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Adaptive Iterative Learning Control for Tracking Trajectories With Non-Equal Trail Lengths and Initial Errors

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ABSTRACT Both non-equal trail lengths and non-zero initial errors are practical challenges to learning control of robotic and mechatronic systems. Iterative learning to update input is still desired, because of the repetitive motion nature of the controlled objects. This paper concerns with the adaptive iterative learning control method for performing non-identical tasks. The time scaling technique is applied to normalize non-equal trail lengths, while the error-tracking approach is adopted for coping with initial errors. Theoretical results for performance analysis are presented in detail. The uniform convergence of the tracking error is examined, while boundedness of the variables in the closed-loop is characterized. It is shown that the fully-saturated learning algorithm plays an important role in assuring uniform boundedness of the control input. The proposed control design method does not require the magnitude transformation, and removes the assumption of identical initial conditions. The time scaling technique is verified to be effective in assuring the expected performance, for tracking tasks with non-equal trail lengths and initial errors.

INDEX TERMS Uniform convergence, initial condition problem, non-equal trial lengths, learning algorithms.

I. INTRODUCTION

Iterative learning control (ILC) is to make full use of repetition of tasks, and by virtue of learning, the control performance is able to be improved effectively. Taking advantage of the historical control experiences to guide the design of controllers, learning strategies depend on the repeatable conditions, instead of accurate knowledge about system dynamics [1]. In the published literature, the contraction mapping based ILCs are relatively rich in theoretical achievements, embodied in forms of D-type and P-type learning algorithms [2], [3]. Recently, based on the Lyapunov synthesis approach, learning control theories have been developed to relax some fundamental assumptions encountered, covering a wide scope of research [4], [5]. Among others, with Adaptive ILC (AILC) parametric uncertainties can be handled effectively. Such learning controls are on the basis of the certainty equivalency principle, and the knowledge about the structure of system dynamics is assumed for control design [6]–[8]. It should be noted that most of the published works assume the linear-in-the-parameter uncertainty, which might not be satisfied in certain applications. For further development, the approximation-based design method are applicable [9]–[11]. Up to now, many efforts have been made for ILC designs for industrial robots and other practical applications [12], [13].

The superiority and efficiency of ILC depend highly on repeatability of the system operation. Yet, as often as not, such strict requirements (e.g., the identical tracking target, identical initial condition, identical trial length) may have potential implementation problems, which are difficult to be guaranteed in the implementation and limit the practical application of ILCs.

Tracking iteration-varying tasks is challenging to the conventional ILC. The situation where the tracking tasks slowly vary with respect to iteration was tackled earlier in [2], and the robustness has been characterized for D-type, PD-type and PID-type learning algorithms. The non-equal task problem was addressed in the framework of AILC [14]–[16]. It is found that for the parameter learning, the reference signals are allowed to be vary with iteration, and the controller design can be carried out with ease. The coinciding results

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of AILC designs in discrete-time domain were reported in [17]–[20]. Recently, the robustness with respect to iteration-dependent references has been established for discrete learning algorithms [21].

A common assumption for ILC is the identical initial condition that the actual initial state is required to be reposited to the desired one at each iteration. Using a 2-D analysis approach [22], sufficient conditions were derived to guarantee both convergence of the learning process for a fixed initial condition and boundedness for variable initial conditions. Two treatment techniques were reported to cope with the arbitrary initial condition issue. One is the time-varying boundary layer method [23], [24] and the other is the trajectory rectifying strategy [25]–[27]. Utilizing the initial rectifying strategy to adjust the desired trajectory that the initial segment of the formed trajectory is jointed smoothly to the desired trajectory at the selected position, it requires that both the actual trajectory for each iteration and the desired trajectory take the same initial value. As an alternative, instead of the state/output tracking, an error-tracking based ILC method was proposed in [28]. This method does not pose any requirement for the initial value of the practical error trajectory, and arbitrary initial shifts are allowed. As can be seen from the published results, how effective can the error-tracking approach be in dealing with problems arising from arbitrary initial shifts [29].

Conventional AILC designs allow iteration-varying tasks with equal lengths. In fact, the actual trial lengths of tasks may vary from iteration to iteration in many applications, and the conventional designs are no longer being applicable. The main results reported in the recent literature considered the statistical property of trial lengths, i.e., a stochastic variable with uniform probability distribution. However, the probability is only used for the analysis, and the learning control design requires none prior information on it [30], [31]. An iteration-average operator was introduced for the ILC controller design in [30] such that control information of previous trials can be useful for performance improvement of current trial. The convergence both in almost sure and mean square senses was established in [31], as long as the probability of full-length iteration is not zero. In [32], the assumption describes the random variable of the varying-length iterations. By Lyapunov synthesis, it was shown that the adaptive ILC scheme is effective for parametric nonlinear systems in which the operation lengths vary randomly, under identical initial conditions.

Learning can be more efficient when the knowledge acquired is applied to the task that is similar to the learned ones. In [33], a time-scale interpolation method was proposed for the input torque profiles of robots obtained by learning control, finding the input profile for a new reference trajectory directly from the learned inputs corresponding to other trajectories. With the use of time/magnitude scaling, direct learning control strategies have been developed in [34], [35]. It was shown that the learning efficiency can be improved significantly by utilizing the existing control experiences. The main idea is that through both time and magnitude scaling, the trial-varying reference trajectories can be normalized to the trial-invariant ones, with equal trial lengths. This feature was borrowed for conducting the AILC control design, in [36], where the magnitude scaling is applied to make the trajectories be transformed into those with the same magnitude with respect to the same time scale, while the time scaling is used to normalize the non-equal trial lengths. For the contraction-mapping learning algorithms, in [37], the time-scale transformation was shown to work well for a class of nonlinear, control-affine systems where the trial duration is bounded, but unknown *a priori*. It is well known that one distinguished feature of AILC is that parameter learning, instead of input learning, allows trail-varying reference trajectories. Therefore, it is especially important to explore the AILC design method without the magnitude-scaling transformation, and characterize the convergence performance of the parameter learning algorithms. In addition, in the related works there is no effort made to address both non-equal trail lengths and non-zero initial errors at the same time. However, practical situations exist, e.g., a robotic manipulator which draws circles in Cartesian space with different radii and periods.

In this paper, we focus our attention on the time scaling technique for AILC control designs for tracking tasks with non-equal trial lengths, where the error-tracking approach is adopted in order to cope with initial errors. The performance analysis is conducted and theoretical results are presented in detail, by which the uniform convergence of the tracking error is established, and boundedness of the variables in the closed-loop is characterized. Numerical simulation is carried out to demonstrate effectiveness of the proposed learning control scheme.

The rest of this paper is organized as follows. In Section II, the ILC problem to be addressed is formulated, which allows non-equal trail lengths in the presence of initial errors. In Section III, we present the time scaling technique, the error-tracking control strategy, and the useful lemmas for convergence analysis. In Section IV, both the ILC control design and performance analysis are presented. In Section V, the learning control design method is extended to the high order nonlinear systems. In Section VI, the control performance in dealing with the non-identical tasks is verified by the numerical example. The conclusion is finally drawn in Section VII.

II. PROBLEM FORMULATION

For simplicity of presentation, we address the problem of trajectory tracking of a simple class of uncertain systems, as an illustrative example. We shall show that the proposed ILC strategy is able to learn from the non-identical tasks, and the obtained results will be extended to the higher-order systems.

Consider the following nonlinear system with parametric uncertainties:

$$\frac{1}{g}\frac{dx(t)}{dt} = u(t) + \theta^T \phi(x(t)), \tag{1}$$

where x is the scalar measurable state, u is the scalar control signal, g is the control gain of the system undertaken, $\theta \in R^m$ denotes a vector of unknown parameters, $\phi \in R^m$ represents a known state-dependent vector-valued function, satisfying that ϕ is bounded as the state is bounded.

The following assumption about system (1) is made.

Assumption 1: The sign of g is known, and without loss of generality, it is assumed that $g > 0$.

