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Decentralized Estimation of Two-Port Equivalents for Power Transmission Lines

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ABSTRACT Two-port equivalent model of the system around a transmission line is commonly used for various line protection related analyses. The main aim of the paper is to present methods that can be adopted in a decentralized manner at the substation level for estimating such an equivalent model. Firstly, the paper contributes to methods for estimation of simple two-source equivalents based on events such as fault on the line or shunt element switching at its terminals. The paper extends a discussion towards limitations of such models in the presence of a transfer path across the line terminals and proposes an extension of the estimation techniques to overcome the issue. Lastly, the paper proposes methods for updating the two-port equivalent model when network topology in the vicinity of the line of concern changes. This contribution differs in the aspect that it only seeks to update the two-port equivalent model, unlike the previous contributions wherein the aim is to estimate the equivalent model without assuming the availability of an initial solution. The methods for updating utilize the equivalent model parameters in a base network scenario and the measurements excited from a topology event. The proposed methods use only limited measurements of bus voltages and line currents, and model parameters of the line of concern and other neighboring apparatus (such as other incident lines or shunt reactors/capacitors). This is an advantage as the availability of the complete network model at any substation is always a challenge. Therefore, the methods can be deployed directly at substations in a decentralized manner. The proposed solutions are computationally easy and amenable for implementation in the framework of digital substations with advanced communication infrastructure. Testing and analysis are done using a simple two source transmission system and the IEEE 39 bus test system.

INDEX TERMS Network equivalencing, protection relaying, source impedances, transmission line, two-port Thevenin equivalent, transmission line fault.

I. INTRODUCTION

Various analyses in power transmission line protection depend on an equivalent model of the system as seen from the line terminals. Fig. 1(a) shows the schematic of a transmission line connected between the terminal buses M and N. Typically, in interconnected high voltage transmission systems, each such line will be connected to the rest of the network through other transmission lines. Depending on the operational requirement, shunt elements such as a reactor or capacitor may also be connected to the buses. The load element may represent the downstream lower voltage level network and/or a direct HT load tapping.

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An equivalent model of the system is shown in Fig. 1(b), in which all components of the system (except the line of concern) are captured in the two-port Thevenin equivalent. The equivalent system consists of two sources with corresponding impedances Z_{sM} and Z_{sN} and a transfer path with impedance Z_{tr}^{MN}. Often, we come across a simple two-source equivalent in which the transfer path is not present. This could be due to a simplifying assumption that the transfer path is negligible. However, this assumption may not hold for all cases as shall be described later in the paper.

The equivalent model can be used for various analyses. For example, it can be used for analyzing distance relay reach for setting its operational characteristics and for determining the source to line impedance ratio (SIR) for the relay [1]. The model can also be used for setting the power swing

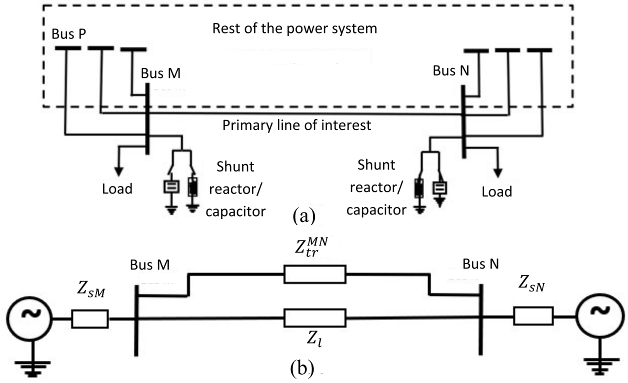

FIGURE 1. (a). Schematic of a typical transmission line network topology (b) Two-port Thevenin equivalent across the primary line of interest.

blinders and out of step logic [2]. Also, the model parameters can help determine system non-homogeneity and facilitate fault location algorithms [3]; especially, algorithms that use one-terminal voltage and current data and methods which use two-ended current measurements only [4]. Note that the parameters of the two-port equivalent depend on the network topology and may need to be updated from time to time as the network topology changes.

It is a well-known concept that short circuit analysis based on the complete network information can yield the equivalent model parameters [2] for any line of interest. The equivalent model can also be calculated if the full network topology and/or the network impedance matrix is available as an input. If we assume that the network (with nb buses) impedance matrix Z'_{bus} not including the line of interest is given by (1a), then we can calculate the equivalent model parameters using only the impedance submatrix of interest, i.e., the elements of Z'_{bus} which correspond only to buses M and N as depicted in (1b) and (1c).

$$Z'_{bus} = \begin{bmatrix} Z'_{11} & Z'_{12} & \cdots & \cdots & \cdots & \cdots & Z'_{1n} \\ Z'_{21} & Z'_{22} & \cdots & \cdots & \cdots & \cdots & Z'_{2n} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & Z'_{mm} & Z'_{mn} & \vdots & \vdots \\ \vdots & \cdots & \cdots & Z'_{nm} & Z'_{nn} & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z'_{nb1} & Z'_{nb2} & \cdots & \cdots & \cdots & \cdots & Z'_{nbnb} \end{bmatrix} \tag{1a}$$

$$\begin{bmatrix} \frac{1}{Z_{sM}} + \frac{1}{Z_{tr}^{MN}} & \frac{-1}{Z_{tr}^{MN}} \\ \frac{-1}{Z_{tr}^{MN}} & \frac{1}{Z_{sN}} + \frac{1}{Z_{tr}^{MN}} \end{bmatrix} = \begin{bmatrix} Z'_{mm} & Z'_{mn} \\ Z'_{nm} & Z'_{nn} \end{bmatrix}^{-1} \tag{1b}$$

$$\Rightarrow Z_{sM} = \frac{Z'_{mm}Z'_{nn} - Z'_{mn}Z'_{nm}}{Z'_{nn} - Z'_{mn}},\ Z_{sN} = \frac{Z'_{mm}Z'_{nn} - Z'_{mn}Z'_{nm}}{Z'_{mm} - Z'_{mn}}$$
$$\text{and } Z_{tr}^{\text{MN}} = \frac{Z'_{mm}Z'_{nn} - Z'_{mn}Z'_{nm}}{Z'_{mn}}. \tag{1c}$$

It immediately follows that for any change in the network topology, the updated Z'_{bus} can be used to re-evaluate or update the equivalent model information.

However, such analyses may not be feasible at a substation level due to a lack of information of the entire network topology and topology updates from all over the system. Decentralized analyses using only locally available information without depending on repetitive updates from the central control center of the grid is a practical requirement for substation automation. In [2], a method using the information of the short circuit current levels at the line terminals and contribution to bolted fault currents at each terminal from the line to approximately compute the equivalent model. Although the method is suitable for a substation level implementation, it might be difficult to obtain the required data at both the buses concurrently unless one stage short circuit experiments at each bus every time a topology change occurs.

There are other substation level approaches that can be considered as close prior art. These methods [5], [6] suggest placing bolted short circuits on a terminal of the line and estimating source impedance by calculating the ratio of bus voltage drop from the nominal value to the change in current seen by the source. Such approaches solve the limited problem of obtaining the source impedance magnitude as will be seen by a protective relay placed at a terminal of the line. Moreover, they face the same challenge as in the previous method that a fault must be created in the system during normal operation. In a broader sense, they fall under the umbrella of techniques using superimposed quantities [7] in which incremental voltage and current phasor measured at a line terminal due to a fault event are used to obtain a source impedance at that terminal.

This paper contributes to solution methodologies for the problem of estimation of two-port equivalent which can be adopted in a decentralized manner at the substation level. Specifically, the contributions can be summarized as follows: (i) based on measurements related to events such as line faults and shunt injection at the line terminals, we present methods for estimating a simple two-source equivalent, (ii) a discussion on the limitation of two-source equivalents, based on which we propose a concept for extending the methods for estimating the two-port Thevenin equivalent of Fig. 1(b), (iii) lastly, we propose methods for updating the equivalent two-port model in response to topology changes occurring in the immediate neighbourhood of the line. While in contributions (i) and (ii) we do not assume the availability of any initial guess for the equivalent model parameters, in the last contribution, we propose to use the equivalent model parameters in a base network scenario, to update the model after a topology change event.

The significance of contribution (i) lies in the understanding that with the amount of information limited to measurements at line terminals and line parameter data, a two-source equivalent might appeal as a plausible solution. However, such simplified equivalents may pose problems in cases where the transfer path is not negligible. This leads us to contribution (ii) in which we attempt to account for the transfer path as well. In doing so we also assume the availability of bus Thevenin impedances at the line terminals. The third contribution of this paper differentiates it from the existing literature in a significant way as it enables an automatic update of the two-port equivalent following a topology event, thus avoiding the need to re-estimate it for the modified network topology. It is based on the consideration that although we may not assume the availability of the entire network model at a substation level, information of the adjacent lines and buses, i.e., network information of the adjacent topology can be made available at a substation. In such a case, the effect of topology changes at least in the neighborhood of the line of interest can be accounted for by updating the two-port equivalent. The lack of a network model is compensated for by using accurate and time-synchronized measurements from neighbouring substations. Sharing of such data is encouraged in the paradigm of wide-area measurements, or other data mechanisms such as R-GOOSE, R-SV, etc., and digital substations with advanced communication infrastructure [8], [9]. The framework of the proposed solutions is depicted in Fig. 2. The proposed solutions are computationally easy. They form potentially useful tools for substation automation and reduce dependence on regular updates of the two-port equivalent from the main control center of the grid. Testing and analysis are done using a simple two source transmission system and the IEEE 39 bus test system. Application of solution for updating the two-port network equivalent in adaptive distance relay setting is demonstrated.

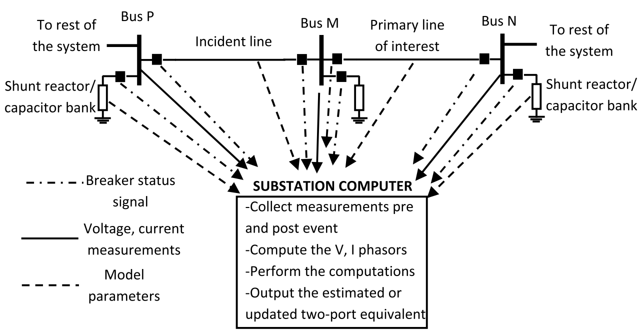

FIGURE 2. Framework of the proposed methodologies for estimating and updating the two-port equivalent.

II. PROPOSED METHOD FOR ESTIMATION OF TWO-PORT EQUIVALENT

In this section, we present methodologies to estimate a two-port equivalent across a transmission line based on events such as faults on the line and shunt injection events at its terminals. The main aim of these methods is to obtain a two-port equivalent without assuming any *a priori* information of the equivalent model parameters. The approach is to estimate them using only a limited amount of substation level measurements and data. Specifically, they are the terminal bus voltage and line current measurements at both ends of the line, currents on lines incident on the terminal buses, current flow through the shunt elements at the buses, and model parameters of the line of concern. We discuss these methods in two stages: (a) estimation of simple two-source equivalent, neglecting the effect of the transfer path, (b) extending the methodologies to estimate the two-port Thevenin equivalent by using the additional information of terminal bus Thevenin impedances.

A. ESTIMATION OF TWO-SOURCE EQUIVALENT

The methods described here utilize measurements following a fault on the line or a shunt element switching event to estimate source impedances of a two-source equivalent. The measured changes in bus voltages due to the extra current injection resulting from the event are utilized. We divide the discussion into two parts: (i) when the event is a line fault, (ii) when the event is a shunt element switching at a terminal of the line.

