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Fault Estimation Filter Design for Two-Dimensional Fornasini-Marchesini Dynamical Systems in Low-Frequency Domain

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ABSTRACT This paper discusses the problem of fault estimation for two-dimensional (2-D) Fornasini-Marchesini (FM) dynamical systems. The objective of this paper is to design a fault estimation filter to reconstruct the characteristics of faults in low-frequency domain and satisfy different performance levels. Utilizing the generalized Kalman-Yakubovich-Popov (GKYP) lemma, this problem is transformed into a multi-objective optimization problem, which is non-convex in essence. On this basis, an optimization algorithm is proposed to solve the non-convex optimization problem. Sufficient conditions are derived for the proposed fault estimation filter. Finally, simulation results show the effectiveness of the theoretical results.

INDEX TERMS Two-dimensional (2-D) systems, fault estimation, finite frequency, generalized Kalman-Yakubovich-Popov (GKYP) lemma.

I. INTRODUCTION

Recent years have witnessed extensive attention on two-dimensional (2-D) systems due to the massive amount of applications in practice, for instance, multi-dimensional digital image processing [1], signal processing [2], digital filtering [3] and repetitive process [4]. Due to its important engineering background, 2-D systems are still one of the research hotspots in the field of control. In recent years, many research results on 2-D systems have been developed. In [5], the authors have investigated the stability analysis of positive 2-D systems with time delays. The authors in [6] have studied the stability analysis of the 2-D nonlinear systems. On this basis, an H_∞ state feedback controller has been designed to solve the stabilization problem. The model reduction problem of 2-D systems over finite-frequency ranges has been studied in [7], where a novel finite-frequency method has been proposed to replace the full frequency method. The authors in [8] have investigated the problem of asynchronous H_∞ control for 2-D Markov jump systems, where the controller gains have been calculated by solving a convex optimization problem.

On the other hand, industrial processes have increasingly higher requirements for the safety and reliability of control systems. Thus, the fault estimation problem has received widespread attention [9]–[15]. The authors in [11] have studied the problem of fault estimation for nonlinear system, where an intermediate estimator has been designed to estimate states and faults simultaneously. In [14], the authors have designed an adaptive fault-tolerant control protocol by utilizing the online fault estimation to reduce the negative effect of actuator fault. In [15], a novel sliding mode observer has been developed to estimate full states and faults. Notice that the minimum phase condition has been relaxed to detectability. However, the mentioned results have not taken the finite-frequency characteristics into consideration, which are potentially more conservative due to over-design. Motivated by this point, it is necessary to consider the finite-frequency methods. The authors in [16] have designed a fault estimation observer in the finite-frequency domain

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for Lipschitz nonlinear multiagent systems subject to system components or actuator fault. In [17], the authors have designed a fault detection filter to detect the occurrence of faults in finite-frequency domain, and a similar problem of linear uncertain systems and linear parameter-varying systems has been addressed in [18], [19]. The authors in [20] have addressed the problem of H_-/H_∞ fault detection observer for 2-D systems, where the effect of fault sensitivity is mainly in low frequency domain. It should be noted that few results are available to solve the fault estimation problem of 2-D FM systems in the literature, to our knowledge. This fact motivates us to carry out this work.

This paper proposes a finite-frequency fault estimation method for two-dimensional FM systems. The main contributions of this paper are summarized as follows:

1) A fault estimation filter is designed to estimate faults in low-frequency domain and satisfies certain performance specifications.

2) An optimization algorithm is proposed to calculate the parameters of the fault estimation filter, which is a non-convex problem in essence.

3) Sufficient conditions for the desired fault estimation filter are established in terms of linear matrix inequality (LMI) and could be solved directly.

Notations: Let $\mathbb{R}$ and $\mathbb{R}^\ell$ represent the set of reals and $\ell-$ dimensional Euclidean space, respectively. For any symmetric matrix $\mathcal{M}$, $\mathcal{M}^T$ and $\mathcal{M}^\perp$ represent its conjugate transpose and quadrature complement, respectively. $\lambda_{\min}(\mathcal{M})$ and $\lambda_{\max}(\mathcal{M})$ are the minimum and maximum eigenvalues of $\mathcal{M}$, respectively. $\sigma_{\max}(\mathcal{N})$ represents the maximum singular value of $\mathcal{N}$. The symbol "$*$" will be used in some matrix expressions to represent symmetric structures.

II. PROBLEM FORMULATION

Consider a 2-D discrete-time systems established upon FM model as follows:

$$\begin{aligned} x(i+1,j+1) &= A_1x(i,j+1)+A_2x(i+1,j) \\ &\quad +B_{d1}d(i,j+1)+B_{d2}d(i+1,j) \\ &\quad +B_{f1}f(i,j+1)+B_{f2}f(i+1,j), \\ y(i,j) &= Cx(i,j)+D_dd(i,j)+D_ff(i,j), \end{aligned} \tag{1}$$

where $x(i,j) \in \mathbb{R}^{n_x}$ denotes the state vector, $y(i,j) \in \mathbb{R}^{n_y}$ denotes the measurement output, $d(i,j) \in \mathbb{R}^{n_d}$ denotes the exogenous disturbance input, $f(i,j) \in \mathbb{R}^{n_f}$ is the fault signal in low-frequency domain. A_1, A_2, B_{d1}, B_{d2}, B_{f1}, B_{f2}, C, D_d and D_f are the real system matrices of compatible dimensions.

The purpose of this paper is to design a fault estimation filter as follows:

$$\begin{aligned} \hat{x}(i+1,j+1) &= A_{F1}\hat{x}(i,j+1)+A_{F2}\hat{x}(i+1,j) \\ &\quad +B_{F1}y(i,j+1)+B_{F2}y(i+1,j), \\ \gamma(i,j) &= C_F\hat{x}(i,j)+D_Fy(i,j), \end{aligned} \tag{2}$$

where $\hat{x}(i,j) \in \mathbb{R}^{n_x}$ is the state estimation vector and $\gamma(i,j) \in \mathbb{R}^{n_\ell}$ is the residual signal. Matrices A_{F1}, A_{F2}, B_{F1}, B_{F2}, C_F and D_F are the parameters of fault estimation filter to be determined.

