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Optimization of Economic Efficiency in Distribution Grids Using Distribution Locational Marginal Pricing

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ABSTRACT Distribution locational marginal pricing (DLMP) is an increasingly popular pricing signal that can be used to incentivize grid-friendly behavior of distributed energy resources (DER) to optimize economic efficiency in distribution grids. In this paper, a lossy direct-current optimal power flow (DCOPF) is utilized to obtain the iterative calculation framework for DLMPs. The two-stage algorithm iterates between the transmission system optimal power flow (OPF) and the distribution system OPF until no significant changes in DLMPs are observed. Real power losses are estimated using a static piecewise linear approximation technique. A sampling algorithm is proposed to minimize the possible convergence issues associated with the proposed mathematical model. DLMPs are calculated for the *i)* contemporary, *ii)* enhanced, and *iii)* meshed distribution grids using an IEEE 34-bus test feeder. The test transmission system is modeled using an IEEE 30-bus system. Finally, the calculated DLMPs in the enhanced grid are compared against three existing pricing mechanisms via three case studies to prove its validity and superiority, especially in congested systems with high penetration of price-responsive loads (PRLs).

INDEX TERMS Distribution locational marginal pricing (DLMP), distributed energy resources (DER), price-responsive loads (PRL), enhanced distribution grid, optimal power flow (OPF).

I. INTRODUCTION

Green-energy policies and innovative technological advancements have collectively led to a rapid growth of interest for grid-integration of distributed energy resources (DER) over the last decade. As a result of these advancements, the structure and the control procedures of the future distribution grid is predicted to change. It is expected that closer interactions between transmission and distribution networks will be required and concepts tailored for transmission systems will be extended to distribution networks. Development of smart grids and the increase in penetration levels of DER such as battery energy storage systems (BESSs) and price-responsive loads (PRLs), bring about new challenges to the centralized distribution infrastructure and distribution system operators (DSOs). For example, the future distribution grid is likely to experience significantly higher levels of congestion at various unpredicted locations, due to the higher number of participants and the thermal limits of the generators [1].

Although the decentralized power generation can offer a range of benefits to the electricity grid such as reduced network losses and lower transmission costs, ill-management of DER will cause sharp voltage fluctuations and supply-demand imbalances within the system [2]. To that end, efficient coordination and integration of DER could help to manage system reliability and security issues while also improving the market efficiency [3]. Generating appropriate pricing signals is crucial for efficient DER management, to ensure efficient congestion management and efficacy in overall grid operation [4].

Locational marginal pricing (LMP) is a security-constrained and fair pricing mechanism defined as the incremental cost of serving an infinitesimal change of load at a specific node, while respecting all physical constraints in a transmission system. In fact, the "transmission price" between two nodes in the network is defined as the difference

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in LMPs between the two nodes [5]. It is beneficial to all market participants because of its ability to manage congestion effectively in the transmission system [6], [7]. Recently, LMPs are modified to incorporate factors such as harmonic injection and environmental pollution into the transmission price [8]. However, employing LMPs in the distribution systems is not straightforward due to the substantial differences of the two systems [9].

The current pricing mechanisms used in the distribution networks are inadequate to communicate accurate pricing signals to all participants within the grid, in the presence of high penetration of DER. Furthermore, these methods cannot be consistently applied to different configurations of the distribution grid, ignore vital elements of the system and make impractical model assumptions in derivation of the nodal prices. This paper aims to investigate an extension of LMP concept to various types of distribution systems in order to encourage grid-friendly behavior from DER to benefit system operations at both the distribution and transmission levels. The main contributions of the research are listed below:

- We develop an iterative framework to calculate distribution locational marginal pricing (DLMP) based on solving a lossy direct-current optimal power flow (DCOPF) problem.
- Real power losses are incorporated using a piecewise linear technique and proposal of a sampling algorithm for potential convergence issues that could occur in the calculation framework due to the non-continuity of load bids.
- We compare the calculated DLMP with three existing pricing mechanisms; fixed-rate pricing (FR), time-of-use pricing (ToU) and real-time pricing (RTP), to prove its superiority over these methods in optimizing economic efficiency. In particular, the optimization of aggregate load consumption by the DLMP scheme in the presence of network congestion.

The rest of this paper is organized as follows. Section II provides a literature research on DLMP, to provide the readers with an overview of the existing works, with emphasis on various employed mathematical techniques. Section III provides a description of the methodology applied in solving the lossy-DCOPF problem to obtain the DLMP formulation. This section also highlights the possibility of convergence issues and proposes the solution to minimize the drawback. Section IV presents the numerical illustrations of DLMP prices obtained for three different types of distribution grid. Section V is devoted to the comparison of DLMP pricing with three other widely used pricing mechanisms in the contemporary distribution system. Finally, Section VI highlights the main conclusions derived from the study and provides recommendations on possible future work associated with this research.

II. LITERATURE SURVEY

The DLMP is the cost to optimally deliver an increment of energy to a specific node in a distribution system, while adhering to all system operational constraints. Similar to the LMP, DLMP can be decomposed into three parts as marginal energy cost, marginal loss cost and marginal congestion cost [10]. By proper application and calculation of nodal pricing technique, DER and other involved loads are able to schedule their assets in an efficient and grid-friendly manner [11]. The main advantages of accurately computed DLMPs include:

- It is economically efficient since it reflects the market-clearing price at which market surplus is maximized, i.e., the DLMP will prevent under/overconsumption in a power system.
- Unlike LMP, DLMP is efficient in the long run because it acts as a guide for necessary nodal upgrades and optimal allocation of distributed generators.
- Representation of the effect of marginal load cost at a node, based on the system conditions.
- Minimization of cross-subsidies between customers.
- Increased system reliability due to load-matching capability of DLMPs.
- Significant reduction in operational costs of the entire power system.

Two main challenges in implementing DLMPs are identified in [12]; *(i) Computational complexity:* the DCOPF assumptions are often invalid in distribution networks with high resistance to reactance (R/X) ratio, while the ACOPF in distribution grids with a large number of nodes might not be computationally feasible. *(ii) Network information privacy and accessibility:* correct calculation of DLMPs requires all the information regarding the transmission, distribution and local networks.

The DLMPs are necessary in two-sided markets created on the basis of fundamental economic theories [13], [14]. They allow modifications on both the supply and demand side of the market, unlike most other existing pricing mechanisms. Hence, DLMP formulation has drawn worldwide interest to be a potential asset that could be used to incentivize DER to behave in a beneficial manner for the distribution network [5], [15], [16]. A common approach used by most existing works, for calculation of DLMPs can be summarized in a 3-step process, without the loss of generality [17]:

- Transmission LMPs (TLMPs) are determined by clearing transmission markets.
- This information is passed to the DSOs.
- DSOs clear local markets in the low-voltage grid to determine DLMPs at the individual distribution node level.

