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Effect of Misaligned Relay on Output Power and Efficiency in Wireless Power Transfer

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ABSTRACT In magnetic resonance-based wireless power transfer (WPT), any misalignment between resonators degrades the strength of the magnetic coupling, and the use of a relay does not always improve performance if it is misaligned with a transmitter or a receiver. It is therefore necessary to determine the proper activation of a relay depending on the degree of misalignment. In this paper, we analyze the effect of a misaligned relay on two important performance metrics in WPT, namely output power and efficiency. In particular, we derive the achievable output power and efficiency with optimal load resistance based on an equivalent circuit model, and determine whether it is beneficial to utilize the relay for a given lateral and angular misalignment. By simulation and experimental verifications under various environments, we validate the exactness of our analysis, and we also confirm two interesting observations, namely 1) with regard to performance improvement in terms of output power and efficiency, it is better to deactivate the relay if it is seriously misaligned with the other resonators and 2) output power is more vulnerable to the misalignment of a relay than efficiency.

INDEX TERMS Wireless power transfer, magnetic resonance, misalignment, relay activation, output power, efficiency.

I. INTRODUCTION

The attractiveness of wireless power transfer (WPT) as a means of enabling the delivery of power to electrical devices has led to its emergence as a promising alternative to the use of power cables. Ever since Tesla established the principle of WPT in the last century [1], the possibility of nonradiative WPT via strongly coupled magnetic resonance was demonstrated theoretically and experimentally by a research group at MIT [2]. Inspired by early research, there have been many attempts to investigate such characteristics as operating frequency [3], [4], mutual coupling [5], [6], and frequency splitting [7]–[9] in WPT.

In addition to the characterization of WPT, a number of research avenues have been pursued regarding its improvement with respect to the two key performance metrics of i) efficiency and ii) output power. For the first, the efficiency is the ratio of the power dissipated in a load resistor divided by the power transmitted by a power source.

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A variety of methods have been proposed to improve efficiency, including antenna fabrication [10], [11], impedance matching [12]–[14], circuit design [15], [16], and optimization of inductive links [17]. For the second, the output power represents the power dissipated at the load on the receiving side, and optimal coupling [18], load resistance [19], and system parameters [20] have all been derived to maximize this. Although efficiency and output power are both important performance metrics in WPT, they have different characteristics and implications as shown in [21], and their values cannot be maximized simultaneously at the same operational point [19], [20]. A different approach is therefore required to optimize each performance metric in turn.

Recently, the use of relays has been considered for WPT in order to improve both the system efficiency and the operating distance [22]–[27]. Specifically, the optimal impedance matching and improved stability using an intermediate relay were analyzed [22], [23], and an optimal configuration of a relay was proposed to enhance the efficiency [24], [25]. In addition, the authors of [26], [27] discussed the effects

TABLE 1. Literature survey.

Literature	Configuration	Metric	Challenge	Method
[2]	4-coil	Efficiency	Possibility	Show that 60 watts can be transferred with 40% efficiency over distances at 2 meters.
[3], [4]	2-coil	Efficiency	Operating frequency	Optimal frequencies for dispersive tissue and maximum matched gain.
[5], [6]	2-coil	Efficiency	Inductive coupling	Analyze the relationship between mutual coupling and WPT performances.
[7]–[9]	2-, 4-coil	Output power	Frequency splitting	Analyze the critical coupling where the maximum WPT can be achieved.
[10], [11]	2-, multi-coil	Efficiency	Antenna	Coil antenna designs for electric vehicles.
[12]–[14]	2-, 4-coil	Efficiency	Impedance matching	Impedance matching for high efficiency.
[15], [16]	Multi-coil	Efficiency	Circuit design	Design of class EF-type voltage-fed multi-resonant inverter and SăĂŞŞăĂŞLCLCC compensation.
[17]	Multi-coil	Efficiency	Inductive coupling	Optimal inductive coupling for WPT performances.
[18]–[20]	2-, 4-coil	Output power	System parameter	Optimal system parameters including coupling and load for maximizing output power.
[21]	Multi-coil	Both	Review	Reveal the difference between efficiency and output power.
[22]	Multi-coil	Efficiency	Impedance matching	Optimal impedance matching condition for a relay system.
[23]	Multi-coil	Efficiency	Stability	Show that use of an intermediate relay achieves higher efficiency and stability.
[24], [25]	Multi-coil	Efficiency	Configuration	Optimal configuration of a relay for maximizing efficiency.
[26], [27]	Multi-coil	Efficiency	Configuration	Analyze the effect of mutual couplings for a multiple-relay system.
[28]–[30]	2-coil	Output power	Analysis	Provide analytical models for coil misalignments.
[31]	2-coil	Efficiency	System parameter	A design method to improve a misalignment tolerance.

of operating frequency and spacing between resonators on system efficiency in domino-form relay systems.

Although the use of relays in WPT has been investigated extensively, in most previous works it has been assumed that the resonators are perfectly aligned, because the performance of WPT can be severely degraded due to lateral and angular misalignments between resonators [28]–[31]. Accordingly, in most applications including electric vehicles [32], [33], wireless chargers for electrical devices [34], [35], and medical implants [36], fixed frames or other means are utilized to keep the perfect alignment between the transmitter and receiver. However, users can add the relay between the aligned transmitter and receiver according to the need for performance improvement. In this case, the relay may be out of alignment with the transmitter and receiver due to the mobile nature of the devices or a carelessness of users. The literature survey is summarized in Table 1.