Augmenting system (1) to include fault estimation filter (2), one gets

$$\begin{aligned} \bar{x}(i+1,j+1) &= \bar{A}_1\bar{x}(i,j+1)+\bar{A}_2\bar{x}(i+1,j) \\ &\quad +\bar{B}_{d1}d(i,j+1)+\bar{B}_{d2}d(i+1,j) \\ &\quad +\bar{B}_{f1}f(i,j+1)+\bar{B}_{f2}f(i+1,j), \\ e(i,j) &= \bar{C}\bar{x}(i,j)+\bar{D}_dd(i,j)+\bar{D}_ff(i,j), \end{aligned} \tag{3}$$

where $e(i,j) = \gamma(i,j) - f(i,j)$, $\bar{x}(i,j) = [x^T(i,j)\ \hat{x}^T(i,j)]^T$, and

$$\begin{aligned} \bar{A}_1 &= \begin{bmatrix} A_1 & 0 \\ B_{F1}C & A_{F1} \end{bmatrix}, \quad \bar{B}_{d1} = \begin{bmatrix} B_{d1} \\ B_{F1}D_d \end{bmatrix}, \\ \bar{A}_2 &= \begin{bmatrix} A_2 & 0 \\ B_{F2}C & A_{F2} \end{bmatrix}, \quad \bar{B}_{d2} = \begin{bmatrix} B_{d2} \\ B_{F2}D_d \end{bmatrix}, \\ \bar{C} &= \begin{bmatrix} D_FC & C_F \end{bmatrix}, \quad \bar{B}_{f1} = \begin{bmatrix} B_{f1} \\ B_{F1}D_f \end{bmatrix}, \\ \bar{D}_d &= D_FD_d,\ \bar{D}_f = D_FD_f - I, \quad \bar{B}_{f2} = \begin{bmatrix} B_{f2} \\ B_{F2}D_f \end{bmatrix}. \end{aligned}$$

Note that the residual signal $e(i,j)$ needs to be sensitive to faults and robust to disturbances. It is necessary to make the estimation error as small as possible under certain constraint conditions. More precisely, system (3) should meet the following performance specifications:

$$\sup_{\omega_1,\omega_2} \sigma_{\max}(G_{fe}(\omega_1,\omega_2)) < \gamma_1, \forall\, |\omega_1| \le \bar{\omega}_{11}, |\omega_2| \le \bar{\omega}_{12}, \tag{4}$$

$$\sup_{\omega_1,\omega_2} \sigma_{\max}(G_{de}(\omega_1,\omega_2)) < \gamma_2, \forall\, |\omega_1| \le \bar{\omega}_{21}, |\omega_2| \le \bar{\omega}_{22}, \tag{5}$$

where γ_1 and γ_2 are given positive scalars, $\bar{\omega}_{k1}, \bar{\omega}_{k2} \in [0,\ \pi]$, $k = 1,\ 2$ denote the frequency bounds of $f(i,j)$ and $d(i,j)$, respectively. $G_{fe}(\omega_1,\omega_2)$ and $G_{de}(\omega_1,\omega_2)$ denote the transfer functions of the fault and the disturbance with respect to the estimation error, respectively. Then, these two functions can be accurately characterized as follows:

$$\begin{aligned} G_{fe}(\omega_1,\omega_2) &= \bar{C}(z_1z_2I - z_2\bar{A}_1 - z_1\bar{A}_2)^{-1} \\ &\quad \times (z_2\bar{B}_{f1} + z_1\bar{B}_{f2}) + \bar{D}_f, \end{aligned} \tag{6}$$

$$\begin{aligned} G_{de}(\omega_1,\omega_2) &= \bar{C}(z_1z_2I - z_2\bar{A}_1 - z_1\bar{A}_2)^{-1} \\ &\quad \times (z_2\bar{B}_{d1} + z_1\bar{B}_{d2}) + \bar{D}_d, \end{aligned} \tag{7}$$

where $z_1 = e^{j\omega_1}$ and $z_2 = e^{j\omega_2}$ represent two z-transform operators in different directions, respectively.

Then the fault estimation problem in the finite frequency domain is formulated as: Given two positive constants γ_1 and γ_2, design a fault estimation filter (2) such that the augmented system (3) is asymptotically stable and the finite frequency performance specifications (4) and (5) are satisfied.

Remark 1: The purpose of introducing finite-frequency H_∞ indices in (4) and (5) is to minimize the inherent effect of the fault and disturbance on the estimation error. Moreover, the finite-frequency indices mentioned in this article can be regarded as an extensions of the standard indices, where $\bar{\omega}_{k1}$, $\bar{\omega}_{k2} \in [0, \pi]$, $k = 1,\ 2$. In other words, if $\bar{\omega}_{k1} = \bar{\omega}_{k2} = \pi$, the above indices can be reduced to the standard indices as shown in [21].

Next, it is necessary to introduce several useful lemmas used in the subsequent theoretical derivation.

Lemma 1 ([22]): For a symmetric matrix $\Phi \in \mathbb{R}^{n\times n}$ and two matrices $\Gamma \in \mathbb{R}^{n\times m}$ and $\Lambda \in \mathbb{R}^{n\times m}$, there exists a matrix $X \in \mathbb{R}^{m\times m}$ satisfying

$$\Phi + \Gamma X \Lambda^T + \Lambda X^T \Gamma^T < 0, \tag{8}$$

if and only if the following inequalities hold:

$$\Gamma^{\perp} \Phi \Gamma^{\perp^T} < 0,\ \ \Lambda^{\perp} \Phi \Lambda^{\perp^T} < 0, \tag{9}$$

where Φ and X are required to be full rank matrices, and Γ and Λ are column full rank matrices.