A number of mathematical definitions for the DLMP are reviewed in [18], where the advantages and shortcomings of the formulations are also provided. Most works including [19]–[21] use the contemporary Lagrangian multipliers of the optimal power flow problem. A novel linearized power flow approach was proposed in [22] for derivation of marginal energy and loss components of the DLMP, without considering the congestion component. However, the trust-region based solution methodology built around a first-order

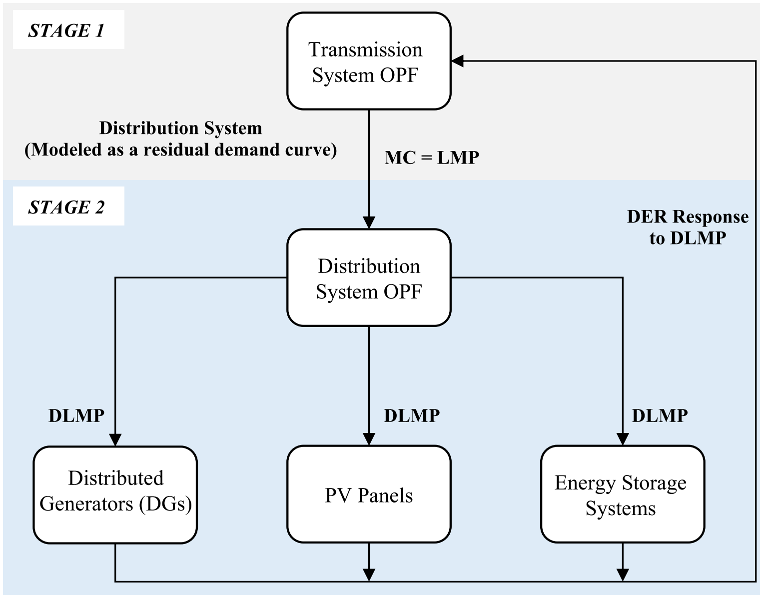

FIGURE 1. **Two-stage process for DLMP calculation.**

approximation of the ACOPF in [23], showed better performance than [22].

Both [24] and [25] used DLMPs for analyzing a transactive day-ahead market model while emphasizing on the impact of variable renewable energy generation uncertainty and DSO interaction for demand response, respectively. DLMPs were calculated based on a novel loss reduction allocation policy in [26] while [27] extended this work by incorporating emissions of the network. Distributed generators (DGs) with lower losses were remunerated utilizing game theory in the former and nucleolus theory in the latter. In [28], Benders' cuts were used to simulate generalized bid functions and a convexified ACOPF problem was solved for computing DLMPs. Similar to [28], [29] used a modified Benders decomposition as the foundation concept to minimize dispatch complexity, when a large number of DER were involved.

Studies on multi-phase DLMPs have recently gained popularity although the extension of DLMPs from single-phase to three-phase is not straightforward. The importance of three-phase schemes was emphasized in [30], in which it is suggested that DLMPs could be useful in power balancing across phases. In [31], a convexified OPF problem was solved to derive DLMPs for a three-phase distribution network. Although [31]–[33] calculated three-phase DLMPs, a linearized distribution grid model was used. The transactive energy trading framework presented in [34] was specified such that all phases at a specific node have equal DLMPs. It should be noted that all these works assumed balanced three-phase loads and ignored market state or conditions.

Currently, DLMPs are explored in a wide range of applications [35]. Authors of [9], [36], [37] showed different methods by which system congestion, caused by electric vehicles charging, can be minimized (or even eliminated) when DLMPs are applied, while [38] analyzed how electric vehicles charging may be honed using DLMP application. In [32], a linearized ACOPF model was used to prove how significant cost reductions can be achieved when electricity trading is based on DLMPs. In [39], DLMPs were applied using a Genetic algorithm to solve the distribution network configuration problem.

However, all of the prior works have primarily focused on a one-shot approach. Moreover, the aforementioned works do not examine the true price responsive behavior of flexible loads and distributed resources on the distribution system. Hence, they are unable to unlock the potential values that can be provided by the optimal coupling between the transmission and distribution systems through the extension of the LMP to the distribution system. To this end, there is a need for an integrated framework for the transmission and distribution systems that provides a closed loop solution with due consideration to conventional demand elasticity in both systems. In this regard, this paper proposes an iterative approach to integrate the transmission and distribution systems together such that efficient scheduling of the resources are incentivized across the entire grid structure.

III. METHODOLOGY

A. SYNOPSIS

Ideally, the OPF problem should analyze the transmission and distribution systems in a single integrated model, as proposed in [33]. However, the overall advantages of such an analysis is overwhelmed by the computational complexity of the bi-system integration. Hence, a two-stage process is used for DLMP calculation, as shown in Figure 1. The first stage of the optimization considers the transmission system OPF in which the distribution system is modeled by its total residual demand curve. This curve, which is derived in a similar method as the one used in [40], represents the change in demand of the distribution system with respect to LMP changes in the transmission system. This ensures that the features of the DER are incorporated in the distribution system. The solution of the first stage is the equilibrium LMP and the cleared quantity at which the transmission system could operate with maximum economic efficiency. The operating point is achieved such that the total cost to supply the demand in the distribution system is minimized.

The second stage comprises of the distribution system OPF in which the transmission system is represented as a residual supply curve [41]. This curve represents the changes in price of the transmission system with respect to changes in demand of the distribution system. The proposed system solves the transmission system OPF multiple times, unlike 'one-shot' approaches. Furthermore, one-shot approaches cannot encapsulate the interactions between the two systems adequately which leads to deviations of DER in the distribution network from the representation used in the transmission system problem.

The separation of the process into two stages makes the system vulnerable to model mismatch of the distribution system in the transmission system and vice-versa. For example, it is extremely difficult to predict the aggregate demand for an enhanced distribution system. Single-shot approaches could cause deviations of DER in the distribution network from the representation employed in the transmission system problem. Consequently, the two-stage paradigm would not be able to produce the required results for both systems.

To overcome the mismatch problem stated above, the calculation framework can iterate between the two stages until no changes in DLMPs or the aggregate load (in the distribution system) are observed. Iterations between the stages are analyzed with respect to the whole system. At each iteration, the newest update from the other optimization process will be used as an input to the problem that is currently solved. The iterative framework is able to optimally couple the two stages of optimization, such that the distribution system network conditions can be accurately modeled in the transmission system OPF and the transmission system network conditions act as a control signal for the DER. Optimal coupling of the two systems provide sufficient opportunities for utilizing the distribution resources for ancillary services [34]. Although [12], [35], [38] attempt to divide the OPF problem, no iteration between the two problems is used to attain an optimal solution.

