Quantum Fisher Information of a Teleported State in Heisenberg XYZ Chain With Magnetic Field and Kaplan–Shekhtman–Entin-Wohlman–Aharony Interaction

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ABSTRACT One of the teleportation versions is applied to convey an entangled two-qubit system via a class of a two-qubit XYZ Heisenberg chain model. At thermal equilibrium, the exact solution of the XYZ model with an external magnetic field and the Kaplan–Shekhtman–Entin-Wohlman–Aharony interaction (KSEWA-interaction) is proposed in X, and Z direction. The teleported quantum Fisher information is studied, where we estimate the weight parameter θ, temperature and the strength parameter of KSEWA-interaction. It is found that, the strength of the KSEWA-interaction and the magnetic field be of benefit in the case of estimating the weight function. Meanwhile, the temperature decreases the teleported information.

INDEX TERMS Teleportation, KSEWA-interaction, quantum Fisher information, XYZ-Heisenberg model, thermal equilibrium.

I. INTRODUCTION

Quantum teleportation plays a significant role in the quantum communication theory. It allows the transfer of an unknown entangled state between users via many protocols, and it has been proposed by Bennett *et al.* [1]. Recently, quantum teleportation has very popular in the scientific community for its study, whether theoretically or experimentally [2]. Hence, several protocols have emerged to study the teleportation. For instance, Bowen and Bose reconstructed a protocol (T_0) using the Bell states, Pauli matrices, and an arbitrary two-qubit as a resource [3]. By doubling protocol T_0, Lee and Kim have discussed a protocol to transmit an entangled two-qubit system [4]. Moreover, quantum teleportation is investigated using several channels as, thermal entangled states [5]–[7], the cavity of quantum electrodynamics [8], [9], and the relativistic accelerated frame [10]–[12]. Via the thermal mixed state in *XXX* Heisenberg chain models, the quantum

entanglement teleportation and fidelity are investigated [13]. Through the entanglement, the usefulness of a two-qubit XY model affected by an external magnetic field on the standard teleportation protocol has studied [14]. Also, the thermal entanglement and teleportation of XXZ chain with different Dzyaloshinskii–Moriya (DM) interaction have discussed [15]. On the other hand, the entanglement control in an anisotropic two-qubit Heisenberg XYZ model with external magnetic fields and DM-interaction are investigated [16], [17]. Necessary and sufficient conditions for local creation of quantum correlation is obtained [18]. However, the evolution equation of the quantum entanglement for bipartite systems is studied [19], and the dependence of spin squeezing in a generalized one-axis twisting model is investigated [20]. The orthogonality time of two qubit XX chain model in the presence of DM-interaction is examined for different initial states [21].

Furthermore, the quantum Fisher information (QFI) play a role in estimating information for any parameter in the quantum system [22]. The QFI is applied as an estimator

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in many quantum systems as QED, non-inertial frame, thermal nonequilibrium [23]–[25]. The dynamics of QFI under two different decoherence channels for a single qubit in terms of the Bloch representation is investigated [26]. The effects of vacuum fluctuations on the teleported QFI of the phase parameter of the atomic state is discussed [27]. The QFI and teleportation in the Ising-Heisenberg chain of a heterotrimetallic coordination compound Fe-Mn-Cu with non-uniform magnetic fields is studied [28]. The dynamics of QFI and nonequilibrium thermal quantum correlation in Heisenberg chain models is investigated [25], [29]. Via the partial measurements as a quantum technique, the QFI of two schemes of teleportation for a bipartite system under the amplitude damping noise channel is enhanced [30]. While, the effect of the DM-interaction in the presence of the intrinsic phase decoherence on the teleportation via a two-qubit Heisenberg XYZ chain model is investigated [31], [32]. The three direction of DM-interaction in the presence of XYZ chain model is used to generate entangled network from partial entangled-state [33].

