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Mean-Square Admissibility Analysis and Controller Design for Itô-Type Stochastic Singular Systems

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ABSTRACT The issues of mean-square admissibility and synthesis of Itô-type stochastic singular systems (SSSs) under Brownian parameter perturbations are introduced in this article. For ease of computation, a novel sufficient condition is given to guarantee autonomous systems are mean-square admissible via strict linear matrix inequalities(LMIs). Furthermore, own to the measurable of the system states, both state feedback controller and observer-based controller (OBC) for Itô-type SSSs are investigated. However, in Itô-type SSSs, because the state of the system and the observer can be affected by Brownian fluctuation, it is not feasible that the observer and control gains design are completely separate. To this end, an innovative design approach is also proposed to solve the controller and observer parameters simultaneously in form of strict LMIs. Finally, three examples are introduced to demonstrate the effectiveness of the proposed method.

INDEX TERMS Itô-type SSSs, mean-square admissibility, Brownian parameter perturbations, OBC, strict LMIs.

I. INTRODUCTION

Singular systems are a kind of dynamic systems, which are more general and natural to represent and to handle the behaviour of practical models than the normal ones, for instance, power systems, electrical circuit systems, constrained mechanical systems, bioeconomic systems, and more other field [1]–[3]. The study of singular systems has important theoretical and practical significance, so there are many research achievements on admissibility analysis and synthesis [4], [5], H_∞ control [6], [7], observer and filter design [8], [9], fault diagnosis and fault tolerance control [10], [11], and so on. In practical control systems, there exist stochastic environment noises inevitably. So, the researches about stochastic systems have been paid more attentions due to their widespread applications in numerous actual systems [12]–[14].

Combining the advantages of singular models and stochastic models, stochastic singular models can describe the features of physical systems more conveniently and accurately. Therefore, many attractive results and a large variety of control problems have been investigated and solved. Based on H-representation approach, SSSs with Brownian motion can be converted into the standard singular system, then the stability theorem of this kind of systems is given via LMIs in [15]. In [16], based on nonparallel distributed compensation sliding mode control method, robust control issues of nonlinear SSSs are discussed. Using stochastic Lyapunov functional, the dissipative controller design method for fuzzy Itô-type SSSs is proposed in [17]. Using the stochastic analysis techniques, the passivity analysis and state-feedback controller for fuzzy delay SSSs is discussed in [18]. By multiple

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Lyapunov functions and matrix decomposition approach, mean square admissibility conditions for the SSSs with Poisson switching are proposed [19]. In [20], the unmatched controller design method is given to guarantee IT2 uncertain fuzzy SSSs stochastic admissibility. In [21], the mean-square (MS) exponential stability of SSSs with Brownian motion is proved strictly, which establishes the firm foundation for the subsequent analysis and synthesis for such system. The observer-based MS admissible controller design approach for Markov Itô-type SSSs is discussed via LMIs in [22]. In [21], [22], the sequential design method is given to obtain controller and observer gains and this calculation method is comparatively conservative. Thus, a novel method to deduce the conservatism during the design of controller and observer is become an important challenge.

According to the background hereinbefore, MS admissibility and controller design problem of Itô-type SSSs is discussed in this article. First of all, using Lyapunov functional and stochastic analysis techniques, sufficient conditions for Itô-type SSSs to be MS admissibility are given and proven completely via LMIs. Secondly, both state feedback controller (SFC) and OBC are investigated to ensure the MS admissibility of the considered systems. The parameters of controller and observer are solved simultaneously and equality constraint conditions are cancelled by matrix operations. Finally, the validity and applicability of our proposed method is illustrated by three examples. The contributions of the study are summarized below.

1) The sufficient condition of Itô-type SSSs to be mean-square admissible is proposed with a complete proof process. Meanwhile, this admissibility condition can be solved by strict LMIs, which removes the equality constraint condition. The obtained admissibility theorem contains related results in deterministic singular systems and stochastic normal systems.

2) Both the design method of SFC and OBC are given to ensure the mean-square admissibility of closed-loop system. In the OBC design technique, the parameters of controller and observer are solved simultaneously, which is different from the sequential design method in [21]. Form illustrative example, the proposed method in this article is less conservativeness than the relevant results in [21].

The structure of this article is organized as follows. In Section II, Preliminary results are provided. Section III provides main results of MS admissibility and controller design for Itô-type SSSs. Finally, Section IV and V contain three illustration examples and conclusions, respectively.

Notations:

$Q \succeq 0 (Q > 0)$: positive semi-definite (positive definite) matrix;

$Q \preceq 0 (Q < 0)$: semi-definite (negative definite) matrix;

A^{T}: transpose of A;

$\mathfrak{E}()$: expectation operator;

$\mathtt{Det}(A)$: determinant of the matrix A.

$\mathfrak{R}^n$: n-dimensional Euclidean space;

$\mathfrak{R}^{m\times n}$: $m \times n$ real matrices set;

$\mathtt{Deg}()$: degree of the polynomial.

A^+: Moore-Penrose pseudo inverse of A;

$\mathtt{Rank}(A)$: rank of the matrix A.

II. PRELIMINARIES

Consider the Itô-type SSSs defined as follow.

$$\begin{aligned} \mathbb{E}dx(t) &= \mathcal{A}x(t)dt + \mathcal{B}u(t)dt + \mathcal{J}x(t)d\varpi(t) \\ y(t) &= \mathcal{C}x(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathfrak{R}^n$ and $u(t) \in \mathfrak{R}^m$ denote state / input vector, $y(t) \in \mathfrak{R}^q$ denote output vector; Matrix $\mathbb{E}$ might be singular and $\mathtt{Rank}\mathbb{E} = n_r \leq n$; $\mathbb{E}, \mathcal{A}, \mathcal{B}, \mathcal{J}$, and $\mathcal{C}$ are known constant matrices. $\varpi(t)$ is one-dimensional Brownian motion with $\mathfrak{E}(d\varpi(t)) = 0$ and $\mathfrak{E}(d\varpi^2(t)) = 0$.

Next, some lemmas and definitions are given as follow.

Definition 2.1 ([15]):

- Unforced SSSs (1) is impulse free if
$$\mathtt{Deg}(\mathtt{Det}(s\mathbb{E} - \mathcal{A})) = \mathtt{Rank}(\mathbb{E})$$
- Unforced SSSs (1) is MS asymptotically stable if $\forall x(0) \in \mathfrak{R}^n$,
$$\lim_{t\to\infty} \mathfrak{E}\{\|x(t)\|^2\} = 0$$
- Unforced SSSs (1) is MS admissible if the system has the unique solution, impulse free and MS asymptotically stable.

Assumption 2.1 ([21]): $\mathtt{Rank}(\mathbb{E}, \mathcal{J}) = \mathtt{Rank}(\mathbb{E})$.