DCOPF formulation is used to solve the transmission system problem as well as the distribution system problem. However, the DCOPF formulation is based on a set of assumptions [36] which are generally invalid for the distribution system because of the higher proportion of energy losses, arising from the higher *R/X* ratio in the distribution system. To overcome this issue, a DCOPF model that incorporates real power losses (lossy DCOPF) is used in this paper. Table 1 provides a summary of the notations used in this paper.

TABLE 1. Notation definition.

Symbol	Description
$t, \mathcal{T}$	Index/set of time
$d, \mathcal{D}$	Index/set of loads
$g, \mathcal{G}$	Index/set of generators
$i, \mathcal{I}$	Index/set of line segments
ϵ	Price elasticity
η_{DG}	Efficiency of DG
η_{PRL}	Load efficiency of PRL
bp_d	Price of load bid d
P, P_0	Observed/reference price
Q, Q_0	Observed/reference quantity
P_{PRL}	Load output of PRL
g_{PRL}	Cost to supply PRL
B_x	Susceptance in line x
G_x	Conductance in line x
C_g	Operating cost of generator g
D_n	Real power demand at bus n
G_n	Generator set at bus n
P_g	Real power output of generator g
P_x	Real power flow in line x
P_n^{Loss}	Real losses on all lines connected to bus n
P_n^R	Real power fed in at bus n and drawn out at bus R
P_{DG}	Real power output of DG unit
$\underline{P}_g, \overline{P}_g$	Min/Max real power output capacity of generator g
$\underline{P}_x, \overline{P}_x$	Min/Max power limit of line x
$\underline{P}_{DG}, \overline{P}_{DG}$	Min/Max real power output capacity of DG
$\underline{\theta}_{nm}, \overline{\theta}_{nm}$	Min/Max voltage angle difference between bus n and m
$\overline{\phi}_h^+$	Max length of line segment h
V_m, V_n	Voltage of sending bus m, receiving bus n
PDF_x	Power transfer Distribution factor
β_j	Slope of segment j
ϕ_h^+, ϕ_h^-	Length of positive/negative orthant line segment h
θ_m, θ_n	Voltage angle of sending bus m, receiving bus n
θ_{nm}	Voltage angle difference between bus n and m

B. DCOPF FOR CALCULATING DLMPs

The traditional DCOPF does not consider system losses and does not produce accurate solutions when real power losses are considered. Real power losses need to be incorporated for distribution system applications because the distribution system circuits have high resistances and operate at low voltage levels. As a result, a DCOPF with losses formulation (lossy-DCOPF) is developed for calculating DLMPs. A similar model is used in [42] for optimizing the allocation of energy storage systems in an enhanced power grid. The lossy-DCOPF is superior to its unmodified counterpart because of three reasons; *(i)* loss approximation is internal to the formulation [37], *(ii)* the formulation does not require a slack bus [43], *(iii)* calculated DLMPs are suitable for usage in multiple market frequencies.

1) LOSSLESS DCOPF

The DCOPF is the linear approximation of the complicated ACOPF problem. The assumptions stated in [36] are used to develop the DCOPF. By principle, all calculations involving real power losses and reactive power are ignored in the DCOPF formulation shown below:

$$\min_{P_g} \sum_g C_g P_g \tag{1}$$

subject to.

$$\sum_{\forall x(n,;)} P_x - \sum_{\forall x(;,n)} P_x - D_n - \sum_{g \in G_n} P_g = 0 \quad \forall n \tag{2}$$

$$B_x(\theta_n - \theta_m) - P_x = 0 \quad \forall x \tag{3}$$

$$\underline{P}_x \leq P_x \leq \overline{P}_x \quad \forall x \tag{4}$$

$$\underline{\theta}_{nm} \leq \theta_n - \theta_m \leq \overline{\theta}_{nm} \quad \forall n, m \tag{5}$$

$$\underline{P}_g \leq P_g \leq \overline{P}_g \quad \forall g \tag{6}$$

The objective function in (1) aims to minimize operation costs subject to constraints of node balance (2), line flow (3), real power branch flow limit (4), transient stability limit (5), and generator real power output limit (6). The DCOPF formulation shown above can be further approximated using power transfer distribution factors (PDFs). PDFs describe the fraction of real power fed in at each bus n and drawn out at a reference bus R, on line x [44].

$$\min_{P_g} \sum_g C_g P_g \tag{7}$$

subject to.

$$P_n^R - D_n - \sum_{g \in \mathcal{G}} P_g = 0 \quad \forall n \tag{8}$$

$$\sum_{n \in \mathcal{N}} P_n^R = 0 \tag{9}$$

$$\underline{P}_x \leq \sum_{n \in \mathcal{N}} PDF_{x,n}^R \leq \overline{P}_x \quad \forall x \tag{10}$$

$$\underline{P}_g \leq P_g \leq \overline{P}_g \quad \forall g \tag{11}$$

LMPs are calculated as the dual variables of the node balance constraint in the standard DCOPF problem. However, LMPs obtained from above formulations does not capture the marginal loss component, since it is a lossless model.

2) LOSSY DCOPF

The lossy DCOPF formulation is developed by modifying the standard DCOPF model such that it includes linear real-power losses. These losses and its effect on the price solutions are internally approximated by the DCOPF formulation. Linearization of losses is performed via a piecewise linear technique [41], so that the linear properties of the standard DCOPF are retained. Linear approximation of the real-power losses is required for the lossy DCOPF formulation. The approximation that bus voltages V_m and V_n are 1 p.u is applied to (12), for the derivation of (13) so that the latter is linearized over $\theta_n - \theta_m$, the bus angle difference across line x.

$$P_n^{loss} = G_x(|V_m^2| + |V_n^2| - 2|V_m||V_n| \times \cos(\theta_n - \theta_m)) \tag{12}$$

$$P_n^{loss} = 2\, G_x(1 - \cos(\theta_n - \theta_m)) \tag{13}$$

$$\theta_n - \theta_m = \sum_{i \in \mathcal{I}} (\phi_i^+ - \phi_i^-) \tag{14}$$

$$0 \leq \phi_i^+ \leq \overline{\phi}_i \quad \forall i \tag{15}$$

$$0 \leq \phi_i^- \leq \overline{\phi}_i \quad \forall i \tag{16}$$

$$1 - \cos(\theta_n - \theta_m) = \sum_{i \in \mathcal{I}} \beta_i (\phi_i^+ + \phi_i^-) \tag{17}$$

$$P_n^{loss} = \sum_{\forall x(n,;)} 2G_x(\sum_{i \in \mathcal{I}} \beta_i \phi_i^-) + \sum_{\forall x(;,n)} 2G_x(\sum_{i \in \mathcal{I}} \beta_i \phi_i^+) \tag{18}$$

Eq. (12) denotes AC power losses, while (13) expresses the linear power losses. Approximation for bus angle difference using curve segment lengths is modeled using (14)-(16). (17) expresses the piecewise linear curve representation, and (18) denotes linear approximation of real power losses.