Our motivation is applying the Lee and Kim scheme to teleport an entangled state from lab Alice to Bob. We use two copies of a two-qubit Heisenberg XYZ chain model with an external magnetic field and KSEWA-interaction as a quantum channel. The KSEWA-interaction restrain the local minimum of quantum correlation [34]. Where, KSEWA-interaction is a symmetric exchange interaction, this interaction can be unpretentious with time growing compared with the antisymmetric DM-interaction [35]. Kaplan [36], and Shekhtman *et al.* [37], [38] are debated the significance of the symmetric KSEWA-interaction, where it can reconstruct in O(3) of the isotropic Heisenberg system which is not achieved by the DM-interaction. Hence, we use the KSEWA-interaction, and study the effect of the symmetric interaction in the presence of the external field on the teleportation. Moreover, The QFI is employed to quantify the amount of the teleported information in the existence of the external parameters and the wight function. The paper is organized as: In the following section we defined the mathematical form of the QFI, and shown how to apply the scheme of Lee and Kim. Sec. (III) is devoted to proposing the solution of the system in X-direction and Z-direction, also we get the final output state after the teleportation process for the unknown state in the channel of XYZ state. As well, we estimated the parameter of the weight function, the strength of the KSEWA-interaction, and the temperature via the QFI. Finally, we deduced our summary.

II. PRELIMINARIES
In this section, we briefly introduce the mathematical form of quantum Fisher information and teleportation protocol that we use in this work.

A. QUANTUM FISHER INFORMATION
The quantum Fisher information (QFI) is employed to estimate the information resulting from the quantum system.

It gives an easy way to esteem the parameters that created the quantum system. The general form of QFI to estimate the parameter ϵ is given by [22]:

$$\mathcal{F}_\epsilon = Tr[\partial_\epsilon \rho_\epsilon L_\epsilon] \tag{1}$$

where $2\partial_\epsilon \rho_\epsilon = \{\rho_\epsilon, L_\epsilon\}$, and L_ϵ is the symmetric logarithmic derivative. By the diagonalization of the density matrix $\rho_\epsilon = \sum_i \chi_i |\Psi_i\rangle\langle\Psi_i|$, the QFI is reformulated as [39]:

$$\mathcal{F}_\epsilon = \sum_{i=1}^n \frac{(\partial_\epsilon \chi_i)^2}{\chi_i} - \sum_{i\neq m}^n \frac{8\chi_i\chi_m}{\chi_i + \chi_m}|\langle\Psi_i|\partial_\epsilon\Psi_m\rangle|^2$$
$$+ 4\sum_{i=1}^n \chi_i(\langle\partial_\epsilon\Psi_i|\partial_\epsilon\Psi_i\rangle - |\langle\Psi_i|\partial_\epsilon\Psi_i\rangle|^2), \tag{2}$$

where $\chi_i \neq 0$ and $\chi_i + \chi_m \neq 0$. In the following, we quantify the QFI of the two-qubit system to esteem the parameters in the present system.

B. TELEPORTATION PROCESS
The main idea of quantum state teleportation is transferring an unknown quantum state between two users. Lee and Kim have studied teleportation via two independent quantum channels of an unknown entangled state [4], [40]. We consider the input state $|\psi_{in}\rangle = \cos\frac{\theta}{2}|00\rangle + \sin\frac{\theta}{2}|11\rangle, 0 \leq \theta \leq \pi$, as that Alice want to send it into his partner Bob. The final teleported state can be obtained by [4]:

$$\hat{\rho}_{out} = \sum_{n,m} P_{nm}(\sigma_n \otimes \sigma_m)\hat{\rho}_{in}(\sigma_n \otimes \sigma_m), n, m = 0, 1, 2, 3 \tag{3}$$

where $P_{nm} = Tr(K_n\hat{\rho}_{ab})Tr(K_m\hat{\rho}_{ab})$, with the operator $\hat{\rho}_{ab}$ is represented an arbitrary channel that generated from our solving of a two-qubit XYZ-Heisenberg model. σ_i are the Pauli Matrices, while the operators K_i in terms of Bell states are defined as,

$$K_{1,2} = |\Phi^\mp\rangle\langle\Phi^\mp|, \quad K_{0,3} = |\Psi^\mp\rangle\langle\Psi^\mp|,$$
$$|\Psi^\pm\rangle = \frac{|10\rangle \pm |01\rangle}{\sqrt{2}}, \quad |\Phi^\pm\rangle = \frac{|11\rangle \pm |00\rangle}{\sqrt{2}}. \tag{4}$$

The protocol considered that Alice teleported an initial state $\rho_{in} = |\psi_{in}\rangle\langle\psi_{in}|$ to Bob, where the two partners are shared the states $\rho_{34}(T)$, and $\rho_{56}(T)$ as a channel. Let the particles 3,5 are given to Alice, while 4,6 are shared by Bob. The particles 1,3 and 2,5 are interacting with the source. As a result of the interaction of 3 with 4, and 5 with 6, we can transform 1 with 4 and 2 with 6 respectively. Fig.(1) shows how we can teleport the initial state between two labs.