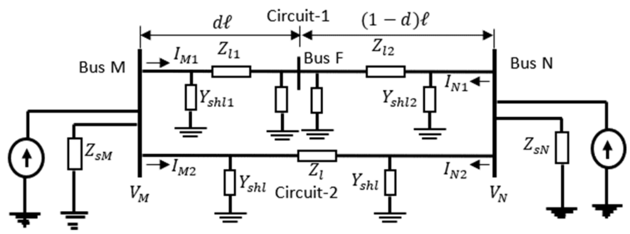

FIGURE 3. Two-source equivalent model with the primary transmission line of concern between buses M and N represented by pi-model.

1) FAULT ON THE LINE

Let us consider the network shown in Fig. 3 which depicts a simple two source equivalent across the line between buses M and N. Node 'F' is a fictitious node that represents the point of occurrence of a fault on the line. Note that a double-circuit line is used for the ensuing analysis and discussion as it is very common in high voltage transmission. However, the analysis is applicable for a single circuit line also as will be indicated later in the discussion. A fault at Bus F can be considered as injection of fault current I_{flt} at the same bus due to which, the voltages at terminal buses M and N undergo change. Let us define the impedances Z_{MF} and Z_{NF} as shown below in (2).

$$Z_{MF} := \frac{V_M^0 - V_M^f}{I_{flt}}, \quad Z_{NF} := \frac{V_N^0 - V_N^f}{I_{flt}} \tag{2}$$

Symbols V and I represent voltage and current phasors respectively, which can be obtained by applying any fundamental phasor extraction technique such as discrete Fourier transform or the method of least squares. A '0' in superscript denotes a steady state and 'f' denotes faulted state quantity measured at the bus (M or N) indicated by the subscript. Neglecting currents in the shunt admittance branches of the line, changes in the bus voltages due to the fault current injection can be expressed as:

$$V_M^0 - V_M^f = I_{flt} Z_{sM} \frac{\frac{Z_{l2}}{(Z_{sum}/Z_l)} + Z_{sN}}{Z_{par} + (Z_{sM} + Z_{sN})} \tag{3a}$$

$$V_N^0 - V_N^f = I_{flt} Z_{sN} \frac{\frac{Z_{l1}}{(Z_{sum}/Z_l)} + Z_{sM}}{Z_{par} + (Z_{sM} + Z_{sN})} \tag{3b}$$

where $Z_{sum} = Z_l + Z_{l1} + Z_{l2}$ and $Z_{par} = \frac{Z_l(Z_{l1}+Z_{l2})}{Z_{sum}}$. From (3a) and (3b) we obtain the following expressions:

$$Z_{MF} = Z_{sM} \frac{\frac{Z_{l2}}{(Z_{sum}/Z_l)} + Z_{sN}}{Z_{par} + (Z_{sM} + Z_{sN})} \tag{4a}$$

$$Z_{NF} = Z_{sN} \frac{\frac{Z_{l1}}{(Z_{sum}/Z_l)} + Z_{sM}}{Z_{par} + (Z_{sM} + Z_{sN})} \tag{4b}$$

Equations (4a) and (4b) when solved to find Z_{sM} and Z_{sN} yield the following result as shown in (5) below.

$$Z_{sM} = \frac{-b + \sqrt{b}^2 - 4ac}{2a},$$
$$Z_{sN} = \frac{(Z_{par} + Z_{sM})Z_{MF} - \frac{Z_{l2}}{(Z_{sum}/Z_l)} Z_{sM}}{Z_{sM} - Z_{MF}} \tag{5}$$

where, $a = Z_{MF} - Z_{NF} - \frac{Z_{l2}}{(Z_{sum}/Z_l)}$, $b = Z_{par}Z_{MF} + \frac{Z_{l1}}{(Z_{sum}/Z_l)}(Z_{MF} - Z_{NF}) - \frac{Z_{l1}Z_{l2}}{(Z_{sum}/Z_l)^2}$, $c = \frac{Z_{l1}}{(Z_{sum}/Z_l)} Z_{par} Z_{MF}$.

Assuming equivalent pi circuit model of the transmission line [10], $Z_l = Z_c \sinh(\gamma l)$, $Z_{l1} = Z_c \sinh(\gamma dl)$, $Z_{l2} = Z_c \sinh(\gamma(1-d)l)$, where Z_c is the characteristic impedance and γ is the propagation constant of the line. As evident, Z_{l1} and Z_{l2} and all related quantities such as Z_{sum}, Z_{par}, a, b, and c are dependent on the fault location 'd', which is therefore required to be estimated first. It can be found by using the principle that the fault point voltage calculated using measurements from either end of the line should be the same [11]. This leads to the following formula for 'd':

$$d = \frac{1}{\gamma l} \tanh^{-1} \left(\frac{V_N^f \cosh(\gamma l) + (I_{N1}^f Z_c) \sinh(\gamma l) - V_M^f}{V_N^f \sinh(\gamma l) - (I_{N1}^f Z_c) \cosh(\gamma l) - Z_c I_{M1}^f} \right) \tag{6}$$

Fault current I_{flt} (considered positive away from the bus 'F') can be calculated once the fault location is obtained. To do this, we obtain fault current contribution from each terminal bus M and N and then add them up to find the total fault current as shown below in (7).

$$I_{flt}^M = -\frac{1}{Z_c} \sinh(\gamma dl) V_M^f + \cosh(\gamma dl) I_{M1}^f \tag{7a}$$

$$I_{flt}^N = -\frac{1}{Z_c} \sinh(\gamma(1-d)l) V_N^f + \cosh(\gamma(1-d)l) I_{N1}^f \tag{7b}$$

$$I_{flt} = I_{flt}^M + I_{flt}^N \tag{7c}$$

Below we provide a summary of steps for this solution which we shall call SE1 in the rest of the paper:

1. Obtain fault location d by using two-ended voltage, line current measurements, and line parameters as in (6).
2. Obtain the fault current I_{flt} by using the fault location d and the two ended voltage and line current measurements as in (7).
3. Obtain Z_{MF} and Z_{NF} as shown in (2) with measured change in respective bus voltage and the calculated fault current.
4. Calculate impedances Z_l, Z_{l1}, Z_{l2} using the line parameters and the fault location d.
5. Solve for Z_{sM} and Z_{sN} using (5).

Once impedances of the two-source equivalent are determined, the equivalent source voltages can be calculated using the appropriate KVL formulations. For instance, in this case, they can be determined as:

$$E_{sM} = V_M^0 + \left(\frac{V_M^0 - V_N^0}{Z_l} \right) Z_{sM};$$
$$E_{sN} = V_N^0 - \left(\frac{V_M^0 - V_N^0}{Z_l} \right) Z_{sN}$$

Remarks: In deriving (3), currents in the shunt admittance branches were ignored. In order to take them into account, the following corrections need be done: To Z_{MF} and Z_{NF} as obtained in step 3 above, multiply $\xi = \frac{1}{1 - Z_{FF} Y_{shd}}$ as a correction factor where, $Z_{FF} = \frac{V_F^0 - V_F^f}{I_{flt}}$ and $Y_{shd} = Y_{shl1} + Y_{shl2} = \frac{1}{Z_c} \tanh\left(\frac{\gamma dl}{2}\right) + \frac{1}{Z_c} \tanh\left(\frac{\gamma(1-d)l}{2}\right)$. Note that once the fault location d is calculated, the fault point voltage V_F can be found by using V_M and I_{M1} measurements in the pi-equivalent model equations for the line section MF of the faulted circuit.

For a single circuit transmission line, we consider impedance Z_l of the unfaulted circuit of Fig. 3 as $Z_l = \infty$. This makes the term $Z_{sum}/Z_l = 1$ and $Z_{par} = Z_{l1} + Z_{l2}$. Substitute these values in (5) to obtain Z_{sM} and Z_{sN}.

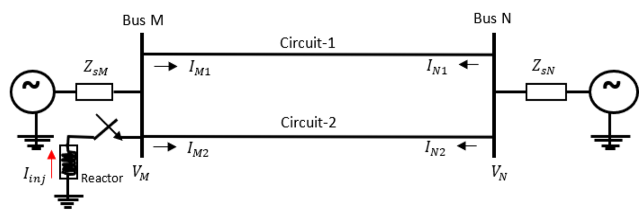

FIGURE 4. Two-source equivalent model with the primary transmission line of concern between buses M and N and shunt reactor switching.

2) SHUNT INJECTION AT A TERMINAL

Let us now consider current injection due to a shunt element (reactor/capacitor/load) switching event on a terminal bus, say bus M as shown in Fig. 4. Let us define the following impedances:

$$Z_{MF} := \frac{V_M^0 - V_M^I}{I_{inj}}, \quad Z_{NF} := \frac{V_N^0 - V_N^I}{I_{inj}} \tag{8}$$

Here, I_{inj} represents the shunt current injection at the bus (considered positive when measured in the direction *into* the bus). An 'I' in the superscript denotes quantity after the injection event occurred. Changes in the bus voltages due to the current injection can be expressed as:

$$V_M^0 - V_M^I = I_{inj} Z_{sM} \frac{(Z_l/2) + Z_{sN}}{(Z_l/2) + (Z_{sM} + Z_{sN})} \tag{9a}$$

$$V_N^0 - V_N^I = I_{inj} Z_{sN} \frac{Z_{sM}}{(Z_l/2) + (Z_{sM} + Z_{sN})} \quad (9b)$$

From (9a) and (9b) we obtain the following expressions:

$$Z_{MF} = Z_{sM} \frac{(Z_l/2) + Z_{sN}}{(Z_l/2) + (Z_{sM} + Z_{sN})} \quad (10a)$$

$$Z_{NF} = Z_{sN} \frac{Z_{sM}}{(Z_l/2) + (Z_{sM} + Z_{sN})} \quad (10b)$$

Equations (10a) and (10b) when solved to find Z_{sM} and Z_{sN} yield the result as shown in (11), which can be further simplified to the expressions shown in (12).

$$Z_{sM} = \frac{(Z_l/2)\, Z_{MF}}{Z_l/2 + Z_{NF} - Z_{MF}}, \quad Z_{sN} = \frac{(Z_{sM} + Z_l/2)\, Z_{NF}}{Z_{sM} - Z_{NF}} \quad (11)$$

$$Z_{sM} = \frac{Z_{MF}}{1 + \left(\frac{I_M^o - I_M^I}{I_{inj}}\right)}, \quad Z_{sN} = \frac{Z_{NF}}{\left(\frac{I_N^o - I_N^I}{I_{inj}}\right)} \quad (12)$$

In (12), $I_M = I_{M1} + I_{M2}$, $I_N = I_{N1} + I_{N2}$ (see Fig. 4). Implementation using the expressions in (12) is free of line model parameters. Equations similar to (9)-(12) can also be derived for shunt element switching at bus N and not shown here for brevity. Below we provide a summary of steps for this solution which we shall call SE2 in the rest of the paper:

1. Obtain the injected current I_{inj} directly from the substation measurements.
2. Obtain Z_{MF} and Z_{NF} as the ratio of the change in respective bus voltage and the injected current as in (8).
3. Calculate line impedance Z_l using the line parameters and solve for Z_{sM} and Z_{sN} using obtained Z_{MF} and Z_{NF} in (11) OR solve for Z_{sM} and Z_{sN} by using Z_{MF}, Z_{NF} and measured change in line currents at both ends as in (12).