Unlike previous works which focus on the benefit of the relay, our study begins with the question, “Is the use of a relay always favorable for enhancing the performance of WPT even when it is severely misaligned with the other resonators?”. Accordingly, we investigate the effect of misalignment of a relay on the achievable WPT performances in terms of output power and efficiency. To our best knowledge, our study is the first attempt to provide a policy for determining the activation of a relay depending on the degree of misalignment. Our contributions can be summarized as follows.

- We model a WPT system with a misaligned relay using an equivalent circuit model (ECM) and derive closed-form expressions for achievable output power and efficiency with optimal load resistance.
- In comparison with a two-coil system, we find the following rules of interest for relay activation. 1) the use of a relay can reduce the achievable output power and efficiency if it is severely misaligned with a transmitter and a receiver. 2) the misalignment of a relay has a more negative impact on output power than it does on efficiency. These findings could lead to practical guidelines for determining whether or not to activate a relay in the presence of a misalignment.

- In order to validate the accuracy of our analysis, we fabricate resonators with a spiral shape, and perform simulations using an advanced design system (ADS) for a variety of scenarios based on the measured parameters of the resonators. By means of measurements undertaken using a vector network analyzer (VNA), we also verify that the analytical results are in good agreement with the measured ones.

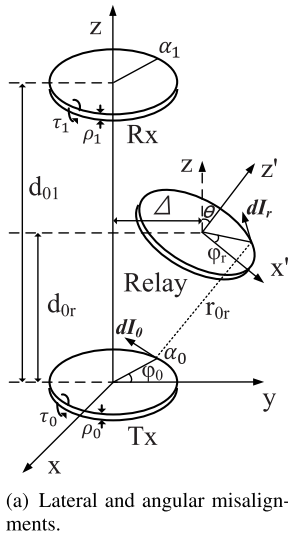
The remainder of this paper is organized as follows. In Section II, we describe the system model of WPT with a misaligned relay, and derive basic equations using the ECM. In Section III, we provide the optimal activation rules for a relay to improve output power and efficiency. In Section IV, simulations and experiments are described to allow evaluation of the accuracy of our analysis under various scenarios. Finally, we conclude our paper in Section V.

II. SYSTEM MODEL

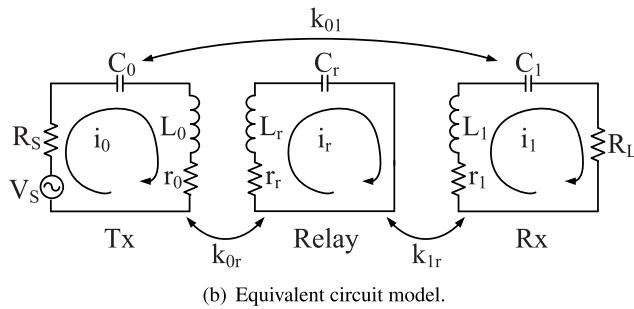
Fig. 1 (a) shows the configuration of a WPT relay system with lateral and angular misalignments; the system consists of a transmitter (Tx), a relay, and a receiver (Rx). The parameters α_i , τ_i , and ρ_i , denote the outer radius, the number of turns, and the pitch of resonator i , respectively, where $i \in \{0, r, 1\}$. Here, the subscripts, 0, r , and 1, indicate Tx, the relay, and Rx, respectively. In addition, d_{ij} is the distance between resonators i and j , and Δ and θ respectively represent the degree of lateral and angular misalignment between Tx and the relay. The WPT system can be represented by an ECM, as shown in Fig. 1 (b), in which an alternating voltage source, V_S , and a source resistor, R_S , are linked to Tx, while a load resistor, R_L , is connected to Rx. A self-inductance and a parasitic resistance for resonator i are denoted by L_i and r_i , respectively, and a lumped capacitance, C_i , is connected to resonator i in series to ensure that all resonators have the same resonant frequency, as follows.

$$\omega_o = 2\pi f_o = \frac{1}{\sqrt{L_0 C_0}} = \frac{1}{\sqrt{L_r C_r}} = \frac{1}{\sqrt{L_1 C_1}}. \quad (1)$$

A coupling coefficient, $k_{ij} = \frac{M_{ij}}{\sqrt{L_i L_j}}$, is used to indicate the strength of the magnetic coupling between the two resonators



(a) Lateral and angular misalignments.



(b) Equivalent circuit model.

