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Nonlinear Characteristic of Clamp Loosing in Aero-Engine Pipeline System

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ABSTRACT Long service time and excessive vibration of aero-engine will cause failure of clamps, which usually fix pipelines on the outside of an aero-engine. In this paper, the nonlinear vibration characteristics caused by loosening clamps in aero-engine pipeline are determined. According to the real parameters of an aero-engine clamp-pipeline test rig, the clamp is simplified into a nonlinear spring-damping. Thus, a clamp-pipeline nonlinear model based on cubic stiffness is established. The multi-scale method is used to obtain the analytical solution of the nonlinear model. Based on the real test data, the model parameters of the clamp-pipeline nonlinear system are identified, and then the nonlinear vibration characteristics of the system are obtained. Finally, using the vibration transmissibility method, the influence of the severity of clamp loosening on the nonlinear vibration characteristics of the system is obtained.

INDEX TERMS Aero-engine, clamp-pipeline system, duffing equation, nonlinear vibration.

I. INTRODUCTION

The pipelines system is an important external accessory to ensure normal operation of an aero-engine. Due to installation factors or the impact of external environmental vibration, there is greater possibility that the clamp in the pipeline system become loose. The main reasons contain: (1) The tightening force of bolts is not enough, resulting in a relative displacement between clamp and pipeline; (2) The vibration of an aero-engine is too large, causing the clamp to loosen; (3) The deviation of clamp's structural size causes a clearance between clamp and pipeline after installation; (4) The performance of elastic material is degraded, resulting in a clearance between clamp and pipeline [1]. In vibrating environment, the loosening of a clamp will increase the risk of other faults. Therefore, the research on the nonlinear vibration characteristics caused by loosening of a clamp has important engineering value.

The external pipeline system of aero-engines generally has a long span. According to the design and installation requirements of aero pipeline system, long-span pipeline must be clamped at regular intervals. In a long-span pipeline, for a certain section, especially an important section that needs to analyze or deal with vibration problems, the clamps at both ends and the pipeline form a typical clamp-pipeline system [2]. The analysis of a clamp-pipeline system avoids the difficulty of global modeling and analysis [3]. In many cases, with reasonable truncation and simplification of a pipeline system, the analysis results are more reasonable, and can fundamentally reveal the vibration behavior and vibration mechanism of a clamp-pipeline system.

A clamp-pipeline system has typical nonlinear vibration characteristics. The vibration-based nonlinear analysis methods generally include analytical methods and finite element analysis methods [4], [5]. In recent years, new nonlinear analysis methods such as harmonic balance method(HBM) and nonlinear output frequency response functions(NOFRFS) have also been developed [6], [7]. For the nonlinear vibration characteristics of the clamp-pipeline system, scholars have also proposed many new effective analysis methods. Gao *et al.* [8] proposed a model reduction method, and used this method to analyze the vibration of a hydraulic pipeline with long distance and multi-supports with elasticity. The results show that the natural frequencies and mode shapes of the reduced model are the same as full FE, but the calculation is faster. Guo *et al.* [9] proposed

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a non-intrusive multi-dimensional Chebyshev polynomial approximation method (M-CPAM), which is used to study the frequency response of a clamp-pipeline system. The results show that under the same pre-tightening force, the stiffness of a clamp has greater dispersion, and with the increase of tightening torque, the dispersion of the clamp-pipeline system tends to be concentrated. Quyang *et al.* [10] analyzed a clamp-pipeline system using modal analysis methods, and the results showed that the longer the pipeline length between two clamp supports, the higher the resonance frequency of the pipeline. Liu *et al.* [11] proposed a Semi-Analytical Model and Genetic Algorithm, and used this algorithm to optimize the position of the clamps in a pipeline system, thereby effectively reducing the resonance amplitude of a clamp-pipeline system. Rocha and Rachid [12] based on Glimm's method and an operator splitting technique, proposed a new numerical procedure, which was used to solve an eight-equation fluid-structure interaction (FSI) model used in transient analyses of piping systems. Wu *et al.* [13] proposed a ''numerical'' (or vector) mode approach incorporated with the transfer matrix method, and applied the method to a ''aperiodic'' clamp-pipeline system excited by hydraulic. Rong *et al.* [14] based on absolute nodal coordinate (ANC) formulation and transfer matrix method (TMM), proposed a novel efficient Riccati ANC–TMM, which is used in the nonlinear dynamic analysis of pipe conveying fluid with large deformations. The results show that the calculation efficiency of this method is significantly improved compared to the ordinary ANC method. These literatures only use nonlinear analysis methods to describe the vibration characteristics of the clamp system, but does not explain the nature of the clamp loosening fault.