Lemma 2 ([23]): For the 2-D FM model described by (1), it is assumed that $\det(z_1 z_2 I - z_2 \bar{A}_1 - z_1 \bar{A}_2)^{-1} \neq 0$ holds for $\forall(z_1, z_2) \in \{(z_1, z_2) \in \mathbb{C} \times \mathbb{C} : |z_1| \geq 1, |z_2| \geq 1\}$. Given an Hermitian matrix Θ and constants $\bar{\omega}_1, \bar{\omega}_2 \in [0, \pi]$, if there exist matrices P_k and $Q_k \in \mathbb{C}^{n\times n}$, $k = 1, 2$ satisfying $Q_k > 0$ and

$$\begin{bmatrix} A & B_f \\ I & 0 \end{bmatrix}^T \begin{bmatrix} P & Q \\ * & \Delta \end{bmatrix} \begin{bmatrix} A & B_f \\ I & 0 \end{bmatrix} + \Theta < 0, \tag{10}$$

where

$$\begin{aligned} &P = P_1 + P_2,\ Q = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix}, \\ &A = \begin{bmatrix} A_1 & A_2 \end{bmatrix},\ B_f = \begin{bmatrix} B_{f1} & B_{f2} \end{bmatrix}, \\ &\Delta = \mathrm{diag}\{-P_1 - 2\cos\bar{\omega}_1 Q_1, -P_2 - 2\cos\bar{\omega}_2 Q_2\}. \end{aligned}$$

From this, the following finite-frequency condition is satisfied:

$$\begin{bmatrix} G(\omega_1, \omega_2) \\ I(\omega_1, \omega_2) \end{bmatrix}^T \Theta \begin{bmatrix} G(\omega_1, \omega_2) \\ I(\omega_1, \omega_2) \end{bmatrix} < 0, \tag{11}$$

where $\forall(\omega_1, \omega_2) \in \Omega \triangleq [-\bar{\omega}_1, \bar{\omega}_1] \times [-\bar{\omega}_2, \bar{\omega}_2]$ and

$$\begin{aligned} G(\omega_1, \omega_2) &\triangleq \begin{bmatrix} e^{j\omega_2} G(\omega_1, \omega_2) \\ e^{j\omega_1} G(\omega_1, \omega_2) \end{bmatrix},\ I(\omega_1, \omega_2) \triangleq \begin{bmatrix} e^{j\omega_2} I \\ e^{j\omega_1} I \end{bmatrix}, \\ G(\omega_1, \omega_2) &\triangleq (e^{j(\omega_1+\omega_2)} I - e^{j\omega_2} A_1 - e^{j\omega_1} A_2)^{-1} \\ &\quad \times (e^{j\omega_2} B_{f1} + e^{j\omega_1} B_{f2}). \end{aligned}$$

Lemma 3 ([24]): Given a matrix $\bar{A} = \begin{bmatrix} \bar{A}_1 & \bar{A}_2 \end{bmatrix}$, the augmented system (3) is said to be asymptotically stable if there exists a symmetric matrix $P_s > 0$ such that

$$\begin{bmatrix} \bar{A} \\ I \end{bmatrix}^T \begin{bmatrix} P_s & 0 \\ 0 & \mathrm{diag}\{-P_{s1}, -P_{s2}\} \end{bmatrix} \begin{bmatrix} \bar{A} \\ I \end{bmatrix} < 0. \tag{12}$$

Remark 2: By investigating the existing literature [2], [3], [7], [16], the fault often occurs in low-frequency domain due to its slow change. In addition, the stuck fault considered in this paper also belongs to the low-frequency domain. Motivated by these facts, it is necessary to design a fault estimation filter to characterize the fault properties in finite-frequency domain.

III. MAIN RESULTS

First, the following results give the sensitive and robustness conditions for the proposed fault estimation filter, respectively.

Theorem 1: Given constants $\bar{\omega}_{11}$, $\bar{\omega}_{12} \in [0, \pi]$ and $\gamma_1 > 0$, the finite-frequency H_∞ performance index (4) holds, if there exist Hermitian matrices P_{fk1}, P_{fk3}, Q_{fk1} and Q_{fk3}, matrices P_{fk2}, Q_{fk2}, G_1, G_2, G_3, $\bar{A}_{F1}$, $\bar{A}_{F2}$, $\bar{B}_{F1}$, $\bar{B}_{F2}$, $\bar{C}_F$ and $\bar{D}_F$, $k = 1,\ 2$ such that the following conditions are satisfied:

$$\begin{bmatrix} Q_{fk1} & Q_{fk2} \\ * & Q_{fk3} \end{bmatrix} > 0,\ k = 1,\ 2 \tag{13}$$

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \Sigma_{16} & \Sigma_{17} & \Sigma_{18} & 0 & 0 \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} & \Sigma_{26} & \Sigma_{27} & \Sigma_{28} & 0 & 0 \\ * & * & \Sigma_{33} & \Sigma_{34} & 0 & 0 & 0 & 0 & C^T \bar{D}_F^T & 0 \\ * & * & * & \Sigma_{44} & 0 & 0 & 0 & 0 & \bar{C}_F^T & 0 \\ * & * & * & * & \Sigma_{55} & \Sigma_{56} & 0 & 0 & 0 & C^T \bar{D}_F^T \\ * & * & * & * & * & \Sigma_{66} & 0 & 0 & 0 & \bar{C}_F^T \\ * & * & * & * & * & * & -\gamma_1^2 I & 0 & \tilde{D}_f^T & 0 \\ * & * & * & * & * & * & * & -\gamma_1^2 I & 0 & \tilde{D}_f^T \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{14}$$

where

$$\begin{aligned} &\Sigma_{11} = P_{f11} + P_{f21} - G_1 - G_1^T,\ \Sigma_{14} = Q_{f12} + \bar{A}_{F1}, \\ &\Sigma_{12} = P_{f12} + P_{f22} - G_3 - G_2^T,\ \Sigma_{16} = Q_{f22} + \bar{A}_{F2}, \\ &\Sigma_{13} = Q_{f11} + G_1 A_1 + \bar{B}_{F1} C,\ \Sigma_{17} = G_1 B_{f1} + \bar{B}_{F1} D_f, \\ &\Sigma_{15} = Q_{f21} + G_1 A_2 + \bar{B}_{F2} C,\ \Sigma_{18} = G_1 B_{f2} + \bar{B}_{F2} D_f, \\ &\Sigma_{23} = Q_{f12}^T + G_2 A_1 + \bar{B}_{F1} C,\ \Sigma_{24} = Q_{f13} + \bar{A}_{F1}, \\ &\Sigma_{25} = Q_{f22}^T + G_2 A_2 + \bar{B}_{F2} C,\ \Sigma_{26} = Q_{f23} + \bar{A}_{F2}, \\ &\Sigma_{22} = P_{f13} + P_{f23} - G_3 - G_3^T,\ \Sigma_{27} = G_2 B_{f1} + \bar{B}_{F1} D_f, \\ &\Sigma_{33} = -P_{f11} - 2\cos\bar{\omega}_{11} Q_{f11},\ \Sigma_{28} = G_2 B_{f2} + \bar{B}_{F2} D_f, \\ &\Sigma_{34} = -P_{f12} - 2\cos\bar{\omega}_{11} Q_{f12},\ \Sigma_{44} = -P_{f13} \\ &\qquad - 2\cos\bar{\omega}_{11} Q_{f13}, \\ &\Sigma_{55} = -P_{f21} - 2\cos\bar{\omega}_{12} Q_{f21},\ \Sigma_{56} = -P_{f22} \\ &\qquad - 2\cos\bar{\omega}_{12} Q_{f22}, \\ &\Sigma_{66} = -P_{f23} - 2\cos\bar{\omega}_{12} Q_{f23},\ \tilde{D}_f = \bar{D}_F D_f - I. \end{aligned}$$

