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Probability-Based Robust Stochastic Predictive Fault-Tolerant Control for Industrial Processes With Actuator Failures and Interval Time-Varying Delays

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ABSTRACT A stochastic robust predictive fault-tolerant control (SRPFTC) method is proposed for industrial processes with interval time-varying delays and actuator failures occurring under a certain probability. Its main contribution is to propose a new control strategy for time-delay systems where actuator failures meet a certain probability, which improve the traditional fault-tolerant control (FTC) method. First of all, an extended state space model composing of the state and output error is established to describe a type of industrial processes with interval time-varying delays, uncertainties, unknown disturbances and actuator failures occurring under a certain probability. Secondly, based on stochastic Lyapunov function theory, robust model predictive control method, time delay upper and lower bounds and linear matrix inequality theory, the stochastic stability conditions of the above the equivalent model are given. Then the control law updated in real time according to the probability of actuator failure is also given. This not only ensures the tracking control under different conditions (normal or fault), but also reduces the energy consumption of traditional FTC methods. Lastly, the effectiveness and feasibility are verified by the case study of the TTS20 water tank.

INDEX TERMS Stochastic fault-tolerant control, robust predictive control, industrial process, actuator faults, interval time-varying delays.

I. INTRODUCTION

The complex industrial process represented by process industry is a vital pillar industry of national economy and social development. The level of overall industrial equipment has basically reached the international advanced level, but the stability of the product quality is poor, and it is difficult to support the production of high value-added products. As a result, many advanced process control methods [1]–[4] are proposed by domestic and foreign scholars. The emergence of these methods has made up for the shortcomings of traditional control methods and has accelerated the development of industrial control. Among these research results, model predictive control (MPC) method is regarded as the most appropriate method in practical industrial production [5]. Due to its advantage in solving the control problem with multiple input multiple output and constrains as well as offering the improved control performance, MPC method has obtained much more attention from all over the world and applied to various kinds of industries [6]–[9]. However, these studies mainly use the MPC method to optimize the performance of the system. However, in a complex production environment, equipment failures often occur, and how to deal with the impact of failures becomes more and more important.

With the continuous growth of the online shopping and express transportation industry, the production of a large

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number of products in a short period of time is a problem that every enterprise and factory must solve. To this end, modern industry needs more sensors, actuators, field control stations, equipments and the computer control systems. In order to meet these demands, the industrial equipments are usually subject to the strict operating conditions, which is likely to cause the malfunction of system. If the failure of system occurs, it will directly affect the actual industrial production and even lead to the huge damage of equipment and staff. In this work, the actuator fault is studied. The actuator runs well at the beginning and achieves the expected control goal. However, over a period of time, the probability of fault will increase due to the aging of equipment and the complex operating environment. But we always wish the system to be stable even under the condition of failure. As a result, the safe, effective and stable operation of industrial process has widely caused more interests from all over the world.

As we known, the fault-tolerant control (FTC) is a reliable method to solve the above fault. There have been a great number of achievements about FTC [10]–[12]. Among them, Although the researches have successfully used FTC to reduce the effect of actuator failure on the system control effect, they are all basic FTC controllers with poor control effect and greater conservativeness. In recent decades, the related results about combining FTC with MPC have been presented for increasing the control capability of system. MPC method is introduced into FTC scheme. De Almeida and Leißling [13] proposed a fault-tolerant MPC method to control the advanced technologies testing aircraft system. On basis of an extended state space formulation of the process models, Zhang *et al.* [14] proposed an improved predictive functional control algorithm to realize the satisfying closed-loop control performance under the conditions of unknown disturbances and actuator faults. Tao *et al.* [15] presented a novel state space MPC strategy for linear systems with partial actuator faults. Wang *et al.* [16] investigated the tracking and optimized questions for the industrial processes by using the output feedback FTC and MPC method. Xu *et al.* [17] proposed an active fault-tolerant MPC strategy for the system with actuator and sensor failures. In this strategy, fault detection is implemented by utilizing a set-valued observer, fault isolation is performed by setting manipulations, and FTC is carried out based on the designed robust MPC law. Zou *et al.* [18] proposed a model predictive FTC method together with genetic algorithm for batch processes in the presence of disturbances and partial actuator faults.

However, the control performance may also be influenced by the time-delay that usually happens in practical industrial process. In addition, the time-delay is a factor to bring about the instability of system. Therefore, Zhou *et al.* [19] proposed the problem of H-infinity fault detection in the time-delay delta system, which has random two-channel packet loss and limited communication to solve the packet loss problem in network control. And there have been a lot of results [20]–[23] about utilizing MPC to solve the process with time-delay. The emergence of these results proves that MPC can effectively deal with the time delay problem in the system, but these articles do not consider how to deal with the problem of actuator failure in the case of time delay. And rare researches [24], [25] about integrated FTC with MPC have been presented for system with time-delay, failure and other issues. It can be found that the reliable control is applied in references [23], [25], which is quite appropriate for system with frequent failures. Remarkably, as the science technique develops, some advanced devices are designed and developed so that the possibility of fault becomes lower. If the reliable control law is implemented all the time, it will cause the unnecessary loss of energy and raw materials. Moreover, the deterministic process is studied in [23], [25]. In practical industrial production, failures are usually stochastic and unpredictable so that how to select the appropriate controller is quite hard. To this end, the design of a control scheme that can update the control law in real time based on the failure probability is a very important issue.

On basis of the aforementioned problem, this work proposes a stochastic robust predictive fault-tolerant control (SRPFTC) method integrating FTC with MPC for system with actuator fault occurring under a certain probability. Firstly, a state space model is built which considered the random failure with a certain probability in the actuator of system. Secondly, with the purpose of ensuring that the system has better tracking performance, a novel extended model is used when designing the SRPFTC control law. This model introduces the output tracking error into the traditional state space model. Thirdly, in view of the Lyapunov theory, optimization control method as well as stochastic control theory, the stochastic stable conditions are given by utilizing the expression of linear matrix inequality (LMI). The switched control law in the presence of different probabilities is further obtained by solving these LMI conditions. Finally, a case study in the TTS20 water tank under the different probabilities is used to verify whether the method is effective and feasible.

The innovations of this article are as follows:

(1) A new multi-degree-of-freedom extended state-space model with state increment and output tracking error is constructed, which can improve the freedom of system controller adjustment.

(2) Taking into account the randomness of actuator failures in actual industrial processes, different from the method of using fault-tolerant control from beginning to end in reference [23], the proposed method can update the corresponding control law in real time according to the probability of actuator failure, thereby reducing the conservativeness of the traditional FTC methods, improving the tracking performance of the system and reducing energy consumption, so as to increase corporate profits.

The writing idea of this article is as follows. The problem is described in Section 2. Section 3 is to establish a new extended state space model and design a random robust predictive FTC method in view of probability. Section 4 is to simulate on the mathematical model of the TTS20 water tank. Section 5 is a conclusion.

II. DESCRIPTION OF THE PROBLEM

Common industrial processes can be expressed in the following form:

$$\begin{cases} x(k+1) = A(k)x(k) + A_d(k)x(k-d(k)) \\ \qquad +Bu(k) + \varpi(k) \\ y(k) = Cx(k) \end{cases} \tag{1}$$

where $A(k) = A + N_a(k), A_d(k) = A_d + N_d(k), N_a(k)$ and $N_d(k)$ are the internal disturbances caused by the model uncertainties that meet $\left[N_a(k)\ N_d(k) \right] = \chi\gamma(k)\left[\omega\ \omega_d \right]$ and $\gamma^{\mathrm{T}}(k)\gamma(k) \le I, \{A,\ A_d,\ B,\ \chi,\ \omega,\ \omega_d,\ C\}$ are known constant matrices. $x(k) \in \mathbb{R}^{n_x}, u(k) \in \mathbb{R}^{n_u}, y(k) \in \mathbb{R}^{n_y}$ stand for the state, input, output variables of the process, n_x, n_u, n_y are referred to the dimension of variables. $\varpi(k)$ is the external interference that changes with the environment and other factors. $d(k)$ is a time delay that changes over time that meets $d_{\min} \le d(k) \le d_{\max}$, $d_{\max}$ and $d_{\min}$ the upper and lower bounds of the time delay. k denotes the current discrete-time and $k+1$ is the next time.

Generally, the control input and output are imposed at or near the following constraints in order to obtain the most economic benefits in industrial production.

$$\begin{cases} \|u(k)\| \le u_{\max} \\ \|y(k)\| \le y_{\max} \end{cases} \tag{2}$$

where $u_{\max}$ and $y_{\max}$ are the boundary value of the control input and output of the system. Combining formula (1) with formula (2), the robust performance index is described as

$$\min_{u(k+\kappa),m\ge 0} \max_{[A(k+\kappa)A_d(k+\kappa)B(k+\kappa)]\in\Omega,\kappa\ge 0} J_\infty$$

$$J_\infty(k) = \sum_{f=0}^{\infty}[(y(k+\kappa|k) - y_r(k+\kappa))^{\mathrm{T}}Q(y(k+\kappa|k) \\ -y_r(k+\kappa)) + u(k+\kappa|k)^{\mathrm{T}}Ru(k+\kappa|k)] \tag{3}$$

subject to

$$\begin{cases} \|u(k+\kappa|k)\| \le u_{\max} \\ \|y(k+\kappa|k)\| \le y_{\max} \end{cases}$$

where $y(k+\kappa|k)$ and $u(k+\kappa|k)$ are the expected control input and system output at $k+\kappa$ predicted at discrete time k. Q and R are the free weighting matrices of the output and input of the control system.

As the aging of the equipment and the strict operation, the possibility of faults for interval system, actuator and sensor will greatly increase. Hence, the practical $u(k)$ is very difficult to obtain, i.e. the control input $u^F(k)$ and $u(k)$ are different. Considering the above situation, the actuator fault is studied and the form of fault is expressed as $u^F(k) = \alpha u(k)$.α is an unknown diagonal matrix related to actuator failure that can vary freely within the following range.

$$0 \le \underline{\alpha} \le \alpha \le \overline{\alpha} \tag{4}$$

where $\underline{\alpha} \le I$ and $\overline{\alpha} \ge I$ are known boundaries.

In consequence, the mathematical description of the industrial process with actuator failure is shown in formula (5).

$$\begin{cases} x(k+1) = A(k)x(k) + A_d(k)x(k-d(k)) \\ \quad +B\alpha u(k) + \varpi(k) \\ y(k) = Cx(k) \end{cases} \tag{5}$$

In fact, the fault of system is stochastic but meets a certain probability. The proposed method mainly solves the stochastic control problem by using probability in view of SRPFTC method. The main objective is the control laws $\overline{K}^0$ and $\overline{K}^1$ are updated online through the failure probability of the actuator. The control law $\overline{K}^1$ is used when the failure probability is small, and the control law $\overline{K}^0$ is used when the failure probability is large, which reduces the unnecessary loss of raw materials and energy in traditional fault-tolerant controllers.

In the production process, if the actuator is fault-free at the current moment, there will be two situations in the actuator at the next moment, one is that the actuator continues to be fault-free, and the other is that the actuator fails. The probability of this kind of fault is represented as follows.

$$0 \le P\{\delta(k+1) = 1|\delta(k) = 0)\} = \vartheta \le 1 \tag{6}$$
$$0 \le P\{\delta(k+1) = 0|\delta(k) = 0)\} = 1 - \vartheta \le 1 \tag{7}$$
$$0 \le P\{\delta(k+1) = 1|\delta(k) = 1)\} = 1 \tag{8}$$
$$0 \le P\{\delta(k+1) = 0|\delta(k) = 1)\} = 0 \tag{9}$$

where $\delta(k) = \begin{cases} 0, & normalsystem \\ 1, & faultsystem \end{cases}$, denotes whether or not there is a fault for the system. $P\{\pi|\delta\}$ is referred to the probability of event π happening under the condition of event δ. Similarly, Eq. (6) denotes the probability for the fault in the next time under the normal condition in the current time and the probability is ϑ. Eq. (7) and Eq. (6) are opposite, and the probability is $1-\vartheta$. From Eq. (8), it can be seen that the probability is 1. This is because the fault must occur in the next time under the abnormal condition of the current time. Eq. (9) and Eq. (10) are opposite, and the probability is 0.

Remark 1: In practical industrial production, the above problems about the probability of fault indeed exist. Taking the water tank level system as an example, the control valve may become rusty during long-term operation of the system, which may cause unrecoverable phenomena such as stuck on the switch of the control valve. Distinctly, the above phenomenon obeys a certain probability. The probability is known in this paper. It can be obtained by statistical method, which is not the main research work of this paper. To this end, the above problem that derives from the practical engineering is studied by the proposed method.