FIGURE 1. System model for WPT with a relay.

i and j , where M_{ij} denotes a mutual inductance.¹ In the presence of lateral and angular misalignments, M_{ij} can be expressed as follows [30]:

$$M_{ij} = \frac{\mu_0 \tau_i \tau_j \alpha_i \alpha_j}{4\pi} \times \oint \oint \frac{\sin \varphi_i \sin \varphi_j \cos \theta + \cos \varphi_i \cos \varphi_j}{r_{ij}^{1/2}} d\varphi_i d\varphi_j \quad (2)$$

where r_{ij} is given by

$$r_{ij} = \alpha_i^2 + \alpha_j^2 + d_{ij}^2 + \Delta^2 + 2\Delta\alpha_j \cos \varphi_j \cos \theta - 2\Delta\alpha_i \cos \varphi_i - 2\alpha_i \alpha_j (\cos \varphi_i \cos \varphi_j \cos \theta + \sin \varphi_i \sin \varphi_j) - 2\alpha_j d_{ij} \cos \varphi_j \sin \theta. \quad (3)$$

At the resonant frequency, the reactance term for resonator i , e.g., $j\omega_o L_i + \frac{1}{j\omega_o C_i}$, becomes zero. Therefore, we can formulate the following equations from Kirchhoff's voltage law (KVL):

$$\begin{aligned} V_S &= (R_S + r_0)i_0 + j\omega_o \sqrt{L_0 L_r} k_{0r} i_r + j\omega_o \sqrt{L_0 L_1} k_{01} i_1, \\ 0 &= j\omega_o \sqrt{L_0 L_r} k_{0r} i_0 + r_r i_r + j\omega_o \sqrt{L_r L_1} k_{1r} i_1, \\ 0 &= j\omega_o \sqrt{L_0 L_1} k_{01} i_0 + j\omega_o \sqrt{L_r L_1} k_{1r} i_r + (r_1 + R_L)i_1. \end{aligned} \quad (4)$$

¹Note that the coupling coefficient can be estimated accurately in real-time using the method proposed in [37].

Herein, we assume that the level of cross-coupling between nonadjacent resonators is negligible because it is negligibly small compared to the coupling between adjacent resonators, i.e., $k_{0r}, k_{1r} \gg k_{01}$ [22], [23], [25]–[27].² Then, using (4), the amount of current in each resonator can be obtained as

$$i_0 = \frac{V_S}{R_S + r_0 + \frac{\omega_o^2 L_0 L_r k_{0r}^2}{r_r + \frac{\omega_o^2 L_r L_1 k_{1r}^2}{r_1 + R_L}}}, \quad (5)$$

$$i_r = \frac{j\omega_o \sqrt{L_0 L_r} k_{0r}}{r_r + \frac{\omega_o^2 L_r L_1 k_{1r}^2}{r_1 + R_L}} \cdot i_0, \quad (6)$$

$$i_1 = \frac{\omega_o^2 L_r \sqrt{L_0 L_1} k_{0r} k_{1r}}{r_r (r_1 + R_L) + \omega_o^2 L_r L_1 k_{1r}^2} \cdot i_0. \quad (7)$$

III. OPTIMAL ACTIVATION OF RELAY FOR OUTPUT POWER AND EFFICIENCY

In this section, we analyze the optimal condition to activate the relay by comparing the output power and efficiency of the relay system with those of a two-coil system with Tx and Rx only. We also provide some meaningful observations.

A. ANALYSIS OF OUTPUT POWER

Using (5), the input power at Tx can be found as (8), shown at the bottom of the next page, where $P_S = \frac{V_S^2}{R_S}$, $Q_S = \frac{\omega_o L_0}{R_S}$, $Q_L = \frac{\omega_o L_1}{R_L}$, and $Q_i = \frac{\omega_o L_i}{r_i}$ for $i \in \{0, r, 1\}$. From (7), the output power dissipated in the load of Rx can also be derived as shown in (9), as shown at the bottom of the next page.

The optimal value of Q_L for maximizing the output power can be derived by finding Q_L that satisfies $\frac{\partial P_{out}}{\partial Q_L} = 0$, as follows.

$$Q_L^{P*} = \frac{Q_1 \left(1 + \frac{Q_0}{Q_S} + Q_0 Q_r k_{0r}^2\right)}{\left(1 + \frac{Q_0}{Q_S}\right) (1 + Q_r Q_1 k_{1r}^2) + Q_0 Q_r k_{0r}^2}. \quad (10)$$

In addition, the optimal load resistance that corresponds to Q_L^{P*} can also be calculated as

$$R_L^{P*} = \frac{r_1 \left[\left(1 + \frac{Q_0}{Q_S}\right) (1 + Q_r Q_1 k_{1r}^2) + Q_0 Q_r k_{0r}^2\right]}{1 + \frac{Q_0}{Q_S} + Q_0 Q_r k_{0r}^2}. \quad (11)$$

Given that R_L^{P*} in (11) is proportional to k_{0r} and k_{1r} , R_L^{P*} will decrease as the misalignment becomes more severe.

By substituting (10) into (9), the achievable output power can be found as (12), shown at the bottom of the next page. From the fact that P_{out}^* is mainly affected by P_S , Q_i , and k_{ij} where $(i, j) \in \{0, r, 1\}$ in (12), we note that the achievable output power can be improved if the following conditions are satisfied: i) Tx uses a high voltage source to generate

²The validity of this assumption will be justified through our performance evaluations in Section IV, where the analytical results obtained under this assumption agree well with the simulation and experimental results.

a higher P_S , ii) the resonators are strongly coupled to have a higher k_{ij} , iii) each resonator is designed to have a higher L_i and a lower r_i to achieve a higher Q_i .