Taking into account the non-identical trajectories, in the presence of initial errors and non-equal lengths, the control objective of this paper is to make the system state $x_k(t_k)$ follow the given desired trajectory, $x_{d,k}(t_k), t_k \in [0, T_k]$, as close as possible when k approaches infinity, where t_k is the time index and $T_k > 0$ is the trial length of the kth trial. It is observed that, in comparison with the existing works devoted to ILC methods, the desired trajectories are even totally different between two cycles, since both the trial length T_k and the initial value of $x_{d,k}(t_k)$ are allowed to be iteration-varying.

Remark 1: It is worth noting that for an ILC system, its learning ability is obtained through the update action at each time instant along iteration axis, being able to learn from whatever is invariant with respect to iteration. Obviously, iterative learning algorithms are applicable for estimating constant unknowns involved in system (1), in the absence of initial errors, which allow the tasks to change with iterations, but require them to be equal in length.

III. PRELIMINARIES

In this paper, we focus our attention on developing a time-scaling learning strategy for system (1) to address the tracking control problem with non-equal-length tasks. One more challenge facing such a problem would be the presence of initial errors. We adopt the time scaling technique to handle non-identical tasks, while we use the error-tracking control strategy to solve the problem of initial errors. As such, the proposed learning control scheme works effectively for the trajectory tracking.

A. TIME SCALING

We begin with the description for the time scaling technique that would be useful for our control design to be presented. Let us denote T the virtual trial length, and define the time scaling as follows:

$$t = \frac{T}{T_k} t_k, \quad k = 0, 1, 2, \cdots$$

and

$$t_k = \frac{T_k}{T} t, \quad k = 0, 1, 2, \cdots$$

by which the trial intervals are scaled to be of a fixed duration, $T > 0$. Thereby the iterative learning can be carried out over an identical interval, i.e., $t \in [0, T]$.

Remark 2: For the developed learning schemes, let us choose the virtual length, T. In the case that all trial lengths $T_k, k = 0, 1, 2, \cdots$, are known, actually we can choose the length of any trial as the virtual length. In particular, two typical choices are:

i) $T = \min\{T_k, k = 0, 1, 2, \cdots\}$; and
ii) $T = \max\{T_k, k = 0, 1, 2, \cdots\}$.

For case i), a length contraction is needed such that the time interval is shortened from $[0, T_k]$ to $[0, T]$, whereas for case ii) a length stretch is required for each trial interval. The time scale is definitely a linear mapping for normalizing the non-equal trial lengths, which makes the problem solvable.

By applying the time scaling, the smooth desired trajectory, $x_{d,k}(t_k)$, can be written as

$$x_{d,k}(t_k) = x_{d,k}\left(\frac{T_k}{T} t\right), \quad k = 0, 1, 2, \cdots \tag{2}$$

By denoting $\xi_{d,k}(t) = x_{d,k}(T^{-1}T_k t)$, $\xi_{d,k}(t) = x_{d,k}(t_k)$. Then, system (1) at the kth iteration can be expressed by

$$\frac{1}{g}\frac{dx_k(t_k)}{dt_k} = u_k(t_k) + \theta^T \phi_k\left(x_k(t_k)\right), \tag{3}$$

where $t_k \in [0, T_k]$. Similarly, using the time scaling gives rise to

$$\frac{T}{gT_k}\frac{dx_k\left(\frac{T_k}{T}t\right)}{dt} = u_k\left(\frac{T_k}{T}t\right) + \theta^T \phi_k\left(x_k\left(\frac{T_k}{T}t\right)\right).$$

Denote $\xi_k(t) = x_k\left(T^{-1}T_k t\right)$, $v_k(t) = u_k\left(T^{-1}T_k t\right)$, and $\psi_k(t) = \phi_k\left(x_k\left(T^{-1}T_k t\right)\right)$. Then

$$\frac{T}{gT_k}\frac{d\xi_k(t)}{dt} = v_k(t) + \theta^T \psi_k(t), \tag{4}$$

where $t \in [0, T]$.

With the aid of the time scaling, the original form of system (3) is transformed into the normal form of (4), with an identical trial length $[0, T]$.

B. DESIRED ERROR TRAJECTORIES

Let us define the tracking error as

$$\bar{\xi}_k(t) = \xi_k(t) - \xi_{d,k}(t).$$

It follows from (4) that

$$\frac{T}{gT_k}\frac{d\bar{\xi}_k(t)}{dt} = v_k(t) + \theta^T \psi_k(t) - \frac{T}{gT_k}\frac{d\xi_{d,k}(t)}{dt}. \tag{5}$$

By the conventional design, the objective can be restated as that, through finding $v_k(t)$ for the error system (5), the tracking error $\bar{\xi}_k(t)$ is assured to converge to zero on $[0, T]$, as k increases. Furthermore, to address the initial condition problem, let us denote by $\tilde{\xi}_k(t)$ the error between $\bar{\xi}_k(t)$ and $\xi_k^*(t)$, where $\xi_k^*(t)$ indicates the desired error trajectory. Then the $\tilde{\xi}_k$-dynamics can be given as

$$\frac{T}{gT_k}\frac{d\tilde{\xi}_k(t)}{dt} = v_k(t) + \Theta^T \Psi_k(t), \tag{6}$$

with

$$\Theta = \left[\theta^T, \frac{1}{g}\right]^T,$$

$$\Psi_k = \left[\psi_k^T, -\frac{T}{T_k}\left(\frac{d\xi_{d,k}}{dt} + \frac{d\xi_k^*}{dt}\right)\right]^T.$$

Remark 3: In comparison with (3), the gain T/T_k appears in the left-hand side of system (6), indicates the impact of the non-equal trial lengths.

Now the control objective can be stated as follows: Find the input profile $v_k(t)$, $t \in [0, T]$, such that $\bar{\xi}_k(t)$ follows $\xi_k^*(t)$ as close as possible on $[0, T]$. It is seen that as $\bar{\xi}_k(t)$ converges to $\xi_k^*(t)$, in the presence of arbitrary initial errors, $\xi_k(t)$ will converge to $\xi_{d,k}(t) + \xi_k^*(t)$. The transient and steady-state specifications for the tracking error are the requirement for $\xi_k^*(t)$.

The following assumption is made, which is only a restriction on the setting of initial value of the desired error trajectory, $\xi_k^*(0)$.

Assumption 2: The value of $\xi_k^*(0)$ is set to satisfy

$$\xi_k^*(0) = \bar{\xi}_k(0), \tag{7}$$

where $\bar{\xi}_k(0)$ is the initial value of the tracking error, $\bar{\xi}_k(t)$.

Remark 4: Assumption 2 implies that $\tilde{\xi}_k(0) = 0$, which is only a restriction on the initial value of the desired error trajectory, but not pose any requirement for the tracking error itself. What's more, the initial value of the tracking error is allowed to be given arbitrarily but bounded. In turn, the system state is allowed to take arbitrary but bounded initial value. This gives a significant advantage because there is no restriction on the initial repositioning.

In order to satisfy Assumption 2, the desired error trajectory $\xi_k^*(t)$, $k = 0, 1, 2, \cdots$, can be formed as

$$\xi_k^*(t) = \begin{cases} \bar{\xi}_k(0)\zeta(t), & t \in [0, \Delta] \\ 0, & t \in (\Delta, T] \end{cases} \tag{8}$$

where Δ is the setting moment to connect the beginning position and the desired trajectory, and $\zeta(t)$ is taken as a smooth and monotonically decreasing function on $[0, \Delta]$, which satisfies $\zeta(0) = 1$ and $\zeta(\Delta) = 0$.

Remark 5: From (8), $\xi_k^*(t)$ is continuously differentiable on the interval $[0, T]$, only depending on two factors, $\bar{\xi}_k(0)$ and Δ. The initial error $\bar{\xi}_k(0)$ is determined by the actual state trajectory, while $\zeta(t)$ and Δ can be set to assure the convergence performance of the desired error trajectory. As $\bar{\xi}_k(t)$ completely converges to the pre-specified error trajectory $\xi_k^*(t)$ on $[0, T]$, ξ_k will completely track the given desired trajectory $\xi_{d,k}$, for all $t \in [\Delta, T]$. In turn, x_k completely tracks $x_{d,k}$, for all $t_k \in [T^{-1}T_k\Delta, T_k]$.

