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SBL-Based 2-D DOA Estimation for L-Shaped Array With Unknown Mutual Coupling

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ABSTRACT Based on L-shaped array (LSA), a variety of algorithms for direction of arrival (DOA) estimation have been developed in recent years. However, conventional methods are sensitive to the mutual coupling which may cause the performance of the DOA estimation to degrade dramatically or even fail. In order to solve this problem, a new algorithm for DOA estimation is proposed in this paper. Different from the existing algorithms, the DOA estimation of LSA with mutual coupling is achieved from the perspective of sparse Bayesian learning. A hierarchical form of the Student t prior is used to enhance the sparsity of source signal and achieve excellent performance for angle estimation. In the meanwhile, the mutual coupling effect is compensated blindly by sacrificing some sensors as auxiliary components and the hyperparameters and parameters updates are obtained by the expectation-maximization method. Simulation results verify that the performance of our method is superior than that of the state-of-art methods, particularly in the scenario of highly correlated and coherent signals.

INDEX TERMS DOA estimation, L-shaped array (LSA), mutual coupling, sparse Bayesian learning (SBL), Student t prior.

I. INTRODUCTION

With the increasing complexity of modern signal processing data, the direction of arrival (DOA) estimation for multiple narrowband and wideband source has attracted widespread attention in various military and national economic fields, such as biomedical imaging, sonar, communication systems, and radio astronomy [1], [2]. At present, many algorithms for DOA estimation are proposed for uniform linear array (ULA), but they can only offer one-dimensional angle information [3], [9]–[11]. There are many researches on DOA estimation of 2-D arrays [4]–[7] in recent years, such as circular array, rectangular array, and cross-shaped array. Compared with these 2-D arrays, the structure of L-shaped array (LSA) is simpler and easy to implement [4], and it has attracted a mushrooming number of attention [5]–[8].

However, with the miniaturization of mobile devices, the distance between antennas becomes smaller, which may cause the mutual coupling effect [9], [10]. It is proved that the mutual coupling can change the steering vector and deteriorate the performance of DOA estimation [11], [12]. Therefore, the calibration of mutual coupling is crucial to ensure the accuracy of DOA estimation. There are two main approaches to estimate DOA in the presence of mutual coupling, namely, the subspace approach and the sparse signal recovery (SSR) approach. In the subspace approach, there are three typical methods [11], [13]–[19]. The first method is achieved by a cost function and an iterative procedure [11]. In this method, the cost function iterates alternately between the DOA and the array error, then the estimator of DOA and mutual coupling coefficients are obtained by minimizing the cost function. However, the method is time consuming owning to the high computational complexity and obtain non-unique solutions. The second method is called rank reduction (RARE) [13]–[15], it requires a lot of received data and multidimension search which brings a large amount of calculations. The third method is called auxiliary method [16]–[19], which uses some elements at both ends of ULA as auxiliary elements [16]. Although the auxiliary array element can weaken the influence of mutual coupling effect, it sacrifices

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the array aperture and is only applicable in the scenario of few DOA or many array elements.

Recently, the rise of SSR has brought tremendous changes to the field of DOA estimation [20]–[25]. Compared with the subspace approach, it possesses many advantages. For example, it can improve the robustness to noise and has a wonderful performance in the case of few snapshots and correlated source [21]. The SSR method divides the entire spatial domain into dense grid, then the array data model is represented in a sparse form and the source signals is obtained by transforming the optimization problem. Sparse Bayesian learning (SBL) [26]–[35] is a typical method of SSR, which uses the sparse prior information of the source signals to establish a hierarchical probability model and optimizes the posterior probability function to obtain the reconstructed signal. It is proved that SBL can obtain fantastic performance compared with other SSR methods [27], [28], [34]. This method was first proposed in [30], which was addressed to solve the mutual coupling, gain-phase uncertainty, and sensor location error. However, the SBL algorithms have not been applied in the LSA with mutual coupling. Thus, it's worthwhile to demonstrate the performances of the SBL algorithm for LSA with unknown mutual coupling.

In this paper, we estimate the DOA of LSA in the presence of mutual coupling by a new SBL method. Two selection matrices are defined for eliminating the mutual coupling effect between the two subarrays. In the proposed algorithm, a hierarchical form of the Student t priors is utilized to strengthen the sparsity constraint [26], [35] and the expectation-maximization(EM) algorithm is selected to obtain the updates of the parameters. In the scenario of low SNR ratio and few snapshots, the proposed method yields superior performance than the state-of-art methods. The root mean square error (RMSE) of the subspace method is dozens of times higher than our method when the signal is coherent, which shows that our method can obtain exceptional DOA estimation accuracy.

The rest of this paper is organized as follows. We first propose the narrow signal model for LSA in Section II. In Section III, we describe the data model from SBL viewpoint and divide LSA into two parts by using blind mutual coupling effect compensation. Then, there is an SVD step to reduce the size of the measured matrix. Finally, we solve for the updates of parameter and hyperparameters. To verify the superiority of our method, some simulations is conducted in Section IV and concluded in Section V.

In this paper, we introduce the following notations. The uppercase bold letters ($\boldsymbol{Z}$) and lowercase bold letters ($\boldsymbol{z}$) represent matrix and vector, respectively. $(\cdot)^T$, $(\cdot)^{-1}$ and $(\cdot)^H$ denote the transpose, inverse and Hermitian transpose, respectively. $(\cdot)_{i\cdot}$, $(\cdot)_{\cdot j}$ and $(\cdot)_{ij}$ are the i-th row, jth column, and the element at the i-th row and j-th column of the matrix, respectively. $\mathrm{tr}(\cdot)$ denotes the trace operator. $E[\cdot]$ indicates the expectation and $\mathrm{diag}(\cdot)$ is to form a diagonal matrix with entries of a vector.

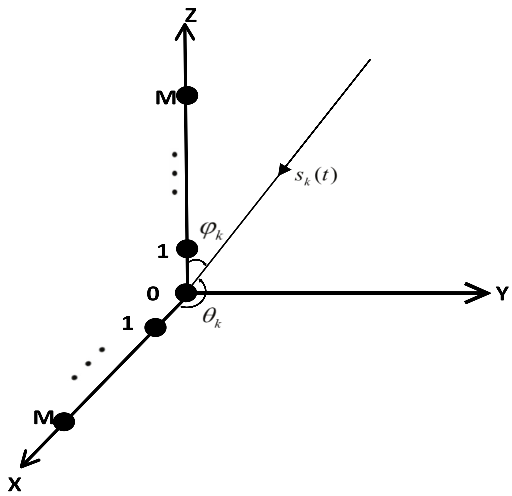


FIGURE 1. DOA estimation of L-shaped array.