For comparison, the optimal load resistance and achievable output power for the two-coil system, respectively $R_{L,2c}^{P*}$ and $P_{out,2c}^*$, are derived using a similar approach, as follows.

$$R_{L,2c}^{P*} = \frac{r_1 \left(1 + \frac{Q_0}{Q_S} + Q_0 Q_1 k_{01}^2\right)}{\left(1 + \frac{Q_0}{Q_S}\right)}, \quad (13)$$

$$P_{out,2c}^* = \frac{P_S \frac{Q_0}{Q_S} Q_0 Q_1 k_{01}^2}{4 \left(1 + \frac{Q_0}{Q_S}\right) \left(1 + \frac{Q_0}{Q_S} + Q_0 Q_1 k_{01}^2\right)}. \quad (14)$$

By comparing P_{out}^* with $P_{out,2c}^*$, we can determine when it is more beneficial to use a relay considering the achievable output power, i.e., the activation condition, as (15), shown at the bottom of the page. Specifically, the terms in (15) including $\frac{1}{k_{01}^2}$, $\frac{1}{Q_r k_{0r}^2}$, and $\frac{1}{Q_r k_{1r}^2}$ will become negligibly small in the

strongly coupled region where the conditions, $Q_i Q_j k_{ij}^2 > 1$ and $Q_i \gg Q_S$ for $(i, j) \in \{0, r, 1\}$, hold [2]. Finally, we can conclude that the use of a relay is preferable for improving the achievable output power when condition (16), as shown at the bottom of the page, is satisfied. From this result, we can make the following remark.

Remark 1 (Activation Condition of Relay Regarding P_{out}^):* If a relay is seriously misaligned with a transmitter or a receiver (i.e., k_{0r} or k_{1r} is small) or a quality factor of the relay is not sufficiently large to compensate for the misalignment (i.e., Q_r is small), it is advantageous to deactivate the relay in order to improve P_{out}^* .

B. ANALYSIS OF EFFICIENCY

Efficiency, which we denote as η , is defined as the ratio of the output power to the input power, thus it is derived as (17), shown at the bottom of the page, using (8) and (9). The optimal value of Q_L to maximize the efficiency can be found

$$\begin{aligned} P_{in} &= i_0 \cdot V_S \\ &= \frac{V_S^2 [r_r(r_1 + R_L) + \omega_o^2 L_r L_1 k_{1r}^2]}{r_r(R_S + r_0)(r_1 + R_L) + (R_S + r_0)\omega_o^2 L_r L_1 k_{1r}^2 + (r_1 + R_L)\omega_o^2 L_0 L_r k_{0r}^2} \\ &= \frac{P_S \frac{Q_0}{Q_S} \left(1 + Q_r Q_1 k_{1r}^2 + \frac{Q_1}{Q_L}\right)}{\left(1 + \frac{Q_0}{Q_S}\right) \left(1 + Q_r Q_1 k_{1r}^2 + \frac{Q_1}{Q_L}\right) + \left(1 + \frac{Q_1}{Q_L}\right) Q_0 Q_r k_{0r}^2}. \end{aligned} \quad (8)$$

$$\begin{aligned} P_{out} &= i_1^2 \cdot R_L \\ &= \frac{V_S^2 \omega_o^4 L_0 L_r^2 L_1 k_{0r}^2 k_{1r}^2 R_L}{[r_r(R_S + r_0)(r_1 + R_L) + (R_S + r_0)\omega_o^2 L_r L_1 k_{1r}^2 + (r_1 + R_L)\omega_o^2 L_0 L_r k_{0r}^2]^2} \\ &= \frac{P_S \frac{Q_0}{Q_S} \frac{Q_1}{Q_L} Q_0 Q_r^2 Q_1 k_{0r}^2 k_{1r}^2}{\left[\left(1 + \frac{Q_0}{Q_S}\right) \left(1 + Q_r Q_1 k_{1r}^2 + \frac{Q_1}{Q_L}\right) + \left(1 + \frac{Q_1}{Q_L}\right) Q_0 Q_r k_{0r}^2\right]^2}. \end{aligned} \quad (9)$$

$$P_{out}^* = \frac{P_S \frac{Q_0}{Q_S} Q_0 Q_r^2 Q_1 k_{0r}^2 k_{1r}^2}{4 \left(1 + \frac{Q_0}{Q_S} + Q_0 Q_r k_{0r}^2\right) \left[\left(1 + \frac{Q_0}{Q_S}\right) \left(1 + Q_r Q_1 k_{1r}^2\right) + Q_0 Q_r k_{0r}^2\right]}. \quad (12)$$

$$\left(\frac{Q_0}{Q_S Q_r k_{0r}^2} + Q_0 + \underbrace{\frac{1}{Q_r k_{0r}^2}}_{\text{negligible}}\right) \left[\left(1 + \frac{Q_0}{Q_S}\right) \left(Q_1 + \underbrace{\frac{1}{Q_r k_{1r}^2}}_{\text{negligible}}\right) + Q_0 \frac{k_{0r}^2}{k_{1r}^2}\right] < \left(1 + \frac{Q_0}{Q_S}\right) \left(\frac{Q_0}{Q_S k_{01}^2} + Q_0 Q_1 + \underbrace{\frac{1}{k_{01}^2}}_{\text{negligible}}\right) \quad (15)$$