Then, the parameters of the proposed fault estimation filter (2) are calculated by

$$\begin{aligned} &A_{F1} = G_3^{-1} \bar{A}_{F1},\ A_{F2} = G_3^{-1} \bar{A}_{F2},\ C_F = \bar{C}_F, \\ &B_{F1} = G_3^{-1} \bar{B}_{F1},\ B_{F2} = G_3^{-1} \bar{B}_{F2},\ D_F = \bar{D}_F. \end{aligned} \tag{15}$$

Proof: By Lemma 2, it follows from (8) that

$$\begin{bmatrix} \tilde{C} & \tilde{D}_f \\ 0 & I \end{bmatrix}^T \begin{bmatrix} I & 0 \\ * & -\gamma_1^2 I \end{bmatrix} \begin{bmatrix} \tilde{C} & \tilde{D}_f \\ 0 & I \end{bmatrix} + \begin{bmatrix} \tilde{A} & \tilde{B}_f \\ I & 0 \end{bmatrix}^T \begin{bmatrix} P_f & Q_f \\ * & \Delta_f \end{bmatrix} \begin{bmatrix} \tilde{A} & \tilde{B}_f \\ I & 0 \end{bmatrix} < 0, \quad (16)$$

where

$$\tilde{A} = \begin{bmatrix} \bar{A}_1 & \bar{A}_2 \end{bmatrix}, \quad \tilde{B}_f = \begin{bmatrix} \bar{B}_{f1} & \bar{B}_{f2} \end{bmatrix}, \quad \tilde{C} = \mathrm{diag}\{\bar{C},\ \bar{C}\},\ \tilde{D}_f = \mathrm{diag}\{\bar{D}_f,\ \bar{D}_f\}.$$

Then, condition (14) can be further rewritten as

$$\begin{bmatrix} \tilde{A} & \tilde{B}_f \\ I & 0 \\ 0 & I \end{bmatrix}^T \left\{ \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_f & Q_f \\ * & \Delta_f \end{bmatrix} \times \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \tilde{C}^T & 0 \\ \tilde{D}_f^T & I \end{bmatrix} \begin{bmatrix} I & 0 \\ * & -\gamma_1^2 I \end{bmatrix} \times \begin{bmatrix} 0 & \tilde{C} & \tilde{D}_f \\ 0 & 0 & I \end{bmatrix} \right\} \begin{bmatrix} \tilde{A} & \tilde{B}_f \\ I & 0 \\ 0 & I \end{bmatrix} < 0. \quad (17)$$

For convenience, define $\Gamma = \begin{bmatrix} -I & \tilde{A} & \tilde{B}_f \end{bmatrix}^T$, and then one has $\Gamma^{\perp} = \begin{bmatrix} \tilde{A}^T & I & 0 \\ \tilde{B}_f^T & 0 & I \end{bmatrix}$. By Lemma 1, the following sufficient condition can be obtained to guarantee that condition (15) holds:

$$\begin{bmatrix} 0 & 0 \\ \tilde{C}^T & 0 \\ \tilde{D}_f^T & I \end{bmatrix} \begin{bmatrix} I & 0 \\ * & -\gamma_1^2 I \end{bmatrix} \begin{bmatrix} 0 & \tilde{C} & \tilde{D}_f \\ 0 & 0 & I \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_f & Q_f \\ * & \Delta_f \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} + \mathrm{He}(\Gamma G^T \Lambda^T) < 0, \quad (18)$$

which is further guaranteed by

$$\begin{bmatrix} P_f - G - G^T & Q_f + G\tilde{A} & G\tilde{B}_f & 0 \\ * & \Delta_f & 0 & \tilde{C}^T \\ * & * & -\gamma_1^2 I & \tilde{D}_f^T \\ * & * & * & -I \end{bmatrix} < 0, \quad (19)$$

where

$$G = \begin{bmatrix} G_1 & G_3 \\ G_2 & G_3 \end{bmatrix},\ P_f = P_{f1} + P_{f2},\ Q_f = \begin{bmatrix} Q_{f1} & Q_{f2} \end{bmatrix},$$
$$\Delta_f = \mathrm{diag}\{-P_{f1} - 2\cos\bar{\omega}_{11} Q_{f1},\ -P_{f2} - 2\cos\bar{\omega}_{12} Q_{f2}\},$$
$$P_{f1} = \begin{bmatrix} P_{f11} & P_{f12} \\ * & P_{f13} \end{bmatrix},\ Q_{f1} = \begin{bmatrix} Q_{f11} & Q_{f12} \\ * & Q_{f13} \end{bmatrix},$$
$$P_{f2} = \begin{bmatrix} P_{f21} & P_{f22} \\ * & P_{f23} \end{bmatrix},\ Q_{f2} = \begin{bmatrix} Q_{f21} & Q_{f22} \\ * & Q_{f23} \end{bmatrix}.$$

Therefore, it is concluded that condition (17) is equivalent to condition (12) with expression (13). This completes the proof.