II. DATA MODEL FOR DOA ESTIMATION

An LSA consisting of two uniform linear orthogonal arrays is established and illustrated in Fig.1. Suppose that there are K narrowband sources $\{s_k(t)\}_{k=1}^{K}$ ($t = 1, \ldots, T$, where T is the snapshots) arrive at the array with $2M + 1$-elements from different directions $\{(\theta_k, \varphi_k)\}_{k=1}^{K}$ $(-90^\circ < \theta_k < 90^\circ, -90^\circ < \varphi_k < 90^\circ)$. φ_k is the elevation angle while θ_k is the azimuth angle of the kth source.

Then the $(2M+1) \times T$ array output matrix $\boldsymbol{Y}$ is given by:

$$\boldsymbol{Y} = \boldsymbol{CAS} + \boldsymbol{N} \tag{1}$$

where $\boldsymbol{Y} = (\boldsymbol{y}(1), \boldsymbol{y}(2), \ldots, \boldsymbol{y}(T))$, $\boldsymbol{y}(t) = [y_1(t), y_2(t), \ldots, y_{2M+1}(t)]^T$, $\boldsymbol{S} = (\boldsymbol{s}(1), \boldsymbol{s}(2), \ldots, \boldsymbol{s}(T))$, $\boldsymbol{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$. $\boldsymbol{N} = (\boldsymbol{n}(1), \boldsymbol{n}(2), \ldots, \boldsymbol{n}(T))$ denotes the noise matrix, $\boldsymbol{n}(t) = \left[n_1(t), n_2(t), \ldots, n_{2M+1}(t)\right]^T$. $\boldsymbol{A} = [\boldsymbol{a}(\theta_1, \varphi_1), \boldsymbol{a}(\theta_2, \varphi_2), \ldots, \boldsymbol{a}(\theta_K, \varphi_K)]$ is the steering matrix, $\boldsymbol{a}(\theta_k, \varphi_k) = \left[e^{-j\varpi\cos\theta_k}, \ldots, e^{-j\varpi M\cos\theta_k}, 1, e^{-j\varpi\cos\varphi_k}, \ldots, e^{-j\varpi M\cos\varphi_k}\right]$, $\varpi = 2\pi d/\lambda$, d denotes the interval and λ is the signal wavelength. The $(2M+1) \times (2M+1)$ mutual coupling matrix(MCM) for LSA is denoted as $\boldsymbol{C}$ and can be given by [16]:

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{D} & \boldsymbol{g} & \boldsymbol{B} \\ \boldsymbol{g}^T & 1 & \boldsymbol{g}^T \\ \boldsymbol{B} & \boldsymbol{g} & \boldsymbol{D} \end{bmatrix} \tag{2}$$

where $\boldsymbol{D} = \mathit{Toeplitz}\{[1, c_1, \ldots, c_p, \mathbf{0}_{1\times(M-p-1)}]\}$, $\boldsymbol{g}$ can be written as $\boldsymbol{g} = \left[c_1, \ldots, c_p, \mathbf{0}_{1\times(M-p)}\right]^T$, $\boldsymbol{B}$ is given by $\left\{\boldsymbol{B}_{i,j} = \boldsymbol{B}_{j,i}, l_{i,j} \le pd; \boldsymbol{B}_{i,j} = 0, \mathit{otherwise}\right\}$, $l_{i,j} = \sqrt{i^2 + j^2}$ and p is the number of mutual coupling coefficients.

To estimate the elevation and azimuth angles separately, two selection matrices are defined

$$\boldsymbol{T}_1 = \left[\mathbf{0}_{(M+1-2p)}, \boldsymbol{I}_{M+1-2p}, \mathbf{0}_{(M+1-2p)\times(M+1+p)}\right] \tag{3}$$

$$\boldsymbol{T}_2 = \left[\mathbf{0}_{(M+1-2p)\times(M+p)}, \boldsymbol{I}_{M+1-2p}, \mathbf{0}_{(M+1-2p)\times p}\right] \tag{4}$$

Applying $\boldsymbol{T}_1$, $\boldsymbol{T}_2$ to (1), respectively, it can be rewritten as [17]

$$\boldsymbol{X} = \boldsymbol{T}_1\boldsymbol{Y} = \boldsymbol{A}_x\boldsymbol{\Delta}_x\boldsymbol{S} + \boldsymbol{N}_x \tag{5}$$

$$\boldsymbol{Z} = \boldsymbol{T}_2\boldsymbol{Y} = \boldsymbol{A}_z\boldsymbol{\Delta}_z\boldsymbol{S} + \boldsymbol{N}_z \tag{6}$$

where $\boldsymbol{\Delta}_x = diag\left\{\delta_1^x, \delta_2^x, \ldots, \delta_K^x\right\}$, $\boldsymbol{\Delta}_z = diag\left\{\delta_1^z, \delta_2^z, \ldots, \delta_K^z\right\}$, $\delta_k^x = e^{-jp2\pi d/\lambda \cos\theta_k} + \sum_{l=1}^{p}\left[c_{p-l+1}e^{-j2\pi d/\lambda(l-1)\cos\theta_k} + c_l e^{-j2\pi d/\lambda(p+l)\cos\theta_k}\right]$, and $\delta_k^z = e^{-jp2\pi d/\lambda \sin\varphi_k} + \sum_{l=1}^{p}\left[c_{p-l+1}e^{-j2\pi d/\lambda(l-1)\sin\varphi_k} + c_l e^{-j2\pi d/\lambda(p+l)\sin\varphi_k}\right]$. Through the above steps, the mutual coupling effect between the subarrays can be eliminated.