It is an effective method by establishing a reasonable physical model to explore the nature of the clamp loosening failure. By establishing a precise clamp-pipeline system model based on physical relationships, the influence of system parameters on vibration response can be explored intuitively. Based on the Euler–Bernoulli beam theory and the nonlinear Lagrange strain theory, Lee and Chung [15] established a new nonlinear vibration model of a pipeline system with fixed ends at both ends. Paidoussis and Li [16] studied the nonlinear dynamic behavior of a cantilever infusion straight pipe with nonlinear spring support, and analyzed the influence of clamp parameters on the instability of the pipeline system. Meng *et al.* [17] proposed nonlinear equations of 3-D motion of pipes conveying incompressible fluid, which was solved by the incremental harmonic balance method. Lee and Park [18] proposed a spectral element model for the uniform straight pipeline conveying internal unsteady fluid based on the Hamilton's principles and the principles of fluid mechanics. Although the simplified model can solve some problems, it still has a certain gap with the real result.

Enriching the model further, by introducing fluid-solid coupling and friction into the model can make the simulation results closer to the real state, and also convenient to determine the impact of these conditions on system stability. Zhang [19] used the wave propagation approach to analyze the coupling frequency generated by fluid- pipeline system, and explored the influence of flow velocity and pipe size on coupling frequency. Ferràs *et al.* [20] established a fluid-structure interaction model in which skin and dry friction were considered, and the influence of friction state on system stability was analyzed. In addition to the simulation method, analyzing the measured data of the test rig is also an effective method to explore the pipeline system. Yan and Chai [21], through accelerated tests, found that excessive vibration and unreasonable pre-tightening force will cause pipeline seal to fail.

The above literatures have done a lot of work on the establishment of the fluid-solid coupling model of a clamp-pipeline system, the method of model solving, and the influence of structural parameters on the vibration response, and the results are remarkable. However, the nonlinear characteristics of a clamp-pipeline system caused by the loosening of clamp are rarely mentioned. Fully understanding of the characteristics can better avoid the occurrence of clamp loosening fault or reduce the impact of the fault on engine vibration. Based on this, this paper aims at the structure of an aero-engine clamp-pipeline system, based on the assumption that the pipe itself is rigid, and the stiffness of loosening clamp is nonlinear. A linear dynamic model of a straight clamp-pipeline system is established, and then a nonlinear dynamic model considering the cubic term of displacement is developed. The multi-scale method in analytical theory is used to obtain the response under main resonance of the nonlinear system. Finally, combined with the measured data of a clamp-pipeline system test rig, the nonlinear characteristics of the straight clamp-pipeline system are obtained.

II. MODELING AND SOLVING

A. DYNAMIC MODEL OF STRAIGHT CLAMP-PIPELINE SYSTEM

The aero-engine pipeline system connects engine components and accessories of engine, and transports the respective specified fluids to keep engine operating [22]. The pipelines are fastened by clamps, and the clamps are fixed on the engine casing by tightening bolts. Due to the special structure of clamp, the contact part between clamp and pipeline can be considered as rigid contact or elastic contact. In general, the support stiffness of a clamp is less than the stiffness of a pipeline. Therefore, the clamp can be defined as a flexible body, and its mechanical model can be simplified as an elastic support, as shown in Fig. 1.

In an aero-engine clamp-pipeline system, generally in order to reduce vibration amplitude, the inner ring of the clamp is equipped with flexible materials, such as rubber or metal felt. When the vibration frequency is high, the vibration of the system is obviously nonlinear due to the damping characteristics of the flexible material. However, the problem of nonlinear damping is extremely complex and has little impact on the system. Generally, in order to simplify the analysis,

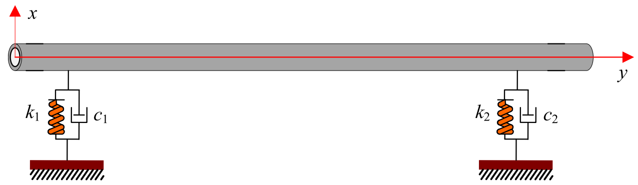

FIGURE 1. Mechanical model of a straight clamp-pipeline system.

it is assumed that the damping has linear characteristics. The commonly used damping is Rayleigh damping, as

$$c = \alpha m + \beta k \tag{1}$$

where, α and β is Rayleigh damping coefficient [23], its specific expression is

$$\begin{cases} \alpha = \dfrac{4\pi f_1 f_2(\zeta_1 f_2 - \zeta_2 f_1)}{f_2^2 - f_1^2} \\ \beta = \dfrac{\zeta_2 f_2 - \zeta_1 f_1}{\pi(f_2^2 - f_1^2)} \end{cases} \tag{2}$$

where, f_1 and f_2 are the first and second natural frequencies of the pipeline system, Hz; ζ_1 and ζ_2 are the modal damping ratio, which can be obtained through experimental tests.

When the pre-tightening force of the clamp can prevent the clamp from loosening in vibrating environment, it can be considered that the stiffness of the clamp is linear.

When the pipeline vibrates near the first-order natural frequency, it mainly consider the clamp stiffness k_x in the x direction, as shown in Fig.1. For linear stiffness, the relationship between force and displacement is

$$f_k(x) = k_x x \tag{3}$$

where k_x is linear stiffness, N/m.

