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Dynamic Thresholds Identification for Green Extension and Red Truncation Strategies for Bus Priority

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ABSTRACT Among the different strategies adopted to improve the efficiency and reliability of the bus services, bus signal priority is a low cost and less infrastructure- demanding solution that has the potential to reduce bus travel times in urban arterials. This paper develops analytical models for finding the thresholds for Green Extension (GE) and Red Truncation (RT) for a four-phase signal system with buses on conflicting phases. The thresholds are developed based on reducing the total person delay after considering buses from the current cycle and unserved buses from the previous cycle for priority decisions. The proposed models aim for zero-delay service for the buses from the current cycle and reducing delay for the unserved buses from the previous cycle. The models reveal that at multi-phase signals, several bus priority alternatives are possible to reduce total person delay and agencies can choose alternatives based on their requirements and constraints. These models are evaluated in VISSIM microscopic simulation environment. The evaluation results indicate a 16.7 to 42.8% reduction in total person intersection delay due to the implemented bus priority.

INDEX TERMS Bus signal priority, green extension, isolated intersection, red truncation, total person delay.

I. INTRODUCTION

One of the most effective solutions to reduce congestion and air pollution on urban streets and highways is to promote public transport by making it more reliable, accessible, and efficient. The strategies that are adopted to improve public transportation performance in countries like the U.S. range from transit priority lanes in the urban network to queue jumpers, curb extensions, and transit signal priority at intersections. Based on the benefit-cost analysis, it is preferable to implement strategies that are confined to the intersection level, as the major source of delay for buses is signalized intersections [1].

Bus Signal Priority (BSP) is an operational strategy that modifies the normal signal operation process to better accommodate transit vehicles. It aims to reduce travel time and delay of buses, thereby reducing the schedule deviation and enhancing the reliability of bus services. Hence, it is an inexpensive way to make public transit more competitive with automobiles. BSP can be viewed as both demand side and supply side traffic management measure. It makes public transit more attractive, which can make people shift from private modes to public transport, thus helping to reduce the vehicular demand on roads. It improves the signal operation to maximize the person throughput, making it a supply management measure too. Despite these benefits, their potential negative impacts on the other vehicles in conflicting directions warrant a thorough validation before implementation.

A variety of BSP strategies were developed and implemented around the world with more benefits and lesser adverse impacts [2]. Depending on the intersection characteristics, availability of equipment and budget, the implementation of BSP can be passive (offline) or active (online). The active strategies are more infrastructure intensive, requiring sensors to detect transit vehicles and advanced controllers to make necessary signal changes in real-time. However, with recent developments in Intelligent Transportation System (ITS) technologies, rule-based active priority strategies such as Red Truncation (RT) and Green Extension (GE) are more commonly adopted in many countries. These conventional strategies consider fixed extension or truncation for a few seconds without quantifying delay, and priority is granted conditionally or unconditionally. The conditional priority can be granted based on the schedule deviation of the

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bus [3]–[6], occupancy [7], headway [8], etc. However, the benefits of these strategies are limited due to its heuristic nature [9].

The scope of the bus priority studies ranges from isolated intersection, to that at arterial level and with signal co-ordination [10]–[15]. The effects of transit facilities like exclusive lane [10], [11], [16], near side bus stop [17], [18], pre-signal strategy [19], BRT [9], [20] were also investigated.

The model-based strategies developed so far can also be called an optimization-based strategy, since they attempt to provide priority based on the optimization of some performance criterion. The real-time characteristics of the intersection and bus are considered as input and the signal timings are optimized with the objective of bus priority. A variety of mathematical models and their solution methods have been proposed for real-time adaptive bus signal priority. For example, a dynamic signal timing optimization model by considering both arrival and departure flows as functions of time was developed in [10]. The bus arrival was represented by giving a weight factor, which is a function of current traffic demand, queuing conditions of the intersection, and bus lateness. The study [20] extended the conventional priority strategies, GE and RT, with suppression strategies like Green Truncation and Red Extension to reduce delays and avoid queue spillback at median BRT stations. An optimization model for adaptive transit signal priority using a parallel genetic algorithm (PGA) was developed in [22]. The optimization objective function minimized the estimated individual vehicular delay at the intersection, with the bus delays weighted by a factor, which was a function of the passenger occupancy of the bus, the queuing condition of all the intersection movements and the schedule lateness of the bus. Mathematical formulation of the total person delay of auto and transit vehicles using the cumulative arrival departure curve was developed in [23]. The optimization problem was to minimize the total person delay at the intersection, subjected to constraints of minimum, maximum green times and cycle time. A stochastic mixed-integer nonlinear model (SMINP) was proposed in [24] for a real-time TSP control system with the objective to minimize the deviations of the resulting phase split times from the optimal background split times, so as to reduce the negative impacts on conflicting traffic. Han *et al* [25] formulated a quadratic optimization problem to minimize the maximum among the approach control delay at the intersection and the moving, queuing and waiting delay of the bus arriving at the intersection during the design cycle.

A passenger delay based optimization method is used in [26], where the performance of both the buses and private vehicles are considered for the green extension strategy. The proposed model was shown to reduce total person delay near saturation conditions. The length of priority time, degree of saturation and the number of lanes were shown to be the most influencing factors that affect the priority. A summary of various techniques used for solving the optimization of bus priority are listed in Table 1 and various objective functions used are summarised in Table 2.

TABLE 1. Summary of popular solution techniques.

Solving Technique	Studies
Branch and Bound method and Fmincon Function in MATLAB	Christofa et al. [23]
Sequential Quadratic Programing Solver in MATLAB	Han et al. [25]
Standard Branch and Bound Routine	Hu et al. [6]
Dynamic Programming Algorithm	Xu and Ye [18]
Enumeration technique	Ma et al. [11], Shu et al. [26]
Genetic Algorithm	Stevanoic et al.[27] , Ghanim and Lebdeh[14]
Parallel Genetic Algorithm	Zhou et al. [22]

TABLE 2. Summary of objective functions used.

Objective Function	Studies
Vehicle delay	Liu et al. [21],Zhou et al. [22]
Vehicle delay and bus delay	Li et al. [29]
Person delay	Christofa et al. [23], Thodi et al. [28]
Delay deviation by priorirty	Ma et al. [11]
Deviations of resulting phase split times	Zeng et al. [24]
Moving, queueing and waiting delay of bus and maximum control delay of traffic	Han el al. [25]
In passenger delay and passenger waiting delay at next bus stops	Xu and Ye [18]
Passenger traveling delay at intersections and bus stops	Li and Jin [15]
Total travel time of all passengers, bus delay and person based delay	Hu et al. [6]
Total passenger delay increment	Shu et al. [26]

However, using real-time optimisation for priority implementations has disadvantages in terms of computational requirements. This makes analytical models, which are computationally less demanding and more attractive for practical implementation. These models help in better understanding of the variables, their relationships, and their effects on the model output. An analytical approach for bus priority strategies was proposed in [28] and derived optimality conditions for the green extension and red truncation. However, the study proposed computation of GE and RT values, which also falls under the optimal priority calculation in real time. In addition, the analysis was limited to a two-phase signal setting. This study addresses these two limitations by deriving closed-form expressions for GE and RT thresholds for four phase signals that require minimal computational infrastructure.

The extension of the analytical approaches to optimal bus priority for higher number of phases leads to cumbersome equations for practical purposes. Therefore, there is a need to develop bus priority strategies for a multi-phase system in a more manageable form. The optimization-based strategies from the literature only consider the buses that arrive in a cycle to determine the bus priority strategies. These strategies can result in buses not getting the priority and have to wait through the whole red phase in the next cycle. To overcome these gaps in the literature, this paper proposes a methodology to determine dynamic thresholds for green extension and red truncation for a four-phase signal. The methodology considers buses that arrive in the current cycle as well as the buses that did not get priority in the previous cycle while ensuring total person delay reduction during a cycle. The closed-form expressions for GE and RT thresholds derived in this paper are simpler and support bus priority implementation with minimal computational infrastructure, computational time, and implementation/O&M cost for large scale field implementation of higher phase signals. Using the proposed thresholds we can provide dynamic real time solution for bus signal priority without a need for an optimizer in the field. This makes our study different from the GE/RT optimisations and the priority models reported so far.