Then, the SVD method is utilized to reduce the size of the receiving data:

$$\boldsymbol{X} = \boldsymbol{U}_S\boldsymbol{\Lambda}_S\boldsymbol{V}_S^H + \boldsymbol{U}_n\boldsymbol{\Lambda}_n\boldsymbol{V}_n^H \tag{7}$$

where $\boldsymbol{U}_S = \boldsymbol{U}(:, 1:K)$, $\boldsymbol{U}_n = \boldsymbol{V}(:, K+1:M)$, $\boldsymbol{V}_S = \boldsymbol{V}(:, 1:K)$ and $\boldsymbol{V}_n = \boldsymbol{U}(:, K+1:M)$, Let $\boldsymbol{X}_{SV} = \boldsymbol{X}\boldsymbol{V}_S$, which possesses most of the signal power, then we have:

$$\boldsymbol{X}_{SV} = \boldsymbol{A}_x\boldsymbol{\Delta}_x\boldsymbol{S}_{SV} + \boldsymbol{N}_{SV}^x \tag{8}$$

In order to apply sparse Bayesian learning algorithm to DOA estimation, $\tilde{\theta}_n\ (n = 1, \ldots, N)$ which uniformly covers the DOA range is introduced and (8) can be rewritten as:

$$\boldsymbol{X}_{SV} = \tilde{\boldsymbol{A}}_x\tilde{\boldsymbol{\Delta}}_x\tilde{\boldsymbol{S}}_{SV} + \boldsymbol{N}_{SV}^x \tag{9}$$

where $\tilde{\boldsymbol{A}}_x = \left[\boldsymbol{a}_x(\theta_1), \boldsymbol{a}_x(\theta_2), \ldots, \boldsymbol{a}_x(\theta_N)\right]$, $\tilde{\boldsymbol{\Delta}}_x = diag(\boldsymbol{\delta}) = diag\left\{\delta_1^z, \delta_2^z, \ldots, \delta_N^z\right\}$, and $\tilde{\boldsymbol{S}}_{SV}$ is an $N \times K$ complex matrix. The problem to be solved by SBL is to estimate $\tilde{\boldsymbol{S}}_{SV}$ based on the known $\boldsymbol{X}_{SV}$ and $\tilde{\boldsymbol{A}}_x$.

III. SPARSE BAYESIAN ALGORITHM USING EXPECTATION-MAXIMIZATION APPROACH

In this section, the updates of parameters and hyperparameters are obtained from the perspective of SBL on the basis of the above methods. First, the prior distribution of the source signal and noise are presented, then the posterior probability density function $p\left(\tilde{\boldsymbol{S}}_{SV}|\boldsymbol{X}_{SV}, \beta, \boldsymbol{\gamma}; \tilde{\boldsymbol{\Delta}}_x\right)$ is derived, and finally the estimation of parameters and hyperparameters have been obtained via the EM algorithm.

A. PRIOR DISTRIBUTION ASSUMPTION

$\tilde{\boldsymbol{S}}_{SV}$ is only non-zero in the true DOA and zero for other redundant rows, so it is of row sparse. Supposed that each row of $\tilde{\boldsymbol{S}}_{SV}$ has a complex Gaussian distribution with zero mean, and the inverse variance vector is denoted by $\boldsymbol{\gamma} = [\gamma_1, \gamma_2 \ldots, \gamma_N]^T$. Let $\boldsymbol{\Gamma} = diag(\boldsymbol{\gamma})$, we have

$$p\left(\tilde{\boldsymbol{S}}_{SV}|\boldsymbol{\gamma}\right) = \prod_{k=1}^{K} \mathcal{CN}\left(\tilde{s}_k^{SV}|\boldsymbol{0}_{N\times 1}, \boldsymbol{\Gamma}^{-1}\right) \tag{10}$$

where $\tilde{s}_k^{SV}$ is the k-th column of $\tilde{\boldsymbol{S}}_{SV}$ and $\boldsymbol{0}_{m\times n}$ is an $m \times n$ zero matrix. Due to the Gamma hyperprior is conjugate to the Gaussian distribution, we assign Gamma distribution for the hyperparameters $\boldsymbol{\gamma}$:

$$p(\boldsymbol{\gamma}) = \prod_{n=1}^{N} \Gamma(\gamma_n; 1, o) \tag{11}$$

where o is a small positive constant. When o approaches 0, the probability density $p(\boldsymbol{\gamma})$ reaches its peak at the origin. Therefore, the two hierarchical priors are sparse priors that satisfy most rows of $\tilde{\boldsymbol{S}}_{SV}$ is $\boldsymbol{0}$. In the presence of Gaussian noise, we found that most $\boldsymbol{\gamma}_i$ tend to infinity and the rows of $\tilde{\boldsymbol{S}}_{SV}$ corresponding to these infinity $\boldsymbol{\gamma}_i$ tend to zero. thus the DOA can be estimated by searching the index where the small $\boldsymbol{\gamma}_i$ is located. Assumed that each elements in $\boldsymbol{N}$ has a complex white Gaussian distribution with a common variance σ^2, *i.e.*, $p\left(\boldsymbol{N}_{i,j}\right) = \mathcal{CN}\left(\boldsymbol{N}_{i,j}|0, \beta^{-1}\right)$, where $\beta = \sigma^{-2}$ denotes the noise precision. In general, the variance σ^2 and the precision β are unknown. Hence, β is modeled as a Gamma hyperprior:

$$p(\beta) = \Gamma(\beta; a, b) \tag{12}$$

where a and b are set close to zero. Thus we have

$$p\left(\boldsymbol{X}_{SV}|\tilde{\boldsymbol{S}}_{SV}, \beta; \tilde{\boldsymbol{\Delta}}_x\right) = \prod_{k=1}^{K} \mathcal{CN}\left(\boldsymbol{x}_{\boldsymbol{k}}^{SV}|\tilde{\boldsymbol{A}}_x\tilde{\boldsymbol{\Delta}}_x\tilde{\boldsymbol{s}}_{\boldsymbol{k}}^{SV}, \beta^{-1}\boldsymbol{I}_{M-2p+1}\right) \tag{13}$$

where $\boldsymbol{x}_{\boldsymbol{k}}^{SV}$ denotes n-th column of $\boldsymbol{X}_{SV}$, $\boldsymbol{I}_{m\times n}$ denotes $m \times n$ identify matrix.

B. PARAMETER AND HYPARAMETER ESTIMATION

Since the posterior probability density function $p\left(\tilde{\boldsymbol{S}}_{SV}|\boldsymbol{X}_{SV}, \beta, \boldsymbol{\gamma}; \tilde{\boldsymbol{\Delta}}_x\right)$ cannot be expressed explicitly, the EM algorithm is used for Bayesian inference.

