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Event-Triggered Vehicle Platoon Control Under Random Communication Noises

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ABSTRACT As an effective way of improving traffic efficiency, vehicle platoon control has attracted extensive interest recently. Communication between vehicles tends to be affected by communication noises. Aimed at improving communication efficiency, an event-triggered vehicle platoon control under random communication noises is studied in this paper. First, for vehicle platoons with linear third-order dynamics, a time-varying consensus gain $c(t)$ is introduced to reduce the effects of the communication noises. Second, with the introduction of the algebraic graph theory and matrix analysis theory, conditions for internal stability and l_p-string stability under random additive communication noises are derived. Third, by utilizing the Lyapunov approach and *Itô* stochastic differential equations, the consensus of vehicle platoon under random additive communication noises is proved. Last, to reduce the frequent communication between vehicles, an event-triggered mechanism is introduced, and the design for the triggering parameter is derived. The effectiveness of the proposed method is verified with some numerical simulations.

INDEX TERMS Vehicle platoon, communication noises, event-triggered, string stability, consensus.

I. INTRODUCTION

Vehicle platoons are important parts of intelligent transportation. From the PATH Project in California to the SARTRE Project in Europe, the Energy ITS Project in Japan, and *et al.*, much attention has been paid to vehicle platoons. The improvement of vehicle platoon technology can effectively increase the road throughput, improve road safety, reduce fuel consumption, and so on [1]–[3]. With the development of intelligent networked vehicles, the Internet of Vehicles, and unmanned driving technology, the adaptive cruise control (ACC) system has been installed in a growing number of vehicles to improve vehicle platoon performance. Vehicles can measure the state information of ahead information through sensors [4]–[6], and cooperative adaptive cruise control (CACC) is extended to the ACC technology. Vehicle-to-Vehicle (V2V) information exchanges and Vehicle-to-Infrastructure (V2I) wireless communication technology are the prospects of CACC [7]. Aimed at different communication structures and different controllers, many researchers have paid extensive attention to vehicle platoons [8]–[11]. Considering engine limits and uncertain dynamics, Baldi *et al.* established platoons of bidirectional cooperative vehicles [8]; while parameter uncertainties were taken into account in [9]. Liu *et al.* in [10] proposed adaptive strategies of cyclic communication, and the non-uniform communication topology was studied in [11].

Most of the researches about vehicle platoons were assumed to be interference-free. Note that noises are unavoidable in real vehicle platoons, and the performance of the vehicle platoon could be affected by noises from communication channels and the external environment. Thus, researches on vehicle platoon control with communication noises are necessary and meaningful. In [12], considering the influence of measurement noises, the average consensus of multiple agents was considered for systems with fixed topology and directed topology, respectively. A time-varying consensus gain was introduced to the consensus protocol, and finally, two critical conditions the consensus gain should satisfy were derived. This method was widely used in [13]–[15]. [16] investigated the mean square consensus for multiple agents that were affected by noises over directed networks, and

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proposed two protocols for driving the agents to converge to the average value of their initial states. In [17], it was proved that, for a multi-agent system with a directed communication graph, the adaptive gain could eventually tend to an ideal estimate when considering the influence of the additive random white noise.

It should be mentioned that those methods mentioned above all adopted time-triggered communication strategies. Time-triggered communication affects both the stability and consensus of vehicle platoons due to packet loss in data transmission [18]–[20], random data missing [21] unexpected changes in communication load, and changes in sampling accuracy [22], [23]. Much attention has been paid to the event-triggered control strategies (ETCS) to reduce the amount of data transmission. With the introduction of the event-triggered method, the controller's continuous workload can be effectively reduced, which is effective in saving passband resources and avoiding passengers' discomfort caused by frequent acceleration or deceleration. Moreover, ETCS could minimize the possibility of collision.

To the best of our knowledge, the first work on event-triggered strategy in a multi-agent system is [24], with several works followed in different settings [16], [25]–[32]. Based on the results in [24], Dimarogonas further studied the first-order distributed event-triggered strategy in [25], where the distributed strategy only needed to know the information of neighboring nodes, which greatly improved the execution efficiency of the controller. Cheng *et al.* in [26] considered distributed event-triggered consensus algorithm from leadless vehicle platoon and leading vehicle platoon, respectively. Considering the heterogeneity of vehicle dynamics, [27] and [28] designed novel distributed event-triggered strategies with high computational efficiency. For nonlinear systems, [29] derived the event-triggered conditions under time-varying transmission delay and [30] analyzed the fixed communication topology and derived conditions for the system to achieve the exponential convergence. In [31], for scenarios with and without external disturbances, an adaptive triggering protocol was constructed, and sufficient conditions for the multi-agent system to meet the consensus were derived. Based on a first-order dynamic model, Liu *et al.* derived the event-triggered conditions, achieved its mean square consensus, and extended the results to systems with switching topologies [16]. [32] generalized the consensus gain function and derived the sufficient condition for stability when the gain was negative. However, most of the researches discussed above were based on the first and second-order models.

In this paper, motivated by [12] and [16], we investigate the event-triggered vehicle platoon under random communication noises. We establish the third-order model to form the vehicle platoon dynamics, rather than the first-order model proposed in [12] and [16]. We also study the vehicle platoon control under random communication noises. Furthermore, we focus on improving the execution efficiency of consensus and utilizing the event-triggered mechanism to avoid frequent real-time communications. [12] didn't consider the event-triggered strategy. In [16], A. Hu *et al.* proposed an event-triggered strategy, but it still needed the global information to construct the triggering function. In this respect, how to design a fully distributed and more applicable event-triggered function is a big challenge. Moreover, with the introduction of a time-varying consensus gain, the information topology will not be a fixed one. The system matrix is time-varying due to random communication noises. Therefore, analyzing the stability and consensus of a vehicle platoon with a time-varying matrix is another urgent problem to be solved. The main contributions of this paper are two-fold.

First, we investigate the vehicle platoon with a third-order dynamic model under random communication noises. By introducing a time-varying consensus gain to the controller design, we derive the conditions for stability of the vehicle platoon. We prove that the vehicle platoon could achieve its mean square consensus.

Second, compared with [16], we obtain a novel event-triggered function, which is decentralized, and no information from any following vehicle is needed. To some extent, the event-triggered strategy we proposed is suitable for vehicle platoon with decreasing frequent communication. Also, compared with [29], who adopted a complex event-triggered strategy to analyze a nonlinear networked control system, in this paper, we proposed a more general event-triggered strategy, which decreased the complexity in designing the controller. Moreover, by using $It\hat{o}$ stochastic differential equations and Lyapunov theory, we derive the triggering parameter design method, which could eventually achieve the mean square consensus.

The remainder of this paper is organized as follows. The basic knowledge of graph theory and the relevant parameters are provided in Section II. The problem formulation is presented in Section III. In Section IV, we give the system design, analyze the system performance under random communication noises and event-triggered communication mechanism. Section V verifies the main results through numerical studies. Conclusion and future works are given in Section VI.

II. PRELIMINARY

Each vehicle in the vehicle platoon is considered as a node of the multi-agent system (MAS). The information flow communicated among nodes of the MAS is modeled by a graph $G = (N, \mathcal{E})$, where $N = \{1, 2, \ldots, n\}$ represents the node sets, and the i-th vehicle can be regarded as the i-th node. $\mathcal{E} \subseteq N \times N$ represents the edge sets in the graph, i.e., the communication links between vehicles. $\mathcal{A} = [a_{ij}]_{n\times n}$ is the adjacency matrix. If $(j, i) \in \mathcal{E}$, one has $a_{ij} = 1$; otherwise $a_{ij} = 0$. Let $deg_{in}(i) = \sum_{j=1}^{n} a_{ij}$ be the in-degree of node i, and $D = diag\{deg_{in}(i)\}$ be the degree matrix. A sequence $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$ of edges is called a directed path from node i_1 to node i_k. G is called a strongly connected digraph, if for any $i, j \in N$, there is a directed path from i to j. A directed tree is a digraph, where every node except the root has exactly one parent, and the root is a source. A spanning tree of G is a directed tree whose node set is N and whose edge set is a subset of $\mathcal{E}$. If G is a strongly connected digraph, then it must contain a spanning tree. Let $L = D - \mathcal{A}$ be the Laplacian matrix of a graph G and $P_L(i, i)$ be the spinning matrix, which indicates whether the following vehicle can receive information from the leader. If the information can be received, one has $P_L(i, i) = 1$; otherwise, $P_L(i, i) = 0$.

