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Distributed Time-Varying Output Formation Tracking Control for General Linear Multi-Agent Systems With Multiple Leaders and Relative Output-Feedback

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ABSTRACT In this paper, a unified distributed swarm intelligence algorithm is developed to study time-varying output formation (TVOF) for a general linear multi-agent system (MAS) with a directed network. New adaptive output-feedback formation protocols are proposed to achieve TVOF stabilization for leaderless directed networks and TVOF tracking for leader-follower networks. For the leaderless case, only agents' outputs are required to achieve the desired time-varying formation. An adaptive observer-type formation protocol is constructed via relative outputs of neighboring agents. No global information of the directed network is used to determine the protocol. A distributed algorithm is developed to solve the TVOF stabilization problem after the observability decomposition. For the leader-follower case, only partial agents have knowledge of the leaders' information. An adaptive formation tracking protocol is constructed using dynamic relative output-feedback for neighboring followers. Based on the distributed algorithm, it is proved that the TVOF tracking problem with multiple leaders can be solved in a fully distributed manner.

INDEX TERMS Output formation tracking, adaptive observer-type protocol, dynamic relative output-feedback, time-varying formation, multi-agent system.

I. INTRODUCTION

Swarm intelligence has been the subject of many studies in the multi-agent system (MAS) community during the past decades. Coordination of the actions by communication of the agents is one of the basic requirements in swarm intelligence. A growing body of literature [1]–[3] shows that the significance of collaborative control cannot be ignored to achieve the requirement. Formation control, as one of the main research branches of collaborative control, has been widely used as a major approach to regulate relative coordinates among unmanned surface vehicles [4], mobile robots [5], unmanned aerial vehicles [6] and so on. A group of agents working together and maintaining the desired formation configuration is more efficient than a single agent when executing challenging missions such as rescue operations, reconnaissance, security patrol, and exploration. Many formation control strategies [7]–[9] have been proposed in addressing the issues for MASs.

Recent developments in the field of consensus control have led to an increased interest in developing consensus-based formation control approaches. In [10] and [11], consensus-based formation controllers were developed for time-invariant formation (TIF) control problems, which requires the MAS to reach and keep a fixed formation. In practical missions, the formation configuration may be required to vary dynamically for avoiding obstacles and increasing the range of exploration. Thus, time-varying formation (TVF) control problems are derived. Without considering the derivative of the desired formation configuration, the approaches in studying consensus control and TIF control problems cannot be directly used to tackle TVF control problems. Several attempts have been made to address time-varying state formation (TVSF) control problem for low-order MASs in [12]–[14]. Distributed TVSF control approaches in [15] and [16] are suitable for large-scale

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general linear MASs with switching undirected network and fixed directed network, respectively. In practical engineering applications, a MAS can be formed by multiple vehicles, such as mobile robots [5] and unmanned aerial vehicles [17], [18]. Position, velocity, attitude, and other unmeasured variables constitute the states of each vehicle. The full state measurements are expensive because lots of sophisticated sensors need to be installed. However, the full state measurements are sometimes unavailable in practice. Using measured output information to design the formation protocol is of significance. For example, multiple vehicles are executing missions in different areas. It requires that each vehicle's positions and velocities reach and keep the desired TVF, and the compulsive formation requirements of its attitudes can be relaxed. The attitudes of each vehicle only need to remain stable to resist disturbances of environmental changes. Hence, it is more meaningful to address the time-varying output formation (TVOF) control problems, which aims to propel only outputs of each agent to achieve the given TVF. Noting that the TVSF is a special case of the TVOF. Reference [19] discussed time-invariant output formation for MASs. TVOF control problems for general linear MASs were studied in [3]. Reference [20] first proposed a fully distributed TVOF control approach that is suitable for large-scale MASs. However, [3], [20] only discussed TVOF stabilization and maintenance. In many applications, it is usually the first step for the MAS to form a formation. Then, formation tracking problems should be addressed for some high-level missions. Due to the macro trajectory of a MAS is not considered in TVOF stabilization problems, the time-varying output formation tracking (TVOFT) control problems for MASs are more practical because the tracking control part in TVOFT control problems provides possibilities for high-level missions. Another reminder is that the previous TVOF results in [3], [20] all required the absolute outputs of each agent and its neighbors. In some circumstances, it is more expensive and difficult to obtain the absolute outputs than the relative outputs. If it is available to obtain neighboring agents' absolute outputs, their relative outputs are accessible, but not vice versa. TVOF protocols using only relative outputs of neighboring agents are more practical. It is meaningful and challenging to address the distributed TVOFT control problems for general linear MASs with relative output-feedback. This problem has not been discussed extensively.

Motivated by the above discussion, fully distributed relative output-feedback formation approaches for general linear MASs with a directed network are developed in this paper, from the leaderless TVOF stabilization to the leader-follower TVOFT. First, an adaptive TVOF protocol is constructed via dynamic relative output-feedback for leaderless TVOF control problems, rather than absolute output-feedback in [20]. After the observability decomposition and output formation decomposition, the problems are transformed into TVOF stabilization problems. A distributed algorithm is developed to determine the TVOF protocol, and the algorithm's stability is proved. Then, another adaptive TVOF protocol is proposed for solving TVOFT control problems with tracking multiple leaders. The stability of the proposed distributed algorithm when solving TVOFT control problems can also be obtained.

Compared with the previous works, the main contributions of this paper are threefold. First, adaptive TVOF protocols and a unified distributed algorithm is proposed to solve TVOF stabilization and tracking problems. No global information is required in achieving the desired TVOF. The definition of general TVOF is given from the perspective of the output space. However, no algorithms were given in [12], [13], [19]. The TVOF results in [3] required the global information of the communication network, and have limited value for large-scale MASs. Only TVOF stabilization problems were discussed in [3], [20]. The distributed TVOFT approaches in this paper are more practical than those in [3], [20] for MASs to execute high-level missions. Second, only the relative outputs between neighboring agents are required in the proposed TVOF protocols. The communication burden between neighboring agents is reduced based on the proposed TVOF protocols. However, the distributed TVOF results in [20], [21] required the absolute outputs of each agent. Compared with the acquisition of the absolute outputs of each agent and its neighbors, it is more convenient and economical to obtain the relative outputs in many applications. The availability of the absolute outputs is a sufficient unnecessary condition for the availability of the relative outputs. Third, a sufficient condition is developed to track the convex combination of leaders' outputs in a fully distributed manner using relative output-feedback. So far, little attention has been paid to the effect of multi-leaders in the field of formation tracking. The TVSF tracking results in [22] were neither fully distributed nor based on output-feedback. Full state-feedback information was required in the TVSF tracking results of [23], which are uneconomical. The approaches proposed in this paper are distributed and more economical to solve TVOFT problems with multi-leaders.

The remainder of this paper is organized as follows. After formulating distributed TVOFT control problems for networked linear MASs with multi-leaders in Section II, Section III reports the approach for solving distributed TVOF control problem for leaderless MASs. Section IV addresses the application of the proposed approach when solving distributed TVOFT problem with multi-leaders. A numerical example is given in Section V. Concluding remarks are presented in Section VI.

Notations: Let $\mathbb{R}^{n\times m}$ denote the set of $n \times m$ real matrices. For any real matrix $\mathcal{P}$, the transpose of $\mathcal{P}$ is $\mathcal{P}^T$. The Kronecker product of matrices $\mathcal{P}$ and $\mathcal{Q}$ is denoted by $\mathcal{P} \otimes \mathcal{Q}$. Denote the identity matrix of dimension n by I_n. Let $\mathcal{I}_m^n$ be the index set of $\{m, m+1, \cdots, n\}$. $diag\{\cdot\}$ is a diagonal matrix from its argument. For simplicity of notation, denote the zero matrices of appropriate size and all-one column vectors of appropriate size by 0 and $\mathbf{1}$, respectively. Let $col\{x_1, x_2, \cdots, x_n\}$ represent the column vector equals

$[x_1^T, x_2^T, \cdots, x_n^T]^T$. Denote the maximum (resp. minimum) eigenvalue of real symmetric matrix $\mathcal{P}$ by $\lambda_{max}^{\mathcal{P}}$ (resp. $\lambda_{min}^{\mathcal{P}}$).

II. PROBLEM FORMULATION

Consider a networked MAS comprising $N + M$ agents with general linear dynamics. The communication network of the MAS is described by a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, where $\mathscr{V} = \{1, 2, \cdots, N+M\}$ is the node set and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is the edge set. Let $\mathscr{W} = [w_{ij}] \in \mathbb{R}^{(N+M)\times(N+M)}$ be the nonnegative adjacency matrix of $\mathscr{G}$. Each agent in the MAS is treated as a node in $\mathscr{G}$. The communication channel between agents i and j can be seen as an edge (i, j) in $\mathscr{G}$ and its strength is represented by w_{ij}. If agent i can receive the information from agent j, $(i, j) \in \mathscr{E}$, $w_{ij} > 0$, and $j \in \mathscr{N}_i$ which is the neighbor set of agent i; otherwise, $(i, j) \notin \mathscr{E}$, $w_{ij} = 0$, and $j \notin \mathscr{N}_i$. For all $i \in \mathscr{V}$, $w_{ii} = 0$. There is a directed path from agent i_1 to agent i_m if there exists a sequence of ordered edges formed by (i_k, i_{k+1}) $(k = 1, 2, \cdots, m-1)$. If there exists an agent having directed paths to every other node, it says that $\mathscr{G}$ contains a directed spanning tree. For a strongly connected $\mathscr{G}$, it always has a directed spanning tree; but not vice versa. For any agents i and j, if agent i has a directed path to agent j, it says that $\mathscr{G}$ is strongly connected. Let $\mathscr{L} = [\mathscr{L}_{ij}] \in \mathbb{R}^{(N+M)\times(N+M)}$ be the Laplacian matrix of $\mathscr{G}$ with $\mathscr{L}_{ij} = -w_{ij}$ $(i \neq j)$ and $\mathscr{L}_{ii} = \sum_{j=1}^{N+M} w_{ij}$.