1) E-STEP

Utilizing the Bayes rule and treating $\tilde{S}_{SV}$ as a hidden variable, the posterior distribution of is also a complex Gaussian:

$$\begin{aligned} &p\left(\tilde{\boldsymbol{S}}_{SV}|\boldsymbol{X}_{SV}, \beta, \boldsymbol{\gamma}; \tilde{\boldsymbol{\Delta}}_x\right) \\ &= p\left(\boldsymbol{X}_{SV}|\tilde{\boldsymbol{S}}_{SV}, \beta; \tilde{\boldsymbol{\Delta}}_x\right) \cdot p\left(\tilde{\boldsymbol{S}}_{SV}|\boldsymbol{\gamma}\right) / p\left(\boldsymbol{X}_{SV}|\beta, \boldsymbol{\gamma}; \tilde{\boldsymbol{\Delta}}_x\right) \\ &= \prod_{k=1}^{K} \mathcal{CN}\left(\tilde{\boldsymbol{s}}_{\boldsymbol{k}}^{SV}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}\right) \end{aligned} \tag{14}$$

where $\boldsymbol{\mu}_k$ is k-th column of $\boldsymbol{\mu}$and

$$\boldsymbol{\mu} = \beta\boldsymbol{\Sigma}\tilde{\boldsymbol{\Delta}}_{\boldsymbol{x}}^{\boldsymbol{H}}\tilde{\boldsymbol{A}}_x^{\boldsymbol{H}}\boldsymbol{x}_{SV} \tag{15}$$

$$\boldsymbol{\Sigma} = \left(\beta\tilde{\boldsymbol{\Delta}}_{\boldsymbol{x}}^{\boldsymbol{H}}\tilde{\boldsymbol{A}}_x^{\boldsymbol{H}}\tilde{\boldsymbol{A}}_x\tilde{\boldsymbol{\Delta}}_x + \boldsymbol{\Gamma}\right)^{-1} \tag{16}$$

The estimates of β, $\boldsymbol{\gamma}$ and $\tilde{\boldsymbol{\Delta}}_x$ are updated by maximizing $\langle \ln p(\boldsymbol{X}_{SV}, \tilde{\boldsymbol{S}}_{SV}, \beta, \boldsymbol{\gamma}; \tilde{\boldsymbol{\Delta}}_x)\rangle_{E^{(i)}}$, which can be expressed as:

$$\left\{\beta^{(i+1)}, \boldsymbol{\gamma}^{(i+1)}, \tilde{\boldsymbol{\Delta}}_x^{(i+1)}\right\} = \arg\max_{\beta,\boldsymbol{\gamma},\boldsymbol{\Delta}_{\boldsymbol{x}}} \langle \ln p(\boldsymbol{X}_{SV}, \tilde{\boldsymbol{S}}_{SV}, \beta, \boldsymbol{\gamma}; \tilde{\boldsymbol{\Delta}}_x)\rangle_{E^{(i)}} \tag{17}$$

or, equivalently:

$$\left\{\beta^{(i+1)}, \boldsymbol{\gamma}^{(i+1)}, \tilde{\boldsymbol{\Delta}}_x^{(i+1)}\right\} = \arg\max_{\beta,\boldsymbol{\gamma},\boldsymbol{\Delta}_{\boldsymbol{x}}} \langle \ln p(\boldsymbol{X}_{SV}, \tilde{\boldsymbol{S}}_{SV}, \beta, \boldsymbol{\gamma}; \tilde{\boldsymbol{\Delta}}_x) p(\tilde{\boldsymbol{S}}_{SV}|\boldsymbol{\gamma}) p(\beta) p(\boldsymbol{\gamma})\rangle_{E^{(i)}} \tag{18}$$

the superscript $(\cdot)^i$ represents the i-th iteration.

2) M-STEP

In this subsection, each unknown variable β, $\boldsymbol{\gamma}$, and $\tilde{\boldsymbol{\Delta}}_x$ are optimize by calculating (18).

i. For β, ignoring the independent terms, (18) can be simplified to:

$$\begin{aligned}&\langle \ln p(\boldsymbol{X}_{SV}, \tilde{\boldsymbol{S}}_{SV}, \beta, \boldsymbol{\gamma}; \tilde{\boldsymbol{\Delta}}_x) p(\beta)\rangle_{E^{(i)}} \\ &\quad = \langle \ln p(\boldsymbol{X}_{SV}, \tilde{\boldsymbol{S}}_{SV}, \beta, \boldsymbol{\gamma}; \tilde{\boldsymbol{\Delta}}_x)\rangle_{E^{(i)}} + p(\beta)\end{aligned} \quad (19)$$

Differentiating the above formula and setting its derivative with respect to β to zero, it can be obtained: ii. For $\boldsymbol{\gamma}$, ignoring the terms irrelevant of $\boldsymbol{\gamma}$, (18) becomes:

$$\langle \ln p(\tilde{\boldsymbol{S}} \mid \boldsymbol{\gamma}) p(\boldsymbol{\gamma})\rangle_{E^{(i)}} = \left\langle \ln p\left(\tilde{\boldsymbol{S}}_{SV} \mid \boldsymbol{\gamma}\right)\right\rangle_{E^{(i)}} + \langle \ln p(\boldsymbol{\gamma})\rangle_{E^{(i)}} \quad (21)$$

Differentiating (21) with respect to $\boldsymbol{\gamma}$, setting these derivatives to zero. the updates of γ_i can be given by

$$\gamma_n^{(i+1)} = \frac{K}{o + \sum_{k=1}^{K} \left[\boldsymbol{\Xi}_k^{(i)}\right]_{nn}}, \quad n = 1, 2, \ldots, N \quad (22)$$

where $\boldsymbol{\Xi}_k^{(i)} = \boldsymbol{\mu}_k^{(i)} \left(\boldsymbol{\mu}_k^{(i)}\right)^H + \boldsymbol{\Sigma}^{(i)}$∘

iii. For $\tilde{\boldsymbol{\Delta}}_x$, ignoring the independent terms, (18) can be written as:

$$\begin{aligned}&\langle \ln p(\boldsymbol{X}_{SV}, \tilde{\boldsymbol{S}}_{SV}, \beta, \boldsymbol{\gamma}; \tilde{\boldsymbol{\Delta}}_{\boldsymbol{x}})\rangle_{E^{(i)}} \\ &= -\beta \sum_{k=1}^{K} \left\| \boldsymbol{x}_k^{SV} - \tilde{\boldsymbol{A}}_x \tilde{\boldsymbol{\Delta}}_x^{(i)} \boldsymbol{\mu}_k^{(i)} \right\|_2^2 \\ &\quad - \beta K \operatorname{tr}\left[\tilde{\boldsymbol{A}}_x \tilde{\boldsymbol{\Delta}}_x^{(i)} \boldsymbol{\Sigma}^{(i)} \left(\tilde{\boldsymbol{A}}_x \tilde{\boldsymbol{\Delta}}_x^{(i)}\right)^H\right] + const.\end{aligned} \quad (23)$$

For the purpose of taking the derivative of the above equation with respect to $\tilde{\boldsymbol{\Delta}}_x$, the following lemma is utilized.