III. PHYSICAL MODEL
The mathematical description of the Hamiltonian H of two-qubit (A and B) Heisenberg XYZ chain model influenced by the KSEWA-interaction [41], and an external magnetic field is given by:

$$\hat{H} = J_x\hat{S}_x^A\hat{S}_x^B + J_y\hat{S}_y^A\hat{S}_y^B + J_z\hat{S}_z^A\hat{S}_z^B + M\hat{S}^A + M\hat{S}^B + \hat{S}^A.\Gamma.\hat{S}^B, \tag{5}$$
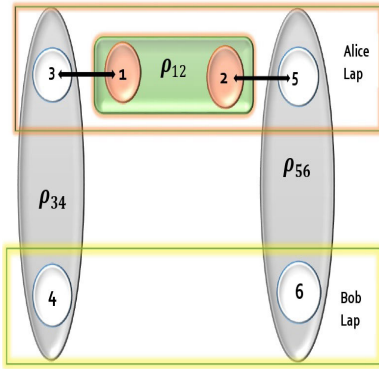
where, J_i are the real exchange coupling between the two qubits in the x,y, and z-direction, if $J_i > 0$ the natural system in antiferromagnetic behaviour, while it in the ferromagnetic for $J_i < 0$. The vector $\hat{S}^i = \{\hat{S}^i_x, \hat{S}^i_y, \hat{S}^i_z\}$, $i = A, B$ described via the Pauli matrices. M, is a vector in x, y, and z-direction, that represented the strength of the magnetic field. While, Γ is a symmetric traceless tensor, it is defined by [34].

$$\Gamma = \begin{pmatrix} 0 & \Gamma_z & \Gamma_y \\ \Gamma_z & 0 & \Gamma_x \\ \Gamma_y & \Gamma_x & 0 \end{pmatrix} \quad (6)$$

At thermal equilibrium, the density operator in terms of eigenvalues and its corresponding eigenstates is given by:

$$\rho(T) = \sum_i \lambda_i |\psi_i\rangle \langle\psi_i|, \quad \text{with the probability} \quad \lambda_i = \frac{1}{Z} e^{\frac{-E_i}{kT}}, \quad (7)$$

where, E_i are the eigenvalues with the related eigenstates $Z = |\psi_i\rangle$. Z is the partition function with $\sum_i e^{\frac{-E_i}{kT}}$. The parameter T, and k are the temperature, and Boltzmann's constant, respectively.

A. THE MAGNETIC FIELD AND KSEWA-INTERACTION IN X-DIRECTION

Let us assume that, the strength of the coupling magnetic field and KSEWA-interaction are in the X-direction. So, the Hamiltonian is obtained by:

$$\hat{H} = J_x \hat{S}^A_x \hat{S}^B_x + J_y \hat{S}^A_y \hat{S}^B_y + J_z \hat{S}^A_z \hat{S}^B_z + M_x \hat{S}^A_x \\ + M_x \hat{S}^B_x + \Gamma_x.(\hat{S}^A_y \hat{S}^B_z + \hat{S}^A_z \hat{S}^B_y), \quad (8)$$
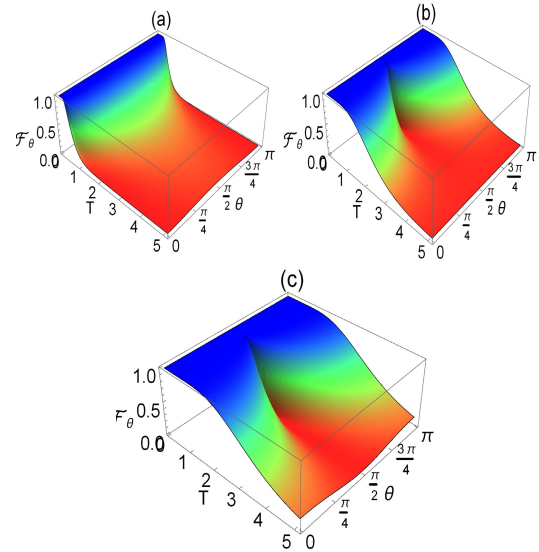
The energy spectrum via the eigenvalues are calculated as;