In the lossy DCOPF framework, the loss placement process depends on the sign of the bus angle difference across the line x. The sign of the bus angle difference is used to determine if the power flow is in the expected direction. A difference greater than zero implies that the power flow occurs in the expected direction and hence, losses are placed at the sending bus. The lossy DCOPF is developed using a linear programming technique [29].

$$\max_{P_g} \sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} bp_{d,t} D_{n,d,t} - \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} C_{g,t} P_{g,t} - \eta_{DG} \sum_{j \in \mathcal{J}} (P_{DG} - \overline{P}_{DG})^2 \tag{19}$$

$$\text{s.t. } B_x(\theta_n - \theta_m) - P_{x,t} = 0 \quad \forall t, x \tag{20}$$

$$\sum_{g \in \mathcal{G}} P_{g,t} + \sum_{\forall x(n,;)} P_{x,t} - \sum_{\forall x(;,n)} P_{x,t} - \sum_{\forall d \in \mathcal{D}} D_{n,d,t} - P_{n,t}^{loss} + \sum_{\forall j(0,R)} P_{DG_j} + \sum_{\forall k(0,NR)} P_{DG_k} = 0 \quad \forall t, n \tag{21}$$

$$P_n^{loss} - \sum_{\forall x(n,;)} 2G_x(\sum_{i \in \mathcal{I}} \beta_i \phi_{i,t}^-) - \sum_{\forall x(;,n)} 2G_x(\sum_{i \in \mathcal{I}} \beta_i \phi_{i,t}^+) = 0 \quad \forall t, n \tag{22}$$

$$\theta_{n,t} - \theta_{m,t} - \sum_{i \in \mathcal{I}} (\phi_{i,t}^{x+} - \phi_{i,t}^{x-}) = 0 \quad \forall t, x \tag{23}$$

$$0 \leq \phi_{i,t}^{x+} \leq \overline{\phi}_{i,t} \quad \forall i, x \tag{24}$$

$$0 \leq \phi_{i,t}^{x-} \leq \overline{\phi}_{i,t} \quad \forall i, x \tag{25}$$

$$\underline{P}_x \leq P_x \leq \overline{P}_x \quad \forall x \tag{26}$$

$$\underline{P}_g \leq P_g \leq \overline{P}_g \quad \forall g \tag{27}$$

$$\underline{P}_{DG} \leq P_{DG} \leq \overline{P}_{DG} \quad \forall j \tag{28}$$

The optimization problem in (19) aims to maximize the market surplus subject to constraints. DC approximation of the line flow is modeled using (20), while (21) and (22) denote the node balance, and variable load constraints, respectively.

Eq (14) is modified as (23) to model angle difference approximation, and (24) and (25) represent analogous line segment length restrictions in (15) and (16). Line capacity, generator output, and DG output are constrained as in (26), (27), and (28), respectively. Equation (23) is the most important equation in the formulation since it

- forms the bridge between the loss approximation (22) and the line flow equation approximation (20).
- enforces a direct proportionality between the magnitudes of the real power flows and approximated losses.
- ensures uni-directionality of the approximated real power losses and the real power flows.

C. CONVERGENCE ISSUES

The two-stage iterative framework proposed in [40] models the transmission system in the distribution OPF using an infinite generator. This approximation is inaccurate for measuring the effect of changes in distribution system demand in the transmission system. Furthermore, it leads to a perfectly elastic supply curve which causes uncertainty in convergence to a correct solution, when calculating the DLMP. This is because the perfect elasticity of the supply curve implies the cost of consumption is fixed in the transmission system, irrespective of the amount of consumption in the distribution network. This will lead to a range of market-clearing quantities in the distribution OPF. In such a scenario, the solver will randomly select the market clearing price (MCP). This problem is solved in this study using the residual supply curve.

Another potential reason for the divergence is the inflexibility of the decoupled system with respect to network congestions. For instance, congestion might cause the DLMP to increase even when the level of consumption decreases. This inverse relationship between the price and the consumption can also lead to convergence issues. In the proposed framework in [40], if the load sets the MCP in the distribution system problem, the approximation of the distribution system (with price-sensitive loads) using a perfectly inelastic demand curve in the transmission system OPF will be inaccurate. This representation implies that the electricity demand is unaffected by the electricity price, however the demand does depend on the proxy LMP at the interconnection point between the transmission and distribution systems. If the model solution differs from the clearing price, the proxy LMPs will fail to converge, leading to a sub-optimal, non-unique MCP. The problem is solved via a more accurate representation of the distribution system using a residual demand curve in the transmission system OPF.

In this paper, we employ a convenient sampling method to derive a starting point for the aggregate demand curve [45]. The method assumes that the conditions of the distribution network will be known in reality. The algorithm uses an artificial generator to obtain price-quantity combinations using the lossy DCOPF formulation. A new combination is obtained and updates the aggregate demand curve, at each iteration.

Algorithm 1 Sampling Algorithm to Overcome Possible Convergence Issues

Set Infinite generator marginal cost as marginal price
Solve lossy DCOPF for distribution system
Sample all prices
Derive aggregate distribution demand curve
Solve lossy DCOPF for transmission system
Solve lossy DCOPF for distribution system using final distribution load consumption

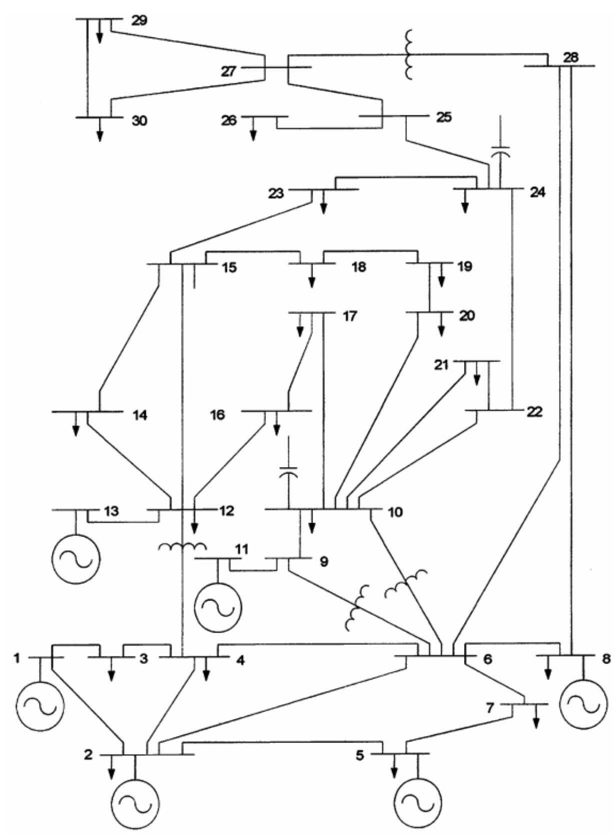

FIGURE 2. One-line diagram of IEEE 30-bus system [46].