Theorem 2: Given constants $\bar{\omega}_{21},\ \bar{\omega}_{22} \in [0, \pi]$ and $\gamma_2 > 0$, the finite-frequency H_∞ performance index (5) holds, if there exist Hermitian matrices $P_{dk1}, P_{dk3}, Q_{dk1}$ and Q_{dk3}, matrices $P_{dk2}, Q_{dk2}, F_1, F_2, G_3, \bar{A}_{F1}, \bar{A}_{F2}, \bar{B}_{F1}, \bar{B}_{F2}, \bar{C}_F$ and $\bar{D}_F$, $k = 1,\ 2$ such that the following conditions are satisfied:

$$\begin{bmatrix} Q_{dk1} & Q_{dk2} \\ * & Q_{dk3} \end{bmatrix} > 0,\ k = 1,\ 2 \quad (20)$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} & \Gamma_{16} & \Gamma_{17} & \Gamma_{18} & 0 & 0 \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} & \Gamma_{26} & \Gamma_{27} & \Gamma_{28} & 0 & 0 \\ * & * & \Gamma_{33} & \Gamma_{34} & 0 & 0 & 0 & 0 & C^T\bar{D}_F^T & 0 \\ * & * & * & \Gamma_{44} & 0 & 0 & 0 & 0 & \bar{C}_F^T & 0 \\ * & * & * & * & \Gamma_{55} & \Gamma_{56} & 0 & 0 & 0 & C^T\bar{D}_F^T \\ * & * & * & * & * & \Gamma_{66} & 0 & 0 & 0 & \bar{C}_F^T \\ * & * & * & * & * & * & -\gamma_2^2 I & 0 & \tilde{D}_d^T & 0 \\ * & * & * & * & * & * & * & -\gamma_2^2 I & 0 & \tilde{D}_d^T \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (21)$$

where

$$\Gamma_{11} = P_{d11} + P_{d21} - F_1 - F_1^T,\ \Gamma_{14} = Q_{d12} + \bar{A}_{F1},$$
$$\Gamma_{12} = P_{d12} + P_{d22} - G_3 - F_2^T,\ \Gamma_{16} = Q_{d22} + \bar{A}_{F2},$$
$$\Gamma_{13} = Q_{d11} + F_1 A_1 + \bar{B}_{F1} C,\ \Gamma_{17} = F_1 B_{d1} + \bar{B}_{F1} D_d,$$
$$\Gamma_{15} = Q_{d21} + F_1 A_2 + \bar{B}_{F2} C,\ \Gamma_{18} = F_1 B_{d2} + \bar{B}_{F2} D_d,$$
$$\Gamma_{23} = Q_{d12}^T + F_2 A_1 + \bar{B}_{F1} C,\ \Gamma_{24} = Q_{d13} + \bar{A}_{F1},$$
$$\Gamma_{25} = Q_{d22}^T + F_2 A_2 + \bar{B}_{F2} C,\ \Gamma_{26} = Q_{d23} + \bar{A}_{F2},$$
$$\Gamma_{22} = P_{d13} + P_{d23} - G_3 - G_3^T,\ \Gamma_{27} = F_2 B_{d1} + \bar{B}_{F1} D_d,$$
$$\Gamma_{33} = -P_{d11} - 2\cos\bar{\omega}_{21} Q_{d11},\ \Gamma_{28} = F_2 B_{d2} + \bar{B}_{F2} D_d,$$
$$\Gamma_{34} = -P_{d12} - 2\cos\bar{\omega}_{21} Q_{d12},\ \Gamma_{44} = -P_{d13} - 2\cos\bar{\omega}_{21} Q_{d13},$$
$$\Gamma_{55} = -P_{d21} - 2\cos\bar{\omega}_{22} Q_{d21},\ \Gamma_{56} = -P_{d22} - 2\cos\bar{\omega}_{22} Q_{d22},$$
$$\Gamma_{66} = -P_{d23} - 2\cos\bar{\omega}_{22} Q_{d23},\ \tilde{D}_d = \bar{D}_F D_d.$$

Then, the parameters of the proposed fault estimation filter (2) are calculated by (13).

Proof: Similar to the proof of Theorem 1, this derivation is omitted here.

Based on Lemma 3, the stability analysis of the augmented system (3) is given in the following theorem.

Theorem 3: Given two positive constants α and β satisfying $\alpha + \beta = 1$, the augmented system (3) is asymptotically stable, if there exist Hermitian matrices P_{s1}, P_{s3}, matrices H_1, H_2, A_{F1}, A_{F2}, B_{F1}, B_{F2}, P_{s2} such that the following conditions are satisfied:

$$\begin{bmatrix} P_{s1} & P_{s2} \\ * & P_{s3} \end{bmatrix} > 0, \tag{22}$$

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} \\ * & * & \Phi_{33} & \Phi_{34} & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & \Phi_{55} & \Phi_{56} \\ * & * & * & * & * & \Phi_{66} \end{bmatrix} < 0, \tag{23}$$

where

$$\begin{aligned} &\Phi_{11} = P_{s1} - H_1 - H_1^T, \ \Phi_{13} = H_1 A_1 + B_{F1} C, \\ &\Phi_{12} = P_{s2} - G_3 - H_2^T, \ \Phi_{15} = H_1 A_2 + B_{F2} C, \\ &\Phi_{33} = -\alpha P_{s1}, \ \Phi_{34} = -\alpha P_{s2}, \ \Phi_{44} = -\alpha P_{s3}, \\ &\Phi_{55} = -\beta P_{s1}, \ \Phi_{56} = -\beta P_{s2}, \ \Phi_{66} = -\beta P_{s3}, \\ &\Phi_{23} = H_1 A_1 + B_{F1} C, \ \Phi_{14} = \Phi_{24} = A_{F1}, \\ &\Phi_{25} = H_2 A_2 + B_{F2} C, \ \Phi_{16} = \Phi_{26} = A_{F2}, \\ &\Phi_{22} = P_{s3} - G_3 - G_3^T. \end{aligned}$$