$$E^x_{1,2} = -J_x \mp (J_y + J_z), \qquad E^x_{3,4} = J_x \mp w^x.$$
$$\text{with, } w^x = \sqrt{4(M_x^2 + \Gamma_x^2) + (J_y - J_z)^2}. \quad (9)$$

The corresponding calculation of the eigenstates are given by,

$$|\psi^x_1\rangle = -|\Psi^-\rangle, \qquad |\psi^x_2\rangle = -|\Phi^-\rangle,$$
$$|\psi^x_3\rangle = \sin\theta_1 |\Phi^+\rangle + \eta\cos\theta_1 |\Psi^+\rangle,$$
$$|\psi^x_4\rangle = \sin\theta_2 |\Phi^+\rangle + \eta\cos\theta_2 |\Psi^+\rangle,$$

FIGURE 2. The behavior of $\mathcal{F}_\theta$ in X- direction, with $J_x = 0.2, J_y = 0.5$ and $J_z = 0.9$. (a) $\Gamma_x = 0 = M_x$, (b) $\Gamma_x = 4$, $M_x = 0$, (c) $\Gamma_x = 4$, $M_x = 4$.

here $\tan\theta_{1,2} = \frac{2\sqrt{M_x^2 + \Gamma_x^2}}{J_y - J_z \mp w^x}$, and the complex parameter $\eta = \frac{M_x + i\Gamma_x}{\sqrt{M_x^2 + \Gamma_x^2}}$, while $|\Phi^\pm\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$, $|\Psi^\pm\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$.

The thermal equilibrium density operator (7) for this case can be directly written as;

$$\hat{\rho}^x(T) = \frac{1}{Z} \begin{pmatrix} A^x_+ & C^x & C^x & A^x_- \\ C^{x*} & B^x_+ & B^x_- & C^{x*} \\ C^{x*} & B^x_- & B^x_+ & C^{x*} \\ A^x_- & C^x & C^x & A^x_+ \end{pmatrix} \quad (10)$$

where,

$$A^x_\pm = \frac{1}{2}\left(\pm e^{\frac{-E_2}{kT}} + \sin^2\theta_1 e^{\frac{-E_3}{kT}} + \sin^2\theta_2 e^{\frac{-E_4}{kT}} \right),$$
$$B^x_\pm = \frac{1}{2}\left(\pm e^{\frac{-E_1}{kT}} + \cos^2\theta_1 e^{\frac{-E_3}{kT}} + \cos^2\theta_2 e^{\frac{-E_4}{kT}} \right),$$
$$C^x = \frac{\eta^*}{4}\left(\sin 2\theta_1 e^{\frac{-E_3}{kT}} + \sin 2\theta_2 e^{\frac{-E_4}{kT}} \right).$$

By applying the teleportation process (3), we can get the final output state,

$$\hat{\rho}^x_{out}(T) = \frac{4}{Z^2}\left\{ \left((A^x_+ \sin\frac{\theta}{2})^2 + (B^x_+ \cos\frac{\theta}{2})^2\right)|00\rangle\langle 00| + \right.$$
$$+ A^x_+ B^x_+(|01\rangle\langle 01| + |10\rangle\langle 10|) + \left((A^x_+ \cos\frac{\theta}{2})^2\right.$$
$$\left. + (B^x_+ \sin\frac{\theta}{2})^2\right)|11\rangle\langle 11| + \left[((A^x_-)^2 + (B^x_-)^2)\right.$$
$$\left.\left. \times \sin\theta|00\rangle\langle 11| + A^x_- B^x_- \sin\theta|01\rangle\langle 10| + h.c.\right]\right\} \quad (11)$$