$$\left(\frac{Q_S Q_0}{Q_S + Q_0}\right) \left(Q_S \frac{k_{0r}^2}{k_{1r}^2} + \frac{1}{Q_r k_{1r}^2}\right) + \frac{Q_1}{Q_r k_{0r}^2} < \frac{1}{k_{01}^2}. \quad (16)$$

$$\begin{aligned} \eta &= \frac{P_{out}}{P_{in}} \\ &= \frac{\frac{Q_1}{Q_L} Q_0 Q_r^2 Q_1 k_{0r}^2 k_{1r}^2}{\left(1 + Q_r Q_1 k_{1r}^2 + \frac{Q_1}{Q_L}\right) \left[\left(1 + \frac{Q_0}{Q_S}\right) \left(1 + Q_r Q_1 k_{1r}^2 + \frac{Q_1}{Q_L}\right) + \left(1 + \frac{Q_1}{Q_L}\right) Q_0 Q_r k_{0r}^2\right]}. \end{aligned} \quad (17)$$

from $\frac{\partial \eta}{\partial Q_L} = 0$, as follows.

$$Q_L^{\eta^*} = \frac{Q_1 \sqrt{1 + \frac{Q_0}{Q_S} + Q_0 Q_r k_{0r}^2}}{\sqrt{(1 + Q_r Q_1 k_{1r}^2) \left[(1 + Q_r Q_1 k_{1r}^2) \left(1 + \frac{Q_0}{Q_S} \right) + Q_0 Q_r k_{0r}^2 \right]}} \quad (18)$$

The optimal load resistance that corresponds to $Q_L^{\eta^*}$ can also be derived as

$$R_L^{\eta^*} = \frac{r_1 \sqrt{(1 + Q_r Q_1 k_{1r}^2) \left[(1 + Q_r Q_1 k_{1r}^2) \left(1 + \frac{Q_0}{Q_S} \right) + Q_0 Q_r k_{0r}^2 \right]}}{\sqrt{1 + \frac{Q_0}{Q_S} + Q_0 Q_r k_{0r}^2}} \quad (19)$$

Similar to (11), $R_L^{\eta^*}$ is proportional to k_{0r} and k_{1r} such that $R_L^{\eta^*}$ decreases as the level of misalignment in the resonators increases. By substituting (18) into (17), the achievable efficiency, which is denoted as η^* , can be found as (20), shown at the bottom of the page. Note that η^* can also be improved using the same conditions as for P_{out}^* except that it is not affected by P_S .

A similar approach can be used to show the optimal load resistance and achievable efficiency for the two-coil system as follows.

$$R_{L,2c}^{\eta^*} = \frac{r_1 \sqrt{\left(1 + \frac{Q_0}{Q_S} + Q_0 Q_1 k_{01}^2 \right)}}{\sqrt{\left(1 + \frac{Q_0}{Q_S} \right)}}, \quad (21)$$

$$\eta_{2c}^* = \frac{Q_0 Q_1 k_{01}^2}{\left(\sqrt{1 + \frac{Q_0}{Q_S}} + \sqrt{1 + \frac{Q_0}{Q_S} + Q_0 Q_1 k_{01}^2} \right)^2}. \quad (22)$$

By comparing η^* with η_{2c}^* , the activation condition of the relay for the achievable efficiency can be obtained as shown in (24). Similar to (15), $\frac{1}{k_{01}^2}$, $\frac{1}{Q_r k_{0r}^2}$, $\frac{1}{Q_r k_{1r}^2}$, and $\frac{Q_0}{Q_r^2 k_{1r}^2}$ in (23), as shown at the bottom of the page, become negligibly small in the strongly coupled region. Note that the condition, $\frac{Q_0}{Q_S^2 k_{ij}^2} \gg \frac{1}{k_{ij}^2}$ for $(i, j) \in \{0, r, 1\}$, also holds because $R_S \gg r_0$ in most of WPT environments [7], [9], [12], [23], [24]. If (24), as shown at the bottom of the page, is satisfied, it is better to activate the relay in order to improve the achievable efficiency; otherwise, the relay must be deactivated to reach the same end. In view of implementation, the method proposed in [14] that has automatic feedback control and communication systems can be adopted, such that the Tx and Rx send the information on coupling coefficient and quality factor through the communication system, and the relay can decide whether to deactivate the relay function using (24) based on this received information.

From the fact that the activation condition for P_{out}^* is tighter than that for η^* , i.e., $\left(\frac{Q_S Q_0}{Q_S + Q_0} \right) \left(Q_S \frac{k_{0r}^2}{k_{1r}^2} + \frac{1}{Q_r k_{1r}^2} \right) + \frac{Q_1}{Q_r k_{0r}^2}$ in (16) is always larger than $\frac{Q_1}{Q_r k_{0r}^2}$ in (24), we make the following remark, as a conjecture.