Lemma 1 ([24]): The necessary and sufficient conditions of the fact that $\mathscr{L}$ has a simple eigenvalue 0 with $\mathbf{1}$ as its right eigenvector, and other non-zero eigenvalues have positive real parts are that $\mathscr{G}$ has a directed spanning tree.

In TVOFT control problems, each agent in the MAS has its roles.

Definition 1: Agent i is called a leader (resp. follower) if $\mathscr{N}_i = \emptyset$ (resp. $\mathscr{N}_i \neq \emptyset$). Denote the followers' set and the leaders' set by $\mathcal{F} = \mathcal{I}_1^N$ and $\mathcal{L} = \mathcal{I}_{N+1}^{N+M}$, respectively. A follower i is well-informed (resp. uninformed) if $\mathcal{L} \subset \mathscr{N}_i$ (resp. $\mathcal{L} \cap \mathscr{N}_i = \emptyset$).

The dynamics of agent i is described by (1).

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t), \\ y_i(t) = Cx_i(t), \end{cases} \quad (i \in \mathcal{F}) \tag{1a}$$

$$\begin{cases} \dot{x}_i(t) = Ax_i(t), \\ y_i(t) = Cx_i(t), \end{cases} \quad (i \in \mathcal{L}) \tag{1b}$$

where $x_i(t) \in \mathbb{R}^r$, $y_i(t) \in \mathbb{R}^p$, and $u_i(t) \in \mathbb{R}^q$ are the states, measured outputs, and control inputs of agent i, respectively. $A \in \mathbb{R}^{r\times r}$, $B \in \mathbb{R}^{r\times q}$ and $C \in \mathbb{R}^{p\times r}$ are system matrices, where $rank(B) = q$ and $rank(C) = p$. The matrices A and B are supposed to satisfy the following assumption.

Assumption 1: (A, B) is stabilizable.

Let the desired TVOF be specified by a vector $f(t) = col\{f_1(t), f_2(t), \cdots f_N(t)\}$ for all follower agents, where each $f_i(t) \in \mathbb{R}^p$ in $f(t)$ is piecewise continuously differentiable.

Definition 2: Under any preset bounded initial states, the follower agents are said to achieve the desired TVOFT

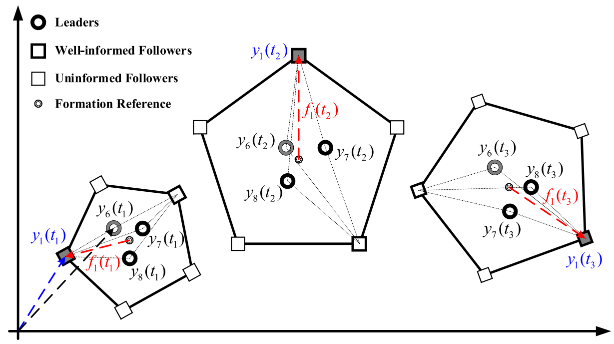


FIGURE 1. The illustration example of the TVOFT control with multi-leaders for different timestamps.

with multi-leaders if

$$\lim_{t\to\infty} (y_i(t) - f_i(t) - \sum\nolimits_{k=N+1}^{N+M} \alpha_k y_k(t)) = 0, \tag{2}$$

where $i \in \mathcal{F}$. $\alpha_k > 0$ $(k \in \mathcal{L})$ and $\sum_{k=N+1}^{N+M} \alpha_k = 1$.

To clarify the roles of $f_i(t)$ in Definition 2, consider the following example.

Illustration example 1: Consider a MAS consisting of five followers and three leaders. Five followers form a pentagon formation that is influenced by the leaders. In FIGURE 1, the follower rotates around leaders. Well-informed followers can obtain information from leaders and other neighbor followers. Uninformed followers can interact with its neighbor followers rather than any leaders. The formation reference is determined by the convex combination of leader k $(k \in \{6, 7, 8\})$. From Definition 2, the formation reference can be represented by $\sum_{k=6}^{8} \alpha_k y_k(t)$. The reference output trajectory of follower i $(i \in \{1, 2, \ldots, 5\})$ is determined by the formation reference and the desired TVOF vector $f_i(t)$, i.e., $\sum_{k=6}^{8} \alpha_k y_k(t) + f_i(t)$. When the MAS achieves (2), $f_i(t)$ is the desired relative offset vector of $y_i(t)$ with respect to the formation reference. If any leader (i.e., leader 8) is endangered by faults, the rest leaders (i.e., leaders 6 and 7) determine the formation reference. From (2), it follows that $\lim_{t\to\infty}((y_i(t) - y_j(t)) - (f_i(t) - f_j(t))) = 0$ $(i, j \in \{1, 2, \ldots, 5\})$, which means that the geometric relationship between the two pentagons formed by $f_i(t)$ and $y_i(t)$ $(i \in \{1, 2, \ldots, 5\})$ is congruent finally. $f_i(t)$ specifies not only the TVOF configuration but also the relative time-varying coordinate of each follower with respect to the formation reference.

Remark 1: If $M = 1$, MAS (1) is said to achieve the TVOFT with one leader. If $M = 1$ and $\sum_{k=1}^{N} f_i(t) = 0$, (2) can be transformed as $\lim_{t\to\infty}(y_{N+1}(t) - \frac{1}{N}\sum_{k=1}^{N} y_k(t)) = 0$; that means the leader's outputs is in the center of the followers' output formation which is specified by $f_i(t)$ $(i \in \mathcal{F})$. In this scenario, the TVOFT problem degenerates into the target enclosing problem as discussed in [26]. If $M = 1$ and $f_i(t) = 0$, the TVOFT problem discussed in this paper degenerates into an output consensus problem.

III. DISTRIBUTED TIME-VARYING OUTPUT FORMATION FOR MASs WITHOUT LEADERS

In this section, the distributed TVOF control problem for MAS (1) without leaders is considered. An adaptive observer-type TVOF protocol is constructed based on the relative outputs of neighboring agents. Then a distributed algorithm is developed to determine the parameters in the TVOF protocol. The algorithm's stability is proved in this section.

If there are no leaders in MAS (1), $M = 0$, $\mathcal{L} = \emptyset$ and $\mathcal{F} = \mathcal{I}_1^N$. All agents in the MAS belong to the followers' set. In this case, the formation reference is determined by the collaborative results of neighboring agents, rather than by the leaders' outputs as defined in (2).

The communication network of the MAS with N agents is described by a directed graph $\mathscr{G}$ with Laplacian matrix $\mathscr{L}$.

Assumption 2: The communication network of the MAS denoted by $\mathscr{G}$ is strongly connected.

Definition 3: Under any preset bounded initial states, MAS (1) is said to achieve the given TVOF specified by $f_i(t)$ ($i \in \mathcal{F}$) without leaders if

$$\lim_{t\to\infty}(y_{ij}(t) - f_{ij}(t)) = 0, \tag{3}$$

where $i, j \in \mathcal{F}$, $y_{ij}(t) = y_i(t) - y_j(t)$, and $f_{ij}(t) = f_i(t) - f_j(t)$.

Remark 2: Because of no leaders in the MAS, all agents should communicate with their neighbors and reach the output formation at last. The evolution of each agent's outputs mainly refers to its neighbors' outputs. The roles of all agents are equal. It mainly focuses on how to achieve the desired TVOF (3) in a fully distributed manner. It should be noted that the TVOF problem discussed in this section degenerates into the TVSF problem in [12], [13], [15], [16] if $C = I_r$. If $f_i(t) = Ch_i(t)$ as considered in [25], where $h_i(t)$ is another piecewise continuously differentiable vector, the TVOF control problem degenerates into a TVSF control problem because (3) equals to $\lim_{t\to\infty}(x_{ij}(t) - h_{ij}(t)) = 0$ in this particular situation.

The following transformation is useful for constructing the TVOF protocol in this section. Since $rank(C) = p$, $\exists \bar{C} \in \mathbb{R}^{(r-p)\times r}$ such that $\mathcal{T} = [C^T, \bar{C}^T]^T \in \mathbb{R}^{r\times r}$ is nonsingular. It has $\mathcal{T}A\mathcal{T}^{-1} = \begin{bmatrix}\mathcal{A}_{11} & \mathcal{A}_{12}\\ \mathcal{A}_{21} & \mathcal{A}_{22}\end{bmatrix}$ and $\mathcal{T}B = \begin{bmatrix}\mathcal{B}_1\\ \mathcal{B}_2\end{bmatrix}$, where $\mathcal{A}_{11} \in \mathbb{R}^{p\times p}$ and $\mathcal{B}_1 \in \mathbb{R}^{p\times q}$. There exists a nonsingular matrix $\bar{\mathcal{T}} \in \mathbb{R}^{(r-p)\times(r-p)}$ such that $\mathcal{A}_{12}\bar{\mathcal{T}} = \begin{bmatrix}\bar{\mathcal{A}}_{12} & 0\end{bmatrix}$ and $\bar{\mathcal{T}}^{-1}\mathcal{A}_{22}\bar{\mathcal{T}} = \begin{bmatrix}\bar{\mathcal{A}}_{22} & 0\\ \hat{\mathcal{A}}_{22} & \tilde{\mathcal{A}}_{22}\end{bmatrix}$, where $(\bar{\mathcal{A}}_{22}, \bar{\mathcal{A}}_{12})$ is completely observable and $\bar{\mathcal{A}}_{22} \in \mathbb{R}^{s\times s}$ $(0 < s < r-p)$. Additionally, define $\bar{\mathcal{T}}^{-1}\mathcal{A}_{21} = \begin{bmatrix}\bar{\mathcal{A}}_{21}^T & \tilde{\mathcal{A}}_{21}^T\end{bmatrix}^T$ and $\bar{\mathcal{T}}^{-1}\mathcal{B}_2 = \begin{bmatrix}\bar{\mathcal{B}}_2^T & \tilde{\mathcal{B}}_2^T\end{bmatrix}^T$. Redefine $\mathcal{A} = \begin{bmatrix}\mathcal{A}_{11} & \bar{\mathcal{A}}_{12}\\ \bar{\mathcal{A}}_{21} & \bar{\mathcal{A}}_{22}\end{bmatrix}$, $\mathcal{B} = \begin{bmatrix}\mathcal{B}_1\\ \bar{\mathcal{B}}_2\end{bmatrix}$, and $\mathcal{C} = \begin{bmatrix}I_p & 0\end{bmatrix}$. There exist nonsingular matrices $\hat{\mathcal{T}} = \begin{bmatrix}\hat{\mathcal{T}}_{11} & \hat{\mathcal{T}}_{12}\\ \hat{\mathcal{T}}_{21} & \hat{\mathcal{T}}_{22}\end{bmatrix}$ and $\tilde{\mathcal{T}}^{-1} = \begin{bmatrix}\tilde{\mathcal{T}}_1^T & \tilde{\mathcal{T}}_2^T\end{bmatrix}^T$, such that $\hat{\mathcal{T}}\mathcal{B}\tilde{\mathcal{T}} = \begin{bmatrix}I_q & 0\\ 0 & 0\end{bmatrix}$. In the following analysis, vector $f_i(t)$ will be notated as f_i for simplicity, so do other vectors.

