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Theoretical and Numerical Studies on Vibratory Synchronization Transmission of a Vibrating Mechanical System Driven by Single Motor Considering Sliding Dry Friction

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ABSTRACT As a continuous work of the previous literatures, a special dynamical model with one cylindrical roller driven by a single exciter and one outer ring, is taken for example to explore the vibratory synchronization transmission (VST) of the system considering sliding dry friction in this paper. The motion differential equations of the system, are given firstly. Using the average method, the theory condition of implementing VST is obtained. The VST characteristics are qualitatively discussed in numerical, which are further quantitatively verified by simulations. It is shown that, the vibration amplitudes of the roller in the horizontal and vertical directions are basically identical, and less affected by the friction coefficient, but the stable phase difference between the exciter and the outer ring is affected too much by it. Based on the present work, some new types of vibrating equipments, such as vibrating crushers/mills, can be designed.

INDEX TERMS Sliding dry friction, vibratory synchronization transmission, cylindrical roller, exciter.

I. INTRODUCTION

There are many vibration or synchronization problems in engineering or real-time system fields, such as the chaotic vibration, bifurcation, stabilization and synchronization control for fractional discrete-time systems, digital chaos and local synchronization for a universal analog-digital hybrid mechanism, and vibration for the gear system, etc. [1]–[3]. Especially in vibration utilization engineering, utilizing the theory of synchronization of exciters (generally eccentric rotors driven by motors), many new vibrating machines can be designed and widely used in the various industrial production process, which can implement the crushing, milling, screening and cooling/drying of materials, and more and more researchers are inspired to study it.

In the theory of synchronization of exciters, there is a particular phenomenon called VST, which can be here defined as exciters without direct power supply, can keep operating synchronously with the other one or multiple exciters (at least one of multiple exciters is driven by power supply). Using this principle, some new types of vibrating machines can be invented.

In the connection with the development of synchronization theories of exciters, we must date back to the findings by Blekhman [4]–[6], who firstly gave the theoretical investigation of synchronization of two identical exciters by using Poincare small parameter method, followed by which he developed this method to be so called the method of direct motion separation, and from then on, researches of synchronization were found to be of great significance in engineering. Inoue *et al.* [7] gave the synchronization of the mechanical system with multiple cycles by using the perturbation method, including the synchronization problem with two (or three) times frequency. Considering the effects of damping of the system, Wen *et al.* [8], [9] not only developed the synchronization theory of exciters, studied the synchronization problems for n ($n \geq 1$) times frequency, but also based on which invented many new vibrating

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machines applied to engineering successfully, such as vibrating mills, vibrating feeders, vibrating dewatering screens, etc. Balthazar *et al.* [10], [11] discussed the synchronization problem of two or four non-ideal sources on a flexible portal frame structure, of which a particular phenomenon, called the Sommerfeld Effect, was revealed.

The great engineering applicable prospects of synchronization theory, have attracted more and more scholars and engineers to pay their attentions to investigate it. Such as Fang and Hou [12] and Fang *et al.* [13] revealed the synchronization characteristics of the rotor-pendulum system. The synchronization of the system with different key parameters of the two vibrators with a common rotational axis in a far-resonant vibrating system, is presented by Chen *et al.* [14]. Zhang *et al.* [15], [16] studied the stability, Sommerfeld effect, and the VST of three rollers in the vibrating system with two exciters; as well as discussed synchronization and stability of the vibrating system with two or multiple rigid frames, driven by two or four reversed exciters [17], [18]. Kong *et al.* [19] gave synchronization analysis in a simply supported beam system excited by two non-ideal induction motors, and also revealed Sommerfeld effect within it.

Different from the abovementioned studies on the synchronization of exciters, in this paper, considering the sliding dry friction effect, a special dynamical model is taken for example to explore another VST manner, where a cylindrical roller driven by a single exciter, can drive an outer ring to realize synchronous circular motion of the system, then further facilitate the energy exchange between the outer ring and the exciter, and finally the VST of the total system is realized. The present dynamical model was proposed firstly in [4] and [5], however, some deep investigations on VST, such as the coupling dynamical characteristics, numerical qualitative analyses and simulation verifications, are less considered. The present work aims at revealing deeply the coupling mechanism of implementing VST of the system by theory and numeric methods, based on the [4] and [5]. This can be also considered as another attempt to develop the theory of VST and expand its application field, base on which a new engineering application approach of VST can be expanded.

The present work is organized as follows: the dynamical model and the corresponding motion differential equations of the considered vibrating system, are given firstly. Then the VST of the system is investigated in theory. Next, some numeric qualitative analyses and simulations are provided, where simulations with different friction coefficients are carried out to quantitatively examine the validity of the theoretical and numerical qualitative results. Finally the conclusions are summarized.

II. DYNAMICAL MODEL AND MOTION DIFFERENTIAL EQUATIONS OF THE SYSTEM

The system dynamic model is shown in Figure1 [4], [5], which includes an exciter, an outer ring and a cylindrical roller. The outer ring is connected to the foundation by coil

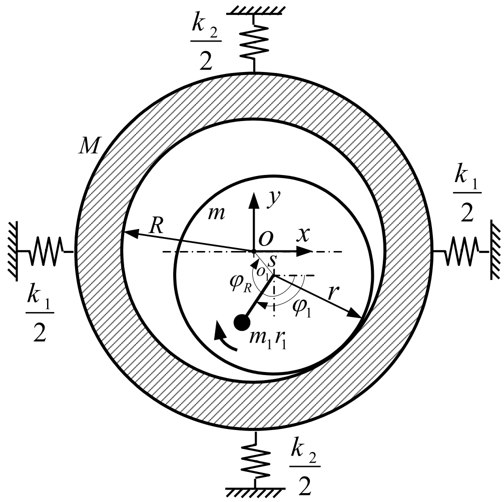

FIGURE 1. A dynamical model of a considered vibrating system with one outer ring and one roller driven by a single exciter.

springs, and a cylindrical roller is placed in the inner cavity of the outer ring. An exciter driven by an induction motor and coaxial with the cylindrical roller, can drive the cylindrical roller to implement a circular motion along with the inwall of the outer ring. At the same time, since the cylindrical roller satisfies the condition of not leaving the inwall of the ring (which will be given later), the outer ring can rotate around the centroid O in Figure.1 following with the cylindrical roller, and the rotational angle and the eccentric radius of the latter are denoted by φ_R and s, respectively.

In this paper, the outer ring can be regarded as an eccentric rotor rotating around the mass center of the system when the ring is stationary, that is, the outer ring can be seen as an exciter without a direct power supply. If the ring can synchronously rotate with the exciter under the condition of a certain dry friction, the VST of the system is implemented. The system embodies two degrees of freedom: the displacements of the centroid of the cylindrical roller in x- and y- directions. The exciter rotates around its axis, and its phase angle is denoted by φ_1, the radius of rotation is r_1.