Remark 2 (Vulnerability of P_{out}^ to the Misalignment of the Relay):* P_{out}^* is more vulnerable to the misalignment of the relay than η^* . Therefore, it is necessary to deactivate the relay at a lower degree of misalignment to avoid the degradation of P_{out}^* , compared to η^* .

$$\eta^* = \frac{Q_0 Q_r^2 Q_1 k_{0r}^2 k_{1r}^2}{\left[\sqrt{\left(1 + \frac{Q_0}{Q_S} + Q_0 Q_r k_{0r}^2 \right) \left(1 + Q_r Q_1 k_{1r}^2 \right)} + \sqrt{\left(1 + \frac{Q_0}{Q_S} \right) \left(1 + Q_r Q_1 k_{1r}^2 \right) + Q_0 Q_r k_{0r}^2} \right]^2}. \quad (20)$$

$$\begin{aligned} \eta^* &> \eta_{2c}^* \\ &\times \left[\sqrt{\left(\underbrace{\frac{1}{Q_r k_{0r}^2}}_{\text{negligible}} + \frac{Q_0}{Q_S Q_r k_{0r}^2} + Q_0 \right) \left(\underbrace{\frac{1}{Q_r k_{1r}^2}}_{\text{negligible}} + Q_1 \right)} + \sqrt{\left(\underbrace{\frac{1}{Q_r k_{0r}^2}}_{\text{negligible}} + \frac{Q_0}{Q_S Q_r k_{0r}^2} \right) \left(\underbrace{\frac{1}{Q_r k_{1r}^2}}_{\text{negligible}} + Q_1 \right) + \frac{Q_0}{\underbrace{Q_r k_{1r}^2}_{\text{negligible}}}} \right]^2 \\ &< \left[\sqrt{\underbrace{\frac{1}{k_{01}^2}}_{\text{negligible}} + \frac{Q_0}{Q_S k_{01}^2}} + \sqrt{\underbrace{\frac{1}{k_{01}^2}}_{\text{negligible}} + \frac{Q_0}{Q_S k_{01}^2} + Q_0 Q_1} \right]^2 \end{aligned} \quad (23)$$

$$\frac{Q_1}{Q_r k_{0r}^2} < \frac{1}{k_{01}^2}. \quad (24)$$

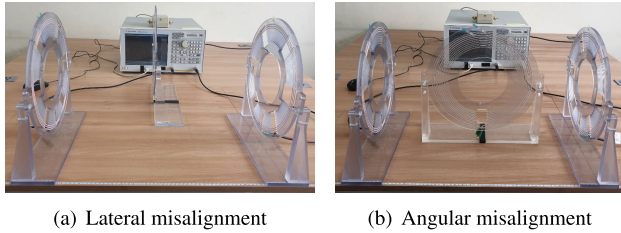


FIGURE 2. Experimental setup.

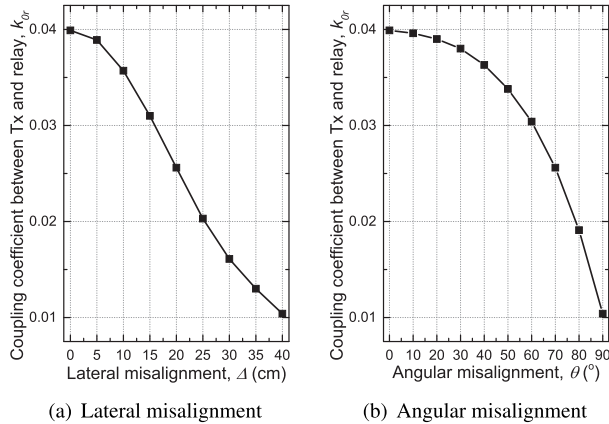


FIGURE 3. Coupling coefficient between Tx and relay vs. degree of misalignment.

IV. PERFORMANCE EVALUATION AND DISCUSSION

In order to validate our results experimentally, we fabricated resonators with a spiral shape using Litz wire. All resonators have an identical structure with the following specifications measured by VNA: $\alpha_i = 15$ cm, $\rho_i = 0.5$ cm, $\tau_i = 3$, $V_S = 10$ V, $R_S = 10 \Omega$, $L_i = 7.35$ μ H, $C_i = 74.97$ pF, $r_i = 2.7 \Omega$, $f_o = 6.78$ MHz, $Q_S = 31.31$, and $Q_i = 115.96$, where $i \in \{0, r, 1\}$. We set $d_{01} = 60$ cm, in which the corresponding $k_{01} = 0.0136$, and the relay was positioned in the middle of d_{01} .³ As shown in Fig. 2, the relay was moved along the y-axis for a distance of up to 40 cm with angular alignment to model lateral misalignment, or alternatively rotated up to 90 degrees with lateral alignment to model angular misalignment. Fig. 3 shows the coupling coefficient between Tx and relay (k_{0r}) versus the degree of misalignment (Δ or θ) for (a) lateral misalignment and (b) angular misalignment. As expected, the coupling coefficient shows a marked decrease as the degree of misalignment is increased in both cases. Given that it is difficult to vary the load resistance and voltage source in experiments using VNA, the results for Figs. 4-6 are obtained by simulations using ADS while that for Fig. 7 is measured by experiments using VNA. It should be noted that the results for Figs. 4-6 contain the physical characteristics of fabricated resonators because all the measured parameters for the resonators including the coupling coefficients are used for

³It was revealed that the optimal position of the relay is near the center of the Tx and Rx if all resonators have the identical structure [24].