C. USEFUL LEMMAS

The following lemmas are helpful for the convergence analysis to be presented, where Lemma 1 is found in literature [38].

Lemma 1: Suppose $\{\sigma_k(t)\}$ is an equicontinuous sequence of function on $[0, T]$, and converges to zero pointwisely for each $t \in [0, T]$, as k approaches to infinity, then $\{\sigma_k(t)\}$ converges to zero uniformly on $[0, T]$, as k increases.

When applying Lemma 1, we need to check the equicontinuity of $\{\sigma_k(t)\}$, for which the following can be used.

Lemma 2: If $\dot{\sigma}_k(t)$ is uniformly bounded on $[0, T]$, then $\{\sigma_k(t)\}$ is equicontinuous on $[0, T]$.

Proof: By virtue of the mean value theorem, for $t', t'' \in [0, T]$,

$$\sigma_k(t') - \sigma_k(t'') = \dot{\sigma}_k(t''')(t' - t''),$$

with

$$t''' = \xi t' + (1 - \xi)t'', \quad \xi \in [0, 1].$$

Since $\dot{\sigma}_k(t)$ is uniformly bounded on $[0, T]$, there exists $M > 0$ such that $|\dot{\sigma}_k(t)| \leq M$ on $[0, T]$ and for all k. Then

$$|\sigma_k(t') - \sigma_k(t'')| = |\dot{\sigma}_k(t''')||t' - t''| \leq M|t' - t''|.$$

For every $\varepsilon > 0$, if we choose $\delta = \varepsilon/M$, then for $|t' - t''| \leq \delta$,

$$|\sigma_k(t') - \sigma_k(t'')| \leq \varepsilon.$$

Hence, $\sigma_k(t)$ is equicontinuous. ■

Lemma 3: If there exists $M > 0$ such that

$$\int_0^t \dot{\sigma}_k^2(s)ds \leq M \tag{9}$$

on $[0, T]$, then $\sigma_k(t)$ is equicontinuous on $[0, T]$.

Proof: By appealing to the Schwarz inequality, for $t', t'' \in [0, T]$,

$$|\sigma_k(t') - \sigma_k(t'')| = \left|\int_{t'}^{t''} \dot{\sigma}_k(s)ds\right| \leq \sqrt{\int_{t'}^{t''} 1^2 ds}\sqrt{\int_{t'}^{t''} \dot{\sigma}_k^2(s)ds}.$$

It follows from (9) that

$$|\sigma_k(t') - \sigma_k(t'')| \leq \sqrt{t'' - t'}\sqrt{M}. \tag{10}$$

Hence, $\sigma_k(t)$ is equicontinuous, if we choose $\delta = \varepsilon^2/M$, for every $\varepsilon > 0$. ■

Using Lemmas 2 and 3, the following convergence results can be established, on the basis of Lemma 1.

Lemma 4: For a sequence of function, $\{\sigma_k(t)\}$, which converges to zero pointwisely for each $t \in [0, T]$, as k approaches to infinity, if $\dot{\sigma}_k(t)$ is uniformly bounded on $[0, T]$, then $\{\sigma_k(t)\}$ converges to zero uniformly on $[0, T]$, as k increases.

Lemma 5: Suppose $\{\sigma_k(t)\}$ converges to zero pointwisely for each $t \in [0, T]$, as k approaches to infinity, and there exists $M > 0$ such that, for $t \in [0, T]$,

$$\int_0^t \dot{\sigma}_k^2(s)ds \leq M, \tag{11}$$

then $\{\sigma_k(t)\}$ converges to zero uniformly on $[0, T]$, as k increases.

IV. AILC DESIGN AND ANALYSIS

Taking into account the ILC problem of system (6) on the fixed time interval $[0, T]$, both the control design and performance analysis are presented in this section.

For the error system (6) at the kth iteration, we propose the following learning control law

$$v_k(t) = -\kappa\tilde{\xi}_k(t) - \Theta_k^T(t)\Psi_k(t), \tag{12}$$

where $\kappa > 0$ is the adjustable gain, and Θ_k is an estimate of the unknown parameter vector Θ, defined in (6). The learning law for updating Θ_k is as follows:

$$\begin{aligned}\Theta_k(t) &= \Theta_{k-1}(t) + \gamma\Psi_k(t)\tilde{\xi}_k(t),\\ \Theta_{-1}(t) &= 0, \quad t \in [0, T]\end{aligned} \tag{13}$$

where $\gamma > 0$ is the adjustable gain. With the control law (12), the $\tilde{\xi}_k$-dynamics can be expressed by

$$\frac{T}{gT_k}\frac{d\tilde{\xi}_k(t)}{dt} = -\kappa\tilde{\xi}_k(t) + \tilde{\Theta}_k^T(t)\Psi_k(t) \tag{14}$$

with $\tilde{\Theta}_k(t) = \Theta - \Theta_k(t)$ being the estimation error.

Now we summarize the stability and convergence results of the proposed AILC controller (12) and (13) in the following theorem.

Theorem 1: Let the control law (12) together with the learning law (13) be applied to system (6), under Assumptions 1 and 2. Then

i) $\tilde{\xi}_k(t)$, $\int_0^t \Theta_k^T(s)\Theta_k(s)ds$ and $\int_0^t v_k^2(s)ds$ are bounded, for each $t \in [0, T]$ and for all k; and

ii) $\tilde{\xi}_k(t)$ is guaranteed to converge to zero uniformly on $[0, T]$, as k goes to infinity.

Proof: The proof includes three parts.

Part A: Boundedness of Lyapunov-like function.

For the kth iteration, let us choose the following form of Lyapunov-like function,

$$L_k(t) = V(\tilde{\xi}_k(t)) + \frac{1}{2\gamma}\int_0^t \tilde{\Theta}_k^T(s)\tilde{\Theta}_k(s)ds \tag{15}$$

with

$$V(\tilde{\xi}_k(t)) = \frac{T}{2\,gT_k}\tilde{\xi}_k^2(t). \tag{16}$$

The difference of $L_k(t)$ between two consecutive iterations, $\Delta L_k(t) = L_k(t) - L_{k-1}(t)$, can be written as

$$\begin{aligned}\Delta L_k(t) &= V(\tilde{\xi}_k(t)) - V(\tilde{\xi}_{k-1}(t))\\ &\quad + \frac{1}{2\gamma}\int_0^t \left(\tilde{\Theta}_k^T(s)\tilde{\Theta}_k(s) - \tilde{\Theta}_{k-1}^T(s)\tilde{\Theta}_{k-1}(s)\right)ds.\end{aligned} \tag{17}$$

It follows from (14) that

$$\begin{aligned}\tilde{\xi}_k^2(t) &= \tilde{\xi}_k^2(0) + 2\int_0^t \tilde{\xi}_k(s)\dot{\tilde{\xi}}_k(s)ds\\ &= \tilde{\xi}_k^2(0) - \frac{2\kappa gT_k}{T}\int_0^t \tilde{\xi}_k^2(s)ds\\ &\quad + \frac{2gT_k}{T}\int_0^t \tilde{\Theta}_k^T(s)\Psi_k(s)\xi_k(s)ds\end{aligned}$$

implying that

$$\begin{aligned}V(\tilde{\xi}_k(t)) &= V(\tilde{\xi}_k(0)) - \kappa\int_0^t \tilde{\xi}_k^2(s)ds\\ &\quad + \int_0^t \tilde{\Theta}_k^T(s)\Psi_k(s)\xi_k(s)ds.\end{aligned} \tag{18}$$

For examining the integral term on the right-hand side of (17), we need the following relationship:

$$\begin{aligned}&\tilde{\Theta}_k^T(s)\tilde{\Theta}_k(s) - \tilde{\Theta}_{k-1}^T(s)\tilde{\Theta}_{k-1}(s)\\ &= -2\tilde{\Theta}_k^T(s)\left(\Theta_k(s) - \Theta_{k-1}(s)\right)\\ &\quad - \left(\Theta_k(s) - \Theta_{k-1}(s)\right)^T\left(\Theta_k(s) - \Theta_{k-1}(s)\right).\end{aligned} \tag{19}$$