Substituting the kinetic energy, the potential energy and the energy dissipation functions of the system into Lagrange's equations, the motion differential equations of the system are presented directly as

$$\begin{aligned}
&(m+m_1+M)\,\ddot{x}+f_1\dot{x}+k_1x\\
&\quad=-m_1r_1\left(\dot{\varphi}_1^2\cos\varphi_1+\ddot{\varphi}_1\sin\varphi_1\right)\\
&\qquad-Ms\left(\dot{\varphi}_R^2\cos\varphi_R+\ddot{\varphi}_R\sin\varphi_R\right),\\
&(m+m_1+M)\,\ddot{y}+f_2\dot{y}+k_2y\\
&\quad=m_1r_1\left(\dot{\varphi}_1^2\sin\varphi_1-\ddot{\varphi}_1\cos\varphi_1\right)\\
&\qquad+Ms\left(\dot{\varphi}_R^2\sin\varphi_R-\ddot{\varphi}_R\cos\varphi_R\right),\\
&J_1\ddot{\varphi}_1+f_{\mathrm{d}}\dot{\varphi}_1\\
&\quad=T_{\mathrm{e}1}-m_1r_1(\ddot{x}\sin\varphi_1+\ddot{y}\cos\varphi_1),\\
&J_R\ddot{\varphi}_R\\
&\quad=-Ms(\ddot{x}\sin\varphi_R+\ddot{y}\cos\varphi_R)-Mgs\cos\varphi_R-f_RNs, \qquad (1)
\end{aligned}$$

where m, m_1 and M are the masses of the cylindrical roller, exciter and the outer ring, respectively; J_1 and J_R are the moment of inertia of the exciter and the outer ring, $J_1 = m_1 r_1^2$, $J_R = m_R s^2 + I_R$; f_1 and f_2 are damping coefficients of the system in x- and y- directions, respectively; k_1 and k_2 are stiffness coefficients of the system in x- and y- directions; R and r are the radius of the outer ring and the cylindrical roller, $s = R - r$; φ_1 and φ_R are rotating phases of the exciter and the outer ring, respectively; f_d is the damping coefficient of the axis of the motor; N is the action force between the cylindrical roller and the inwall of the outer ring, $N = M\left(-g\sin\varphi_R + s\dot{\varphi}_R^2 + \ddot{x}\cos\varphi_R - \ddot{y}\sin\varphi_R\right)$; T_{e1} denotes the electromagnetic output torque of the motor, which detailed expressions and the corresponding relationship with the electrical parameters can be seen in [20].

III. THEORY ANALYSES ON VST OF THE SYSTEM

According to the dynamic model of the system, the average phase between the exciter and the outer ring, is assumed to be φ, and their phase difference is 2α, i.e.,

$$\varphi_1 - \varphi_R = 2\alpha, \quad \varphi_1 + \varphi_R = 2\varphi, \tag{2}$$

From Eq. (2), we have

$$\varphi_1 = \varphi + \alpha, \quad \varphi_R = \varphi - \alpha. \tag{3}$$

The average angular velocity of the exciter and the outer ring is $\dot{\varphi} = \omega_{m0}$ in the steady state, in this case, the angular accelerations of the exciter and the outer ring are all zero, i.e., $\ddot{\varphi}_i = 0$. Substituting Eq. (3) into the first two formulae in Eq. (1), and based on the transfer function method [21], the responses of the cylindrical roller in x- and y- directions, can be obtained as

$$x = \frac{r_m r_1}{\mu_1}\cos(\varphi + \alpha - \gamma_1) + \frac{r_{ms} s}{\mu_1}\cos(\varphi - \alpha - \gamma_1), \tag{4}$$

$$y = -\frac{r_m r_1}{\mu_2}\sin(\varphi + \alpha - \gamma_2) - \frac{r_{ms} s}{\mu_2}\sin(\varphi - \alpha - \gamma_2), \tag{5}$$

where

$$r_m = \frac{m_1}{M + m + m_1}, \quad r_{ms} = \frac{M}{M + m + m_1},$$

$$\mu_1 = 1 - \frac{1}{z_1^2}, \quad \mu_2 = 1 - \frac{1}{z_2^2}, \ z_1 = \frac{\omega_{m0}}{\omega_1}, \ z_2 = \frac{\omega_{m0}}{\omega_2},$$

$$\omega_1 = \sqrt{\frac{k_1}{M + m + m_1}}, \quad \omega_2 = \sqrt{\frac{k_2}{M + m + m_1}},$$

$$\xi_1 = \frac{f_1}{2\sqrt{k_1(M + m + m_1)}}, \quad \xi_2 = \frac{f_2}{2\sqrt{k_2(M + m + m_1)}},$$

$$\gamma_1 = \begin{cases} \arctan\dfrac{2\xi_1 z_1}{1 - z_1^2}, & 1 - z_1^2 > 0 \\ \pi + \arctan\dfrac{2\xi_1 z_1}{1 - z_1^2}, & 1 - z_1^2 < 0 \end{cases},$$

$$\gamma_2 = \begin{cases} \arctan\dfrac{2\xi_2 z_2}{1 - z_2^2}, & 1 - z_2^2 > 0 \\ \pi + \arctan\dfrac{2\xi_2 z_2}{1 - z_2^2}, & 1 - z_2^2 < 0 \end{cases}.$$

A. THEORETICAL CONDITION OF THE CYLINDRICAL ROLLER WITHOUT LEAVING TO THE INWALL OF THE OUTER RING

In the steady state the precondition of ensuring the synchronous operation between the active exciter and the passive outer ring, is that the cylindrical roller cannot leave the inwall surface of the outer ring. So it is necessary to firstly study this theoretical precondition, which means that the action force N received from the outer ring inwall, should be greater than 0, i.e.,

$$N = M\left(-g\sin\varphi_R + s\dot{\varphi}_R^2 + \ddot{x}\cos\varphi_R - \ddot{y}\sin\varphi_R\right) > 0 \tag{6}$$

After the simplification of Eq. (6), there is

$$g\sin\varphi_R - \ddot{x}\cos\varphi_R + \ddot{y}\sin\varphi_R < s\omega_{m0}^2 \tag{7}$$

Substituting the second derivative of x and y in Eqs. (4) and (5) with respect to time t into Eq. (7), the simplified expression is obtained. In practice engineering, generally, there are $\gamma_1 \approx \gamma_2 = \gamma_0$, $\mu_1 \approx \mu_2 = \mu_0$ [9], so Eq. (7) can be further simplified as

$$C_N < 1 \tag{8}$$

with

$$C_N = g\Big/s\omega_{m0}^2 + (r_m r_1/s\mu_0)\cos(2\alpha - \gamma_0) + (r_m/\mu_0)\cos(\gamma_0),$$

which is the conditional coefficient of the cylindrical roller without leaving the inwall of the outer ring. In this case the system can ensure that the cylindrical roller does not leave the inwall surface of the outer ring under the condition of $C_N < 1$.