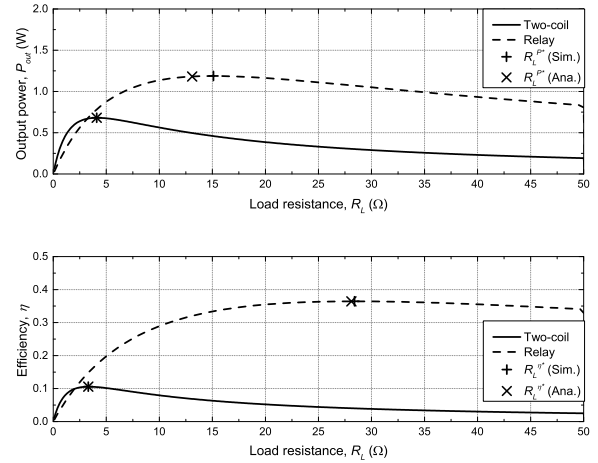


FIGURE 4. Output power and efficiency vs. load resistance.

ADS simulations. In the following, we validate the exactness of our analysis by comparing the analytical results (Ana.) with the simulation (Sim.) and experimental (Exp.) results.

Fig. 4 shows the output power (P_{out}) and efficiency (η) versus the load resistance (R_L) with perfect alignment, i.e., $\Delta = 0$ cm and $\theta = 0^\circ$. The result reveals that both output power and efficiency are concave with respect to the load resistance, implying that there exists an optimal value of R_L for maximizing P_{out} and η . From the fact that the relay system shows a more gentle curve of P_{out} and η against R_L than that obtained for the two-coil system, we note that the use of the relay ensures a stable performance according to the variation in R_L . In addition, we also note that the optimal R_L to maximize η ($R_L^{\eta*}$) is almost identical between the simulation result and the analysis, which justifies our approach. Although the optimal R_L to maximize P_{out} (R_L^{P*}) is different between simulation and analysis, the difference is slight. We provide detailed reasoning for why the gap for optimal R_L is greater for maximizing P_{out} through the following result.

Fig. 5 shows the optimal load resistance (R_L^*) for the relay system versus the degree of (a) lateral misalignment (Δ) and (b) angular misalignment (θ). As analyzed in (11) and (19), R_L^{P*} and $R_L^{\eta*}$ decrease as the level of misalignment, i.e., Δ and θ , increases. The assumption of negligible k_{01} causes some errors when deriving the analytical results of I_0 and I_1 , because the cross-coupling, k_{01} , actually affects the currents in both Tx and Rx, i.e., I_0 and I_1 . However, these errors may be reduced in the analysis of efficiency because η^* is determined by the ratio of I_0 and I_1 . Meanwhile, the error in the analysis of I_1 has a severe effect on P_{out}^* , and as a result there is a relatively large difference in R_L^{P*} between the simulation and analytical results. However, they have the same tendency as the degree of misalignment is varied.

The general performance evaluations taking into account the effects of misaligned relay between Tx and Rx on

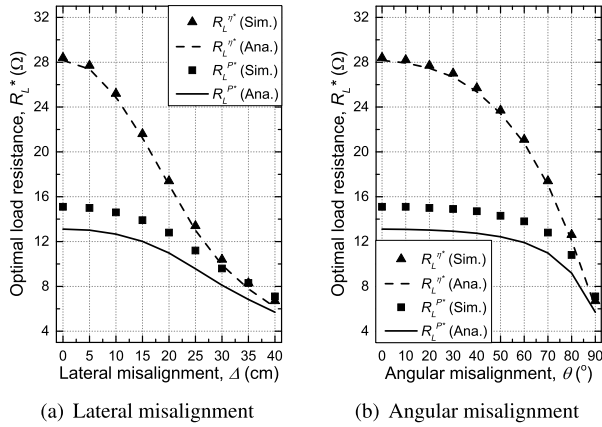


FIGURE 5. Optimal load resistance for the relay system vs. degree of misalignment.

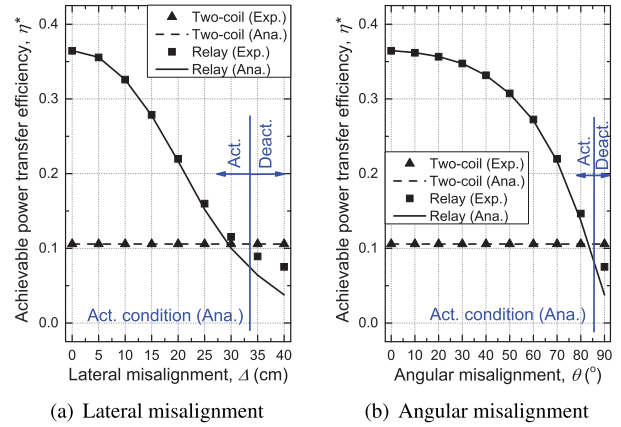


FIGURE 7. Achievable efficiency vs. degree of misalignment.