Consider the following adaptive TVOF protocol using neighboring agents' relative outputs

$$\begin{cases}\dot{\theta}_i = \mathcal{A}\theta_i + K_1(\mathcal{C}\theta_i - \sum_{j\in\mathcal{N}_i} w_{ij}(y_{ij} - f_{ij})) + \mathcal{B}K_2\vartheta_i,\\ \dot{\varphi}_i = (\mathcal{A} + \mathcal{B}K_2)\varphi_i + (\rho_i + \phi_i)K_1\mathcal{C}(\vartheta_i - \theta_i),\\ \dot{\rho}_i = (\vartheta_i - \theta_i)^T\mathcal{C}^T\mathcal{C}(\vartheta_i - \theta_i),\\ u_i = K_2\varphi_i + f_i^c,\end{cases} \tag{4}$$

where $i \in \mathcal{F}$. $\theta_i, \varphi_i \in \mathbb{R}^{p+s}$ are the protocol states. $\vartheta_i = \sum_{j\in\mathcal{N}_i} w_{ij}\varphi_{ij}$ with $\varphi_{ij} = \varphi_i - \varphi_j$. ρ_i denotes the coupling weight for agent i with $\rho_i(0) > 0$. ϕ_i is a continuous monotonically increasing function and $\phi_i(\varepsilon) > 0$ when $\varepsilon > 0$. K_1 and K_2 in (4) are feedback control gains. Corresponding to each f_i, f_i^c is its formation compensation signal.

Remark 3: Two observers θ_i and φ_i are designed in adaptive TVOF protocol (4). The role of θ_i is to estimate the formation error of each agent using relative output-feedback. φ_i is designed to generate feedback for the control input of each agent and realize the desired TVOF. Different from the results in previous work [20], [21], only relative outputs between neighboring agents, rather than absolute outputs of neighboring agents, are required to construct protocol (4). The variable ϑ_i in protocol (4) implies that each agent should transmit observer φ_i through the communication network. However, the approaches in [20], [21] require to transmit two observers. In [25], not only the observer but also the full states have to be transmitted. Compared with the previous works, the proposed TVOF protocol (4) based on relative output-feedback reduces the communication burden. K_1 is employed to configure the motion modes of the first observer. K_2, ρ_i and ϕ_i are used to propel all agents to achieve the given TVOF. f_i^c expands the feasible TVOF set.

Let $\bar{y}_i = \bar{C}x_i$ be the non-output component of x_i. Pre-multiply both sides of the first equation in (1a) by $I_N \otimes \mathcal{T}$. Under protocol (4), the MAS can be described as

$$\begin{cases}\dot{\theta}_i = \mathcal{A}\theta_i + K_1(\mathcal{C}\theta_i - \sum_{j\in\mathcal{N}_i} w_{ij}(y_{ij} - f_{ij})) + \mathcal{B}K_2\vartheta_i,\\ \dot{\varphi}_i = (\mathcal{A} + \mathcal{B}K_2)\varphi_i + (\rho_i + \phi_i)K_1\mathcal{C}(\vartheta_i - \theta_i),\\ \dot{y}_i = \mathcal{A}_{11}y_i + \mathcal{A}_{12}\bar{y}_i + \mathcal{B}_1K_2\varphi_i + \mathcal{B}_1f_i^c,\\ \dot{\bar{y}}_i = \mathcal{A}_{21}y_i + \mathcal{A}_{22}\bar{y}_i + \mathcal{B}_2K_2\varphi_i + \mathcal{B}_2f_i^c.\end{cases} \tag{5}$$

Let $z_i = y_i - f_i$. From (5), it has

$$\begin{cases}\dot{\theta}_i = \mathcal{A}\theta_i + K_1(\mathcal{C}\theta_i - \sum_{j\in\mathcal{N}_i} w_{ij}(z_i - z_j)) + \mathcal{B}K_2\vartheta_i,\\ \dot{\varphi}_i = (\mathcal{A} + \mathcal{B}K_2)\varphi_i + (\rho_i + \phi_i)K_1\mathcal{C}(\vartheta_i - \theta_i),\\ \dot{z}_i = \mathcal{A}_{11}z_i + \mathcal{A}_{12}\bar{y}_i + \mathcal{B}_1K_2\varphi_i + \mathcal{A}_{11}f_i - \dot{f}_i + \mathcal{B}_1f_i^c,\\ \dot{\bar{y}}_i = \mathcal{A}_{21}z_i + \mathcal{A}_{22}\bar{y}_i + \mathcal{B}_2K_2\varphi_i + \mathcal{A}_{21}f_i + \mathcal{B}_2f_i^c.\end{cases} \tag{6}$$

Define $e = col\{e_1, e_2, \cdots e_N\}$ with $e_i = \sum_{j\in\mathcal{N}_i} w_{ij}(z_i - z_j)$ and $\bar{z} = col\{\bar{z}_1, \bar{z}_2, \cdots \bar{z}_N\}$ with $\bar{z}_i = \sum_{j\in\mathcal{N}_i} w_{ij}(\bar{y}_i - \bar{y}_j)$. Let $f = col\{f_1, f_2, \cdots, f_N\}$, $\theta = col\{\theta_1, \theta_2, \cdots, \theta_N\}$, $\varphi = col\{\varphi_1, \varphi_2, \cdots, \varphi_N\}$, $\vartheta = col\{\vartheta_1, \vartheta_2, \cdots, \vartheta_N\}$, and $f^c = col\{f_1^c, f_2^c, \cdots, f_N^c\}$. Denote by $\rho = diag\{\rho_1, \rho_2, \cdots, \rho_N\}$ and $\phi = diag\{\phi_1, \phi_2, \cdots, \phi_N\}$. System (6) can be

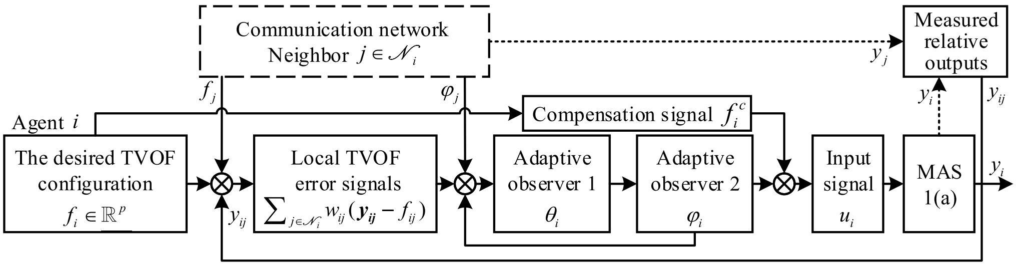


FIGURE 2. The block diagram of TVOF protocol (4).

transformed to the following compact form.

$$\begin{cases}\dot{\theta}=(I_N\otimes(\mathcal{A}+K_1\mathcal{C}))\theta-(I_N\otimes K_1)e)+(I_N\otimes\mathcal{B}K_2)\vartheta,\\ \dot{\vartheta}=(I_N\otimes(\mathcal{A}+\mathcal{B}K_2))\vartheta+(\mathcal{L}(\rho+\phi)\otimes K_1\mathcal{C})(\vartheta-\theta),\\ \dot{e}=(I_N\otimes\mathcal{A}_{11})e+(I_N\otimes\mathcal{A}_{12})\bar{z}+(I_N\otimes\mathcal{B}_1K_2)\vartheta\\ \qquad+(\mathcal{L}\otimes\mathcal{A}_{11})f-(\mathcal{L}\otimes I_p)\dot{f}+(\mathcal{L}\otimes\mathcal{B}_1)f^c,\\ \dot{\bar{z}}=(I_N\otimes\mathcal{A}_{21})e+(I_N\otimes\mathcal{A}_{22})\bar{z}+(I_N\otimes\mathcal{B}_2K_2)\vartheta\\ \qquad+(\mathcal{L}\otimes\mathcal{A}_{21})f+(\mathcal{L}\otimes\mathcal{B}_2)f^c.\end{cases} \tag{7}$$

By the definition of e, the TVOF control problem is solved if e asymptotically converges to zero. In (7), one can see that the observable component of $(\mathcal{A}_{22},\mathcal{A}_{12})$ effects $\bar{z}$. Then, the observation decomposition of (7) is given. Define $\tilde{z}_i=\bar{\mathcal{T}}^{-1}\bar{z}_i$. Based on the feature of $\bar{\mathcal{T}}^{-1}$, $\tilde{z}_i$ comprises two parts which are the observable part $\tilde{z}_{oi}$ and the unobservable part $\tilde{z}_{\bar{o}i}$. Let $\tilde{z}_o=col\{\tilde{z}_{o1},\tilde{z}_{o2},\cdots\tilde{z}_{oN}\}$ and $\tilde{z}_{\bar{o}}=col\{\tilde{z}_{\bar{o}1},\tilde{z}_{\bar{o}2},\cdots\tilde{z}_{\bar{o}N}\}$. The last two equations in (7) can be divided into the following three parts.