Remarks: For a single circuit transmission line, the following adjustment needs to be made: Use (12) to obtain the source impedances with the only change that $I_M = I_{M1}$ and $I_N = I_{N1}$ as the second circuit is not present. Note that in analyses presented above, for double circuit lines both the circuits together constitute the primary line assuming that parameters and measurements from both the circuits are available.

Of course, if the two-port equivalent across one single circuit, say Circuit-1, is of interest then we will have the parallel circuit, i.e., Circuit-2 as the transfer path in addition to the estimated source impedances. In case parameters and/or measurements from the parallel circuit are not available, then it has to be considered as part of the transfer path that also needs to be estimated.

Evidently, the implementation of methods SE1 and SE2 depend on the identification of an event- a line fault or a shunt element switch at a terminal of the line. It could either be a manual or an automated trigger in response to such an event based on communication of intra and inter-substation level signals (as depicted in the framework of Fig. 2.) such as a relay trip output, line, or shunt element breaker/switch status change, etc. which are indicative of the event. A bus fault event can also be considered as shunt current injection, wherein the fault current itself is the injected current. It can be computed as a vector sum of currents on all the feeders associated with that bus. Another important aspect that needs mentioning is that methods SE1 and SE2 do not preclude the idea of low power signal injection to excite changes in voltage and currents using which the estimations can be carried out. Application of such techniques for grid impedance estimation has been reported in the literature [12].

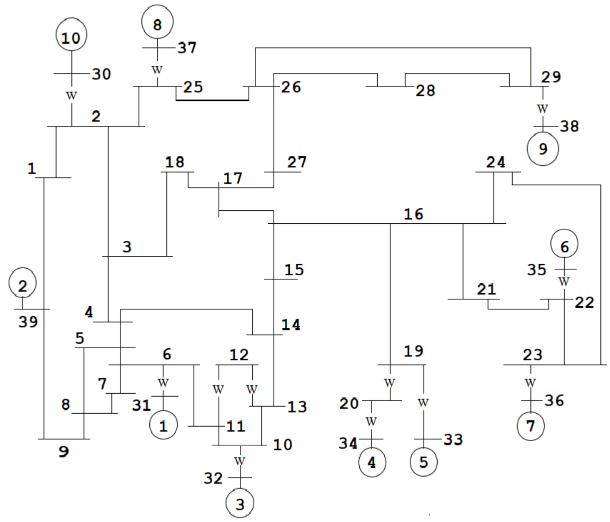

FIGURE 5. **IEEE 39 bus test system.**

B. ESTIMATION OF THE TWO-PORT THEVENIN EQUIVALENT

The methods discussed till now assume that the system around the line of concern can be accurately represented by a simple two source equivalent. This is acceptable or can be considered as a reasonable approximation only when the line of concern is the only path for the flow of incremental current from Bus M to Bus N or the effective transfer path between Buses M and N has a very high impedance in comparison to the primary line of concern. We illustrate this with the following example based on the IEEE 39 bus test system [13] which is depicted in Fig. 5. Particularly, we consider the following three lines: Line 16-19, Line 16-17, and Line 5-8 with impedance Z_l of 0.0016+j0.0195 pu, 0.0007+j0.0089 pu and 0.0008+j0.0112 pu respectively. Table 1. shows the two-port Thevenin equivalents as obtained from a network analysis of the system as described by (1). Line 16-19 clearly corresponds to the case in which there is no transfer path at all. Line 16-17 represents a case in which the transfer path can be neglected with respect to the primary line, as its impedance is much larger in comparison to the line impedance. Line 5-8 corresponds to a case in which the transfer path impedance is comparable to the line impedance and can therefore be considered significant. We consider obtaining the simple two-source equivalent across each of the three lines. To do so, we simulate the following events separately: Line 16-19:

TABLE 1. Actual two-port equivalent parameters obtained by network analysis.

Line M-N	Actual two-port equivalent impedances		
	Z_{sM}(pu)	Z_{sN}(pu)	Z_{tr}^{MN} (pu)
Line 16-19	0.0048+j0.0146	0.0060+j0.0341	∞
Line 16-17	0.0054+j0.0167	0.0075+j0.0288	-0.0186+j0.2039
Line 5-8	0.0036+j0.0214	0.0169+j0.0403	0.0011+j0.0174

TABLE 2. Estimated two-source equivalent parameters obtained by SE2 based on fault at bus N.

Line M-N	Estimated two-port equivalent impedances		
	Z_{sM}(pu)	Z_{sN}(pu)	Comparison to actual parameters (Table I)
Line 16-19	0.0048+j0.0146	0.0060+j0.0341	Identical
Line 16-17	0.0057+j0.0173	0.0069+j0.0276	Approximate
Line 5-8	0.0061+j0.0352	0.0082+j0.0263	Not comparable

TABLE 3. Estimated two-source equivalent parameters obtained by SE2 based on fault at bus M.

Line M-N	Estimated two-source equivalent impedances		
	Z_{sM}(pu)	Z_{sN}(pu)	Comparison to estimated params. (Table II)
Line 16-19	0.0048+j0.0146	0.0060+j0.0341	Identical
Line 16-17	0.0052+j0.0164	0.0080+j0.0300	Similar
Line 5-8	0.0036+j0.0183	0.0281+j0.0662	Different

A fault at Bus 19, Line 16-17: A fault at bus 17, and Line 5-8: A fault at bus 8. Method SE2 is used as the bus fault can be considered as a shunt injection event for which $I_{inj} = -I_{flt}$. Current in the fault, i.e., I_{flt} is calculated as the sum of currents (measured in the direction into the bus) on all the lines and shunt elements incident at the bus.

The results of the analysis are shown in Table 2. For Line 5-8, we observe that the source impedances of the simple two-source equivalent are *not comparable* to that of the exact two-port equivalent as shown in Table 1. This is because the effect of the significant transfer path is transferred on to the estimated source impedances of the two-source equivalent. For this line, the two-source equivalent cannot be considered as an acceptable approximation to the two-port Thevenin equivalent. For Line 16-17, we observe that the source impedances of the simple two-source equivalent *approximate* the values obtained in the exact two-port equivalent. This is because the transfer path, in this case, is insignificant and therefore its effect on the estimated source impedances is negligible. For Line 16-19, we observe that the two-source equivalent is *identical* to the two-port Thevenin equivalent. This is not surprising as for this line there is no transfer path at all.

Mathematically, for a line with a significant transfer path, a simple two-source equivalent is *not unique*. It rather corresponds to the set of equations for the injection event (particularly the location of the injection event) which we try to fit the model into. In other words, the way in which the effect of the transfer path impedance is absorbed into the two-source impedances of the simple two-source equivalent can vary depending on the set of equations we are trying to fit the model. In this sense, a simple two-source equivalent is a *local* or *specific equivalent* of the generic two-port Thevenin equivalent. A similar analysis to this effect can be found in Section 5.2 of reference [7] if we view its parallel line description as a transfer path.

To illustrate this point, we also consider obtaining the two-source equivalent across each of the three lines based on the following events: Line 16-19: A fault at Bus 16, Line 16-17: A fault at bus 16, and Line 5-8: A fault at bus 5. The results are shown in Table 3. Now we compare the values thus obtained with that of Table 2.

For Line 5-8, we observe that the source impedances of the simple two-source equivalent obtained based on the two different fault locations vary significantly from each other. For Line 16-17, we observe the source impedances of the simple two-source equivalent obtained based on the two different fault locations are similar. For Line 16-19, we observe that the source impedances of the simple two-source equivalent obtained based on the two different fault locations are identically the same.

Note that here we have compared the two-source equivalents obtained based on events at terminal bus M and bus N only. Likewise, for each line, we can obtain a two-source equivalent based on a fault anywhere on the line (using SE1). From the above discussions, we can make the following inferences:

a) For a line with a significant transfer path, variations in the two-source equivalents obtained from injection events at different locations on the line can be significant and usage of an estimated two-source equivalent for any application must be done with prudence.
b) For a line with an insignificant transfer path, such variations in two-source equivalents are not significant; estimates can be used for applications with a certain degree of confidence.
c) For a line with no transfer path, there is a unique two-source equivalent which is identical to its two-port Thevenin equivalent.

These are important observations because obtaining a simple two-source equivalent estimate might appear as a plausible option when the amount of information in terms of measured entities and network model is as limited. These inferences can then be considered as guidelines on the usage of such estimates. Having made these observations the next question that arises is: Can methods SE1 and SE2 be extended/modified in order to obtain the two-port Thevenin equivalent? The answer is yes, provided additional information of the bus Thevenin impedances at terminals M and N is available. Let us for simplicity look at the resulting model equations for the scenario of shunt injection at a terminal bus, say bus M, as shown in Fig. 6. They are:

$$Z_{MF} = \frac{Z_{sM}(Z_{par} + Z_{sN})}{Z_{par} + (Z_{sM} + Z_{sN})}, \quad Z_{NF} = \frac{Z_{sN} Z_{sM}}{Z_{par} + (Z_{sM} + Z_{sN})} \tag{13}$$

$$Z_{th}^{M} = \frac{Z_{sM}(Z_{par} + Z_{sN})}{(Z_{sM} + Z_{par} + Z_{sN})}, \quad Z_{th}^{N} = \frac{Z_{sN}(Z_{par} + Z_{sN})}{(Z_{sM} + Z_{par} + Z_{sN})} \tag{14}$$

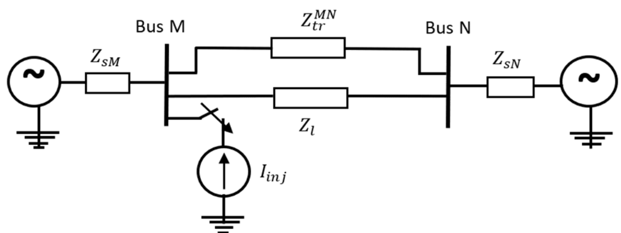

FIGURE 6. Two-port Thevenin equivalent model for shunt injection analysis.

where Z_{MF} and Z_{NF} are defined in (8) and $Z_{par} = \frac{Z_l Z_{tr}^{MN}}{Z_l + Z_{tr}^{MN}}$ denotes the effective impedance between buses M and N. We can solve the set of simultaneous non-linear expressions (13) and (14), to obtain unknowns Z_{sM}, Z_{sN} and Z_{tr}^{MN}. Although feasible, solving such a set of non-linear equations can be cumbersome and requires an initial estimate of the unknowns. A simpler yet effective solution is possible if we consider $Z_l \cong k Z_{tr}^{MN}$ where k is a scalar, which is a reasonable approximation for high voltage transmission systems.

We can then iteratively solve the equations as described below:

1. Choose a value of k; and calculate $Z_{tr}^{MN} = \frac{Z_l}{k}$
2. Solve (13) only for Z_{sM}, Z_{sN}
3. From the obtained value of Z_{sM}, Z_{sN} and Z_{tr}^{MN} calculate the bus Thevenin impedances as in (14)
4. Check if the calculated bus Thevenin impedances match the actual values
5. If the match is poor, then increment k and repeat steps 2-5, else check for convergence and break.