Most of the notations in this paper are fairly standard. For convenience, some notations are explained here. The real domains are denoted by $\mathbb{R}$. The set of $m \times n$ real matrices is denoted by $\mathbb{R}^{m\times n}$. Let 0_n and I_n be the n-order zero and identity matrix, respectively. Let A^T be the transpose of a matrix A, where $\lambda_{min}(A)$ and $\lambda_{max}(A)$ represent the minimum and maximum eigenvalues of matrix A, respectively. $||\cdot||$ denotes the Euclidean norm of vectors or the induced 2-norm of square matrices, and $\|\{x(t)\}\|_{\ell_p} = \sum_{t=0}^{\infty} |x(t)|^p < \infty$. $tr(A)$ denotes the trace of matrix A. For a matrix $A \in \mathbb{R}^{n\times n}$, if and only if all its eigenvalues have the negative real part, it is called Hurwitz. For a given random number X, $E(X)$ represents its mathematical expectation. For matrix $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$, let $A \otimes B$ be the Kronecker product of A and B, shown as

$$A \otimes B = \begin{pmatrix} a_{11}B & \dots & a_{m1}B \\ \vdots & \ddots & \vdots \\ a_{1n}B & \cdots & a_{nn}B \end{pmatrix} \in \mathbb{R}^{mp\times nq}.$$

A continuous function $\alpha : [0, c) \to [0, \infty)$ is said to be of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

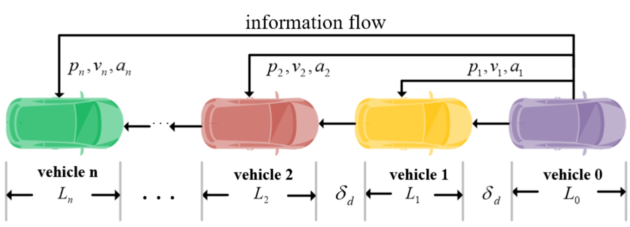


FIGURE 1. **Topology of the Predecessor-leader following (PLF) vehicle platoon.**

III. PROBLEM FORMULATION

A. INFORMATION FLOW TOPOLOGY

The topology of the predecessor-leader following (PLF) vehicle platoon system studied in this paper is shown in Fig. 1. Suppose there are $n + 1$ vehicles with one leading vehicle and n followers. The leading vehicle is denoted as number 0, while the rest are denoted as $1, 2, \cdots, n$, respectively. The exchange of information among n following vehicles is described by a directed graph $G = (N, \mathcal{E})$. The information flow from the leader vehicle 0 to the n followers is described by a diagonal matrix $P_L \in \mathbb{R}^{n\times n}$.

Remark 1: In this paper, graph G is assumed to contain a spanning tree, and all the following vehicles can receive information from the leading vehicle.

B. VEHICLE DYNAMICS

For each vehicle i ($i \in N$), the drive/brake torque is considered as the control input, and the following longitudinal dynamic characteristics are obtained

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \dot{v}_i(t) = \dfrac{1}{m_i}\Big(\eta_{T,i}\dfrac{T_i(t)}{R_i} - C_i v_i^2(t) - m_i g f\Big) \\ \tau_i \dot{T}_i(t) + T_i(t) = T_{i,des}(t), \end{cases} \tag{1}$$

where $p_i(t)$ and $v_i(t)$ are the position and velocity, respectively; m_i is the vehicle mass; $\eta_{T,i}$ is the mechanical efficiency of drive-line; $T_i(t)$ and $T_{i,des}(t)$ are the actual and desired driving/braking torque, respectively; R_i is the tire radius; C_i is the lumped aerodynamic drag coefficient; g is the acceleration due to gravity; f is the coefficient of rolling resistance; τ_i is the inertia delay of longitudinal dynamics.

Remark 2: In this paper, the longitudinal slip of the tire is ignored, the rotation dynamics is integrated as a first-order inertia transfer function, the vehicle body is considered to be rigid and symmetric, and the influence of pitch and yaw motion is ignored.

Motivated by [33]–[35], a linear feedback controller is designed as

$$T_{i,des}(t) = \frac{R_i}{\eta_{T,i}}\big(m_i g f + C_i v_i(t)(2\tau_i \dot{v}_i + v_i) + m_i u_i(t)\big), \tag{2}$$

where $u_i(t)$ is the control input. Then a linear model for vehicle longitudinal dynamics was obtained

$$\tau_i \dot{a}_i(t) + a_i(t) = u_i(t),$$

where $a_i(t) = \dot{v}_i(t)$ denotes the acceleration of vehicle i. With some manipulations, the dynamic model of vehicle i in (1) can be rewritten as

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \dot{v}_i(t) = a_i(t), \quad i = 1, 2 \dots, n, \\ \tau_i \dot{a}_i(t) + a_i(t) = u_i(t). \end{cases} \tag{3}$$

Then we can derive the matrix form of (3), shown as $\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t)$, where

$$x_i(t) = \begin{pmatrix} p_i(t) \\ v_i(t) \\ a_i(t) \end{pmatrix}, \quad A_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{pmatrix}, \quad B_i = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{pmatrix}.$$

In this paper, we assume that the inertial delays of all vehicle are homogeneous and denote τ_i as τ.

C. VEHICLE PLATOON WITH RANDOM COMMUNICATION NOISES

Let the expected distance between two adjacent vehicles be r. The distance error between following vehicle i ($i \in N$) and the leading vehicle is defined as

$$\delta_i(t) = p_i(t) - p_0(t) + \sum_{f=1}^{i} L_f + ir, \tag{4}$$

where $p_i(t)$ and $p_0(t)$ are the position of the i-th and the leading vehicle, respectively; L_i is the length of the i-th vehicle. Let $\omega_{ji}(t)$ and $\omega_{0i}(t)$ be the communication noises between vehicle j, i and 0, i, respectively. In this paper, we assume that $\omega_{ji}(t)$ and $\omega_{0i}(t)$ are independent Laplace variables.

The control input $u_i(t)$ in (3) is designed as

$$\begin{aligned} u_i(t) = c(t)\sum_{j=1}^{n} a_{ij}\Big[k_p\big(p_j(t) - p_i(t) - \sum_{f=j+1}^{i}(L_f + r)\big) \\ +k_v\big(v_j(t) - v_i(t)\big) + k_a\big(a_j(t) - a_i(t)\big) + \omega_{ji}(t)\Big] \\ -c(t)P_L(i,i)\Big[k_p(p_i(t) - p_0(t) + \sum_{f=1}^{i}(L_f + r)) \\ +k_v(v_i(t) - v_0(t)) + k_a(a_i(t) - a_0(t)) + \omega_{0i}(t)\Big], \end{aligned} \tag{5}$$

where k_p, k_v, k_a, and time varying $c(t)$ are the control gains to be designed.

Remark 3: In (5), $c(t): [0, \infty) \rightarrow (0, \infty)$ is piecewise continuous. Specially, when there is no communication noises, $c(t)$ can be designed as a constant, which is the same as the one described in [35]. From the physical point of view, $c(t)$ is actually a decreasing step size. (5) provides a correction term controlled by $c(t)$, which is similar to the recursion in classical stochastic approximation algorithms. Generally speaking, $c(t)$ is a parameter ranged in (0, 1) and is designed as a decreasing function of time t. If $c(t)$ is an increasing number greater than 1, the noises contained in the control input of (5) will be infinitely amplified with the increase of time t, and eventually the asymptotic consensus will not be achieved.

Remark 4: In this paper, we mainly studied the homogeneous vehicle platoons. In a sense, although the length of each vehicle i is different, which belongs to the category of heterogeneous vehicle platoons, we can still analyze the system as a homogeneous vehicle. An important further research issue is to analyze heterogeneous vehicle platoons such as different dynamics and different control gains, which beckons our future work.