$$\begin{cases}\dot{e}=(I_N\otimes\mathcal{A}_{11})e+(I_N\otimes\bar{\mathcal{A}}_{12})\tilde{z}_o+(I_N\otimes\mathcal{B}_1K_2)\vartheta\\ \qquad+(\mathcal{L}\otimes\mathcal{A}_{11})f-(\mathcal{L}\otimes I_p)\dot{f}+(\mathcal{L}\otimes\mathcal{B}_1)f^c,\\ \tilde{z}_o=(I_N\otimes\bar{\mathcal{A}}_{21})e+(I_N\otimes\bar{\mathcal{A}}_{22})\tilde{z}_o+(I_N\otimes\bar{\mathcal{B}}_2K_2)\vartheta\\ \qquad+(\mathcal{L}\otimes\bar{\mathcal{A}}_{21})f+(\mathcal{L}\otimes\bar{\mathcal{B}}_2)f^c,\\ \tilde{z}_{\bar{o}}=(I_N\otimes\tilde{\mathcal{A}}_{21})e+(I_N\otimes\hat{\mathcal{A}}_{22})\tilde{z}_o+(I_N\otimes\tilde{\mathcal{A}}_{22})\tilde{z}_{\bar{o}}\\ \qquad+(I_N\otimes\tilde{\mathcal{B}}_2K_2)\vartheta+(\mathcal{L}\otimes\tilde{\mathcal{A}}_{21})f+(\mathcal{L}\otimes\tilde{\mathcal{B}}_2)f^c.\end{cases} \tag{8}$$

Let $\bar{e}=[e^T,\tilde{z}_o^T]^T$. It gets that $e=\mathcal{C}\bar{e}$. Combine the first two equations of (8) and substitute them into (7), it yields

$$\begin{cases}\dot{\theta}=(I_N\otimes\mathcal{A})\theta+(I_N\otimes K_1\mathcal{C})(\theta-\bar{e})+(I_N\otimes\mathcal{B}K_2)\vartheta,\\ \dot{\vartheta}=(I_N\otimes(\mathcal{A}+\mathcal{B}K_2))\vartheta+(\mathcal{L}(\rho+\phi)\otimes K_1\mathcal{C})(\vartheta-\theta),\\ \dot{\bar{e}}=(I_N\otimes\mathcal{A})\bar{e}+(I_N\otimes\mathcal{B}K_2)\vartheta+\begin{bmatrix}\mathcal{L}\otimes\mathcal{A}_{11}\\ \mathcal{L}\otimes\bar{\mathcal{A}}_{21}\end{bmatrix}f\\ \qquad+(\mathcal{L}\otimes\mathcal{B})f^c-\begin{bmatrix}\mathcal{L}\otimes I_p\\ 0\end{bmatrix}\dot{f}.\end{cases} \tag{9}$$

Because $\mathcal{T},\bar{\mathcal{T}}^{-1}$ are nonsingular and $(\bar{\mathcal{A}}_{22},\bar{\mathcal{A}}_{12})$ is completely observable, the following lemma can be obtained.

Lemma 2 ([3]): Based on Assumption 1, it has that $(\mathcal{A},\mathcal{B})$ is stabilizable and $(\mathcal{A},\mathcal{C})$ is completely observable.

Algorithm 1 Distributed Procedures to Design TVOF Controllers

Step 1: For a given TVOF f, check the following feasibility condition (10) for $\forall i\in\mathcal{F}$ and $j\in\mathcal{N}_i$.

$$\lim_{t\to\infty}((\hat{\mathcal{T}}_{21}\mathcal{A}_{11}+\hat{\mathcal{T}}_{22}\bar{\mathcal{A}}_{21})f_{ij}-\hat{\mathcal{T}}_{21}\dot{f}_{ij})=0. \tag{10}$$

If (10) is satisfied, then continue; otherwise, the algorithm stops.

Step 2: Solve condition (11) to determine f_i^c $(\forall i\in\mathcal{F})$.

$$\lim_{t\to\infty}(\tilde{\mathcal{T}}_1f_{ij}^c+(\hat{\mathcal{T}}_{11}\mathcal{A}_{11}+\hat{\mathcal{T}}_{12}\bar{\mathcal{A}}_{21})f_{ij}-\hat{\mathcal{T}}_{11}\dot{f}_{ij})=0, \tag{11}$$

where $f_{ij}^c=f_i^c-f_j^c$. Noting that f_i^c $(\forall i\in\mathcal{F})$ are not unique. One of the methods is to compute a f_k^c $(k\in\mathcal{F})$ at first, then the rest f_j^c $(j\in\mathcal{F},j\neq k)$ can be determined by (11).

Step 3: Choose $K_1=-P^{-1}\mathcal{C}^T$, where $P>0$ is a solution to the linear matrix inequality (LMI)

$$P\mathcal{A}+\mathcal{A}^TP-2\mathcal{C}^T\mathcal{C}<0. \tag{12}$$

Step 4: Choose K_2 such that $\mathcal{A}+\mathcal{B}K_2$ is Hurwitz. Based on Assumption 1 and Lemma 2, there always exists a K_2 satisfying this condition.

Step 5: Choose $\phi_i=\tau_\phi(\vartheta_i-\theta_i)^TP(\vartheta_i-\theta_i)$ with a given positive constant τ_ϕ.

Next, a distributed algorithm is proposed to determine the parameters in protocol (4).

Remark 4: Algorithm 1 gives a feasible TVOF set which is determined by condition (10). Condition (10) requires the desired TVOF of agent j $(j\in\mathcal{N}_i)$. As the communication network determines the neighbor set of each agent, the feasible TVOF set depends on the communication network. f_i^c $(\forall i\in\mathcal{F})$ determined by (11) is applied to expand the feasible TVOF set. In Algorithm 1, no global information about the communication network is required.

The following lemmas are useful to prove the main result of this section.

Lemma 3 ([27]): If the Nth-order directed graph $\mathcal{G}$ is strongly connected, there exists a matrix $\Xi=diag\{\xi_1,\xi_2,\ldots,\xi_N\}$ with ξ_i $(i=1,2,\ldots,N)$ being the entries of ξ such that $\xi^T\mathcal{L}=0$. Define $\hat{\mathcal{L}}=\Xi\mathcal{L}+\mathcal{L}^T\Xi$.

Then the following inequality holds

$$\min_{\tau^T x=0, x\neq 0} \frac{x^T\bar{\mathscr{L}}x}{x^T x} > \frac{\lambda_{2\bar{\mathscr{L}}}}{N},$$

where $\lambda_{2\bar{\mathscr{L}}}$ represents the smallest nonzero eigenvalue of $\bar{\mathscr{L}}$ and τ is any positive vector.

Lemma 4 ([28]): For any positive numbers p and q satisfying $1/p + 1/q = 1$, it has that $ab < a^p/p + b^q/q$, where a and b are nonnegative real numbers.

In the following, the main result of this section is derived.

Theorem 1: If Assumptions 1, 2 hold and the desired TVOF f satisfies the feasibility condition (10), the MAS (1) can achieve f under protocol (4) determined by Algorithm 1.

Proof: Let $\hat{e} = col\{\hat{e}_1, \hat{e}_2, \cdots, \hat{e}_N\}$ with $\hat{e}_i = \theta_i - \bar{e}_i$ and $\omega = col\{\omega_1, \omega_2, \cdots, \omega_N\}$ with $\omega_i = \vartheta_i - \theta_i$. From (9), it gets

$$\begin{cases} \dot{\theta} = (I_N\otimes(\mathcal{A}+\mathcal{B}K_2))\theta+(I_N\otimes K_1\mathcal{C})\hat{e}+(I_N\otimes\mathcal{B}K_2)\omega, \\ \dot{\omega} = (I_N\otimes\mathcal{A}+\mathscr{L}(\rho+\phi)\otimes K_1\mathcal{C})\omega-(I_N\otimes K_1\mathcal{C})\hat{e}, \\ \dot{\hat{e}} = (I_N\otimes(\mathcal{A}+K_1\mathcal{C}))\hat{e}+\begin{bmatrix}\mathscr{L}\otimes I_p\\ 0\end{bmatrix}\dot{f} \\ \quad -\begin{bmatrix}\mathscr{L}\otimes\mathcal{A}_{11}\\ \mathscr{L}\otimes\bar{\mathcal{A}}_{21}\end{bmatrix}f-(\mathscr{L}\otimes\mathcal{B})f^c. \end{cases} \tag{13}$$

Consider the following Lyapunov function candidate

$$V_1 = \frac{1}{2}\sum_{i=1}^N \xi_i\tau_\phi(2\rho_i+\phi_i)\omega_i^T P\omega_i+\beta\hat{e}^T(I_N\otimes P)\hat{e} + \frac{1}{2}\sum_{i=1}^N \xi_i(\rho_i-\bar{\rho})^2, \tag{14}$$

where β and $\bar{\rho}$ are positive constants to be determined later, and ξ_i $(i \in \mathcal{F})$ are the entries of ξ where $\xi^T\mathscr{L} = 0$. By Lemma 3, let $\Xi = diag\{\xi_1, \xi_2, \ldots, \xi_N\} > 0$ such that λ_{max}^{Ξ} is positive and real. The time derivative of V_1 yields

$$\begin{aligned} \dot{V}_1 &= \frac{1}{2}\sum_{i=1}^N \xi_i[(2\dot{\rho}_i+\dot{\phi}_i)\phi_i+(2\rho_i+\phi_i)\dot{\phi}_i] \\ &\quad +2\beta\hat{e}^T(I_N\otimes P)\dot{\hat{e}}+\sum_{i=1}^N \xi_i(\rho_i-\bar{\rho})\dot{\rho}_i \\ &= 2\omega^T[(\rho+\phi)\Xi\otimes P]\dot{\omega}+2\beta\hat{e}^T(I_N\otimes P)\dot{\hat{e}} \\ &\quad +\sum_{i=1}^N \xi_i(\phi_i+\rho_i-\bar{\rho})\phi_i^T\mathcal{C}^T\mathcal{C}\phi_i. \end{aligned} \tag{15}$$