Remark 2.1: Based on Assumption 2.1, the invertible matrices $\mathbb{R}$ and $\mathbb{S}$ can be obtained such that

$$\mathbb{R}\mathbb{E}\mathbb{S} = \begin{bmatrix} I_{n_r} & 0 \\ 0 & 0 \end{bmatrix}$$
$$\mathbb{R}\mathcal{A}\mathbb{S} = \begin{bmatrix} \mathcal{A}_1 & \mathcal{A}_2 \\ \mathcal{A}_3 & \mathcal{A}_4 \end{bmatrix}$$
$$\mathbb{R}\mathcal{J}\mathbb{S} = \begin{bmatrix} \mathcal{J}_1 & \mathcal{J}_2 \\ 0 & 0 \end{bmatrix}$$

Considering the state transformation

$$\mathbb{S}^{-1}x(t) = \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \end{bmatrix}$$

the system (1) is restricted equivalent with

$$\begin{aligned} d\varepsilon_1(t) &= (\mathcal{A}_1\varepsilon_1(t)dt + \mathcal{A}_2\varepsilon_2(t))dt + (\mathcal{J}_1\varepsilon_1(t) \\ &\quad + \mathcal{J}_2\varepsilon_2(t))d\varpi(t)\ 0 = (\mathcal{A}_3\varepsilon_1(t)dt + \mathcal{A}_4\varepsilon_2(t))dt \end{aligned} \tag{2}$$

So, it is known that the diffusion term is not the impact of system structure. Then, when $(\mathbb{E}, \mathcal{A})$ is impulse-free, by [1], $\mathcal{A}_4$ is invertible. Further, system (1) is restricted equivalent to

$$\begin{aligned} d\varepsilon_1(t) &= (\mathcal{A}_1 - \mathcal{A}_2\mathcal{A}_4^{-1}\mathcal{A}_3)\varepsilon_1(t)dt \\ &\quad + (\mathcal{J}_1 - \mathcal{J}_2\mathcal{A}_4^{-1}\mathcal{A}_3)\varepsilon_1(t)d\varpi(t) \\ \varepsilon_2(t) &= -\mathcal{A}_4^{-1}\mathcal{A}_3\varepsilon_1(t) \end{aligned} \tag{3}$$

So, the pair $(\mathbb{E}, \mathcal{A})$ is impulse free, and $\mathtt{Rank}(\mathbb{E}, \mathcal{J}) = \mathtt{Rank}(\mathbb{E})$, which guarantee the existence and uniqueness of impulse-free solution of the system (1).

Lemma 2.1 ([23]): Let $\mathbb{X} \in \mathfrak{R}^{n\times n}$ be symmetric with $\mathbb{E}_R^T\mathbb{X}\mathbb{E}_R > 0$, $\mathbb{T} \in \mathfrak{R}^{(n-n_r)\times(n-n_r)}$ is invertible. Then, $\mathbb{X}\mathbb{E}^T + S\mathbb{T}M$ is invertible and its inverse is expressed as

$$(\mathbb{X}\mathbb{E}^T + S\mathbb{T}M)^{-1} = X\mathbb{E} + M^TTS^T$$

where full row rank matrices M and S satisfies $M\mathbb{E} = 0$ and $\mathbb{E}S = 0$, respectively; $\mathbb{E} = \mathbb{E}_L\mathbb{E}_R^T$, in which $\mathbb{E}_L$ and $\mathbb{E}_R$ are with full column rank; $X^{\mathrm{T}} = X$ and T is invertible with

$$\begin{aligned} \mathbb{E}_L^TX\mathbb{E}_L &= (\mathbb{E}_R^T\mathbb{X}\mathbb{E}_R)^{-1} \\ T &= (MM^T)^{-1}\mathbb{T}^{-1}(S^TS)^{-1} \end{aligned}$$

Lemma 2.2 ([24]): Considering system (1), let

$$\mathbb{V}(x(t)) = x^{\mathrm{T}}(t)\mathbb{E}^{\mathrm{T}}\mathbb{P}x(t) \tag{4}$$

where the invertible matrix $\mathbb{P}$ satisfies

$$\mathbb{E}^{\mathrm{T}}\mathbb{P} = \mathbb{P}^{\mathrm{T}}\mathbb{E} \succeq 0 \tag{5}$$

By defining the weak infinitesimal operator $\mathcal{L}$, Itô formula can be given as

$$d\mathbb{V}(x(t)) = \mathcal{L}\mathbb{V}(x(t))dt + 2x^{\mathrm{T}}\mathbb{P}^{\mathrm{T}}\mathcal{J}x(t)d\varpi(t) \tag{6}$$

where

$$\mathcal{L}\mathbb{V}(x(t))=x^T(t)(\mathcal{A}^T\mathbb{P} + \mathbb{P}^T\mathcal{A} + \mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T\mathbb{P}\mathbb{E}^+\mathcal{J})x(t)$$

Lemma 2.3 ([25]): If there exist a scalar $\varrho > 0$ and matrices Δ, $\mathbb{U}_\iota$, $\mathbb{V}_\iota$, $\mathbb{W}_\iota$ $(\iota = 1, \ldots, m)$ hold

$$\begin{bmatrix} \Delta & \mathbb{U}_1 + \varrho\mathbb{V}_1 & \cdots & \mathbb{U}_m + \varrho\mathbb{V}_m \\ * & \mathrm{diag}\{-\varrho\mathbb{W}_1 - \varrho\mathbb{W}_1^{\mathrm{T}} & \cdots & -\varrho\mathbb{W}_m - \varrho\mathbb{W}_m^{\mathrm{T}} \end{bmatrix} < 0 \tag{7}$$

we get

$$\Delta + \sum_{\iota=1}^{m}(\mathbb{U}_\iota\mathbb{W}_\iota^{-1}\mathbb{V}_\iota^{\mathrm{T}} + \mathbb{V}_\iota\mathbb{W}_\iota^{-\mathrm{T}}\mathbb{U}_\iota^{\mathrm{T}}) < 0 \tag{8}$$

III. MAIN RESULT

A. ADMISSIBILITY ANALYSIS

Theorem 3.1: Unforced SSSs (1) is MS admissible if there exist a symmetric matrix $X \in \mathfrak{R}^{n\times n}$ and a nonsingular matrix $T \in \mathfrak{R}^{(n-n_r)\times(n-n_r)}$, such that the following LMIs conditions hold.

$$\mathbb{E}_L^TX\mathbb{E}_L > 0 \tag{9}$$

$$\begin{aligned} &\mathcal{A}^T(X\mathbb{E} + M^TTS^T) + (X\mathbb{E} + M^TTS^T)^T\mathcal{A} \\ &\quad + \mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^TX\mathbb{E}\mathbb{E}^+\mathcal{J} < 0 \end{aligned} \tag{10}$$

where row full rank matrices M, S with $M\mathbb{E} = 0$ and $\mathbb{E}S = 0$.

Proof: First of all, we prove that unforced SSSs (1) is impulse free and has an unique solution.