Substituting (18) and (19) into (17) yields

$$\begin{aligned}&\Delta L_k(t)\\ &= -V(\tilde{\xi}_{k-1}(t)) + V(\tilde{\xi}_k(0))\\ &\quad - \kappa\int_0^t \tilde{\xi}_k^2(s)ds + \int_0^t \tilde{\Theta}_k^T(s)\Psi_k(s)\tilde{\xi}_k(s)ds\\ &\quad - \frac{1}{\gamma}\int_0^t \tilde{\Theta}_k^T(s)\left(\Theta_k(s) - \Theta_{k-1}(s)\right)ds\\ &\quad - \frac{1}{2\gamma}\int_0^t \left(\Theta_k(s) - \Theta_{k-1}(s)\right)^T\left(\Theta_k(s) - \Theta_{k-1}(s)\right)ds.\end{aligned}$$

Note that $V(\tilde{\xi}_k(0)) = (2\,gT_k)^{-1}T\tilde{\xi}_k^2(0) = 0$, according to Assumption 2. We obtain

$$\begin{aligned}&\Delta L_k(t)\\ &= -\frac{T}{2gT_{k-1}}\tilde{\xi}_{k-1}^2(t) - \kappa\int_0^t \tilde{\xi}_k^2(s)ds\\ &\quad + \int_0^t \frac{1}{\gamma}\tilde{\Theta}_k^T(s)\left(\gamma\Psi_k(s)\tilde{\xi}_k(s) - \Theta_k(s) + \Theta_{k-1}(s)\right)ds\\ &\quad - \frac{1}{2\gamma}\int_0^t \left(\Theta_k(s) - \Theta_{k-1}(s)\right)^T\left(\Theta_k(s) - \Theta_{k-1}(s)\right)ds.\end{aligned}$$

Applying (13) results in

$$\begin{aligned}&\Delta L_k(t)\\ &= -\frac{T}{2gT_{k-1}}\tilde{\xi}_{k-1}^2(t) - \kappa\int_0^t \tilde{\xi}_k^2(s)ds\\ &\quad - \frac{1}{2\gamma}\int_0^t \left(\Theta_k(s) - \Theta_{k-1}(s)\right)^T\left(\Theta_k(s) - \Theta_{k-1}(s)\right)ds\end{aligned}$$

and leads to

$$\Delta L_k(t) \leq -\frac{T}{2\,gT_{k-1}}\tilde{\xi}_{k-1}^2(t). \tag{20}$$

So far, the non-positivity of $\Delta L(t)$ along the iteration axis is confirmed, which gives rise to

$$L_k(t) \leq L_{k-1}(t). \tag{21}$$

Therefore, $L_k(t)$ is monotonically decreasing with respect to iteration for each $t \in [0, T]$.

In order to prove the boundedness of $L_0(t)$, we calculate the derivative of $L_0(t)$ as

$$\begin{aligned}
&\dot{L}_0(t)\\
&= \dot{V}(\tilde{\xi}_0(t)) + \frac{1}{2\gamma}\tilde{\Theta}_0^T(t)\tilde{\Theta}_0(t) - \frac{1}{2\gamma}\tilde{\Theta}_0^T(0)\tilde{\Theta}_0(0)\\
&= -\kappa\tilde{\xi}_0^2(t) + \tilde{\Theta}_0^T(t)\Psi_0(t)\tilde{\xi}_0(t) + \frac{1}{2\gamma}\tilde{\Theta}_0^T(t)\tilde{\Theta}_0(t)\\
&\quad - \frac{1}{2\gamma}\tilde{\Theta}_0^T(0)\tilde{\Theta}_0(0)\\
&\le \tilde{\Theta}_0^T(t)\Psi_0(t)\tilde{\xi}_0(t) + \frac{1}{2\gamma}\tilde{\Theta}_0^T(t)\tilde{\Theta}_0(t)\\
&= \frac{1}{\gamma}\tilde{\Theta}_0^T(t)\Theta_0(t) + \frac{1}{2\gamma}\tilde{\Theta}_0^T(t)\tilde{\Theta}_0(t)\\
&= \frac{1}{2\gamma}\Theta^T\Theta - \frac{1}{2\gamma}\Theta_0^T(t)\Theta_0(t)\\
&\le \frac{1}{2\gamma}\Theta^T\Theta.
\end{aligned}$$

Since $L_0(0) = V(\tilde{\xi}_0(0)) = 0$, then

$$L_0(t) \le L_0(0) + \frac{T}{2\gamma}\Theta^T\Theta = \frac{T}{2\gamma}\Theta^T\Theta, \tag{22}$$

which implies the finiteness of $L_0(t)$ on $[0, T]$. In turn the uniform boundedness of $L_k(t)$ on $[0, T]$ can be immediately established by (21) and (22).

Part B: Boundedness of the variables.

According to the definition of $L_k(t)$, the uniform boundedness of $\tilde{\xi}_k(t)$ on $[0, T]$ is obtained, and $\int_0^t \tilde{\Theta}_k^T(s)\tilde{\Theta}_k(s)ds$ is bounded on $[0, T]$ for all k. Since

$$\Theta_k^T(t)\Theta_k(t) \le 2\tilde{\Theta}_k^T(t)\tilde{\Theta}_k(t) + 2\Theta^T\Theta,$$

then $\int_0^t \Theta_k^T(s)\Theta_k(s)ds$ is bounded on $[0, T]$ for all k as well. It follows from (12) that

$$\begin{aligned}
|v_k(t)| &\le \kappa\left|\tilde{\xi}_k(t)\right| + \|\Theta_k(t)\|\,\|\Psi_k(t)\|\\
&\le c_1 + c_2\,\|\Theta_k(t)\|,
\end{aligned}$$

where $c_1 = \kappa \sup\left|\tilde{\xi}_k(t)\right|$ and $c_2 = \sup\|\Psi_k(t)\|$. In view of the fact that $(a+b)^2 \le 2\,a^2 + 2\,b^2$, we have

$$v_k^2(t) \le 2c_1^2 + 2\,c_2^2\,\|\Theta_k(t)\|^2.$$

Hence, the boundedness of $\int_0^t v_k^2(s)ds$ follows by noting that

$$\int_0^t v_k^2(s)ds \le 2\,c_1^2\,T + 2\,c_2^2\int_0^t \|\Theta_k(s)\|^2\,ds < +\infty.$$

Part C: Convergence of the tracking error.

Again using (20) we obtain

$$\begin{aligned}
L_k(t) &= \Delta L_k(t) + L_{k-1}(t)\\
&\ \vdots\\
&= \sum_{j=1}^{k}\Delta L_j(t) + L_0(t)\\
&\le L_0(t) - \sum_{j=1}^{k}\frac{T}{2\,gT_{j-1}}\tilde{\xi}_{j-1}^2(t)
\end{aligned}$$

implying that

$$\sum_{j=1}^{k}\frac{T}{2\,gT_{j-1}}\tilde{\xi}_{j-1}^2(t) \le L_0(t).$$

By the necessary condition of convergence of series, the pointwise convergence of $\tilde{\xi}_k^2(t)$ can be established, due to the finiteness of $L_0(t)$. Namely, $\lim_{k\to\infty}\tilde{\xi}_k^2(t) = 0$ pointwisely for each $t \in [0, T]$.