This new combination is representative of the new solution from the distribution network, hence, the derivation of the aggregate demand curve from the algorithm is equivalent to modeling the network conditions. The process ends when adequate combinations are obtained to plot an initial estimate of the aggregate demand curve. Algorithm 1 illustrates the process for overcoming convergence issues. An identical sampling simulation technique is used for RTP, ToU and FR. However, the RTP only considers the transmission network as it cannot model the price-sensitive loads in the distribution system with adequate veracity.

IV. IMPLEMENTATION

A. TEST SYSTEM

The algorithm is implemented on a modified IEEE 30-bus system [46] using MATLAB 2019a and run on an i7 dual-core processor. The one-line diagram of the IEEE 30-bus

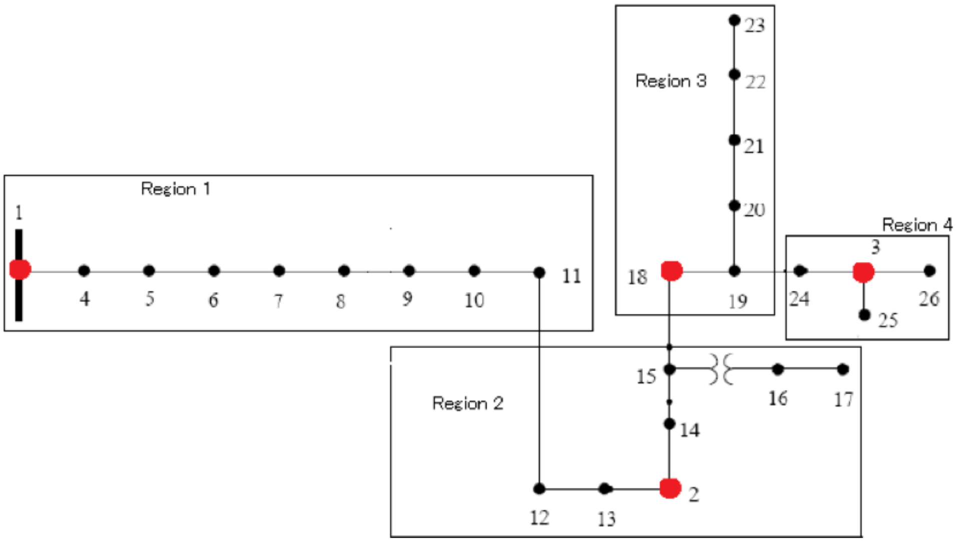

FIGURE 3. One-line diagram of the modified IEEE 34-bus system.

TABLE 2. Summary of distribution system feeders specifications.

Region/ Feeder	Bus location	kV level	Total load (kW)	Total length (km)	Number of (Nodes)
1	1	50	324	35.8	8
2	2	11	230	20.8	6
3	3	11	126	3.6	5
4	18	11	42	1.8	3
	Total		722	62	22

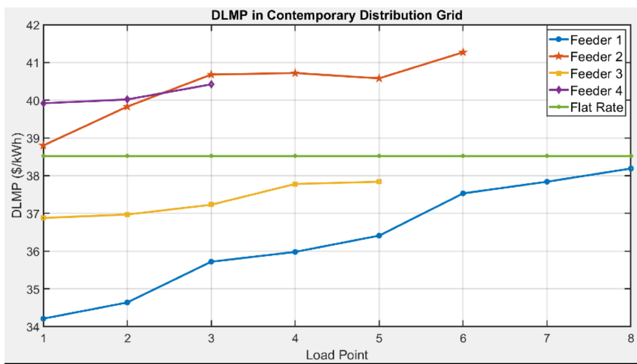

FIGURE 4. DLMPs in a traditional distribution grid.

test system which represents the test transmission system is shown in Figure 2. The test data for the system is extracted from MATPOWER 7.0 interior point solver [47], [48] and then modified to suit the purpose of the relevant study.

A modified IEEE 34-bus testbed is used to represent the test distribution system in this section. Figure 3 shows the one-line diagram of the modified 34-bus system [49]. The system has a peak load of 1.6 MVA distributed among four primary distribution between 22 load points. The distribution system includes four regions and each region has a four-wire multi-grounded feeder. The four-wire multi-grounded feeders are placed at bus 1,2, 3 and 18, represented by the red dots in Figure 3. Feeders placed at bus 2, 3 and 18 are operated at 11 kV while feeder at bus 1 is operated at 50 kV. Subsequently, the testbed is segregated into four regions, each region corresponding to a single feeder. Table 2 summarizes the individual feeder information in the distribution system.

B. DLMP IN CONTEMPORARY DISTRIBUTION GRID

The main objective of this case study is to identify trends in DLMPs with respect to distance from the feeder and creation of cross-subsidies due to average pricing mechanisms. The distribution system is radial and all loads are modeled to be perfectly inelastic. Difference in DLMPs at load points is solely due to real power losses since all four feeders are oversized to avoid congestion. Figure 4 shows the calculated DLMPs at each node for each feeder in the system. The load points are ordered such that the closest to the feeder gets the lowest number, for example, bus 4 in region 1 is the first load point of feeder 1.

Figure 4 highlights the effect of distance between the feeder and the load point on the amount of real power losses. If the load point is located close to a radial feeder, lower real power losses are incurred, leading to a lower nodal DLMP. The only exception to this trend is seen in feeder 2, where a slight reduction in DLMP is observed between the fourth and fifth load points of the second feeder.

The flat rate of 38.52 $ is calculated using the aggregate revenue requirement (for energy loss recovery) of the transmission system. Under FR pricing, the difference in individual contributions of each load to the system losses are not taken into account, which leads to some load points paying more than the actual cost for consumption of energy, while other load points pay less than the actual cost. This concept is economically defined as cross-subsidization.

Cross-subsidization has a broader effect and serious consequences than simple price discrimination in electricity markets.

However, it will not affect the prices sufficiently to cause economic inefficiencies in a contemporary distribution grid. In the following sections, it will become obvious how the adverse effects of these cross-subsidies will be problematic in an enhanced distribution grid, especially in the presence of congestion.

C. DLMP IN ENHANCED DISTRIBUTION GRID

This illustration is similar to the previous section, except for the introduction of PRLs to the system. The main difference between a normal load and a PRL is that the energy to supply the PRL will only be purchased if the load cost is low enough. In general, PRLs improve the efficiency and overall behavior in congested electricity markets, although it may not be able to remove the congestion completely. A total of five PRLs are introduced into the system at node 13, 17, 21, 23 and 26. These interruptible loads are represented as negative real power injections in the simulation with associated costs, modeled in Section V-B.