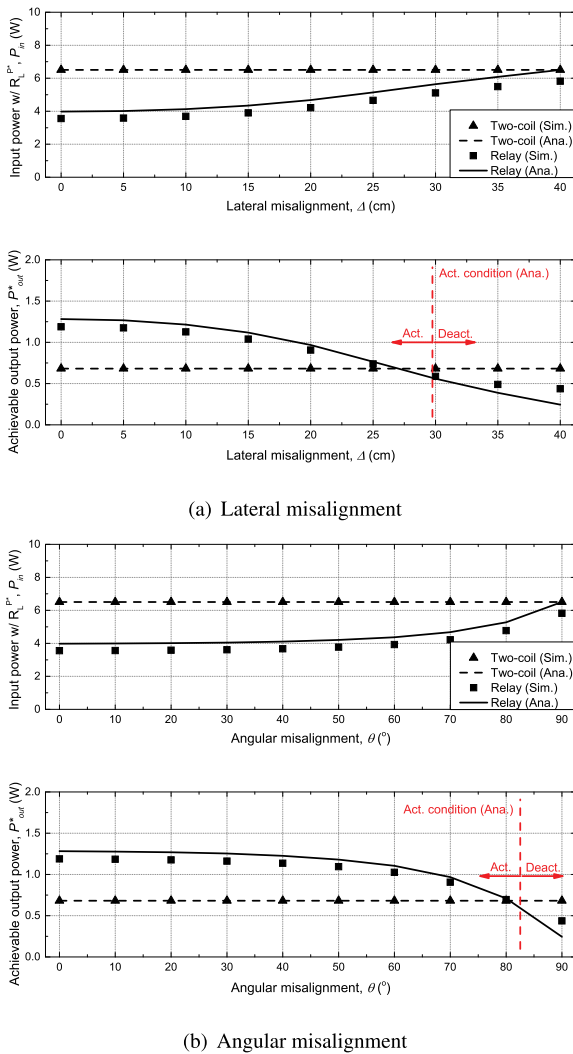


FIGURE 6. Input power with R_L^{P*} and achievable output power vs. degree of misalignment.

output power and efficiency are discussed in Figs. 6 and 7. In particular, Fig. 6 shows the input power (P_{in}) with R_L^{P*} and achievable output power (P_{out}^*) versus the degree

of (a) lateral misalignment (Δ) and (b) angular misalignment (θ). Although the input power of the relay system increases with the degree of misalignment, it is always lower than that of the two-coil system in both cases, which indicates that the use of relay reduces input power. In the result of P_{out}^* , the red dotted line indicates the boundary that determines whether to activate the relay or not to enhance P_{out}^* , which was found analytically in (16). As can be seen from the result, the relay system provides a lower achievable output power than the two-coil system when the degree of misalignment exceeds this activation line, e.g., $\Delta > 27$ cm for lateral misalignment and $\theta > 80^\circ$ for angular misalignment. This result demonstrates the accuracy of our analysis and implies that it is beneficial to deactivate the relay when it is misaligned significantly with Tx and Rx in terms of output power, as per remark 1.

Fig. 7 shows the achievable efficiency (η^*) versus the degree of (a) lateral misalignment (Δ) and (b) angular misalignment (θ).⁴ Here, the blue line represents the analytical activation condition for η^* , which was derived in (24). From the experimental results, we can show that the achievable efficiency of the relay system becomes lower than that of the two-coil system when the degree of misalignment exceeds this activation condition in both cases, i.e., $\Delta > 30$ cm for lateral misalignment and $\theta > 80^\circ$ for angular misalignment. This result confirms that the range of misalignment considered in our experiments is sufficient to show the effect of the misaligned relay on efficiency because a reversal of performance between two-coil and relay systems is observed. From the results of Figs. 6 and 7, we also find that a reversal of performance in P_{out}^* between the relay and two-coil systems occurs at a lower degree of misalignment compared to η^* , which is consistent with remark 2. In addition, at greater degree of misalignment, the cross-coupling is no longer much smaller than the coupling between adjacent resonators, which

⁴It should noted that the achievable output power and efficiency are measured at $d_{01} = 60$ cm, which is twice the diameter of the resonator. Because of such a long measuring distance, it shows inherently low performances as the degree of misalignment increases.

violates the assumption that cross-coupling can be neglected in our analysis. As a result, there is a slight difference in both P_{out}^* and η^* between the experimental and analytical results for the relay system when $\Delta > 30$ cm for lateral misalignment and $\theta > 80^\circ$ for angular misalignment. However, these values show good general agreement with each other.

V. CONCLUSION

In considering the influence of a relay that has lateral or angular misalignment on output power and efficiency, we employed an ECM to derive the closed-form equations for achievable output power and efficiency with optimal load resistances. By comparing the performances of a relay system with those of a two-coil system, we have revealed analytically that 1) it is beneficial for improving the achievable output power and efficiency to deactivate the relay if the activation condition is not satisfied as a result of severe misalignment, and 2) the misalignment of the relay has a greater adverse impact on output power than it does on efficiency. The accuracy of our analysis was also demonstrated experimentally for a variety of cases. We expect our study to provide insightful information for determining the activation of a relay, thereby minimizing the use of key resources in mid-range WPT applications.

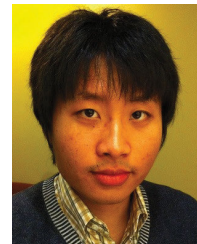
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