Let $\widetilde{p}_i(t) = \delta_i(t) = p_i(t) - p_0(t) + \sum_{f=1}^{i}(L_f + r)$, $\widetilde{v}_i(t) = v_i(t) - v_0(t)$, and $\widetilde{a}_i(t) = a_i(t) - a_0(t)$. Controller in (5) can be rewritten as

$$\begin{aligned} u_i(t) = c(t)\sum_{j=1}^{n} a_{ij}\Big[k_p(\widetilde{p}_j(t) - \widetilde{p}_i(t)) + k_v(\widetilde{v}_j(t) - \widetilde{v}_i(t)) \\ +k_a(\widetilde{a}_j(t) - \widetilde{a}_i(t)) + \omega_{ji}(t)\Big] \\ -c(t)P_L(i,i)\Big(k_p\widetilde{p}_i(t) + k_v\widetilde{v}_i(t) + k_a\widetilde{a}_i(t) + \omega_{0i}(t)\Big). \end{aligned} \tag{6}$$

For vehicle i ($i \in N$), let $\widetilde{x}_i(t) = [\widetilde{p}_i^T(t), \widetilde{v}_i^T(t), \widetilde{a}_i^T(t)]^T$ be its tracking error to the leading vehicle, and the total communication noise be $\omega_i(t) = \sum_{j=1}^{n} a_{ij}\omega_{ji}(t) - P_L(i,i)\omega_{0i}(t)$.

Then one has

$$u(t) = -c(t)(L + P) \otimes K^T\widetilde{x}(t) + c(t)\omega(t), \tag{7}$$

where $\widetilde{x}(t) = [\widetilde{x}_1(t), \ldots, \widetilde{x}_n(t)]^T$, $K = [k_p, k_v, k_a]^T$, $u(t) = [u_1(t), \ldots, u_n(t)]^T$, and $\omega(t = [\omega_1(t), \ldots, \omega_n(t)]^T$.

By combing (3) and (7), one can derive the matrix form for the vehicle platoon, shown as

$$\begin{aligned} \dot{\widetilde{x}}(t) &= \Big(A - c(t)B(L + P) \otimes K^T\Big)\widetilde{x}(t) + c(t)B\omega(t) \\ &= A_c\widetilde{x}(t) + c(t)B\omega(t), \end{aligned} \tag{8}$$

where $A_c = A - c(t)B(L + P) \otimes K^T$, $A = \text{diag}\{A_1, \ldots, A_n\}$, and $B = \text{diag}\{B_1, \ldots, B_n\}$.

Definition 1: A vehicle platoon is said to reach its mean square consensus if it has the following properties:

$$\lim_{t\rightarrow\infty} E(||\widetilde{x}_i(t)||)^2 = 0, \quad i = 1, 2, \ldots n, \tag{9}$$

where $\widetilde{x}_i(t) = [\widetilde{p}_i^T(t), \widetilde{v}_i^T(t), \widetilde{a}_i^T(t)]^T$ is the tracing error vector of following vehicle i to the leading vehicle.

Let $z_i(t) = \tilde{p}_i(t) - \tilde{p}_{i-1}(t)$ be the spacing error between vehicle i and its predecessor $i - 1$. The we can formulate the following linear system model

$$\begin{cases} \dot{\widetilde{x}}(t) = A_c\widetilde{x}(t) + c(t)B\omega(t) \\ z_i(t) = C_{z_i}\widetilde{x}(t) \end{cases} \tag{10}$$

where $C_{z_i} \in R^{1\times 3n}$ is a row vector with its i-th element being 1, the $3i - 2$-th element being -1, and all other elements being zero. To investigate the string stability of the platoon, we assume that the leading vehicle is faced with a bounded disturbance.

Then, we can have the following definition, i.e., l_p-string stability by referring to [36].

Definition 2: The platoon system (8) under random communication noises is l_p-string stable, if there exists functions $\bar{\beta}$ and σ of class $\mathcal{K}$, and constants $\bar{c}$ and $\bar{c}_\omega$ such that for any initial condition $z_i(t_0)$ and communication noises ω_i, $i \in N$, satisfying

$$\sup_{i\in N}|z_i(t_0)| < \bar{c}, \sup_{i\in N}\|\omega_i\| < \bar{c}_\omega,$$

the solution $z_i(t)$, $i \in N$, exists for all $t > t_0$ and satisfies

$$\sup_{i\in N}|z_i(t)| \le \bar{\beta}\Big(\sup_{i\in N}|z_i(t_0)|, t - t_0\Big) + \bar{\sigma}\Big(\sup_{i\in N}\|\omega_i\|\Big).$$

D. EVENT-TRIGGED VEHICLE PLATOON CONTROL WITH COMMUNICATION NOISES

For a vehicle platoon with a communication topology shown as Fig. 1, if the control input is triggered in a continuous form, the communication resources are wasted to some extent. Also, when the acceleration of a vehicle platoon is constantly changing, the use of a time-triggered communication strategy will not only affect the comfort of the passengers, but also cause collisions between vehicles. Therefore, considering random communication noises, an event-triggered communication strategy is further introduced to form the control input $u_i(t)$ in (6).

Let $b_i[k]$ $(i \in N)$ be the k-th triggering moment of vehicle i, where k is a real number. Suppose $b_i[0] = 0$. Let $b[k] = \left[b_1[k], b_2[k], \ldots, b_n[k]\right]^T$. When $b_i[k] \le t < b_i[k+1]$, one has $\hat{\widetilde{p}}_i(t) = \widetilde{p}_i(b_i[k])$, $\hat{\widetilde{v}}_i(t) = \widetilde{v}_i(b_i[k])$, and $\hat{\widetilde{a}}_i(t) = \widetilde{a}_i(b_i[k])$. Let $e_i(t)$ be the measurement error between the actual state error $\widetilde{x}_i(t)$ and the one at the triggering moment, which is denoted as $\hat{\widetilde{x}}_i(b[k])$. One can has

$$e_i(t) = \widetilde{x}_i(t) - \hat{\widetilde{x}}_i(b[k]), (b_i[k] \le t < b_i[k+1]). \quad (11)$$

Note that, in this paper, all the following vehicle i can receive information from the leading vehicle, then $\widetilde{x}_i(t)$ and $\hat{\widetilde{x}}_i(b[k])$ can both be calculated. Denote $\hat{\widetilde{x}}(b[k]) = [\hat{\widetilde{x}}_1(b[k]), \ldots, \hat{\widetilde{x}}_n(b[k])]^T$ and $e(t) = [e_1(t), \ldots, e_n(t)]^T$. Due to the last vehicle in the platoon doesn't need to communicate with others, one thus has $\hat{\widetilde{x}}_n(b_i[k]) = \widetilde{x}_n(t)$ and $e_n(t) = [0, 0, 0]^T$. For $b[k] \le t < b[k+1]$, we can get the event-triggered control input $u(t)$ as

$$u(t) = -c(t)(L+P) \otimes K^T \cdot \hat{\widetilde{x}}(b[k]) + c(t)\omega(t), \quad (12)$$

and the event-triggered vehicle platoon can be written as

$$\dot{\widetilde{x}}(t) = A\widetilde{x}(t) - c(t)B(L+P) \otimes K^T \hat{\widetilde{x}}(b[k]) + c(t)B\omega(t) \quad (13)$$

Motivated by [37], in this paper, we design the following event-triggered function:

$$f_i(t) = ||e_i(t)||^2 - \alpha||\widetilde{x}_i(t)||^2 - \theta e^{-\delta t}, \quad (14)$$

where $\theta > 0$, $\delta > 0$, and $\alpha > 0$ are the triggering parameters to be designed. Specifically, the event times for vehicle i are defined by

$$b_i[k+1] = \min\left\{t \ge b_i[k] \middle| f_i(t) \ge 0\right\}.$$

Remark 5: According to [37], with triggering function designed as $f_i(t, e_i(t)) = |e_i(t)| - (c_0 + c_1 e^{-\delta t})$, when $c_0 \ge 0$, $c_0 + c_1 > 0$, and $0 < \delta < \lambda_2(G)$ hold, the closed-loop system does not exhibit any Zeno behavior. In addition, from (14), one has that the deceases of α and θ will reduce the bounds of inter-event times, and if δ and θ less than or equal to zero, triggering function will be meaningless. Since $||e_i(t)||^2$ represents the measurement error of vehicle i, $\alpha||\widetilde{x}_i(t)||^2 + \theta e^{-\delta t}$ can be regarded as a threshold for the measurement errors. Also, (14) is decentralized since it can be computed locally without using information from any other following vehicles.

