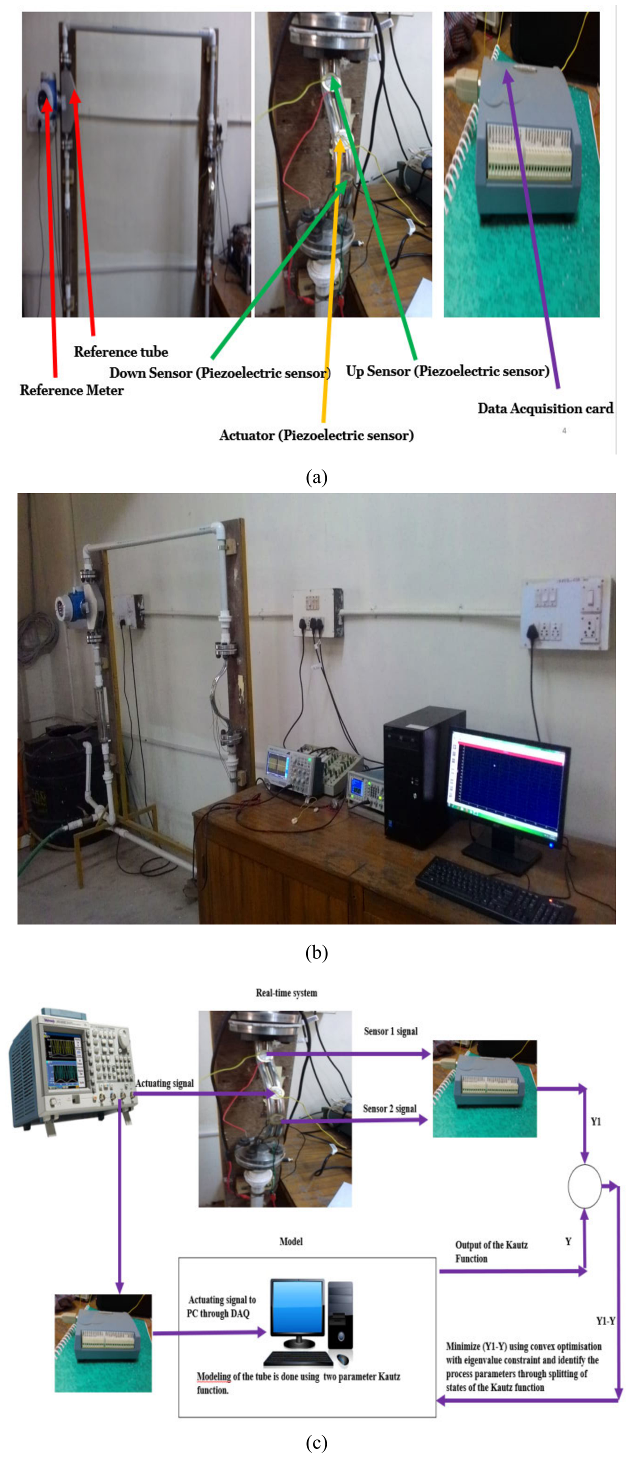

FIGURE 4. a. Components in Experimental Setup. b. Real-Time Experimental Setup. c. Overall Block Diagram of the proposed work.

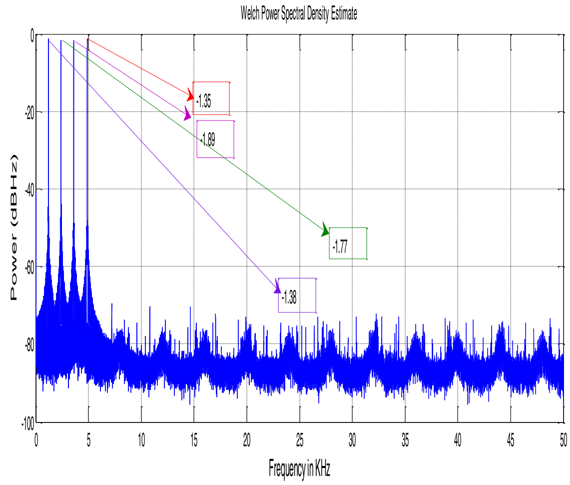

FIGURE 5. Frequency estimation of sensor signal using Welch Power Spectral Density at different frequencies.