When the tightening torque of the clamp is not enough, or external vibration causes the tightening bolts to become loosening during service, the clamp and the pipeline are not completely tightened. It can be considered that the stiffness of the clamp is nonlinearity.

Assuming that the loosening clearance of clamp is x_0, it can be represented by a piecewise linear elastic force model, as shown in Fig. 2.

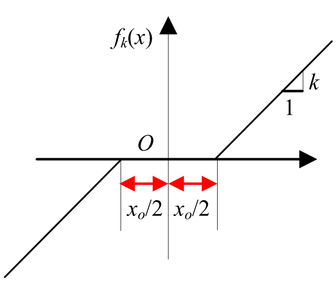

FIGURE 2. Piecewise linear elastic force model.

The piecewise linear elastic force can be divided into three sections

$$f_k(x) = \begin{cases} k_x(x - x_0/2), & x > x_0/2 \\ 0, & -x_0/2 \le x \le x_0/2 \\ k_x(x + x_0/2), & x < -x_0/2 \end{cases} \tag{4}$$

Duffing equations can represent a large class of nonlinear systems. Therefore, the cubic stiffness is used to fit piecewise linear stiffness. And then a Duffing stiffness model is obtained

$$f_k(x) = kx \pm bx^3 \tag{5}$$

where, k is the linear stiffness after fitting, N/m; b is the coefficient of the nonlinear term, N/m^3. The Duffing stiffness model is suitable for describing the stiffness of the clamp-pipeline system when the clamp is loosened.

The nonlinear vibration of the clamp-pipeline system under main resonance is considered in this paper. The corresponding clamp-pipeline system shown in Fig. 1 can be simplified to a single-degree-of-freedom spring-mass model with linear proportional damping and Duffing stiffness under harmonic excitation, as shown in Fig. 3.

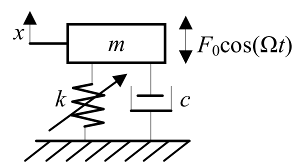

FIGURE 3. The clamp-pipeline system model based on Duffing stiffness.

According to Eq. (1) and Eq. (5), the differential equation based on Duffing stiffness of the model can be established

$$m\ddot{x} + c\dot{x} + kx \pm bx^3 = F_0 \cos(\Omega t) \tag{6}$$

B. ANALYTICAL SOLUTION BASED ON MULTI-SCALE METHOD

To analyze the vibration characteristics of the nonlinear systems, the multi-scale method in analytical theory is used [24].

Rewrite Eq. (6) as

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x \pm bx^3 = F\cos(\Omega t) \tag{7}$$

where, $2\xi\omega_n = \frac{c}{m}$, $\omega_n^2 = \frac{k}{m}$, $F = \frac{F_0}{m}$, $b' = \frac{b}{m}$. For writing convenience, b' will be defined as b below.

Define

$$\xi\omega_n \to \varepsilon\xi\omega_n, \quad b \to \varepsilon b \tag{8}$$

Then Eq. (7) can be rewritten as

$$\ddot{x} + 2\varepsilon\xi\omega_n\dot{x} + \omega_n^2 x \pm \varepsilon bx^3 = F\cos(\Omega t) \tag{9}$$

where, $\ddot{x} = \frac{d^2x}{dt^2}$, $\dot{x} = \frac{dx}{dt}$, ε is a small positive dimensionless parameter, $\xi\omega_n > 0$, ξ is the damping ratio coefficient, ω_n is the natural angular frequency of the system, b is the cubic stiffness term coefficient. When the symbol before b is "+", the spring is hardening spring. On the contrary, it is softening spring. F is the amplitude of the exciting force, and Ω is the angular frequency of the exciting force.

When the external excitation frequency Ω is close to the natural frequency ω_n of the system, namely, $\Omega \approx \omega_n$, a excitation with small amplitude will cause the system to produce a large amplitude response. Therefore, in the case of main resonance, the external excitation can be regarded as a small parameter

$$F\cos(\Omega t) \rightarrow \varepsilon F\cos(\Omega t) \tag{10}$$

Assume

$$\Omega = \omega_n + \varepsilon\sigma \tag{11}$$

where, σ is a detuning parameter.

With the multi-scale method, the solution of the Eq. (9) can be expressed in different time scales as

$$x(t,\varepsilon) = x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) + \cdots \tag{12}$$

where, $T_0 = t$, $T_1 = \varepsilon t$.

From Eq. (10) and Eq. (11), the external excitation under main resonance can be expressed as

$$F(t) = \varepsilon F\cos(\omega_n T_0 + \sigma T_1) \tag{13}$$

Substituting Eq. (12) and Eq. (13) into Eq. (9), make the coefficients of the ε^0 and ε^1 terms at both ends of the relative equation equal.

For ε^0

$$D_0^2 x_0 + \omega_n^2 x_0 = 0 \tag{14}$$

For ε^1

$$D_0^2 x_1 + \omega_n^2 x_1 = -2D_0 D_1 x_0 - 2\xi\omega_n D_0 x_0 - b x_0^3 + F\cos(\omega_n T_0 + \sigma T_1) \tag{15}$$

where, $D_n = \frac{\partial}{\partial T_n}$.