By the above description, Eq. (5) can transform into

$$\begin{cases} x(k+1) = A(k)x(k) + A_d(k)x(k-d(k)) \\ \quad +(1-\delta(k))Bu(k) + \delta(k)B\alpha u(k) + \varpi(k) \\ y(k) = Cx(k) \end{cases} \tag{10}$$

To make the consequent design and proof easier, the following expression is applied.

$$\beta = \frac{\overline{\alpha} + \underline{\alpha}}{2}, \beta_0 = \frac{\overline{\alpha} - \underline{\alpha}}{\overline{\alpha} + \underline{\alpha}} \quad (11)$$

Based on Eqs. (4) and (10), by introducing the unknown matrix α_0, the following formula can be satisfied.

$$\alpha = (I + \alpha_0)\beta \quad (12)$$

where $|\alpha_0| \le \beta_0 \le I$.

Remark 2: In the actual production environment, the actuator is fully open or fully closed is a complete failure, which will cause the system to be out of control. At this time, the controller cannot realize the control of the controlled object. Therefore, this article considers the case of partial actuator failure, which means that although the actuator fails, it can still operate within a certain range.

Remark 3: The main difficulty in this paper is how to reduce the conservativeness that traditional fault-tolerant control brings to the system by improving the traditional fault-tolerant control, reduce the production cost of the enterprise, and increase the profit of the enterprise.

III. PROBABILITY BASED SRPFTC

A. EXTENDED STATE SPACE MODEL

On the basis of Eq. (10), let $\Delta x(k+1) = x(k+1) - x(k)$, formula (13) is obtained.

$$\begin{cases} \Delta x(k+1) = A(k)\Delta x(k) + A_d(k)\Delta x(k-d(k)) \\ +(1-\delta(k))B\Delta u(k) + \delta(k)B\alpha\Delta u(k) + \overline{\varpi}(k) \\ \Delta y(k) = C\Delta x(k) \end{cases} \quad (13)$$

where $\Delta = 1 - z^{-1}$, $\overline{\varpi}(k) = (N_a(k) - N_a(k-1)x(k-1) + (N_d(k) - N_d(k-1)x(k-1-d(k-1)) + \Delta\varpi(k)$.

The output tracking error of the system is as follows

$$e(k) = y(k) - y_r(k) \quad (14)$$

where $y_r(k)$ is the set value.

Synthesizing Eqs. (13) and (14), $e(k+1)$ can be expressed as

$$\begin{aligned} e(k+1) &= e(k) + C(A(k)\Delta x(k) + C(A_d(k))\Delta x(k-d(k)) \\ &+ (1-\delta(k))CB\Delta u(k) + \delta(k)CB\alpha\Delta u(k) + C\overline{\varpi}(k) \end{aligned} \quad (15)$$

Combining formula (13) with formula (15), the new model can be obtained as following:

$$\begin{cases} \overline{x}_1(k+1) = \overline{A}(k)\overline{x}_1(k) + \overline{A}_d(k)\overline{x}_1(k-d(k)) \\ \quad +(1-\delta(k))\overline{B}\Delta u(k) + \delta(k)\overline{B}\alpha\Delta u(k) + \overline{G}\overline{\varpi}(k) \\ \Delta y(k) = \overline{C}_1\overline{x}_1(k) \\ z(k) = e(k) = \overline{E}\overline{x}_1(k) \end{cases} \quad (16)$$

where $\overline{x}_1(k) = \begin{bmatrix} \Delta x(k) \\ e(k) \end{bmatrix}$, $\overline{x}_1(k-d(k)) = \begin{bmatrix} \Delta x(k-d(k)) \\ e(k-d(k)) \end{bmatrix}$, $\overline{A}(k) = \begin{bmatrix} A + N_a(k) & 0 \\ CA + CN_a(k) & I \end{bmatrix} = \overline{A} + \overline{N}_a(k)$, $\overline{A} = \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix}$, $\overline{N}_a(k) = \overline{\chi}\gamma(k)\omega$, $\overline{A}_d(k) = \begin{bmatrix} A_d + N_d(k) & 0 \\ CA_d + CN_d(k) & 0 \end{bmatrix} = \overline{A}_d + \overline{N}_d(k)$, $\overline{A}_d = \begin{bmatrix} A_d & 0 \\ CA_d & 0 \end{bmatrix}$, $\overline{N}_d(k) = \overline{\chi}\gamma(k)\overline{\varpi}_d$, $\overline{B} = \begin{bmatrix} B \\ CB \end{bmatrix}$, $\overline{\chi} = \begin{bmatrix} \chi \\ C\chi \end{bmatrix}$, $\overline{\omega} = \begin{bmatrix} \omega & 0 \end{bmatrix}$, $\overline{\varpi}_d = \begin{bmatrix} \varpi_d & 0 \end{bmatrix}$, $\overline{G} = \begin{bmatrix} I \\ C \end{bmatrix}$, $\overline{C}_1 = \begin{bmatrix} C & 0 \end{bmatrix}$, $\overline{E} = \begin{bmatrix} 0 & I \end{bmatrix}$.

Remark 4: The control law is designed with a new model that introduces the output tracking error into the traditional state space model, which ensures the tracking performance and increases the degree of freedom with the system.

The SRPFTC law is expressed as

$$\Delta u^F(k) = (1-\delta(k))\overline{K}^0\overline{x}_1(k) + \delta(k)\overline{K}^1\overline{x}_1(k) \quad (17)$$

where $\delta(k) = 0$ stands for the normal system and $\Delta u^F(k) = \overline{K}^0\overline{x}_1(k)$. $\delta(k) = 1$ stands for the fault system and $\Delta u^F(k) = \overline{K}^1\overline{x}_1(k)$. $\overline{K}^0$ and $\overline{K}^1$ are the gains that can be computed by the sequence theorem or corollary.

Remark 5: For the previous FTC methods, no matter whether the fault of system occurs or not, a FTC control law is always used, which leads to the waste of resources. However, in formula (17), the SRPFTC law is deigned to update the corresponding control law according to the probability of failure of the actuator and to ensure that the system has better tracking performance under different probabilities. This scheme applied in practical industrial production can save the waste of resources and energy.

Remark 6: When the failure random number generated is less than the actuator failure probability, then $\delta(k) = 1$. When the failure random number generated is greater than the actuator failure probability, then $\delta(k) = 0$.

Using the $\Delta u^F(k)$ in formula (17) update formula (16), the updated stochastic closed-loop state space model can be obtained.

$$\begin{cases} \overline{x}_1(k+1) = \sum_{i=0}^{1}\widehat{A}^i(k)\overline{x}_1(k) + \overline{A}_d(k)\overline{x}_1(k-d(k)) \\ \quad +\overline{G}\overline{\varpi}(k)\Delta y(k) = \overline{C}_1\overline{x}_1(k) \\ z(k) = e(k) = \overline{E}\overline{x}_1(k) \end{cases} \quad (18)$$

where $\widehat{A}^0(k) = (1-\delta(k))(\overline{A}(k) + \overline{B}\overline{K}^0)$, $\widehat{A}^1(k) = \delta(k)(\overline{A}(k) + \overline{B}\alpha \cdot \overline{K}^1)$.

Definition 1 (Robust MPC Problem) [23]: For the stochastic systems (18), the robust MPC problem is feasible if the control law $\Delta u^p(k+\kappa|k)$ can be solved by the following "min-max" optimization problem (19).

$$\min_{\Delta u(k+\kappa)} \max J_\infty^p$$

$$\begin{aligned} \overline{J}_\infty^p(k) &= \sum_{\kappa=0}^{\infty}[(\overline{x}_1^p(k+\kappa|k))^{\mathrm{T}}\overline{Q}_1^p(\overline{x}_1^p(k+\kappa|k)) \\ &+ \Delta u^p(k+\kappa|k)^{\mathrm{T}}\overline{R}_1^p\Delta u^p(k+\kappa|k)] \end{aligned} \quad (19)$$

subject to

$$\begin{cases} \|\Delta u^p(k+\kappa|k)\| \le \Delta u^p_{\max} \\ \|\Delta y^p(k+\kappa|k)\| \le \Delta y^p_{\max} \end{cases}$$

where $\overline{Q}^p_1$ and $\overline{R}^p_1$ are the weighting matrices for system state variables and the control input, respectively. $\overline{x}^p_1(k+\kappa|k)$, $\Delta u^p(k+\kappa|k)$ are the state and input of the system at the $k+\kappa$ time predicted at the time k.

Definition 2 [26]: Given a scalar $\varsigma > 0$, the stochastic system (18) is to be robust H_∞ performance, if it is stochastically asymptotically stable under the zero initial condition and any unknown bounded disturbances $\overline{\omega}(k)$, and the system output $z(k)$ satisfies $\|z\|_{\overleftrightarrow{E}_2} \le \varsigma \|\overline{\omega}\|_{\overleftrightarrow{E}_2}$, where $\|z\|^2_{\overleftrightarrow{E}_2} = \overleftrightarrow{\mathrm{E}}(\sum_{k=0}^{\infty} \|z(k)\|^2)$, $\overleftrightarrow{\mathrm{E}}$ is referred to the expectation.

B. MAIN RESULTS

Firstly, the lemmas that will be used in the following developments, are recalled below.

Lemma 1 (Schur Complements Lemma) [27]: When W, J are real matrices and P is a matrix of appropriate dimensions, if the following formula holds

$$P^{\mathrm{T}}JP - W < 0 \tag{20}$$

then there are

$$\begin{bmatrix} -W & P^{\mathrm{T}} \\ P & -J^{-1} \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -J^{-1} & P \\ P^{\mathrm{T}} & -W \end{bmatrix} < 0 \tag{21}$$

Lemma 2 [28]: For any vector $\delta(k) \in R^n$, when there are two positive integers $\wp_0, \wp_1$ and a matrix $0 < \overline{R} \in \mathbb{R}^{n\times n}$, it satisfies the following formula

$$-(\wp_1 - \wp_0 + 1)\sum_{i=\wp_0}^{\wp_1} \Gamma^{\mathrm{T}}(i)\overline{m}\Gamma(i) \le -\sum_{i=\wp_0}^{\wp_1} \Gamma^{\mathrm{T}}(i)\overline{m}\sum_{i=\wp_0}^{\wp_1} \Gamma^{\mathrm{T}}(i) \tag{22}$$

Lemma 3 [29]: When $\Psi = \Psi^{\mathrm{T}}, \gamma^{\mathrm{T}}\gamma \le I$ and $\chi, \gamma, \varpi, \Psi$ are known real matrixes, if it has the following form:

$$\Psi + \chi\gamma\omega + \chi^{\mathrm{T}}\gamma^{\mathrm{T}}\omega^{\mathrm{T}} < 0 \tag{23}$$

then there is $\rho > 0$, it has

$$\Psi + \rho^{-1}\chi\chi^{\mathrm{T}} + \rho\omega^{\mathrm{T}}\omega < 0 \tag{24}$$