In Fig.(2) we estimate the weight function θ via the QFI, where we consider the magnetic and KSEWA-interaction in X-direction. It is clear that, the behavior of $\mathcal{F}_\theta$ depends on the temperature parameter T and the weight θ. In absence of the magnetic and KSEWA-interaction Fig.(2.a) shows

that, by increasing the temperature, the Fisher information is rapidly vanished and the information remains slightly dependent on θ. The behaviour of the function $\mathcal{F}_\theta$ in the presence of the KSEWA-interaction ($\Gamma_x = 4$) is displayed in Fig.(2.b), where the information area is limited at any $T < 4.5$, and the maximum values of QFI are exhibited at the initial T. Moreover, the function is suddenly damping at $\theta = \pi/2$. By adding the external magnetic field and the KSEWA-interaction ($M_x = 4$, ($\Gamma_x = 4$)) to the quantum system, Fig.(2.c) depicts the QFI is improved. However, a sudden collapse of the estimation degree is generated at $\theta = \frac{\pi}{2}$. In general, the temperature is damped the information, while the teleported information improved by taken the parameters Γ_x, and M_x into account. Physically, on one hand, the interaction parameters M_x, and Γx may slow the decrease rate of the entanglement by raising the critical temperature, which mean the teleported system converted from a separable state to a partial entangled state. On the other hand, the unknown state cannot be teleported in a thermal environment beside M_x, and Γx at $\theta = \pi/2$, where the maximum entangled state turns into a separable state.

The effect of temperature on the teleported state is estimated in Fig.(3), where the output state ρ_{out} is subject to the same conditions as in the previous figure. Fig.(3.a) shows that, the function $\mathcal{F}_T$ has a maximum estimation degree after the onset of the interaction period, then it suddenly has no effect on the teleported state. Different behaviour is demonstrated in Fig.(3.b), where we set $\Gamma_x = 4$, $M_x = 0$. The maximum bounds of $\mathcal{F}_T$ are centered around $\theta = \frac{\pi}{2}$ and $T = 2$. Moreover, by adding the magnetic field with $M_x = 4$ the maximum bounds of $\mathcal{F}_T$ decreases decrease to 0.1. In general, the effect of temperature on the teleported system is limited at $T > 0$, and it decreases as Γ_x and M_x are increasing. Physically, the effect of temperature is after the onset interaction, which means that the temperature separates the state and the external parameters are slowing the decoherence. Hence,
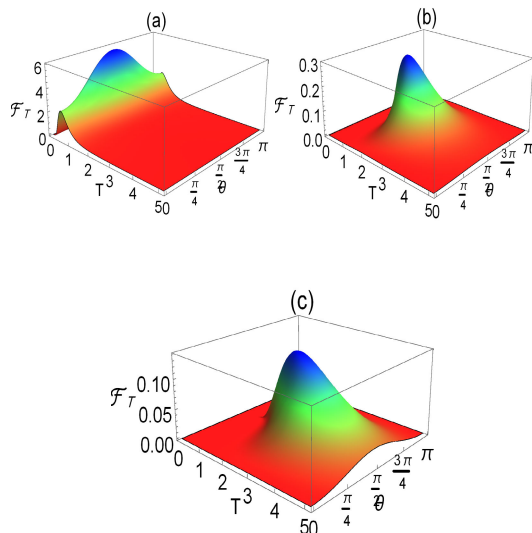
the damage caused by the temperature can be controlled via the magnetic field and KSEWA-interaction.