It follows from (14) that

$$\begin{aligned}
\left|\frac{d\tilde{\xi}_k(t)}{dt}\right| &\le \frac{\kappa gT_k}{T}|\tilde{\xi}_k(t)| + \frac{gT_k}{T}\|\tilde{\Theta}_k(t)\|\,\|\Psi_k(t)\|\\
&\le c_3 + c_4\|\tilde{\Theta}_k(t)\|,
\end{aligned}$$

where $c_3 = \kappa gT_kT^{-1}\sup|\tilde{\xi}_k(t)|$ and $c_4 = gT_kT^{-1}\sup\|\Psi_k(t)\|$. Hence,

$$\left|\frac{d\tilde{\xi}_k(t)}{dt}\right|^2 \le 2c_3^2 + 2c_4^2\|\tilde{\Theta}_k(t)\|^2$$

implying that

$$\int_0^t\left|\frac{d\tilde{\xi}_k(s)}{ds}\right|^2 ds \le 2c_3^2\,T + 2c_4^2\int_0^t\|\tilde{\Theta}_k(s)\|^2\,ds.$$

Therefore, $\tilde{\xi}_k(t)$ is equicontinuous on $[0, T]$. In virtue of Lemma 5, $\tilde{\xi}_k(t)$ converges to zero uniformly $[0, T]$, as k increases, i.e.,

$$\lim_{k\to\infty}\tilde{\xi}_k(t) = 0 \ \text{ uniformly on } [0, T].$$

This completes the proof. ■

Remark 6: By Theorem 1, due to the time scaling technique, the uniform boundedness of $\tilde{\xi}_k(t)$ and $x_k(t)$ can be also derived. $\Theta_k(t)$ is assured to be bounded in the sense of $L_2[0, T]$. So $v_k(t)$ and $u_k(t)$ are. It should be noted that both $\Theta_k(t)$ and $v_k(t)$ are not guaranteed to be bounded in the sense of $L_\infty[0, T]$. So $u_k(t)$ is. Nevertheless, by exploiting the boundedness results, the uniform convergence of $\tilde{\xi}_k(t)$ on $[0, T]$ is established in Theorem 1, which leads to the uniform convergence of $\bar{\xi}_k(t)$ on $[\Delta, T]$. Moreover, $x_k(t_k)$ is assured to converge to $x_{d,k}(t_k)$ uniformly on $[T_k/T\Delta, T_k]$, as $k \to \infty$.

V. AILC FOR HIGHER-ORDER SYSTEMS

In this section, the design method is extended to the following class of nth-order SISO nonlinear systems

$$\begin{cases}
\dot{x}_i = x_{i+1},\ i = 1, 2, \cdots, n-1\\
\frac{1}{g}\dot{x}_n = u + \theta^T\phi(x)\\
y = x_1
\end{cases} \tag{23}$$

where $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the state vector, u and y are the scalar input and output of the system, respectively, and g is the control gain, $\theta \in R^m$ denotes the unknown parameter vector, $\phi \in R^m$ represents the known state-dependent vector-valued function. The dynamics at the kth iteration is given by, for $t_k \in [0, T_k]$,

$$\begin{cases} \frac{dx_{i,k}(t_k)}{dt_k} = x_{i+1,k}(t_k), i = 1, 2, \cdots, n-1 \\ \frac{1}{g}\frac{dx_{n,k}(t_k)}{dt_k} = u_k(t_k) + \theta^T \phi_k(x_k(t_k)) \\ y_k(t_k) = x_{1,k}(t_k) \end{cases} \tag{24}$$

Let us apply the time scaling technique for addressing the non-repeatable learning control problem. The given desired trajectory $y_{d,k}(t_k)$, $t_k \in [0, T_k]$, can be written as

$$y_{d,k}(t_k) = y_{d,k}\left(\frac{T_k}{T}t\right). \tag{25}$$

Defining $\xi_{d,k}(t) = y_{d,k}(T^{-1}T_k t) = y_{d,k}(t_k)$, $\xi_{i,k}(t) = x_{i,k}(T^{-1}T_k t)$, $\bar{y}_k(t) = y_k(T^{-1}T_k t)$, $v_k(t) = u_k(T^{-1}T_k t)$, and $\psi_k(t) = \phi_k\left(x_k\left(T^{-1}T_k t\right)\right)$, the error system (24) can be rewritten as

$$\begin{cases} \frac{T}{T_k}\frac{d\xi_{i,k}(t)}{dt} = \xi_{i+1,k}(t), i = 1, 2, \cdots, n-1 \\ \frac{T}{gT_k}\frac{d\xi_{n,k}(t)}{dt} = v_k(t) + \theta^T \psi_k(t) \\ \bar{y}_k(t) = \xi_{1,k}(t) \end{cases} \tag{26}$$

where $t \in [0, T]$.

To handle the control design for the high-order system (26), the filtered error $\xi_{f,k}$ is defined as

$$\xi_{f,k}(t) = [\Lambda^T \; 1]\bar{\xi}_k(t) \tag{27}$$

and the state error is defined as $\bar{\xi}_k(t) = \xi_k(t) - \bar{\xi}_{d,k}(t) = [\bar{\xi}_{1,k}(t), \bar{\xi}_{2,k}(t), \cdots, \bar{\xi}_{n,k}(t)]^T$, with $\xi_k(t) = [\xi_{1,k}(t), \xi_{2,k}(t), \cdots, \xi_{n,k}(t)]^T$ and $\bar{\xi}_{d,k}(t) = [\xi_{d,k}(t), T_k^{-1}T \dot{\xi}_{d,k}(t), \cdots, (T/T_k)^{n-1}\xi_{d,k}^{(n-1)}(t)]^T$. And $\Lambda = [\lambda_1, \lambda_2, \cdots, \lambda_{n-1}]^T$ is chosen such that polynomial $s^{n-1} + \lambda_{n-1}s^{n-2} + \cdots + \lambda_1$ is Hurwitz. Besides, let us denote by $\tilde{\xi}_{f,k}$ the error between $\xi_{f,k}$ and $\xi^*_{f,k}$, where $\xi^*_{f,k}$ is the desired filtered error trajectory. As such, the $\tilde{\xi}_{f,k}$-dynamics can then be expressed as

$$\frac{T}{gT_k}\frac{d\tilde{\xi}_{f,k}(t)}{dt} = v_k(t) + \Theta^T \Psi_k(t), \tag{28}$$

where

$$\Theta = [\theta^T, \frac{1}{g}]^T,$$
$$\Psi_k = [\psi_k^T, \nu_k]^T,$$
$$\nu_k(t) = [0 \;\; \Lambda^T]\bar{\xi}_k(t) - \left(\frac{T}{T_k}\right)^n \xi_{d,k}^{(n)}(t) - \frac{T}{T_k}\frac{d\xi^*_{f,k}(t)}{dt}.$$

In order to achieve the control objective, we need to make the following assumption, a restriction on the setting of initial value of the desired filtered error trajectory.

Assumption 3: The initial value of the desired filtered error trajectory, $\xi^*_{f,k}(0)$ is set to satisfy

$$\xi^*_{f,k}(0) = \xi_{f,k}(0), \tag{29}$$

where $\xi_{f,k}(0)$ is the initial value of the filtered error $\xi_{f,k}$.

According to the definition of $\xi_{f,k}$, a similar formulation of the desired filtered error trajectory $\xi^*_{f,k}$ to (8) is obtained:

$$\xi^*_{f,k}(t) = \begin{cases} \xi_{f,k}(0)\zeta(t), & t \in [0, \Delta] \\ 0, & t \in (\Delta, T] \end{cases} \tag{30}$$

Remark 7: In fact, practical systems usually keep static at the beginning of operation so that the initial setting can be simplified. For the case where $\bar{\xi}_{i,k}(0) = 0, i = 2, 3, \cdots, n$, the desired filtered error trajectory $\xi^*_{f,k}$ can be formed as

$$\xi^*_{f,k}(t) = \begin{cases} \bar{\xi}_{1,k}(0)\zeta(t), & t \in [0, \Delta] \\ 0, & t \in (\Delta, T] \end{cases}$$

where $\zeta^{(i-1)}(0) = 0, i = 2, 3, \cdots, n$.

Theorem 2: Consider the adaptive iterative learning system described by the error system (28) and the controller,

$$v_k(t) = -\kappa\tilde{\xi}_{f,k}(t) - \Theta_k^T(t)\Psi_k(t) \tag{31}$$

together with the fully-saturated learning law given as, for $t \in [0, T]$,

$$\Theta_k(t) = \text{sat}(\Theta_k^*(t))$$
$$\Theta_k^*(t) = \Theta_{k-1}^*(t) + \gamma\Psi_k(t)\tilde{\xi}_{f,k}(t) \tag{32}$$

and $\Theta_{-1}(t) = 0$. Under Assumptions 1 and 3, the error $\tilde{\xi}_{f,k}$ is guaranteed to converge to zero uniformly, as the iteration number k goes to infinity, while $\tilde{\xi}_{f,k}(t)$, $\Theta_k(t)$ and $v_k(t)$ are uniformly bounded on $[0, T]$ and for all k.