B. FEASIBILITY ANALYSIS OF REGARDING THE OUTER RING AS AN EXCITER

It is known that the cylindrical roller can move along the inwall of the outer ring without leaving it during the steady operation of the system, at this situation the cylindrical roller driven by the exciter, can realize circular motion around the inwall of the outer ring, and drive the outer ring to move, the rotation angle of the outer ring, φ_R, being as an important freedom degree of the system, should satisfy the following balance equation

$$J_R\ddot{\varphi}_R = -Mgs\cos\varphi_R - Ms(\ddot{x}\sin\varphi_R + \ddot{y}\cos\varphi_R) - R(\dot{\varphi}_R, \varphi_R, \varphi_1) \tag{9}$$

where $R(\dot{\varphi}_R, \varphi_R, \varphi_1)$ is the resistance torque between the outer ring and the cylindrical roller, and it can be expressed as follows

$$R(\dot{\varphi}_R, \varphi_R, \varphi_1) = f_R s N \operatorname{sgn}(\dot{\varphi}_R) \tag{10}$$

Here N is the positive pressure from the outer ring to the roller, $N = M\left(-g\sin\varphi_R + s\dot{\varphi}_R^2 + \ddot{x}\cos\varphi_R - \ddot{y}\sin\varphi_R\right)$, $\operatorname{sgn}(\dot{\varphi}_R)$ can adjust the positive and negative values of the resistance torque according to the direction of the positive pressure.

Substituting Eq. (10) and the second derivative of x and y of Eqs. (4) and (5) into Eq. (9), the following expression can be obtained:

$$\begin{aligned} J_R\ddot{\varphi}_R &= -Mgs\cos(\varphi_R) \\ &\quad - Ms\omega_{\mathrm{m0}}^2\left[G\sin(\varphi_1-\varphi_R-\gamma_0) - H\sin(\gamma_0)\right] \\ &\quad - f_R Ms[-\omega_{\mathrm{m0}}^2 G\cos(\varphi_1-\varphi_R-\gamma_0) \\ &\quad - \omega_{\mathrm{m0}}^2 H\cos(\gamma_0) - g\sin(\varphi_R) + s\dot{\varphi}_R^2] \end{aligned} \tag{11}$$

with $G = r_m r_1 / \mu_0$, $H = r_m s / \mu_0$.

According to [4] and [5], the fast and slow separation for φ_R in the above formula, can be performed, the above expression may become

$$J_R\ddot{\varphi}_R + f\dot{\varphi}_R = V_{1/1}(\alpha) - f_R Ms^2\omega_{\mathrm{m0}}^2 \tag{12}$$

where $f = 2f_R Ms^2\omega_{\mathrm{m0}}$, which is the damping constant of the system, $V_{1/1}(\alpha)$ is the vibration torque, and its expression is expressed as

$$V_{1/1}(\alpha) = -\frac{1}{2}Ms\omega_{\mathrm{m0}}^2\left\{\begin{array}{l} G[\sin(2\alpha-\gamma_0) \\ +f_R\cos(2\alpha-\gamma_0)] \\ +H[\sin(\gamma_0)+f_R\cos(\gamma_0)] \end{array}\right\} \tag{13}$$

When the angle ρ' is introduced into the system, and considering $\tan\rho' = f_R$, the above formula in Eq. (13), can be reduced to

$$V_{1/1}(\alpha) = -\frac{MsA\omega_{\mathrm{m0}}^2}{\sin(\rho')}\cos(2\alpha+\rho'-\chi) \tag{14}$$

with χ is the additional auxiliary angle when simplifying the motion expression of the outer ring, and A is respect to the amplitude of its motion trajectory, and $A = (r_{\mathrm{m}}/2\mu_0)\sqrt{r_1^2+s^2+2r_1 s\cos(2\alpha-2\gamma_0)}$.

In the case of $\alpha = \alpha_* =$ constant, it corresponds to the stationary state of the system, there is $V_{1/1}(\alpha) - f_R Ms^2\omega_{\mathrm{m0}}^2 = 0$, i.e.,

$$-\frac{MsA\omega_{\mathrm{m0}}^2}{\sin(\rho')}\cos(2\alpha+\rho'-\chi) - f_R Ms^2\omega_{\mathrm{m0}}^2 = 0 \tag{15}$$

This leads to

$$\cos(2\alpha+\rho'-\chi) = -\frac{s}{A}\sin(\rho') \tag{16}$$

Since $\left|-s\sin(\rho')/A\right| < 1$ in Eq. (16), we can obtain the following expression:

$$\frac{s}{A} < \frac{\sqrt{1+(f_R)^2}}{f_R} \tag{17}$$

The above expression can be described as: the eccentric radius of the outer ring rotating around the system centroid must not exceed $\sqrt{1+(f_R)^2}/f_R$ times than the amplitude of its motion trajectory. When the above conditions are satisfied, under the effect of the sliding dry friction between the cylindrical roller (driven by the exciter) and the outer ring, the outer ring centroid can be driven by the rotating roller to move around the stationary centroid O of the system, and the outer ring can be regarded as an 'exciter' rotating around O, which means this 'exciter' can follow the cylindrical roller to realize a circular motion with a certain frictional force.

C. THEORETICAL CONDITION OF IMPLEMENTING VST OF THE SYSTEM

Differentiating Eqs. (4) and (5), and inserting the results into the last two formulae of Eq. (1), then integrating them over $\varphi = 0 \sim 2\pi$ and taking the mean, finally the average torque balance equations of the exciter and the outer ring, are expressed as

$$\overline{T}_{L1} = T_{\mathrm{e01}} - f_{\mathrm{d}}\omega_{\mathrm{m0}} = T_{\mathrm{u}}(ls_1 + lc_1), \tag{18}$$

$$\overline{T}_{L2} = T_{\mathrm{e02}} - T_{\mathrm{u}} f_R \eta_{\mathrm{m}} \eta_r^2 (s_1+2) = \eta_m T_{\mathrm{u}}(ls_2 + lc_2), \tag{19}$$

with

$$\begin{aligned} ls_1 &= \eta_r s_1\sin(2\overline{\alpha}), \quad lc_1 = \eta_r c_1\cos(2\overline{\alpha}) + c_2, \\ ls_2 &= \eta_r(-s_2 + c_2 f_R)\sin(2\overline{\alpha}), \\ lc_2 &= \eta_r(c_2 + s_2 f_R)\cos(2\overline{\alpha}) + \eta_r^2 c_1, \\ s_1 &= r_{ms}\left(-\frac{\cos(\gamma_1)}{\mu_1} - \frac{\cos(\gamma_2)}{\mu_2}\right), \\ c_1 &= r_{ms}\left(-\frac{\sin(\gamma_1)}{\mu_1} - \frac{\sin(\gamma_2)}{\mu_2}\right), \\ s_2 &= r_{\mathrm{m}}\left(-\frac{\cos(\gamma_1)}{\mu_1} - \frac{\cos(\gamma_2)}{\mu_2}\right), \\ c_2 &= r_{\mathrm{m}}\left(-\frac{\sin(\gamma_1)}{\mu_1} - \frac{\sin(\gamma_2)}{\mu_2}\right), \quad \eta_r = s/r_1, \\ \eta_m &= M/m_1,\ T_{\mathrm{u}} = m_1 r^2\omega_{\mathrm{m0}}^2/2. \end{aligned}$$