Substituting $\dot{\omega}$ and $\dot{\hat{e}}$ from (13) into (15), one gets

$$\begin{aligned} &\dot{V}_1 \\ &= \omega^T[(\rho+\phi)\Xi\otimes(P\mathcal{A}+\mathcal{A}^TP)+2(\rho+\phi)\Xi\mathscr{L}(\rho+\phi)\otimes PK_1\mathcal{C} \\ &\quad +(\rho+\phi-\bar{\rho}I_N)\Xi\otimes\mathcal{C}^T\mathcal{C}]\omega-2\omega^T[(\rho+\phi)\Xi\otimes PK_1\mathcal{C}]\hat{e} \\ &\quad +2\beta\hat{e}^T[I_N\otimes P(\mathcal{A}+K_1\mathcal{C})]\hat{e}+2\beta\hat{e}^T[I_N\otimes P]F, \end{aligned} \tag{16}$$

where $F = \begin{bmatrix}\mathscr{L}\otimes I_p\\ 0\end{bmatrix}\dot{f} - \begin{bmatrix}\mathscr{L}\otimes\mathcal{A}_{11}\\ \mathscr{L}\otimes\bar{\mathcal{A}}_{21}\end{bmatrix}f - (\mathscr{L}\otimes\mathcal{B})f^c$. Based on Algorithm 1, substituting $K_1 = -P^{-1}\mathcal{C}^T$ into (16), it gets

$$\begin{aligned} &\dot{V}_1 \\ &= \omega^T[(\rho+\phi)\Xi\otimes(P\mathcal{A}+\mathcal{A}^TP)-(\rho+\phi)\bar{\mathscr{L}}(\rho+\phi)\otimes\mathcal{C}^T\mathcal{C} \\ &\quad +(\rho+\phi-\bar{\rho}I_N)\Xi\otimes\mathcal{C}^T\mathcal{C}]\omega+2\omega^T[(\rho+\phi)\Xi\otimes\mathcal{C}^T\mathcal{C}]\hat{e} \\ &\quad +\beta\hat{e}^T[I_N\otimes(P\mathcal{A}+\mathcal{A}^TP-2\mathcal{C}^T\mathcal{C})]\hat{e}+2\beta\hat{e}^T[I_N\otimes P]F, \end{aligned} \tag{17}$$

where $\bar{\mathscr{L}} = \Xi\mathscr{L}+\mathscr{L}^T\Xi$. Denote $\bar{\omega} = ((\rho+\phi)\otimes I_{p+s})\omega$. In light of Lemma 3, it leads to

$$\bar{\omega}^T(\bar{\mathscr{L}}\otimes I_{p+s})\bar{\omega} \geq \frac{\lambda_{2\bar{\mathscr{L}}}}{N}\bar{\omega}^T\bar{\omega}. \tag{18}$$

By Lemma 4, it can be obtained that

$$\begin{aligned} 2\omega^T[(\rho+\phi)\Xi\otimes\mathcal{C}^T\mathcal{C}]\hat{e} &\leq \frac{\lambda_{2\bar{\mathscr{L}}}}{2N}\omega^T[(\rho+\phi)^2\otimes\mathcal{C}^T\mathcal{C}]\omega \\ &\quad +\frac{2N}{\lambda_{2\bar{\mathscr{L}}}}\hat{e}^T[\Xi^2\otimes\mathcal{C}^T\mathcal{C}]\hat{e}. \end{aligned} \tag{19}$$

It should be noted that

$$\begin{aligned} 3\omega^T[(\rho+\phi)\Xi\otimes\mathcal{C}^T\mathcal{C}]\omega &\leq \frac{\lambda_{2\bar{\mathscr{L}}}}{2N}\omega^T[(\rho+\phi)^2\otimes\mathcal{C}^T\mathcal{C}]\omega \\ &\quad +\omega^T[\bar{\rho}\Xi\otimes\mathcal{C}^T\mathcal{C}]\omega, \end{aligned} \tag{20}$$

where $\bar{\rho} \geq (9N\lambda_{max}^{\Xi})/(2\lambda_{2\bar{\mathscr{L}}})$. Substituting (18), (19), and (20) into (17), one gets

$$\begin{aligned} \dot{V}_1 &\leq \omega^T[(\rho+\phi)\Xi\otimes(P\mathcal{A}+\mathcal{A}^TP-2\mathcal{C}^T\mathcal{C})]\omega \\ &\quad +\beta\hat{e}^T[I_N\otimes(P\mathcal{A}+\mathcal{A}^TP-2\mathcal{C}^T\mathcal{C})]\hat{e} \\ &\quad +\frac{2N}{\lambda_{2\bar{\mathscr{L}}}}\hat{e}^T[\Xi^2\otimes\mathcal{C}^T\mathcal{C}]\hat{e}+2\beta\hat{e}^T[I_N\otimes P]F. \end{aligned} \tag{21}$$

In light of Lemma 4, it has $2\beta\hat{e}^T[I_N\otimes P]F \leq \beta\sigma\hat{e}^T(I_N\otimes P^2)\hat{e}+\frac{\beta}{\sigma}F^TF$, where σ is a positive constant to be determined later. Let $\Gamma = -(P\mathcal{A}+\mathcal{A}^TP)+2\mathcal{C}^T\mathcal{C}$ and $\Pi = \frac{2N}{\lambda_{2\bar{\mathscr{L}}}}\hat{e}^T[\Xi^2\otimes\mathcal{C}^T\mathcal{C}]\hat{e}+\beta\sigma\hat{e}^T(I_N\otimes P^2)\hat{e}-\beta\hat{e}^T[I_N\otimes\Gamma]\hat{e}$. Since P and Γ are real symmetric matrices, λ_{max}^P and λ_{min}^Γ are real. Choose sufficiently small σ such that $\lambda_{min}^\Gamma-\sigma(\lambda_{max}^P)^2 > 0$ and sufficiently large $\beta > \frac{2N\lambda_{max}^{\Xi}\lambda_{max}^{\mathcal{C}^T\mathcal{C}}}{\lambda_{2\bar{\mathscr{L}}}(\lambda_{min}^\Gamma-\sigma(\lambda_{max}^P)^2)}$. One obtains that $\Pi < 0$. Since f_i satisfies condition (10) and f_i^c is determined by condition (11), it has

$$\lim_{t\to\infty}\left(\begin{bmatrix}\tilde{\mathcal{T}}_1\\ 0\end{bmatrix}f_{ij}^c+\begin{bmatrix}\hat{\mathcal{T}}_{11} & \hat{\mathcal{T}}_{12}\\ \hat{\mathcal{T}}_{21} & \hat{\mathcal{T}}_{22}\end{bmatrix}\begin{bmatrix}\mathcal{A}_{11}\\ \bar{\mathcal{A}}_{21}\end{bmatrix}f_{ij}-\begin{bmatrix}\hat{\mathcal{T}}_{11} & \hat{\mathcal{T}}_{12}\\ \hat{\mathcal{T}}_{21} & \hat{\mathcal{T}}_{22}\end{bmatrix}\begin{bmatrix}I_p\\ 0\end{bmatrix}\dot{f}_{ij}\right)=0. \tag{22}$$

According to the definitions of $\hat{\mathcal{T}}$ and $\tilde{\mathcal{T}}$, (22) can be transformed as

$$\lim_{t\to\infty}\left(\hat{\mathcal{T}}\mathcal{B}f_{ij}^c+\hat{\mathcal{T}}\begin{bmatrix}\mathcal{A}_{11}\\ \bar{\mathcal{A}}_{21}\end{bmatrix}f_{ij}-\hat{\mathcal{T}}\begin{bmatrix}I_p\\ 0\end{bmatrix}\dot{f}_{ij}\right)=0. \tag{23}$$

As $\hat{\mathcal{T}}$ is a nonsingular matrix, one obtains that (24) is satisfied from (23).

$$\lim_{t\to\infty}\left(\mathcal{B}f_{ij}^c+\begin{bmatrix}\mathcal{A}_{11}\\ \bar{\mathcal{A}}_{21}\end{bmatrix}f_{ij}-\begin{bmatrix}I_p\\ 0\end{bmatrix}\dot{f}_{ij}\right)=0. \tag{24}$$

It further implies that for $\forall i \in \mathcal{F}$ and $j \in \mathcal{N}_i$,

$$\lim_{t\to\infty}\left(\begin{bmatrix}\mathscr{L}\otimes I_p\\ 0\end{bmatrix}\dot{f}-\begin{bmatrix}\mathscr{L}\otimes\mathcal{A}_{11}\\ \mathscr{L}\otimes\bar{\mathcal{A}}_{21}\end{bmatrix}f-(\mathscr{L}\otimes\mathcal{B})f^c\right)=0, \quad (25)$$

which means that $\lim_{t\to\infty}F = 0$. Thus, there exists a finite time $\bar{t}$ such that $\lim_{t\in[\bar{t},\infty)}(\Pi+\frac{\beta}{\sigma}F^TF)\leq 0$. From (21), it has

$$\dot{V}_1 \leq \omega^T[(\rho+\phi)\Xi\otimes(P\mathcal{A}+\mathcal{A}^TP-2\mathcal{C}^T\mathcal{C})]\omega, \quad (26)$$

when $t\in[\bar{t},\infty)$. Based on (12) in Algorithm 1, $\rho\geq\rho(0)>0$ and $\phi>0$, one gets that

$$\begin{aligned}&\omega^T[(\rho+\phi)\Xi\otimes(P\mathcal{A}+\mathcal{A}^TP-2\mathcal{C}^T\mathcal{C})]\omega\\ &\quad\leq\omega^T[\rho(0)\Xi\otimes(P\mathcal{A}+\mathcal{A}^TP-2\mathcal{C}^T\mathcal{C})]\omega\\ &\quad\leq 0. \end{aligned}\quad (27)$$

It can be obtained that $\lim_{t\in[\bar{t},\infty)}\dot{V}_1\leq 0$, and V_1 is bounded. Moreover, $\dot{V}_1\equiv 0$ implies that $\omega\equiv 0$. By LaSalle's invariance principle [29], it holds that ω asymptotically converges to zero.