The main objective of this study is to exhibit the contribution of DLMP towards improving economic efficiency. It does so by incentivizing the flexible loads to behave in a way that benefits the whole system. The radial distribution grid is represented in the transmission system by aggregating demand bids at the PRL nodes. For simplicity, all price sensitive loads are assumed to have approximately 70% of their peak load demand to be essentially price inelastic and they have a constant bid value of 50 \$. The bids of loads points at nodes 13, 17, 21, 23 and 26 are considered to be 40.2, 42.2, 46.6, 41.8, 39.2 \$, respectively. The bid values are chosen based on the flexibility of the load, with higher flexibility resulting in higher bid value.

The DLMP is a precise pricing signal that ensures the PRLs behave in an economically optimal manner, hence the behavior optimized by the DLMP is assumed to be optimal for the overall system efficiency. The behavior incentivized by the DLMP at the PRLs is determined by comparing the bid value of the flexible part of the load to the DLMP. If the DLMP is lower than the bid value, the PRL consumes and vice versa. The DLMPs calculated at the load points for the whole system are plotted in Figure 5.

The fixed rate from the previous section is reproduced in Figure 5. For PRLs at node 17 and 26, the bid values lie in between the FR and the DLMP. This means that the flexible part of the load is used for consumption, contradicting the optimal behavior incentivized by the DLMP. Hence, the FR leads to sub-optimal behavioral patterns of the PRLs since it produces incorrect pricing signals for an enhanced distribution grid. This will worsen the cost of congestion in the system and defeat the purpose of having PRLs in the system, in the first place. Table 3 summarizes the behavior incentivized by both FR and DLMP at each load point.

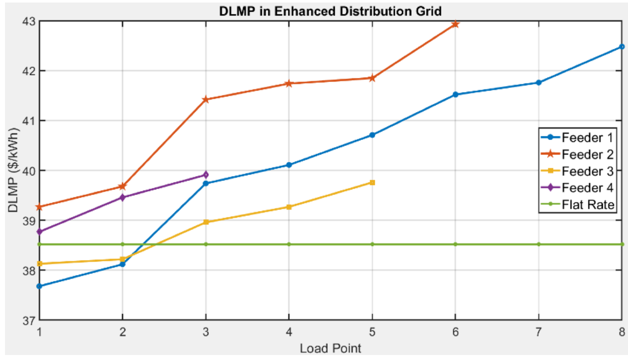

FIGURE 5. DLMPs in an enhanced distribution grid.

TABLE 3. Summary of PRL behavior under FR and DLMP.

Node No.	Behavior under FR	Behavior under DLMP
13	Consumption	No Consumption
17	Consumption	Consumption
21	No Consumption	No Consumption
23	Consumption	No Consumption
26	No Consumption	No Consumption

D. DLMP IN CONGESTED DISTRIBUTION GRID

In general, distribution grids have a 50% utilization radial configuration design, with normally open points in between feeders to allow connection with other feeders during outages or maintenance. The increasing penetration levels of DER in the distribution grid increases the possibility of congestion events, primarily because they are located at remote locations from the high demand regions. Such congestion events can have adverse effects on the grid stability and can cause blackouts in extreme situations. The study in [50] investigated the pressing need for congestion management in distribution grids.

The main aim of this case study is to analyze the ability of DLMPs to manage/alleviate congestion in the distribution system. Throughout this section, it is assumed that there is no limitation in the active and reactive power generation. This section builds on from the previous section to introduce congestion in a system segment by limiting the power flow from bus 8 to 9 in the distribution system. The PRLs from the previous section are removed and a 350 kW fuel cell with an active power-dependent marginal fuel cost of 12 \$/kWh is placed at LP6 of the second feeder. For simplicity, it is assumed that the efficiency of the DG unit is a constant 90%, irrespective of the nodal voltage. The DLMPs calculated for the radial distribution grid in the presence of congestion and in the absence of congestion are shown in Figure 6 and Figure 7, respectively. It must be noted that the only difference between the contemporary grid and the uncongested case is the extra generation by the DG unit.

The trend in DLMP from one load point to another is affected due to congestion, and the variation in DLMPs for each individual feeder is much higher. The range of DLMPs

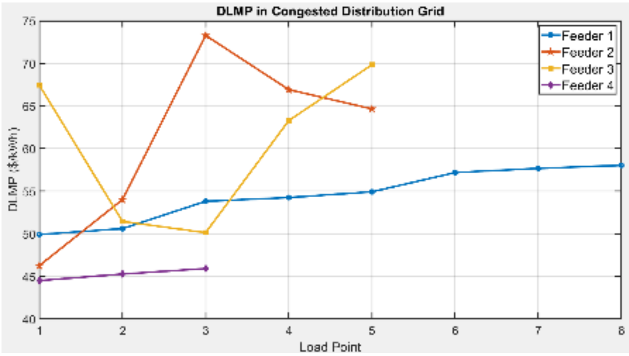

FIGURE 6. DLMPs in congested distribution grid.

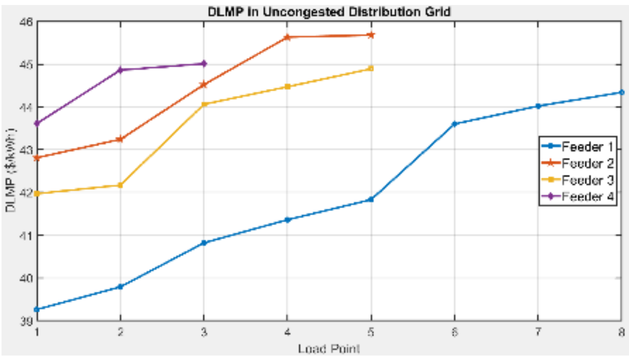

FIGURE 7. DLMPs in uncongested distribution grid.

in the congested system (44 to 74 $) is much higher than the range observed in the uncongested system (39 to 46 $). Another interesting observation is that the highest price occurs at the same load point for both the congested and uncongested systems, whereas the lowest price occurs at different locations for the two systems. For the congested system, the lowest price is observed at the first load point of the forth feeder, whereas the uncongested system has the lowest price at the first load point of the first feeder. Moreover, the second and third feeders were more affected by the application of congestion than the other feeders. All these changes observed in the system are due to the requirement of adhering to the system constraints.

The variation in DLMP due to congestion is an important indicator of the need for system updates. It also provides general insight into how well the DGs are placed in the system. The graphs suggest that another DG should have been placed at LP2 or LP3 of feeder 2 or LP1 or LP3 of feeder 3 to further reduce the impact of congestion on the system because these nodes have the highest variation in their prices in the congested system.