In Eqs. (18) and (19): $T_u = m_1 r^2\omega_{\mathrm{m0}}^2/2$ denotes the kinetic energy of the standard exciter; $\overline{T}_{Li}$ ($i = 1, 2$) represents the load torque of the motor i (motor 2 is a virtual motor and does not actually exist); T_{e01} denotes the electromagnetic output torque of the motor when the system operates at the average angular velocity ω_{m0}, and $T_{\mathrm{e02}} = 0$. Besides, during the process of integral above, compared with changes of φ by time t, 2α changes very small, so the phase difference 2α might be considered as the slow-changing parameter [4]–[6], which has been replaced by its integral mean value $2\overline{\alpha}$.

According to Eqs. (18) and (19), the difference between the dimensionless residual torque $\tau_{\mathrm{c12}}(\overline{\alpha})$ between the exciter and the outer ring, is obtained as

$$\begin{aligned} \tau_{\mathrm{c12}}(\overline{\alpha}) &= \frac{\bar{T}_{L1} - \bar{T}_{L2}}{T_{\mathrm{u}}} - c_2 + \eta_{\mathrm{m}}\eta_r^2 c_1 \\ &= [\eta_r s_1 - \eta_{\mathrm{m}}\eta_r(-s_2 + c_2 f_R)]\sin(2\overline{\alpha}) \\ &\quad + [\eta_r c_1 - \eta_{\mathrm{m}}\eta_r(c_2 + s_2 f_R)]\cos(2\overline{\alpha}) \end{aligned} \tag{20}$$

Besides, in the super-resonant state of a small damping vibrating system, the changing of γ_i and μ_i ($i = 1, 2$) is so small that usually regarded as a constant [8], [9]. Thus the constraint function for $2\overline{\alpha}$ can be expressed as

$$|\tau_{\mathrm{c12}}(\overline{\alpha})| \le \tau_{\mathrm{c12max}} \tag{21}$$

According to Eqs. (18)-(21), the theory criterion of implementing VST of the system, can be derived as

$$\left|\begin{array}{l} \frac{(T_{\mathrm{e01}} - f_{\mathrm{d}}\omega_{\mathrm{m0}}) - (T_{\mathrm{e02}} - T_{\mathrm{u}} f_R \eta_{\mathrm{m}}\eta_r^2(s_1+2))}{T_{\mathrm{u}}} \\ -c_2 + \eta_{\mathrm{m}}\eta_r^2 c_1 \end{array}\right| \le \tau_{\mathrm{c12\,max}} \tag{22}$$

The synchronous solutions for the phase difference $2\overline{\alpha}$ and the operating frequency ω_{m0} in Eqs. (18) and (19), denoted by $2\overline{\alpha}_0$ and ω^*_{m0}, can be solved if the VST criterion (i.e., Eq. (22)) is satisfied. Since the system has only one exciter driven by the motor, when the cylindrical roller is driven by the exciter along the inwall of the outer ring and drives the outer ring to operate synchronously with the exciter, the system can naturally achieve stable operation. Therefore, there is no need to further discuss the stability of the system.

Adding the two formulae in Eqs. (18) and (19), the average dimensionless loading torque of the two motors (where motor 2 is a virtual motor) is obtained, which can be expressed as

$$\begin{aligned}\tau_{\mathrm{a}}(\bar{\alpha}) &= \frac{1}{2T_{\mathrm{u}}}(\overline{T}_{L1}+\overline{T}_{L2})\\ &= \frac{1}{2T_{\mathrm{u}}}(ls_1+lc_1+\eta_m ls_2+\eta_m lc_2)\\ &= \frac{1}{2}\{[\eta_r s_1+\eta_{\mathrm{m}}\eta_r(-s_2+c_2 f_R)]\sin(2\overline{\alpha})\\ &\quad +[\eta_r c_1+\eta_{\mathrm{m}}\eta_r(c_2+s_2 f_R)]\cos(2\overline{\alpha})+c_2+\eta_{\mathrm{m}}\eta_r^2 c_1\}\end{aligned} \tag{23}$$

Similarly, the constraint function of the average dimensionless load torque of the two motors (where motor 2 is a virtual motor) is

$$\tau_{\mathrm{a}}(\bar{\alpha}) < \tau_{\mathrm{amax}} \tag{24}$$

The coupling relationship between the exciter and the outer ring is an important factor for implementing the VST of the system. The larger the coupling torque between them, the easier the system to achieve synchronization. In order to reveal intuitively the synchronization ability of the system, the ratio of the difference of the dimensionless residual torque between the exciter and the outer ring $\tau_{\mathrm{c12\,max}}$ to the dimensionless maximum average torque τ_{amax}, can be defined as the synchronization capability coefficient ζ between the exciter and the outer ring, i.e.,

$$\zeta = \frac{\tau_{\mathrm{c12\,max}}}{\tau_{\mathrm{a\,max}}} \tag{25}$$

IV. NUMERIC QUALITATIVE ANALYSES ON VST OF THE SYSTEM

According to the above theory investigated results, in this section some numerical qualitative discussions are given to further reveal the dynamical characteristics of the system. The type of motor is a three-phase squirrel-cage (50 Hz, 380 V, 6-pole, 0.75 kW, rated speed 980 r/min); the rotor resistance $R_r = 3.40\Omega$, the stator resistance $R_s = 3.35\Omega$, the mutual inductance $L_m = 164\mathrm{mH}$, the rotor inductance $L_r = 170\mathrm{mH}$, the stator inductance $L_s = 170\mathrm{mH}$, and damping coefficient of the axis of the motor is $f_{\mathrm{d}} = 0.05$. The other parameters of the system: $k_1 = 8400\,\mathrm{kN/m}$, $k_2 = 1524\,\mathrm{kN/m}$, $m_1 = 10\mathrm{kg}$, $m_2 = 1200\mathrm{kg}$, $M = 10\mathrm{kg}$, $r_1 = 0.15\mathrm{m}$, $r = 0.03\mathrm{m}$, $s = 0.02\mathrm{m}$, $\xi_1 = 0.02$, $\xi_2 = 0.07$, $f_1 = f_2 = 7.66\mathrm{kN\cdot s/m}$.