Lemma 1 (see [11]): For $N \times N$ complex matrix $\boldsymbol{E}$ which is diagonal and $N \times 1$ complex vector $\boldsymbol{X}$, we can obtain $\boldsymbol{E} \cdot \boldsymbol{X} = \boldsymbol{Q}_1(\boldsymbol{X}) \cdot \boldsymbol{e}$, where $\boldsymbol{e}_i = \boldsymbol{E}_{ii}, i = 1, 2, \ldots, N$; $\left\{[\boldsymbol{Q}(\boldsymbol{X})]_{ij} = \boldsymbol{X}_i, i = j; [\boldsymbol{Q}(\boldsymbol{X})]_{ij} = 0, otherwise\right\}$, $i = 1, 2, \ldots, N, j = 1, 2, \ldots, N$.

Due to the application of Lemma 1, there are:

$$\begin{aligned}&\left\| \boldsymbol{x}_k^{SV} - \tilde{\boldsymbol{A}}_x \tilde{\boldsymbol{\Delta}}_x^{(i)} \boldsymbol{\mu}_k^{(i)} \right\|_2^2 \\ &\quad = \left\| \boldsymbol{x}_k^{SV} - \tilde{\boldsymbol{A}}_x \boldsymbol{Q}\left(\boldsymbol{\mu}_k^{(i)}\right) \boldsymbol{\delta}_x \right\|_2^2\end{aligned} \quad (24)$$

$$\begin{aligned}&tr\left(\tilde{\boldsymbol{A}}_x \tilde{\boldsymbol{\Delta}}_x^{(i)} \boldsymbol{\Sigma}^{(i)} \left(\tilde{\boldsymbol{A}}_x \tilde{\boldsymbol{\Delta}}_x^{(i)}\right)^H\right) = \left\| \tilde{\boldsymbol{A}}_x \tilde{\boldsymbol{\Delta}}_x^{(i)} \boldsymbol{G}^{(i)} \right\|_2^2 \\ &\quad = \sum_{n=1}^{N} \left\| \tilde{\boldsymbol{A}}_x \tilde{\boldsymbol{\Delta}}_x^{(i)} \boldsymbol{g}_n^{(i)} \right\|_2^2 = \sum_{n=1}^{N} \left\| \tilde{\boldsymbol{A}}_x \boldsymbol{Q}\left(\boldsymbol{g}_n^{(i)}\right) \boldsymbol{\delta}_x \right\|_2^2\end{aligned} \quad (25)$$

where $\boldsymbol{G}^{(i)} \left(\boldsymbol{G}^{(i)}\right)^H$ is a Cholesky decomposition of $\boldsymbol{\Sigma}^{(i)}$ and $\boldsymbol{g}_n^{(i)}$ is the n-th column of $\boldsymbol{G}^{(i)}$. Substituting (24) and (25) into (23), calculating the derivative of $\boldsymbol{\delta}_x$ and setting these derivatives to zero, the updates of $\boldsymbol{\delta}_x$ can be given by:

$$\begin{aligned}\boldsymbol{\delta}_x = &\left\{ \sum_{k=1}^{K} \left[\tilde{\boldsymbol{A}}_x \boldsymbol{Q}\left(\boldsymbol{\mu}_k^{(i)}\right)\right]^{\boldsymbol{H}} \tilde{\boldsymbol{A}}_x \boldsymbol{Q}\left(\boldsymbol{\mu}_k^{(i)}\right) \right. \\ &\left. + K \sum_{\boldsymbol{n}=1}^{\boldsymbol{N}} \left[\tilde{\boldsymbol{A}}_x \boldsymbol{Q}\left(\boldsymbol{g}_n^{(i)}\right)\right]^{\boldsymbol{H}} \tilde{\boldsymbol{A}}_x \boldsymbol{Q}\left(\boldsymbol{g}_n^{(i)}\right) \right\}^{-1} \\ &\cdot \left\{ \sum_{\boldsymbol{k}=1}^{\boldsymbol{K}} \left[\tilde{\boldsymbol{A}}_x \boldsymbol{Q}\left(\boldsymbol{\mu}_k^{(i)}\right)\right]^{\boldsymbol{H}} \boldsymbol{x}_{\boldsymbol{k}}^{SV} \right\}\end{aligned} \quad (26)$$

The EM algorithm is carried out by repeating (20), (22), and (26) until the error reaches a certain value. Once the algorithm is valid, the DOA can be estimated from the updates $\boldsymbol{\gamma}$. It is observed that most $\boldsymbol{\gamma}$ tend to infinity, and the indexes of small $\boldsymbol{\gamma}$ correspond to the real DOAs. The estimation of the elevation angles of z-axis is the same as the one of the azimuth angles of x-axis and the angle pair-matching can be done by using the method of [33], it does not be derived additionally in this paper.

C. COMPUTATIONAL COMPLEXITY COMPARISON

After the previous derivations, the computational complexity of the proposed algorithm is analyzed. In this paper, the data is complexed. As the complexity of complex multiplication is greater than that of complex addition, we only discuss the complexity of multiplication. According to Section III.B, it is known that the complexity of our method is mainly determined by (15), (16), (20), (22), (26), and the SVD process. The complexity of the calculation of the multiplication of $\tilde{\boldsymbol{A}}_x \tilde{\boldsymbol{\Delta}}_x$ is $O\left\{N^2 (M - 2p + 1)\right\}$. Then, the computational complexity which includes the calculation of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ in (15) and (16) is: $O\left\{N^3 + \left(2N^2 + NK\right)(M - 2p + 1)\right\}$; the construction of β, $\boldsymbol{\gamma}$, and $\boldsymbol{\delta}_x$ in (20), (22), and (26) is: $O\left\{N^3 + 2N^2 + 3N^2 (M - 2p + 1) + N(M - 2p + 1)(M - 2p + 2)\right\}$; and the complexity of the SVD in (7) is: $O\left\{(M - 2p + 1)^3\right\}$.

For the conventional SBL method without SVD, the computational complexity of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ is $O\{N^3 + (2N^2 + NT)(M - 2p + 1)\}$ which is larger than the proposed algorithm. The computational complexity of calculating other updates is the same as those of the algorithm in this paper. In (26), the parameter K is replaced by T. As $T \gg K$, the conventional SBL method is more time-consuming than our algorithm. Under normal circumstances, $N \gg M \gg K$ for 2-D DOA estimation. Therefore, the total computational complexity in this paper can be approximated as $O\left\{2N^3 + 6N^2 (M - 2p + 1)\right\}$ per iteration. In addition, the computational complexity of the augmented PM in [18] is about $O\left\{4(M - 2p + 1)^2 T + 28(M - 2p)^2 K\right\}$. The computational complexity of the method in [19] is about $O\left\{2(M - 2p + 1)^2 T + 2(M - 2p)^2 K\right\}$, and that of the RARE method in [14] is calculated as $O\left\{(2M + 1)^2 T + 2N^\theta N^\varphi (p + \zeta + 1)(2M + 1)^2\right\}$ (where N^θ and N^φ represent the search times of azimuth and elevation, we often set them equal to N). As $N \gg M$ and the SBL method requires about 50 iterations, the computational complexity of the proposed method is larger than that of the other methods.