Theorem 1: The considered stochastic system (18) is robustly stable, and it has a H_∞ performance if there are some known scalars $\varsigma > 0, \theta^i > 0, 0 \le d_{\min} \le d_{\max}$, unknown symmetric positive matrices $\overline{P}^i_1, \overline{T}^i_1, \overline{M}^i_1, \overline{G}^i_1, \overline{L}^i_1, \overline{S}^i_1, \overline{S}^i_2, \overline{M}^i_3, \overline{M}^i_4, \overline{X}^i_1, \overline{X}^i_2 \in \mathbb{R}^{(n_x+n_e)}$, unknown matrices $\overline{Y}^i_1 \in \mathbb{R}^{n_u\times(n_x+n_e)}$ and unknown positive scalars $\overline{\rho}^i_1, \overline{\rho}^i_2, i = 0, 1$, so that the following LMIs hold, (25)–(29), as shown at the bottom of the next page, and the control law of the controller is $\overline{K}^i = \overline{Y}^i_1(\overline{L}^i_1)^{-1}$, where $\widehat{\phi}^i_1 = -\overline{L}^i_1 + \overline{M}^i_3 + \overline{D}^i_1\overline{S}^i_2 + \overline{S}^i_2 - \overline{X}^i_2, \overline{D}_1 = (d_{\max} - d_{\min} + 1)I, \cap^{00} = \sqrt{1-\vartheta}, \cap^{01} = \sqrt{\vartheta}$, $\cap^{11} = I, \overline{D}_2 = d_{\max}I$, $\circledast$ represents the transposition of elements symmetrical to the main diagonal,

$$\overline{\Pi}^0_{11} = \begin{bmatrix} \widehat{\phi}^0_1 & 0 & \overline{L}^0_1 & 0 \\ 0 & -\overline{S}^0_1 & 0 & 0 \\ \overline{L}^0_1 & 0 & -\overline{M}^0_4 - \overline{X}^0_1 & 0 \\ 0 & 0 & 0 & -\varsigma^2 I \end{bmatrix},$$

$$\overline{\Pi}^1_{11} = \begin{bmatrix} \widehat{\phi}^1_1 & 0 & \overline{L}^1_1 & 0 \\ 0 & -\overline{S}^1_1 & 0 & 0 \\ \overline{L}^1_1 & 0 & -\overline{M}^1_4 - \overline{X}^1_1 & 0 \\ 0 & 0 & 0 & -\varsigma^2 I \end{bmatrix},$$

$$\overline{\Pi}^0_{12} = \begin{bmatrix} \overline{L}^{0}_1\overline{A}^{\mathrm{T}} + \overline{Y}^{0\mathrm{T}}_1\overline{B}^{\mathrm{T}} & \overline{L}^0_1\overline{A}^{\mathrm{T}} + \overline{Y}^{0\mathrm{T}}_1\overline{B}^{\mathrm{T}} - \overline{L}^0_1 \\ \overline{S}^0_1\overline{A}_d & \overline{S}^0_1\overline{A}_d \\ 0 & 0 \\ \overline{G}^{\mathrm{T}} & \overline{G}^{\mathrm{T}} \end{bmatrix},$$

$$\overline{\Pi}^1_{12} = \begin{bmatrix} \overline{L}^{1}_1\overline{A}^{\mathrm{T}} + \overline{Y}^{1\mathrm{T}}_1\beta\overline{B}^{\mathrm{T}} & \overline{L}^1_1\overline{A}^{\mathrm{T}} + \overline{Y}^{1\mathrm{T}}_1\beta\overline{B}^{\mathrm{T}} - \overline{L}^1_1 \\ \overline{S}^1_1\overline{A}_d & \overline{S}^1_1\overline{A}_d \\ 0 & 0 \\ \overline{G}^{\mathrm{T}} & \overline{G}^{\mathrm{T}} \end{bmatrix},$$

$$\overline{\Pi}^0_{13} = \begin{bmatrix} \overline{L}^0_1\overline{E}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \overline{\Pi}^1_{13} = \begin{bmatrix} \overline{L}^1_1\overline{E}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\overline{\Pi}^0_{14} = \begin{bmatrix} \overline{Y}^{0\mathrm{T}}_1\overline{R}^{\frac{1}{2}}_1 & \overline{L}^0_1\overline{Q}^{\frac{1}{2}}_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \overline{\Pi}^1_{14} = \begin{bmatrix} \overline{Y}^{1\mathrm{T}}_1\overline{R}^{\frac{1}{2}}_1 & \overline{L}^1_1\overline{Q}^{\frac{1}{2}}_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\overline{\Pi}^0_{15} = \begin{bmatrix} \overline{L}^0_1\overline{\omega}^{\mathrm{T}} & \overline{L}^0_1\overline{\omega}^{\mathrm{T}} \\ \overline{S}^0_1\overline{\omega}^{\mathrm{T}}_d & \overline{S}^0_1\overline{\omega}^{\mathrm{T}}_d \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \overline{\Pi}^1_{15} = \begin{bmatrix} \overline{L}^1_1\overline{\omega}^{\mathrm{T}} & \overline{L}^1_1\overline{\omega}^{\mathrm{T}} \\ \overline{S}^1_1\overline{\omega}^{\mathrm{T}}_d & \overline{S}^1_1\overline{\omega}^{\mathrm{T}}_d \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\overline{\Pi}^1_{16} = \begin{bmatrix} \overline{Y}^{1\mathrm{T}}_1\beta & \overline{Y}^{1\mathrm{T}}_1\beta \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \overline{\Pi}^0_{22} = \mathrm{diag}\begin{bmatrix} -\overline{L}^0_1 + \overline{\rho}^0_1\overline{\chi}\overline{\chi}^{\mathrm{T}} \\ -\overline{X}^0_1\overline{D}^{-2}_2 + \overline{\rho}^0_1\overline{\chi}\overline{\chi}^{\mathrm{T}} \end{bmatrix},$$

$$\overline{\Pi}^1_{22} = \mathrm{diag}\begin{bmatrix} -\overline{L}^1_1 + \overline{\rho}^1_1\overline{\chi}\overline{\chi}^{\mathrm{T}} + \overline{\rho}^1_2\overline{B}\beta^2_0\overline{B}^{\mathrm{T}} \\ -\overline{X}^1_1\overline{D}^{-2}_2 + \overline{\rho}^1_1\overline{\chi}\overline{\chi}^{\mathrm{T}} + \overline{\rho}^1_2\overline{B}\beta^2_0\overline{B}^{\mathrm{T}} \end{bmatrix}, \overline{\Pi}^0_{33}$$

$$= \overline{\Pi}^1_{33} = -I, \overline{\Pi}^0_{44} = \mathrm{diag}\left[-\theta I \ -\theta I\right],$$

$$\overline{\Pi}^1_{44} = \mathrm{diag}\left[-\theta I \ -\theta I\right], \overline{\Pi}^0_{55} = \mathrm{diag}\left[-\overline{\rho}^0_1 I \ -\overline{\rho}^0_1 I\right],$$

$$\overline{\Pi}^1_{55} = \mathrm{diag}\left[-\overline{\rho}^1_1 I \ -\overline{\rho}^1_1 I\right], \overline{\Pi}^1_{66} = \mathrm{diag}\left[-\overline{\rho}^1_2 I \ -\overline{\rho}^1_2 I\right].$$

Proof: First, the asymptotical stability and the optimized performance for the stochastic system (18) with $\overline{\omega}(k) = 0$ is proved. Assume $\overline{x}_1(k)$ meets the following conditions:

$$\begin{aligned} &V(\overline{x}_1(k+\kappa+1|k)) - V(\overline{x}_1(k+\kappa|k)) \le \\ &-[(\overline{x}_1(k+\kappa|k))^{\mathrm{T}}\overline{Q}_1(\overline{x}_1(k+\kappa|k)) \\ &+\Delta u(k+\kappa|k)^{\mathrm{T}}\overline{R}_1\Delta u(k+\kappa|k)](32) \end{aligned} \tag{30}$$

Adding up both sides of formula (30) from $p = 0$ to ∞ and requiring that $V(\bar{x}_1(\infty)) = 0$ or $\bar{x}_1(\infty) = 0$, it can be obtained as

$$\bar{J}_\infty(k) \le V(\bar{x}_1(k)) \le \theta \tag{31}$$

where θ is the upper bound of $\bar{J}_\infty(k)$. The Lyapunov-Krasovskii function is constructed as follows.

$$V(\bar{x}_1(k+\kappa)) = \sum_{n=1}^{5} V_n(\bar{x}_1(k+\kappa)) \tag{32}$$

And in order to the presentation, it has

$$\begin{aligned} \bar{x}_{1d}(k+\kappa) &= \bar{x}_1(k+\kappa-d(k)), \bar{x}_{1d_{\max}}(k+\kappa) \\ &= \bar{x}_1(k+\kappa-d_{\max}), \varepsilon_1(k+\kappa) \\ &= \bar{x}_1(k+\kappa+1) - \bar{x}_1(k+\kappa), \\ \bar{\varphi}_1(k+\kappa) &= \left[\bar{x}_1^{\mathrm{T}}(k+\kappa)\ \bar{x}_{1d}^{\mathrm{T}}(k+\kappa)\ \bar{x}_{1d_{\max}}^{\mathrm{T}}(k+\kappa)\right]^{\mathrm{T}}. \end{aligned} \tag{33}$$

where

$$\begin{aligned} V_1(\bar{x}_1(k+\kappa)) &= \bar{x}_1^{\mathrm{T}}(k+\kappa)\bar{P}_1^i\bar{x}_1(k+\kappa) \\ &= \bar{x}_1^{\mathrm{T}}(k+\kappa)\theta(\bar{L}_1^i)^{-1}\bar{x}_1(k+\kappa), \end{aligned}$$

$$\begin{aligned} V_2(\bar{x}_1(k+\kappa)) &= \sum_{f=k-d(k)}^{k-1} \bar{x}_1^{\mathrm{T}}(f+\kappa)\bar{T}_1^i\bar{x}_1(f+\kappa) \\ &= \sum_{f=k-d(k)}^{k-1} \bar{x}_1^{\mathrm{T}}(f+\kappa)\theta(\bar{S}_1^i)^{-1}\bar{x}_1(f+\kappa), \\ V_3(\bar{x}_1(k+\kappa)) &= \sum_{f=k-d_{\max}}^{k-1} \bar{x}_1^{\mathrm{T}}(f+\kappa)\bar{M}_1^i\bar{x}_1(f+\kappa) \\ &= \sum_{f=k-d_{\max}}^{k-1} \bar{x}_1^{\mathrm{T}}(f+\kappa)\theta(\bar{M}_2^i)^{-1}\bar{x}_1(f+\kappa), \\ V_4(\bar{x}_1(k+\Omega)) &= \sum_{l=-d_{\max}}^{-d_{\min}}\sum_{f=k+l}^{k-1} \bar{x}_1^{\mathrm{T}}(f+\kappa)\bar{T}_1^i\bar{x}_1(f+\kappa) \\ &= \sum_{l=-d_{\max}}^{-d_{\min}}\sum_{f=k+l}^{k-1} \bar{x}_1^{\mathrm{T}}(f+\kappa)\theta(\bar{S}_1^i)^{-1}\bar{x}_1(f+\kappa), \\ V_5(\bar{x}_1(k+\Omega)) &= d_{\max}\sum_{l=-d_{\max}}^{-1}\sum_{f=k+l}^{k-1} \varepsilon_1^{\mathrm{T}}(f+\kappa)\bar{G}_1^i\varepsilon_1(f+\kappa) \\ &= d_{\max}\sum_{l=-d_{\max}}^{-1}\sum_{f=k+l}^{k-1} \varepsilon_1^{\mathrm{T}}(f+\kappa)\theta(\bar{X}_1^i)^{-1} \\ &\quad \times\varepsilon_1(f+\kappa). \end{aligned}$$

$$\begin{bmatrix} \bar{\Pi}_{11}^0 & \cap^{00}\bar{\Pi}_{12}^0 & \cap^{01}\bar{\Pi}_{12}^1 & \cap^{00}\bar{\Pi}_{13}^0 & \cap^{01}\bar{\Pi}_{13}^1 & \cap^{00}\bar{\Pi}_{14}^0 & \cap^{01}\bar{\Pi}_{14}^1 & \cap^{00}\bar{\Pi}_{15}^0 & \cap^{01}\bar{\Pi}_{15}^1 & \cap^{01}\bar{\Pi}_{16}^1 \\ * & \bar{\Pi}_{22}^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \bar{\Pi}_{22}^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\Pi}_{33}^0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \bar{\Pi}_{33}^1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \bar{\Pi}_{44}^0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \bar{\Pi}_{44}^1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \bar{\Pi}_{55}^0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \bar{\Pi}_{55}^1 & 0 \\ * & * & * & * & * & * & * & * & * & \bar{\Pi}_{66}^1 \end{bmatrix} < 0 \tag{25}$$

$$\begin{bmatrix} \bar{\Pi}_{11}^1 & \cap^{11}\bar{\Pi}_{12}^1 & \cap^{11}\bar{\Pi}_{13}^1 & \cap^{11}\bar{\Pi}_{14}^1 & \cap^{11}\bar{\Pi}_{15}^1 & \cap^{11}\bar{\Pi}_{16}^1 \\ * & \bar{\Pi}_{22}^1 & 0 & 0 & 0 & 0 \\ * & * & \bar{\Pi}_{33}^1 & 0 & 0 & 0 \\ * & * & * & \bar{\Pi}_{44}^1 & 0 & 0 \\ * & * & * & * & \bar{\Pi}_{55}^1 & 0 \\ * & * & * & * & * & \bar{\Pi}_{66}^1 \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} -1 & \bar{x}_l^{\mathrm{T}}(k|k) \\ \bar{x}_l(k|k) & -\bar{\varphi}_l^i \end{bmatrix} \le 0 \tag{27}$$

$$\begin{bmatrix} -\Delta u_{\max}^2 & \bar{Y}_1^i \\ \bar{Y}_1^{i\mathrm{T}} & -\bar{\varphi}_l^i \end{bmatrix} \le 0 \tag{28}$$

$$\begin{bmatrix} -\Delta y_{\max}^2(\bar{\varphi}_l^i) & \bar{C}\bar{\varphi}_l^i \\ (\bar{C}\bar{\varphi}_l^i)^{\mathrm{T}} & -I \end{bmatrix} \le 0 \tag{29}$$