We shall call this method as SE3. This simple work-around allows us to deal only with linear equations, even though it involves iterations. Moreover, iterating is not difficult since we know that k can practically lie within a confined range of values. For example, we can iterate for values of k in the range of 0.05 to 1.0. A value of k smaller than 0.05 means the impedance of the transfer path is more than 20 times that of the line, which means that the transfer path is not significant and can be ignored for all practical purposes. A value of k greater than 1.0 means that the impedance of the transfer path is less than that of the line, which is not commonly observed in practical high voltage systems. Although not described here, a similar approach can be adopted for the scenario of line faults to estimate the two-port Thevenin equivalent. An important point to note here is that the method assumes the availability of bus Thevenin impedance at the line terminals. The problem of estimating bus Thevenin impedances is not new. It has been dealt with in a variety of ways in literature and in practice [14]–[16] including approaches based on locally available measurements as well.

C. COMPARISON WITH EXISTING METHODS

We now compare methods SE1, SE2, and SE3 with some of the methods existing in literature and practice. The method described in [2] uses the below equations to compute the parameters of the two-port equivalent:

$$K_M = \frac{I_{fNM}}{I_{fM}}, \quad K_N = \frac{I_{fMN}}{I_{fN}}$$

$$X = \frac{K_M Z_l}{1 - (K_M + K_N)}, \quad Q = \frac{K_N Z_l}{1 - (K_M + K_N)}$$

$$W = \frac{1}{I_{fM}} - X(1 - K_M)$$

Here, entities X, Q, and W are the corresponding impedance parameters of a wye-system equivalent of the network (except the primary line of interest) shown in Fig. 1(b). Entities I_{fN} and I_{fMN} are the total fault current and fault contribution over the primary line for a 3ph fault at bus N. Entities I_{fM} and I_{fNM} are defined similarly for fault at bus M.

The method described in [5] follows a very simplistic approach to estimate the source impedance behind the relay. Assuming that relay is located at bus M, the method suggests applying a three-phase fault at bus M, removing fault current contribution from the line MN, and dividing the nominal system voltage by the adjusted fault current. On similar lines, reference [6] suggests improvements by instead placing the fault at the remote bus, i.e., bus N in this case, and dividing the drop of voltage at bus M from the nominal system voltage by the current on line MN seen by the relay. Reference [7] suggests dividing the change in voltage phasor at the relay by the change in the current seen by the relay following a fault on the line.

A detailed comparison is provided in Table 4 by considering factors such as the inputs needed by the methods, the estimated outputs, scenarios under which the methods are applicable, their treatment of transfer path, and their applicability for practical analyses. The method presented in [2] provides an estimate of the complete two-port equivalent, however, the inputs required by the method limit its applicability to a scenario in which they can be obtained by concurrently staged faults at both the terminals of the primary line. Methods [5] and [6] aim at estimating an approximate magnitude of the source impedance behind the relay for the purpose of obtaining SIR. In doing so, they do not consider obtaining all the complex impedance parameters of the two-port equivalent and are therefore limited in application. Method [7] is particularly suitable for fault location application. The source impedances obtained using this method can only be *specific,* as discussed in Section II-B. Proposed methods SE1 and SE2 have limitations in their treatment of transfer path (discussed previously in Section II-B). However, they provide complex source impedance values and

TABLE 4. A comparative analysis with existing approaches.

Method	Required inputs	Estimated outputs	Treatment of transfer path	Applicability	Evaluation scenario(s)
[2]	Total fault current for 3-ph fault at buses M and N, fault current contribution over the line for 3-ph fault at buses M and N	Complex source and transfer path impedances	Transfer path considered and estimated	Generic	Based on concurrently obtained input data through staged faults
[5]	Voltage at the relay bus, fault current, current over the line measured at relay bus	Source impedance magnitude at the bus	Transfer path ignored	Limited/specific, (e.g., relay SIR calculation)	Based on staged or chance occurrence of bolted short circuit at the relay bus
[6]	Voltage at the relay bus, current over the line measured at relay bus	Source impedance magnitude at the bus	Effect of transfer path captured, but its impedance is not estimated	Limited/specific (e.g., relay SIR calculation)	Based on line fault events
[7]	Voltage at the relay bus, current over the line measured at relay bus	Complex source impedance at the bus	Transfer path ignored	Limited/specific (e.g., fault location)	Based on line fault, shunt element switching and bus fault events
SE1, SE2	Voltage at terminal buses M and N, current over the line measured at buses M and N, currents on all the incident feeders at each terminal bus	Complex source impedance at both the buses	Transfer path ignored	Limited/specific (as discussed in Section II-B and IV-B)	Based on line fault, shunt element switching and bus fault events
SE3	Same as SE1, SE2 plus bus Thevenin impedances at buses M and N	Complex source and transfer path impedances	Transfer path considered and estimated	Generic	Based on line fault, shunt element switching and bus fault events

are applicable over a wider range of event scenarios such as line faults, bus faults, shunt switching events, etc. Proposed method SE3 overcomes limitations of SE1 and SE2, as it computes the complete two-port equivalent parameters including the transfer path impedance. It is also advantageous in comparison to the method in [2], as it does not require the data for current contributions over the line for terminals bus faults.

III. PROPOSED METHOD FOR UPDATE OF TWO-PORT THEVENIN EQUIVALENT

Another important and useful tool for the substation operator could be one which updates the impedance parameters of the two-port equivalent when a network topology change takes place in close vicinity of the line of concern. The idea here differs completely from the existing and proposed methods for estimating the equivalent. For example, a method for estimation such as SE3 will have to be re-applied on the modified network after the topology change event in order to update the two-port equivalent. This means, one has to wait for a favorable evaluation scenario to occur in order to synthesize the necessary inputs for the estimation process. In contrast, we propose methods for an automatic update of the equivalent by simply using its parameters in the base network scenario and the measurements excited due to the topology event itself. This approach is novel in the context of computing two-port network equivalent across a transmission line.

The methods proposed here consider the following topology change scenarios: (i) tripping-off of a line incident (e.g. Line M-P of Fig. 1(a)) on one of the terminal buses of the line of concern, (ii) tripping-off of a shunt reactor/capacitor at a bus one level up (e.g. Bus P of Fig. 1(b)), i.e. an adjacently connected bus in the topology. Again, acknowledging the fact that as a substation level exercise information of the entire network topology may not always be available, our approach is to find a method that uses only local topology update information and related measurements. The proposed methods envisage utilization of measurements at both ends of a transmission line and nodes one level up in the network. Specifically, with reference to Fig. 2, the measurements required are voltage measurements at buses M, N and P before and after the topology change event, the current through the element before it tripped (in case of line MP trip, the current measured on the line at bus M and the current drawn by the element in case of a shunt element trip at bus P). Model information needed is the impedance parameters of the tripped element, impedance parameters of the primary line, and the initial two-port equivalent impedance parameters across the primary line and the line which trips.

Let us define the impedance submatrix of the full network Z_{bus} which consists of only the nine elements corresponding to nodes M, N and P by notation $Z\,(mnp)$ shown in (16) below.

$$Z\,(mnp) = \begin{pmatrix} Z_{mm} & Z_{mn} & Z_{mp} \\ Z_{nm} & Z_{nn} & Z_{np} \\ Z_{pm} & Z_{pn} & Z_{pp} \end{pmatrix} \tag{15}$$

We can similarly define submatrix $Z\,(mnp)'$ (with elements denoted by $'$ in superscript) corresponding to the network from which the branch between node M and N is removed, and submatrix $Z\,(mnp)''$(with elements denoted by $''$ in superscript) corresponding to the network from which the branch between nodes M and N and the tripped element are removed. Then the updated impedance parameters of the two-port equivalent can be obtained as

$$Z_{sM}=\frac{Z''_{mm}Z''_{nn}-Z''_{mn}Z''_{nm}}{Z''_{nn}-Z''_{mn}},\quad Z_{sN}=\frac{Z''_{mm}Z''_{nn}-Z''_{mn}Z''_{nm}}{Z''_{mm}-Z''_{mn}} \tag{16}$$

$$\text{and } Z_{tr}^{\mathrm{MN}}=\frac{Z''_{mm}Z''_{nn}-Z''_{mn}Z''_{nm}}{Z''_{mn}} \tag{17}$$

which follows directly from the result of (1c). Therefore, if we know the submatrix $Z(mnp)''$, then the problem of updating is a straightforward arithmetic manipulation of its elements as given by (17).

The elements of $Z(mnp), Z(mnp)'$ and $Z(mnp)''$ are related. This is because $Z(mnp)'$ can be obtained from $Z(mnp)$ mathematically by doing series and shunt branch (corresponding to the series impedance and shunt admittances of line MN) modifications [10] on $Z(mnp)$. The formulae for series and shunt branch modifications are provided in the Appendix. A similar argument holds for the transition from $Z(mnp)'$ to $Z(mnp)''$ with respect to line MP. Thus, if $Z(mnp)$ is known, then we can obtain $Z(mnp)''$ provided parameters of the lines MN and MP are available. Now, the question arises that whether we know all the elements of $Z(mnp)$ to begin with? We analyze this aspect next.

1) ELEMENTS OF $Z(mnp)$

Proposition: We can obtain all elements of $Z(mnp)$ except for $Z_{pn}(=Z_{np})$, provided impedance parameters of the two-port equivalents across lines MN and MP in the base network (i.e. the initial two-port equivalents) are given.

Proof: One important corollary of the relationship (1b) is that if we know the parameter set $\{Z_{sM}, Z_{sN}, Z_{tr}^{MN}\}$ for the base network, then we can obtain elements $Z'_{mm}, Z'_{mn}, Z'_{nm}$ and Z'_{nn} as shown in (18) below.

$$Z(mn)' = \begin{pmatrix} Z'_{mm} & Z'_{mn} \\ Z'_{nm} & Z'_{nn} \end{pmatrix} = \begin{pmatrix} \frac{1}{Z_{sM}}+\frac{1}{Z_{tr}^{MN}} & \frac{-1}{Z_{tr}^{MN}} \\ \frac{-1}{Z_{tr}^{MN}} & \frac{1}{Z_{sN}}+\frac{1}{Z_{tr}^{MN}} \end{pmatrix}^{-1}$$

$$\Rightarrow Z'_{mm}=\frac{Y_{sN}+Y_{tr}^{MN}}{Y_{sM}Y_{sN}+Y_{tr}^{MN}(Y_{sM}+Y_{sN})},$$

$$Z'_{nn}=\frac{Y_{sM}+Y_{tr}^{MN}}{Y_{sM}Y_{sN}+Y_{tr}^{MN}(Y_{sM}+Y_{sN})} \text{ and}$$

$$Z'_{mn}=Z'_{nm}=\frac{Y_{tr}^{MN}}{Y_{sM}Y_{sN}+Y_{tr}^{MN}(Y_{sM}+Y_{sN})} \tag{18a}$$

where $Y_{sM}=\frac{1}{Z_{sM}}, Y_{sN}=\frac{1}{Z_{sN}}, Y_{tr}^{MN}=\frac{1}{Z_{tr}^{MN}}$.

Once these elements are obtained, the elements Z_{mm}, Z_{mn}, Z_{nm} and Z_{nn} can be obtained by carrying out the modification calculations on $Z(mn)'$ pertaining to addition of the line MN between buses M and N as shown below:

$$Z(mn)=\begin{pmatrix} Z_{mm} & Z_{mn} \\ Z_{nm} & Z_{nn} \end{pmatrix}=Z(mn)'-\frac{\{Z'_{mn}\}\{Z'_{mn}\}^T}{\Delta_{mn}} \tag{18b}$$

where $\{Z'_{mn}\}=\begin{bmatrix} Z'_{mm}-Z'_{mn} \\ Z'_{nm}-Z'_{nn} \end{bmatrix}$ and $\Delta_{mn}=Z'_{mm}+Z'_{nn}-2Z'_{mn}+Z_l$, Z_l being the series impedance of the branch MN.