By $\mathbb{E}_L^TX\mathbb{E}_L > 0$ and $\mathbb{E} = \mathbb{E}_L\mathbb{E}_R^T$, we can obtain

$$\mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^TX\mathbb{E}\mathbb{E}^+\mathcal{J} \succeq 0 \tag{11}$$

then, (10) results in

$$\Phi = \mathcal{A}^T(X\mathbb{E}+M^TTS^T)+(X\mathbb{E}+M^TTS^T)^T\mathcal{A} < 0 \tag{12}$$

Using Assumption 2.1, the invertible matrices $\mathbb{R}$ and $\mathbb{S}$ can be obtained such that

$$\begin{aligned} \hat{E} &= \mathbb{R}\mathbb{E}\mathbb{S} = \begin{bmatrix} I_{n_r} & 0 \\ 0 & 0 \end{bmatrix} \\ \hat{A} &= \mathbb{R}\mathcal{A}\mathbb{S} = \begin{bmatrix} \mathcal{A}_1 & \mathcal{A}_2 \\ \mathcal{A}_3 & \mathcal{A}_4 \end{bmatrix}, \\ \mathbb{R}\mathcal{J}\mathbb{S} &= \begin{bmatrix} \mathcal{J}_1 & \mathcal{J}_2 \\ 0 & 0 \end{bmatrix}, \end{aligned}$$

Accordingly, we get

$$\begin{aligned} \mathbb{R}^{-\mathrm{T}}X\mathbb{R}^{-1} &= \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix}, \\ M\mathbb{M}^{-1} &= H\begin{bmatrix} 0 & I \end{bmatrix} \\ \mathbb{N}^{\mathrm{T}}S &= \begin{bmatrix} 0 \\ I \end{bmatrix}G \end{aligned}$$

where H and G are nonsingular matrices.

Then, Pre- and post-multiplying Φ with $\mathbb{S}^{\mathrm{T}}$ and $\mathbb{S}$, one has

$$\begin{bmatrix} \oplus & \oplus \\ \oplus & \mathcal{A}_4^TH^TTG^T + GT^TH\mathcal{A}_4 \end{bmatrix} < 0 \tag{13}$$

where $\oplus$ is independent from the results discussed below, the real expressions are omitted. So, we get

$$\mathcal{A}_4^TH^TTG^T + GT^TH\mathcal{A}_4 < 0 \tag{14}$$

which means $\mathcal{A}_4$ is invertible.

Then, we have

$$\begin{aligned} &\mathtt{Deg}(\mathtt{Det}(s\mathbb{E} - \mathcal{A})) \\ &\quad = \mathtt{Deg}(\mathtt{Det}(\mathbb{R}^{-1})\mathtt{Det}(s\hat{E} - \hat{A})\mathtt{Det}(\mathbb{S}^{-1})) \\ &\quad = \mathtt{Deg}(\mathtt{Det}(-\mathcal{A}_4)\mathtt{Det}(sI_{n_r} - (\mathcal{A}_{11} - \mathcal{A}_2\mathcal{A}_4^{-1}\mathcal{A}_3)) \\ &\qquad \times \mathtt{Det}(\mathbb{R}^{-1})\mathtt{Det}(\mathbb{S}^{-1})) \\ &\quad = n_r \\ &\quad = \mathtt{Rank}(\mathbb{E}) \end{aligned}$$

According to Definition 2.1 and Remark 2.1, unforced SSSs (1) is impulse-free and has an unique solution.

Secondly, we prove that this system is MS asymptotically stable. Since unforced system (1) is impulse free, then invertible matrices $\mathbb{W}$ and $\mathbb{G}$ can be obtained for satisfying

$$\begin{aligned} \mathbb{W}\mathbb{E}\mathbb{G} &= \begin{bmatrix} I_{n_r} & 0 \\ 0 & 0 \end{bmatrix} \\ \mathbb{W}\mathcal{A}\mathbb{G} &= \begin{bmatrix} \mathcal{A}_1 & 0 \\ 0 & I \end{bmatrix}, \\ \mathbb{W}\mathcal{J}\mathbb{G} &= \begin{bmatrix} \mathcal{J}_1 & \mathcal{J}_2 \\ 0 & 0 \end{bmatrix}, \\ \mathbb{W}^{-\mathrm{T}}X\mathbb{W}^{-1} &= \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix}, \\ M\mathbb{W}^{-1} &= N\begin{bmatrix} 0 & I \end{bmatrix} \end{aligned}$$

$$\mathbb{G}^{\mathrm{T}}S = \begin{bmatrix} 0 \\ I \end{bmatrix} Q$$

where N and Q are the nonsingular matrices.

By defining

$$\mathbb{G}^{-1}x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

this system is restricted equivalent with

$$\begin{aligned} dx_1(t) &= \mathcal{A}_1 x_1(t)dt + \mathcal{J}_1 x_1(t)d\varpi(t) \\ x_2(t) &= 0 \end{aligned} \tag{15}$$

Then, pre- and post-multiplying (10) with $\mathbb{G}^T$ and $\mathbb{G}$, implies

$$\begin{bmatrix} \mathcal{A}_1^{\mathrm{T}}X_1 + X_1\mathcal{A}_1 + \mathcal{J}_1^{\mathrm{T}}X_1\mathcal{J}_1 & \oplus \\ \oplus & \oplus \end{bmatrix} < 0 \tag{16}$$

Thus, we have

$$\mathcal{A}_1^{\mathrm{T}}X_1 + X_1\mathcal{A}_1 + \mathcal{J}_1^{\mathrm{T}}X_1\mathcal{J}_1 < 0 \tag{17}$$

Furthermore, by $\mathbb{E}_L X \mathbb{E}_L > 0$ and

$$\mathbb{W}\mathbb{E}_L = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

we can deduce $X_{11} > 0$. Then, by [15], we have

$$\lim_{t\to\infty} \mathfrak{E}\|x_1(t)\|^2 = 0$$

Further, from $x_2(t) = 0$, one has

$$\lim_{t\to\infty} \mathfrak{E}\|x(t)\|^2 = 0$$

Thus, on the basis of Definition 2.1, this systems is MS asymptotically stable.

As a result, this system is MS admissible.