$\overline{P}_1^i$, $\overline{T}_1^i$, $\overline{M}_1^i$, $\overline{M}_2^i$ and $\overline{G}_1^i$ are positive definite matrices. Let

$$\overline{\xi}(k+\kappa) = \begin{bmatrix} \overline{x}_1(k+\kappa)^{\mathrm{T}} & \overline{x}_1 (k+\kappa-d(k))^{\mathrm{T}} & \cdots \\ \overline{x}_1 (k+\kappa -d_{\max})^{\mathrm{T}} & \cdots & \varepsilon_1 (k+\kappa-1)^{\mathrm{T}} \end{bmatrix},$$

$$\overline{\psi}_1^i = \mathrm{diag}\left[\overline{P}_1^i \; \overline{T}_1^i \cdots \overline{M}_1^i \cdots d_M \overline{G}_1^i \right],$$

$$(\overline{\Pi}_1^i)^{-1} = \mathrm{diag}\left[(\overline{L}_1^i)^{-1} \; (\overline{S}_1^i)^{-1} \cdots (\overline{M}_2^i)^{-1} \cdots d_M (\overline{X}_1^i)^{-1} \right].$$

It has

$$V(\overline{x}_1(k+\kappa)) = \overline{\xi}^{\mathrm{T}}(k+\kappa)\overline{\psi}_1^i\overline{\xi}(k+\kappa) = \overline{\xi}^{\mathrm{T}}(k+\kappa)\theta(\overline{\Pi}_1^i)^{-1}\overline{\xi}(k+\kappa) \tag{34}$$

For each $\delta(k)=i,\ i=0,1$, it has

$$\begin{aligned}\overleftrightarrow{\mathrm{E}}\{(\Delta V(\overline{x}_1(k+\kappa))\} &= \overleftrightarrow{\mathrm{E}}\{V(\overline{x}_1(k+\kappa+1), \delta(k+1)|\overline{x}_1(k+\kappa), \delta(k)) \\ &\quad -V(\overline{x}_1(k+\kappa), \delta(k)=i)\} \\ &= \overleftrightarrow{\mathrm{E}}\left\{\sum_{n=1}^{5}\Delta V_n(\overline{x}_1(k+\kappa))\right\}\end{aligned} \tag{35}$$

where

$$\begin{aligned}&\overleftrightarrow{\mathrm{E}}\{\Delta V_1(\overline{x}_1(k+\kappa))\} \\ &= \overline{x}_1^{\mathrm{T}}(k+\kappa+1)\sum_{j=0}^{1}(\cap^{ij})^2\theta(\overline{L}_1^i)^{-1}\overline{x}_1(k+\kappa+1) \\ &\quad -\overline{x}_1^{\mathrm{T}}(k+\kappa)\theta(\overline{L}_1^i)^{-1}\overline{x}_1(k+\kappa),\end{aligned}$$

$$\begin{aligned}&\overleftrightarrow{\mathrm{E}}\{\Delta V_2(\overline{x}_1(k+\kappa))\} \\ &= \sum_{f=k+1-d(k+1)}^{k}\overline{x}_1^{\mathrm{T}}(f+\kappa)\theta(\overline{S}_1^i)^{-1}\overline{x}_1(f+\kappa) \\ &\quad -\sum_{f=k-d(k)}^{k-1}\overline{x}_1^{\mathrm{T}}(f+\kappa)\theta(\overline{S}_1^i)^{-1}\overline{x}_1(f+\kappa) \\ &\le \overline{x}_1^{\mathrm{T}}(k+\kappa)\theta(\overline{S}_1^i)^{-1}\overline{x}_1(k+\kappa) \\ &\quad -\overline{x}_{1d}^{\mathrm{T}}(k+\kappa)\theta(\overline{S}_1^i)^{-1}\overline{x}_{1d}(k+\kappa) \\ &\quad +\sum_{f=k+1-d_{\max}}^{k-d_{\min}}\overline{x}_1^{\mathrm{T}}(f+\kappa)\theta(\overline{S}_1^i)^{-1}\overline{x}_1(f+\kappa),\end{aligned}$$

$$\begin{aligned}&\overleftrightarrow{\mathrm{E}}\{\Delta V_3(\overline{x}_1(k+\kappa))\} \\ &= \sum_{f=k+1-d_{\max}}^{k}\overline{x}_1^{\mathrm{T}}(f+\kappa)\theta(\overline{M}_2^i)^{-1}\overline{x}_1(f+\kappa) \\ &\quad -\sum_{f=k-d_{\max}}^{k-1}\overline{x}_1^{\mathrm{T}}(f+\kappa)\theta(\overline{M}_2^i)^{-1}\overline{x}_1(f+\kappa) \\ &\le \overline{x}_1^{\mathrm{T}}(k+\kappa)\theta\overline{M}_2^{-1}\overline{x}_1(k+\kappa) \\ &\quad -\overline{x}_{1d_{\max}}^{\mathrm{T}}(k+\kappa)\theta(\overline{M}_2^i)^{-1}\overline{x}_{1d_{\max}}(k+\kappa),\end{aligned}$$

$$\begin{aligned}&\overleftrightarrow{\mathrm{E}}\{\Delta V_4(\overline{x}_1(k+\kappa))\} \\ &= \sum_{l=-d_{\max}}^{-d_{\min}}\sum_{f=k+1+l}^{k}\overline{x}_1^{\mathrm{T}}(f+\kappa)\theta(\overline{S}_1^i)^{-1}\overline{x}_1(f+\kappa) \\ &\quad -\sum_{l=-d_{\max}}^{-d_{\min}}\sum_{f=k+l}^{k-1}\overline{x}_1^{\mathrm{T}}(f+\kappa)\theta(\overline{S}_1^i)^{-1}\overline{x}_1(f+\kappa) \\ &< (d_{\max}-d_{\min}+1)\overline{x}_1^{\mathrm{T}}(k+\kappa)\theta(\overline{S}_1^i)^{-1}\overline{x}_1(k+\kappa) \\ &\quad -\sum_{f=k+1-d_{\max}}^{k-d_{\min}}\overline{x}_1^{\mathrm{T}}(f+\kappa)\theta(\overline{S}_1^i)^{-1}\overline{x}_1(f+\kappa),\end{aligned}$$

$$\begin{aligned}&\overleftrightarrow{\mathrm{E}}\{\Delta V_5(\overline{x}_1(k+\kappa))\} \\ &= d_{\max}\sum_{l=-d_{\max}}^{-1}\sum_{f=k+1+l}^{k-1}\varepsilon_1^{\mathrm{T}}(f+\kappa)\theta(\overline{X}_1^i)^{-1}\varepsilon_1(f+\kappa) \\ &\quad +d_{\max}^2\varepsilon_1^{\mathrm{T}}(k+\kappa)\sum_{j=0}^{1}(\cap^{ij})^2\theta(\overline{X}_1^i)^{-1}\varepsilon_1(k+\kappa) \\ &\quad -d_{\max}\sum_{l=-d_{\max}}^{-1}\sum_{f=k+l}^{k-1}\varepsilon_1^{\mathrm{T}}(f+\kappa)\theta(\overline{X}_1^i)^{-1}\varepsilon_1(f+\kappa) \\ &= d_{\max}^2\varepsilon_1^{\mathrm{T}}(k+\kappa)\sum_{j=0}^{1}(\cap^{ij})^2\theta(\overline{X}_1^i)^{-1}\varepsilon_1(k+\kappa) \\ &\quad -d_{\max}\sum_{f=k-d_{\max}}^{k-1}\varepsilon_1^{\mathrm{T}}(f+\kappa)\theta(\overline{X}_1^i)^{-1}\varepsilon_1(f+\kappa).\end{aligned}$$

According to Lemma 2, the above inequation is expressed as follows.

$$\begin{aligned}&\overleftrightarrow{\mathrm{E}}\{\Delta V_5(\overline{x}_1(k+\kappa))\} \\ &\le d_{\max}^2\varepsilon_1^{\mathrm{T}}(k+\kappa)\sum_{j=0}^{1}(\cap^{ij})^2\theta(\overline{X}_1^i)^{-1}\varepsilon_1(k+\kappa) \\ &\quad -\sum_{f=k-d_{\max}}^{k-1}\varepsilon_1^{\mathrm{T}}(f+\kappa)\theta(\overline{X}_1^i)^{-1}\sum_{f=k-d_{\max}}^{k-1}\varepsilon_1(f+\kappa) \\ &= d_{\max}^2(\overline{x}_1(k+\kappa+1)-\overline{x}_1(k+\kappa))^{\mathrm{T}}\sum_{j=0}^{1}(\cap^{ij})^2\theta\cdot \\ &\quad (\overline{X}_1^i)^{-1}(\overline{x}_1(k+\kappa+1)-\overline{x}_1(k+\kappa))-(\overline{x}_1(k+\kappa) \\ &\quad -\overline{x}_{1d_{\max}}(k+\kappa))^{\mathrm{T}}\theta(\overline{X}_1^i)^{-1}(\overline{x}_1(k+\kappa)-\overline{x}_{1d_{\max}}(k+\kappa))\end{aligned} \tag{36}$$

From formula (29), it has

$$\theta^{-1}\Delta V(\overline{x}_1(k+\kappa|k))+\theta^{-1}\overline{J}_1^i(k)\le 0 \tag{37}$$

where

$$\overline{J}_1^i(k) = (\overline{x}_1(k+\kappa|k))^{\mathrm{T}}\overline{Q}_1(\overline{x}_1(k+\kappa|k)) +\Delta u^F(k+\kappa|k)^{\mathrm{T}}\overline{R}_1\cdot$$

$\Delta u^F(k+\kappa|k)$ is the optimized performance index, in which $\Delta u^F(k+\kappa|k) = (1-\delta(k))\overline{K}^0\overline{x}_1(k+\kappa|k) + \delta(k)\overline{K}^1\overline{x}_1(k+\kappa|k)$, $\delta(k) = 0$ stands for the normal system, $\Delta u^F(k+\kappa|k)\overline{x}_1(k+\kappa|k)$, $\delta(k) = 1$ stands for the fault system and $\Delta u^F(k+\kappa|k) = \overline{K}^1\overline{x}_1(k+\kappa|k)$.

Synthesizing Eqs. (35)-(37), it has

$$\begin{aligned} & \mathrm{E}\left\{\theta^{-1}\Delta V(\overline{x}_1(k+\kappa)) + \theta^{-1}\overline{J}_1^i(k)\right\} \\ & \quad \le \mathrm{E}\left\{\overline{\varphi}_1^{\mathrm{T}}(k)\overline{\Phi}_1^i\overline{\varphi}_1(k)\right\} \end{aligned} \tag{38}$$

$$\begin{aligned} \overline{\Phi}_1^i = & \begin{bmatrix} \overline{\phi}_1^i & 0 & (\overline{X}_1^i)^{-1} \\ \ast & -(\overline{S}_1^i)^{-1} & 0 \\ \ast & \ast & -(\overline{M}_2^i)^{-1} - (\overline{X}_1^i)^{-1} \end{bmatrix} \\ & + \overline{\Lambda}_1^{iT}(\overline{L}_1^i)^{-1}\overline{\Lambda}_1^i + \overline{\Lambda}_2^{iT}(\overline{D}_2^i)^2(\overline{X}_1^i)^{-1}\overline{\Lambda}_2^i \\ & + \overline{\lambda}_1^{iT}\theta^{-1}\overline{\lambda}_1^i + \overline{\lambda}_2^{iT}\theta^{-1}\overline{\lambda}_2^i \end{aligned} \tag{39}$$

where $\overline{\phi}_1^i = -(\overline{L}_1^i)^{-1} + (\overline{M}_2^i)^{-1} + \overline{D}_1(\overline{S}_1^i)^{-1} + (\overline{S}_1^i)^{-1} - (\overline{X}_1^i)^{-1}$, $\overline{\Lambda}_1^i = \sum_{j=0}^{1}(\cap^{ij})^2\left[\widehat{A}^j(k) \;\; \overline{A}_d(k) \;\; 0\right]$, $\overline{\Lambda}_2^i = \sum_{j=0}^{1}(\cap^{ij})^2\left[\widehat{A}^j(k) - I \;\; \overline{A}_d(k) \;\; 0\right]$, $\overline{\lambda}_1^i = \sum_{j=0}^{1}(\cap^{ij})^2\left[\overline{Q}_1^{\frac{1}{2}} \;\; 0 \;\; 0\right]$, $\overline{\lambda}_2^i = \sum_{j=0}^{1}(\cap^{ij})^2\left[\overline{R}_1^{\frac{1}{2}}\overline{Y}_1^i\left(\overline{L}_1^i\right)^{-1} \;\; 0 \;\; 0\right]$.