IV. MAIN RESULTS

In this section, we will illustrate the stability analysis and the triggering parameter design for vehicle platoon with random communication noises.

Before illustrating the results of this paper, two Lemmas are introduced.

Lemma 1 ([38]Young's Inequality): Given $x, y \in \mathbb{R}$, for any $\rho \in \mathbb{R} > 0$, one has

$$xy \le \frac{x^2}{2\rho} + \frac{\rho y^2}{2}.$$

Lemma 2 ([39]Cauchy-Schwartz Integral Inequality): Let $f(x)$ and $g(x)$ be integrable on $[0, \infty]$, then

$$\left[\int_0^\infty f(x)g(x)dx\right]^2 \le \int_0^\infty f^2(x)dx \cdot \int_0^\infty g^2(x)dx$$

holds.

A. STABILITY ANALYSIS

In this section, we will illustrate the internal and string stability analysis of (8) under random communication noises.

1) INTERNAL STABILITY ANALYSIS

Results on internal stability are shown as Theorem 1.

Theorem 1: For vehicle platoon system (8) *with random communication noises, if control gains k_p, k_v, k_a, and $c(t)$ satisfy*

$$k_p c(t) > 0, k_a c(t) > 0, k_v > \frac{k_p \tau}{1 + c(t)k_a} \quad (15)$$

and

$$\begin{cases} \int_0^\infty c(t)dt = \infty, \\ \int_0^\infty c^2(t)dt < \infty, \end{cases} \quad (16)$$

then vehicle platoon (8) *is internal stable, indicating that the noise-interfered vehicle platoon can achieve its mean square consensus.*

Proof: Let's first prove that A_c is Hurwiz if (15) is satisfied. In (8), A_c can be written as

$$A_c = \begin{pmatrix} 0_n & I_n & 0_n \\ 0_n & 0_n & I_n \\ -\frac{c(t)(L+P)k_p}{\tau} & -\frac{c(t)(L+P)k_v}{\tau} & -\frac{I_n + c(t)(L+P)k_a}{\tau} \end{pmatrix}.$$

Write $L + P$ as H for simplicity. Then the eigenvalues of A_c are the roots of the following equation,

$$\begin{aligned} &\det(\lambda I_{3n} - A_c) \\ &= \begin{vmatrix} \lambda I_n & -I_n & 0_n \\ 0_n & \lambda I_n & -I_n \\ \frac{c(t)k_p}{\tau}H & \frac{c(t)k_v}{\tau}H & \lambda I_n + \frac{1}{\tau}(I_n + c(t)k_a H) \end{vmatrix} \\ &= \begin{vmatrix} \lambda I_n & -I_n & 0_n \\ 0_n & \lambda I_n & -I_n \\ \frac{c(t)k_p}{\tau}H & \frac{c(t)k_v}{\tau}H & \lambda I_n + \frac{1}{\tau}(I_n + c(t)k_a H) \end{vmatrix} \\ &\quad \cdot \begin{vmatrix} I_n & \frac{1}{\lambda}I_n & \frac{1}{\lambda^2}I_n \\ 0_n & I_n & \frac{1}{\lambda}I_n \\ 0_n & 0_n & I_n \end{vmatrix} \\ &= \lambda^2 \det\left(\left(\frac{k_p}{\tau\lambda^2} + \frac{k_v}{\tau\lambda}\right)c(t)H + \lambda I_n + \frac{1}{\tau}(I_n + c(t)k_a H)\right) \\ &= \prod_{i=1}^n \left(\lambda^3 + \frac{1}{\tau}(1 + c(t)k_a\lambda_i)\lambda^2 + \frac{k_v}{\tau}c(t)\lambda_i\lambda + \frac{k_p}{\tau}c(t)\lambda_i\right), \end{aligned} \quad (17)$$

where λ_i is the i-th eigenvalue of matrix H. Let $g_i(\lambda) = \lambda^3 + a_i\lambda^2 + b_i\lambda + c_i$, where $a_i = \frac{1}{\tau}(1 + c(t)k_a\lambda_i)$, $b_i = \frac{k_v}{\tau}c(t)\lambda_i$, and

$c_i = \frac{k_p}{\tau}c(t)\lambda_i$. The eigenvalues of matrix A_c are the solutions of equation $\prod_{i=1}^{n} g_i(\lambda_i) = 0$. With the use of Routh-Hurwitz stability criterion, one has

$$\begin{cases} a_i > 0, b_i > 0, c_i > 0, \\ a_i b_i > c_i. \end{cases}$$

Therefore, one can get $k_p c(t) > 0$, $k_a c(t) > 0$, and $k_v > \max_{1\le i\le n} \frac{k_p\tau}{1+c(t)k_a\lambda_i(H)}$. With information flow topology designed as Fig. 1, one has that $\lambda_1(H) = 1$ and $\lambda_2(H) = \lambda_3(H) = \dots \lambda_n(H) = 2$. Therefore, one has

$$\max_{1\le i\le n} \frac{k_p\tau}{1+c(t)k_a\lambda_i(H)} = \frac{k_p\tau}{1+c(t)k_a} = \frac{k_p\tau}{1+c(t)k_a},$$

indicating that A_c is Hurwitz when (15) is satisfied.

Let's continue to prove that for a Hurwitz A_c, the mean square consensus of the noise-interfered vehicle platoon (8) is achieved when control gain $c(t)$ further satisfies (16). Rewrite (8) into the following $It\hat{o}$ stochastic differential form

$$d\widetilde{x}(t) = \left(A - c(t)BH \otimes K^T\right)\widetilde{x}(t) + c(t)Bd\omega(t). \tag{18}$$

Since A_c is Hurwitz when (15) is satisfied, then, for any positive definite matrix $Q_{3n\times 3n}$, there exists a positive definite matrix $M_{3n\times 3n}$ such that $MA_c + A_c^T M < -Q$ holds. Choose the Lyapunov function as $V(t) = \widetilde{x}(t)^T M\widetilde{x}(t)$. By taking the derivative of $V(t)$, one has

$$\begin{aligned} dV(t) \le\; &\widetilde{x}^T(MA + A^T M)\widetilde{x}(t)dt \\ &-c(t)\widetilde{x}^T\left(MBH \otimes K^T + H^T B^T \otimes KM\right)\widetilde{x}(t)dt \\ &+c^2(t)Z_0 dt + 2c(t)\widetilde{x}^T MBd\omega(t) \end{aligned} \tag{19}$$

where $Z_0 = tr(MBB^T)$. Let $J = BH \otimes K^T$. In (19), according to [12],

$$E\left(2c(t)\widetilde{x}(t)^T MBd\omega(t)\right) = 2\int_0^t c(t)\widetilde{x}(\tau)^T MBd\omega(\tau)d\tau = 0$$

holds, then one can have

$$\begin{aligned} E(dV(t)) \le\; &E\left(\widetilde{x}(t)^T(MA + A^T M)\widetilde{x}(t)\right)dt + c^2(t)Z_0 dt \\ &- E\left(c(t)\widetilde{x}(t)^T(MJ + J^T M)\widetilde{x}(t)\right)dt. \end{aligned} \tag{20}$$

One can further gets

$$\begin{aligned} E\left(\frac{dV(t)}{dt}\right) &\le E\left(\widetilde{x}(t)^T(MA + A^T M)\widetilde{x}(t)\right) + c^2(t)Z_0 \\ &\quad -E\left(c(t)\widetilde{x}(t)^T(MJ + J^T M)\widetilde{x}(t)\right) \\ &= E\left(\tilde{x}(t)^T(MA_c + A_c^T M)\tilde{x}(t)\right) + c^2(t)Z_0 \\ &\le -\lambda_{\min}(Q)\mathrm{E}\left(\tilde{x}(t)^T\tilde{x}(t)\right) + c^2(t)Z_0 \\ &\le -\lambda_{\min}(Q)\frac{E(V(t))}{\lambda_{\max}(M)} + c^2(t)Z_0 \end{aligned} \tag{21}$$