Based on the above given parameters, the main natural frequencies of the system are calculated as: $\omega_1 \approx 73.5\mathrm{rad/s}$ and $\omega_2 \approx 36.4\mathrm{rad/s}$.

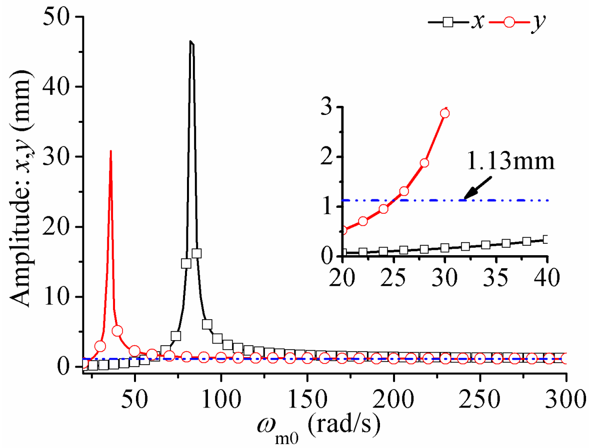

FIGURE 2. Frequency-amplitude curves of roller in *x*- and *y*- directions.

A. RESPONSE CHARACTERISTICS OF THE CYLINDRICAL ROLLER

Inserting the above given parameters of the system into Eqs. (4) and (5), the frequency-amplitude curves of the roller in x- and y- directions, are obtained, see Figure 2.

Before the resonance point, the amplitude of the cylindrical roller increases monotonically with the increasing frequency, and the natural frequency can be adjusted by changing the stiffness of the spring and the mass of the cylindrical roller. When the operating frequency is greater than the natural frequency, the amplitude of the roller is basically maintained at a constant value. As shown in Figure 2, in the super resonant state, the vibration amplitudes of the roller in x- and y-directions are all about 1.13 mm.

B. SYNCHRONIZATION CHARACTERISTICS OF THE SYSTEM

According to Eq. (8), only in the case of $C_N < 1$, the cylindrical roller does not leave the inwall surface of the outer ring during the steady operation process, which provides a precondition for the realization of the VST. From Figure 3(a), we can see that, the coefficient C_N of the roller against the inwall of the ring is always less than 1, which means that the cylindrical roller can always move circularly along the inwall of the ring in the super-resonant region.

In the case that the cylindrical roller can move circularly along the inwall of the outer ring and without leaving it, the outer ring can move following with the roller. According to Eq. (17), it can be known that when the eccentric radius s of the outer ring rotating surround the mass center does not exceed $\sqrt{1+(f_R)^2}\Big/f_R$ times than its motion amplitude A under the condition of the dry friction, the outer ring can be regarded as an exciter rotating around the center of the system.

In Figure 3(b), with the increasing friction coefficient, $\sqrt{1+(f_R)^2}\Big/f_R$ is getting smaller, and tends to be a roughly stable value after $f_R > 0.10$. As shown in the partial enlarged view, the ratio between the rotating radius of the outer ring

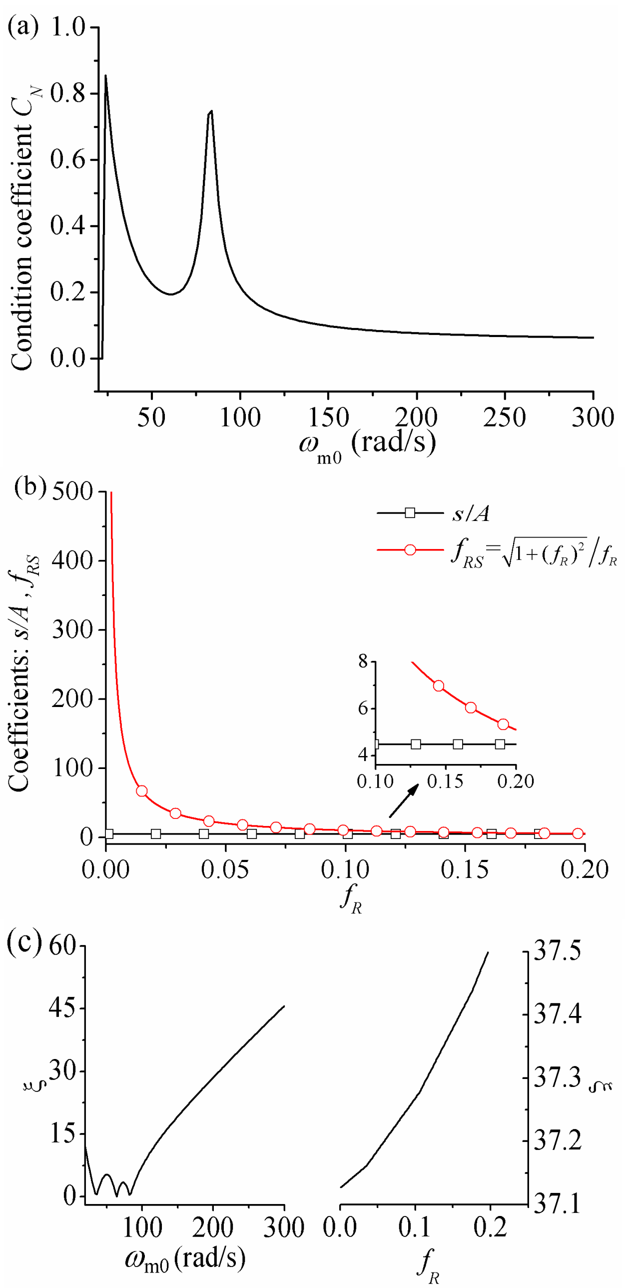

FIGURE 3. **The characteristic coefficients of the system: (a) The coefficient of the roller without leaving to the inner wall of the ring; (b) The feasibility curve of the outer ring can be regarded as an exciter; (c) Curve of synchronization ability coefficient with the different value of ω_{m0} and f_R.**