Remark 3.1: (1) If $\mathcal{J} = 0$, Theorem 3.1 is translated to the criterion of deterministic singular system, which is expressed as

$$\mathbb{E}_L^T X \mathbb{E}_L > 0 \tag{18}$$

$$\mathcal{A}^T(X\mathbb{E} + M^T TS^T) + (X\mathbb{E} + M^T TS^T)^T\mathcal{A} < 0 \tag{19}$$

(2) If $\mathbb{E} = I$, we get $M = 0$, $S = 0$, Theorem 3.1 reduces to the stochastic stability criterion for normal Itô-type systems, which can be expressed as

$$X > 0 \tag{20}$$

$$\mathcal{A}^T X + X^T\mathcal{A} + \mathcal{J}^T X \mathcal{J} < 0 \tag{21}$$

B. CONTROLLER DESIGN

1) STATE FEEDBACK CONTROLLER

When the system states are completely accessible, the following SFC is considered.

$$u(t) = K_s x(t) \tag{22}$$

Then, the closed-loop system can be described by

$$\begin{aligned} \mathbb{E}dx(t) &= (\mathcal{A} + \mathcal{B}K_s)x(t)dt + \mathcal{J}x(t)d\varpi(t) \\ &= \bar{\mathcal{A}}x(t)dt + \mathcal{J}x(t)d\varpi(t) \end{aligned} \tag{23}$$

Theorem 3.2: The system (23) is MS admissible if there exists symmetric matrix $\mathbb{X} \in \mathfrak{R}^{n\times n}$, nonsingular matrix $\mathbb{T} \in \mathfrak{R}^{(n-n_r)\times(n-n_r)}$ and matrices $\mathcal{L} \in \mathfrak{R}^{m\times n}$ and $\mathcal{H} \in \mathfrak{R}^{m\times(n-n_r)}$, such that the following LMIs conditions hold

$$\mathbb{E}_R^T \mathbb{X} \mathbb{E}_R > 0 \tag{24}$$

$$\begin{bmatrix} \Sigma & \bar{X}^T\mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}_R \\ * & -\mathbb{E}_R^T\mathbb{X}\mathbb{E}_R \end{bmatrix} < 0 \tag{25}$$

where

$$\begin{aligned} \Sigma &= \bar{X}^T A^T + A\bar{X} + \mathcal{B}(\mathcal{L}\mathbb{E}^T + \mathcal{H}M) + (\mathcal{L}\mathbb{E}^T + \mathcal{H}M)^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} \\ \bar{X} &= \mathbb{X}\mathbb{E}^T + S\mathbb{T}M \end{aligned}$$

Then, the controller parameter K_s is given by

$$K_s = (\mathcal{L}\mathbb{E}^T + \mathcal{H}M)(\mathbb{X}\mathbb{E}^T + S\mathbb{T}M)^{-1} \tag{26}$$

Proof: Using Schur complement lemma [2], LMI (25) is equivalent to

$$\begin{aligned} &\bar{X}^T\mathcal{A}^T + \mathcal{A}\bar{X} + \mathcal{B}(\mathcal{L}\mathbb{E}^T + \mathcal{H}M) + (\mathcal{L}\mathbb{E}^T + \mathcal{H}M)^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} \\ &\quad + \bar{X}^T\mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}_R(\mathbb{E}_R^T\mathbb{X}\mathbb{E}_R)^{-1}\mathbb{E}_R^T\mathbb{E}^+\mathcal{J}\bar{X} < 0 \end{aligned} \tag{27}$$

Then, by Lemma 2.1, we know that $\bar{X} = \mathbb{X}\mathbb{E}^T + S\mathbb{T}M$ is nonsingular and its inverse is

$$\bar{X}^{-1} = (\mathbb{X}E^{\mathrm{T}} + S\mathbb{T}M)^{-1} = XE + M^T TS^T$$

Meanwhile, by (24), we have

$$E_L^T X E_L = (E_R^T\mathbb{X}E_R)^{-1} > 0$$

Let $\mathcal{L} = K_s\mathbb{X}$ and $\mathcal{H} = K_s S\mathbb{T}$, we get

$$\begin{aligned} &\bar{X}^T\bar{\mathcal{A}}^T + \bar{\mathcal{A}}\bar{X} + \bar{X}^T\mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T X\mathbb{E}\mathbb{E}^+\mathcal{J}\bar{X} \\ &= \bar{X}^T\mathcal{A}^T + \mathcal{A}\bar{X} + \mathcal{B}K_s(\mathbb{X}\mathbb{E}^T + S\mathbb{T}M) \\ &\quad + (\mathbb{X}\mathbb{E}^T + S\mathbb{T}M)^{\mathrm{T}}K_s^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} \\ &\quad + \bar{X}^T\mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}_R\mathbb{E}_L^T X\mathbb{E}_L\mathbb{E}_R^T\mathbb{E}^+\mathcal{J}\bar{X} \\ &= \bar{X}^T\mathcal{A}^T + \mathcal{A}\bar{X} + \mathcal{B}(\mathcal{L}\mathbb{E}^T + \mathcal{H}M) + (\mathcal{L}\mathbb{E}^T + \mathcal{H}M)^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} \\ &\quad + \bar{X}^T\mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}_R\mathbb{E}_L^T X\mathbb{E}_L\mathbb{E}_R^T\mathbb{E}^+\mathcal{J}\bar{X} \\ &= \bar{X}^T\mathcal{A}^T + \mathcal{A}\bar{X} + \mathcal{B}(\mathcal{L}\mathbb{E}^T + \mathcal{H}M) + (\mathcal{L}\mathbb{E}^T + \mathcal{H}M)^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} \\ &\quad + \bar{X}^T\mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}_R^T(\mathbb{E}_R\mathbb{X}\mathbb{E}_R)^{-1}\mathbb{E}_R^T\mathbb{E}^+\mathcal{J}\bar{X} < 0 \end{aligned} \tag{28}$$

Further, pre- and post-multiplying (28) with $\bar{X}^{-\mathrm{T}}$ and $\bar{X}^{-1}$, imply

$$\begin{aligned} &\bar{\mathcal{A}}^T(XE + M^T TS^T) + (XE + M^T TS^T)^T\bar{\mathcal{A}} \\ &\quad + \mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T X\mathbb{E}\mathbb{E}^+\mathcal{J} < 0 \end{aligned} \tag{29}$$

Based on Theorem 3.1, the system (23) is MS admissible.

2) DESCRIPTOR OBSERVER-BASED CONTROLLER

If the state variables are not measured, SFC cannot be achieved to realize system stabilization. then, the following OBC is given.

$$\begin{aligned} \mathbb{E}d\hat{x}(t) &= \mathcal{A}\hat{x}(t)dt + \mathcal{B}u(t)dt + \mathcal{L}[\hat{y}(t) - y(t)]dt \\ \hat{y}(t) &= C\hat{x}(t) \\ u(t) &= K_o\hat{x}(t) \end{aligned} \tag{30}$$

where $\hat{x}(t)$ the estimation of $x(t)$, $\mathcal{L}$, K_o are observer and controller parameters, respectively.