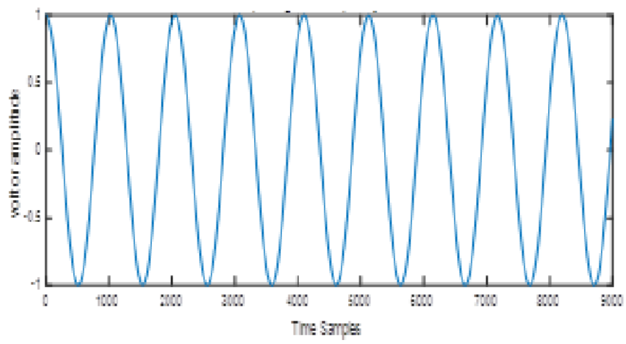

FIGURE 6. Input or Excitation signal.

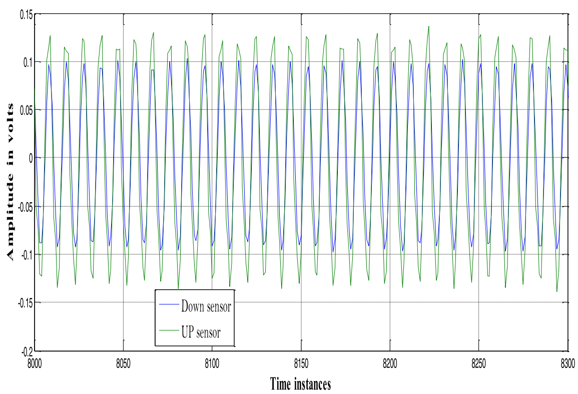

FIGURE 7. Sensor readings.

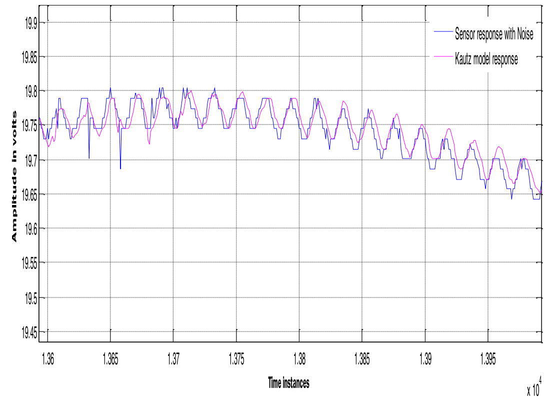

FIGURE 8. Tracking of sensor signal using Kautz function with Noise.

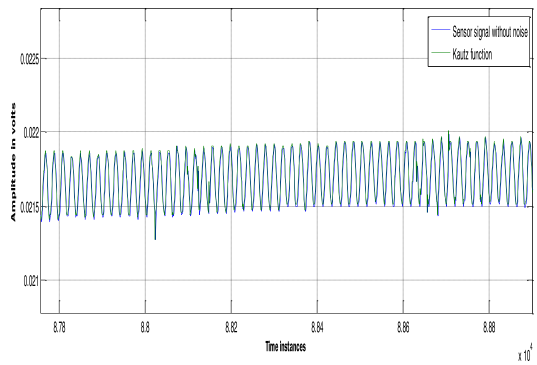

FIGURE 9. Tracking of sensor signals using Kautz function without Noise.

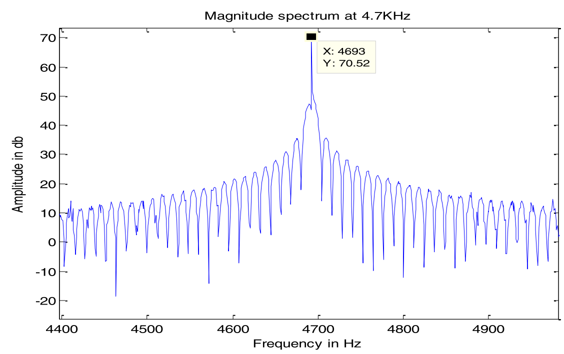

FIGURE 10. Estimated Natural frequency of the sensor signal.

guess values is based on the simple stability criteria for pole locations in discrete functions.

Since, this work utilizes the discrete Kautz function, the initial guess should be less than unity irrespective of its sign.

Fig 10 shows the natural frequency of the tube from the collected sensor readings depicted in Fig. 7. The frequency of the tube is found to be 4.7 KHz against the source or excitation frequency of 4.772 KHz which is found from the internal properties of the Piezo sensors. This difference is negligible and will not deviate the system from the resonance condition, which is the basic for the operation of CMFM.