In (21), let η be $\frac{\lambda_{\min}(Q)}{\lambda_{\max}(M)}$. Then $\eta > 0$ holds. According to the Comparison Principle [40],

$$E(V(t)) \le I_0(t) + Z_0 I_1(t) \tag{22}$$

holds, where $I_1(t) = \int_0^t e^{-\eta t}c^2(\xi)d(\xi)$ and $I_0(t) = E(V(0))e^{-\eta t}$. Since for any given $\epsilon > 0$, there exists a $\xi_0 > 0$ such that $\int_0^\infty c^2(\xi)d\xi < \epsilon$ holds [13]. Therefore, for any $t > \xi$, one has

$$\begin{aligned} I_1(t) &= \int_0^t e^{-\eta t}c^2(\xi)d\xi \\ &= \int_0^{\xi_0} e^{-\eta t}c^2(\xi)d\tau + \int_{\xi_0}^t e^{-\eta t}c^2(\xi)d\xi \\ &\le e^{-\eta t}\int_0^{\xi_0} c^2(\xi)d\xi + \int_{\xi_0}^t c^2(\xi)d\xi \\ &\le e^{-\eta t}\int_0^{\infty} c^2(\xi)d\xi + \int_{\xi_0}^\infty c^2(\xi)d\xi \\ &\le e^{-\eta t}\int_0^{\infty} c^2(\xi)d\xi + \varepsilon. \end{aligned} \tag{23}$$

Hence, we can get $\lim_{t\to\infty} Z_0 I_1 = 0$, indicating that $\lim_{t\to\infty} E(V(t)) = 0$. Then $\lim_{t\to\infty} E(||\widetilde{x}_i(t)||)^2 = 0$ holds. Therefore, the mean square consensus of vehicle platoon (8) is achieved when (15) and (16) are both satisfied. This concludes the proof. □

Remark 6: In this paper, we mainly studied the predecessor leader following (PLF) topology. According to [33], $\lambda_{\min}(H) = 1$ also holds in Predecessor following (PF) topology, Bidirectional-leader (BDL) topology, Two-predecessors following (TPF) topology, and Two-predecessor-leader following (TPLF) topology. Thus, $\max_{1\le i\le n} \frac{k_p\tau}{1+c(t)k_a\lambda_i(H)} = \frac{k_p\tau}{1+c(t)k_a}$ holds and Theorem 1 can be extended to topologies mentioned above.

Remark 7: In (16), the first condition is called the convergence condition, and it can make all vehicles' states reach an agreement with a reasonable rate. The other one is called the robustness condition, which makes the closed-loop system's static error to be finite regardless of the measurement noises. Then the consensus protocol is robust against communication noises. There all lots of selections for $c(t)$, such as $c(t) = \frac{1}{1+t}$ and $c(t) = \frac{\log(1+t)}{1+t}$. Different $c(t)$ results in different convergence rates.

2) STRING STABILITY ANALYSIS

We will analyze the string stability of the platoon system under communication noises by utilizing H_∞ norm. Motivated by [41] and [42], (10) can be expressed in the following discrete-time form,

$$\begin{cases} \widetilde{x}(k+1) = A_c(k)\widetilde{x}(k) + c(k)B(k)\omega(k) \\ z_i(k) = C_{z_i}(k)\widetilde{x}(k) \end{cases} \tag{24}$$

To ease the presentation, the following notations $A_c(k) = A_{ci}$, $B(k) = B_{ci}$, $C_{z_i}(k) = C_{z_i}$ are adopted, and system (24) is denoted by $\mathbb{G}$. Then the H_∞ norm from the input ω_i to output z_i is given by

$$\|\mathbb{G}\|_\infty^2 = \sup_{0\ne\omega_i\in\mathcal{L}_2^p} \frac{\|z_i\|_2^2}{\|\omega_i\|_2^2} \tag{25}$$

where $\|z_i\|_2^2 = \sum_{t=0}^\infty E\left(z_i^2(t)\right)$ and $\|\omega_i\|_2^2 = \sum_{t=0}^\infty E\left(\omega_i^2(t)\right)$.

Lemma 3 [42]: The system $\mathbb{G}$ is stable and satisfies the norm constraint $\|\mathbb{G}\|_\infty^2 < \gamma$ if and only if there exist positive definite matrices $M_i \in \mathbb{R}^{3n\times 3n}$ such as

$$\begin{bmatrix} A_{ci} & c(t)B_{ci} \\ C_{zi} & 0 \end{bmatrix}^T \begin{bmatrix} M_{ci} & c(t)B_{ci} \\ C_{zi} & I \end{bmatrix} \begin{bmatrix} A_{ci} & c(t)B_{ci} \\ C_{zi} & 0 \end{bmatrix} - \begin{bmatrix} M_i & 0 \\ 0 & \gamma I \end{bmatrix} < 0 \tag{26}$$

holds, for all $i \in N$, where the positive definite matrix M_{ci} is satisfied by the following matrix inequality

$$\begin{bmatrix} M_i & A_{ci}^T \\ A_{ci} & M_{ci}^{-1} \end{bmatrix} < 0. \tag{27}$$

Then the string stability of the vehicle platoon can be ensured if the following optimization problem has feasible solutions:

$$\|\mathbb{G}\|_\infty^2 = \inf_{(\gamma, M_i)\in\Gamma} \gamma, \tag{28}$$

where Γ is the set of all positive definite matrices γ and M_i.

Therefore, the platooning system with random communication noises can achieve its l_p-string stability.

Remark 8: From the string stability analysis mentioned above, the disturbance will not be amplified with the length of the vehicle platoon. Note that the system designed in this paper is stochastic, so H_∞ norm is also applied in this case. Though this analysis method is a useful tool to verify string stability, it may be increase the complexity of the solution.

B. TRIGGERING PARAMETERS DESIGN

In this section, we will analyze the design for the triggering parameters α and the results are shown as Theorem 2.

Theorem 2: For event-triggered vehicle platoon (13) *with triggering function* (14)*, given that k_p, k_v, k_a, and $c(t)$ satisfy* (15) *and* (16)*, if parameter α further satisfies*

$$0 < \alpha < \frac{2\lambda_{min}(Q)}{c(t)\lambda_{max}(MJ + J^T M)} - 1, \tag{29}$$

then the event-triggered vehicle platoon is asymptotically stable, indicating that the event-triggered vehicle platoon can achieve its mean square consensus.

Proof: With $e_i(t)$ defined as (11), on the basis of (13), for $b[k] \le t < b[k+1]$, the event-triggered vehicle platoon can be rewritten as

$$\dot{\tilde{x}}(t) = \big(A - c(t)J\big)\tilde{x}(t) + c(t)J\cdot e(t) + c(t)B\omega(t). \tag{30}$$

Then the stochastic differential can be written as

$$d\dot{\tilde{x}}(t) = \Big[\big(A - c(t)J\big)\tilde{x}(t) + c(t)J\cdot e(t)\Big]dt + c(t)Bd\omega(t). \tag{31}$$

Note that k_p, k_v, k_a, and $c(t)$ satisfy (15) and (16), then A_c is Hurwitz. Design the same Lyapunov function as Sec.IV.A, i.e., $V(t) = \tilde{x}(t)^T M\tilde{x}(t)$. Then one has

$$\begin{aligned} dV(t) \le{}& \tilde{x}(t)^T \Big(MA + A^T M\Big)\tilde{x}(t)dt \\ &-c(t)\tilde{x}(t)^T\Big(MJ + J^T M\Big)\tilde{x}(t)dt \\ &+c(t)\big(\tilde{x}(t)^T MJ\cdot e(t)dt + e(t)^T J^T M\tilde{x}(t)\big)dt \\ &+2c(t)\tilde{x}(t)^T MBd\omega(t) + c^2(t)Z_0 dt. \end{aligned} \tag{32}$$