Similarly, using the parameter set $\{Z_{sM}, Z_{sP}, Z_{tr}^{MP}\}$ for the base network we can obtain Z_{mp}, Z_{pm} and Z_{pp} following the same steps. Thus, it is clear that all elements of $Z(mnp)$ except for Z_{np} and Z_{pn} can be obtained using the initial parameter sets of $\{Z_{sM}, Z_{sN}, Z_{tr}^{MN}\}$ and $\{Z_{sM}, Z_{sN}, Z_{tr}^{MP}\}$ in the relationships described by (18). The main challenge lies in obtaining Z_{np} and Z_{pn} from the available substation measurements. This constitutes the primary objective of the proposed solution. Note that the network Z_{bus} and any of its submatrix involving a few buses are always symmetric and therefore Z_{np} and Z_{pn} are equal.

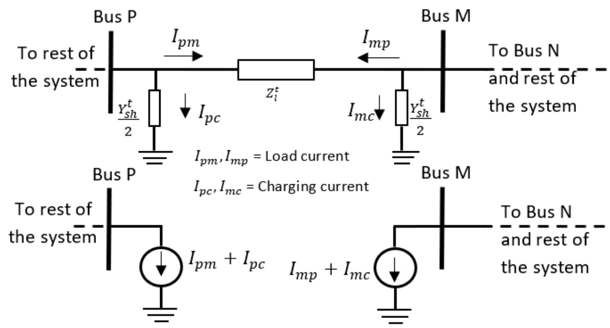

FIGURE 7. Equivalence of current flow on line and current injections at terminal buses.

A. UPDATE ON INCIDENT LINE TRIPS

This scenario corresponds to the situation in which line MP trips leading the system into a new operating condition. At this new operating condition, the two-port equivalent parameters for the primary line, i.e., line MN would have changed, which are to be estimated. For the description ahead, let us (for the sake of convenience in the description) consider a minor change in definitions of the impedance submatrices as provided earlier in this section. Let us now consider that the submatrix $Z(mnp)'$ corresponds to the network from which the line MP is removed, with the primary line MN intact. The submatrix $Z(mnp)''$ corresponds to the network from which the line MN is also removed. Note that this change in definition relates only to a change in the sequence in which the two lines are removed from the base network. The elements of $Z(mnp)''$ remain the same as before. With this understanding let us proceed with description of the method. The current flow on the line MP can be considered as current injections at buses M and P of the system without the line MP physically present in the network as depicted in Fig. 7. Therefore, tripping of line MP can be considered as a change in current injections at Bus M and Bus P for the system without the line MP. Thus, for the line MP trip, we can write the voltage change at bus N as follows:

$$\Delta V_n=(I_{mp}+I_{mc})Z'_{mn}+(I_{pm}+I_{pc})Z_{pn}{}'$$

$$\Rightarrow\frac{\Delta V_n}{(I_{mp}+I_{mc})}=(Z'_{mn}-Z'_{pn})+\frac{(I_{mc}+I_{pc})}{(I_{mp}+I_{mc})}Z'_{pn} \tag{19}$$

We know that $Z(mnp)'$ and $Z(mnp)$ are related by means of series and shunt branch modifications pertaining to the line MP. Hence, the right-hand side of the expression (19) can be written completely in terms of elements of $Z(mnp)$. This will lead us to an equation with Z_{pn} as the variable since the quantity on the left-hand side can be obtained from measurements. The resulting equation is linear in nature and can be solved conveniently to find Z_{pn}, the final formula for which is as shown in (20).

$$Z_{pn} = \frac{\left(D\left(Z_{nm} - A + BE\right) + \frac{\alpha E Z^t_{sh}}{Z_d - Z^t_{sh}}\right) - \frac{\Delta V_n}{(I_{mp}+I_{mc})}}{D\left(1+B\right) + \alpha Z^t_{sh}/(Z_d - Z^t_{sh})} \quad (20)$$

where, $A = \frac{Z_{nm}(Z_{mm}-Z_{mp})}{Z_{mm}-Z^t_{sh}}$, $F = Z_{mp} - Z_{pp} - \frac{Z_{mp}(Z_{mm}-Z_{mp})}{Z_{mm}-Z^t_{sh}}$;

$$Z_d = Z_{pp} - \frac{Z^2_{mm}}{Z_{mm} - Z^t_{sh}}, \quad x_d = Z_{mm}\left(-\frac{Z^t_{sh}}{Z_{mm} - Z^t_{sh}}\right);$$

$$\bar{X} = F\left(\frac{-Z^t_{sh}}{Z_{mm} - Z^t_{sh}}\right),$$

$$\bar{Z} = \frac{-Z^t_{sh}\left(Z_{mm} - Z_{mp}\right)}{Z_{mm} - Z^t_{sh}} - \frac{x_d F}{Z_d - Z^t_{sh}};$$

$$B = \frac{F}{Z_d - Z^t_{sh}}, \quad C = -\frac{Z^t_l}{\delta},$$

$$D = C - \frac{\alpha \bar{X}}{\delta}, \delta = \bar{Z} - \bar{X} - Z^t_l;$$

$$\alpha = \frac{(I_{mc} + I_{pc})}{(I_{mp} + I_{mc})}, \quad E = Z_{mp}Z_{mn}/(Z_{mm} - Z^t_{sh})$$

Note that the charging currents on line MP before it had tripped need to be calculated as shown in (21) below. The current on line MP (before it had tripped) at bus M end can either be directly measured or be calculated as shown in 22.

$$I_{mc} = V^0_m \frac{Y^{MP}_{sh}}{2}, \quad I_{pc} = V^0_p \frac{Y^t_{sh}}{2} \quad (21)$$

$$I_{mp} + I_{mc} = \frac{V^0_m - V^0_p}{Z^t_l} + V^0_m \frac{Y^t_{sh}}{2} \quad (22)$$

where V^0_m and V^0_p are the voltages measured at bus M and bus P respectively before the occurrence of the trip event. Z^t_l and Y^t_{sh} are the series impedance and the shunt admittance associated with the incident line MP.

We shall call this method as SU1, the steps for which are summarized below:

1. Obtain $Z(mn)$ by using the impedance parameters of the two-port equivalents across lines MN in the base network and modification calculations pertaining to addition of line MN
2. Obtain $Z(mp)$ by using the impedance parameters of the two-port equivalents across lines MP in the base network and modification calculations pertaining to the addition of line MP
3. Obtain the measured voltages at buses M, P before the line trip and calculate the charging currents on the incident line MP using (21) and therefore the sum of charging currents; obtain the current measured on the incident line MP at bus M before it tripped or calculate the same using (22).
4. Obtain the measured voltage change at bus N due to the line trip and use (20) to determine Z_{pn}; once Z_{pn} is obtained, all elements of $Z\ (mnp)$ are identified.
5. Sequentially remove the effects of line MN and MP from $Z(mnp)$ using the necessary series and shunt modification calculations to obtain $Z\ (mnp)''$.
6. Use (17) to obtain the modified/updated equivalent impedances.

1) AN APPROXIMATE ALTERNATIVE SU1_APX

Alternatively, this problem can also be approached without assuming that we know the two-port equivalent across the incident line MP. Like the expression in (19), we can write equations for buses M and P as follows:

$$\frac{\Delta V_m}{(I_{mp} + I_{mc})} = (Z'_{mm} - Z'_{pm}) + \frac{(I_{mc} + I_{pc})}{(I_{mp} + I_{mc})} Z'_{pm} \quad (23)$$

$$\frac{\Delta V_p}{(I_{mp} + I_{mc})} = (Z'_{mp} - Z'_{pp}) + \frac{(I_{mc} + I_{pc})}{(I_{mp} + I_{mc})} Z'_{pp} \quad (24)$$

We observe that if the ratio of the charging current to the load current on line MP is negligible, then (19), (23), and (24) simplify to the following:

$$\frac{\Delta V_n}{(I_{mp} + I_{mc})} \approx (Z'_{mn} - Z'_{pn}), \frac{\Delta V_m}{(I_{mp} + I_{mc})} \approx (Z'_{mm} - Z'_{pm})$$

$$\text{and} \ \frac{\Delta V_p}{(I_{mp} + I_{mc})} \approx (Z'_{mp} - Z'_{pp}). \quad (25)$$

Now, considering line MP as only a series branch of impedance Z^t_l(since current through shunt admittances are assumed negligible), we obtain the following relationships between elements of $Z\ (mnp)$ and $Z\ (mnp)'$:

$$Z'_{mm} - Z'_{mp} = \frac{-Z^t_l(Z_{mm} - Z_{mp})}{Z_{mm} + Z_{pp} - 2Z_{mp} - Z^t_l}$$

$$Z'_{nm} - Z'_{np} = \frac{-Z^t_l(Z_{nm} - Z_{np})}{Z_{mm} + Z_{pp} - 2Z_{mp} - Z^t_l}$$

$$Z'_{pm} - Z'_{pp} = \frac{-Z^t_l(Z_{pm} - Z_{pp})}{Z_{mm} + Z_{pp} - 2Z_{mp} - Z^t_l}. \quad (26)$$

Equations (25) and (26) can be solved to obtain $(Z_{mm} - Z_{mp})$, $(Z_{nm} - Z_{np})$ and $(Z_{pm} - Z_{pp})$ as shown below:

$$(Z_{mm} - Z_{mp}) = \eta \frac{\Delta V_m}{(I_{mp} + I_{mc})}, (Z_{nm} - Z_{np}) = \eta \frac{\Delta V_n}{(I_{mp} + I_{mc})}$$

$$\text{and} \ (Z_{pm} - Z_{pp}) = \eta \frac{\Delta V_p}{(I_{mp} + I_{mc})} \quad (27)$$

where

$$\frac{1}{\eta} = \left(1 + \left(1 - \frac{\Delta V_p}{\Delta V_m}\right)\frac{\Delta V_m}{(I_{mp} + I_{mc})}\right)/Z^t_l.$$

Using already computed values of Z_{mm} and Z_{nm}, we can then obtain Z_{mp}, Z_{np} and Z_{pp}. This constitutes an alternative to

SU1. However, as mentioned earlier it involves the simplifying approximation that the charging current of line MP is negligible in comparison to its loading current. The steps of SU1_apx are summarized below:

1. Same as step 1 of SU1
2. Obtain the change in voltage of buses M, N, and P due to the line MP trip. Also, obtain the current on the line MP at bus M end before it tripped.
3. Solve (27), use $Z(mn)$ to obtain Z_{mp}, Z_{np} and Z_{pp} and construct $Z(mnp)$.
4. Sequentially remove the effects of line MN and MP from $Z\,(mnp)$ using the necessary series and shunt modification calculations to obtain $Z\,(mnp)''$.
5. Use (17) to obtain the modified/updated equivalent impedances.

B. UPDATE ON SHUNT ELEMENT TRIP AT ADJACENT BUS

This scenario corresponds to the situation in which a shunt reactor/capacitor trips at bus P leading the system into a new operating condition. At this new operating condition, the two-port equivalent parameters for the line MN would have changed, which are to be estimated.