Eq. (14) is the 0-order approximation of Eq. (9), with only free vibration terms. Therefore, its solution is expressed as

$$x_0 = A(T_1)\exp(i\omega_n T_0) + CC \tag{16}$$

where, CC is the conjugate of the previous term.

Substituting Eq. (16) into Eq. (15),

$$\begin{aligned} &D_0^2 x_1 + \omega_n^2 x_1 \\ &= -[2i\omega_n(D_1 A + \xi\omega_n A) + 3bA^2\bar{A}]\exp(i\omega_n T_0) \\ &\quad -bA^3\exp(3i\omega_n T_0) + \frac{1}{2}F\exp[i(\omega_n T_0 + \sigma T_1)] + CC \end{aligned} \tag{17}$$

In the main resonance state, make the sum of the coefficients including the term $\exp(i\omega_n T_0)$ be zero

$$2i\omega_n(D_1 A + \xi\omega_n A) + 3bA^2\bar{A} - \frac{1}{2}F\exp(i\sigma T_1) = 0 \tag{18}$$

Eq. (18) is a first-order differential equation about amplitude A. Its solution can be defined as

$$A = \frac{1}{2}a\exp(i\varphi) \tag{19}$$

where, a and φ are amplitude and phase difference angle respectively.

Substituting Eq. (19) into Eq. (18), and dividing it into real part and imaginary part,

$$D_1 a = -\xi\omega_n a + \frac{F}{2\omega_n}\sin(\sigma T_1 - \varphi) \tag{20}$$

$$a D_1\varphi = \frac{3ba^3}{8\omega_n} - \frac{F}{2\omega_n}\cos(\sigma T_1 - \varphi) \tag{21}$$

Substituting Eq. (19) into Eq. (16), the solution of the ε^0 term equation can be obtained as x_0, and then substituting x_0 into Eq. (12), the 0-th order approximate solution of the system is

$$x = a\cos(\omega_n t + \varphi) + O(\varepsilon) \tag{22}$$

The amplitude a and the phase difference angle φ are determined by Eq. (20) and (21).

Define $\gamma = \sigma T_1 - \varphi$, then Eq. (20) and (21) can be converted to

$$D_1 a = -\xi\omega_n a + \frac{F}{2\omega_n}\sin\gamma \tag{23}$$

$$a D_1\gamma = \sigma a - \frac{3ba^3}{8\omega_n} + \frac{F}{2\omega_n}\cos\gamma \tag{24}$$

Therefore, the 0-th order approximate solution of Eq. (22) can be transformed into

$$\begin{aligned} x &= a\cos(\omega_n t + \sigma T_1 - \gamma) + O(\varepsilon) \\ &= a\cos(\Omega t - \gamma) + O(\varepsilon) \end{aligned} \tag{25}$$

In order to obtain the solution of steady-state, let $D_1 a = D_1\gamma = 0$, then from Eq. (23) and Eq. (24), the following equations can be obtained

$$\xi\omega_n a = \frac{F}{2\omega_n}\sin\gamma \tag{26}$$

$$\sigma a - \frac{3ba^3}{8\omega_n} = -\frac{F}{2\omega_n}\cos\gamma \tag{27}$$

Add the squares of equations (23) and (24) to get the relationship between the response amplitude a, the external excitation amplitude F, and external excitation amplitude Ω, namely amplitude frequency characteristics

$$4\omega_n^2\left[(\xi\omega_n)^2 + \left(\sigma - \frac{3ba^2}{8\omega_n}\right)^2\right]a^2 = F^2 \tag{28}$$

Divide (27) by equation (26) to get the expression about the phase angle, namely phase frequency characteristic

$$\tan\gamma = \frac{-\xi\omega_n}{\sigma - \frac{3ba^2}{8\omega_n}} \tag{29}$$

III. EXPERIMENT

It is mainly focused on the nonlinear characteristics of the clamp-pipeline system under the main resonance state in this paper. The frequency sweeping method of the hydraulic system is achieved by changing the speed of the plunger pump.

It is considered that the required main resonance frequency cannot be obtained with the limitations of the hydraulic system. In order to facilitate the study of the nonlinear vibration characteristics of the clamp-pipeline system, an electromagnetic vibrating table is used to perform basic excitation.

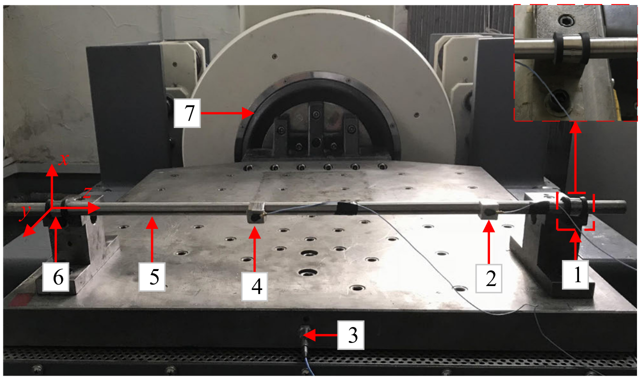

FIGURE 4. Pipeline test rig (1) Clamp 1; (2) Accelerometer 1; (3) Accelerometer 2; (4) Accelerometer 3; (5) Pipeline; (6) Clamp 2; (7) Vibration shaker.