The paper is structured as follows. Section 2 presents analytical model development in an under-saturated four-phase scenario. Section 3 explains the implementation process. Section 4 and 5 show the results of the proposed strategies based on simulation.

II. DELAY FORMULATION

The present study considers total person delay of all vehicles that arrive at an intersection in a signal cycle. In the majority of the earlier literature, the delay functions were multiplied by an appropriate weighting factor which will usually be a bus characteristic. The weighing factor considered here is the occupancy of the vehicles. As buses have the highest occupancy, the weighing factor will prioritize buses over other vehicles in the signalized intersection.

The assumptions made while formulating the model are given below:

1) Under-saturation conditions prevail in all approaches. Hence, the vehicles that arrive during red, will be discharged during the next green of that approach.
2) Arrival rates and saturation flow rates are constant within a period (eg. Morning off-peak period) for each approach.
3) The bus occupancies are known.
4) Cycle length and phase sequences are fixed.
5) The ratios of green time of non-priority approaches remain the same even after the execution of the priority strategy.

TABLE 3. Notations used in the formulation.

Symbol	Description
C	Cycle length
g	Effective green time
r	Effective red time
y	Amber time
q	Uniform arrival rate of vehicles
s	Uniform departure rate of vehicles (saturation flow)
tb	Bus arrival time in the queue with respect to start of the red interval
T	Period between the start and end of queue
d_{bj}	Stopped delay of the bus j in the queue
d_{ai}	Aggregate delay for other vehicles during phase i
O_{bj}	Passenger occupancy of bus j
O_a	Average occupancy of all other vehicles
N_p	Number of vehicles in the queue at the beginning of the cycle for approach 'p'
D	Total person delay of the intersection
D_g	Total person delay of the intersection after green extension
D_r	Total person delay of the intersection after red truncation
x	Proportion at which green time of non-priority phase is deducted/increased due to green extension/red truncation in the priority phase
r^*	The red time after priority strategy
g^*	The green time after priority strategy
$g_{\min}$	Minimum green required for under-saturation
e	The extended amount of green time or truncated amount of red time

The notations used in the derivation are given in Table 3. Any additional numerical attached to the given symbol indicates the phase.

The objective is to develop a bus priority strategy with lesser impact on the conflicting traffic. For this, the function selected is the total person delay at the intersection. The generalized formula for total person delay considered is:

$$D = O_a \sum_{i=1}^{n} d_a^i + \sum_{j=1}^{k} O_b^j d_b^j, \tag{1}$$

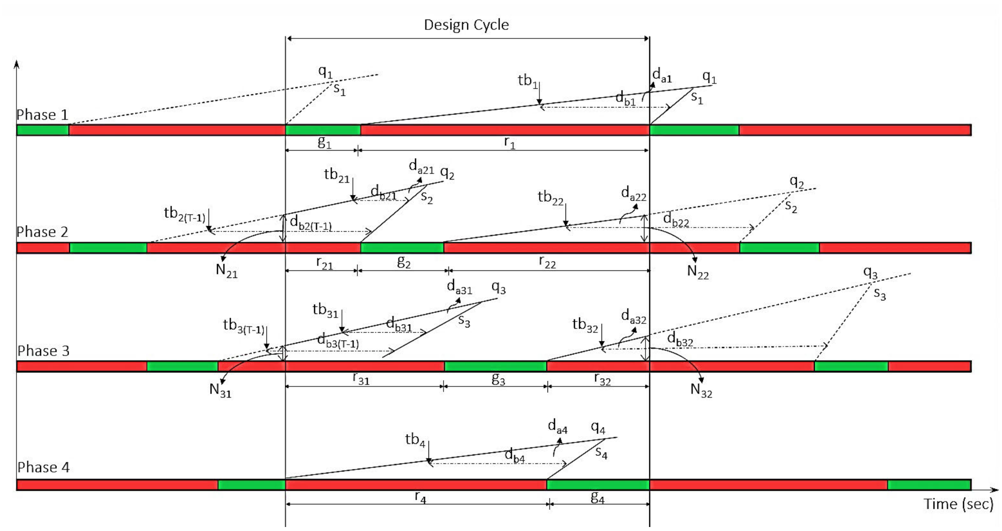


FIGURE 1. Cumulative arrival and departure curve for the four phase signal system.

where 'n' is the number of phases in the cycle and 'k' is the total number of buses considered for analysis. The other vehicles' delay function ($\boldsymbol{d_a^i}$) and the bus delay function ($\boldsymbol{d_b^j}$) should be formulated as a function of signal timing and are discussed below.

A. AGGREGATED VEHICLE DELAY (d_a)

Consider the cumulative arrival departure curve for a four-phase signal as shown in Figure 1. Note that r_{ij} indicates the j^{th} red of the i^{th} phase. The lost times are not considered and hence g denotes the effective green time.

Here, the delay of all vehicles arriving during the red interval 'r' of the design cycle is given by 'd_a', which is the area under the curves and can be computed using Webster's uniform delay equation as given in (2). Here, the delay in the design cycle alone is considered to compute the signal timings of the same.

$$d_{a1}\ or\ d_{a4} = \frac{0.5qr^2}{1-\frac{q}{s}}. \tag{2}$$

The equation can be reduced as:

$$d_{a1}\ or\ d_{a4} = pr^2, \tag{3}$$

where,

$$p = \frac{0.5q}{1-\frac{q}{s}}. \tag{4}$$

The ratio 'p' is defined for each phase using the respective flow rates.

In the 2nd phase, there are two red intervals in each cycle, r_{21} and r_{22} respectively. The residual queue at the beginning of red interval is denoted as N_{21} and can be expressed as:

$$N_{21T} = r_{22(T-1)}q_2. \tag{5}$$

where $r_{22(T-1)}$ is the 2nd red in the previous cycle, which is known. Thus, the vehicle delay in the first red (d_{a21}) is the area under the trapezoid, which can be expressed as:

$$d_{a21} = \frac{N_{21}}{2}(2r_{21} + \frac{N_{21}}{s_2}) + p_2(r_{21} + \frac{N_{21}}{s_2})^2. \tag{6}$$

Similarly, the delay caused in the r_{22} (d_{a22}) is,

$$d_{a22} = \frac{q_2 r_{22}^2}{2}. \tag{7}$$

The delay expressions in the third phase can be developed in the same manner as in the second phase. Thus, the aggregated delay of vehicles arrived during the design cycle, d_a can be expressed as:

$$\begin{aligned} d_a = {} & p_1 r_1^2 + \frac{N_{21}}{2}(2r_{21} + \frac{N_{21}}{s_2}) + p_2(r_{21} + \frac{N_{21}}{s_2})^2 \\ & + \frac{q_2 r_{22}^2}{2} + \frac{N_{31}}{2}(2r_{31} + \frac{N_{31}}{s_3}) + p_3(r_{31} + \frac{N_{31}}{s_3})^2 \\ & + \frac{q_3 r_{32}^2}{2} + p_4 r_4^2. \end{aligned} \tag{8}$$

B. BUS DELAY

The delay of the bus (d_b) that arrives at the intersection during the red interval at 'tb' can be computed from Figure 1. Given 'tb', the time required for the bus to be served in green (t_g) can be expressed as:

$$t_g = tb(\frac{q}{s}). \tag{9}$$

Then, the delay of the bus, 'd_b' can be obtained as:

$$d_b = r - tb(1 - \frac{q}{s}). \tag{10}$$

Thus, both the auto and bus delay can be expressed in terms of the red interval (r), which is our decision variable. With these expressions, auto and bus delay for any signal conditions can be formulated.

C. BASE MODEL FOR FOUR PHASE SIGNAL SYSTEM

The base signal system is developed based on vehicle delay minimization, as below.