IV. SIMULATIONS AND DISCUSSION

In this section, several simulations are carried out in order to illustrate the superiority of our method in comparison with the RARE method in [14], the augmented PM in [18], and the method in [19]. Unless stated otherwise, $o = 0.01$, $a = b = 0.001$. The iteration is stop when $\|\hat{\gamma}_{i+1} - \hat{\gamma}_i\|_2 / \|\hat{\gamma}_{i+1}\|_2 < \varepsilon$ or the proposed method reaches the maximum number of iterations, where ε represents the convergence threshold. ε is set to 0.001 and the maximum number of iterations is set to 1000. The grid interval is chosen as $r = 1°$. The interval of adjacent element of the LSA is $d = \lambda/2$. The power of each source is equal to σ_s^2 and the signal to noise ratio (SNR) is equal to $10log_{10}\left(\sigma_s^2/\sigma_n^2\right)$. The search range of angle is $[0°, 180°]$. To verify the overall performance of DOA estimation, the RMSE of 2-D angle estimation is given as

$$RMSE = \sqrt{\frac{1}{KP}\sum_{w=1}^{W}\sum_{k=1}^{K}\left(\left(\hat{\theta}_k^{(w)} - \theta_k\right)^2 + \left(\hat{\varphi}_k^{(w)} - \varphi_k\right)^2\right)}$$

where W denotes Monte Carlos trials. In all simulations, M is fixed at $M = 8$. The mutual coupling coefficients are c_1, c_2 and c_3 with distances $d, 2d, \sqrt{2}\,d$. The settings of the mutual coupling coefficients is limited by the following rules:

i. As the distance of the array element increases, the mutual coupling coefficient gradually decreases and it can be approximated to zero when the distance exceeds a certain threshold.

ii. If the distance between the adjacent elements is equal, the mutual coupling coefficient between the neighboring elements with same distance is almost the same.

Experiment 1: DOA estimation of different methods

First, the estimated DOAs of the proposed method is compared with the RARE method in [14], the augmented PM method in [18], and the method in [19]. The sources are placed at $(20°, 25°)$ and $(50°, 45°)$ with $SNR = 10$dB and $T = 100$. The mutual coupling coefficients are $c_1 = 0.2 - 0.2i$, $c_2 = 0.1 + 0.09i$ and $c_3 = 0.1 - 0.2i$.

As shown in Fig.2 (a), Fig.2 (c) and Table 1, it is shown that the PM method has the largest error in DOA estimation. The RARE method, the augmented PM method and the method in [19] can estimate elevation and azimuth angle jointly. The RARE method gives a three-dimensional spatial spectrum, which requires a 2-D spectral peak search while the augmented PM method and the method in [19] does not need. Although the RARE method and our method can estimate DOA perfectly, the RARE method will produce false peaks, which proves that our method has a more exceptional estimation performance than the other methods. Then, the DOA estimation of the SBL algorithm is also studied for a single beam in Fig.2 (b). The source is placed at $(20°, 25°)$ and other settings are the same as before. The result shows that our method can estimate DOA accurately.

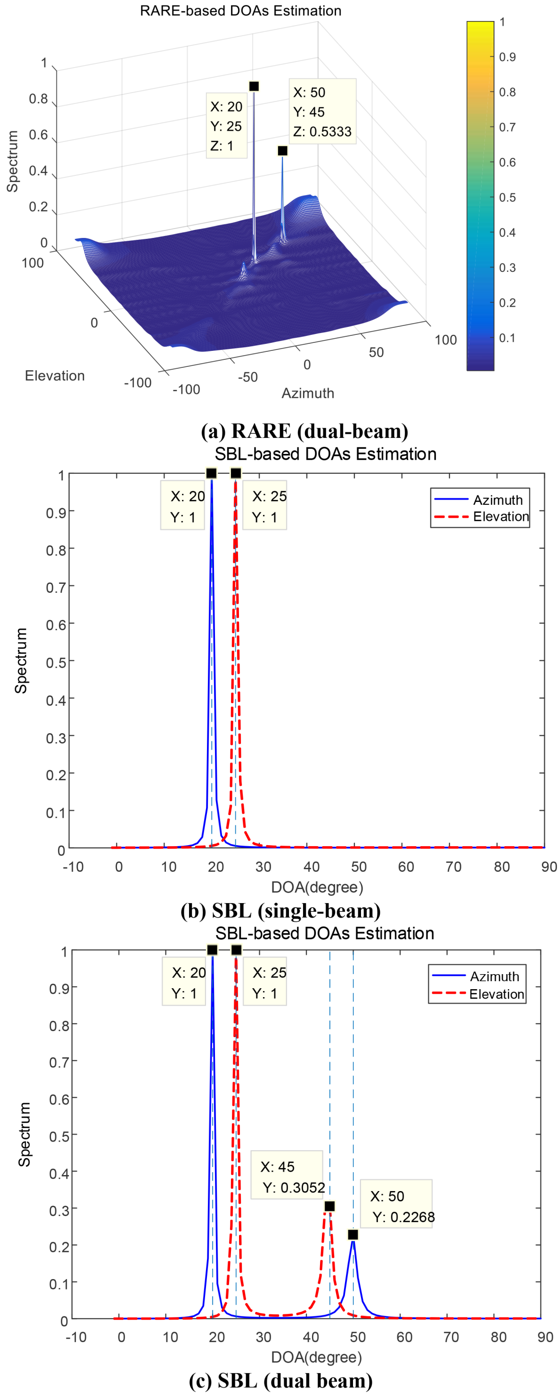


FIGURE 2. Spatial spectrum of different methods.

$$\beta^{(i+1)} = \frac{KM + (a-1)}{b + \sum_{k=1}^{K}\left\|\boldsymbol{x}_k^{SV} - \tilde{\boldsymbol{A}}_x\tilde{\boldsymbol{\Delta}}_x^{(i)}\boldsymbol{\mu}_k^{(i)}\right\|_2^2 + K\,\mathrm{tr}\left[\tilde{\boldsymbol{A}}_x\tilde{\boldsymbol{\Delta}}_x^{(i)}\boldsymbol{\Sigma}^{(i)}\left(\tilde{\boldsymbol{A}}_x\tilde{\boldsymbol{\Delta}}_x^{(i)}\right)^H\right]} \tag{20}$$

TABLE 1. Estimated DOAs.