Fig(4) estimates the parameter Γ_x to quantify the efficiency of KSEWA-interaction on the teleportation process. It depicts that, the general behaviour of $\mathcal{F}_{\Gamma_x}$ with different values of T and M_x is similar to that displayed in Fig.(3). As well, the estimation degree is predicted at the same intervals of θ and Γ_x. However, the maximum bounds in Fig.(3) is greater than that shown in Fig(4). Also, a squeezing peak is generated in the presence of M_x, it decreases by increasing temperature.
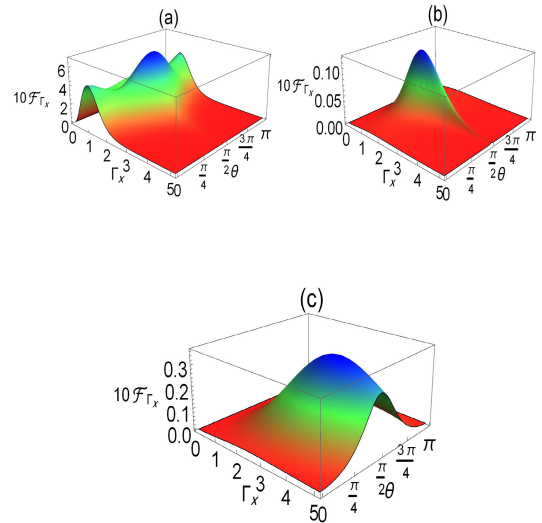

FIGURE 4. The behavior of $\mathcal{F}_{\Gamma_x}$ in X- direction, with $J_x = 0.2$, $J_y = 0.5$ and $J_z = 0.9$. (a) $T = 1$, $M_x = 0$, (b) $T = 1$, $M_x = 3$, (c) $T = 3$, $M_x = 3$.

B. THE MAGNETIC FIELD AND KSEWA-INTERACTION IN Z-DIRECTION

In Z-direction for the coupling magnetic field and KSEWA-interaction, the Hamiltonian can be written as:

$$\hat{H} = J_x \hat{S}_x^A \hat{S}_x^B + J_y \hat{S}_y^A \hat{S}_y^B + J_z \hat{S}_z^A \hat{S}_z^B + M_z \hat{S}_z^A + M_z \hat{S}_z^B + \Gamma_z(\hat{S}_x^A \hat{S}_y^B + \hat{S}_y^A \hat{S}_x^B), \quad (12)$$

The thermal equilibrium density operator (7) in Z-direction is calculated as;

$$\hat{\rho}(T) = \frac{1}{Z} \begin{pmatrix} A_{11} & 0 & 0 & A_{14} \\ 0 & A_{22} & A_{23} & 0 \\ 0 & A_{23} & A_{33} & 0 \\ A_{14}^* & 0 & 0 & A_{44} \end{pmatrix} \quad (13)$$

with the non-zero elements,

$$A_{11} = \sin^2 \phi_1\, e^{\frac{-E_3^z}{kT}} + \sin^2 \phi_2\, e^{\frac{-E_4^z}{kT}},$$

$$A_{22} = e^{\frac{J_z}{kT}} \cosh(\frac{J_x + J_y}{kT}) = A_{33},$$

$$A_{44} = \cos^2 \phi_1\, e^{\frac{-E_3^z}{kT}} + \cos^2 \phi_2\, e^{\frac{-E_4^z}{kT}},$$

$$A_{23} = -e^{\frac{J_z}{kT}} \sinh(\frac{J_x + J_y}{kT}) = A_{32}$$

$$A_{14} = \zeta \sin 2\phi_1\, e^{\frac{-E_3^z}{kT}} + \zeta \sin 2\phi_2 e^{\frac{-E_4^z}{kT}} = A_{41}^*$$

FIGURE 3. The behavior of $\mathcal{F}_T$ in X- direction, with $J_x = 0.2$, $J_y = 0.5$ and $J_z = 0.9$. (a) $\Gamma_x = 0 = M_x$, (b) $\Gamma_x = 4$, $M_x = 0$, (c) $\Gamma_x = 4$, $M_x = 4$.

Hear,

$$E_{1,2}^z = -J_z \mp (J_x + J_y), \quad E_{3,4}^z = J_z \mp w^z,$$

with, $w^z = \sqrt{4(M_z^2 + \Gamma_z^2) + (J_x - J_y)^2}$

where, $\tan \phi_{1,2} = \frac{2M_z - w^z}{4\Gamma_z^2 + (J_x - J_y)^2}$, and the complex parameter $\zeta = \frac{J_x - J_y - 2i\Gamma_z}{\sqrt{(J_x - J_y)^2 + 4\Gamma_z^2}}$.