Substituting K_1 into the third equation in (13), it gets $(\mathcal{A}+K_1\mathcal{C})$ is Hurwitz [30]. Combining with (25), it obtains the fact that $\hat{e}$ asymptotically converges to zero. Since $\hat{e}$ and ω converge to zero asymptotically, and $(\mathcal{A}+\mathcal{B}K_2)$ is Hurwitz, it follows from the first equation in (13) that θ also asymptotically converges to zero. In virtue of the definitions of $\hat{e}$ and $\bar{e}$, it concludes that e asymptotically converges to zero, i.e., the distributed TVOF control problem is solved. ■

Remark 5: It can be obtained from (22)-(25) that condition (10) is sufficient for (25). Noting that condition (10) is also necessary for (25). The detailed proof of the necessity can be found in [20]. A given TVOF $f(t)$ can be achieved by MAS (1) under protocol (4) if and only if $f(t)$ satisfies condition (10). In contrast to the previous TVOF control results in [3], which are applicable to all agents with knowing the global information in advance, the distributed approach in this section doesn't require any agent knowing the global information. When solving TVOF control problem for large-scale MASs, the distributed approach in this section is more advantageous than those in [3]. Compared with the adaptive TVOF protocol in [20], which requires the absolute outputs of each agent, protocol (4) in this section only depends on the relative outputs between neighboring agents. In practical applications, the approach in this section is more advantageous than that in [20] because the relative outputs are much easier to obtain.

IV. DISTRIBUTED TIME-VARYING OUTPUT FORMATION TRACKING FOR MASs WITH MULTI-LEADERS

This section extends the analysis to the TVOF control problems for the MAS with multi-leaders. Consider a cluster of $N+M$ agents in MAS (1), which means that $\mathcal{L}=\mathcal{I}_{N+1}^{N+M}$ and $\mathcal{F}=\mathcal{I}_1^N$. In this case, the formation reference is determined by the trajectories of the leaders' outputs. The communication network of the MAS is described by a directed graph $\mathscr{G}$ with Laplacian matrix $\mathscr{L}$.

Assumption 3: The graph $\mathscr{G}$ has a spanning tree with its root node being the leader.

Assumption 4: $\forall i\in\mathcal{F}$, agent i is either uninformed or well-informed. $\forall i,j\in\mathcal{F}$, there exists at least one directed path from a well-informed agent i to an uninformed agent j.

By Assumptions 3 and 4, the form of $\mathscr{L}$ changes to $\mathscr{L}=\begin{bmatrix}\mathscr{L}_{\mathcal{F}} & \mathscr{L}_{\mathcal{L}}\\ 0 & 0\end{bmatrix}$, where $\mathscr{L}_{\mathcal{F}}\in\mathbb{R}^{N\times N}$ and $\mathscr{L}_{\mathcal{L}}\in\mathbb{R}^{N\times M}$. Consider the following protocol for each follower.

$$\begin{cases}\dot{\bar{\theta}}_i=\mathcal{A}\bar{\theta}_i+K_1(\mathcal{C}\bar{\theta}_i-\tilde{e}_i)+\mathcal{B}K_2\bar{\vartheta}_i,\\ \dot{\bar{\varphi}}_i=(\mathcal{A}+\mathcal{B}K_2)\bar{\varphi}_i+(\rho_i+\phi_i)K_1\mathcal{C}(\bar{\vartheta}_i-\bar{\theta}_i),\\ \dot{\rho}_i=(\bar{\vartheta}_i-\bar{\theta}_i)^T\mathcal{C}^T\mathcal{C}(\bar{\vartheta}_i-\bar{\theta}_i),\\ u_i=K_2\bar{\varphi}_i+f_i^c,\end{cases}\quad (28)$$

where $i\in\mathcal{F}$, $\tilde{e}_i=\sum_{j=1}^N w_{ij}(y_{ij}-f_{ij})+\sum_{k=N+1}^{N+M}w_{ik}(y_{ik}-f_i)$, $\bar{\theta}_i,\bar{\varphi}_i\in\mathbb{R}^{p+s}$ are the protocol states. $\bar{\vartheta}_i=\sum_{j=1}^N w_{ij}(\bar{\varphi}_i-\bar{\varphi}_j)+\sum_{k=N+1}^{N+M}w_{ik}(\bar{\varphi}_i-\bar{\varphi}_k)$ with $\bar{\varphi}_k=0$. $\phi_i=\bar{\tau}_\phi(\bar{\vartheta}_i-\bar{\theta}_i)^TP(\bar{\vartheta}_i-\bar{\theta}_i)$. ρ_i denotes the coupling weight for agent i with $\rho_i(0)>0$. K_1 and K_2 are feedback control gains. f_i^c is the formation compensation signal.

Let $\tilde{e}=col\{\tilde{e}_1,\tilde{e}_2,\cdots\tilde{e}_N\}$ and $\tilde{\bar{z}}=col\{\tilde{\bar{z}}_1,\tilde{\bar{z}}_2,\cdots\tilde{\bar{z}}_N\}$ with $\tilde{\bar{z}}_i=\sum_{j=1}^N w_{ij}(\bar{y}_i-\bar{y}_j)+\sum_{k=N+1}^{N+M}w_{ik}(\bar{y}_i-\bar{y}_k)$. When $j\in\mathcal{L}$, $z_j=y_j$ and $\bar{y}_j=\bar{C}x_j$. Denote $\bar{\theta}=col\{\bar{\theta}_1,\bar{\theta}_2,\cdots,\bar{\theta}_N\}$, $\bar{\varphi}=col\{\bar{\varphi}_1,\bar{\varphi}_2,\cdots,\bar{\varphi}_N\}$, and $\bar{\vartheta}=col\{\bar{\vartheta}_1,\bar{\vartheta}_2,\cdots,\bar{\vartheta}_N\}$. It has that $\tilde{e}=(\mathscr{L}_{\mathcal{F}}\otimes I_p)z+(\mathscr{L}_{\mathcal{L}}\otimes I_p)z_{\mathcal{L}}$ with $z_{\mathcal{L}}=col\{z_{N+1},z_{N+2},\cdots,z_{N+M}\}$. Substitute protocol (28) into MAS (1), one can obtain the closed-loop dynamics (29).

$$\begin{cases}\dot{\bar{\theta}}=(I_N\otimes(\mathcal{A}+K_1\mathcal{C}))\bar{\theta}-(I_N\otimes K_1)\tilde{e})+(I_N\otimes\mathcal{B}K_2)\bar{\vartheta},\\ \dot{\bar{\vartheta}}=(I_N\otimes(\mathcal{A}+\mathcal{B}K_2))\bar{\vartheta}+(\mathscr{L}_{\mathcal{F}}(\rho+\phi)\otimes K_1\mathcal{C})(\bar{\vartheta}-\bar{\theta}),\\ \dot{\tilde{e}}=(I_N\otimes\mathcal{A}_{11})\tilde{e}+(I_N\otimes\mathcal{A}_{12})\tilde{\bar{z}}+(I_N\otimes\mathcal{B}_1K_2)\bar{\vartheta}\\ \qquad+(\mathscr{L}_{\mathcal{F}}\otimes\mathcal{A}_{11})f-(\mathscr{L}_{\mathcal{F}}\otimes I_p)\dot{f}+(\mathscr{L}_{\mathcal{F}}\otimes\mathcal{B}_1)f^c,\\ \dot{\tilde{\bar{z}}}=(I_N\otimes\mathcal{A}_{21})\tilde{e}+(I_N\otimes\mathcal{A}_{22})\tilde{\bar{z}}+(I_N\otimes\mathcal{B}_2K_2)\bar{\vartheta}\\ \qquad+(\mathscr{L}_{\mathcal{F}}\otimes\mathcal{A}_{21})f+(\mathscr{L}_{\mathcal{F}}\otimes\mathcal{B}_2)f^c.\end{cases}\quad (29)$$

The TVOFT control problem is solved if the formation tracking error $\tilde{e}$ asymptotically converges to zero. After the observation decomposition, it has that $\tilde{e}=\mathcal{C}\tilde{\bar{e}}$. Let $\hat{\bar{e}}=\bar{\theta}-\tilde{\bar{e}}$ and $\bar{\omega}=\bar{\vartheta}-\bar{\theta}$. It can be transformed from (29) that

$$\begin{cases}\dot{\bar{\theta}}=(I_N\otimes(\mathcal{A}+\mathcal{B}K_2))\bar{\theta}+(I_N\otimes K_1\mathcal{C})\hat{\bar{e}}+(I_N\otimes\mathcal{B}K_2)\bar{\omega},\\ \dot{\bar{\omega}}=(I_N\otimes\mathcal{A}+\mathscr{L}_{\mathcal{F}}(\rho+\phi)\otimes K_1\mathcal{C})\bar{\omega}-(I_N\otimes K_1\mathcal{C})\hat{\bar{e}},\\ \dot{\hat{\bar{e}}}=(I_N\otimes(\mathcal{A}+K_1\mathcal{C}))\hat{\bar{e}}+\begin{bmatrix}\mathscr{L}_{\mathcal{F}}\otimes I_p\\ 0\end{bmatrix}\dot{f}\\ \qquad-\begin{bmatrix}\mathscr{L}_{\mathcal{F}}\otimes\mathcal{A}_{11}\\ \mathscr{L}_{\mathcal{F}}\otimes\bar{\mathcal{A}}_{21}\end{bmatrix}f-(\mathscr{L}_{\mathcal{F}}\otimes\mathcal{B})f^c.\end{cases}\quad (30)$$

The following lemma is useful to analyze the main result of this section.