If $r_1, r_{21}, r_{22}, r_{31}, r_{32}$ and r_4 are the red intervals for the four phases, (8) represents the total vehicle delay of the signalized intersection (D_v), where $p_1 = \frac{0.5q_1}{1-\frac{q_1}{s_1}}$, $p_2 = \frac{0.5q_2}{1-\frac{q_2}{s_2}}$, $p_3 = \frac{0.5q_3}{1-\frac{q_3}{s_3}}$, and $p_4 = \frac{0.5q_4}{1-\frac{q_4}{s_4}}$.

In order to reduce the number of decision variables, the following relationships are used.

$$r_{21} = C - r_1, \tag{11}$$

$$r_{31} = C - r_{22}, \tag{12}$$

and

$$r_{32} = C - r_4. \tag{13}$$

These relations were substituted in (8), and the independent decision variables were reduced in terms of r_1, r_{22}, and r_4. The residual queue at the beginning of the design cycle, N_{21} and N_{31} were considered as constants since they are independent of signal timings of the current cycle. The objective was to arrive at the base signal timing that minimizes the vehicle delay at the intersection. Thus, the solution is:

$$r_1 = \frac{2p_2\left(C + \frac{N_{21}}{s_2}\right) + N_{21}}{2(p_1 + p_2)}, \tag{14}$$

$$r_{22} = \frac{2p_3\left(C + \frac{N_{31}}{s_3}\right) + N_{31}}{2p_3 + q_2}, \tag{15}$$

and

$$r_4 = \frac{q_3 C}{2p_4 + q_3}. \tag{16}$$

Solving for other variables using relations 11 to 13 yields the following equations.

$$r_{21} = \frac{2Cp_1 - N_{21}(2\frac{p_2}{s_2} + 1)}{2(p_1 + p_2)}, \tag{17}$$

$$r_{31} = \frac{q_2 C - N_{31}(2\frac{p_3}{s_3} + 1)}{2p_3 + q_2}, \tag{18}$$

and

$$r_{32} = \frac{2p_4 C}{2p_4 + q_3}. \tag{19}$$

The base signal intervals are considered as constant over the signal cycles. To solve (14)-(15) and (17)-(18), r_{32} was used. Then,

$$N_{31} = q_3 r_{32}. \tag{20}$$

Now, r_{22} can be computed, which helps in determining N_{21} as:

$$N_{21} = q_2 r_{22}. \tag{21}$$

Next, minimum greens for each phase are computed for maintaining under-saturation. The minimum green required for under-saturation can be written as:

$$g_{min} = \frac{q}{s}\,\text{(C-y)}\,. \tag{22}$$

Using these equations, the total person delay was formulated, which is used for developing bus priority solutions, as discussed in the next section.

D. TOTAL PERSON DELAY

Based on Figure 1, the total person delay of the intersection (D) of the four-phase signal system can be taken as the sum of vehicle delay (8) and bus delay (10) multiplied by their respective occupancies as:

$$\begin{aligned}
D(T) = {} & O_a\{p_1 r_1^2 + \frac{N_{21}}{2}(2r_{21} + \frac{N_{21}}{s_2}) + p_2(r_{21} + \frac{N_{21}}{s_2})^2 \\
& + \frac{q_2 r_{22}^2}{2} + \frac{N_{31}}{2}(2r_{31} + \frac{N_{31}}{s_3}) + p_3(r_{31} + \frac{N_{31}}{s_3})^2 \\
& + \frac{q_3 r_{32}^2}{2} + p_4 r_4^2\} + O_{b1}(r_1 - tb_1(1 - \frac{q_1}{s_1})) \\
& + O_{b22(T-1)}(r_{21} + r_{22(T-1)} - tb_{22(T-1)}(1 - \frac{q_2}{s_2})) \\
& + O_{\mathbf{b}21}(r_{21} + r_{22(T-1)} - tb_{21}(1 - \frac{q_2}{s_2})) \\
& + O_{b22}(r_{22} + g_{1(T+1)}) - tb_{22}(1 - \frac{q_2}{s_2}))) + \\
& O_{b32(T-1)}(r_{31} + g_{4(T-1)} - tb_{32(T-1)}(1 - \frac{q_3}{s_3})) \\
& + O_{b31}(r_{31} + g_{4(T-1)} - tb_{31}(1 - \frac{q_3}{s_3})) + O_{b32}(r_{32} \\
& + r_{31(T+1)} - tb_{32}(1 - \frac{q_3}{s_3})) + O_{b4}(r_4 - tb_4(1 - \frac{q_4}{s_4})).
\end{aligned} \tag{23}$$

where O_a is the average occupancy of all vehicles, except bus, that arrive at the intersection during the design cycle, and O_{bij} denotes the occupancy of buses arriving in the respective red durations.

Following the traditional practice in the literature, Red Truncation (RT) and Green Extension (GE) are selected as strategies for bus and analytical expressions for maximum truncation/extension to reduce total person delay are derived.

For the four-phase signal system as shown in Figure 1, the RT is possible in phases 2, 3 and 4 and GE is possible in phases 1,2 and 3. The priority is granted only if the total person delay after the priority action is less than or equal to that with base signal timings (D_T of priority signal timing $\leq$ D_T of base signal timing).

Substituting the base and priority signal timings in (23) and solving above condition will lead to a quadratic function of extension or truncation limit, as shown below:

$$\alpha e^2 + \beta e - \Upsilon \leq 0, \tag{24}$$

where 'e' is the extension or truncation amount in the respective phase where priority action is proposed. The above inequality is solved for maximum allowable extension or truncation limit by equating it to zero. The maximum allowable extension or truncation amount can hence be defined as:

$$e = \frac{-\beta + \sqrt{\beta^2 + 4\alpha\Upsilon}}{2\alpha}. \tag{25}$$

The expressions for α, β and Υ in each of the priority strategy is discussed in the following sections.

E. LIMIT IDENTIFICATION – RED TRUNCATION

As stated earlier, for the four-phase signal system the RT can be done in the 2nd, 3rd and 4th phases of a cycle. To achieve zero-delay service to buses that arrive during red, the existing queue, should be dissipated before the bus reaches the intersection. Thus, the red should be truncated ahead for the queue dissipation time of $\frac{qtb}{s}$.

1) RED TRUNCATION IN PHASE 2 (RT2)

In the case of long cycle lengths, it is possible to have the red truncation in phase 2 that allows the busses (tb_{21}) that arrive at the intersection during the first red interval (r_{21} in Figure 1) to have zero delay at the intersection. Thus, the decision of RT2 should be made at the beginning of the cycle. Here, the truncated amount of red causes the addition of the same amount of red in the first phase (r_1). Hence, it is necessary to ensure minimum green in the first approach while providing this red truncation.

Let the red in phase 2 be truncated by an amount of 'e'. The resulting increased amount of green in phase 2 can be shared between phases 2, 3 and 4. The changed green intervals after red truncation can be written as follows:

$$g_1^* = g_1 - e, \tag{26}$$

$$g_2^* = g_2 + (1 - x_1)e, \tag{27}$$

$$g_3^* = g_3 + (x_1 - x_2)e, \tag{28}$$

and

$$g_4^* = g_4 + x_2 e. \tag{29}$$

where, x_1 and x_2 are the ratio of green times of non-priority approaches, which remain the same even after priority.

$$\text{i.e. } \frac{g_2^*}{g_3^*} = \frac{g_2}{g_3} \text{ and } \frac{g_3^*}{g_4^*} = \frac{g_3}{g_4}. \tag{30}$$

Substituting for the changed intervals in the above equation and solving for $x_1$andx_2 yields:

$$x_1 = \frac{g_3 + g_4}{g_2 + g_3 + g_4}, \tag{31}$$

And

$$x_2 = \frac{g_4}{g_2 + g_3 + g_4}. \tag{32}$$

These priority signal timings can be substituted in (24) to obtain the total person delay after RT2.