According to Lemma 1, the following LMI conditions can be acquired by letting $\overline{\Phi}_1^i < 0$, (40)–(43), as shown at the bottom of the page.

Multiplying the left and right hand sides of LMI of formula (40) by diag$[\overline{L}_1^0\overline{S}_1^0\overline{X}_1^0 II\ IIIII]$, multiplying the left and right hand sides of LMI of formula (40) by diag$[\overline{L}_1^1\ \overline{S}_1^1\ \overline{X}_1^1\ I\ I\ I\ I]$ and making $\overline{L}_1^i(\overline{M}_2^i)^{-1}\overline{L}_1^i = \overline{M}_3^i$, $\overline{L}_1^i\overline{S}_1^{-1}\overline{L}_1^i = \overline{S}_2$, $\overline{L}_1^i(\overline{X}_1^i)^{-1}\overline{L}_1^i = \overline{X}_2^i$, $\overline{X}_1^i(\overline{M}_2^i)^{-1}\overline{X}_1^i = \overline{M}_4$, $\overline{K}^i = \overline{Y}_1^i(\overline{L}_1^i)^{-1}$, $\overline{L}_1^0 = \overline{L}_1^1$ then it can be obtained the following sufficient conditions (42) and (43).

Second, the following H_∞ performance index is used to overcome the disturbance $\overline{\varpi}(k)$.

$$J = \sum_{k=0}^{\infty}\overset{\leftrightarrow}{\mathrm{E}}[z^{\mathrm{T}}(k)z(k) - \varsigma^2\overline{\varpi}^{\mathrm{T}}(k)\overline{\varpi}(k)] \tag{44}$$

For any $\overline{\varpi}(k) \in l_2[0,\infty]$ with nonzero, it can be known that $V(\overline{x}_1(0)) = 0$, $V(\overline{x}_1(\infty)) \ge 0$, $\overline{J}_\infty > 0$. Hence, the

$$\begin{bmatrix} \overline{\phi}_1^0 & 0 & (\overline{X}_1^0)^{-1} & \cap^{00}\widehat{A}^{0\mathrm{T}}(k) & \cap^{00}\widehat{A}^{0\mathrm{T}}(k)-I & \cap^{01}\widehat{A}^{1\mathrm{T}}(k) & \cap^{01}\widehat{A}^{1\mathrm{T}}(k)-I & \cap^{00}(\overline{L}_1^0)^{-1}\overline{Y}_1^{0\mathrm{T}}\overline{R}_1^{\frac{1}{2}} & \cap^{00}\overline{Q}_1^{\frac{1}{2}} & \cap^{01}(\overline{L}_1^1)^{-1}\overline{Y}_1^{1\mathrm{T}}\overline{R}_1^{\frac{1}{2}} & \cap^{01}\overline{Q}_1^{\frac{1}{2}} \\ \ast & -(\overline{S}_1^0)^{-1} & 0 & \cap^{00}\overline{A}_d^{\mathrm{T}}(k) & \cap^{00}\overline{A}_d^{\mathrm{T}}(k) & \cap^{01}\overline{A}_d^{\mathrm{T}}(k) & \cap^{01}\overline{A}_d^{\mathrm{T}}(k) & 0 & 0 & 0 & 0 \\ \ast & \ast & -(\overline{M}_2^0)^{-1}-(\overline{X}_1^0)^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & -\overline{L}_1^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & -\overline{D}_2^{-2}\overline{X}_1^0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & -\overline{L}_1^1 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & -\overline{D}_2^{-2}\overline{X}_1^1 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\theta I & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\theta I & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\theta I & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\theta I \end{bmatrix} < 0 \tag{40}$$

$$\begin{bmatrix} \overline{\phi}_1^1 & 0 & (\overline{X}_1^1)^{-1} & \cap^{11}\widehat{A}^{1\mathrm{T}}(k) & \cap^{11}\widehat{A}^{1\mathrm{T}}(k)-I & \cap^{11}(\overline{L}_1^1)^{-1}\overline{Y}_1^{1\mathrm{T}}\overline{R}_1^{\frac{1}{2}} & \cap^{11}\overline{Q}_1^{\frac{1}{2}} \\ \ast & -(\overline{S}_1^1)^{-1} & 0 & \cap^{11}\overline{A}_d^{\mathrm{T}}(k) & \cap^{11}\overline{A}_d^{\mathrm{T}}(k) & 0 & 0 \\ \ast & \ast & -(\overline{M}_2^1)^{-1}-(\overline{X}_1^1)^{-1} & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & -\overline{L}_1^1 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & -\overline{D}_2^{-2}\overline{X}_1^1 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & -\theta I & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & -\theta I \end{bmatrix} < 0 \tag{41}$$

$$\begin{bmatrix} \overline{\phi}_1^0 & 0 & (\overline{X}_1^0)^{-1} & \cap^{00}(\overline{L}_1^0)\widehat{A}^{\mathrm{T}}(k) & \cap^{00}(\overline{L}_1^0)\widehat{A}^{\mathrm{T}}(k)-\cap^{00}\overline{L}_1^0 & \cap^{01}(\overline{L}_1^0)\overline{A}^{\mathrm{T}}(k)+\cap^{01}\overline{Y}_1^{1\mathrm{T}}\overline{B}^{\mathrm{T}}\alpha & \cap^{01}(\overline{L}_1^0)\overline{A}^{\mathrm{T}}(k)+\cap^{01}\overline{Y}_1^{1\mathrm{T}}\overline{B}^{\mathrm{T}}\alpha-\cap^{01}\overline{L}_1^0 & \cap^{00}\overline{Y}_1^{0\mathrm{T}}\overline{R}_1^{\frac{1}{2}} & \cap^{00}\overline{L}_1^0\overline{Q}_1^{\frac{1}{2}} & \cap^{01}\overline{Y}_1^{1\mathrm{T}}\overline{R}_1^{\frac{1}{2}} & \cap^{01}\overline{L}_1^0\overline{Q}_1^{\frac{1}{2}} \\ \ast & -\overline{S}_1^0 & 0 & \cap^{00}\overline{S}_1^0\overline{A}_d^{\mathrm{T}}(k) & \cap^{00}\overline{S}_1^0\overline{A}_d^{\mathrm{T}}(k) & \cap^{01}\overline{S}_1^0\overline{A}_d^{\mathrm{T}}(k) & \cap^{01}\overline{S}_1^0\overline{A}_d^{\mathrm{T}}(k) & 0 & 0 & 0 & 0 \\ \ast & \ast & -\overline{M}_4^0-\overline{X}_1^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & -\overline{L}_1^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & -\overline{D}_2^{-2}\overline{X}_1^0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & -\overline{L}_1^1 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & -\overline{D}_2^{-2}\overline{X}_1^1 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\theta I & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\theta I & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\theta I & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\theta I \end{bmatrix} < 0 \tag{42}$$

$$\begin{bmatrix} \overline{\phi}_1^1 & 0 & \overline{L}_1^1 & \cap^{11}\overline{L}_1^1\widehat{A}^{\mathrm{T}}(k)+\cap^{11}\overline{Y}_1^{1\mathrm{T}}\overline{B}^{\mathrm{T}}\alpha & \cap^{11}\overline{L}_1^1\widehat{A}^{\mathrm{T}}(k)+\cap^{11}\overline{Y}_1^{1\mathrm{T}}\overline{B}^{\mathrm{T}}\alpha-\cap^{11}\overline{L}_1^1 & \cap^{11}\overline{Y}_1^{1\mathrm{T}}\overline{R}_1^{\frac{1}{2}} & \cap^{11}\overline{L}_1^1\overline{Q}_1^{\frac{1}{2}} \\ \ast & -\overline{S}_1^1 & 0 & \cap^{11}\overline{S}_1^1\overline{A}_d^{\mathrm{T}}(k) & \cap^{11}\overline{S}_1^1\overline{A}_d^{\mathrm{T}}(k) & 0 & 0 \\ \ast & \ast & -\overline{M}_4^1-\overline{X}_1^1 & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & -\overline{L}_1^1 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & -\overline{D}_2^{-2}\overline{X}_1^1 & 0 & 0 \\ \ast & \ast & \ast & \ast & \ast & -\theta I & 0 \\ \ast & \ast & \ast & \ast & \ast & \ast & -\theta I \end{bmatrix} < 0 \tag{43}$$

following inequation holds.

$$\begin{aligned} J \leq &\sum_{k=0}^{\infty} \overset{\leftrightarrow}{\mathrm{E}}[z^{\mathrm{T}}(k)z(k) - (\varsigma)^2\overline{\varpi}^{\mathrm{T}}(k)\overline{\varpi}(k)] \\ &+\theta^{-1}\Delta V(\overline{x}_1(\infty)) + \theta^{-1}\overline{J}_{\infty}(k) \\ = &\sum_{k=0}^{\infty} \overset{\leftrightarrow}{\mathrm{E}}[z^{\mathrm{T}}(k)z(k) - (\varsigma)^2\overline{\varpi}^{\mathrm{T}}(k)\overline{\varpi}(k) \\ &+\theta^{-1}\Delta V(\overline{x}_1(k)) + (\theta)^{-1}\overline{J}(k)] \end{aligned} \tag{45}$$

where $\overline{J}(k) = \overline{x}_1(k)^{\mathrm{T}}\overline{Q}_1\overline{x}_1(k) + \Delta u^F(k)^{\mathrm{T}}\overline{R}_1\Delta u^F(k)$.

Combining with formula (38), it has

$$\begin{aligned} &\overset{\leftrightarrow}{\mathrm{E}}\Big\{z^{\mathrm{T}}(k)z(k) - \tau^2\overline{\omega}^{\mathrm{T}}(k)\overline{\omega}(k) \\ &+\theta^{-1}\Delta V(\overline{x}_1(k)) + \theta^{-1}\overline{J}(k)\Big\} = \begin{bmatrix} \overline{\varphi}_1(k) \\ \overline{\varpi}(k) \end{bmatrix}^{\mathrm{T}} \\ &\left\{ \begin{aligned} &\begin{bmatrix} \overline{\phi}_1^i & 0 & (\overline{X}_1^i)^{-1} & 0 \\ * & -(\overline{S}_1^i)^{-1} & 0 & 0 \\ * & * & -(\overline{M}_2^i)^{-1} - (\overline{X}_1^i)^{-1} & 0 \\ * & * & * & -\varsigma^2 \end{bmatrix} \\ &+ \begin{bmatrix} \overline{\Lambda}_1^{i\mathrm{T}} \\ \overline{G}^{\mathrm{T}} \end{bmatrix} \overline{L}_1^{-1} \begin{bmatrix} \overline{\Lambda}_1^i & \overline{G} \end{bmatrix} \overline{\Lambda}_1 \\ &+ \begin{bmatrix} \overline{\Lambda}_2^{i\mathrm{T}} \\ \overline{G}^{\mathrm{T}} \end{bmatrix} \overline{D}_2^2\overline{X}_1^{-1} \begin{bmatrix} \overline{\Lambda}_2^i & \overline{G} \end{bmatrix} \\ &+ \begin{bmatrix} \overline{E}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \overline{E} & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \overline{\lambda}_1^i & 0 \end{bmatrix}^{\mathrm{T}} \\ &\theta^{-1} \begin{bmatrix} \overline{\lambda}_1^i & 0 \end{bmatrix} \\ &+ \begin{bmatrix} \overline{\lambda}_2^i & 0 \end{bmatrix}^{\mathrm{T}} \theta^{-1} \begin{bmatrix} \overline{\lambda}_2^i & 0 \end{bmatrix} \end{aligned} \right\} \begin{bmatrix} \overline{\varphi}_1(k) \\ \overline{\varpi}(k) \end{bmatrix} \end{aligned} \tag{46}$$

Using the sufficient condition (25) and (26), the following inequation holds.

$$\begin{aligned} &\begin{bmatrix} \overline{\phi}_1^i & 0 & (\overline{X}_1^i)^{-1} & 0 \\ * & -(\overline{S}_1^i)^{-1} & 0 & 0 \\ * & * & -(\overline{M}_2^i)^{-1} - (\overline{X}_1^i)^{-1} & 0 \\ * & * & * & -\varsigma^2 \end{bmatrix} \\ &\quad+ \begin{bmatrix} \overline{\Lambda}_1^{i\mathrm{T}} \\ \overline{G}^{\mathrm{T}} \end{bmatrix} \overline{L}_1^{-1} \begin{bmatrix} \overline{\Lambda}_1^i & \overline{G} \end{bmatrix} \overline{\Lambda}_1 \\ &\quad+ \begin{bmatrix} \overline{\Lambda}_2^{i\mathrm{T}} \\ \overline{G}^{\mathrm{T}} \end{bmatrix} \overline{D}_2^2\overline{X}_1^{-1} \begin{bmatrix} \overline{\Lambda}_2^i & \overline{G} \end{bmatrix} \\ &\quad+ \begin{bmatrix} \overline{E}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \overline{E} & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \overline{\lambda}_1^i & 0 \end{bmatrix}^{\mathrm{T}} \theta^{-1} \begin{bmatrix} \overline{\lambda}_1^i & 0 \end{bmatrix} \\ &\quad+ \begin{bmatrix} \overline{\lambda}_2^i & 0 \end{bmatrix}^{\mathrm{T}} \theta^{-1} \begin{bmatrix} \overline{\lambda}_2^i & 0 \end{bmatrix} < 0 \end{aligned} \tag{47}$$

As a result, the system meets H_∞ performance index.