Lemma 5 ([22]): If Assumptions 3 and 4 hold, all eigenvalues' real parts of $\mathscr{L}_{\mathcal{F}}$ are positive. $-\mathscr{L}_{\mathcal{F}}^{-1}\mathscr{L}_{\mathcal{L}}$ has nonnegative entries and identical rows. The structure of $-\mathscr{L}_{\mathcal{F}}^{-1}\mathscr{L}_{\mathcal{L}}$

is as below

$$-\mathscr{L}_{\mathcal{F}}^{-1}\mathscr{L}_{\mathcal{L}} = \mathbf{1}_M \otimes \frac{[\bar{\alpha}_{N+1}, \bar{\alpha}_{N+2}, \cdots, \bar{\alpha}_{N+M}]}{\sum_{k=N+1}^{N+M} \bar{\alpha}_k}.$$

Lemma 6 ([31]): For a nonsingular M-matrix $\mathscr{L}_{\mathcal{F}}$, there exists a positive diagonal matrix G such that $G\mathscr{L}_{\mathcal{F}} + \mathscr{L}_{\mathcal{F}}^T G > 0$.

The following theorem guarantees that the desired TVOFT can be achieved by MAS (1) with multi-leaders.

Theorem 2: If Assumptions 1, 3, 4 hold and the feasibility condition (10) is satisfied, the TVOFT control problem for MAS (1) with multi-leaders can be solved in a fully distributed manner under protocol (28) and Algorithm 1.

Proof: According to the design in Algorithm 1, $\mathcal{A} + K_1\mathcal{C}$ is Hurwitz. From (22), (23), (24) and the fact that $\mathscr{L}_{\mathcal{F}}$ is nonsingular, one can obtain that

$$\lim_{t\to\infty}\left(\begin{bmatrix}\mathscr{L}_{\mathcal{F}} \otimes I_p\\ 0\end{bmatrix}\dot{f} - \begin{bmatrix}\mathscr{L}_{\mathcal{F}} \otimes \mathcal{A}_{11}\\ \mathscr{L}_{\mathcal{F}} \otimes \bar{\mathcal{A}}_{21}\end{bmatrix} f - (\mathscr{L}_{\mathcal{F}} \otimes \mathcal{B})f^c\right) = 0. \quad (31)$$

Thus, the convergence of $\hat{\tilde{e}}$ in (30) is obtained.

Let $\bar{\Xi} = diag\{\bar{\xi}_1, \bar{\xi}_2, \cdots, \bar{\xi}_N\}$. By Lemma 6, there exists a positive definite $\bar{\Xi}$ such that $\bar{\mathscr{L}}_{\mathcal{F}} = \bar{\Xi}\mathscr{L}_{\mathcal{F}} + \mathscr{L}_{\mathcal{F}}^T\bar{\Xi} > 0$. Consider the following Lyapunov function candidate

$$V_2 = \frac{1}{2}\sum_{i=1}^{N}\bar{\xi}_i\bar{\tau}_\phi(2\rho_i + \phi_i)\bar{\omega}_i^T P\bar{\omega}_i + \tilde{\beta}\hat{\tilde{e}}^T(I_N \otimes P)\hat{\tilde{e}} + \frac{1}{2}\sum_{i=1}^{N}\bar{\xi}_i(\rho_i - \tilde{\rho})^2, \quad (32)$$

where $\tilde{\beta}$ and $\tilde{\rho}$ are positive numbers to be determined later. The time derivative of V_2 yields

$$\begin{aligned}\dot{V}_2 = {} & \bar{\omega}^T[(\rho+\phi)\bar{\Xi}\otimes(P\mathcal{A}+\mathcal{A}^T P) - (\rho+\phi)\bar{\mathscr{L}}_{\mathcal{F}}(\rho+\phi)\otimes\mathcal{C}^T\mathcal{C} \\ & + (\rho+\phi-\tilde{\rho}I_N)\bar{\Xi}\otimes\mathcal{C}^T\mathcal{C}]\bar{\omega} + 2\bar{\omega}^T[(\rho+\phi)\bar{\Xi}\otimes\mathcal{C}^T\mathcal{C}]\hat{\tilde{e}} \\ & + \tilde{\beta}\hat{\tilde{e}}^T[I_N\otimes(P\mathcal{A}+\mathcal{A}^T P - 2\mathcal{C}^T\mathcal{C})]\hat{\tilde{e}} + 2\tilde{\beta}\hat{\tilde{e}}^T[I_N\otimes P]\bar{F},\end{aligned} \quad (33)$$

where $\bar{F} = \begin{bmatrix}\mathscr{L}_{\mathcal{F}}\otimes I_p\\ 0\end{bmatrix}\dot{f} - \begin{bmatrix}\mathscr{L}_{\mathcal{F}}\otimes\mathcal{A}_{11}\\ \mathscr{L}_{\mathcal{F}}\otimes\bar{\mathcal{A}}_{21}\end{bmatrix} f - (\mathscr{L}_{\mathcal{F}}\otimes\mathcal{B})f^c$. Using Lemma 4, it can be found that

$$\begin{aligned}2\bar{\omega}^T[(\rho+\phi)\bar{\Xi}\otimes\mathcal{C}^T\mathcal{C}]\hat{\tilde{e}} \le {} & \frac{\lambda_{min}^{\bar{\mathscr{L}}_{\mathcal{F}}}}{2}\bar{\omega}^T[(\rho+\phi)^2\otimes\mathcal{C}^T\mathcal{C}]\bar{\omega} \\ & + \frac{2}{\lambda_{min}^{\bar{\mathscr{L}}_{\mathcal{F}}}}\hat{\tilde{e}}^T[\bar{\Xi}^2\otimes\mathcal{C}^T\mathcal{C}]\hat{\tilde{e}},\end{aligned} \quad (34)$$

and $2\tilde{\beta}\hat{\tilde{e}}^T[I_N \otimes P]\bar{F} \le \tilde{\beta}\tilde{\sigma}\hat{\tilde{e}}^T(I_N \otimes P^2)\hat{\tilde{e}} + \frac{\tilde{\beta}}{\tilde{\sigma}}\bar{F}^T\bar{F}$, where $\tilde{\sigma}$ is a positive constant to be determined later. Let $\bar{\Pi} = \frac{2}{\lambda_{min}^{\bar{\mathscr{L}}_{\mathcal{F}}}}\hat{\tilde{e}}^T[\bar{\Xi}^2 \otimes \mathcal{C}^T\mathcal{C}]\hat{\tilde{e}} + \tilde{\beta}\tilde{\sigma}\hat{\tilde{e}}^T(I_N \otimes P^2)\hat{\tilde{e}} - \tilde{\beta}\hat{\tilde{e}}^T[I_N \otimes \Gamma]\hat{\tilde{e}}$. Choose $\tilde{\rho} \ge (9\lambda_{max}^{\bar{\Xi}})/(2\lambda_{min}^{\bar{\mathscr{L}}_{\mathcal{F}}})$, $0 < \tilde{\sigma} < \lambda_{min}^{\Gamma}/(\lambda_{max}^{P})^2$, and $\tilde{\beta} > \frac{2\lambda_{max}^{\bar{\Xi}}\lambda_{max}^{\mathcal{C}^T\mathcal{C}}}{\lambda_{min}^{\bar{\mathscr{L}}_{\mathcal{F}}}(\lambda_{min}^{\Gamma} - \tilde{\sigma}(\lambda_{max}^{P})^2)}$. One can get

$$\lim_{t\in[\tilde{t},\infty)}(\bar{\Pi} + \frac{\tilde{\beta}}{\tilde{\sigma}}\bar{F}^T\bar{F}) \le 0. \quad (35)$$

Substituting $\tilde{\rho}$, $\tilde{\sigma}$, $\tilde{\beta}$, (34), and (35) into (33), it can be obtained that

$$\dot{V}_2 \le \bar{\omega}^T[(\rho+\phi)\bar{\Xi}\otimes(P\mathcal{A}+\mathcal{A}^T P - 2\mathcal{C}^T\mathcal{C})]\bar{\omega}\,(t \in [\tilde{t}, \infty)). \quad (36)$$

Since $\rho \ge \rho(0) > 0$ and $\phi > 0$, it has $\lim_{t\in[\tilde{t},\infty)}\dot{V}_2 \le 0$, and V_2 is bounded. Similar to the proof for Theorem 1, then the convergence of $\bar{\theta}$ can be obtained. Further, it has $\lim_{t\to\infty}\tilde{e} = 0$. According to the definition of $\tilde{e}$, it can be obtained that $\lim_{t\to\infty}(z - (-\mathscr{L}_{\mathcal{F}}^{-1}\mathscr{L}_{\mathcal{L}} \otimes I_p)z_{\mathcal{L}}) = 0$. In light of Lemma 5, it has $\lim_{t\to\infty}(y_i - f_i - \frac{1}{\sum_{k=N+1}^{N+M}\bar{\alpha}_k}\sum_{k=N+1}^{N+M}\bar{\alpha}_k y_k) = 0$, which means that (2) is satisfied. The distributed TVOFT control problem using relative output-feedback for MAS (1) with multi-leaders is solved. ■

Remark 6: Based on the structure of $\mathscr{L}_{\mathcal{F}}$, condition (10) is necessary and sufficient for (31). MAS (1) achieves the desired TVOFT under protocol (28) if and only if $f(t)$ satisfies condition (10). However, in the case of only formation control, the communication network should satisfy Assumption 2 and the strongly connected relation between neighboring agents is significant for achieving the given TVOF. In the case of formation tracking control, only Assumption 3 is required for the communication network. In practical applications, if a given nonlinear MAS can be preprocessed by feedback linearization methods, the approach in this section can be used to handle the formation tracking problem for the nonlinear MAS.