Following is a description of the proposed method where the submatrix $Z\,(mnp)'$ corresponds to the network from which the shunt element (of impedance Z_{sh}) at bus P is removed from the base network. The submatrix $Z\,(mnp)''$ corresponds to the network from which the line MN is also removed. The current flow (I_p^{sh}) on the shunt element at bus P can be considered as a current injection at bus P of the network without the shunt element being physically present. Therefore, tripping of the shunt element can be considered as a change in current injections at bus P for the system without the shunt element at bus P. Thus, for the trip of the shunt element at bus P, we can write the voltage change at bus N as follows:

$$\Delta V_n = I_p^{sh} Z'_{pn} \Rightarrow Z'_{pn} = \frac{\Delta V_n}{I_p^{sh}} \tag{28a}$$

Thus, Z'_{pn} can be obtained from measurements of voltage drop at bus N due to the shunt element trip at bus P and the current carried by the shunt element before it tripped. Now, we know that the elements of the submatrix $Z\,(mnp)'$ are related to the elements of $Z(mnp)$ by means of shunt branch modifications pertaining to the shunt element at bus P. Specifically, we are interested in the relationship of (28b)

$$Z'_{pn} = Z_{pn}\left(\frac{-Z_{sh}}{Z_{pp} - Z_{sh}}\right) \tag{28b}$$

which is obtained by modification calculations pertaining to the removal of the shunt branch at bus P. Solving (28a) and (28b), we can find Z_{pn} as shown in (29) below.

$$Z_{pn} = \left(\frac{Z_{pp} - Z_{sh}}{-Z_{sh}}\right)\frac{\Delta V_n}{I_p^{sh}} \tag{29}$$

Note that the current I_p^{sh} can either be measured or computed as: $I_p^{sh} = V_p^0/Z_{sh}$. We shall call this method as SU1, the steps for which are summarized below:

1. Same as step 1 of SU1
2. Same as step 2 of SU1
3. Obtain the change in voltage of bus N due to the shunt element trip at bus P from measurements. Obtain the current through the shunt element before it tripped.
4. Solve (29) to obtain Z_{pn}; construct matrix $Z\,(mnp)$
5. Sequentially remove the effects of shunt element at P and line MN from $Z(mnp)$ using necessary series and shunt modification calculations to obtain $Z\,(mnp)''$.
6. Use (17) to obtain the modified/updated equivalent impedances.

1) AN ALTERNATIVE SU2_ALT

Alternatively, this problem can also be approached without assuming that we know the two-port equivalent across the incident line MP. In this case, in addition to Z_{pn} we will need to estimate Z_{mp} and Z_{pp} as well, for which we can make use of the following relationships:

$$Z'_{pp} = \frac{\Delta V_p}{I_p^{sh}}, \quad Z'_{pp} = Z_{pp}\left(\frac{-Z_{sh}}{Z_{pp} - Z_{sh}}\right) \tag{30}$$

$$Z'_{pm} = \frac{\Delta V_m}{I_p^{sh}}, \quad Z'_{mp} = Z_{mp}\left(\frac{-Z_{sh}}{Z_{pp} - Z_{sh}}\right) \tag{31}$$

First, we will need to solve (30) for Z_{pp}. Then using the obtained value of Z_{pp}, we solve (29) and (31) for Z_{np} and Z_{mp} respectively. Thus, the updated source impedances can be obtained with fewer initial information, specifically that of the two-port equivalent across line MP. However, we will need to have voltage measurements from buses P and M also in this case. We shall call this method as SU2_alt, the steps for which are summarized below:

1. Same as step 1 of SU2
2. Obtain the change in voltage of buses M, N and P due to the shunt element trip at bus P. Also obtain the current through the shunt element before it tripped.
3. Solve (29)-(31) to obtain Z_{mp}, Z_{np} and Z_{pp}, and construct the matrix $Z(mnp)$.
4. Sequentially remove the effects of shunt element at P and line MN from $Z(mnp)$ using the necessary series and shunt modification calculations to obtain $Z\,(mnp)''$.
5. Use (17) to obtain the modified/updated equivalent impedances.

Once the updated two-port equivalent impedances are obtained, the source voltages can be calculated as:

$$E_{sM} = V_M^E + (V_M^E - V_N^E)\left(\frac{1}{Z_l} + \frac{1}{Z_{tr}^{MN}}\right)Z_{sM};$$

$$E_{sN} = V_N^E - (V_M^E - V_N^E)\left(\frac{1}{Z_l} + \frac{1}{Z_{tr}^{MN}}\right)Z_{sN}$$

Remarks: Deployment of methods for updating require identification of a corresponding topology change event.

It could either be a manual or an automated trigger in response to occurrence of such an event based on intra and inter-substation level communication of signals (as depicted in the framework of Fig. 2.) such as a relay trip output, line or shunt element breaker/switch status change, etc. which are indicative of the event.

IV. RESULTS AND ANALYSES

In this section, we present the results of detailed testing of the various methods discussed in the previous sections. Specifically, we present the results in the following manner: (i) test results for solutions SE1 and SE2 based on a test system similar to that shown in Fig. 3 (ii) test results for SE3 based on the IEEE 39 bus test system, (iii) test results for SU1 and SU2 based on the IEEE 39 bus test system.

A. TEST RESULTS FOR SE1 AND SE2

The test system which is similar to that shown in Fig. 3 is simulated in PSCAD [17] environment. The nominal line-to-line rms voltage at the system buses is 220 kV. The lines (120 km each) are represented with frequency dependent (phase) models, effective series impedance of each line being approximately equal to $3.9+j31.42\Omega$. The sources each are modelled as voltage source behind source impedance. Values of source impedances, i.e., Z_{sM} and Z_{sN} are varied in the simulation to produce scenarios with the different source to line impedance ratios on both sides of the line (denoted by $\text{SIR}_{\text{M:N}}$). The results produced in this section have test cases in which $\text{SIR}_{\text{M:N}}$ assumes the following values: 0.1:1, 1:0.1, 2:5. An $\text{SIR}_{\text{M:N}}$ of 0.1:1 means that the source at bus M has a source impedance which is 0.1 times the line impedance in magnitude, so on and so forth. In each case, the stronger source has an impedance angle of 84.8056°, while the weaker source has an angle of 79°.

Other fault related parameters such as fault location (0.5 km, 30 km, 60 km, 90 km, 119.5 km from bus M end) and inception angle (0°, 90°) are also varied to generate different test cases. The testing results of SE1 is summarized in the plot of Fig. 8. The source impedances were successfully estimated in all the cases. The maximum error observed in the numerical testing is less than 4% and the average error is less than 1%.

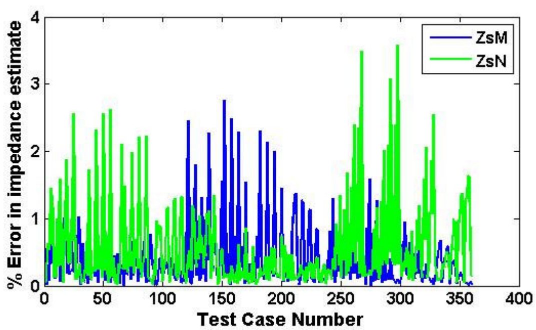

FIGURE 8. Test results for SE1.

For SE2, the same system is used with the switching of shunt reactors of different ratings at each terminal bus

TABLE 5. Estimated two-port equivalent parameters obtained by SE2.

Case $\text{SIR}_{\text{M:N}}$ -Bus-SRV	Estimated two-port equivalent impedances $Z_{sM}(\Omega)$ (% *error*)	$Z_{sN}(\Omega)$ (% *error*)
0.1:1.0-M-100	0.259 +j2.930 **(7.1)**	4.593 +j28.758 **(8.638)**
0.1:1.0-M-200	0.268 +j3.040 **(3.63)**	5.868 +j30.386 **(2.255)**
1.0:0.1-M-100	5.632 +j30.769 **(1.618)**	0.392 +j2.881 **(9.211)**
1.0:0.1-M-200	5.828 +j30.875 **(0.931)**	0.315 +j3.012 **(4.545)**
2.0:5.0-M-100	6.119 +j62.854 **(0.691)**	28.020 +j154.911 **(1.413)**
2.0:5.0-M-200	5.846 +j62.881 **(0.335)**	29.759 +j155.232 **(0.299)**
0.1:1.0-N-100	0.283 +j2.869 **(8.985)**	5.879 +j30.504 **(1.885)**
0.1:1.0-N-200	0.273 +j2.985 **(5.311)**	6.077 +j30.884 **(0.624)**
1.0:0.1-N-100	4.550 +j28.832 **(8.517)**	0.333 +j2.964 **(6.137)**
1.0:0.1-N-200	6.099 +j29.557 **(4.81)**	0.291 +j3.067 **(2.716)**
2.0:5.0-N-100	5.509 +j62.770 **(0.578)**	31.295 +j154.644 **(0.835)**
2.0:5.0-N-200	5.918 +j62.754 **(0.565)**	29.680 +j155.974 **(0.496)**

to create a set of test cases. A test case indicated as 0.1:1.0-M-100 means switching of a shunt reactor of rating 100 MVar at bus M in the system with $\text{SIR}_{\text{M:N}}$ of 0.1:1, so on and so forth. The results of testing are summarized in Table 5. The source impedances were successfully estimated in all the cases. The maximum error observed in the numerical testing is less than 10% and the average error is less than 5%.

B. UTILITY OF TWO SOURCE EQUIVALENT

It is always sensible to use the two-port Thevenin equivalent model. However, obtaining the same may not always be a possibility due to the paucity of information. Thus, many times we depend on a simple two-source equivalent for our analyses. Usage of any algorithm (such as SE1 and SE2) which estimates a two-source equivalent must be done with prudence. They are ideally suited for lines which connect isolated systems as for such lines the two-port Thevenin equivalent is identically the same as the two-source equivalent. They can also be employed with some level of confidence if it is known *a priori* that the primary line is typically associated with an insignificant transfer path. This is because for such lines, a two-source equivalent closely resembles the two-port Thevenin equivalent as has been shown through examples in Section II-B.

The main limitation of such algorithms is in dealing with lines with strong transfer path. In such cases, one needs to consider the application of the estimated equivalent model. Before proceeding further, we introduce some nomenclature for easy reference. For the line MN, let us call the two-source equivalent model obtained based on an injection event at bus N as TS_NEqv and that obtained based on an injection event at bus M as TS_MEqv. Therefore, the entries in the last row of Table 2 represent TS_NEqv for Line 5-8, while the entries in the last row of Table 3 represent TS_MEqv for the same line as they are obtained based on fault events at bus N (i.e. bus 8) and bus M (i.e. bus 5) respectively.

For the purpose of illustration, let us now consider distance relay reach setting application. In order to determine the resistive and reactive reach settings for the relay, we must by using the available equivalent model analyze the apparent impedance seen by the relay for faults (with varying fault resistances) at the remote bus (which is the point we do not want to overreach) [4]. To illustrate, we consider line 5-8 as our primary line of interest. Let us assume the relay on this line placed on bus 5. We simulated faults at the remote bus 8 with varying fault resistance.

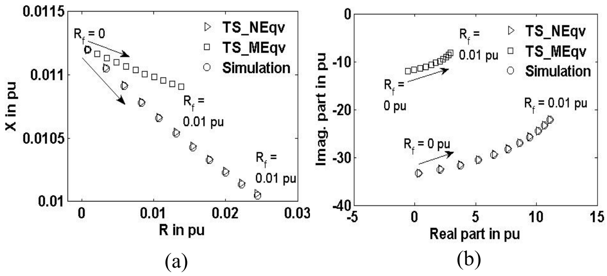

FIGURE 9. Apparent impedance seen by the relay at bus 5, (b) Remote infeed contribution to fault current for fault on bus 8.