Methods	Source1	Source2
Our method	$(20°, 25°)$	$(50°, 45°)$
RARE	$(20°, 25°)$	$(50°, 45°)$
Augmented PM	$(36°, 25°)$	$(48°, 44°)$
Method in [19]	$(20.1°, 24.8°)$	$(50.6°, 44.8°)$

Experiment 2: estimated performance vs. the mutual coupling

In this experiment, some researches about the effect of mutual coupling on spatial spectrum are done in Fig.3. The sources are placed at $(20°, 25°)$ and $(50°, 45°)$ with $T = 300$ and $SNR = 10$dB. The mutual coupling coefficients are $c_1 = 0.1 - 0.1i$, $c_2 = 0.04 + 0.03i$, $c_3 = 0.06 - 0.04i$ in Fig.3(a), which represents the mutual coupling effect is weak. The mutual coupling effect is medium with $c_1 = 0.2 - 0.2i$, $c_2 = 0.1 + 0.1i$, $c_3 = 0.2 - 0.1i$ in Fig.3(b). As shown in Fig.3(c), the effect is strong and $c_1 = 0.9 - 0.4i$, $c_2 = 0.5 + 0.5i$, $c_3 = 0.7 - 0.5i$.

As shown in these figures, no matter the mutual coupling effect is weak, medium or strong, our method can still estimate DOA accurately. It indicates that the proposed algorithm is robust to mutual coupling effects, since we can estimate $\boldsymbol{\delta}_x$ and DOA estimation is hardly affected by mutual coupling.

Experiment 3: estimated performance vs. the initial value

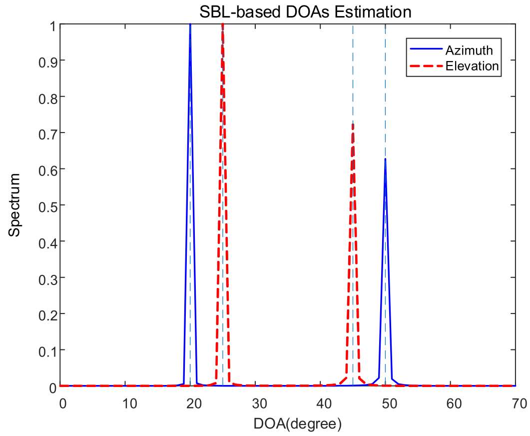
Here, the simulation explores the influence of the initialization of the EM algorithm on the DOA estimation. The settings are the same as Experiment 1 except for the initial values of SBL algorithm. In Fig.4(a), $a = 0.001$, $b = 0.001$, and $o = 0.01$. As shown in Fig.4(b), $a = 0.1$, $b = 0.001$, and $o = 0.01$. $a = 0.001$, $b = 0.1$, $o = 0.01$ in Fig.4(c) and $a = 0.001$, $b = 0.001$, $o = 0.1$ in Fig.4(d).

As shown in these figures, if b and o are close to zero, the proposed method can estimate the DOA effectively when a, b, or o changes. Otherwise, $\boldsymbol{\Sigma}$ is non-positive definite and the algorithm is fail to work. In addition, if all elements of $\boldsymbol{\gamma}$ are equal to 0, the Cholesky decomposition of $\boldsymbol{\Sigma}$ cannot be obtained, which causes the interruption of the iteration.

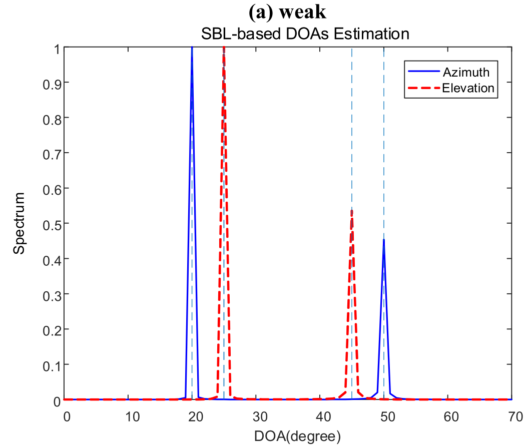
Experiment 4: estimated performance vs. SNR

Then, the performance of the four method against SNR under uncorrelated signals is investigated. The incident angles are $(20°, 25°)$ and $(50°, 45°)$. The snapshots is $T = 100$ and SNR is set from -5dB to 10dB, the mutual coupling coefficients are $c_1 = 0.2 - 0.2i$, $c_2 = 0.1 + 0.09i$ and $c_3 = 0.1 - 0.2i$ and the Monte Carlo trials is set to 100. Fig.5 shows the RMSEs of the three method.

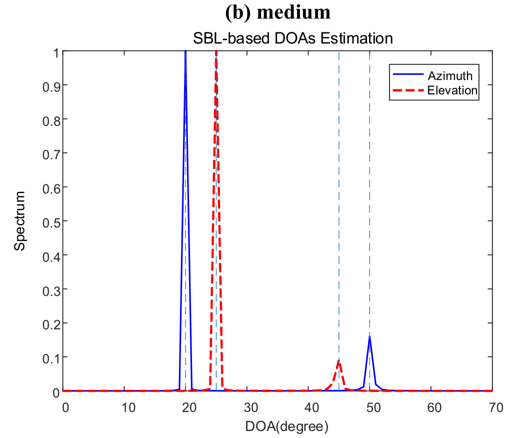
This figure demonstrates that the proposed method possesses the most excellent performance at low SNR and the RMSE converges to zero fastest. The RMSE of out method is close to zero when the SNR is -3dB. This is because we make use of the Student t prior with a hierarchical form, which can enhance the sparsity of source signals, and we utilize the SVD method to speed up the SBL procedure and reduce the sensitivity to noise.



(a) weak



(b) medium



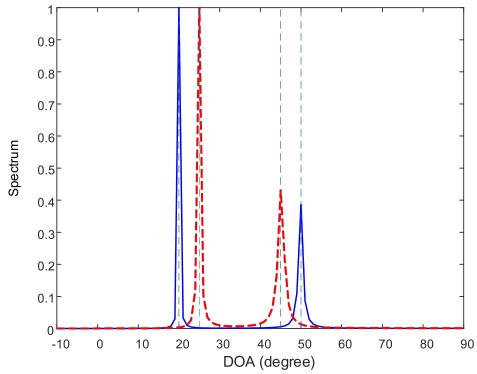
(c) strong

FIGURE 3. Spatial spectrum with different mutual coupling.