Define

$$\begin{aligned}\epsilon(t) &= x(t) - \hat{x}(t)\\ \eta(t) &= [x^{\mathrm{T}}(t), \epsilon^{\mathrm{T}}(t)]^{\mathrm{T}}\end{aligned}$$

Form (1) and (30), we can obtain the following closed-loop system

$$\tilde{\mathbb{E}}d\eta(t) = \tilde{\mathcal{A}}\eta(t)dt + \tilde{\mathcal{J}}\eta(t)d\varpi(t) \tag{31}$$

where

$$\begin{aligned}\tilde{\mathbb{E}} &= \begin{bmatrix}\mathbb{E} & 0\\ 0 & \mathbb{E}\end{bmatrix},\\ \tilde{\mathcal{A}} &= \begin{bmatrix}\mathcal{A}+\mathcal{B}K_o & -\mathcal{B}K_o\\ 0 & \mathcal{A}+\mathcal{L}\mathcal{C}\end{bmatrix},\\ \tilde{\mathcal{J}} &= \begin{bmatrix}\mathcal{J} & 0\\ \mathcal{J} & 0\end{bmatrix}\end{aligned}$$

Theorem 3.3: System (31) is MS admissible if there are a scalar $\varrho > 0$, symmetric matrix $X \in \Re^{n\times n}$, $Y \in \Re^{n\times n}$, nonsingular matrix $T_s \in \Re^{(n-n_r)\times(n-n_r)}$, $T_o \in \Re^{(n-n_r)\times(n-n_r)}$ and matrices $X_K \in \Re^{m\times m}$, $Y_K \in \Re^{m\times n}$, $Y_L \in \Re^{n\times q}$, such that the following LMIs conditions hold.

$$\mathbb{E}_L^T X \mathbb{E}_L > 0 \tag{32}$$

$$\mathbb{E}_L^T Y \mathbb{E}_L > 0 \tag{33}$$

$$\begin{bmatrix}\Phi_{11} & -BY_K & -\mathcal{B}X_K + X_s^T\mathcal{B} + \rho Y_K^{\mathrm{T}}\\ * & \Phi_{22} & -\rho Y_K^{\mathrm{T}}\\ * & * & -\rho X_K - \rho X_K^T\end{bmatrix} < 0 \tag{34}$$

where

$$\begin{aligned}\Phi_{11} &= \mathcal{A}^{\mathrm{T}}X_s + X_s^{\mathrm{T}}\mathcal{A} + \mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T X_s\mathbb{E}\mathbb{E}^+\mathcal{J}\\ &\quad + \mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T X_o\mathbb{E}\mathbb{E}^+\mathcal{J} + \mathcal{B}Y_K + Y_K^T\mathcal{B}^T\\ \Phi_{22} &= \mathcal{A}^{\mathrm{T}}X_o + X_o^{\mathrm{T}}\mathcal{A} + Y_L\mathcal{C} + \mathcal{C}^{\mathrm{T}}Y_L^{\mathrm{T}}\\ X_s &= X\mathbb{E} + M^T T_s S^T\\ X_o &= Y\mathbb{E} + M^T T_o S^T\end{aligned}$$

Then, the gains can be given by

$$\begin{aligned}K_o &= X_K^{-1}Y_K\\ \mathcal{L} &= X_o^{-\mathrm{T}}Y_L\end{aligned}$$

Proof: Let

$$\begin{aligned}\tilde{X} &= \begin{bmatrix}X & 0\\ 0 & Y\end{bmatrix}, \quad \tilde{M} = \begin{bmatrix}M & 0\\ 0 & M\end{bmatrix},\\ \tilde{T} &= \begin{bmatrix}T_s & 0\\ 0 & T_o\end{bmatrix}, \quad \tilde{S} = \begin{bmatrix}S & 0\\ 0 & S\end{bmatrix}\end{aligned}$$

then, one has

$$\begin{aligned}\tilde{M}\tilde{\mathbb{E}} &= \begin{bmatrix}M & 0\\ 0 & M\end{bmatrix}\begin{bmatrix}\mathbb{E} & 0\\ 0 & \mathbb{E}\end{bmatrix} = \begin{bmatrix}M\mathbb{E} & 0\\ 0 & M\mathbb{E}\end{bmatrix} = 0\\ \tilde{\mathbb{E}}\tilde{S} &= \begin{bmatrix}\mathbb{E} & 0\\ 0 & \mathbb{E}\end{bmatrix}\begin{bmatrix}S & 0\\ 0 & S\end{bmatrix} = \begin{bmatrix}\mathbb{E}S & 0\\ 0 & \mathbb{E}S\end{bmatrix} = 0\end{aligned}$$

It is noted that

$$\begin{aligned}\tilde{\mathbb{E}}^+ &= \begin{bmatrix}\mathbb{E}^+ & 0\\ 0 & \mathbb{E}^+\end{bmatrix},\\ \tilde{\mathbb{E}}_L &= \begin{bmatrix}\mathbb{E}_L & 0\\ 0 & \mathbb{E}_L\end{bmatrix}, \quad \tilde{\mathbb{E}}_R = \begin{bmatrix}\mathbb{E}_R & 0\\ 0 & \mathbb{E}_R\end{bmatrix}\end{aligned}$$

and by (32)-(33), we get

$$\tilde{\mathbb{E}}_L^T\tilde{X}\tilde{\mathbb{E}}_L > 0 \tag{35}$$

Next, it can be derived that

$$\begin{aligned}\Pi &= \tilde{\mathcal{A}}^T(\tilde{X}\tilde{\mathbb{E}} + \tilde{M}^T\tilde{T}\tilde{S}^T) + (\tilde{X}\tilde{\mathbb{E}} + \tilde{M}^T\tilde{T}\tilde{S}^T)^T\tilde{\mathcal{A}}\\ &\quad + \tilde{\mathcal{J}}^T(\tilde{\mathbb{E}}^+)^T\tilde{\mathbb{E}}^T\tilde{X}\tilde{\mathbb{E}}\tilde{\mathbb{E}}^+\tilde{\mathcal{J}}\\ &= \begin{bmatrix}\Delta_1 + \mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T X_o\mathbb{E}\mathbb{E}^+\mathcal{J} & -X_s^T\mathcal{B}K_o\\ * & \Delta_2\end{bmatrix}\\ &= \begin{bmatrix}\Theta_1 + \mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T X_o\mathbb{E}\mathbb{E}^+\mathcal{J} & 0\\ * & \Theta_2\end{bmatrix}\\ &\quad + \mathcal{I}_1X_s^T\mathcal{B}K_o\mathcal{I}_2 + (\mathcal{I}_1X_s^T\mathcal{B}K_o\mathcal{I}_2)^T\end{aligned} \tag{36}$$

where

$$\begin{aligned}\Delta_1 &= (\mathcal{A}+\mathcal{B}K_o)^T X_s + X_s^T(\mathcal{A}+\mathcal{B}K_o)\\ &\quad + \mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T X_s\mathbb{E}\mathbb{E}^+\mathcal{J}\\ \Delta_2 &= (\mathcal{A}+\mathcal{L}\mathcal{C})^{\mathrm{T}}X_o + X_o^T(\mathcal{A}+\mathcal{L}\mathcal{C})\\ \Theta_1 &= \mathcal{A}^T X_s + X_s^T\mathcal{A} + \mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T X_s\mathbb{E}\mathbb{E}^+\mathcal{J}\\ \Theta_2 &= \mathcal{A}^{\mathrm{T}}X_o + X_o^{\mathrm{T}}\mathcal{A} + Y_L\mathcal{C} + \mathcal{C}^{\mathrm{T}}Y_L^{\mathrm{T}}\\ \mathcal{I}_1 &= \begin{bmatrix}I\\ 0\end{bmatrix}, \quad \mathcal{I}_2 = \begin{bmatrix}I & -I\end{bmatrix}\end{aligned}$$