The clamp-pipeline system vibration test rig is shown in Fig. 4. The electromagnetic vibration shaker is used to provide the exciting force. The pipeline is installed on the fixture through the clamp, and the fixture and the vibration shaker are rigidly installed. The lightweight acceleration sensors are used to measure the vibration response in the x direction.

Define the origin of the coordinates is on the left, the location of acceleration sensor 1 is installed at $z = 0.475$m, and the location of acceleration sensor 2 is installed at $z = 0.25$m. The purpose of this experiment is to test the vibration response of the clamp-pipeline system with sine sweep frequency excitation and sine fixed frequency excitation under different excitation frequencies and different tightening torques.

The diameter of the pipeline used in the test is 12mm, and other specific parameters are shown in Table 1.

TABLE 1. Geometrical and material parameters of the pipeline.

Parameter	Value
Length of the pipeline (m) / l	0.6
Outer diameter (m) / D	0.012
Inner diameter (m) / d	0.01
Elastic modulus (Pa) / E	2.01×10^{11}
Density (kg/m^3) /ρ	7800
Poisson's ratio	0.3

Generally, the first-order natural frequency of the pipeline system is below 200Hz [25]. Based on this, the specific test arrangement is arranged as follows:

1) Conduct a 20-200Hz fast sine frequency sweep excitation (frequency change rate 1Hz/s) on the pipeline system to initially determine its first-order natural frequency value;

2) Perform fine frequency sweep excitation within the range of plus or minus 20 Hz of the determined natural frequency value to determine the precise value of the first-order natural frequency;

3) Finally, perform sine fixed-frequency excitation near the precise natural frequency to test the vibration response of the clamp-pipeline system, and collect test data for corresponding preprocessing and signal analysis.

A. PARAMETER IDENTIFICATION FOR THE CLAMP-PIPELINE SYSTEM

Generally, the tightening torque of clamps on aero-engine is about 10Nm [26]. To study the clamp loosening state, the tightening torque of the tightening bolt can be decreased. In this test, the clamp tightening torque is adjusted from 10 Nm to 5Nm and 1Nm, which are assumed to be two different loosening states of the clamp.

First, set tightening torque to 1Nm, vibration excitation amplitude of 1g, frequency sweep range of 20-200Hz, and the relative three-dimensional waterfall diagram according to measured data, as shown in Fig. 5.

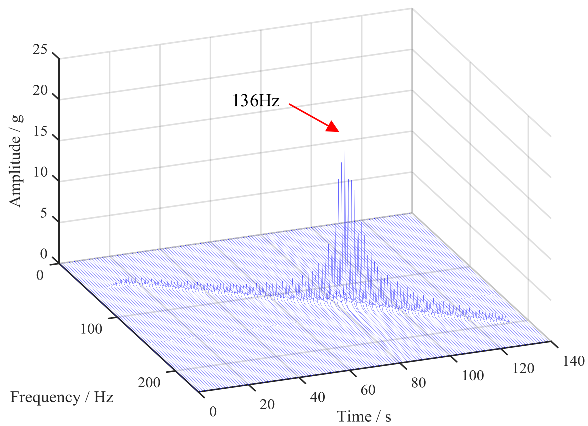

FIGURE 5. Three dimensional waterfall diagram of vibration response under sweep frequency 20-200Hz with 1Nm tightening torque.

It can be seen from Fig. 5 that the first-order natural frequency of the clamp-pipeline system under 1Nm tightening torque is 136Hz. Generally, when the tightening torque is changed, the first-order natural frequency of the system will not change much. Therefore, assuming that the first-order natural frequency under 5Nm tightening torque is around 135Hz, fine frequency sweeping can be performed. Therefore, adjusting the tightening torque to 5Nm, determine the frequency sweep range from 110 Hz to 150Hz, and the excitation amplitude is 2g ($1g = 1 \times 9.8m/s^2$). The relative vibration response is shown in Fig. 6, obtain that the first-order natural frequency under 5Nm tightening torque is 133Hz.

The sinusoidal fixed-frequency excitation under a tightening torque of 5Nm is performed, with excitation amplitude of 2g and excitation frequency of $f = 136$Hz. The relative vibration response is shown in Fig. 7.

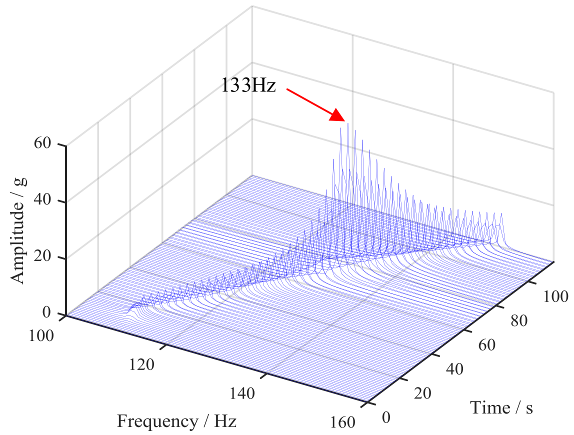

FIGURE 6. Three dimensional waterfall diagram of vibration response of system under sweep frequency 110-150Hz with 5Nm tightening torque.