In Fig. 9(a) there are three different plots: (i) the actual impedance seen by the relay at bus 5 as obtained from fault simulations, (ii) the apparent impedance as estimated using TS_NEqv, (iii) the apparent impedance as estimated using TS_MEqv. It is clearly seen that TS_NEqv gives accurate estimates of the apparent impedance seen by the relay, while TS_MEqv fails to do so. Fig. 9(b) shows a similar comparison for the remote infeed contribution to the fault. To conclude, the two-source equivalent obtained based on an injection event at the remote bus N is better suited for analyses pertaining to relay reach setting. This observation is in agreement with the method proposed in [1] for calculating SIRs to determine line length with respect to relay location. Therefore, in summary, a two-source equivalent is limited in its utility and any such equivalent must be used with prudence when dealing with practical relaying and/or monitoring applications.

C. TEST RESULTS FOR SE3

For validating SE3, we use the IEEE 39 bus test system shown in Fig. 5. We choose a set of lines around which the two-port equivalent are to be estimated. The actual two-port Thevenin equivalents for these lines are provided in Table 6. To illustrate, let us consider the case of a fault on line 5-8 at 50% of the line length. We use the measurements around this event and run the steps of SE3 as indicated in Section II-B. For Line 5-8, the actual bus Thevenin impedances are $Z_{th}^{M} = 0.0152\angle 77.16°$ and $Z_{th}^{N} = 0.0174\angle 76.00°$ which

TABLE 6. Actual two-port equivalent parameters obtained from full network analysis.

Case	Actual two-port equivalent impedances		
	Z_{sM}(pu)	Z_{sN} (pu)	Z_{tr}^{MN}(pu)
Line 5-8	0.0036 +j 0.0214	0.0169 +j 0.0403	0.0011 +j 0.0174
Line 6-7	0.0030 +j 0.0191	0.0271 +j 0.0526	0.0010 +j 0.0203
Line 23-24	0.0054 +j 0.0322	0.0064 +j 0.0215	-0.0009 +j 0. 0625
Line 4-14	0.0069 +j 0.0225	0.0057 +j 0.0295	0.0004 +j 0.0474
Line 16-21	0.0042 +j 0.0139	0.0137 +j 0.0555	0.0026 +j 0.0893

for the purpose of these illustrations are obtained from an analysis of the network impedance matrix. After step 3 for each value of k, we obtain the errors in the estimated bus Thevenin impedances as:

$$Z_{th}^{M}err = \frac{|Z_{th}^{M}est - Z_{th}^{M}|}{|Z_{th}^{M}|}, \quad Z_{th}^{N}err = \frac{|Z_{th}^{N}est - Z_{th}^{N}|}{|Z_{th}^{N}|}.$$

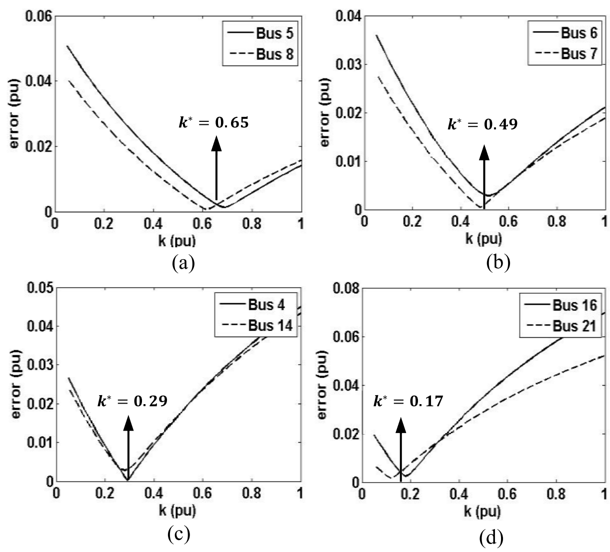

FIGURE 10. Variation of errors in estimated bus Thevenin impedances with variation in $k = |Z_l/Z_{tr}|$ for method SE3.

Fig. 10(a) plots these error values over the variation in k for line 5-8. It is seen that the errors are minimum in the range 0.6-0.7. At about $k^* = 0.65$, their sum, i.e., $Z_{th}^{M}err + Z_{th}^{N}err$ reaches the minimum and therefore that can be considered as the optimal value of k. At this value of k, the solution is $Z_{s5} = 0.0036 + j0.0214\,(pu)$, $Z_{s8} = 0.0169 + j0.0404(pu)$, $Z_{tr}^{58} = \frac{Z_l}{k^*} = 0.0012 + j0.0172(pu)$. Estimations for all the remaining lines are done in a similar manner, the results of which are populated in Table 7 and remaining plots of Fig. 10. A comparison of Tables 6 and 7 shows that the method satisfactorily estimates the two-port Thevenin equivalent in all the cases.

D. TEST RESULTS FOR SU1 AND SU2

For validating SU1 and SU1_apx, we choose a set of "primary line-tripping line" pair. For example, we denote

TABLE 7. Estimated two-port equivalent parameters obtained by SE3.

	Estimated two-port equivalent impedances		
Case	Z_{sM}(pu) (% *error*)	Z_{sN} (pu) (% *error*)	Z_{tr}^{MN}(pu) (% *error*)
Line 5-8	0.0036 +j 0.0214 **(0)**	0.0169 +j 0.0404 **(0.2288)**	0.0012 +j 0.0172 **(1.2825)**
Line 6-7	0.0029 +j 0.0189 **(1.1565)**	0.0281 +j 0.0536 **(2.3901)**	0.0012 +j 0.0188 **(7.4455)**
Line 23-24	0.0052 +j 0.0322 **(0.6126)**	0.0065 +j 0.0215 **(0.4458)**	0.0039 +j 0.0625 **(7.6792)**
Line 4-14	0.0069 +j 0.0225 **(0)**	0.0056 +j 0.0296 **(0.4707)**	0.0028 +j 0.0445 **(7.9413)**
Line 16-21	0.0042 +j 0.0138 **(0.6887)**	0.0137 +j 0.0565 **(1.7493)**	0.0047 +j 0.0794 **(11.3281)**

TABLE 8. Actual two-port equivalent parameters obtained by network analysis.

	Actual two-port equivalent impedances		
Case Prim line, trip line	Z_{sM}(pu)	Z_{sN} (pu)	Z_{tr}^{MN}(pu)
16-17, 16-15	0.0074 +j 0.0230	0.0061 +j 0.0261	∞
16-19, 16-24	0.0049 +j 0.0160	0.0060 +j 0.0341	∞
3-2, 3-18	0.0147 +j 0.0416	0.0013 +j 0.0128	-0.1241 +j 0.6234
17-16, 17-27	0.0096 +j 0.0447	0.0053 +j 0.0166	-0.0219 +j 0.2221
5-8, 5-6	0.0081 +j0.0389	0.0075 +j 0.0257	-0.0015 +j 0.1341
14-13,14-15	0.0100 +j 0.0434	0.0050 +j 0.0341	-0.0015 +j 0.0738
26-25, 26-28	0.0107 +j 0.0398	0.0061 +j 0.0174	0.0464 +j 0.2227
5-8, 5-4	0.0036 +j 0.0340	0.0154 +j 0.0386	0.0011 +j 0.0178

the case in which the primary line is the line between buses 16 and 17 and the line which trips is the one between buses 16 and 15 as Case 16-17, 16-15. The two-port equivalent around the primary line is to be updated after the chosen incident line trips. The actual two-port equivalents after the incident line trip event for each case are shown in Table 8. The estimated equivalents are populated in Table 9. It is observed that the source impedances are estimated to a good level of accuracy (maximum error $< 7\%$). Errors in transfer impedance estimation are observed to be fairly high in some instances, e.g. Case 3-2, 3-18, Case 17-16, 17-27, and Case 5-8, 5-6 of Table 9. However, in these cases, it should also be noted that the transfer impedance is much large in comparison to the impedance of the primary line itself (please refer [13] for values of line impedances). Values of $k(= \ |Z_l|/|Z_{tr}^{MN}|)$ for the cases are 0.024, 0.04, and 0.084 respectively. This means for these cases the transfer path is insignificant for all practical purposes and therefore the high estimation errors bear no significance.

Case 16-17, 16-15, and Case 16-19, 16-24, are ones in which the actual transfer path impedance is infinite. As we can observe, the estimated transfer path impedances for these cases are also very large in comparison to the primary line impedance and therefore transfer path can be neglected in these cases. For the remaining cases, the estimation of transfer path impedances is fairly accurate. This observation is important, particularly for the reason that for these cases the transfer impedance is comparable to the primary line impedance, and therefore it needs to be estimated as correctly as possible. Values of k for the cases are 0.137, 0.143, and 0.63 respectively

TABLE 9. Estimated two-port equivalent parameters obtained by SU1.

	Estimated two-port equivalent impedances		
Case Prim line, trip line	Z_{sM}(pu) (% *error*)	Z_{sN} (pu) (% *error*)	Z_{tr}^{MN}(pu) (% *error*)
16-17, 16-15	0.0075 +j0.0231 **(0.738)**	0.0060 +j0.0260 **(0.433)**	2.1379 – j2.9312 **(99.998)**
16-19, 16-24	0.0044 +j0.0159 **(3.153)**	0.0067 +j0.0342 **(1.959)**	-1.374 +j1.0446 **(100.001)**
3-2, 3-18	0.0147 +j0.0415 **(0.239)**	0.0013 +j0.0128 **(0.327)**	-0.154 +j0.7283 **(17.161)**
17-16, 17-27	0.0100 +j0.0441 **(1.535)**	0.0052 +j0.0167 **(0.774)**	0.0054 +j0.2246 **(12.3)**
5-8, 5-6	0.0056 +j0.0384 **(6.311)**	0.0084 +j0.0257 **(3.373)**	-0.0377 +j0.0829 **(46.759)**
14-13,14-15	0.0112 +j0.0441 **(3.078)**	0.0040 +j0.0334 **(3.513)**	0.0028 +j0.0786 **(8.647)**
26-25, 26-28	00.0106 +j0.0399 **(0.345)**	0.0061 +j0.0174 **(0.031)**	0.0462 +j0.2217 **(0.439)**
5-8, 5-4	0.0037 +j0.0335 **(1.445)**	0.0152 +j0.0394 **(1.924)**	0.0013 +j0.0177 **(1.023)**

TABLE 10. Estimated two-port equivalent parameters obtained by SU1_apx.