By $K_o = X_K^{-1}Y_K$, it can be established that

$$\mathcal{I}_1X_s^T\mathcal{B}K_o\mathcal{I}_2 = \mathcal{I}_1\mathcal{B}Y_K\mathcal{I}_2 + \mathcal{I}_1(-\mathcal{B}X_K + X_s^T\mathcal{B})X_K^{-1}Y_K\mathcal{I}_2$$

Further, (36) can be rearranged

$$\begin{aligned}\Pi &= \Theta + UX_K^{-1}V + V^TX_K^{-T}U^T\\ \Theta &= \begin{bmatrix}\Theta_1 + \mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T X_o\mathbb{E}\mathbb{E}^+\mathcal{J} & 0\\ * & \Theta_2\end{bmatrix}\\ U &= \mathcal{I}_1(-\mathcal{B}X_K + X_s^T\mathcal{B})\\ V &= Y_K\mathcal{I}_2\end{aligned} \tag{37}$$

By Lemma 2.3, we can obtain that (34) guarantees (37). So, $\Pi < 0$. Further, Using Theorem 3.1, this system is MS admissible.

Remark 3.2: Due to the existence of the term $\mathcal{J}^T(\mathbb{E}^+)^T\mathbb{E}^T X_o\mathbb{E}\mathbb{E}^+\mathcal{J}$ in (36), the calculation method of controller and observer parameters for the closed-loop system cannot completely separate. There are fundamental difference between the OBC controller design of deterministic and stochastic singular systems.

Remark 3.3: In [21], [22], the solutions of controller and observer parameters are given by the sequential design method. In this method, the controller gain is solved firstly. Then, by bringing the obtained controller parameter value into the other LMIs, the observer parameters can be solved, which causes this method more complex and conservative.

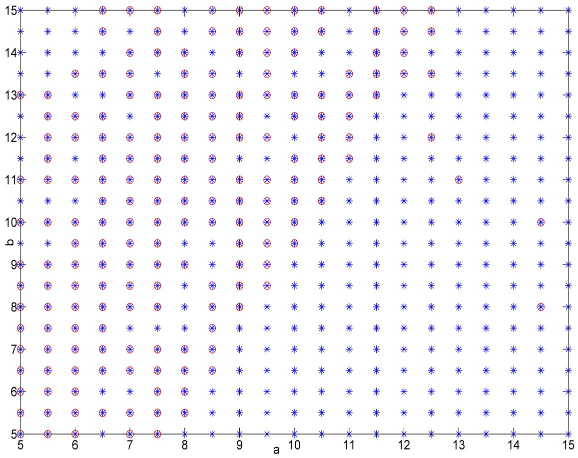

FIGURE 1. Mean-square admissibility analysis by theorem 3.1 (∗) and [21] (∘).

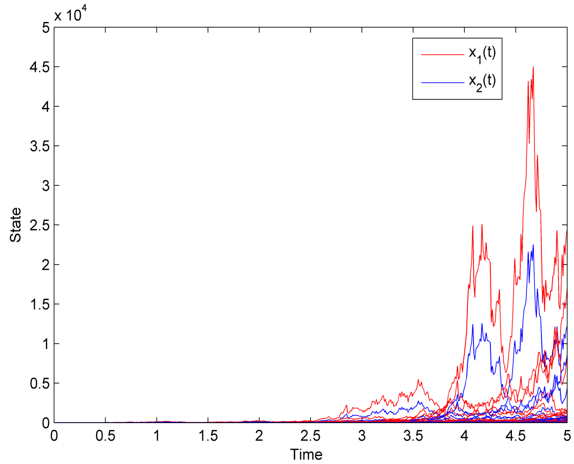

FIGURE 2. State $x_1(t)$ and $x_2(t)$.

To resolve this issue, a novel design method is given in this article, in which the gains can be obtained simultaneously in terms of strict LMIs. Thus, the conservativeness can be significantly decreased.

IV. ILLUSTRATIVE EXAMPLES

Example 1: Consider the following SSSs:

$$\mathbb{E}dx(t) = \mathcal{A}x(t)dt + \mathcal{B}u(t)dt + \mathcal{J}x(t)d\varpi(t) \tag{38}$$
$$y(t) = \mathcal{C}x(t) \tag{39}$$

where

$$\mathbb{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} 10 & 3 & 1 \\ a & 6 & 1 \\ -b & -1 & 0 \end{bmatrix}$$
$$\mathcal{J} = \begin{bmatrix} 0.5 & 0 & 1 \\ -2 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \\ 0 & 2 \end{bmatrix}$$
$$\mathcal{C} = \begin{bmatrix} 1 & 0 & 1 \\ -3 & 1 & 1 \end{bmatrix}$$

The MS admissibility for this system is checked by using Theorem 3.1 and Theorem 4 in [21] for several values of pairs (a,b) in Fig. 1, where $a \in [5, 15]$ and $b \in [5, 15]$. One can find that Theorem 3.1 is less conservativeness than Theorem 4 in the [21].

Example 2 ([15]): Consider the oil catalytic cracking process with influenced by the random environment and the following simplified model is given as

$$\begin{aligned} dx_1(t) &= [W_1x_1(t) + W_2x_2(t) + B_1u(t) + D_1f]dt \\ &\quad + [F_1x_1(t) + F_2x_2(t)]d\varpi \\ 0 &= [W_3x_1(t) + W_3x_2(t) + B_2u(t) + D_2f]dt \end{aligned} \tag{40}$$

where $x_1(t)$, $x_2(t)$, $u(t)$ and f indicate regulated vector, the business benefits vector, regulation value, and nonlinear disturbance, respectively.

Choose the following parameters:

$$W_1 = 1, \quad W_2 = 1.5, \ W_3 = 0.5, \ W_4 = -1$$
$$B_1 = B_2 = 1, \quad D_1 = D_2 = 0, \ F_1 = F_2 = 1$$

the following systems can be obtained.

$$\mathbb{E}dx(t) = \mathcal{A}x(t)dt + \mathcal{B}u(t)dt + \mathcal{J}x(t)d\varpi(t) \tag{41}$$

where

$$\mathbb{E} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} 1 & 1.5 \\ 0.5 & -1 \end{bmatrix}$$
$$\mathcal{J} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

When the initial condition $x_0 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$, Fig. 2 shows state responses (60 tests) of this system. Then, one can find that this system is unstable.