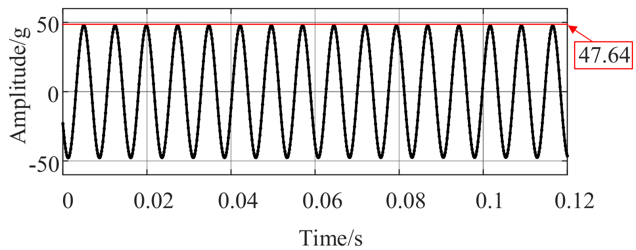

FIGURE 7. Vibration response waveform of fixed frequency excitation under a tightening torque of 5Nm.

It can be seen from Fig. 7 that the vibration response amplitude of the clamp-pipeline system under a tightening torque of 5Nm is $a = 47.64$g. Through Eq. (6), the specific parameters of the system can be identified, as shown in Table 2.

TABLE 2. Geometrical and material parameters of the pipeline.

Parameter	Value
First-order natural angular frequency (rad/s) /ω_n	835.664
Exciting frequency (rad/s) /Ω	854.513
Response amplitude (m/s^2) / a	466.872
Quality of the pipeline (kg) / m_p	0.162
Damping coefficient /α	55.06
Damping coefficient /β	9.66×10^{-6}
Total support stiffness (N/m) / k	1.129×10^{5}
Damping (Ns/m) / c	9.996
Vibration response amplitude (m) / A	6.394×10^{-4}
Exciting force amplitude (N) / F_0	6.378
Phase angle (rad) /θ	-1.028
Frequency ratio / z	1.023
Damping ratio /ξ	0.037

Therefore, under the tightening torque of 5Nm, the linear differential equation of aφ12mm straight clamp-pipeline system is

$$0.162\ddot{x} + 9.996\dot{x} + 1.129 \times 10^5 x = 6.378 \cos(854.513t) \tag{30}$$

B. DETERMINATION OF DUFFING STIFFNESS

In this paper, Duffing stiffness is used to describe the approximate nonlinear stiffness of the system when the clamp is loosened. The main steps of the Duffing stiffness fitting method are: 1) determine the clamp clearance caused by the loosening of the clamp. 2) convert the linear stiffness curve of the clamp into a segmented curve containing the clearance. 3) through the fitting calculation, the Duffing stiffness model is finally obtained.

From Eq. (30), it can be seen that the linear stiffness of the system in the x direction is $k = 1.129 \times 10^5$N/m. The relationship between linear force and displacement can be obtained as $F = 1.129 \times 10^5 x$.

In order to obtain the loosing clearance between the inner sleeve of the clamp and the pipeline under the tightening torque of 5Nm, the difference between the vibration response displacement under the fixed frequency excitation of 136Hz with the excitation amplitude of 2g when the tightening torques are 10Nm and 5Nm can be used to approximate, the relative vibration response displacements can be obtained by quadratic integration of acceleration, as shown in Fig. 8.

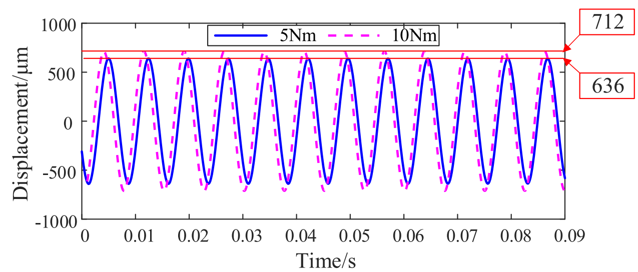

FIGURE 8. Vibration response waveform of fixed frequency excitation.

According to Fig. 8, the clearance caused by the loosening of the clamp under the tightening torque of 5Nm is $s = 7.6 \times 10^{-5}$m, then the $\Delta x = 3.8 \times 10^{-5}$m is obtained along the symmetrical zero, and the piecewise linear force can be obtained as

$$F = \begin{cases} 1.129 \times 10^5 x + 4.29, & x < -3.8 \times 10^{-5} \\ 0, & -3.8 \times 10^{-5} \le x \le 3.8 \times 10^{-5} \\ 1.129 \times 10^5 x - 4.29, & x > 3.8 \times 10^{-5} \end{cases} \tag{31}$$

The corresponding piecewise linear force and displacement curve is shown in Fig. 9.