Remark 7: Theorem 2 shows that the convex combination of multi-leaders' outputs can be tracked by the followers' TVOF in a fully distributed manner. In the one leader case, the leader's failure has a devastating impact on the MAS. While in the multi-leaders case, if some leaders fail, the macro trajectory of the MAS still guided by the rest leaders. The formation tracking with multi-leaders in this section has stronger robustness than that with one leader. The formation tracking results with multi-leaders in [22] assume that full states are observable and available for feedback, the communication network is capable of transmitting high dimensional full states, and each follower is aware of the global information. From protocol (28) and Theorem 2, only observable outputs are available for feedback, the communication network only transmits the lower dimensional observer $\bar{\varphi}_i$, and no global information is required in this paper. The approach in this section is more practical for large scale MASs.

Remark 8: Compared with distributed output containment control problems, which require that the outputs of followers converge to the convex hull formed by the outputs of leaders, the distributed TVOFT control problem considered in this section requires that the outputs of followers converge to the desired formation configuration $f_i(t)$. There is no requirement for relative coordinates between leader and follower in distributed containment control. While $f_i(t)$ describes the changing requirements of relative coordinates between leader and follower in the distributed TVOFT control problem. From (2), the convex hull formed by the outputs of leaders influence the

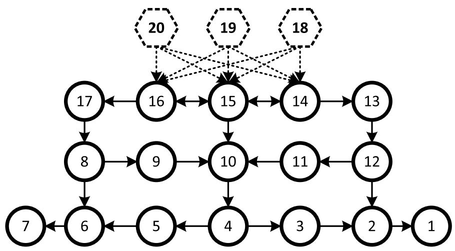


FIGURE 3. The communication network for the MAS with multiple leaders.

formation reference trajectory of followers but is independent of choosing $f_i(t)$. If choosing $f_i(t) = 0$ for every follower, the problem studied in this section degenerates to an output containment control problem.

V. NUMERICAL SIMULATIONS

A numerical example is presented to illustrate the effectiveness of the developed formation approaches in previous sections. Consider a MAS consisting of 20 agents whose dynamics are as shown in MAS (1) with $x_i = [x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i}]^T$, $y_i = [y_{1i}, y_{2i}, y_{3i}]^T$ and

$$A = \begin{bmatrix} 0.75 & 1.5 & -1.5 & -1.25 & 1.0 & -1.0 \\ -0.25 & -2.5 & -0.5 & 0.75 & 2.0 & -1.0 \\ 1.5 & 2.0 & 0 & -0.5 & -1.0 & 0 \\ -1.25 & -2.5 & 0.5 & 0.75 & 3.0 & 1.0 \\ 1.25 & 1.5 & -0.5 & -0.75 & -2.0 & -1.0 \\ 4.75 & 7.5 & -1.5 & -3.25 & -6.0 & -2.0 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.5 \\ 0.5 \\ 0 \\ 1.5 \\ 0.5 \\ -0.5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}.$$

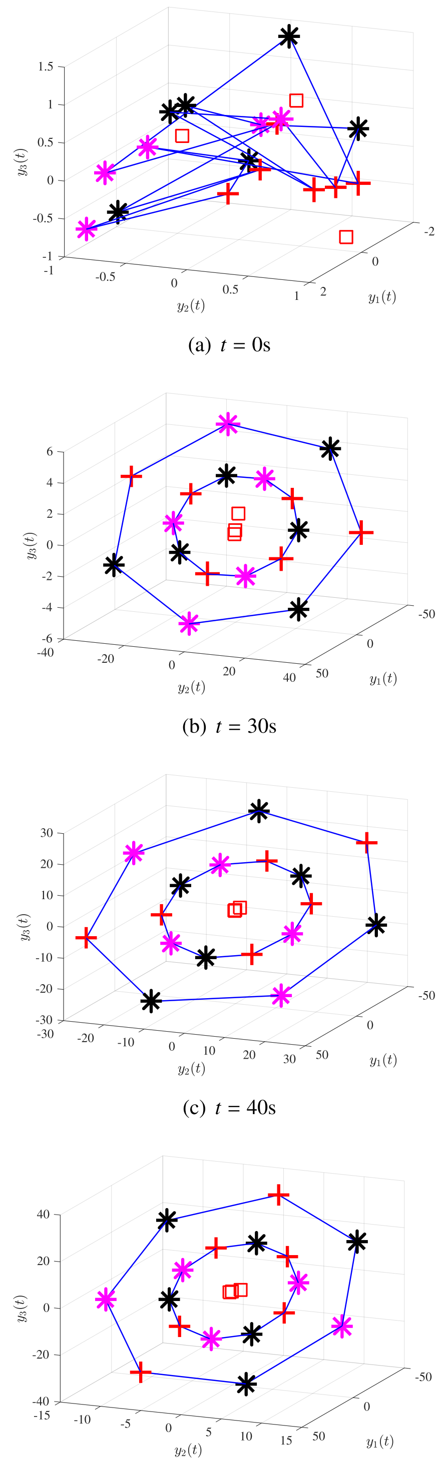
Choose

$$\bar{C} = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

It can be obtained that (A, B) is stabilizable for the MAS and $(\mathcal{A}_{22}, \mathcal{A}_{12})$ is not completely observable. Choose a nonsingular matrix $\bar{\mathcal{T}} = [-1, 1, 0; 1, 0, -1; 0, 1, 1]$. It can be verified that $(\bar{\mathcal{A}}_{22}, \bar{\mathcal{A}}_{12})$ is completely observable. Then, the new system matrices $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ can be calculated.

The given MAS has multiple leaders; that is, $\mathcal{L} = \mathcal{I}_{18}^{20}$ and $\mathcal{F} = \mathcal{I}_1^{17}$. FIGURE 3 gives the communication network of the MAS with 3 leaders. The desired TVOF for all followers is specified by

$$f_i = \begin{bmatrix} 10k\cos(t)\sin(t+\frac{2\pi(i-g)}{n}) + 10k\sin(t)\cos(t+\frac{2\pi(i-g)}{n}) \\ 10k\sin(t)\sin(t+\frac{2\pi(i-g)}{n}) \\ 10k\cos(t)\cos(t+\frac{2\pi(i-g)}{n}) \end{bmatrix},$$



(a) $t = 0$s

(b) $t = 30$s

(c) $t = 40$s

(d) $t = 50$s

FIGURE 4. The output snapshots of the MAS with multiple leaders.

where $n = 10$, $k = 2$, $g = 1$ for $i \in \mathcal{I}_1^{10}$ and $n = 7$, $k = 4$, $g = 11$ for $i \in \mathcal{I}_{11}^{17}$. Choose $\rho_i(0) = 0.1$ $\hat{\mathcal{T}}_{11} = [0, 0, 1]$, $\hat{\mathcal{T}}_{12} = [0, 0]$, $\hat{\mathcal{T}}_{21} = [0, 1, 0; 1, 0, 0; 0, 0, -1; 0, 0, 0]$, $\hat{\mathcal{T}}_{22} = [0, 0; 0, 0; 0, 1; 1, -1]$, and $\tilde{\mathcal{T}} = 1$, then the satisfaction of condition (10) can be verified and the solution of

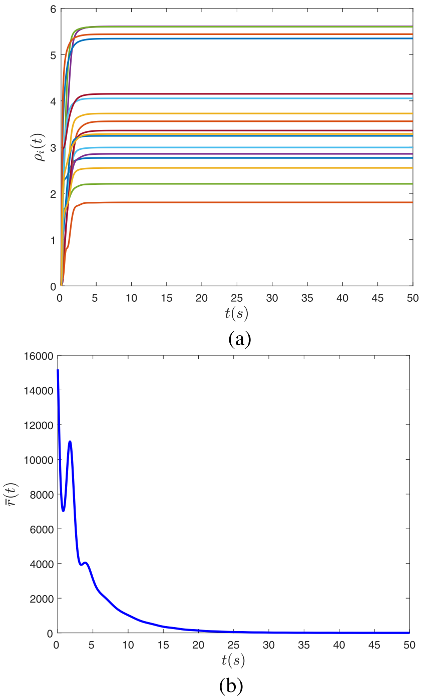


FIGURE 5. **(a) The coupling weights $\rho_i(t)$ and (b) the TVOFT error curve.**

condition (11) can be obtained as

$$f_i^c = -10k\sin(t)\sin(t+\frac{2\pi(i-g)}{n}) + 10k\cos(t) \times \cos(t+\frac{2\pi(i-g)}{n}).$$

According to Algorithm 1, the gain matrices in protocol (28) can be calculated as

$$K_1 = \begin{bmatrix} -1.6389 & -0.0164 & 0.6313 \\ -0.0164 & -0.8890 & 0.5991 \\ 0.6313 & 0.5991 & -6.7454 \\ 0.6171 & 0.3997 & -3.3652 \\ 0.4483 & -0.1324 & 1.1078 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.2408 & 0.1748 & -0.5893 & 0.3600 & -1.5557 \end{bmatrix}.$$

Define $\bar{r}(t) = r^T(t)r(t)$ as the formation error signal of the MAS, where $r(t) = col\{r_2(t), r_3(t), \cdots, r_{17}(t)\}$ with $r_i(t) = z_i(t) - z_1(t)$ $(i \in \mathcal{I}_2^{17})$. The snapshots of the outputs $y_i(t)$ for different timestamps are shown in FIGURE 4. The coupling weights $\rho_i(t)$ and the formation error $\bar{r}(t)$ under protocol (28) are displayed in FIGURE 5. It can be seen that the desired TVOF is achieved and the adaptive coupling weights converge to finite values.

VI. CONCLUSION

Distributed TVOFT control problems were addressed for general MAS using only relative output-feedback. From the leaderless case to the multi-leader case, two adaptive formation protocols composed of local observers were developed, respectively. Both proposed adaptive formation protocols required no global information about the communication network and were fully distributed. Also, no absolute outputs were employed in all proposed formation protocols. These points are the main contributions with respect to the previous related results. Based on these results, it is of interest to further study TVOFT problems for nonlinear MAS with multi-leaders. Moreover, how to address TVOFT control problems with parameter uncertainties and external disturbances is another interesting topic for future work.

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