	Estimated two-port equivalent impedances		
Case Prim line, trip line	Z_{sM}(pu) (% *error*)	Z_{sN} (pu) (% *error*)	Z_{tr}^{MN}(pu) (% *error*)
16-17, 16-15	0.0070 +j0.0231 **(1.543)**	0.0061 +j0.0261 **(0.127)**	-9.8602 -j23.5136 **(100.01)**
16-19, 16-24	0.0055 +j0.0156 **(4.27)**	0.0058 +j0.0339 **(0.776)**	2.1246 +j-3.6937 **(99.998)**
5-8, 5-6	0.0085 +j0.0388 **(1.04)**	0.0074 +j0.0258 **(0.492)**	-0.0019 +j0.1297 **(3.317)**
5-8, 5-4	0.0024 +j0.0344 **(3.778)**	0.0148 +j0.0386 **(1.423)**	0.0013 +j0.0179 **(0.95)**
3-2, 3-18	0.0172 +j0.0336 **(19.02)**	0.0013 +j0.0128 **(0.103)**	-0.4317 +j0.5031 **(51.961)**
17-16, 17-27	0.0057 +j0.0543 **(22.629)**	0.0052 +j0.0167 **(0.725)**	0.0025 +j0.2211 **(10.942)**
14-13,14-15	-0.0050 +j0.0541 **(41.488)**	0.0050 +j0.0348 **(1.931)**	-0.0008 +j0.0708 **(4.111)**
26-25, 26-28	0.0161 +j0.0369 **(14.978)**	0.0061 +j0.0174 **(0.026)**	0.0459 +j0.2227 **(0.215)**

When the same set of test scenarios are presented to SU1_apx, we observe that the estimation accuracies are reasonably good for cases in which the ratio of the charging current to the line loading current for the incident line are low, e.g. the first four cases shown in Table 10. For these cases, the magnitude of $\alpha \left(= \frac{I_{mc}+I_{pc}}{I_{mp}+I_{mc}}\right)$ ranges in between 0.009-0.24. For other remaining cases, α assumes higher magnitudes in between 0.4-0.9 and the estimation errors tend to be large. As mentioned earlier, the near 100% error figures for transfer path estimates in the first two cases of Table 10 do not hold any practical significance.

For validating SU2 and SU2_alt, we consider four cases. For example, we denote the case in which the primary line

TABLE 11. Actual two-port equivalent parameters obtained by network analysis.

Case Prim line, trip bus, shunt element value	Actual two-port equivalent impedances		
	Z_{sM}(pu)	Z_{sN} (pu)	Z_{tr}^{MN}(pu)
17-16, 27, -200	0.0076 +j0.0290	0.0054 +j0.0167	-0.0182 +j0.2004
5-8, 6, 200	0.0036 +j0.021	0.0170 +j0.0402	0.0011 +j0.0174
15-16, 14, 200	0.0155 +j0.0452	0.0042 +j0.0147	-0.0211 +j0.2215
39-9, 1, -200	0.0004 +j0.0055	0.0123 +j0.0642	-0.2919 +j1.1175

TABLE 12. Estimated two-port equivalent parameters obtained by SU2.

Case Prim line, trip bus, shunt element value	Estimated two-port equivalent impedances		
	Z_{sM}(pu) (% *error*)	Z_{sN} (pu) (% *error*)	Z_{tr}^{MN}(pu) (% *error*)
17-16, 27, -200	0.0077 +j0.0288 **(0.707)**	0.0054 +j0.0167 **(0.333)**	-0.0175 +j0.1995 **(0.574)**
5-8, 6, 200	0.0036 +j0.021 **(0.39)**	0.0167 +j0.0405 **(1.007)**	0.0011 +j0.0174 **(0.046)**
15-16, 14, 200	0.0153 +j0.0455 **(0.705)**	0.0042 +j0.0147 **(0.207)**	-0.0206 +j0.2215 **(0.207)**
39-9, 1, -200	0.0004 +j0.0055 **(0.528)**	0.0123 +j0.0642 **(0.013)**	-0.2975+j1.1150 **(0.529)**

TABLE 13. Estimated two-port equivalent parameters obtained by SU2_alt.

Case Prim line, trip bus, shunt element value	Estimated two-port equivalent impedances		
	Z_{sM}(pu) (% *error*)	Z_{sN} (pu) (% *error*)	Z_{tr}^{MN}(pu) (% *error*)
17-16, 27, -200	0.0077 +j0.0288 **(0.585)**	0.0054 +j0.0167 **(0.375)**	-0.0183 +j0.2001 **(0.126)**
5-8, 6, 200	0.0036 +j0.0214 **(0.199)**	0.0167 +j0.0406 **(1.214)**	0.0011 +j0.0174 **(0.019)**
15-16, 14, 200	0.0153 +j0.0454 **(0.629)**	0.0042 +j0.0147 **(0.188)**	-0.0203 +j0.2212 **(0.376)**
39-9, 1, -200	0.0004 +j0.0055 **(0.513)**	0.0123 +j0.0642 **(0.005)**	-0.2965 +j1.1122 **(0.606)**

is the line between buses 17 and 16 and a shunt capacitor of rating 200 MVAr trips at bus 27 as Case 17-16, 27, -200. The actual two-port equivalents after the shunt element trip event for each case is shown in Table 11. The estimated equivalents based on SU2 and SU2_alt are populated in Table 12 and Table 13 respectively. The two-port Thevenin equivalent around the primary line were successfully updated in all the cases. Also, SU2_alt is observed to be a good alternative to SU2 because of its comparable performance.

E. ADAPTIVE RELAY SETTING APPLICATION

Adaptation of the relay characteristics in response to structural and operational changes in the system can mitigate relay malfunction [18]. Such analyses require re-evaluation of the two-port equivalent around the primary line which is protected by the relay. Methods such as SU1 and SU2 become useful in such scenarios. To illustrate, we consider the line between buses 19 and 16 of the IEEE 39 bus test system as the primary line. The relay in consideration is assumed to be placed on bus 19. We consider the scenario in which the system transitions from the base topology to a new topology in which lines 16-21 and 16-17 have been outaged.

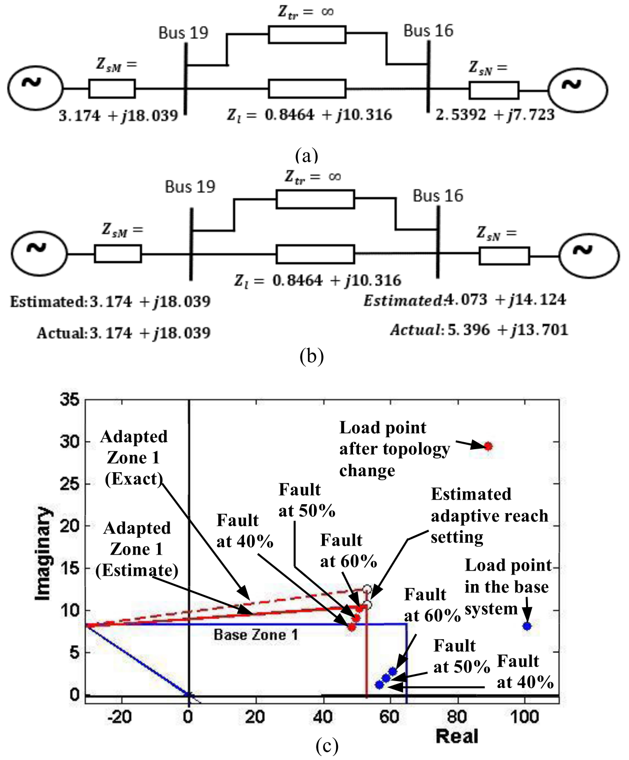

FIGURE 11. Two port Thevenin equivalent across line 19-16 in (a) the base case (b) after the structural changes respectively, (c) Base and adapted Zone-1 characteristics, and relay performance in the base and outaged cases.

This structural change in the system leads to the weakening of the source impedance on the bus 16 side by approximately 45% (see Fig. 11(a) and (b)). This leads to adverse effects on the performance of the relay for high resistance faults on line 19-16 as the base Zone-1 characteristic fails to detect faults at or above 50% of the line length as shown in Fig. 11(c). This is depicted by the position of the red dots with respect to the base Zone-1 characteristic which is depicted in blue. The red dots indicate the impedance measured at the relay location due to the high resistance faults. Note that for the same faults, the base Zone 1 characteristic worked satisfactorily in the base case scenario as depicted by the blue dots. The relay performance gets restored when the Zone-1 resistive and reactive reach settings are modified using the updated two-port equivalent which is shown in Fig. 11(b). The adapted Zone 1 depicted in red in Fig. 11(c) is able to identify the faults.

V. CONCLUSION

In this paper, we have proposed methods for estimating the two-port equivalent of the system around a power transmission line. The proposed methods are aimed for decentralized deployment at substations and can be broadly categorized into methods for estimating two-port equivalent and methods for updating the two-port equivalent in response to a change in the network topology in the vicinity of the line. The estimation methods are based on events such as line faults and shunt element switching at its terminals. When tested for various line fault and shunt element switching scenarios in a simple two-source transmission system, they were found to estimate the source impedances within error of 10%.

Limitations and practical considerations in using the estimated two-source equivalents for larger inter-connected networks are thoroughly discussed and illustrated based on applications such relay reach analysis and calculation of remote infeed. The estimation algorithms are extended to include the effects of transfer path impedance by considering additional input of the bus Thevenin impedances. This approach is found to estimate the two-port Thevenin equivalent satisfactorily with a maximum error of 11% in the transfer path. A comparative analysis with the existing methods is also provided to bring out the unique features and limitations of the proposed approaches.

The methods for updating the two-port Thevenin equivalent consider scenarios such as tripping-off of an incident line and switching of a shunt element at an adjacent bus. The solutions were tested for various topology change scenarios in the IEEE 39 bus test system and found to be reasonably accurate, with maximum errors being less than 10%. Utility of such algorithms in adaptive relay reach setting application is demonstrated using the same test system. In conclusion, the proposed methods perform satisfactorily under simulated test conditions. They have low computational burden and require only a limited amount of model data and measurements. They can potentially form useful tools in the framework of digital substations equipped with reliable communication infrastructure.

APPENDIX

SERIES AND SHUNT BRANCH MODIFICATIONS ON Z_{bus}

Let us consider the network impedance submatrix $Z\,(ijk)$ corresponding to nodes i, j and k as shown below.

$$Z\,(ijk) = \begin{pmatrix} Z_{ii} & Z_{ij} & Z_{ik} \\ Z_{ji} & Z_{jj} & Z_{jk} \\ Z_{ki} & Z_{kj} & Z_{kk} \end{pmatrix}$$

Let us now consider the case in which a new branch with impedance Z_{br} is added to the network in series between buses i and j. The following formula can be used to obtain the new $Z\,(ijk)$ submatrix:

$$Z\,(ijk)_{new} = Z\,(ijk) - \frac{\{Z_{ij}\}\,\{Z_{ij}\}^T}{\Delta_{\mathrm{ij}}} \tag{A1}$$

where $\{Z_{ij}\}$ represents the vector difference between the i_{th} and j_{th} columns of $Z\,(ijk)$ and the entity $\Delta_{\mathrm{ij}} = Z_{ii} + Z_{jj} 2Z_{ij} + Z_{br}$. The same formula can be used in case a branch with impedance Z_{br} between buses i and j is removed from the network, by keeping $-Z_{br}$ in place of $+Z_{br}$ in the formula for Δ_{ij}.

Let us now consider the case in which a shunt element of impedance Z_{sh} is added to the network at bus i. The following formula can be used to obtain the new $Z\,(ijk)$ submatrix:

$$Z\,(ijk)_{new} = Z\,(ijk) - \frac{\{Z_i\}\,\{Z_i\}^T}{\Delta_{\mathrm{ii}}} \tag{A2}$$

where $\{Z_i\}$ represents the i_{th} column of $Z\,(ijk)$ and the entity $\Delta_{\mathrm{ii}} = Z_{ii}+Z_{sh}$. The same formula can be used in case a shunt element with impedance Z_{sh} at bus i is removed from the network, by keeping $-Z_{sh}$ in place of $+Z_{sh}$ in the formula for Δ_{ii}.

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