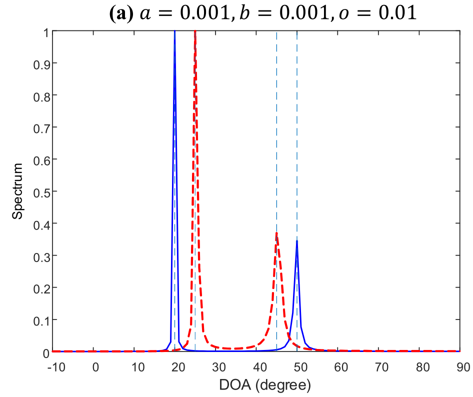
Experiment 5: estimated performance vs. snapshots

Here, the 2D DOA estimation performance versus the snapshots is examined. SNR is set to 0dB and the snapshots ranges from 100 to 1000 with 100 Monte Carlos trials. Other settings are the same as those in Experiment 4. The result is shown in Fig.6. It is indicated that our method retains a perfect estimation performance and is still outstanding than RARE method, the augmented PM method and the method in [19] though the loss in array aperture.

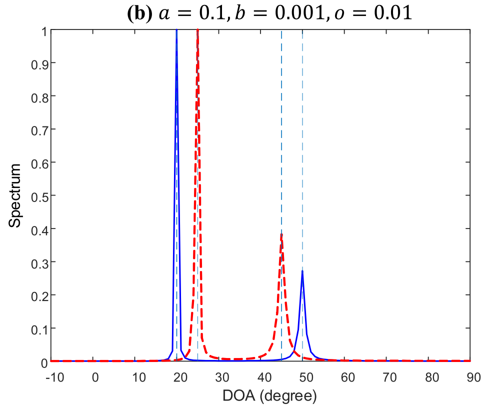
Experiment 6: estimated performance vs. correlation factor



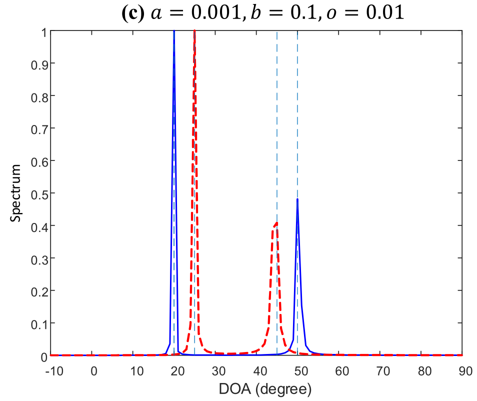
(a) $a = 0.001, b = 0.001, o = 0.01$



(b) $a = 0.1, b = 0.001, o = 0.01$



(c) $a = 0.001, b = 0.1, o = 0.01$



(d) $a = 0.001, b = 0.001, o = 0.001$

FIGURE 4. Spatial spectrum with different initialization.

In this experiment, a simulation about the relationship between the correlation coefficient ρ and the RMSE of DOA estimation is done. The correlation coefficient ranges from 0 (uncorrelated) to 1 (coherent) and the other settings are similar to those in Experiment 4 except that SNR is set to 5dB. The results are illustrated in Fig.7.

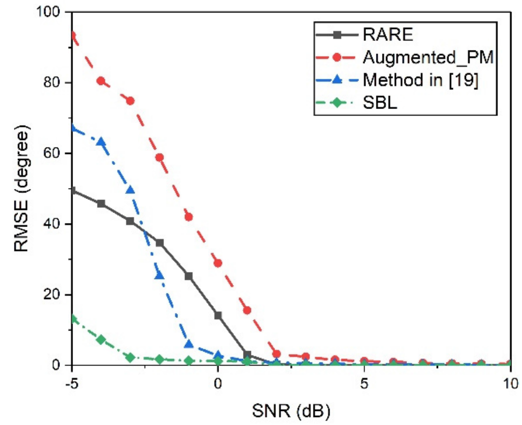


FIGURE 5. RMSE of the DOA estimation against SNR.

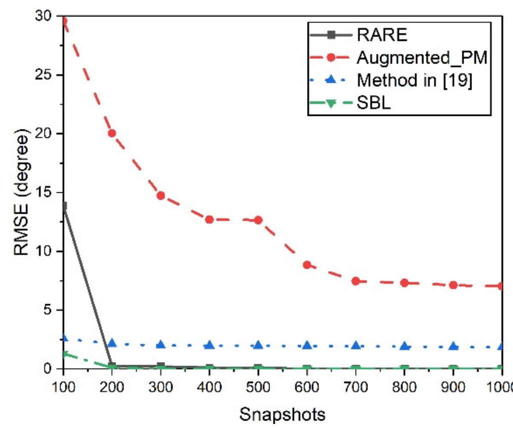


FIGURE 6. RMSE of the DOA estimation against Snapshot.

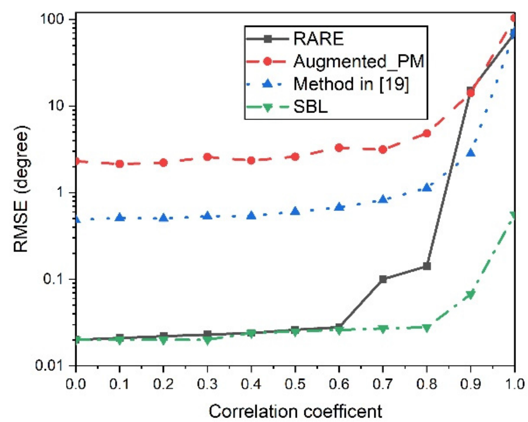


FIGURE 7. RMSE of the DOA estimation against correlation coefficient.

It is noticed that SBL algorithm is more robust than the other three methods when the signals are strongly correlated ($\rho = 0.9$). What's more, when the signals are coherent

($\rho = 1$), the proposed method can still estimate DOAs effectively while the other methods cannot work.

V. CONCLUSION

In this paper, the paper propose a new method to estimate DOA for L-shaped with unknown mutual coupling. It is achieved by using SBL algorithm. Two select matrices are used to split the LSA into two ULAs and to eliminate the mutual coupling between the subarrays. In comparison with the state-of-art methods, the RMSE of our algorithm is smaller, and it is more robust to noise and more suitable in the scenario of few snapshots. Simulation results have verified the efficiency of the proposed algorithm with different mutual coupling effect and demonstrated that it can handle highly correlated and coherent signals.

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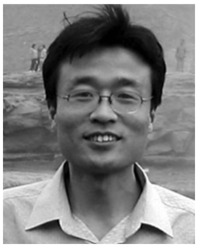

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