Moreover, in order to get the invariant set of the stochastic system (18), taking the maximum value of $\overline{x}_l(k) = \max\ (\overline{x}_1(r)\ \varepsilon_1(r))$, $r \in (k - d_{\max}, k)$, one has

$$V(\overline{x}_1(k)) \leq \overline{x}_l^{\mathrm{T}}(k)\overline{\psi}_l^i\overline{x}_l(k) \leq \theta \tag{48}$$

where $\overline{\psi}_l^i = \overline{P}_1^i + \overline{d}_{\max}\overline{T}_1^i + d_{\max}\overline{M}_1 + \frac{d_{\min}+d_{\max}}{2}(d_{\max} - d_{\min} + 1)\overline{T}_1^i + d_{\max}^2\frac{1+d_{\max}}{2}\overline{G}_1^i$. Making $\overline{\varphi}_l^i = \theta(\overline{\psi}_l^i)^{-1}$, the sufficient condition (27) is obtained by the Lemma 1.

In terms of input constraints, it has

$$\begin{aligned} \|\Delta u(k+\kappa)|k\|^2 &= \left\|\overline{Y}_1^i(\overline{L}_1^i)^{-1}\overline{x}_1(k+\kappa|k)\right\|^2 \\ &= \left\|\overline{Y}_1^i\theta^{-1}\overline{P}_1^i\overline{x}_1(k+\kappa|k)\right\|^2 \\ &\leq \left\|\overline{Y}_1^i\theta^{-1}\overline{\psi}_l^i\overline{x}_l(k+\kappa|k)\right\|^2 \\ &= \left\|\overline{Y}_1^i(\overline{\varphi}_l^i)^{-1}\overline{x}_l(k+\kappa|k)\right\|^2 \\ &\leq \overline{Y}_1^i(\overline{\varphi}_l^i)^{-1}\overline{Y}_1^{i\mathrm{T}} \leq \Delta u_{\max} \end{aligned} \tag{49}$$

From Lemma 1, we can get formula (28).

In terms of output constraints, it has $\|\Delta y(k+\kappa)\|^2 = \left\|\overline{C}\overline{x}_1(k+\kappa|k)\right\|^2$ which is less than $\Delta y_{\max}^2\overline{x}_1(k+\kappa|k)(\overline{\varphi}_l^i)^{-1}\cdot \overline{x}_1^{\mathrm{T}}(k+\kappa|k)$ by condition (29). Taking the maximum value of $\overline{x}_l(k+\kappa) = \max(\overline{x}_1(k+\kappa))$, it has $\Delta y_{\max}^2\overline{x}_1(k+\kappa)(\overline{\varphi}_l^i)^{-1}\cdot \overline{x}_1^{\mathrm{T}}(k+\kappa) \leq \Delta y_{\max}^2\overline{x}_l(k+\kappa)(\overline{\varphi}_l^i)^{-1}\overline{x}_l^{\mathrm{T}}(k+\kappa) \leq \Delta y_{\max}^2$.

Hence, the constrains for input variable and output variable are obtained by using condition (28) and (29). At this point, the theorem 1 is already proved.

Remark 7: In the proof, in order to reduce the conservativeness of the system and fully consider the impact of time delay on the system, the stability of the system is studied by using the augmented Lyapunov functional combined with the free weight matrix $\overline{Q}_1$ and $\overline{R}_1$. The uncertainty $\begin{bmatrix} N_a(k) & N_d(k) \end{bmatrix}$ in the system is expressed as $\chi\gamma(k)\begin{bmatrix} \omega & \omega_d \end{bmatrix}$. Then Lemma 3 is used and the robust performance index in Definition 1 is combined to effectively deal with the uncertainty of the system. Similar to the processing of uncertainty, the actuator failure is expressed in the form of equation (12), and the scaling process is performed through Lemma 3, so as to ensure that the controller can effectively compensate for the actuator failure. In addition, the H infinity performance index is introduced to solve the problem of external interference in the system. To sum up, we treat uncertainty, external interference, and actuator failure as separate entities and deal with them separately. Finally, the stable conditions for the time-delay system with lower conservativeness based on the LMI form in Theorem 1 are obtained.

Corollary 1: The considered stochastic system (18) is robustly stable, and it has a H_∞ performance if there are some known scalars $\varsigma > 0, \theta^i > 0, \overline{d}_1 \geq 0$ unknown symmetric positive matrices $\overline{P}_1^i$, $\overline{T}_1^i$, $\overline{M}_1^i$, $\overline{G}_1^i$, $\overline{L}_1^i$, $\overline{S}_1^i$, $\overline{S}_2^i$, $\overline{M}_3^i$, $\overline{M}_4^i$, $\overline{X}_1^i$, $\overline{M}_4^i$, $\overline{X}_1^i$, $\overline{X}_2^i \in \mathbb{R}^{(n_x+n_e)}$ unknown matrices $\overline{Y}_1^i \in \mathbb{R}^{n_u\times(n_x+n_e)}$ and unknown positive scalars $\overline{\rho}_1^i$, $\overline{\rho}_2^i$, $i = 0, 1$, so that the following LMIs hold, (50), as shown at the bottom of the page:

$$\begin{bmatrix} \aleph_{11}^1 & \cap^{11}\aleph_{12}^1 & \cap^{11}\aleph_{13}^1 & \cap^{11}\aleph_{14}^1 & \cap^{11}\aleph_{15}^1 \\ * & \overline{\Pi}_{22}^1 & 0 & 0 & 0 \\ * & * & \overline{\Pi}_{33}^1 & 0 & 0 \\ * & * & * & \overline{\Pi}_{44}^1 & 0 \\ * & * & * & * & \overline{\Pi}_{55}^1 \end{bmatrix} < 0 \quad (51)$$

$$\begin{bmatrix} -1 & \overline{x}_l^{\mathrm{T}}(k|k) \\ \overline{x}_l(k|k) & -\tilde{\varphi}_l^i \end{bmatrix} \le 0 \quad (52)$$

$$\begin{bmatrix} -\Delta u_{\max}^2 & \overline{Y}_1^i \\ \overline{Y}_1^{i\mathrm{T}} & -\tilde{\varphi}_l^i \end{bmatrix} \le 0 \quad (53)$$

$$\begin{bmatrix} -\Delta y_{\max}^2 (\tilde{\varphi}_l^i)^{-1} & \overline{C} \\ \overline{C}^{\mathrm{T}} & -I \end{bmatrix} \le 0 \quad (54)$$

and the control law of the controller is $\overline{K}^i = \overline{Y}_1^i \overline{L}_1^i$, where $\tilde{\phi}_1^i = -\overline{L}_1^i + \overline{S}_2^i - \overline{X}_2^i$, $\overline{X}_1^i (S_1^i)^{-1} \overline{X}_1^i = \overline{X}_3^i$, $\overline{D} = \overline{d}_1 I$, $\tilde{\varphi}_l^i = \theta(\tilde{\psi}_l^i)^{-1}$, $\tilde{\psi}_l^i = \overline{P}_1^i + \overline{d}_1 \overline{T}_1^i + \overline{d}_1^2 \frac{1+\overline{d}_1}{2} \overline{G}_1^i$,

$$\aleph_{11}^0 = \begin{bmatrix} \tilde{\phi}_1^0 & \overline{L}_1^0 & 0 \\ 0 & -\overline{X}_3^0 - \overline{X}_1^0 & 0 \\ 0 & 0 & -\tau^2 I \end{bmatrix},$$

$$\aleph_{11}^1 = \begin{bmatrix} \tilde{\phi}_1^1 & \overline{L}_1^1 & 0 \\ 0 & -\overline{X}_3^1 - \overline{X}_1^1 & 0 \\ 0 & 0 & -\tau^2 I \end{bmatrix},$$

$$\aleph_{12}^0 = \begin{bmatrix} \overline{L}_1^0 \overline{A}^{\mathrm{T}} + \overline{Y}_1^{0\mathrm{T}} \overline{B}^{\mathrm{T}} & \overline{L}_1^0 \overline{A}^{\mathrm{T}} + \overline{Y}_1^{0\mathrm{T}} \overline{B}^{\mathrm{T}} - \overline{L}_1^0 \\ \overline{X}_1^0 \overline{A}_d & \overline{X}_1^0 \overline{A}_d \\ \overline{G}^{\mathrm{T}} & \overline{G}^{\mathrm{T}} \end{bmatrix},$$

$$\aleph_{12}^1 = \begin{bmatrix} \overline{L}_1^1 \overline{A}^{\mathrm{T}} + \overline{Y}_1^{1\mathrm{T}} \beta \overline{B}^{\mathrm{T}} & \overline{L}_1^1 \overline{A}^{\mathrm{T}} + \overline{Y}_1^{1\mathrm{T}} \beta \overline{B}^{\mathrm{T}} - \overline{L}_1^1 \\ \overline{X}_1^1 \overline{A}_d & \overline{X}_1^1 \overline{A}_d \\ \overline{G}^{\mathrm{T}} & \overline{G}^{\mathrm{T}} \end{bmatrix},$$

$$\aleph_{13}^0 = \begin{bmatrix} \overline{L}_1^0 \overline{E}^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}, \aleph_{13}^1 = \begin{bmatrix} \overline{L}_1^1 \overline{E}^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix},$$

$$\aleph_{14}^0 = \begin{bmatrix} \overline{Y}_1^{0\mathrm{T}} \overline{R}_1^{\frac{1}{2}} & \overline{L}_1^0 \overline{Q}_1^{\frac{1}{2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \aleph_{14}^1 = \begin{bmatrix} \overline{Y}_1^{1\mathrm{T}} \overline{R}_1^{\frac{1}{2}} & \overline{L}_1^1 \overline{Q}_1^{\frac{1}{2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\aleph_{15}^0 = \begin{bmatrix} \overline{L}_1^0 \overline{\omega}^{\mathrm{T}} & \overline{L}_1^0 \overline{\omega}^{\mathrm{T}} \\ \overline{X}_1^0 \overline{\omega}_d^{\mathrm{T}} & \overline{X}_1^0 \overline{\omega}_d^{\mathrm{T}} \\ 0 & 0 \end{bmatrix}, \aleph_{15}^1 = \begin{bmatrix} \overline{L}_1^1 \overline{\omega}^{\mathrm{T}} & \overline{L}_1^1 \overline{\omega}^{\mathrm{T}} \\ \overline{X}_1^1 \overline{\omega}_d^{\mathrm{T}} & \overline{X}_1^1 \overline{\omega}_d^{\mathrm{T}} \\ 0 & 0 \end{bmatrix},$$

$$\aleph_{16}^1 = \begin{bmatrix} \overline{Y}_1^1 \beta & \overline{Y}_1^1 \beta \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Proof: Due to the difference between constant time delay and time-varying time delay, the Lyapunov function that we choose will also change accordingly. The changed formula is as follows:

$$V(\overline{x}_1(k+\kappa)) = \sum_{i=1}^{3} V_i(\overline{x}_1(k+\kappa)) \quad (55)$$

where

$$\begin{aligned} V_1(\overline{x}_1(k+\kappa)) &= \overline{x}_1^{\mathrm{T}}(k+\kappa)\overline{P}_1 \overline{x}_1(k+\kappa) \\ &= \overline{x}_1^{\mathrm{T}}(k+\kappa)\theta \overline{L}_1^{-1} \overline{x}_1(k+\kappa), \\ V_2(\overline{x}_1(k+\kappa)) &= \sum_{f=k-\overline{d}_1}^{k-1} \overline{x}_1^{\mathrm{T}}(f+\kappa)\overline{T}_1 \overline{x}_1(f+\kappa) \\ &= \sum_{f=k-\overline{d}_1}^{k-1} \overline{x}_1^{\mathrm{T}}(f+\kappa)\theta \overline{S}_1^{-1} \overline{x}_1(f+\kappa), \\ V_3(\overline{x}_1(k+\kappa)) &= \overline{d}_1 \sum_{l=-\overline{d}_1}^{-1} \sum_{f=k+l}^{k-1} \varepsilon_1^{\mathrm{T}}(f+\kappa)\overline{G}_1 \varepsilon_1(f+\kappa) \\ &= \overline{d}_1 \sum_{l=-\overline{d}_1}^{-1} \sum_{f=k+l}^{k-1} \varepsilon_1^{\mathrm{T}}(f+\kappa)\theta \overline{X}_1^{-1} \varepsilon_1(f+\kappa). \end{aligned}$$

The subsequent proof is the same as Theorem 1, so it will not be proven in detail in the text.