E. COMPUTATIONAL COMPLEXITY ANALYSIS

To investigate the computational complexity in implementing the proposed calculation framework, we compare the computational time and the number of iterations required for convergence in different systems. It should be noted that an unattained upper limit of 5000 iterations is used in all simulations. This indicates that the system has a reasonable convergence speed. Results in Table 4 show that the congested system has the highest computational complexity, while the contemporary distribution grid needs the lowest number of iterations and computation time for convergence. This trend is consistent throughout this analysis for all regions. Another comprehensible observation is that increasing the number of nodes in a region raises the required computational effort.

TABLE 4. Evaluating computational complexity.

Type of Distribution Grid	Region Number	Computation Time (ms)	No. of Iterations
Contemporary	1	228	790
	2	167	618
	3	152	549
	4	31	5
Enhanced	1	576	1214
	2	297	823
	3	217	895
	4	119	11
Congested	1	1067	3396
	2	753	1763
	3	681	1470
	4	311	19

V. COMPARISONS

A. TEST SYSTEM

In this section, a modified IEEE 30-bus system is considered in order to compare the DLMP with other types of pricing. The data for the system characteristics of the modified IEEE 30-bus system can be found in [51] and the modified generator values are taken from [47]. The distribution system used in the first two case studies are identical to the previous section with all PRLs placed at the same locations. All transmission loads are assumed to be perfectly inelastic. These loads have a 24-hour load profile as shown in Figure 8.

Calculation of other prices are based on the method provided in [52]. The FR is calculated as the weighted average

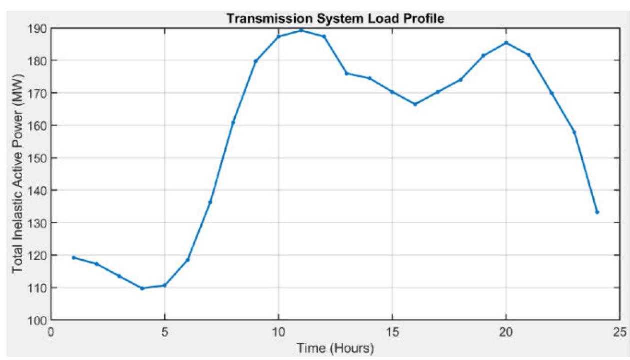

FIGURE 8. 24-hour load profile of the test transmission system.

of the total payment to the transmission system, including the cost of losses. ToU rates are based on a peak period from 6 am to 10 am and 4 pm to 8 pm. The RTP is determined via an aggregate demand curve of the distribution system and the aggregate individual load consumptions at each sample price.

B. LOAD MODELING

In the contemporary market, loads submit bids to purchase/consume electricity at any given price. In the calculation framework, all PRLs also follow this procedure. It is highly probable that loads will be more elastic in the presence of DLMPs in the enhanced distribution system than the loads in the traditional distribution system. However, accurate determination of a load's bid curve is a difficult process, as shown in [53]. A demand curve is a simpler substitute to the bid curve, which can be easily approximated using a series of step functions. Among various functional forms, the power form of the demand curve has an attractive property of constant elasticity [54]. The general form of the power demand curve is given as

$$Q = Q_0(\frac{P}{P_0})^{\epsilon} \tag{29}$$

where ϵ denotes the price elasticity of demand and measures the responsiveness of the demand to prices. Estimation of this coefficient is extremely complicated because it is dependent on multiple factors such as time and the external load conditions. ϵ is generally less than zero because the demand function is an inverse relationship between the price and quantity. The magnitude of the coefficient provides insight about the amount of sensitivity of the demand with respect to prices. If the coefficient is exactly -1, the demand is said to be 'unit elastic' and a coefficient less than -1 represents 'elastic' demand. ϵ can be considered in the OPF objective function because the cost functions for the PRLs are known. For example, if the PRL is modeled as a fuel cell or a micro-turbine, it would have the following standard cost function.

$$Cost = \frac{P_{PRL} \times g_{PRL}}{\eta_{PRL}} \tag{30}$$

where P_{PRL} is the load output, g_{PRL} is the cost to supply the PRL and η_{PRL} is the efficiency of the load.

C. CASE STUDY I

In this case study, it is assumed that the distribution feeders are sized such that there is no congestion in the system and there is no involvement of DGs. The main aim is to observe the variations among different pricing mechanisms for predominantly inelastic loads. Hence, a coefficient of elasticity of -0.1 is used in this simulation. The prices obtained for the various methods under consideration for node 842 (renumbered to 20 in Figure 3), over a 24-hour period are shown in Figure 9. As expected, the inelasticity of the loads leads to negligible impact on the inaccuracy of the other pricing mechanisms. This node is specifically chosen for illustration purposes since it recorded the highest price deviations from

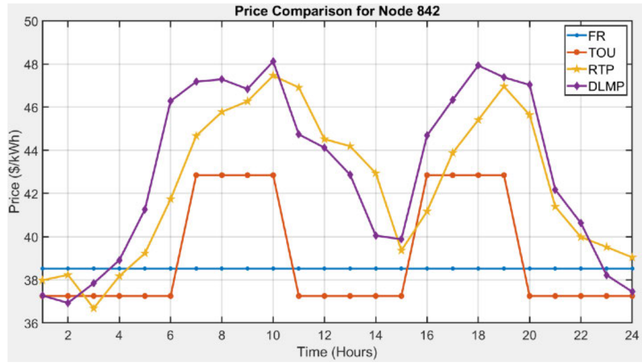

FIGURE 9. Prices for node 842 at $\epsilon = -0.1$.

TABLE 5. Percentage of deviation from optimal DLMPs for node 842 at $\epsilon = -0.1$.

Pricing Mechanism	Deviation (%)
FR	10.71
ToU	8.96
RTP	3.62

optimal DLMPs, on average. Table 5 provides a summary of percentage of deviation from the optimal DLMPs.

D. CASE STUDY II

In this study, the coefficient of elasticity is increased in magnitude to -1.2 in order to highlight the impact caused due to the pricing inaccuracies of the other mechanisms. Similar to the previous case study, no congestion is involved in this study. The prices obtained for the various methods under consideration for node 842, over a 24-hour period are shown in Figure 10. A summary of the percentage of deviation from the optimal DLMP is provided in Table 6.

The higher coefficient of elasticity leads to higher price deviations from the optimal prices obtained using DLMP. The FR and ToU show large deviation from the optimal prices, throughout the 24-hour period. The RTP is still able to track the DLMP price trend, although not as accurate as before.

E. CASE STUDY III

Although RTP was inaccurate in the previous two case studies, it followed a similar trend to the optimal DLMP throughout the 24-hour period. This is due to the fact that those case studies did not involve any congestion in the network. The main aim of this study is to prove that using RTP in a congested network will lead to suboptimal economic solutions that could adversely affect reliability and stability of the network. Congestion is enforced by enforcing a limit on the line rating between buses 8 and 9 in the distribution system, similar to Section IV-D no PRLs are involved in this study.