Proof: Let us choose the following Lyapunov-like function

$$L_k(t) = V(\tilde{\xi}_{f,k}(t)) + \frac{1}{2\gamma}\int_0^t \tilde{\Theta}_k^T(s)\tilde{\Theta}_k(s)ds,$$

and

$$V\left(\tilde{\xi}_{f,k}(t)\right) = \frac{T}{2\,gT_k}\tilde{\xi}_{f,k}^2(t).$$

Differentiating $V\left(\tilde{\xi}_{f,k}(t)\right)$ with respect to time along (28), we obtain

$$\frac{dV(\tilde{\xi}_{f,k}(t))}{dt} = \tilde{\xi}_{f,k}(t)\left(v_k(t) + \Theta^T\Psi_k(t)\right). \tag{33}$$

Substituting (31) into (33) results in

$$\frac{dV(\tilde{\xi}_{f,k}(t))}{dt} = -\kappa\tilde{\xi}_{f,k}^2(t) + \tilde{\Theta}_k^T(t)\Psi_k(t)\tilde{\xi}_{f,k}(t). \tag{34}$$

Integrating both sides of (34) from 0 to t yields, by noting that $V(\tilde{\xi}_{f,k}(0)) = (2\,gT_k)^{-1}T\tilde{\xi}_{f,k}^2(0) = 0$, according to Assumption 3,

$$V(\tilde{\xi}_{f,k}(t))$$
$$= V(\tilde{\xi}_{f,k}(0)) - \kappa\int_0^t \tilde{\xi}_{f,k}^2(s)ds + \int_0^t \tilde{\Theta}_k^T(s)\Psi_k(s)\tilde{\xi}_{f,k}(s)ds$$
$$= -\kappa\int_0^t \tilde{\xi}_{f,k}^2(s)ds + \int_0^t \tilde{\Theta}_k^T(s)\Psi_k(s)\tilde{\xi}_{f,k}(s)ds. \tag{35}$$

Using (19) gives rise to

$$\tilde{\Theta}_k^T(s)\tilde{\Theta}_k(s) - \tilde{\Theta}_{k-1}^T(s)\tilde{\Theta}_{k-1}(s) = -2\tilde{\Theta}_k^T(s)\left(\Theta_k(s) - \Theta_{k-1}(s)\right). \quad (36)$$

By (35) and (36), the difference $\Delta L_k(t)$ can be expressed by

$$\begin{aligned}\Delta L_k(t) \leq& -\frac{T}{2gT_{k-1}}\tilde{\xi}_{f,k-1}^2(t) - \kappa\int_0^t \tilde{\xi}_{f,k}^2(s)ds \\ &+ \int_0^t \tilde{\Theta}_k^T(s)\Psi_k(s)\tilde{\xi}_{f,k}(s)ds \\ &- \int_0^t \frac{1}{\gamma}\tilde{\Theta}_k^T(s)\left(\Theta_k(s) - \Theta_{k-1}(s)\right)ds \\ =& -\frac{T}{2gT_{k-1}}\tilde{\xi}_{f,k-1}^2(t) - \kappa\int_0^t \tilde{\xi}_{f,k}^2(s)ds \\ &+ \int_0^t \tilde{\Theta}_k^T(s)\Psi_k(s)\tilde{\xi}_{f,k}(s)ds \\ &- \int_0^t \frac{1}{\gamma}\tilde{\Theta}_k^T(s)\left(\Theta_k^*(s) - \Theta_{k-1}(s)\right)ds \\ &- \int_0^t \frac{1}{\gamma}\tilde{\Theta}_k^T(s)\left(\Theta_k(s) - \Theta_k^*(s)\right)ds.\end{aligned}$$

Applying learning law (32), we obtain

$$\begin{aligned}\Delta L_k(t) \leq& -\frac{T}{2gT_{k-1}}\tilde{\xi}_{f,k-1}^2(t) - \kappa\int_0^t \tilde{\xi}_{f,k}^2(s)ds \\ &+ \frac{1}{\gamma}\int_0^t \tilde{\Theta}_k^T(s)\left(\Theta_k^*(s) - \Theta_k(s)\right)ds.\end{aligned}$$

By the following relationship [7]:

$$\left(\Theta - \Theta_k\right)^T\left(\Theta_k - \Theta_k^*\right) \geq 0, \quad (37)$$

the difference $\Delta L_k(t)$ can be given as

$$\Delta L_k(t) \leq -\frac{T}{2\,gT_{k-1}}\tilde{\xi}_{f,k-1}^2(t) \quad (38)$$

from which it is concluded that $L_k(t)$ is monotonically decreasing.

The derivative of $L_0(t)$ can be calculated as, in order to prove the finiteness of $L_0(t)$,

$$\begin{aligned}\dot{L}_0(t) =& \dot{V}(\tilde{\xi}_{f,0}(t)) + \frac{1}{2\gamma}\tilde{\Theta}_0^T(t)\tilde{\Theta}_0(t) - \frac{1}{2\gamma}\tilde{\Theta}_0^T(0)\tilde{\Theta}_0(0) \\ =& -\kappa\tilde{\xi}_{f,0}^2(t) + \tilde{\Theta}_0^T(t)\Psi_0(t)\tilde{\xi}_{f,0}(t) + \frac{1}{2\gamma}\tilde{\Theta}_0^T(t)\tilde{\Theta}_0(t) \\ &- \frac{1}{2\gamma}\tilde{\Theta}_0^T(0)\tilde{\Theta}_0(0) \\ \leq& \tilde{\Theta}_0^T(t)\Psi_0(t)\tilde{\xi}_{f,0}(t) + \frac{1}{2\gamma}\tilde{\Theta}_0^T(t)\tilde{\Theta}_0(t).\end{aligned}$$

Applying learning law (32), for $k = 0$,

$$\begin{aligned}\dot{L}_0(t) \leq& \frac{1}{\gamma}\tilde{\Theta}_0^T(t)(\Theta_0^*(t) - \Theta_0(t)) \\ &+ \frac{1}{\gamma}\tilde{\Theta}_0^T(t)\Theta_0(t) + \frac{1}{2\gamma}\tilde{\Theta}_0^T(t)\tilde{\Theta}_0(t) \\ \leq& \frac{1}{\gamma}\tilde{\Theta}_0^T(t)\Theta_0(t) + \frac{1}{2\gamma}\tilde{\Theta}_0^T(t)\tilde{\Theta}_0(t),\end{aligned}$$

where inequality (37) is used. It follows that

$$\begin{aligned}\dot{L}_0(t) \leq& \frac{1}{\gamma}\Theta^T\Theta_0(t) - \frac{1}{\gamma}\Theta_0^T(t)\Theta_0(t) \\ &+ \frac{1}{2\gamma}\left(\Theta^T\Theta - 2\Theta^T\Theta_0(t) + \Theta_0^T(t)\Theta_0(t)\right) \\ =& \frac{1}{2\gamma}\Theta^T\Theta - \frac{1}{2\gamma}\Theta_0^T(t)\Theta_0(t) \\ \leq& \frac{1}{2\gamma}\Theta^T\Theta\end{aligned} \quad (39)$$

by which the finiteness of $L_0(t)$ on $[0, T]$ can be established. Hence $L_k(t)$ is uniformly boundedness on $[0, T]$, due to its monotonically decreasing property.