From Young's inequation in Lemma 1, by letting $\rho = 1$, one can get

$$\begin{cases} \tilde{x}(t)^T MJe(t) \le \frac{1}{2}\tilde{x}(t)^T MJ\tilde{x}(t) + \frac{1}{2}e(t)^T MJe(t) \\ e(t)^T J^T M\tilde{x}(t) \le \frac{1}{2}\tilde{x}(t)^T J^T M\tilde{x}(t) + \frac{1}{2}e(t)^T J^T Me(t), \end{cases}$$

then (32) can be rewritten as

$$\begin{aligned} dV(t) \le{}& \tilde{x}(t)^T\Big(MA + A^T M\Big)\tilde{x}(t)dt \\ &-\frac{c(t)}{2}\tilde{x}(t)^T\Big(MJ + J^T M\Big)\tilde{x}(t)dt \\ &+\frac{c(t)}{2}e(t)^T\Big(MJ + J^T M\Big)e(t)dt \\ &+2c(t)\tilde{x}(t)^T MBd\omega(t) + c^2(t)Z_0 dt. \end{aligned} \tag{33}$$

According to event triggering function (14), one has

$$\begin{aligned} e(t)^T\Big(MJ + J^T M\Big)e(t) \le{}& \alpha\tilde{x}(t)^T\lambda_{\max}\Big(MJ + J^T M\Big)\tilde{x}(t) \\ &+n\theta\lambda_{\max}\Big(MJ + J^T M\Big)e^{-\delta t}. \end{aligned}$$

Hence, (33) can be simplified into

$$\begin{aligned} dV(t) \le{}& \tilde{x}(t)^T\Big(MA + A^T M\Big)\tilde{x}(t)dt \\ &+\frac{\alpha-1}{2}c(t)\Big(MJ + J^T M\Big)\tilde{x}(t)dt \\ &+\frac{n\theta}{2}c(t)\lambda_{\max}\Big(MJ + J^T M\Big)e^{-\delta t}dt \\ &+c^2(t)Z_0 dt + 2c(t)\tilde{x}(t)^T MBd\omega(t). \end{aligned} \tag{34}$$

Similarity to the processing in (21), one has

$$\begin{aligned} &E\left(\frac{dV(t)}{dt}\right) \\ &\le E\Big(\tilde{x}(t)^T\big(MA + A^T M + \frac{\alpha-1}{2}c(t)(MJ + J^T M)\big)\tilde{x}(t)\Big) \\ &\quad+\frac{n\theta}{2}c(t)\lambda_{\max}\Big(MJ + J^T M\Big)e^{-\delta t} + c^2(t)Z_0 \\ &= E\left(\tilde{x}(t)^T\left(MA_c + A_c^T M + \frac{\alpha+1}{2}c(t)\Big(MJ + J^T M\Big)\right)\tilde{x}(t)\right) \\ &\quad+\frac{n\theta}{2}c(t)\lambda_{\max}\Big(MJ + J^T M\Big)e^{-\delta t} + c^2(t)Z_0 \\ &\le \left(-\lambda_{\min}(Q) + \frac{\alpha+1}{2}c(t)\lambda_{\max}\Big(MJ + J^T M\Big)\right)\frac{E(dV)}{\lambda(M)} \\ &\quad+\frac{n\theta}{2}c(t)\lambda_{\max}\Big(MJ + J^T M\Big)e^{-\delta t} + c^2(t)Z_0. \end{aligned} \tag{35}$$

In (35), let $\eta'(t)$ be $\frac{\lambda_{\min}(Q) - \frac{\alpha+1}{2}c(t)\lambda_{\max}(MJ + J^T M)}{\lambda_{\max}(M)}$. When $-\lambda_{\min}(Q) + \frac{\alpha+1}{2}c(t)\lambda_{\max}\big(MJ + J^T M\big) < 0$ is satisfied, i.e., $0 < \alpha < \frac{2\lambda_{\min}(Q)}{c(t)\lambda_{\max}(MJ + J^T M)} - 1$ holds, (35) can be simplified as

$$\begin{aligned} E\left(\frac{dV(t)}{dt}\right) \le{}& -\eta'(t)E(dV) + c^2(t)Z_0 \\ &+\frac{n\theta}{2}c(t)\lambda_{\max}\Big(MJ + J^T M\Big)e^{-\delta t}. \end{aligned} \tag{36}$$

According to the Comparison Principle, (36) can be rewritten as

$$E(V(t)) \leq U_0(t) + U_1(t)Z_0 + U_2(t),$$

where $U_0(t) = V(0)e^{-\int_0^t \eta'(\xi)\mathrm{d}\xi}$. Similarly, one can have $\lim_{t\to\infty} U_0(t) = 0$, $U_1(t) = \int_0^t e^{-\int_\xi^t \eta'(v)\mathrm{d}v} c^2(\xi)d\xi$, and $U_2(t) = \frac{n\theta}{2}\lambda_{\max}\left(MJ+J^TM\right)\int_0^t e^{-\int_\xi^t \eta'(v)\mathrm{d}v} c(\xi)e^{-\delta\xi}d\xi$. Similar to the proof to Theorem 1 in Sec.IV.A, one can have $\lim_{t\to\infty} U_1(t) = 0$, and

$$\begin{aligned}
U_2(t) &= \frac{n\theta}{2}\lambda_{\max}\left(MJ+J^TM\right)\int_0^t e^{-\int_\xi^t \eta'(v)\mathrm{d}v} c(\xi)e^{-\delta\xi}d\xi \\
&= \frac{n\theta}{2}c(\xi)\lambda_{\max}\left(MJ+J^TM\right)e^{-\int_\xi^t \eta'(v)\mathrm{d}v} \\
&\quad \times\left(\int_0^{\xi_0} c(\xi)e^{-\delta\xi}d\xi + \int_{\xi_0}^t c(\xi)e^{-\delta\xi}d\xi\right) \\
&\leq \frac{n\theta}{2}\lambda_{\max}\left(MJ+J^TM\right)e^{-\int_{\xi_0}^t \eta'(v)\mathrm{d}v}\int_0^{\xi_0} c(\xi)e^{-\delta\xi}d\xi \\
&\quad + \int_{\xi_0}^t c(\xi)e^{-\delta\xi}d\xi \\
&\leq \frac{n\theta}{2}\lambda_{\max}\left(MJ+J^TM\right)e^{-\int_{\xi_0}^t \eta'(v)\mathrm{d}v}\int_0^{\infty} c(\xi)e^{-\delta\xi}d\xi \\
&\quad + \int_{\xi_0}^\infty c(\xi)e^{-\delta\xi}d\xi. \qquad (37)
\end{aligned}$$

In (37), since $\theta > 0$, $\delta > 0$, then

$$\lim_{t\to\infty}\frac{n\theta}{2}\lambda_{\max}\left(MJ+J^TM\right)e^{-\int_{\xi_0}^t \eta'(v)\mathrm{d}v}\int_0^\infty c(\xi)e^{-\delta\xi}d\xi = 0$$

holds, and from Lemma 2, one can get that

$$\begin{aligned}
\left(\int_{\xi_0}^\infty c(\xi)e^{-\delta\xi}d\xi\right)^2 &\leq \int_{\xi_0}^\infty c^2(\xi)d\xi \cdot \int_{\xi_0}^\infty e^{-2\delta\xi}d\xi \\
&\leq \varepsilon\cdot\frac{1}{2\delta}e^{-2\delta\xi_0} = \varepsilon_1,
\end{aligned}$$

where ε_1 is the same infinitesimal number as ε in Section.IV.A. Then $\int_{\xi_0}^\infty c(\xi)e^{-\delta\xi}d\xi \leq \sqrt{\varepsilon_1}$ holds. Therefore, $\lim_{t\to\infty} U_2(t) = 0$. Hence, $\lim_{t\to\infty} E(V(t)) = 0$, and $\lim_{t\to\infty} E(||\widetilde{x}_i(t)||)^2 = 0$, indicating that the mean square consensus of vehicle platoon (13) is achieved. The proof is completed. □

V. SIMULATION RESULTS

The correctness of our analysis is illustrated by extensive simulations. Simulation parameters are set as: $n = 8$, $\tau = 0.5$, $D_i = [4.1, 4.2, 4.3, 4.5, 4.8, 4.8, 4.7, 4.3]$ m, $c(t) = 1/(t+1)$, and $r = 10$ m. Random communication noises are designed as the Laplace noise with zero mean and variance 2. The sampling period is set as 10 ms. The initial position and acceleration of the leading vehicle are set as $p_0(0) = (n+1)*r$ and $a_0(t) = 0$, and the initial states of the following vehicles are set as $p_i(0) = (n-i)*3r$, $v_i(0) = 0$, and $a_i(0) = 0$. Table 1 shows the set of gain k_p, k_v, k_a under two different scenarios.