Fig. 11 shows the step response of Kautz model states in which the states mimic the stability of underdamped or lightly damped response.

Fig. 12 and 13 show the comparison of sensor signal that senses vibration in the CMFM tube with different types of signal identification methods along with the Kautz model. The quantitative results are shown in Table 1. In Table 1, the values are obtained for different norms for various black box models. The norms having the least value show the effectiveness of the

TABLE 1. Comparison of Kautz Function with other System Approximation Methods.

Type of System Identification	Order or Number of Scaling Parameters	Root Mean Square (V)	1-Norm or Manhattan Norm (V) $\|\mathcal{H}\|_1$	2-Norm or Euclidean Norm (V) $\|\mathcal{H}\|_2$	∞-Norm (V) $\|\mathcal{H}\|_\infty$	SNR Ratio
Kautz Function	2	$\approx 10^{-5}$	0.00213	$\approx 10^{-7}$	$\approx 10^{-5}$	92.13
FIR	2	1.3212	11.654	1.1432	1.987	76.51
ARX	2	2.1212	24.765	9.5842	5.497	49.88
OE	2	21.222	39.318	17.661	31.22	61.23
Laguerre Function	2	<1000	<1000	<1000	<1000	Not Determined

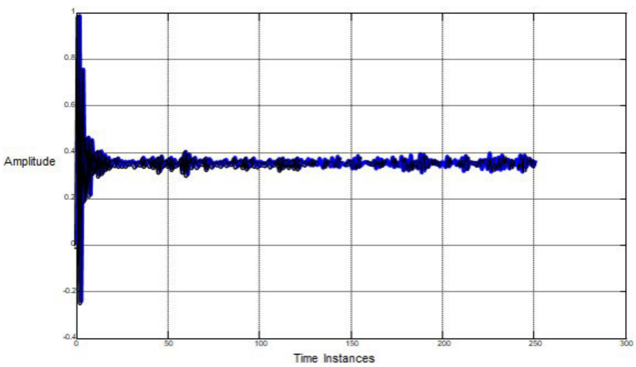

FIGURE 11. Step response of optimal states of CMFM.

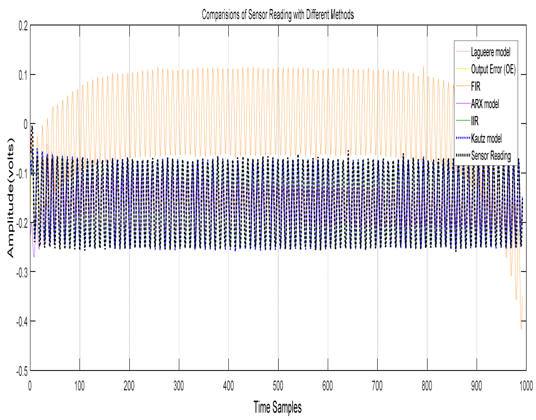

FIGURE 12. Comparison of different system identification methods with sensor signals.

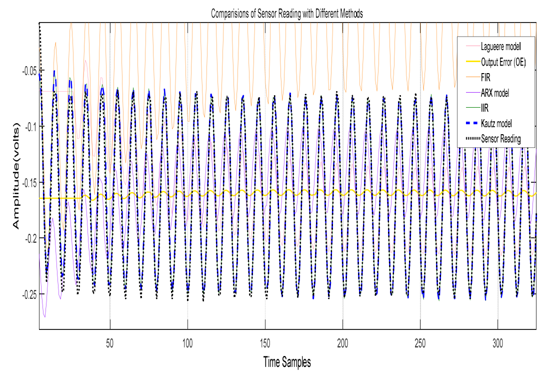

FIGURE 13. Zoomed image of Figure 12.

black box model to mimic the real-time system. The Signal to Noise Ratio (SNR) is the indication of the noise immunity of the models and values of high SNR show that the model has high noise immunity. Hence, the Kautz model is suggested. Table 2 shows the computational efficiency of MATLAB inbuilt optimization solver 'fmincon' with the third-party solver 'SeDuMi' of YALMIP toolbox. The readings in the Table imply that the YALMIP toolbox provides better computational efficiency and this toolbox is highly recommended for solving convex optimization.

V. NUMERICAL RESULTS

This work utilizes two parameter Kautz function for approximating CMFM and a system with order two has been generated.