With similar lines to those in the proof for Theorem 1, we can prove the conclusions. ■

Remark 8: By Theorem 2, not only the uniform boundedness of $\bar{\xi}_k(t)$ and $x_k(t)$ can be derived, but also both $\Theta_k(t)$ and $v_k(t)$ are guaranteed to be uniform bounded. This benefits from the use of the saturated-learning algorithm. By exploiting the uniform boundedness results, in Theorem 2 the uniform convergence of $\tilde{\xi}_k(t)$ on $[0, T]$ is established, resulting in the uniform convergence of $\bar{\xi}_k(t)$ on $[\Delta, T]$. Moreover, $x_k(t_k)$ is assured to converge to $x_{d,k}(t_k)$ uniformly on $[T_k/T\Delta, T_k]$, and $y_k(t_k)$ is guaranteed to converge to $y_{d,k}(t_k)$ on $[T_k/T\Delta, T_k]$, as $k \to +\infty$.

Remark 9: Both problems, the non-equal trial lengths and the non-zero initial errors, are practical challenges. In the published works, the equal path is usually assumed, when addressing the initial condition problem; the zero-error initial errors are assumed when addressing the non-equal trial lengths. In this paper, we explore the AILC design method without the magnitude-scaling transformation, for verifying the applicability of only the time scaling technique for the AILC controller designs. The current results are the convergence of learning control schemes in the sense of L_2. In this paper, the uniform convergence of the proposed learning algorithms is established. It is shown that the fully-saturated learning plays an important role in establishing uniform convergence. In addition, the novel form of Lyapunov function is proposed, and it is shown to be efficient to solve the problem due to such iteration-dependent dynamics.

VI. NUMERICAL SIMULATION

In order to verify effectiveness of the error-tracking AILC method, the numerical simulation is carried out, and the control performance in dealing with the non-identical tasks is illustrated in this section.

For the purpose of simulation, the illustrative system is described in the form of system (3), with $g = 0.5$, $\theta = [0.5\ 0.1\ 0.3]^T$, and $\phi_k(x_k(t_k)) = [0.2\sin(x_k(t_k))\ 0.1x_k(t_k)\ 0.2 - 0.3\cos(x_k^2(t_k))]^T$. The initial value of the system state x_k is randomly set between $[-0.8, 0.8]$. The control objective is to make the output $x_k(t_k)$ follow the iteration-varying control tasks $x_{d,k}(t_k) = A_k(1 - \cos(\pi t_k)) + B_k t_k^2, k = 0, 1, 2, \cdots, N$, where

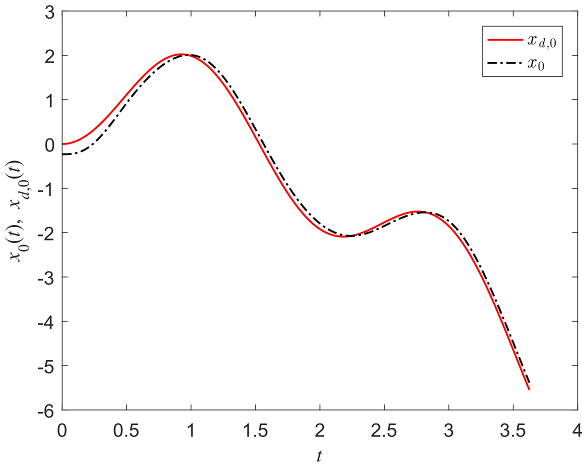

FIGURE 1. State and the desired trajectory as $k = 0$.

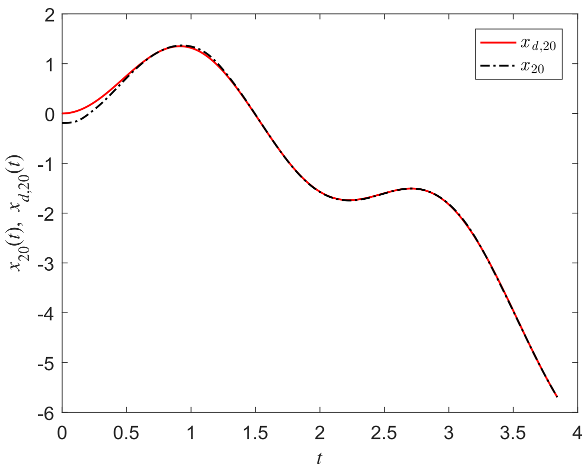

FIGURE 2. State and the desired trajectory as $k = 20$.

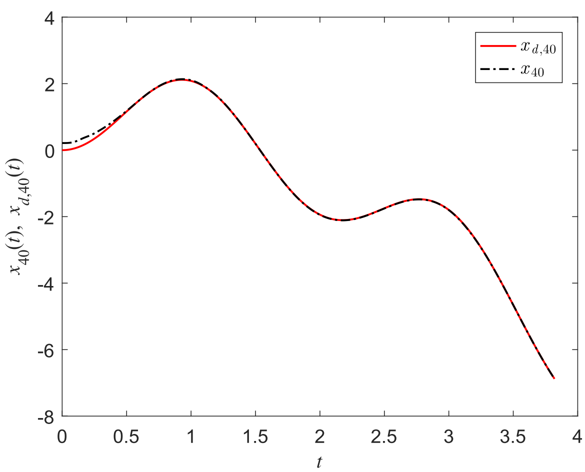

FIGURE 3. State and the desired trajectory as $k = 40$.

$A_k \in [0.4, 2.4]$ and $B_k \in [-1, 1]$ take values which are randomly generated. $N = 100$ is the number of trials, the trial length of kth operation is distributed on $[3.5, 4]$.

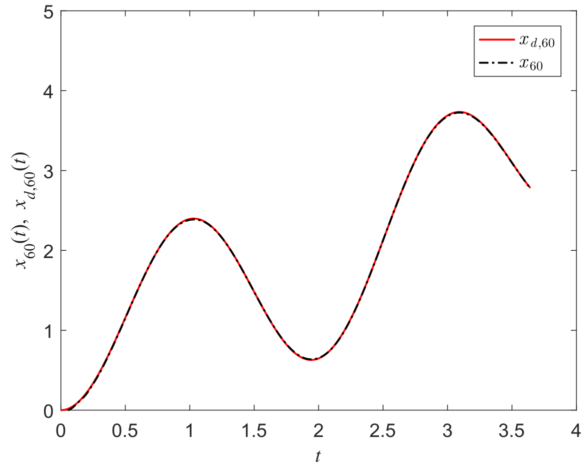

FIGURE 4. State and the desired trajectory as $k = 60$.

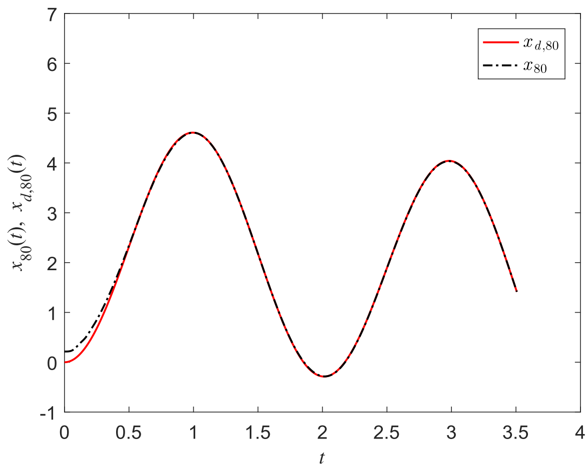

FIGURE 5. State and the desired trajectory as $k = 80$.

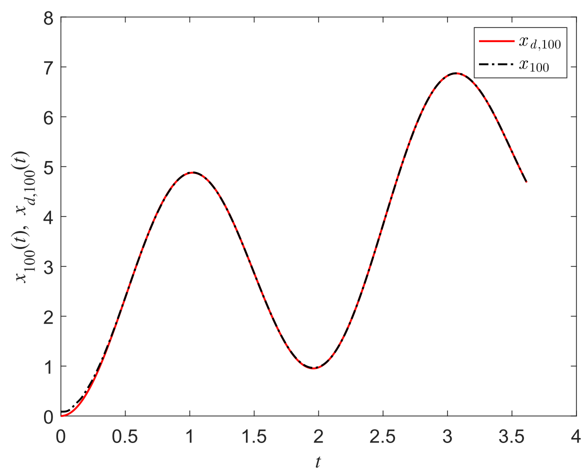

FIGURE 6. State and the desired trajectory as $k = 100$.