Next, By solving LMIs (24)-(25), the following state feedback controller gains are obtained.

$$K_s = \begin{bmatrix} -2.8505 & -3.5000 \end{bmatrix}$$

Then, Fig. 3 shows the states responses (60 tests) and means of closed-loop system. So, this system is MS admissibility under state feedback controller.

Example 3: The circuit system is shown in Fig.4. Variables R, L, and $C_{1,2}$ stand for the resistance, inductor, and capacitances, respectively; $V_{C_1}(t)$, $V_{C_2}(t)$ stand for the voltages of C_1, C_2; $I_{C_1}(t)$ and $I_{C_1}(t)$ stand for the currents flowing over them; and $V_e(t)$ is the voltage source. Based on the principle of circuits, this system can be given as follows:

$$\dot{V}_{C_1}(t) = \frac{1}{C_1} I_{C_1}(t) \tag{42}$$
$$\dot{V}_{C_2}(t) = \frac{1}{C_2} I_{C_2}(t) \tag{43}$$
$$\dot{I}_{C_2}(t) = \frac{1}{L}(V_{C_1}(t) - V_{C_2}(t)) \tag{44}$$
$$0 = V_{C_1}(t) + I_{C_1}(t)R + I_{C_2}(t)R - V_e(t) \tag{45}$$

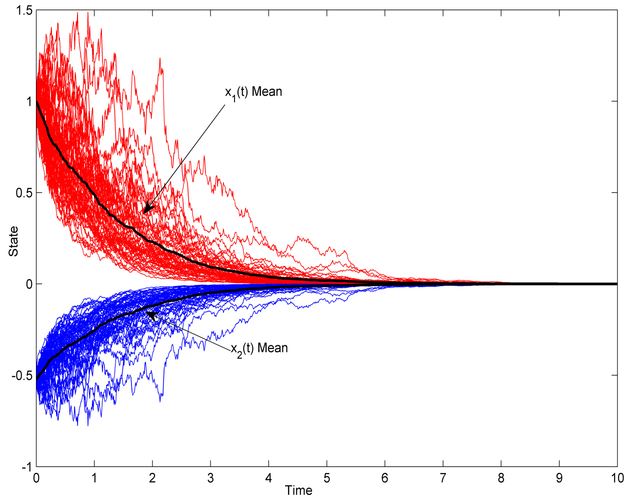

FIGURE 3. State $x_1(t)$ and $x_2(t)$: State feedback controller.

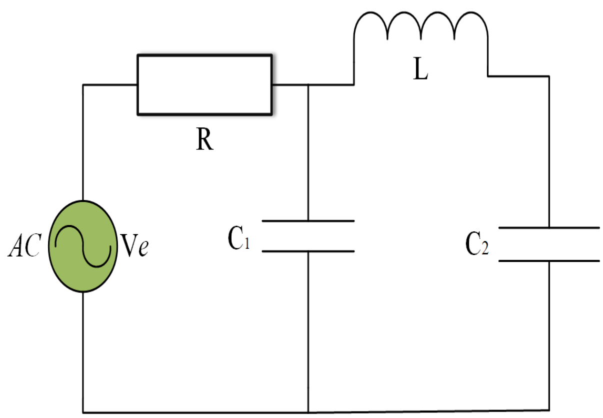

FIGURE 4. Circuit system.

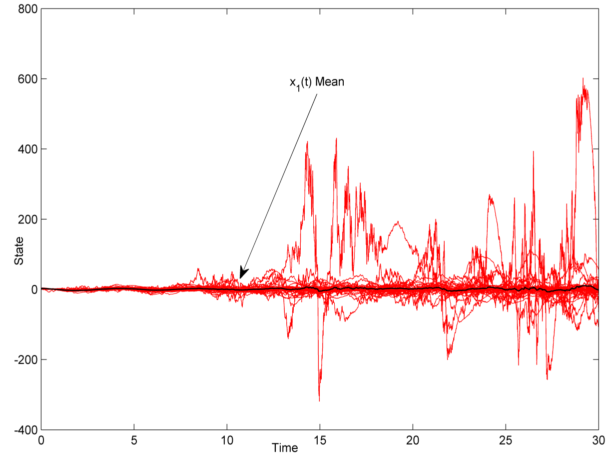

FIGURE 5. State $x_1(t)$.

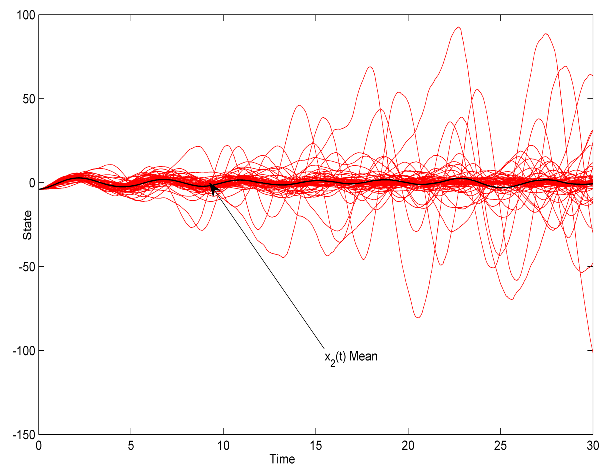

FIGURE 6. State $x_2(t)$.

Further, choose the state $x(t)$, control input $u(t)$, and output $y(t)$ as

$$x(t) = \begin{bmatrix} V_{C_1}(t) \\ V_{C_2}(t) \\ I_{C_2}(t) \\ I_{C_1}(t) \end{bmatrix}$$
$$u(t) = V_e, y(t) = V_{C_2}$$

Equations (42)-(45) can be written as:

$$\begin{aligned} \mathbb{E}\dot{x}(t) &= Ax(t) + \mathcal{B}u(t) \\ y(t) &= \mathcal{C}x(t) \end{aligned} \tag{46}$$

where

$$\mathbb{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & R & R \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$$

Matrix A is influenced by the random environment such as $A = \mathcal{A} + J$"noise". This "noise" term can be described effectively by white noise $\varpi(t)$. Then, system (46) becomes

$$\begin{aligned} \mathbb{E}dx(t) &= \mathcal{A}x(t)dt + \mathcal{B}u(t)dt + \mathcal{J}x(t)d\varpi(t) \\ y(t) &= \mathcal{C}x(t) \end{aligned} \tag{47}$$

Next, Let $C_1 = 1, C_2 = 2, L = 1, R = 2,$

$$J = \begin{bmatrix} 0 & 1.0000 & 0 & -0.5000 \\ 0 & 0.0100 & 0 & 0.0100 \\ 0 & 0.4000 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

the initial states are $x_1(0) = 3$, $x_2(0) = -4$ and $x_3(0) = 1$, stochastic state responses (60 tests) and mean of this system are given in each figure (see Figs. 5-8). So, this system is unstable.