Solving the condition of total person delay after RT2 less than or equal to that during base signal timing leads to (24) with α, β and Υ as given below:

$$\alpha = O_a\left(p_1 + p_2 + x_1^2\left(0.5q_2 + p_3\right) + x_2{}^2\left(0.5q_3 + p_4\right)\right), \tag{33}$$

$$\begin{aligned}\beta = O_a\Bigg(&2p_1r_1 - 2p_2\left(r_{21} + \frac{N_{21}}{s_2}\right)\\ &-N_{21} - x_1\left(2p_3\left(r_{31} + \frac{N_{31}}{s_3}\right) - q_2r_{22} + N_{31}\right)\\ &-x_2\left(2p_4r_4 - q_3r_{32}\right)\Bigg) + \frac{Ob_1q_1}{s_1} - Ob_{22(T-1)}\\ &-x_1\left(Ob_{32(T-1)} + Ob_{31} - \frac{Ob_{22}q_2}{s_2}\right)\\ &-x_2(Ob_4 - \frac{Ob_{32}q_3}{s_3}),\end{aligned} \tag{34}$$

$$\gamma = Ob_{21}\left(r_{21} + r_{22(T-1)}tb_{21}\left(1 - \frac{q_2}{s_2}\right)\right). \tag{35}$$

Thus,

$$ir_{2max1} = \frac{-\beta + \sqrt{\beta^2 + 4\alpha\Upsilon}}{2\alpha}. \tag{36}$$

where ir_{2max1} is the maximum truncation limit, which is a function of traffic flows at the intersection, residual queue length at the beginning of the cycle, base signal timings, bus occupancy, and other vehicles' occupancy.

However, to ensure under-saturated condition in the 1st phase, additional condition for maximum truncation can be written as:

$$ir_{2max2} = g_1 - g_{1min}. \tag{37}$$

Priority will be granted only if:

$$\min\left\{ir_{2max1}, ir_{2max2}\right\} \geq r_{21} - tb_{21}\left(1 - \frac{q_2}{s_2}\right). \tag{38}$$

where, ($r_{21} - -tb_{21} + \frac{q2\ tb21}{s2}$) is the actual truncated amount of red required in the 2nd phase and $\frac{q2\ tb21}{s2}$ is the time required for queue dissipation. Thus, the queue will be dissipated before the bus reaches the intersection and zero delay service will be ensured to the bus.

2) RED TRUNCATION IN PHASE 3(RT3)

The model for red truncation in the third phase is similar to that in the second phase. Here, the truncated amount of red is completely added to the second phase and the increased green times are shared between the 3rd and the 4th phase. The changed green intervals after red truncation in the third phase can be written as:

$$g_1^* = g_1, \tag{39}$$

$$g_2^* = g_2 - e, \tag{40}$$

$$g_3^* = g_3 + (1 - x_1)e, \tag{41}$$

and

$$g_4^* = g_4 + x_1 e. \tag{42}$$

where, x_1 can be defined by assuming that the ratio of green times of 3rd and 4th approaches remain the same even after priority. Substituting for the changed interval and solving for x_1 yields:

$$x_1 = \frac{g_4}{g_3 + g_4}. \tag{43}$$

Here, the decision for RT3 can be taken only after minimum green in the second phase. The expressions for α, β and Υ in (24) for this case is obtained as:

$$\alpha = O_a\left(0.5q_2 + p_3 + {x_1}^2\left(0.5q_3 + p_4\right)\right), \tag{44}$$

$$\begin{aligned}\beta = O_a\Bigg(&-2p_3\left(r_{31} + \frac{N_{31}}{s_3}\right) + q_2 r_{22} \\ &-N_{31} + x_1\left(2p_4 r_4 + q_3 r_{32}\right)\Bigg) + \frac{Ob_{22}q_2}{s_2} \\ &-Ob_{32(T-1)} - x_1(Ob_4 - \frac{Ob_{32}q_3}{s_3}),\end{aligned} \tag{45}$$

and

$$\Upsilon = Ob_{31}\left(r_{31} + g_{4(T-1)}tb_{31}\left(1 - \frac{q_3}{s_3}\right)\right). \tag{46}$$

Thus,

$$ir_{3max1} = \frac{-\beta + \sqrt{\beta^2 + 4\alpha\Upsilon}}{2\alpha} \tag{47}$$

where ir_{3max1} is the maximum limit of truncation in the third phase, which is a function of traffic and bus characteristics as detailed before.

To ensure under-saturated condition in the 2nd phase, additional condition for maximum truncation is:

$$ir_{3max2} = g_2 - g_{2min}, \tag{48}$$

Here also, priority will be granted only if:

$$\min\{ir_{3max1}, ir_{3max2} \geq r_{31} - tb_{31}\left(1 - \frac{q_3}{s_3}\right). \tag{49}$$

The expression on the right hand side of the above inequality is the actual amount of truncated red.

3) RED TRUNCATION IN PHASE 4 (RT4)

Red truncation is the only strategy available to the buses that arrive in the fourth approach to grant priority. Since phase skipping is not allowed, the model for RT4 can cause changes only after the minimum green in the third phase. Thus, the benefit of this model is limited to the buses that arrive in red after the minimum green in the third phase. However, it will lead to a reduced delay for buses in the queue, if any. The development of the model is the same as before except the change in the decision point shifts to the end of minimum green in the third phase. Here, an increased amount of green is the only strategy that can be made available to the fourth phase, since it is the last phase of the cycle.

The changed green intervals after red truncation in the fourth phase can be written as:

$$g_1^* = g_1, \tag{50}$$

$$g_2^* = g_2, \tag{51}$$

$$g_3^* = g_3 - e, \tag{52}$$

$$g_4^* = g_4 + e. \tag{53}$$

The decision for RT4 is taken only at the end of the minimum green in the third phase. The expressions for α, β, and Υ in (24) is obtained as:

$$\alpha = O_a(0.5q_3 + p_4), \tag{54}$$

$$\beta = O_a(-2p_4 r_4 + q_3 r_{32}) + \frac{Ob_{32}q_3}{s_3}, \tag{55}$$

and

$$\Upsilon = Ob_4\left(r_4 - tb_4\left(1 - \frac{q_4}{s_4}\right)\right). \tag{56}$$

Thus,

$$ir_{4max1} = \frac{-\beta + \sqrt{\beta^2 + 4\alpha\Upsilon}}{2\alpha} \tag{57}$$

To ensure under-saturated condition in the 3rd phase, additional condition for maximum truncation is:

$$ir_{4max2} = g_3 - g_{3min}. \tag{58}$$

Priority will be granted only if:

$$\min\{ir_{4max1}, ir_{4max2} \geq r_4 - tb_4\left(1 - \frac{q_4}{s_4}\right). \tag{59}$$

The expression on the right-hand side of the above inequality is the actual amount of truncated red required in the fourth phase when the bus arrives at tb_4.

F. LIMIT IDENTIFICATION – GREEN EXTENSION (GE)

Green extension is implemented when a bus arrives at the intersection just after the green in that approach. Similar to the RT model, in the GE model also, the green is either extended or not extended depending on the resulting person delay. The green will be extended only if the resulting total person delay of the intersection is less than or at least equal to that in base signal timing. Here also the implementation is to achieve zero-delay service at the intersection. For a four-phase signal system (Figure 1), the green extension is possible in phases 1, 2 and 3. The computation of maximum GE limit is done in the same manner as that for maximum RT limit, the difference lies in the signal timings and for the actual extension required in the field.