Taking the initial guess values of a and b as,

$$\epsilon = \begin{pmatrix} 0.345 \\ 0.617 \end{pmatrix}$$

The Kautz parameters generated using (12)-(39) are,

$$AC = \begin{pmatrix} -0.6700 & -0.2292 \\ 1 & 0 \end{pmatrix}$$

$$BC = \begin{pmatrix} 0.437 \\ 0 \end{pmatrix}$$

$$C = (-0.34500.6426)$$

TABLE 2. Computational Efficiency Analysis.

Optimization Toolbox	No of Iterations	CPU Time (Sec)	Relative Error $\left(\frac{Y1-Y}{Y}\right)$	Euclidian Norm $\|\mathcal{H}\|_2$
MATLAB Inbuilt Optimization Toolbox	15	3.5	10^{-3}	0.000265
YALMIP Toolbox	9	0.2	10^{-7}	0.00000178

$$\ominus = \begin{pmatrix} -0.7120 \\ 1.4312 \end{pmatrix}$$

The estimated parameters using SM are

$$\widehat{AC} = \begin{pmatrix} 0.8759 & 0.4288 \\ 0.2112 & 0.0198 \end{pmatrix}$$
$$\widehat{BC} = \begin{pmatrix} -0.5799 \\ 1.3848 \end{pmatrix}$$

The numeric values of coefficients and parameters regarding ARE are,

$$P = \begin{pmatrix} 0.5932 & 0.2988 \\ 0.2988 & 0.0198 \end{pmatrix}$$
$$Q = \begin{pmatrix} 0.4161 & 0.2955 \\ 0.2955 & 0.1959 \end{pmatrix}$$
$$R = 50$$

The control gain matrix is given as,

$$K_{opt} = \begin{pmatrix} 0.00178 \\ 0.0093 \end{pmatrix}$$

Similarly, the Luenberger Observer gain matrix is,

$$L = \begin{pmatrix} 0.0103 \\ 0.0053 \end{pmatrix}$$

and the closed loop transition matrix is

$$(\widehat{AC} - \widehat{BC} * K_{\text{opt}}) = \begin{pmatrix} 0.8685 & 0.4342 \\ 0.1865 & 0.0072 \end{pmatrix}$$

The Eigenvalues (poles) of the transition matrices are obtained as,

$$AC = -0.3350 \pm 0.3420j$$
$$\widehat{AC} = 0.9715 \pm 0.0758j$$
$$(\widehat{AC} - \widehat{BC} * K_{opt}) = 0.9802 \pm 0.0741j$$

The estimated eigenvalues are stable. The size of input vector D is (10000×1) and the size of the output vector Y is (10000×1). The size of the optimal states X_{opt} is (2×10000). The size of the control vector U_{opt} is (2×10000), where 2 is the order of the system.

VI. CONCLUSION

This paper claims the importance of the Kautz model for lightly damped systems as it uses few numbers of parameters and has fast convergence which is proved through illustrative results on the real-time application of Coriolis mass flow meter (CMFM). The system is analyzed at no flow condition using the Kautz model and the result obtained using the Kautz model is superior to deal with the lightly damped systems effectively as compared to other system identification methods. After obtaining the recursive Kautz model in state space form, the states of the model are visualized through convex optimization technique with a constraint in terms of eigenvalues and the system parameters are identified effectively. The optimal states obtained using the state feedback technique (using the obtained optimal control gain) guarantees the improved noise immunity of the system with high signal to noise ratio (SNR). The future scope of this work is to study 1) the sensitivity and robustness of the model in terms of parametric uncertainty, 2) analyze the CMFM at flow condition, 3) to design a robust controller to control the flow rate.

APPENDIX I

The performance metrices in Table 1 are defined as follows,

The norm is defined as the distance between two vectors in space.

1-norm or Manhattan norm: It is sum or difference of magnitude of vectors in space. Norm is denoted as

$$\| \mathcal{H} \|_1 = |Y| - |Y1|$$

2-norm or Euclidean norm: It denotes the shortest distance between two vectors in space.

$$\| \mathcal{H} \|_2 = \sqrt{|Y|^2 - |Y1|^2}$$

∞- norm: It is used to obtain the largest element in a vector

$$\| \mathcal{H} \|_\infty = \max_y(|Y| - |Y1|)$$

Frobenius norm: It is represented in the similar fashion as Euclidean form but it is generally used for matrices rather than vectors.

$$\| \mathcal{H} \|_F = \sqrt{trace\left|AA^H\right|^2}$$

H is the complex conjugate of matrix A

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