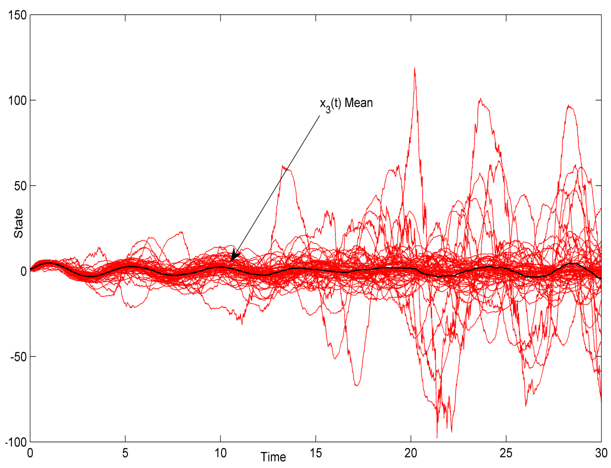

FIGURE 7. State $x_3(t)$.

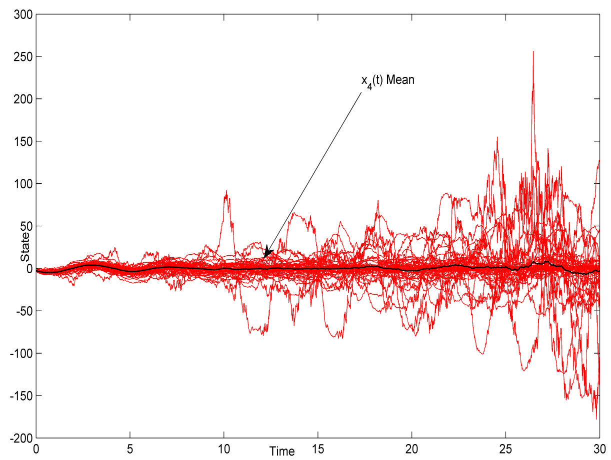

FIGURE 8. State $x_4(t)$.

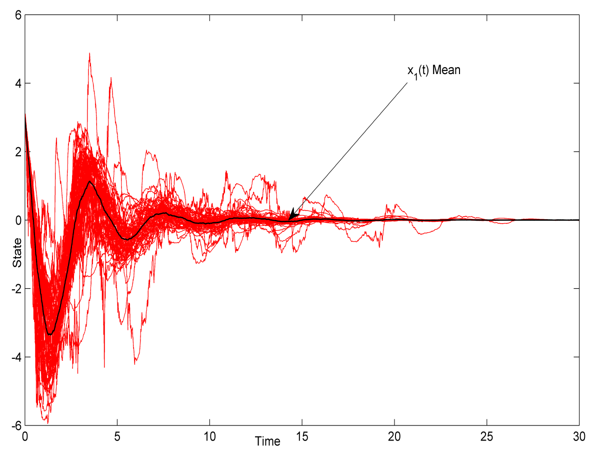

FIGURE 9. State $x_1(t)$: observer-based controller.

Next, using the LMIs Theorems 3.3, the controller and observer parameters are given by

$$K_o = \begin{bmatrix} -0.5596 & -0.1453 & -0.1901 & 0.976 \end{bmatrix}$$

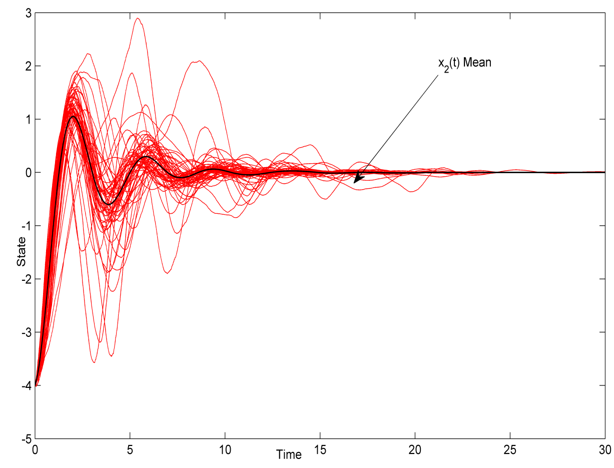

FIGURE 10. State $x_2(t)$: observer-based controller.

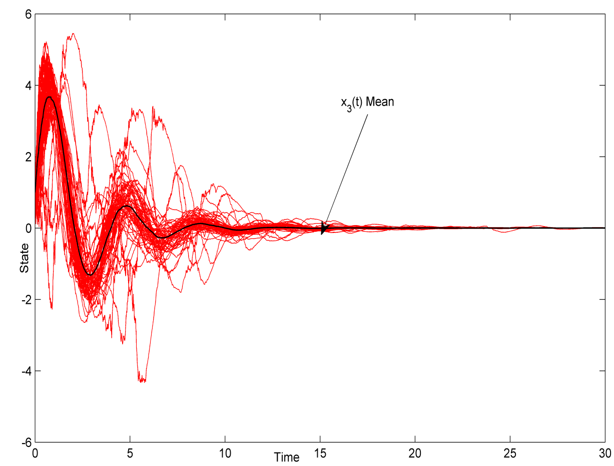

FIGURE 11. State $x_3(t)$: observer-based controller.

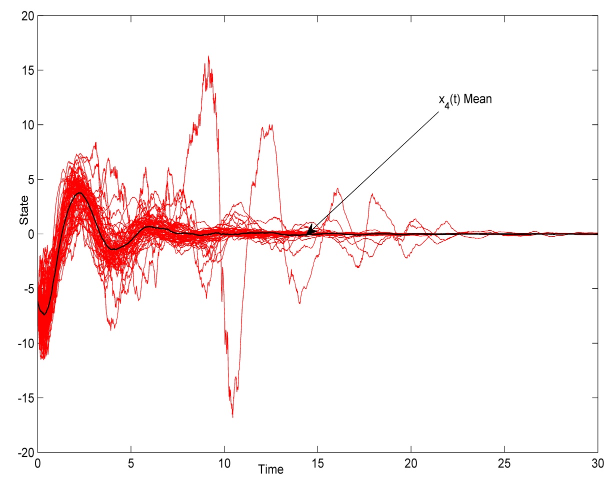

FIGURE 12. State $x_4(t)$: observer-based controller.

$$\mathcal{L} = 10^3 \times \begin{bmatrix} -1.3769 \\ -0.0985 \\ -0.6953 \\ -0.0378 \end{bmatrix}$$

Then, the simulations of closed-loop system under OBC is obtained in Figs. 9-12. It is obvious that the system is MS admissibility under observer-based controller.

V. CONCLUSION

The issues of mean-square admissibility and controller design for SSSs with Brownian parameter perturbations is studied. Both SFC and OBC design method are given to guarantee closed-loop systems are MS admissible via strict LMIs. Different from the previous results, the novel OBC design scheme is introduced to compute the controller and observer gains simultaneously and the conservation is reduced in the method proposed. Three examples are given to demonstrate the feasibility and efficiency of the proposed results.

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