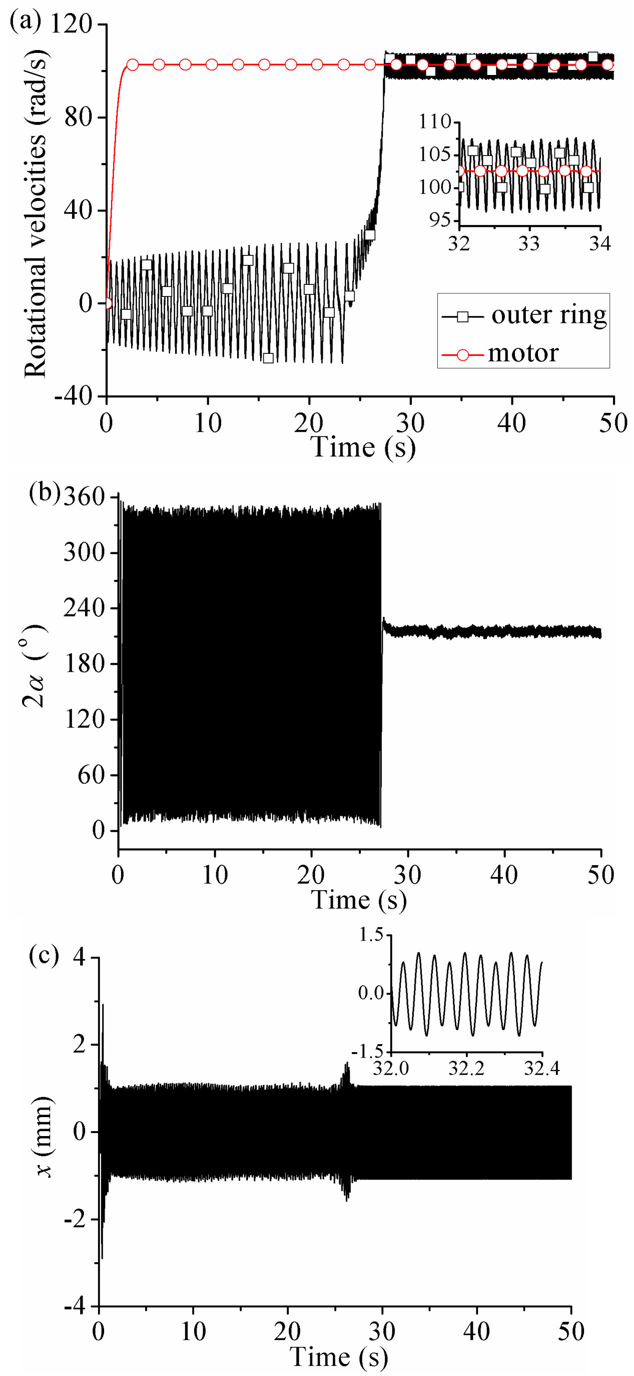

FIGURE 4. **Simulation results for $f_R = 0.06$: (a) Speed of exciter and outer ring; (b) Phase difference between exciter and outer ring; (c) Response of roller in x-direction; (d) Response of roller in y-direction; (e) Coupling force between exciter and outer ring.**

to its amplitude is less than $\sqrt{1+(f_R)^2}\Big/f_R$, which means the ring can be regarded as an exciter that rotates around O, and follows the cylindrical roller to realize a circular motion with a certain dry friction.

The synchronization capability coefficient ζ can be seen as an important index for the achievement of VST between the exciter the outer ring. As shown in Figure 3(c), the synchronization ability coefficient changes irregularly at the near resonance point of the system, and the synchronization ability at this point is relatively small. In general, the synchronization capacity coefficient of the system is greater than 0 in the whole resonance interval, especially in the super resonant region.

Besides, the influences of different friction coefficients on the synchronization ability of the system are discussed here. From Figure 3(c), one can see that, the system synchronization ability coefficient keeps rising with the increasing friction coefficient f_R, that is to say, the greater the friction

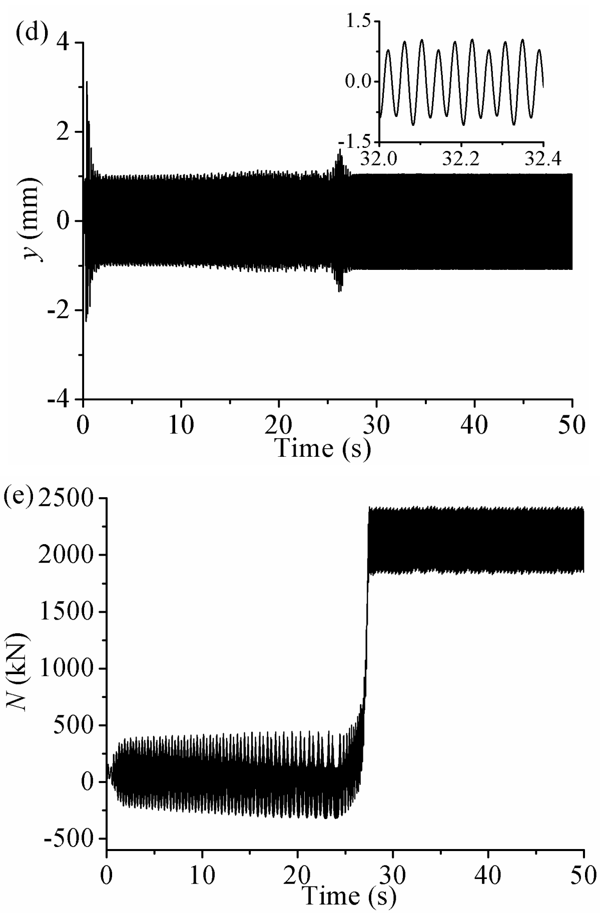

FIGURE 4. Simulation results for $f_R = 0.06$: (a) Speed of exciter and outer ring; (b) Phase difference between exciter and outer ring; (c) Response of roller in x-direction; (d) Response of roller in y-direction; (e) Coupling force between exciter and outer ring.

between the cylindrical roller and the outer ring, the easier of achieving VST.

V. SIMULATIONS

In order to further verify the feasibility of the results of the previous theory and numerical qualitative discussions, another numeric method called the fourth order Runge-Kutta routine, is directly applied to Eq. (1). It is worth mentioning that, according to the practical experiences, the friction coefficient between the cylindrical roller and the inwall of the outer ring is not fixed during the operation process, and will exhibit irregular and random small changes. Usually, the range of fluctuation is between $f_R \approx 0.05 \sim 0.2$, but this does not affect the availability of the simulation results. The other parameters are the same as that in section 4 except for friction coefficients. The detailed simulations are given as follows.

A. SIMULATION RESULTS WITH THE FRICTION COEFFICIENT $f_R \approx 0.06$

As shown in Figure 4(a), here $f_R = 0.06$, the system reaches the steady state after about 27s, the rotational speed of the