For ease of comparison, only RTP and DLMPs at node 808 (renumbered to 6 in Figure 3) are used in the investigation. Node 808 is chosen because it is the most affected node due to introduction of congestion to the distribution system.

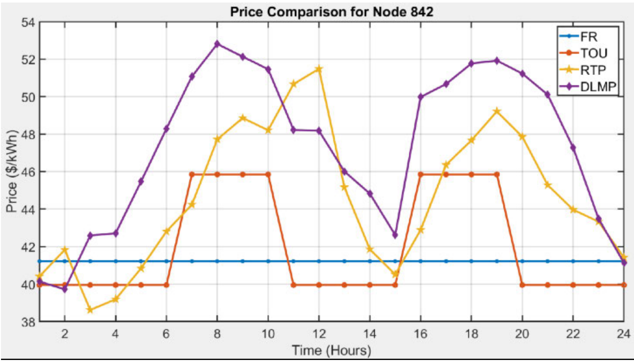

FIGURE 10. Prices for node 842 at $\epsilon = -1.2$.

TABLE 6. Percentage of deviation from Optimal DLMPs for node 842 at $\epsilon = -1.2$.

Pricing Mechanism	Deviation (%)
FR	12.61
ToU	10.99
RTP	6.92

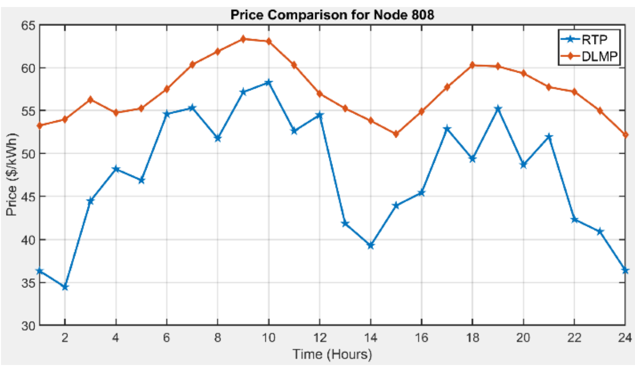

FIGURE 11. Prices for node 808 with congestion.

Figure 11 illustrates the resulting RTP and DLMP, while Figure 12 compares the percentage of deviation in prices with and without congestion at node 808, over a 24-hour period.

Figure 11 illustrates that the RTP is lower than the DLMP when congestion is introduced. Furthermore, the difference between the two prices are highest during the first five hours and the last three hours of the day. For example, the RTP is approximately 20 $ lower than the DLMP during the second hour mark. RTP is no longer able to track the DLMP price trend, unlike in previous case studies. This significant difference in prices for these mechanisms arises from how the RTP is calculated. The calculation process of the RTP does not appropriately consider the distribution system because it does not consider marginal loss or marginal congestion component, unlike the DLMP. Thus, the RTP fails to effectively capture the impact of congestion on the distribution network, which leads to suboptimal prices.

Figure 12 shows that RTP deviates from the optimal DLMPs by over 15% over majority of the observation period,

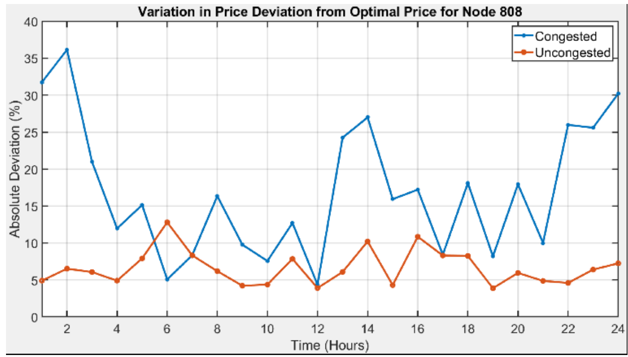

FIGURE 12. Comparison of % deviation of RTPs from optimal DLMP under different network conditions.

when the network is congested. The average percentage of deviation for congested network is 17.23% and 10.28% for the uncongested network. The figure also illustrates the impact of congestion on the RTP. The percentage deviations of RTP in uncongested network is never above 15% over the 24-hour period and is lower than the congested percentage throughout the day except at the sixth hour.

F. DISCUSSION

The superiority of DLMP over existing pricing mechanisms arises from its high robustness under different network conditions and its capability to adequately reflect the time dependency of energy prices. Moreover, DLMPs can reflect both the network and generation conditions in both the transmission and the distribution systems. The FR pricing cannot reflect any system state or the time dependence of energy prices. It also causes cross-subsidization between customers. The ToU rates do not reflect any system state either and inadequately reflect energy prices' time dependence. Contemporary RTP, while reflecting the transmission system state, does not reflect the distribution system state. Hence, it is evident that the existing pricing mechanisms provide insufficiently accurate pricing signals in the distribution systems with high DER penetration. The comparison of DLMP with other pricing methods is summarized in Table 7.

TABLE 7. Comparison of DLMP with other pricing methods.

Criteria	FR	ToU	RTP	DLMP
Reflection of Distribution System State	✗	✗	✗	✓
Reflection of Transmission System State	✗	✗	✓	✓
Reflection of Time Dependence	✗	∼	✓	✓
No Cross Subsidization	✗	∼	✓	✓
High Robustness	✗	✗	✓	✓

VI. CONCLUSION AND FUTURE WORKS

The paper recommends the use of the DLMP as pricing signal in both traditional and enhanced distribution systems. The OPF problem is solved in two stages to reduce the processing stress exerted by the calculation. The calculation framework solves the lossy-DCOPF problem while using a piecewise linear approximation technique for real power loss approximation. The methodology relied on efficient iteration between these two systems for effective coupling and accurate representation of both the transmission and distribution systems. A sampling approach is proposed to overcome possible convergence issues with the paradigm.

DLMPs were calculated for the traditional distribution grid, an enhanced distribution grid and a congested distribution grid. The results extracted via case studies exhibited the superiority of DLMP over existing pricing mechanisms in incentivizing DER to behave in economically efficient method. The dominance of DLMPs is seen to increase substantially with the presence of price-sensitive loads, such as DGs in the distribution network. This is due to its capability to capture the effect of time on energy prices and the real-time network state in the coupled systems, relative to other pricing mechanisms.

For future work, it is recommended to further investigate the economic optimization of the iteration mechanism within the developed calculation framework via consideration of more features of the distribution network. Another promising area for future research could be to find more effective solutions for the existing convergence issues in the model, either via modifications to the proposed solution concept or via a novel solution method. Another line of future work is exploring the scalability of the proposed framework by implementing the presented calculation methodology in large-scale systems.

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