1) GREEN EXTENSION IN PHASE 1 (GE1)

The changed green intervals after green extension can be written as:

$$g_1^* = g_1 + e, \tag{60}$$

$$g_2^* = g_2 - x_1 e, \tag{61}$$

$$g_3^* = g_3 - x_2 e, \tag{62}$$

and

$$g_4^* = g_4 - \left(1 - x_1 - x_2\right)e. \tag{63}$$

where, x_1 and x_2 can be defined by assuming the ratio of green times of non-priority approaches remain the same even after priority.

$$\text{i.e. } \frac{g_2^*}{g_3^*} = \frac{g_2}{g_3} \text{ and } \frac{g_3^*}{g_4^*} = \frac{g_3}{g_4}. \tag{64}$$

Substituting for the changed intervals in the above equation and solving for x_1 and x_2 yields:

$$x_1 = \frac{g_2}{g_2 + g_3 + g_4} \text{ and } x_2 = \frac{g_3}{g_2 + g_3 + g_4}. \tag{65}$$

The expressions for α, β and Υ in (24) is obtained as:

$$\alpha = O_a\Big(p_1 + p_2 + (1 - x_1)^2 (0.5q_2 + p_3) + (1 - x_1 - x_2)^2 (0.5q_3 + p_4)\Big), \tag{66}$$

$$\begin{aligned}\beta = O_a\Big(&-2p_1r_1 + 2p_2\left(r_{21} + \frac{N_{21}}{s_2}\right) \\ &+N_{21} + (1 - x_1)\left(2p_3\left(r_{31} + \frac{N_{31}}{s_3}\right) - q_2r_{22} + N_{31}\right) \\ &+ (1 - x_1 - x_2)(2p_4r_4 - q_3r_{32})) + Ob_{22(T-1)} \\ &+ Ob_{21} + (1 - x_1)\left(Ob_{32(T-1)} + Ob_{31} - \frac{Ob_{22}q_2}{s_2}\right) \\ &+ (1 - x_1 - x_2)(Ob_4 - \frac{Ob_{32}q_3}{s_3}),\end{aligned} \tag{67}$$

and

$$\Upsilon = Ob_1\left(r_1 - tb_1\left(1 - \frac{q_1}{s_1}\right)\right). \tag{68}$$

Thus,

$$ig_{1max1} = \frac{-\beta + \sqrt{\beta^2 + 4\alpha\Upsilon}}{2\alpha} \tag{69}$$

where ig_{1max1} is the maximum limit for green extension in the 1st phase, which is obtained as a function of traffic condition at the intersection and the bus characteristics.

The GE1 causes reduction in green times in all other approaches. Hence, to ensure under-saturated condition in other approaches, additional conditions for maximum extension are formulated as follows:

$$ig_{1max2} = \frac{g_2 - g_{2min}}{x_1}, \tag{70}$$

$$ig_{1max3} = \frac{g_3 - g_{3min}}{x_2}, \tag{71}$$

and

$$ig_{1max4} = \frac{g_4 - g_{4min}}{1 - x_1 - x_2}. \tag{72}$$

Priority will be granted only if:

$$\min\{ig_{1max1}, ig_{1max2}, ig_{1max3}, ig_{1max4} \geq tb_1. \tag{73}$$

In that case, the actual extended amount of green is tb_1.

2) GREEN EXTENSION IN PHASE 2(GE2)

The changed green intervals for this phase can be written as:

$$g_1^* = g_1, \tag{74}$$

$$g_3^* = g_3 - (1 - x)e, \tag{75}$$

$$g_2^* = g_2 + e, \tag{76}$$

and

$$g_4^* = g_4 - xe, \tag{77}$$

where,

$$x = \frac{g_4}{g_3 + g_4}. \tag{78}$$

The expressions for α, β and Υ in (25) is obtained as:

$$\alpha = O_a\Big(0.5q_2 + p_3 + x^2 (0.5q_3 + p_4)\Big), \tag{79}$$

$$\begin{aligned}\beta = O_a\Big(&2p_3\left(r_{31} + \frac{N_{31}}{S_3}\right) - q_2r_{22} \\ &+N_{31} + x(2p_4r_4 - q_3r_{32})) + Ob_{32(T-1)} + Ob_{31} \\ &+ x(Ob_4 - \frac{Ob_{32}q_3}{s_3}),\end{aligned} \tag{80}$$

and

$$\Upsilon = Ob_{22}\left(r_{22} + g_{1(T+1)} - tb_{22}\left(1 - \frac{q_2}{s_2}\right)\right). \tag{81}$$

Thus,

$$ig_{2max1} = \frac{-\beta + \sqrt{\beta^2 + 4\alpha\Upsilon}}{2\alpha}. \tag{82}$$

where ig_{2max1} is the dynamic limit of extension in the second phase.

The maximum extension conditions for ensuring undersaturation in the 3rd and 4th phase can be written as follows:

$$ig_{2max2} = \frac{g_3 - g_{3min}}{1 - x}, \tag{83}$$

and

$$ig_{2max3} = \frac{g_4 - g_{4min}}{x}. \tag{84}$$

Here also, priority will be granted only if:

$$min\{ig_{2max1}, ig_{2max2}, ig_{2max3} \geq tb_{22}. \tag{85}$$

Then, the extended amount of green is tb_{22}.

3) GREEN EXTENSION IN PHASE 3(GE3)

The changed green intervals after priority action can be written as:

$$g_1^* = g_1, \tag{86}$$

$$g_2^* = g_2, \tag{87}$$

$$g_3^* = g_3 + e, \tag{88}$$

and

$$g_4^* = g_4 - e. \tag{89}$$

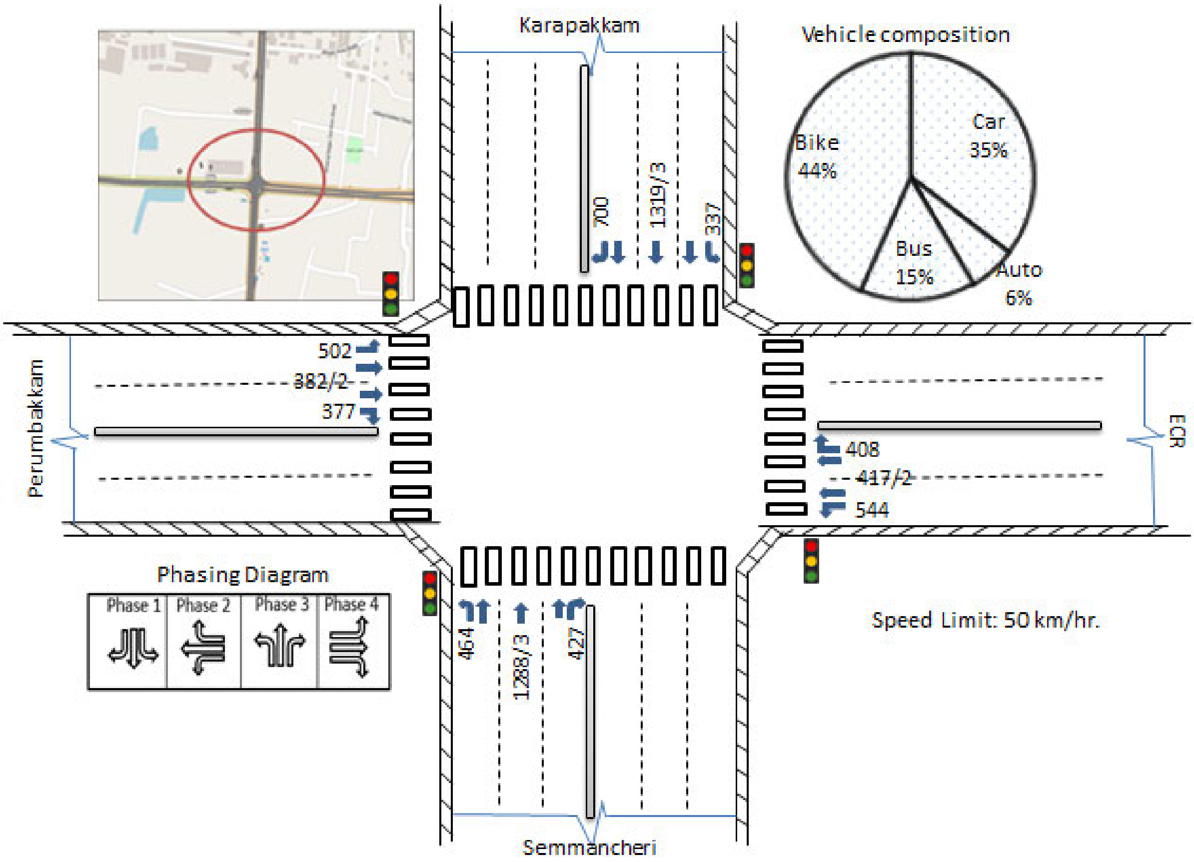


FIGURE 2. Sholinganallur intersection modelled in VISSIM (Volume in veh/hr).