Remark 8: When the system state is unknown, the current system output, historical system output and historical control

$$\begin{bmatrix} \aleph_{11}^0 & \cap^{00}\aleph_{12}^0 & \cap^{01}\aleph_{12}^1 & \cap^{00}\aleph_{13}^0 & \cap^{01}\aleph_{13}^1 & \cap^{00}\aleph_{14}^0 & \cap^{01}\aleph_{14}^1 & \cap^{00}\aleph_{15}^0 & \cap^{01}\aleph_{15}^1 & \cap^{01}\aleph_{16}^1 \\ * & \overline{\Pi}_{22}^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \overline{\Pi}_{22}^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \overline{\Pi}_{33}^0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \overline{\Pi}_{33}^1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \overline{\Pi}_{44}^0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \overline{\Pi}_{44}^1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \overline{\Pi}_{55}^0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \overline{\Pi}_{55}^1 & 0 \\ * & * & * & * & * & * & * & * & * & \overline{\Pi}_{66}^1 \end{bmatrix} < 0 \quad (50)$$

input can be used to construct the state of the system, thereby avoiding the design of a state observer.

Remark 9: With the continuous development of the computer, the computing power of the computer has been greatly improved, which can be solved in a short time. And because the computer control is discrete control, there will be a certain sampling period. For example, in the temperature control, the sampling is usually taken every 10 to 20 seconds, so this period of time is enough for the computer to solve the control law.

A block diagram for the proposed SRPFTC design as Fig. 1.

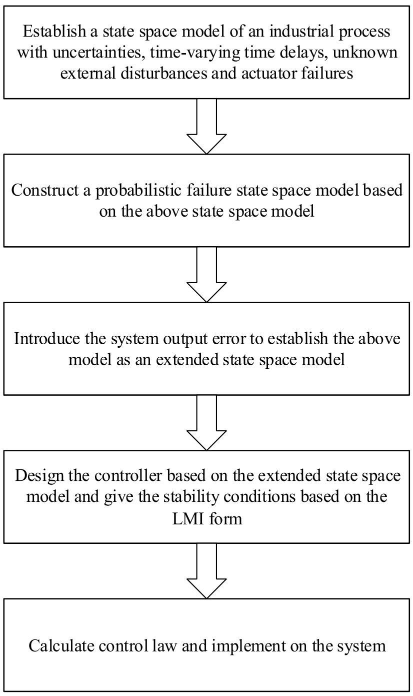

FIGURE 1. Diagram of implementation steps.

IV. SIMULATION RESEARCH ON WATER TANK SYSTEM

A. SYSTEM DESCRIPTION AND PROCESS MODEL

In modern industrial production, liquid level is a common object in process control, and the control effect of liquid level directly affects whether the production process can be carried out smoothly. The water tank is a typical process for many controlled objects in industrial processes, which has a wide range of representativeness in the study of process control with uncertainty and large time-delay. In this paper, the mathematical model of TTS20 water tank in [23] is used as the simulation object. The model is as follows:

$$\begin{cases} x(k+1) = A(k)x(k) + A_d(k)x(k-d(k)) \\ \quad +(1-\delta(k))Bu(k) + \delta(k)B\alpha u(k) + \varpi(k) \\ y(k) = Cx(k) \end{cases} \tag{56}$$

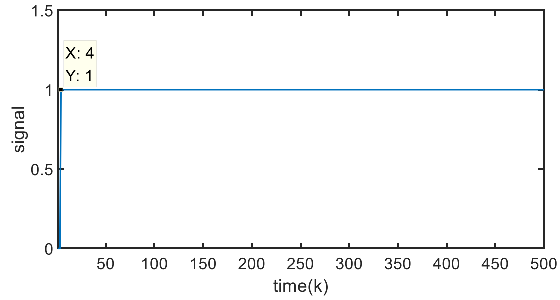

FIGURE 2. Signal of actuator failure under probability as 0.5.

where $\delta(k) = 0$ stands for the normal system, $\delta(k)=1$ stands for the fault system, $A = \begin{bmatrix} 0.9850 & 0.0107 \\ 0.0078 & 0.9784 \end{bmatrix}$, $B = \begin{bmatrix} 64.4453 \\ 0.2559 \end{bmatrix}$, $A_d = \begin{bmatrix} 0.1057 & 0.0004 \\ 0.0002 & 0.0207 \end{bmatrix}$, $\overline{\chi} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $\overline{\varpi} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$, $\overline{\varpi}_d = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $\gamma(k) = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}$, $\varpi(k) = (0.0005\gamma_3 \quad 0.0005\gamma_4)^{\mathrm{T}}$, in which γ_1, γ_2, γ_3, γ_4 are random numbers within $[-1, 1]$. Since the maximum liquid level of the water tank is 0.6 meters, this article takes one third of the maximum liquid level as the set value in the simulation. Assuming that the actuator failure factor is α, the failure boundary is $0.4 = \underline{\alpha} \leq \alpha \leq \overline{\alpha} = 1.2$. Using formula (8), we can obtain $\beta = 0.8$, $\beta_0 = 0.5$. In addition, the input and output constraints are:

$$\begin{cases} |y(k+\kappa|k)| \leq 0.22 \\ |u(k+\kappa|k)| \leq 0.0015 \end{cases} \tag{57}$$

The tracking performance of the system is described by introducing the following formula.

$$D(k) = \sqrt{e^{\mathrm{T}}(k)e(k)} \tag{58}$$

B. SIMULATION RESULTS

In this part, a state space model in the form of formula (56) is used to simulate and examine the effectivity and feasibility of the designed probabilistic fault controller. The controller parameters $Q_1 = \mathrm{diag}[500, 5, 1]$, $R_1 = 1$, are determined through repeated tests.

In order to facilitate the comparison, the simulation results under three different probability situations are given in this paper. When the probability of actuator failure is 0.5, the results of actuator failure signal, the system output response, the control input and the tracking performance are shown in Fig. 2-5. In addition, the results under the probability of actuator failure as 0.05 and 0.005 are shown in Fig. 6-9 and Fig. 10-13, respectively. The simulation result of the time-varying delay is shown in Fig. 14.

Fig. 2, Fig. 6 and Fig. 10 show the signal points of actuator failure under different failure probabilities, respectively. The ordinate "signal" represents the running state of the controller. When "signal" =0, it means that the probability of

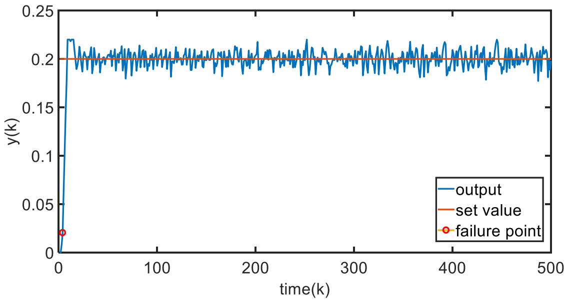

FIGURE 3. Output response of the system under probability as 0.5.

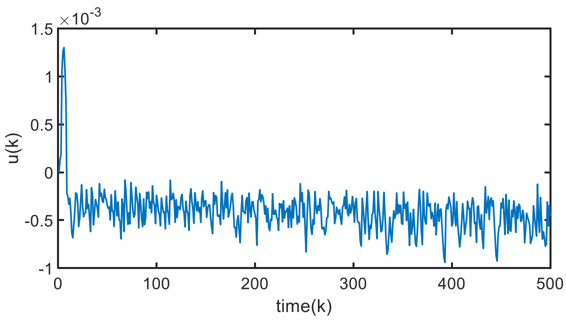

FIGURE 4. Control input of the system under probability as 0.5.

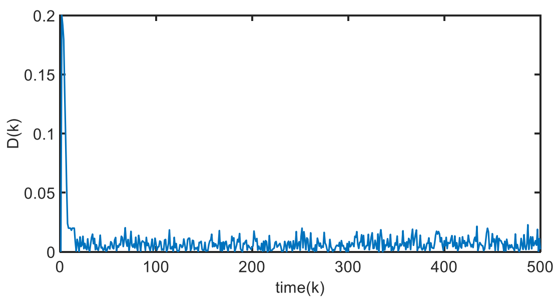

FIGURE 5. Tracking performance of the system under probability as 0.5.

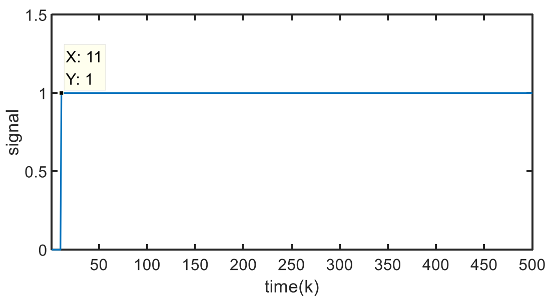

FIGURE 6. Signal of actuator failure under probability as 0.05.

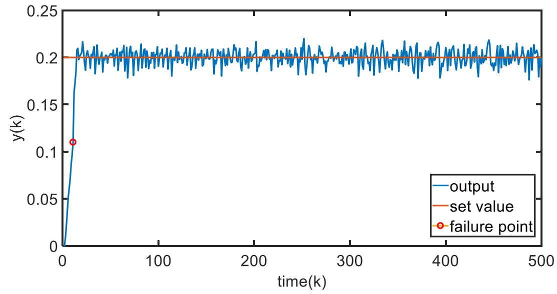

FIGURE 7. Output response of the system under probability as 0.05.

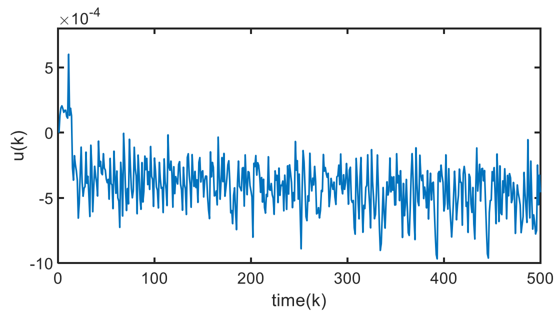

FIGURE 8. Control input of the system under probability as 0.05.

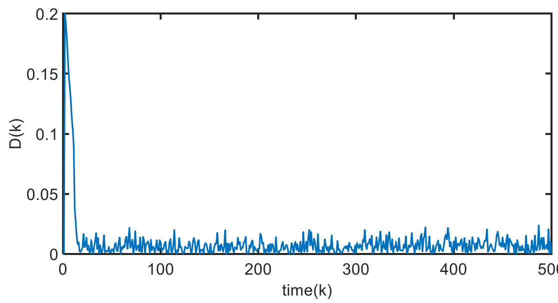

FIGURE 9. Tracking performance of the system under probability as 0.05.