In order to fit the nonlinear force and displacement curve, a cubic curve fitting is performed on Eq. (31), and the relationship between nonlinear force and displacement after fitting is shown in Eq. (32).

$$F = 7.947 \times 10^4 x \pm 1.736 \times 10^{11} x^3 \tag{32}$$

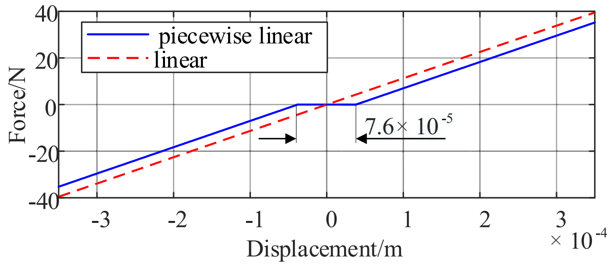

FIGURE 9. Piecewise linear force and displacement curve.

The corresponding nonlinear force and displacement curve is shown in Fig. 10.

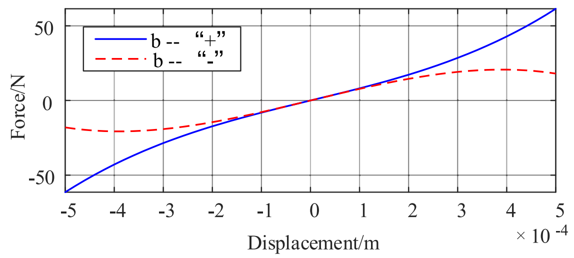

FIGURE 10. Curve of nonlinear force and displacement.

When the symbol of b is "+", as the displacement x increases, the increasing rate of the force F increases, and the Duffing stiffness exhibits a hardening nonlinearity. When the symbol of b is "-", as the displacement x increases, the increasing rate of the force F slows down, and the Duffing stiffness is softening nonlinearity.

After fitting, the linear stiffness changed from $k = 1.129 \times 10^5$N/m to $k = 7.947 \times 10^4$N/m, and the cubic stiffness coefficient $b = 1.736 \times 10^{11}$N/m^3. It is found that the linear stiffness of the system considering the nonlinearity has decreased, and the decrease ratio is about 29.61%. This shows that the nonlinear stiffness of the system shares the total stiffness of the linear system, resulting in a decrease in the linear stiffness of the nonlinear system.

Therefore, according to Eq. (6), the nonlinear differential equation of the system based on Duffing stiffness can be obtained as

$$0.162\ddot{x} + 9.996\dot{x} + 7.947 \times 10^4 x \pm 1.736 \times 10^{11} x^3 = 6.378 \cos(854.513t) \quad (33)$$

Rewrite it as

$$\ddot{x} + 61.7\dot{x} + 4.906 \times 10^5 x \pm 10.716 \times 10^{11} x^3 = 39.37 \cos(854.513t) \quad (34)$$

According to Eq. (34), the system response amplitude and frequency response function curve under the main resonance state can be obtained. From Eq. (28), the detuning parameter expression can be obtained as

$$\sigma = \frac{3ba^2}{8\omega_n} \pm \sqrt{\frac{F^2}{4\omega_n^2 a^2} - (\xi\omega_n)^2} \quad (35)$$

Bring in the relevant parameters in Table 2 to obtain the relationship between the response excitation frequency and the response amplitude, as shown in Fig. 11.

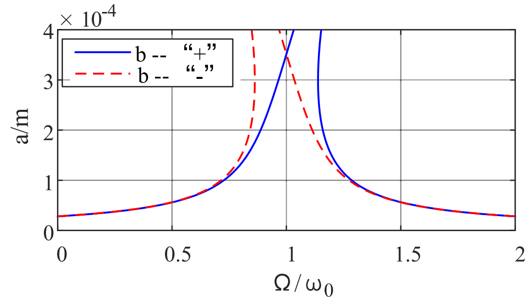

FIGURE 11. Frequency response function curve.

It can be seen from Fig. 11 that when $b = +10.716 \times 10^{11}$, the function curve inclined to the direction of increasing frequency, that is, as the excitation frequency increases, the total stiffness of the system shows an upward trend, reflecting the characteristics of hardening stiffness. When $b = -10.716 \times 10^{11}$, the function curve inclined to the direction of decreasing frequency, that is, as the excitation frequency increases, the total stiffness of the system shows a downward trend, reflecting the characteristics of softening stiffness. The determination of the nonlinear stiffness characteristics of the system requires further analysis.

C. NONLINEAR STIFFNESS CHARACTERISTICS

In order to verify the softening and hardening characteristics of the loosed clamp pipeline system based on Duffing stiffness, the method of vibration transmissibility is adopted in this paper. The transmissibility of a linear system is not affected by the excitation intensity, while the transmissibility of a nonlinear system is affected by the excitation intensity [27]. The transmissibility is defined as the ratio of response amplitude to excitation intensity

$$Transmissibility(f) = \frac{A(f)}{F_0} \quad (36)$$

where f represents the excitation frequency, Hz. $A(f)$ represents the response amplitude with the excitation frequency f, g. F_0 represents the excitation intensity, g.

The influence of different excitation intensities on vibration transmissibility is studied under the tightening torque of 1Nm, 5Nm, and 10Nm. Carry out sine frequency sweep excitation with excitation amplitudes of 1g and 2g respectively, and the sweep frequency range is from 110Hz to 150Hz.