The expressions for α, β and Υ in (24) is obtained as:

$$\alpha = O_a\left(0.5q_3 + p_4\right), \tag{90}$$

$$\beta = O_a\left(2p_4r_4 - q_3r_{32}\right) + Ob_4, \tag{91}$$

and

$$\Upsilon = Ob_{32}\left(r_{32} + r_{31(T+1)} - tb_{32}\left(1 - \frac{q_3}{s_3}\right)\right). \tag{92}$$

Thus,

$$ig_{3max1} = \frac{-\beta + \sqrt{\beta^2 + 4\alpha\Upsilon}}{2\alpha}. \tag{93}$$

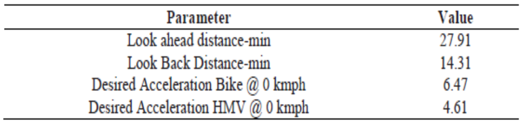
where ig_{3max1} is the maximum limit of extension in the third phase. The maximum extension conditions for ensuring under-saturation in the 4th phase can be written as:

$$ig_{3max2} = g_4 - g_{4min}. \tag{94}$$

As before, priority will be granted only if:

$$min\{ig_{3max1}, ig_{3max2} \geq tb_{32}. \tag{95}$$

Then, the extended amount of green is $tb_{32.}$



Parameter	Value
Look ahead distance-min	27.91
Look Back Distance-min	14.31
Desired Acceleration Bike @ 0 kmph	6.47
Desired Acceleration HMV @ 0 kmph	4.61

FIGURE 3. Changed values of driver behaviour parameters in Vissim simulation model [35].

It is to be noted that in the four-phase signal system, there are six possibilities of priority action. The models are formulated in such a way that, three priority decisions can be taken in the same cycle. For better implementation of the above models, phasing should be done in the decreasing order of arrival volume. Thus minor approaches should be served by 4th phase. The decision points in a signal cycle include the starting of the cycle (GE1/RT2), after minimum green in the second phase (GE2/RT3) and after minimum green in the third phase (GE3/RT4). Evaluation of these strategies using simulation is discussed in the next section.

III. IMPLEMENTATION AND EVALUATION OF BUS PRIORITY MODELS

Evaluation of the performance of the bus priority strategy was done using microsimulation models and analytical models. Analytical approaches used basic delay equations from HCM [30], [31] and queuing theory [32], [33]. However,

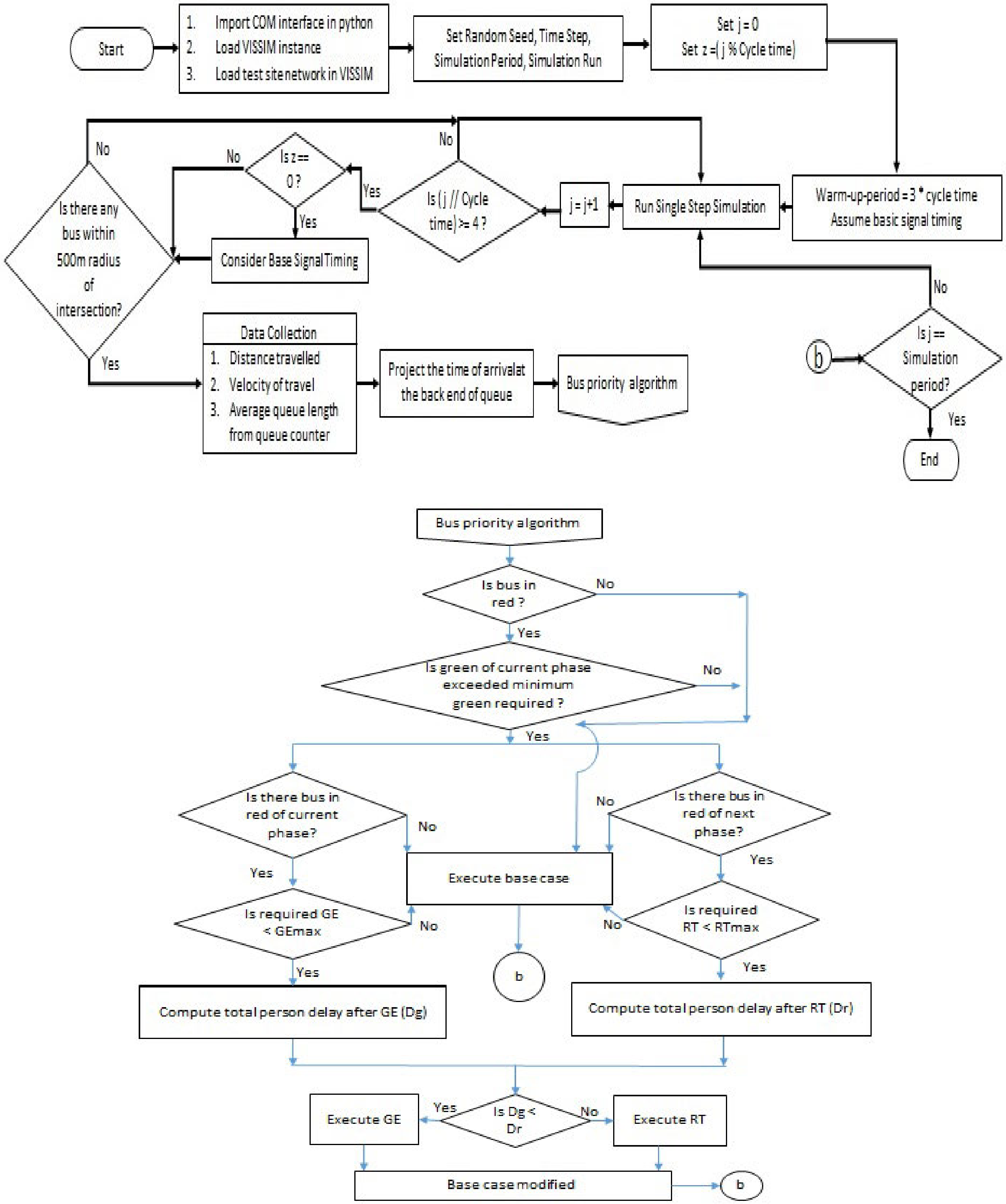


FIGURE 4. **Implementation flowchart for bus priority.**

majority of the studies on BSP used micro-simulation models. Several studies [18], [22], [25], [2] and [24] used VISSIM micro-simulator, mostly in integration with an optimization solver through Component object model (COM) interface or by using vehicle actuated programming (VAP) application. Advanced Interactive Microscopic Simulator for Urban and non-urban Networks (AIMSUN) [23] and CORridor SIMulation (CORSIM) micro-simulator [34] are also used. The reason for the wide usage of VISSIM is due to its COM interface possibility, which allows users to develop and implement their applications on the VISSIM network using a computer programming language such as C++, Visual Basic, Matlab or Python. Based on this, the present study also uses a VISSIM environment with COM interface to the Python programs for evaluating the performance of the developed models.

The test site selected for the implementation of the proposed strategies is the Sholinganallur intersection located on the IT corridor in Chennai. It is an important junction in the southern part of Chennai and is well connected with the city with public transport buses. The intersection consists of Karapakkam in the North, East Coast Road in the West, Perumbakkam in the East and Semmancheri in the South. The North-South road is a six-lane divided highway and the East-West road is a four-lane divided carriageway. The North-South corridor is a major bus route with headway ranging from 1-3 minutes. The intersection developed in VISSIM is shown in Figure 2.

The inputs required for simulating public transport bus system in VISSIM were bus route information, bus passenger occupancy and dwell time distribution at the bus stop. Information on various bus routes passing through the test site was taken from Metropolitan Transport Corporation (MTC), Chennai website (MTC, Chennai Ltd.). The bus passenger capacity was taken as 70 passengers. The base signal timings and the phase plan used for the intersection are given below.

Four Phase Signal

Cycle time: 200 sec

Lost time: 4 sec/phase

The simulated model was calibrated by changing the default values of minimum look ahead and look back distance and desired acceleration of bike and HMV at 0 km/h [35]. Validation error of 7.79% was reported using these values for 1 hour simulation of an intersection in the same corridor of the test intersection. The changed parameter values are shown in Figure 3.