Proof: By Lemma 3, one can yield

$$\Gamma_s^{\perp} \begin{bmatrix} P_s & 0 \\ * & -\text{diag}\{\alpha P_s, \beta P_s\} \end{bmatrix} (\Gamma_s^{\perp})^T < 0, \tag{24}$$

where $\Gamma_s^{\perp} = \begin{bmatrix} \bar{A}^T & I \end{bmatrix}$, $\alpha > 0$, $\beta > 0$ and $P_s > 0$. On this basis, it is further obtained that

$$\Lambda_s^{\perp} \begin{bmatrix} P_s & 0 \\ * & -\text{diag}\{\alpha P_s, \beta P_s\} \end{bmatrix} (\Lambda_s^{\perp})^T = -\text{diag}\{\alpha P_s, \beta P_s\} < 0, \tag{25}$$

where $\Lambda_s^{\perp} = \begin{bmatrix} 0 & I \end{bmatrix}$. Based on [7], it is derived from (24) and (25) that

$$\begin{bmatrix} P_s & 0 \\ * & -\text{diag}\{\alpha P_s, \beta P_s\} \end{bmatrix} + \text{He}(\Gamma_s H^T \Lambda_s^T) < 0, \tag{26}$$

which implies that

$$\begin{bmatrix} P_s & H\bar{A} \\ * & -\text{diag}\{\alpha P_s, \beta P_s\} \end{bmatrix} < 0, \tag{27}$$

where $H = \begin{bmatrix} H_1 & G_3 \\ H_2 & G_3 \end{bmatrix}$. Substituting the correlation matrix into (27), it is concluded that (27) makes (23) hold. The proof is completed.

IV. SOLUTION

A. OPTIMIZATION ALGORITHM

The following optimization algorithm is proposed to calculate the parameters of the proposed fault estimation filter (2):

$$\begin{aligned} &\min \alpha\gamma_1 + \beta\gamma_2, \\ &s.t. \ (14), (15), (18), (20), (21), (22), (23), \end{aligned} \tag{28}$$

where α and β are given constants, and γ_1 and γ_2 represent the H_∞ disturbance attenuation levels, respectively. Then, the parameters of the proposed fault estimation filter (2) are calculated by (15).

Remark 3: Note that the positive real numbers α and β reflect the weights of H_∞ disturbance attenuation levels γ_1 and γ_2, respectively, where $\alpha + \beta = 1$. By investigating the analytical hierarchy process (AHP) mentioned in [26], it is assumed that these performances are equally significant, that is, the ratio is 1 : 1. Therefore, both α and β are given as 0.5 in advance. Obviously, the size of the performance weight directly affects its role in evaluating solution.

B. RESIDUAL EVALUATION FUNCTION AND THRESHOLD

For the 2-D systems, the residual evaluation function should reflect the properties of the two-dimensional variables from the horizontal and vertical directions. Inspired by [24], the residual evaluation function and the threshold are selected as follows:

$$J_r(i,j) = \sqrt{\frac{\sum_{p=0}^{s} \sum_{q=0}^{t} r^T(i-p, j-q)}{(s+1)(t+1) r(i-p, j-q)}}, \tag{29}$$

$$J_{\text{th}} = \sup_{f=0, d \neq 0} J_r(i,j). \tag{30}$$

Then, the occurrence of fault signal $f(i,j)$ can be detected by the following logical relationships:

$$\begin{aligned} &J_r(i,j) \leq J_{\text{th}} \Rightarrow \text{no fault} \Rightarrow \text{no alarm}, \\ &J_r(i,j) > J_{\text{th}} \Rightarrow \text{with fault} \Rightarrow \text{alarm}. \end{aligned} \tag{31}$$

V. NUMERICAL SIMULATION

In this section, we will give two examples to demonstrate the effectiveness of the developed method.

A. EXAMPLE 1

Consider the 2-D discrete-time systems (1) with the following system parameters:

$$\begin{aligned} &A_1 = \begin{bmatrix} 0.0853 & 0.0351 \\ 0.0622 & 0.0513 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0.0402 & 0.0240 \\ 0.0076 & 0.0123 \end{bmatrix}, \\ &B_{d1} = \begin{bmatrix} 0.1839 \\ 0.2400 \end{bmatrix}, \ B_{f1} = \begin{bmatrix} 0.9027 \\ 0.9448 \end{bmatrix}, \ D_d = 0.1112, \\ &B_{d2} = \begin{bmatrix} 0.4173 \\ 0.0497 \end{bmatrix}, \ B_{f2} = \begin{bmatrix} 0.4909 \\ 0.4893 \end{bmatrix}, \ D_f = 0.3692, \\ &C = \begin{bmatrix} 0.3377 & 0.9001 \end{bmatrix}. \end{aligned}$$

Suppose that there exists a stuck fault signal in the above 2-D systems [7], [16]. The objective is to design a fault estimation filter (2) to estimate the fault properties and satisfy certain control specifications simultaneously. Given the weighting coefficients $\alpha = 0.5$, $\beta = 0.5$, the frequency bounds $\bar{\omega}_{11} = \bar{\omega}_{12} = \pi/12$ and $\bar{\omega}_{21} = \bar{\omega}_{22} = \pi/10$, the parameters of the proposed fault estimation filter can be calculated as

$$A_{F1} = \begin{bmatrix} -0.2883 & -0.9788 \\ -0.1235 & -0.0064 \end{bmatrix}, \quad B_{F1} = \begin{bmatrix} -1.2398 \\ -0.7525 \end{bmatrix},$$

$$A_{F2} = \begin{bmatrix} -0.6576 & -0.3862 \\ 0.2072 & -0.2038 \end{bmatrix}, \quad B_{F2} = \begin{bmatrix} -1.0604 \\ 0.0531 \end{bmatrix},$$
$$C_F = \begin{bmatrix} 0.9424 & 1.3202 \end{bmatrix}, \quad D_F = 1.8902$$

with the performance levels $\gamma_1 = 0.3373$ and $\gamma_2 = 0.3037$. In order to demonstrate the effectiveness of the fault estimation filter (2), some simulation results will be given below. First of all, two stuck fault signals are assumed to be

$$f_1(i,j) = \begin{cases} 0.8, & 40 \le i \le 45, \ j \ge 50 \\ 0, & \text{otherwise} \end{cases}$$
$$f_2(i,j) = \begin{cases} 0.6, & 20 \le i \le 27, \ 20 \le j \le 120 \\ 0, & \text{otherwise} \end{cases}$$