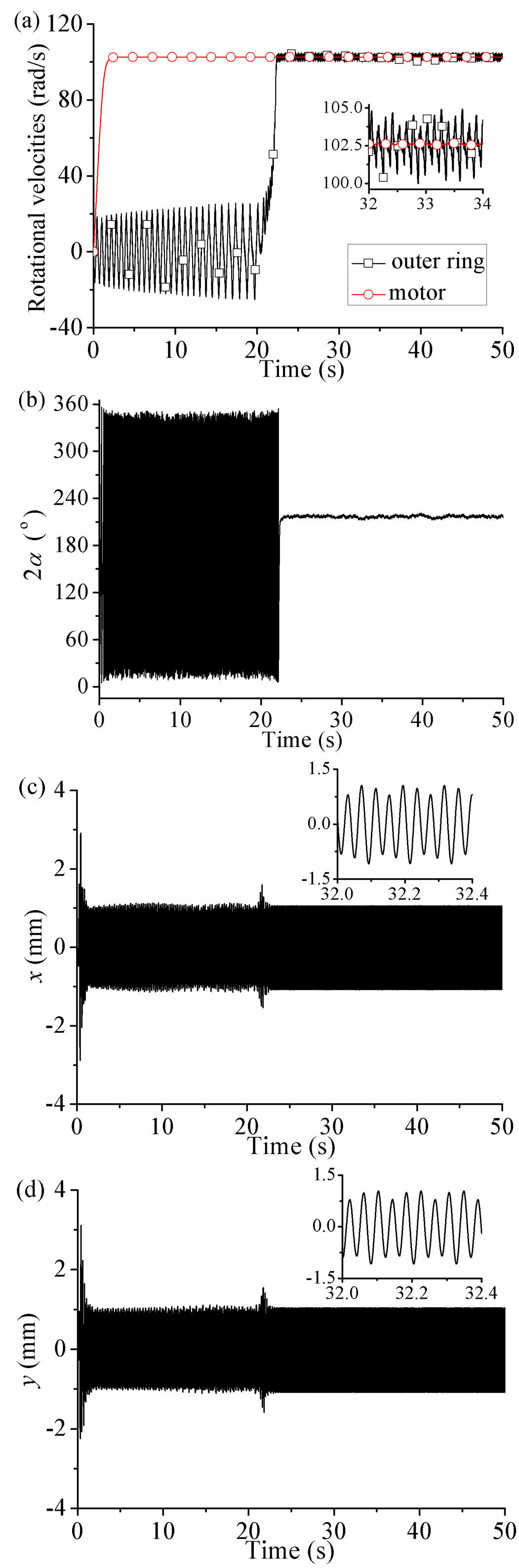

FIGURE 5. Simulation results for $f_R = 0.1$: (a) Speed of exciter and outer ring; (b) Phase difference between exciter and outer ring; (c) Response of roller in x-direction; (d) Response of roller in y-direction; (e) Coupling force between exciter and outer ring.

outer ring is basically equal to that of the exciter. Although it exhibits a certain fluctuation, it can still be considered to

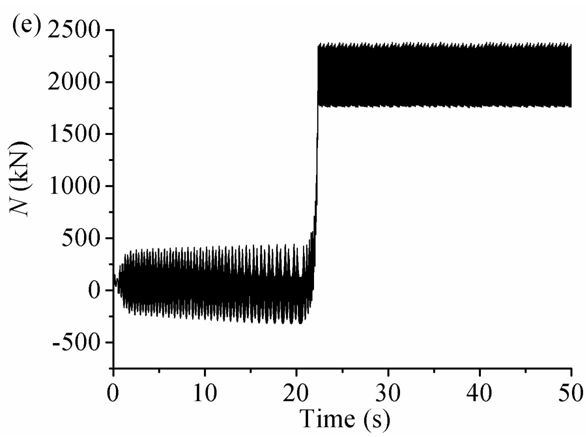

FIGURE 5. Simulation results for $f_R = 0.1$: (a) Speed of exciter and outer ring; (b) Phase difference between exciter and outer ring; (c) Response of roller in x-direction; (d) Response of roller in y-direction; (e) Coupling force between exciter and outer ring.

be approximately stable at 102.8rad/s. At the same time, as shown in Figure 4(e), the acting force between the cylindrical roller and the outer ring is greater than 0, which satisfies the condition that the roller does not leave the inwall surface of the outer ring, and provides a precondition for implementing the VST of the system.

From the above qualitative analyses in Figure 3(c) of section IV.B, we know that the synchronization ability coefficient of the system is always greater than 0, indicating that the exciter and the outer ring can implement the VST, which can be further verified by the fact that the phase difference between them is stabilized in the vicinity of 220° in Figure 4(b). As shown in Figures 4(c) and (d), the maximum displacements of the cylindrical roller in the x-, y- directions are all about 1.2mm, which is basically consistent with what is shown in Figure. 2, and these results also reflect that the motion trajectory of the roller is close to the circular motion.

B. SIMULATION RESULTS WITH THE FRICTION COEFFICIENT $f_R \approx 0.1$

As shown in Figures 5(a) and (e), here $f_R = 0.1$, it takes the system about 23s to reach a steady state, then the rotational velocities of the exciter and the outer ring are basically stabilized about 102.8rad/s, and the acting force between the cylindrical roller and the outer ring is about 2100N, which satisfies the precondition that the roller does not leave the inwall surface of the outer ring. Combined with the numerical qualitative results of section IV, it can be considered that the exciter and the outer ring can achieve the VST.

Compared with the simulation results with $f_R = 0.06$ in Figure 4, in Figures 5(b)-(d), the effect of changing the friction coefficient from 0.06 to 0.1 on the vibration amplitude of the cylindrical roller and the phase difference with $f_R = 0.1$, is almost negligible. in other words, under this condition, the stable phase difference of the system is still substantially stabilized about 220° in Figure 5(b), which is the same as that in Figure 4(b).