The obtained dynamic characteristic curve of the system under the 1Nm, 5Nm, and 10Nm tightening torque is shown in Fig. 12, Fig. 13, and Fig. 14, and the corresponding first-order natural frequency and relative transmissibility are shown in Table 3, Table 4, and Table 5.

As shown in Table 3, under 1Nm tightening torque, as the excitation amplitude value increases from 1g to 2g, the

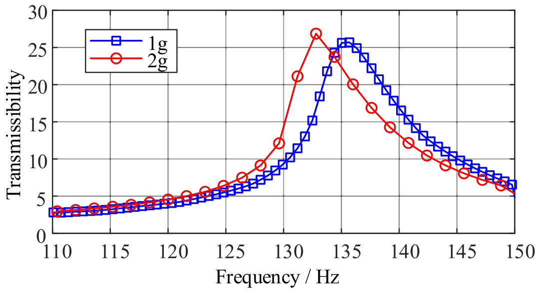

FIGURE 12. Curve of system dynamic characteristic with 1Nm tightening torque.

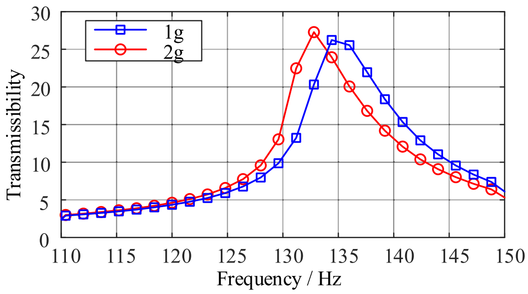

FIGURE 13. Curve of system dynamic characteristic with 5Nm tightening torque.

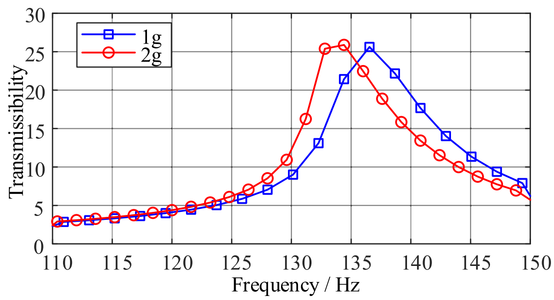

FIGURE 14. Curve of system dynamic characteristic with 10Nm tightening torque.

TABLE 3. Curve parameter with 1Nm tightening torque.

Excitation amplitude	First-order natural frequency (Hz)	Transmissibility
1g	135.7	25.7
2g	132.8	26.87

TABLE 4. Curve parameter with 5Nm tightening torque.

Excitation amplitude	First-order natural frequency (Hz)	Transmissibility
1g	134.4	26.24
2g	132.8	27.28

transmissibility increases from 25.7 to 26.87, and the natural frequency decreases by 2.9Hz. With a tightening torque of 5Nm, as the excitation amplitude increases from 1g to 2g,

TABLE 5. Curve parameter with 10Nm tightening torque.

Excitation amplitude	First-order natural frequency (Hz)	Transmissibility
1g	136.5	25.62
2g	134.4	25.88

the transmissibility increases from 26.24 to 27.28, and the natural frequency decreases by 1.6Hz in Table 4. The change of the transmissibility indicates the nonlinearity of the loosening clamp system. The reduction of the first-order natural frequency shows that the system has the characteristics of softening nonlinearity, but the reduction is small, indicating that the system has weak softening nonlinearity.

As shown in Table 5, under the tightening torque of 10Nm, as the excitation amplitude increases from 1g to 2g, the transmissibility increases from 25.62 to 25.88. Compared to the tightening torque of 1Nm and 5Nm, the transmissibility is almost constant, indicating that the system under this torque can be regarded as a linear system. The natural frequency under the tightening torque of 10Nm decreases by 2.1Hz with the increase of the excitation amplitude. This may be caused by the uncertainty of clamp-pipeline system responses [9].

IV. CONCLUSION

The aero-engine pipeline system is taken as the research object in this paper. Aiming at the loosening faults of the clamp, a nonlinear system model based on Duffing stiffness is proposed. Using the measured data of the test rig, the nonlinear system of the loosening clamp is identified. Multi-scale method is used to analyze the vibration characteristics of the nonlinear system under main resonance, and the frequency responses of the nonlinear system are obtained [28], [29]. Finally, the test of different severity of looseness of the clamp is performed. Some conclusions are summarized as follows:

(1) After considering the cubic stiffness, the linear stiffness decreases from $k = 1.129 \times 10^5$N/m to $k = 7.947 \times 10^4$N/m, and the decrease ratio is about 29.61%.

(2) In loosening state of the clamp, with the increase of the excitation amplitude, the vibration transmissibility increases. And the increase of the severity of looseness makes the natural frequency have a slight decreasing trend.

(3) The straight clamp-pipeline nonlinear system based on Duffing stiffness has the characteristics of weak softening nonlinearity.

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