and the unknown disturbance is assumed to be

$$d(i,j) = 0.05\sin(i)e^{-0.02i} + 0.03\cos(j)e^{-0.03j}.$$

Fig. 1 and Fig. 2 show the stuck fault signals and the disturbance signal, respectively. Under the initial conditions $x_i(k,1) = 0, \hat{x}_i(k,1) = 0, x_i(1,k) = 0, \hat{x}_i(1,k) = 0, i = 1, 2$, by applying the fault estimation filter (2), Fig. 3 presents the result of fault estimation in three-dimensional space. Fig. 4 and Fig. 5 show two stuck fault signals in two-dimensional space, respectively. Fig. 6 and Fig. 7 show the estimation of $f_1(i,j)$ and $f_2(i,j)$ in two-dimensional space, respectively. Based on [24], the threshold value can be computed as $J_{\text{th}} = 0.3125$. On this basis, Fig. 8 shows the residual evaluation function $J_r(i,j)$ and the threshold J_{th} in three-dimensional space. For clarity, Fig. 9 shows the residual evaluation function and the threshold in two-dimensional space. Simulation results show that the faults have been estimated precisely.

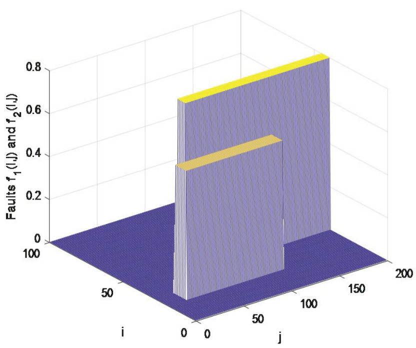

FIGURE 1. The stuck faults $f_1(i,j)$ and $f_2(i,j)$.

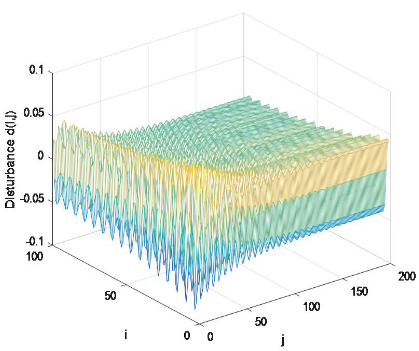

FIGURE 2. The unknown disturbance $d(i,j)$.

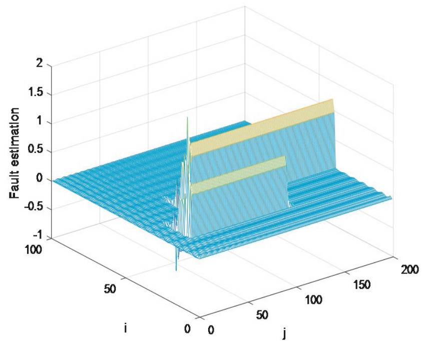

FIGURE 3. Fault estimation in three-dimensional space.

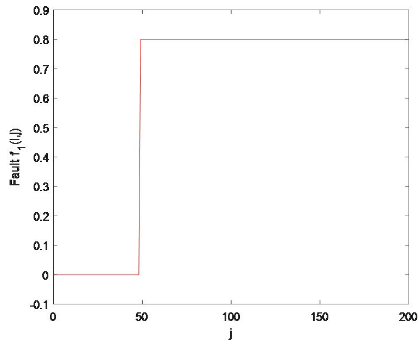

FIGURE 4. $f_1(i,j)$ in two-dimensional space.

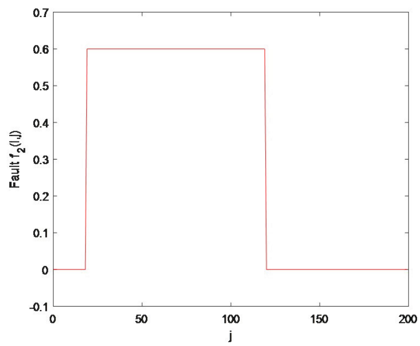

FIGURE 5. $f_2(i,j)$ in two-dimensional space.

B. EXAMPLE 2

Consider the static random field model [25], which can be described by the following two-dimensional system:

$$\eta(i+1,j+1) = a_1\eta(i+1,j) + a_2\eta(i,j+1) - a_1a_2\eta(i,j) + \omega(i,j),$$

where $\eta(i,j)$ represents the state of the random field in space coordinates, a_1 and a_2 denote the coefficients of the horizontal and vertical states, respectively. Denote two stuck fault signals by $f(i+1,j)$ and $f(i,j+1)$, then the system model can be further rewritten as

$$\eta(i+1,j+1) = a_1\eta(i+1,j) + a_2\eta(i,j+1) - a_1a_2\eta(i,j) + A_1 f(i,j+1) + A_2 f(i+1,j) + d(i,j),$$

where A_1 and A_2 represent known system parameter matrices. Letting $x^T(i,j) = \begin{bmatrix} \eta^T(i+1,j) - a_2\eta^T(i,j) & \eta^T(i,j) \end{bmatrix}$ be the

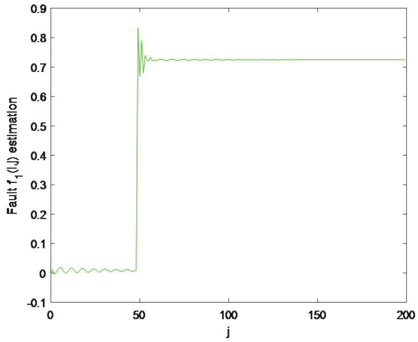

FIGURE 6. Estimation of $f_1(i,j)$ in two-dimensional space.

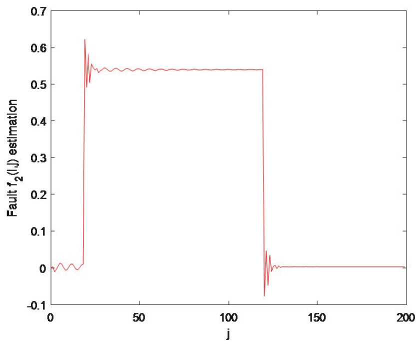

FIGURE 7. Estimation of $f_2(i,j)$ in two-dimensional space.