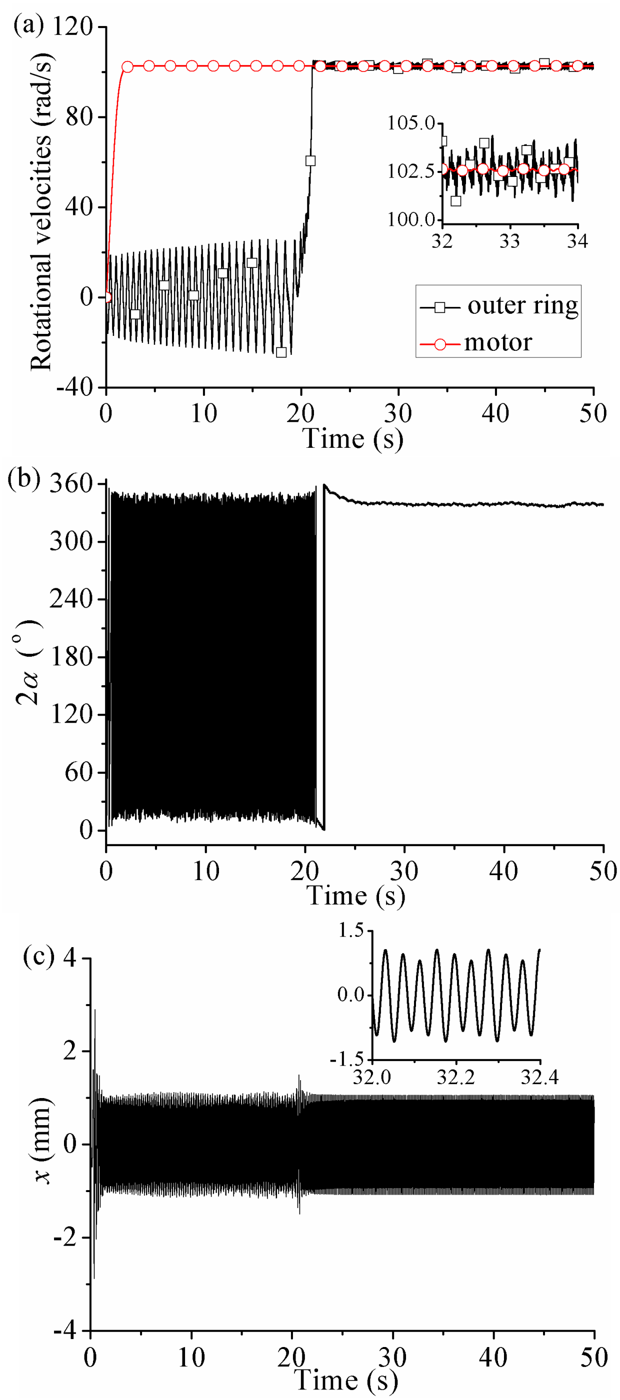

FIGURE 6. Simulation results for $f_R = 0.12$: (a) Speed of exciter and outer ring; (b) Phase difference between exciter and outer ring; (c) Response of roller in x-direction; (d) Response of roller in y-direction; (e) Coupling force between exciter and outer ring.

C. SIMULATION RESULTS WITH THE FRICTION COEFFICIENT $f_R \approx 0.12$

As shown in Figures 6(a) and (e), here the friction coefficient $f_R = 0.12$, the rotational speed of the exciter and the outer ring is still stabilized about 102.8 rad/s, and the acting force between the cylindrical roller and the outer ring is greater than 0, which satisfies the precondition that the roller does not leave the inwall of the outer ring.

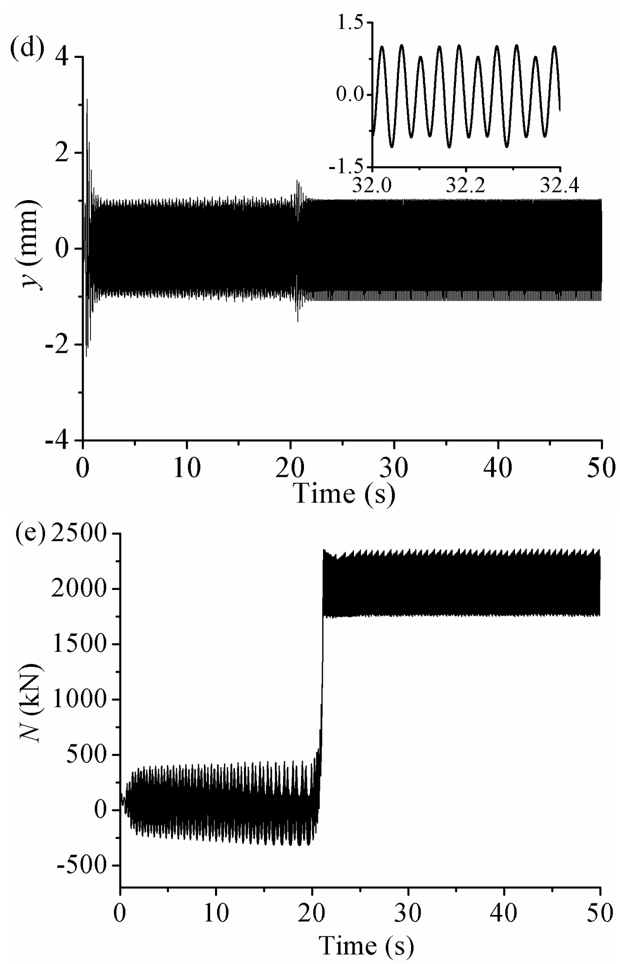

FIGURE 6. Simulation results for $f_R = 0.12$: (a) Speed of exciter and outer ring; (b) Phase difference between exciter and outer ring; (c) Response of roller in x-direction; (d) Response of roller in y-direction; (e) Coupling force between exciter and outer ring.

Different from the above results in Figures 4 and 5, the phase difference between the exciter and the outer ring in Figure 6 (b), is nearly stabilized at nearing 0° (or 360°). The maximum displacements of the cylindrical roller in the x-, y- directions are about 1.2mm in Figures 6(c) and (d), which is similar to the amplitude with $f_R = 0.06$ and $f_R = 0.1$ in Figures 4 and 5.

It can be seen from the above three groups of graphs, no matter what the friction coefficient is, the condition that the roller does not leave the outer ring can be always satisfied, and the friction coefficient has no obvious influence on the vibration amplitude of the cylindrical roller. It is worth noting that when the friction coefficient is greater than 0.1, the stable phase difference of the system changes from 220° to about 0° (or 360°), indicating that changing friction coefficient can affect the stable phase difference between the exciter and the outer ring.

VI. CONCLUSIONS

The following conclusions are stressed:

(1) The cylindrical roller satisfies the fact that does not leave the inwall of the outer ring during the steady operation process, which can lay a precondition for the outer ring to realize circular motion around the mass center of the system.

(2) Through the analyses for assuming the outer ring as an exciter, it is proved that under the condition that the cylindrical roller run against the inwall of the outer ring, if the eccentric radius s of the outer ring rotating surround the mass center does not exceed $\sqrt{1+(f_R)^2}\Big/f_R$ times than its motion amplitude A, the outer ring can be regarded as an exciter who can run around the center of the system, which provides strong support for implementing the VST of the system.

(3) The VST mechanism between the exciter and the outer ring, can be described as the fact that the cylindrical roller driven by the active-drive exciter, can drive the passive-drive outer ring to rotate synchronously following with the exciter around the mass center of the system to realize the VST. The greater the synchronization ability coefficient, the stronger the ability of the system to achieve VST, and the system synchronization capability increases with the increasing friction coefficient.

(4) The vibration amplitudes of the cylindrical roller in the x-, y- directions are basically identical, and less affected by the friction coefficient. Therefore, it can be determined that the circular motion for the cylindrical roller can be achieved during the steady operation process.

(5) The stable phase difference between the exciter and the outer ring, can be affected by the friction coefficient in the case of $f_R > 0.1$, otherwise, there is basically no impact.

(6) Based on the present work, the VST between the exciter and outer ring can be realized, and the materials (such as ores) placed in the cavity between the roller and the outer ring, can be crushed during the steady operation process, which can provide a reference for designing some new vibrating crushers and vibrating mills.

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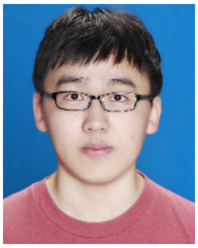

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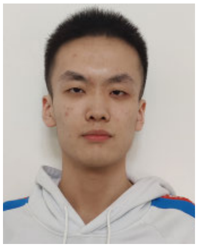

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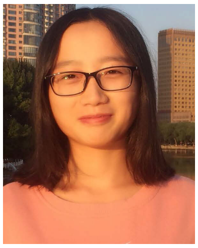

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