The virtual length is selected as $T = \min\{T_k\}$. The controller and the learning law applied in this example are given by (12) and (13), with the parameter settings: $\kappa = 30, \gamma = 20$ and $\Delta = 0.6$.

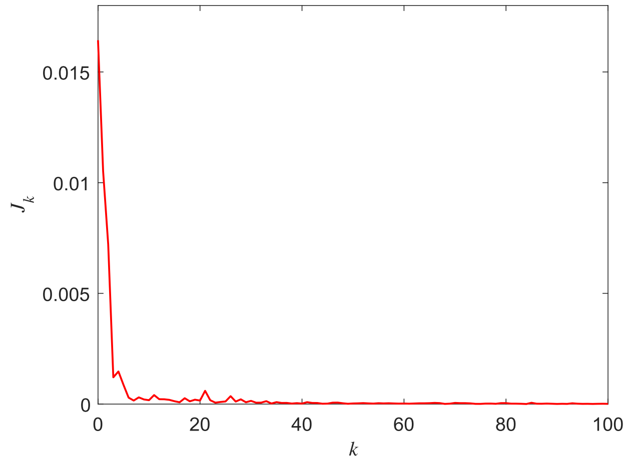

FIGURE 7. Learning histories.

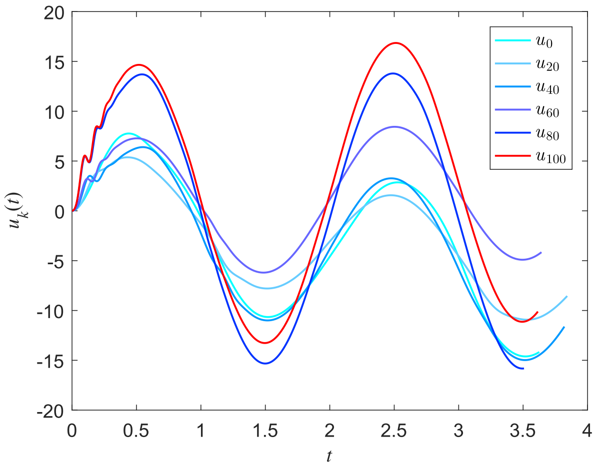

FIGURE 8. Input profiles.

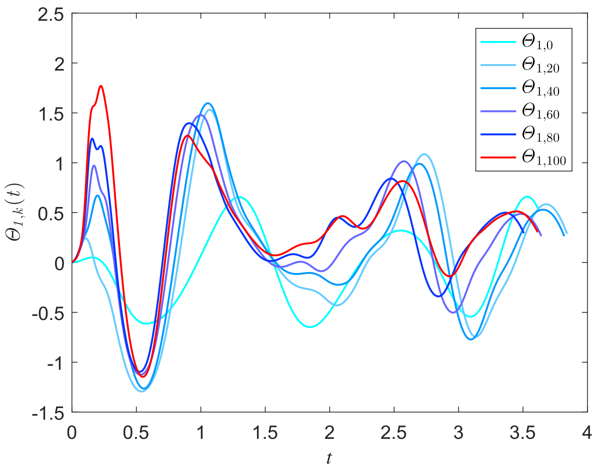

FIGURE 9. Parameter estimate $\Theta_{1,k}$.

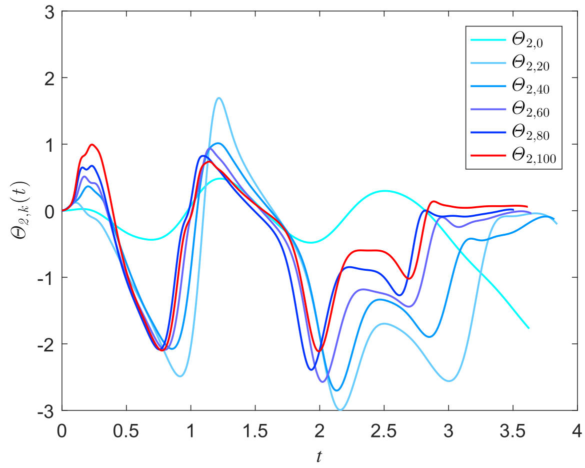

FIGURE 10. Parameter estimate $\Theta_{2,k}$.

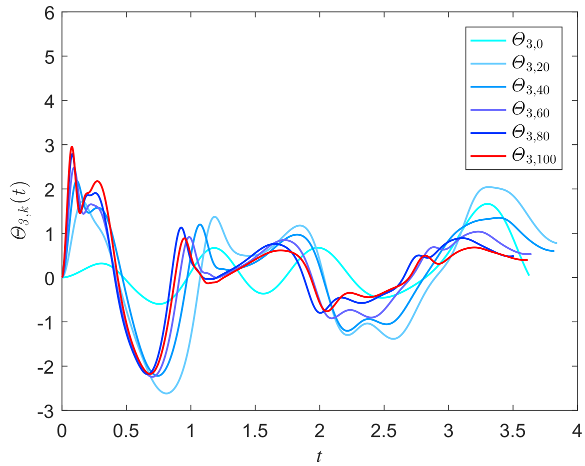

FIGURE 11. Parameter estimate $\Theta_{3,k}$.

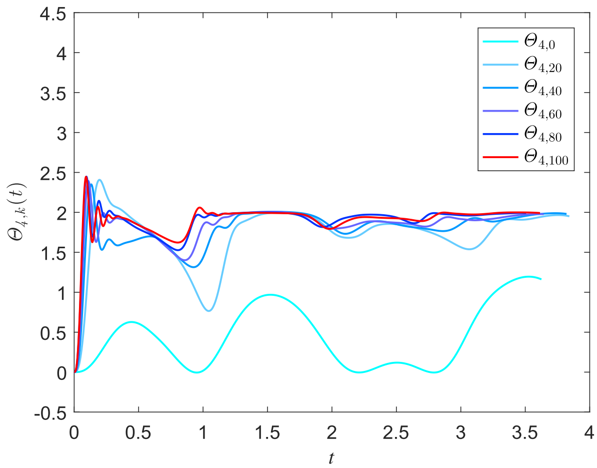

FIGURE 12. Parameter estimate $\Theta_{4,k}$.

The simulation results are obtained and shown in Figs. 1-12. Figs. 1-6 depict the output profiles when $k = 0, 20, 40, 60, 80$, and 100, respectively, showing the convergence history of iteration. The mean squared error $J_k = \frac{1}{M}\sum_{i=1}^{M}(e_k(i))^2$ is adopted as the performance index, shown in Fig. 7, where $e_k = x_k - x_{d,k}$ on the time interval $[T_k/T\Delta, T_k]$.

It is seen that in the presence of initial errors, the actual outputs track the desired trajectories after the pre-specified initial intervals, even if the tasks vary from iteration to iteration in both magnitude and time scales. In addition, the resultant input profiles are shown in Fig. 8. The parameter estimates ($\Theta_k = [\Theta_{1,k}, \Theta_{2,k}, \Theta_{3,k}, \Theta_{4,k}]^T$) are shown in Figs. 9-12.

VII. CONCLUSION

In this paper, the adaptive iterative learning control design method is presented for systems performing tasks with non-equal trial lengths and initial errors. The time scaling technique is used for normalization of non-equal paths, by which the tasks are transformed to those with the same time scale. The error-tracking approach is adopted for coping with initial errors, by which the assumption of identical initial condition is removed. Note that identical initial condition is common in the conventional learning control designs. The proposed control schemes assure the uniform convergence of the tracking error over the operation interval, being different from the pointwise convergence results reported in the published works. Theoretical results of establishing boundedness of the variables in the closed-loop are presented in detail. It has been shown that the fully-saturated learning algorithm plays an important role in assuring uniform boundedness of the control input. The proposed control schemes do not require the magnitude transformation, and have been shown to work well, with only the time scaling technique, by which the expected tracking performance can be obtained for the non-equal length tasks in the presence of initial errors.

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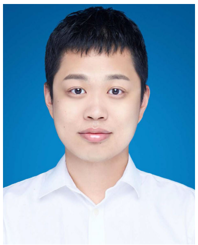

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