One of the main data to be collected from the simulation in this study is the equivalent data that can be collected using DSRC devices that will make V2I communication possible for bus arrival information. To simulate the condition similar to DSRC, the buses were continuously tracked in VISSIM from a distance of 500m. From this data, the arrival time of the bus at the back of the queue is projected from a distance of 500m. The projected time will also be updated every second using COM-Interface programming. The data on the number of vehicles in the residual queue was also collected at the beginning of every cycle. The bus priority models were implemented in the Python programming language. The arrival time of bus in each approach, taken from VISSIM is an input to the python program and the output of the python program, which is the priority signal timings, is fed back to VISSIM, using the COM interface.

TABLE 4. **Simulation parameters considered for evaluation of the four-phase signal system.**

Simulation Period	4000 seconds (20 signal cycles)
Simulation Resolution	10 Time steps/Simulation Second
Random Seed	42
Bus Occupancy	70 (assumed same in all approaches)
Other Vehicles Occupancy	2 (average)
Base signal Timing	$g_1 = 55$, $g_2 = 48$, $g_3 = 52$ and $g_4 = 45$ seconds.

TABLE 5. **Priority strategies executed in each signal cycle in the four-phase signal.**

Cycle No. (1)	Criteria 1 (2)	Criteria 2 (3)	Criteria 3 (4)	G1 (5)	G2 (6)	G3 (7)	G4 (8)
3	GE1	GE2	NO	56	53	52	39
4	GE1	GE2	NO	56	51	51	42
5	GE1	GE2	NO	63	51	47	39
6	NO	GE2	NO	55	60	46	39
7	GE1	GE2	NO	56	53	49	42
8	GE1	RT3	NO	60	42	52	46
9	GE1	GE2	NO	56	49	51	44
10	GE1	GE2	NO	56	55	48	41
11	GE1	RT3	NO	56	41	55	48
12	RT2	RT3	NO	48	42	59	51
13	GE1	RT3	RT4	56	41	50	53
14	GE1	RT3	NO	56	42	55	47
15	RT2	NO	GE3	48	50	62	40
17	RT2	GE2	RT4	48	60	46	46
18	NO	NO	GE3	55	48	55	42
19	GE1	NO	NO	62	46	49	43
20	RT2	GE2	NO	48	59	50	43

An implementation flowchart of the proposed bus priority system is shown in Figure 4. The bus priority flowchart is executed only when the minimum green is achieved in respective active phase. Also note that the base signal timings will be executed, if no bus arrives to give priority.

When a cycle starts, the simulation runs normally until the minimum green in approach 1 has reached. When it is achieved, the bus priority program starts searching for buses in approach 1 and approach 2 and the possibility of GE1 or RT2. The arrival time of buses at the intersection was projected at every simulation second and the computed priority signal timings were updated accordingly. When green starts in the second approach, the simulation continues normally for the period of minimum green in the second approach. When it is reached, the bus priority program starts checking for buses in approach 2 and approach 3 and the possibility of GE2 and RT3. Similarly, after the minimum green in the third approach, the possibility of GE3 and RT4 were checked. If both the priority strategies were found possible at a time,

TABLE 6. The percentage reduction in delay after priority execution in the four-phase signal.

Cycle No. (1)	Total Person Delay Reduction in Percentage **(%)**										
	At Intersection (2)	Bus Delay						Other Vehicles Delay			
		Approach 1 (3)	Approach 2		Approach 3		Approach 4 (8)	Approach 1 (9)	Approach 2 (10)	Approach 3 (11)	Approach 4 (12)
			1st Red (4)	2nd Red (5)	1st Red (6)	2nd Red (7)					
3	42.83	100.00	-6.02	100.00	-62.43	-1.72	-18.17	1.37	2.62	-6.45	-7.89
4	36.37	100.00	-2.51	100.00	-4.94	-8.86	-4.63	1.37	1.44	-4.60	-3.91
5	24.62	100.00	-14.46	100.00	-17.13	-17.05	-43.35	10.73	-3.59	-13.71	-7.89
6	26.84	0.00	0.00	100.00	-14.19	-34.71	-10.66	0.00	8.19	-15.33	-7.89
7	30.95	100.00	-27.77	100.00	-9.69	-19.39	-28.43	1.37	2.94	-7.46	-3.91
8	16.70	100.00	-9.07	-21.04	33.40	16.91	8.79	6.78	-7.39	1.02	1.29
9	31.25	100.00	-22.55	100.00	-1.31	-5.32	-16.11	1.37	0.01	-2.39	-1.29
10	34.45	100.00	-20.18	100.00	-12.01	-0.68	-45.65	1.37	3.94	-9.89	-5.23
11	24.15	100.00	-3.95	-1.30	9.76	1.33	5.32	0.00	-4.41	7.00	3.83
12	18.22	-7.77	19.99	-1.14	23.79	5.80	18.83	-9.89	-0.03	14.62	7.59
13	22.25	100.00	-4.19	-6.36	20.11	-3.42	100.00	6.78	-3.86	5.12	10.06
14	23.98	100.00	-16.65	-8.37	16.64	1.71	-4.41	1.37	-4.66	5.86	2.56
15	26.63	-6.52	25.47	3.84	0.00	100.00	-24.63	-9.89	5.65	8.10	-6.56
17	27.51	-4.08	31.49	100.00	0.00	-8.83	100.00	-4.18	1.85	-20.06	6.35
18	33.96	0.00	0.00	0.00	0.00	100.00	-18.33	0.00	0.00	4.58	-3.91
19	22.39	100.00	0.00	-4.50	0.00	-4.29	0.00	9.42	-6.12	-6029	-2.60
20	29.60	-6.90	36.87	100.00	-2.04	-4.31	-4.56	-9.89	11.72	-4.87	-2.60

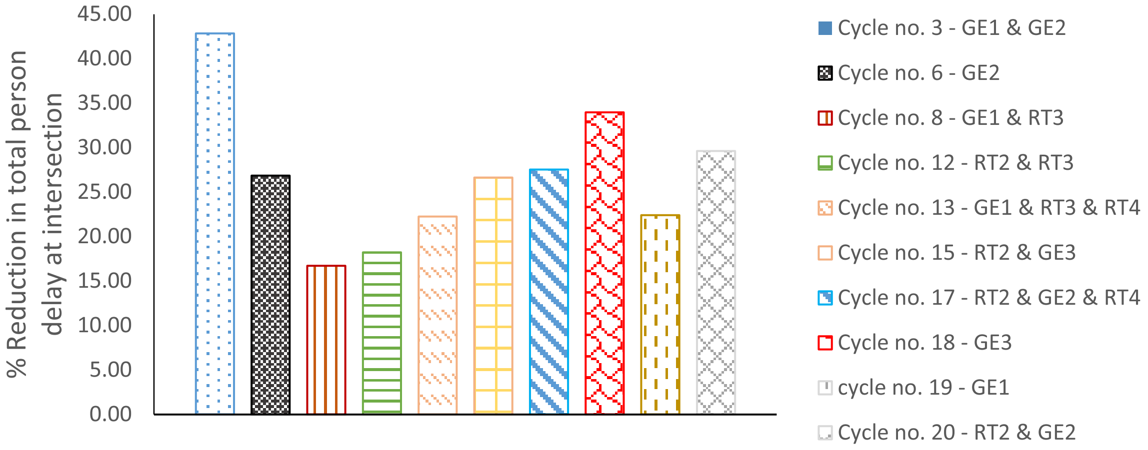


FIGURE 5. Effect of different priority strategies executed.

the priority strategy that causes the least total person delay at the intersection is selected.