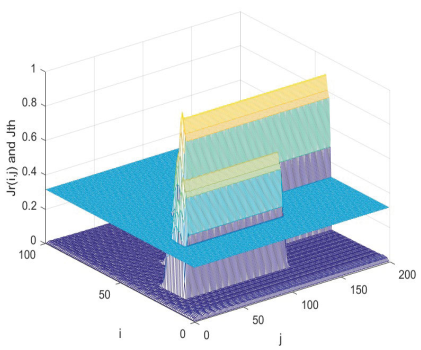

FIGURE 8. $J_r(i,j)$ and J_{th} in three-dimensional space.

augmented vector, the system output is described as follows:

$$y(i,j) = Cx(i,j) + D_f f(i,j) + D_d d(i,j).$$

The system parameter matrices are given as

$$A_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0.03 \end{bmatrix}, \quad B_{d1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_{f1} = \begin{bmatrix} 0.8314 \\ 0.8034 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.06 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{d2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, B_{f2} = \begin{bmatrix} 0.0605 \\ 0.3993 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.03 & 1 \end{bmatrix}, \ D_d = 0.4168, \ D_f = 0.5269.$$

By using the optimization algorithm proposed in Section IV, the parameters of the fault estimation filter can be calculated as

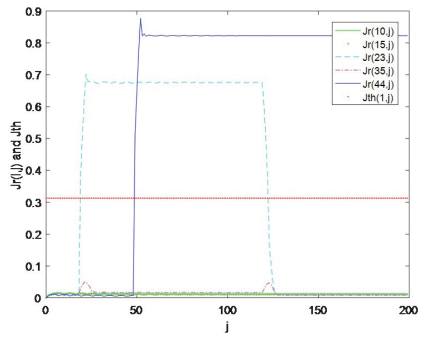

FIGURE 9. $J_r(i,j)$ and J_{th} in two-dimensional space.

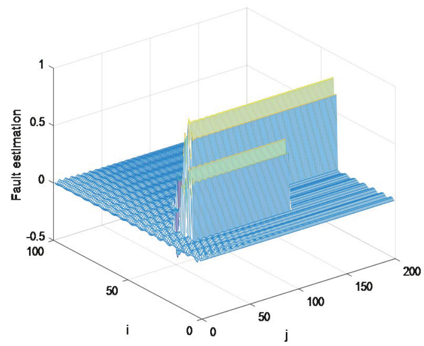

FIGURE 10. Fault estimation in three-dimensional space.

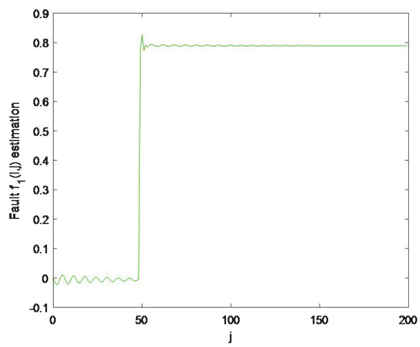

FIGURE 11. Estimation of $f_1(i,j)$ in two-dimensional space.

$$A_{F1} = \begin{bmatrix} 0.1738 & -2.4781 \\ 0.1117 & -0.8260 \end{bmatrix}, \quad B_{F1} = \begin{bmatrix} -2.1564 \\ -1.2843 \end{bmatrix},$$
$$A_{F2} = \begin{bmatrix} 0.0897 & -0.3006 \\ 0.0407 & -0.2679 \end{bmatrix}, \quad B_{F2} = \begin{bmatrix} -0.3056 \\ -0.2730 \end{bmatrix},$$
$$C_F = \begin{bmatrix} 0.0521 & 1.1190 \end{bmatrix}, \quad D_F = 1.1368.$$

Under the same fault and disturbance signals, Fig. 10 presents the result of fault estimation in three-dimensional space. Fig. 11 and Fig. 12 show the estimation of $f_1(i,j)$ and $f_2(i,j)$ in two-dimensional space, respectively. Then, the threshold value can be computed as $J_{th} = 0.3201$. On this basis, Fig. 13 shows the residual evaluation function $J_r(i,j)$ and threshold J_{th} in three-dimensional space. For clarity, Fig. 14 shows the residual evaluation function and threshold in two-dimensional space. It is easily seen from Fig. 11 - Fig. 14 that the faults have been estimated precisely.

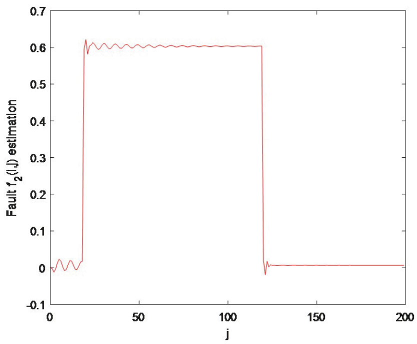

FIGURE 12. Estimation of $f_2(i,j)$ in two-dimensional space.

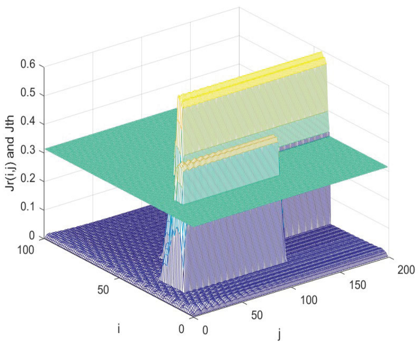

FIGURE 13. $J_r(i,j)$ and J_{th} in three-dimensional space.

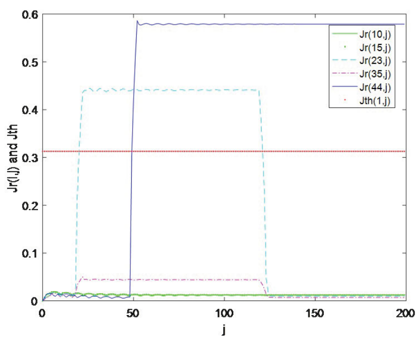

FIGURE 14. $J_r(i,j)$ and J_{th} in two-dimensional space.

VI. CONCLUSION

In this paper, the problem of fault estimation for two-dimensional FM model has been investigated. The design of fault estimation filter can meet certain performance indices and reconstruct the characteristics of faults. Utilizing the generalized KYP lemma, this problem has been recast into a multi-objective optimization problem, which is non-convex in essence. An optimization algorithm has been introduced to solve the difficulties caused by the non-convexity. Finally, two numerical simulations have been presented to verify the effectiveness of the theoretical results.

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