TABLE 1. Parameters setting for k_p, k_v, k_a.

Parameters	Scenario 1	Scenario 2
k_p	0.5	0.5
k_v	0.1	2
k_a	1	1
Theorem 1	Dissatisfied	Satisfied

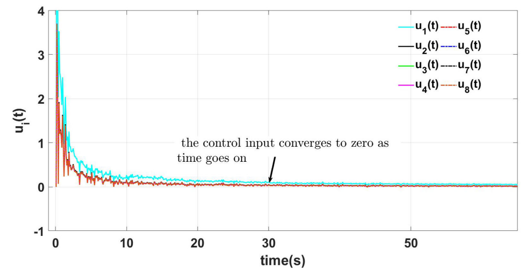
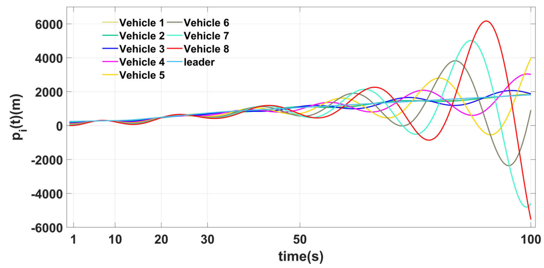
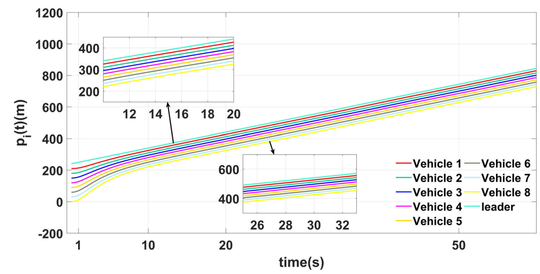


FIGURE 2. The control input $u_i(t)$.



(a)



(b)

FIGURE 3. Positions comparisons with different parameters, where (a) $k_p = 0.5$, $k_v = 0.1$, $k_a = 1$; (b) $k_p = 0.5$, $k_v = 2$, $k_a = 1$.

A. LEADER WITH A CONSTANT VELOCITY

The velocity of the leading vehicle is set as a fixed value $v_0(t) = 10$ (m/s). Simulations are carried out for the two scenarios set in Table 1 to compare the position of the vehicles in the platoon. The control input $u_i(t)$ is illustrated in Fig. 2. The iteration of positions are shown in Fig.3 (a) and (b).

It is shown in Fig. 2 that the control input $u_i(t)$ converges to zero as time goes on, which also reflects that the consensus gain $c(t)$ can mitigate the interference of communication noises in Remark 3. It can be seen from Fig. 3 (a) that, with the continuous progress of sampling time, all the following vehicles diverge; while from Fig. 3 (b), the rear vehicle can

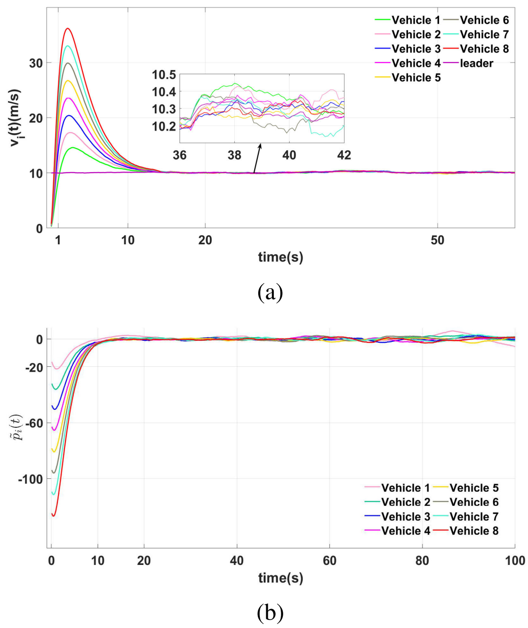


FIGURE 4. Performance of the vehicle platoon with a constant leading velocity and $k_p = 0.5$, $k_v = 2$, $k_a = 1$. (a) $v_i(t)$; (b) $\tilde{p}_i(t)$.

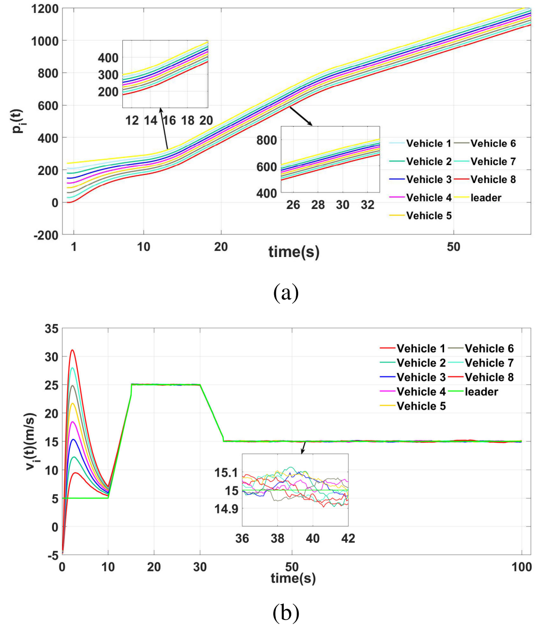


FIGURE 6. Performance of the vehicle platoon with a time-varying leading vehicle and time-triggered communication strategy: (a) $p_i(t)$; (b) $v_i(t)$.

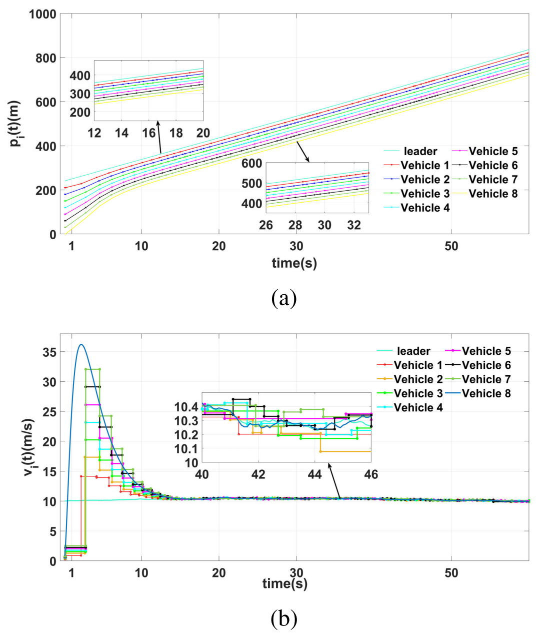


FIGURE 5. Performance of the vehicle platoon with a constant leading vehicle and event-triggered communication strategy: (a) $p_i(t)$; (b) $v_i(t)$.