The impact of the proposed signal priority strategies on both prioritized and non-prioritized vehicles needs to be assessed separately to evaluate its performance at intersection and approach levels. The performance measure used in this study is percentage reduction in the person delay, which is quantified as:

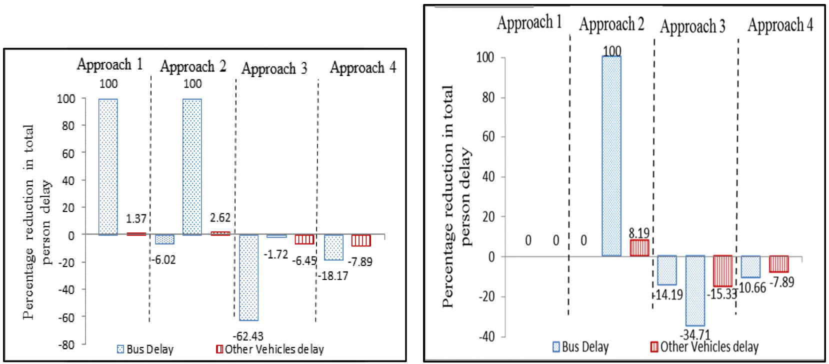
$$\begin{aligned}&\textit{Percentage change in the delay}\\&\quad=\frac{\textit{Delay after priority} - \textit{Delay before priority}}{\textit{Delay before priority}}\times 100.\end{aligned} \tag{96}$$

IV. SIMULATION TEST RESULTS

The simulation parameters and other data used for simulation are given in Table 4.

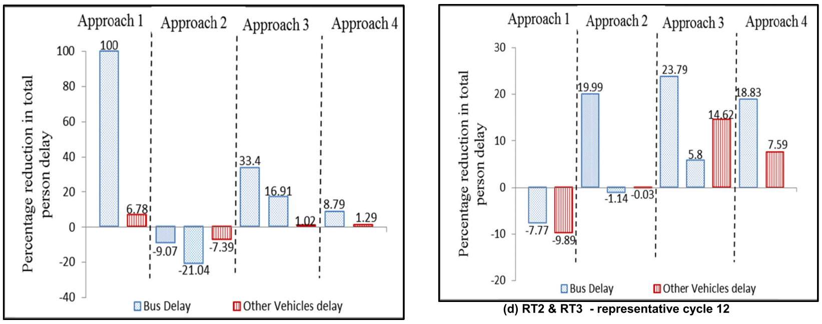
The first two signal cycles were used to warm up. Priority strategies were implemented from the 3rd cycle. The priority strategies implemented in each cycle and the corresponding signal timings are given in Table 5.

From the above table, it can be noted that priority to buses was given in all cycles except the 16th cycle. However, priority given to the 4th phase buses were only twice in 18 cycles. This is because of the implementation process, where the buses in the 4th phase are considered last in the cycle and the chances of them getting priority are the least. This may be



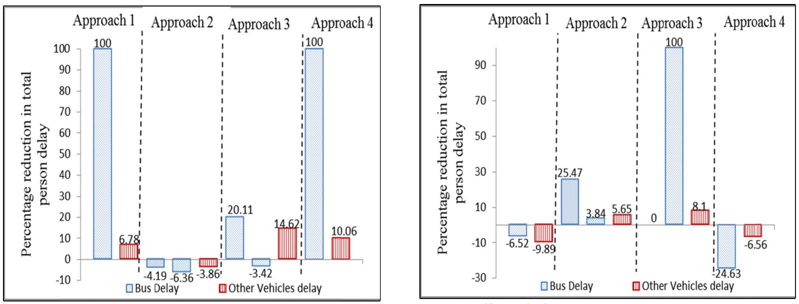
(a) GE1 & GE2 – representative cycle 3

(b) GE2 – representative cycle 6



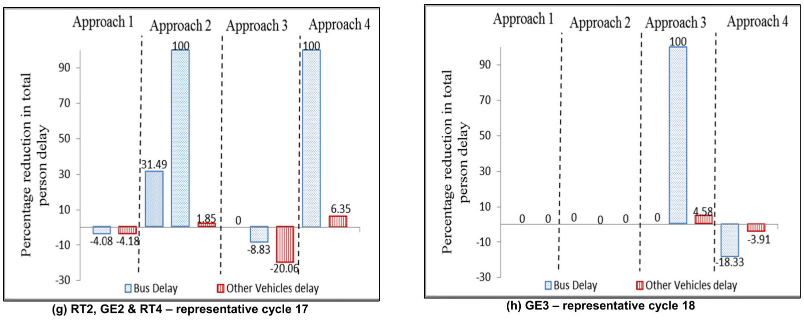
(c) GE1 & RT3 – representative cycle 8

(d) RT2 & RT3 - representative cycle 12



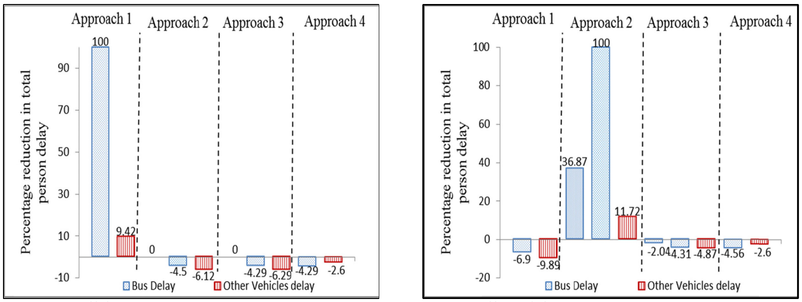
(e) GE1, RT3 and RT4 – representative cycle 13

(f) RT2 & GE3 – representative cycle 15



(g) RT2, GE2 & RT4 – representative cycle 17

(h) GE3 – representative cycle 18



(i) GE1 – representative cycle 19

(j) RT2 & GE2 – representative cycle 20

FIGURE 6. Effect of priority strategies in each approach.

meaningful when there are some major approaches and some minor approaches. Here in our example, 4th phase is considered as a minor approach. The percentage reduction in delay for both buses and other vehicles after priority execution in each cycle is given in Table 6.

The second column shows the percentage reduction in priority at the intersection level in each cycle.

Sample cycles for each scenario were selected and the relative variation in the benefit of each scenario at the intersection is shown in Figure 5.

From the figure, it can be noted that the highest reduction (42.8%) is observed when GE1 and GE2 were executed in the 3rd cycle. The least percentage reduction (16.7%) is spotted in the 8th cycle where GE1 and RT3 were performed. The percentage reduction is not too high as expected when priority strategies were implemented thrice in a cycle. This can be due to the influence of Red Truncation (RT2 & RT3) where 100% priority may not be given to buses.

Columns 3 to 8 of Table 6 shows the percentage reduction in person delay for buses and columns 9 to 12 shows that for other vehicles in each approach. In order to analyze the impact of priority strategies at approach level, each combination of strategies obtained can be considered separately. Representative cycles with each priority strategies are selected from Table 6 and are shown in Figure 6.

From Figure 6, it can be noted that all the vehicles in the prioritized approach experience reduced total person delay, when compared to no priority condition. Overall, it can be seen that almost all the cycles were able to provide signal priority with reductions up to 42% in the intersection delay.

V. SUMMARY AND CONCLUSION

This study proposed analytical models for dynamic thresholds for Green Extension and Red Truncation bus priority strategies for an under-saturated four-phase signal system. These thresholds were derived as functions of traffic flow and transit variables in each approach, base signal timings, other vehicles' average occupancy, and bus passenger occupancies. Following findings were made from the present study:

i. Priority strategies, implemented in feasible cycles, reduced total person delay at the intersection compared to the base case.
ii. Majority of the cycles were found to be feasible in under-saturated conditions providing several bus priority alternatives to reduce total person delay.

Overall, a methodology to determine dynamic thresholds for green extension and red truncation for a four-phase signal was developed. The methodology considered buses that arrive in the current cycle as well as the buses that did not get priority in the previous cycle while ensuring total person delay reduction during a cycle. The closed-form expressions for GE and RT thresholds derived in this paper are simpler and support bus priority implementation with minimal computational infrastructure, computational time, and implementation/O&M cost for large scale field implementation of higher phase signals.

The proposed work can be extended to oversaturated intersection and then to an arterial with signal co-ordination. Another potential future direction is to consider the movement of buses beyond the stop bar, i.e. within the intersection and ensuring that the buses safely cross the intersection as expected [36]. The thresholds and models can also be extended for a simultaneous implementation along with a bus prediction algorithm.

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