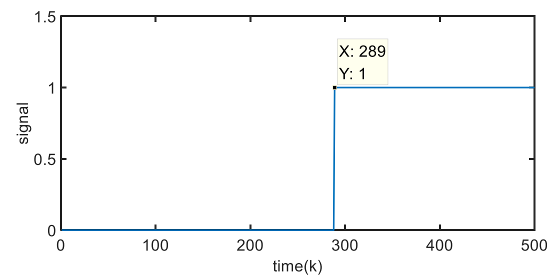

FIGURE 10. Signal of actuator failure under probability as 0.005.

actuator failure is low, and $\overline{K}^0$ is used to update the controller. When "signal" =1, it means that the probability of actuator failure is high, and $\overline{K}^1$ is used to update the controller. It can be seen from that the controller switching time becomes more and more late as the probability of actuator failure decreases.

In Fig. 3, Fig. 7 and Fig. 11, the orange circle represents the failure point where the actuator fails. The blue line is the output response of the system. And the red line is the set value of the system. According to these figures, it can be seen, the designed controller can make the system run stably and make the output response track the set value quickly under the conditions of three different actuator failure probability. When the probability of actuator failure is

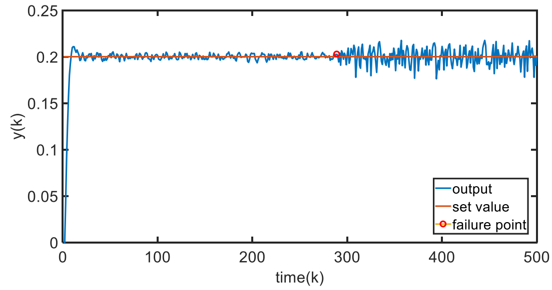

FIGURE 11. Output response of the system under probability as 0.005.

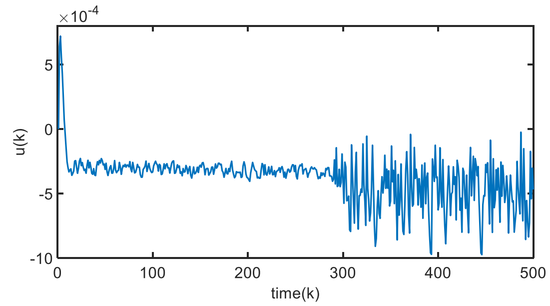

FIGURE 12. Control input of the system under probability as 0.005.

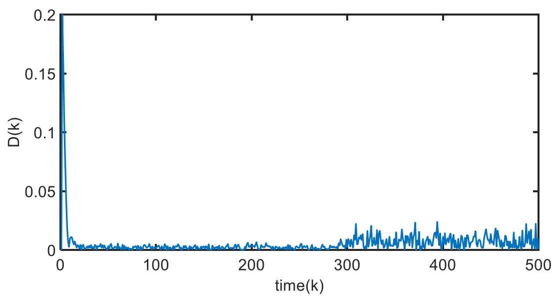

FIGURE 13. Tracking performance of the system under probability as 0.005.

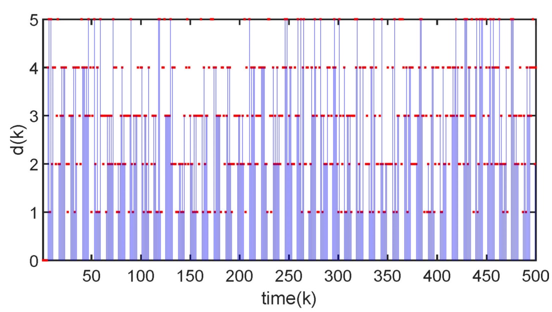

FIGURE 14. Time-varying delay of the system.

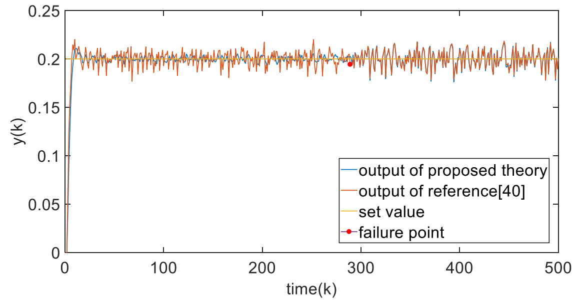

FIGURE 15. Output response comparison between the proposed meth of under probability as 0.005 and reference [23].

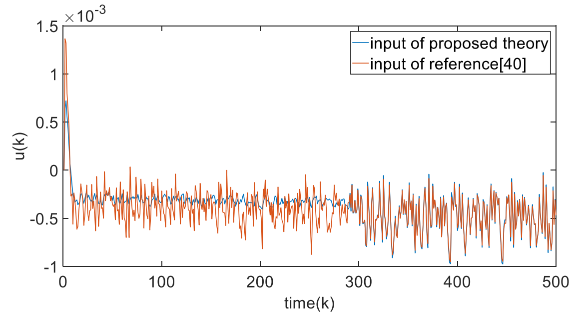

FIGURE 16. Control input comparison between the proposed method under probability as 0.005 and reference [23].

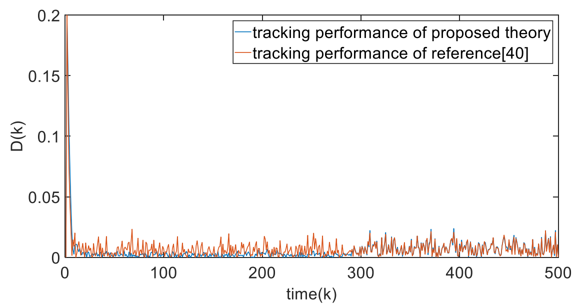

FIGURE 17. Tracking performance comparison between the proposed method under probability as 0.005 and reference [23].

higher such as 0.5 and 0.05, the system is quickly switched to FTC. On the contrary, when the probability of actuator failure is lower such as 0.005, the system is slowly switched to FTC. In addition, as shown in Fig. 11, before the actuator fails, the output response of the system will show a small oscillation around the set value for the influence of factors such as uncertainty, time-varying delays and unknown external disturbances. After the actuator fails, it is switched to a fault-tolerant controller. Under this circumstance, although the oscillation amplitude of the output response increases due to the effect of the actuator failure, the system can still run stably and track the set value.

Fig. 4, Fig. 8 and Fig. 12 show the control inputs of the system with different probability of actuator failure, respectively. It can be known from Fig. 4 and 8, the system switches to fault-tolerant controller earlier and causes the larger maximum value of the control input under the condition of failure probability 0.5. According to Fig. 12, it can be seen, the control input of the system has changed significantly. Before the fault occurs, the control input operated around -3×10^{-4} with less fluctuation because the FTC method was not used. After the fault occurs, the control method is switched to FTC to ensure system operate stably. Therefore, the control input

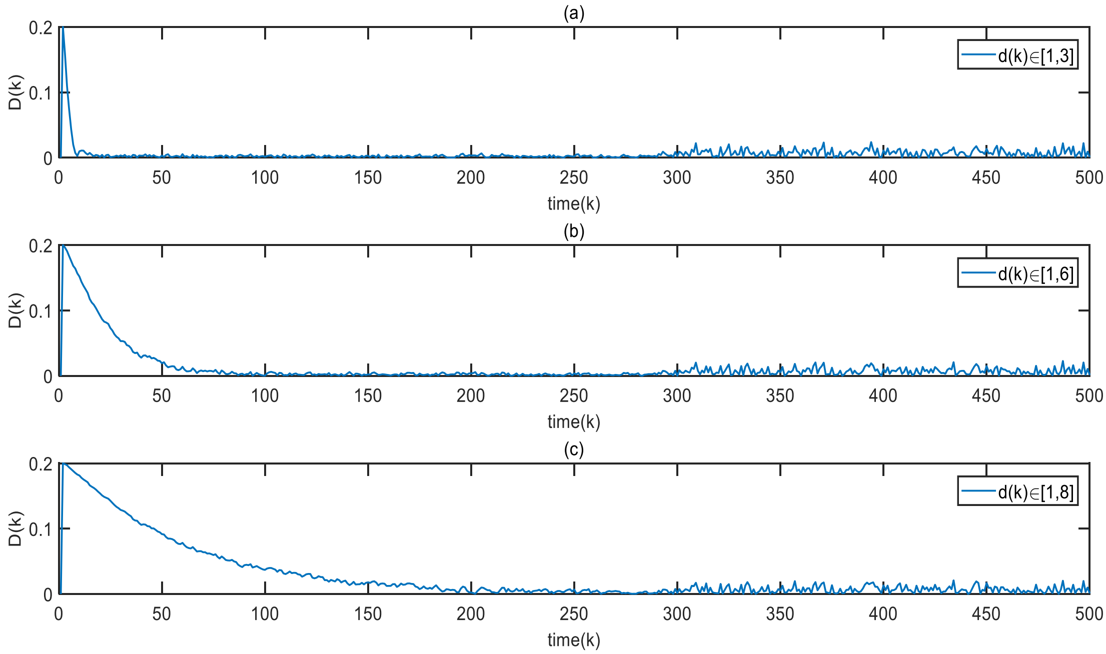

FIGURE 18. Tracking performance of the system with different upper bounds of delay.

operates around -5×10^{-4} and the fluctuation is significantly greater than before the fault.

Fig. 5, Fig. 9 and Fig. 13 show the tracking performances of the system, respectively. According to these figures, the output tracking error of the system can be well controlled no matter what the actuator failure probability is. In addition, from Fig. 13, it will be faster to track the set value under the low probability of actuator failure. This is because the fault-free controller with better effect is adopted. Combined with Fig. 3, Fig. 7 and Fig. 11, when the time delays change with time within a range, the proposed method can ensure that the system can operate stably even if the system is affected by the time delays.

In Fig. 15, Fig. 16 and Fig. 17, we compare the proposed method with the fault-tolerant control method in reference [23] in terms of output response, control input and tracking performance. It can be seen from Fig. 15 that when the probability of actuator failure is 0.005, the method proposed in this paper has significantly smaller output response fluctuations before the failure point, and when the actuator fails, the stable operation of system can also be guaranteed within the allowable range. It can be seen in Fig. 16 that the fluctuation range of the control input before the fault point of the method proposed in this paper is significantly smaller than that of the reference [23], which prevents the large-scale action of the controller and reduces the energy consumption caused by the action of the controller. As shown in Fig. 17, the tracking performance of the proposed method before the point of failure is significantly better than the tracking performance of reference [23], which effectively improves the control accuracy of the system and lays the foundation for the production of better quality products.

In addition, the range of time-varying delay in presence of the actuator failure probability being 0.005 is changed in order to verify the tolerant ability for system time delay using the designed controller. Fig. 18(a), Fig. 18(b) and Fig. 18(c) show the tracking performances of the system, respectively when the ranges of time delay are $[1, 3]$, $[1, 6]$, and $[1, 8]$. According to these figures, the tracking performance of the system deteriorates with the increasing range of the time delay. But the system remains stable in different time delay range.

TABLE 1. Comparison of the amount of raw materials consumed by the fault-tolerant controller and the designed controller.

Probability of actuator failure	Amount of raw materials consumed by fault-tolerant controller in reference [23]	Amount of raw materials consumed by the designed controller
0.5	500a	3b+497a
0.05	500a	10b+490a
0.005	500a	288b+212a

The amount of raw materials (kg) consumed is used to show the advantages of the designed controller compared with the fault-tolerant controllers under different probabilities, as shown in Table 1. In this table, "a" represents the amount of material consumption quantity (kg) by the fault-tolerant controller in each step and "b" is the amount of material consumption quantity (kg) by the conventional controller in each step in which a > b. Accordingly, the designed controller can decrease the consumption of the amount of raw materials, save energy and reduce the cost of production.

V. CONCLUSION

In this paper, a SRPFTC method is proposed for industrial processes with interval time-varying delays and actuator failures occurring under a certain probability. By combining the output error with the traditional state-space model, a new extended state space model that describes a class of industrial processes with interval time-varying delay and actuator failures is established. According to the extended model, using the robust predictive control and stochastic control theory, a stochastic robust predictive fault-tolerant controller is designed. Then the theorem and the corollary to satisfy the LMI constraints are given. In addition, in order to deal with the actuator failure in industrial production with a certain probability, the corresponding control law is updated in real time according to the probability of actuator occurrence, so as to prevent unnecessary economic losses in production. Finally, simulation verification is performed on the basis of the simulation model established by the TTS20 water tank, which verifies whether the proposed method is feasible and effective. According to the simulation results, the designed controller can not only effectively suppress the influence of uncertainty, interval time-varying delay and external disturbance on the system, but also effectively switch the controller under the different probability of actuator failure, which ensure system stability and good tracking performance.

In addition, the probability of actuator failure in this paper is a known variable obtained through statistical methods. In the future, we will continue to optimize the controller by considering the probability distribution that conforms to Markov's prediction.

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