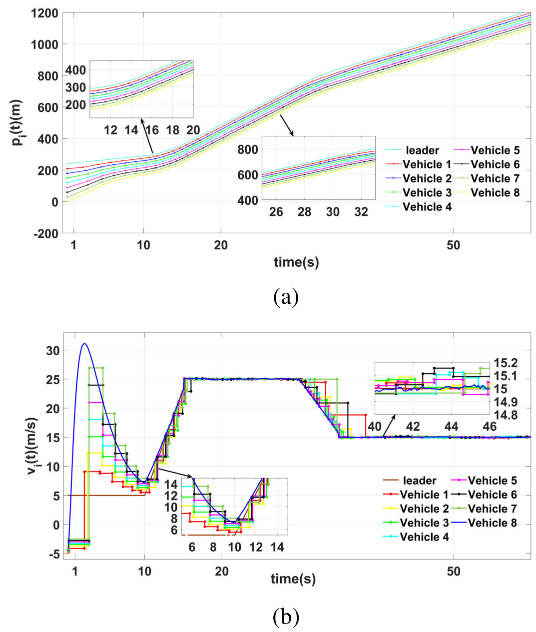


FIGURE 7. Performance of the vehicle platoon with a time-varying leading vehicle and event-triggered communication strategy: (a) $p_i(t)$; (b) $v_i(t)$.

keep a safe distance from the front vehicle, which avoids the occurrence of collision. This verifies the correctness of the conditions for maintaining the internal stability in Theorem 1.

For the one that has inter-stability, the performance of the vehicle platoon is shown in Fig. 4. It can be seen from Fig. 4 (a) that, the states of all the vehicles asymptotically achieve their consensus. The reason why the convergence value still has tiny fluctuations is that there are random communication noises that existed in the platoon. Also, we can observe that $|\widetilde{v}_i(t)| < 0.3\ m/s$ when t is sufficient large, which also verify the correctness of Theorem 1. In addition, it can be shown that the spacing errors in Fig. 4 (b) will not be amplified by the communication noises, which verifies the string stability of the vehicle platoon.

For the second parameter setting scenario in Table 1, simulations are carried out to verify the results of the

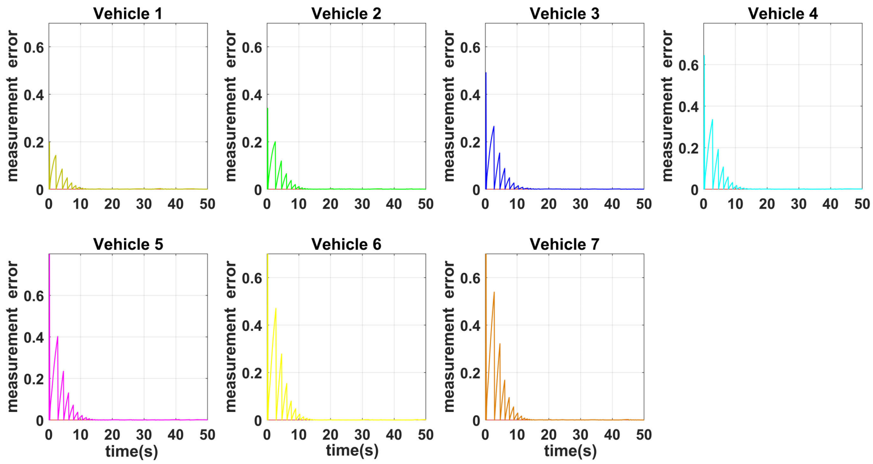


FIGURE 8. Evolution of the measurement error of the following vehicle in norm form.

event-triggered mechanism in Theorem 2. Select $Q = I_{3n}$, thus $\lambda_{\min} = 1$, $\lambda_{\max}(MJ + J^T M) = 1.094$, and then the range for α is (0,0.828). So triggering parameters can be set as: $\alpha = 0.5$, $\theta = 1.1$, $\delta = 1$, then (29) is satisfied. Simulation results on position, velocity are shown in Fig. 5 (a) and 5 (b), respectively. It can be seen from Fig. 5 that, the performance of the vehicle platoon with a constant leading vehicle and event-triggered communication strategy can achieve its mean square consensus, and the protocol updates the relative information in a discrete-time manner. Furthermore, with the consensus algorithm being iterated over 50 times, errors $\widetilde{v}_i(t)$ maintain below 0.5 m/s, which can satisfy the actual requirement of the vehicle platoon system.

B. LEADER WITH A TIME-VARYING VELOCITY

The above experiment is based on the constant velocity of the leading vehicle. However, such a situation is too special to be consistent with the actual situation of the vehicles driving on the expressway. Simultaneously, the acceleration/deceleration of the leading vehicle during the driving process is considered on this basis, i.e., when the leading-vehicle's velocity is a transient variable value, specifically:

$$v_0(t) = \begin{cases} 5\ (m/s), & t \leq 10s \\ 4t - 35\ (m/s), & 10 < t \leq 15s \\ 25\ (m/s), & 15 < t \leq 30s \\ -2t + 85\ (m/s), & 30 < t \leq 35s \\ 15\ (m/s), & t > 35s \end{cases}$$

The consensus of a vehicle platoon under time-triggered condition is considered, and the performance of the vehicle platoon of vehicles are shown in Fig. 6. In Fig. 6, we can see that, i) all vehicles can keep a safe distance without collision from the former one; ii) the velocity error $|\widetilde{v}_i(t)| < 0.25\ m/s$, when t is sufficient large.

The effectiveness of vehicle platoon under the event-triggered mechanism is considered. Fig. 7 show the performance of the following vehicle, Fig. 8 illustrates the evolution of the norm of measurement error of each vehicle, and the triggering events of the following vehicles are shown in Fig. 9, respectively, where '•' indicates the happening of a communication event.

It can be seen from Fig. 7 that, under the influence of the time-varying velocity of the leading vehicle, and considering random communication noises, the vehicle platoon can still maintain a safe distance without collision, and the mean square consensus is reached. From Fig. 8 one can has that, the measurement error of each vehicle approaches zero about 10 s. Fig. 9 shows that the triggering event is sporadic rather than consecutive. Regarding the executing efficiency, we further calculate communication amount and communication rate for each vehicle which is described in Table 2.

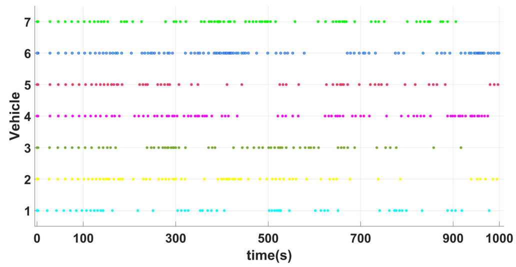


FIGURE 9. Triggering events of following vehicles.

TABLE 2. Communication rates of vehicles in the platoon.

The vehicle number	Communication amount (Sampling times=1000)	Communication rates ($Cr_i, i = 1, 2, ...n-1$)
1	52	5.2%
2	62	6.2%
3	60	6.0%
4	86	8.6%
5	58	5.8%
6	95	9.5%
7	70	7.0%

Note: The average communication rate is ($Ave = \frac{1}{n-1}\sum_{i=1}^{n-1} Cr_i =$ 6.9%).

The average communication rate of the n vehicles is about 6.9%, which is far less than the one of the time-triggered one. Moreover, from Fig. 8 and Fig. 9, Zeno behavior is excluded due to finite event-triggered times which verifies Remark 5 in this paper.

VI. CONCLUSION

Aimed at improving the communication efficiency of the vehicle platoon, and considering the existence of communication noises among vehicles, this paper proposed an event-triggered vehicle platoon control under random communication noises. Based on a third-order vehicle dynamic model, a time-varying consensus gain function was introduced to attenuate the random communication noises. The internal stability of the vehicle platoon was analyzed by using the matrix eigenvalue perturbation theory and block matrix polynomials. In addition, string stability was analyzed under random communication noises. Then the consensus of the vehicle platoon in mean square was analyzed by utilizing the *Itô* stochastic differential formula and Lyapunov theory. To further improve the communication efficiency, an event-triggered communication mechanism was introduced, and the condition for the triggering parameter was derived. Lastly, simulations were conducted to illustrate the efficiency of the main results of this paper.

Communication impairments such as communication losses, fading channels, and time delays beckons our future work.

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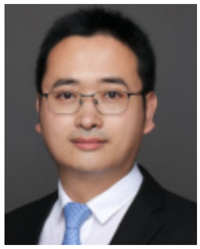

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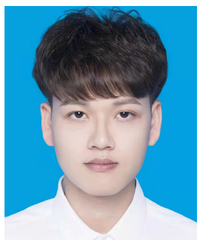

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