According to the equation (3), the output teleported state in the computational basis is given by,

$$\hat{\rho}_{out}^z(T) = \frac{4}{Z^2} \left\{ \left(e^{\frac{-2J_z}{kT}} \cosh^2 \frac{w^z}{kT} \sin^2 \frac{\theta}{2} + A_{22}^2 \cos^2 \frac{\theta}{2} \right) |00\rangle \langle 00| \right.$$
$$+ e^{-\frac{J_z}{kT}} A_{22} \cosh\left(\frac{w^z}{kT}\right) \left(|01\rangle \langle 01| + |10\rangle \langle 10| \right)$$
$$+ \left(e^{\frac{-2J_z}{kT}} \cosh^2 \frac{w^z}{kT} \cos^2 \frac{\theta}{2} + A_{22}^2 \sin^2 \frac{\theta}{2} \right) |11\rangle \langle 11|$$
$$+ \sin\theta \left[\frac{1}{2} \left(\Re(A_{14})^2 + (A_{23})^2 \right) |00\rangle \langle 11| \right.$$
$$\left. \left. + A_{23} \Re(A_{14}) |01\rangle \langle 10| + h.c. \right] \right\} \quad (14)$$

Fig.(5) studies the effect of the parameters Γ_z and M_z on the estimation degree of the parameter θ in Z-direction, where we assume that the system prepared under the same conditions as that displayed in Fig.(2). In the absence of Γ_z and M_z, the function starts from the extreme state and continues for a small period, and it decreases gradually as the temperature T increases. The dependence on the weight function θ is weak compared to temperature T and the sudden death areas are exclusive at $\theta = 0$ and π. The parameter Γ_z leads to improve in $\mathcal{F}_\theta$. This improvement in the estimation degree is more pronounced after adding the external parameters Γ_z and M_z. In general, the temperature decreases the QFI of the weight function θ. Physically, it means that the initial entangled state suffering from decoherence. Nevertheless, the outcome of the
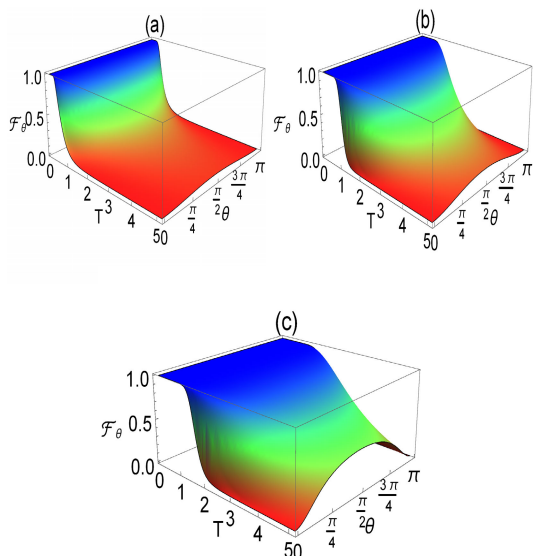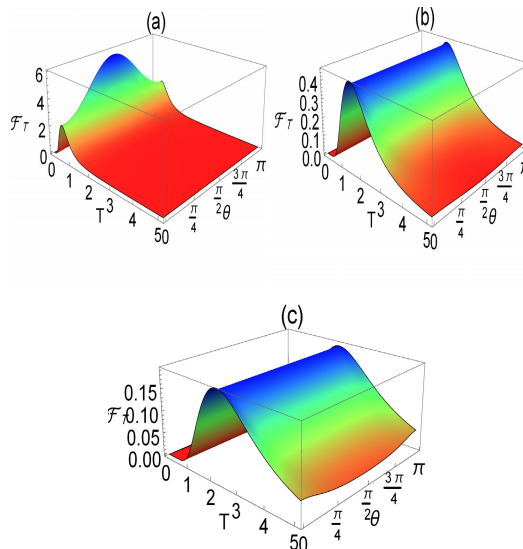

FIGURE 6. The behavior of $\mathcal{F}_T$ in Z- direction, with $J_x = 0.2$, $J_y = 0.5$ and $J_z = 0.9$. (a) $\Gamma_z = 0 = M_z$, (b) $\Gamma_z = 4$, $M_z = 0$, (c) $\Gamma_z = 4$, $M_z = 4$.
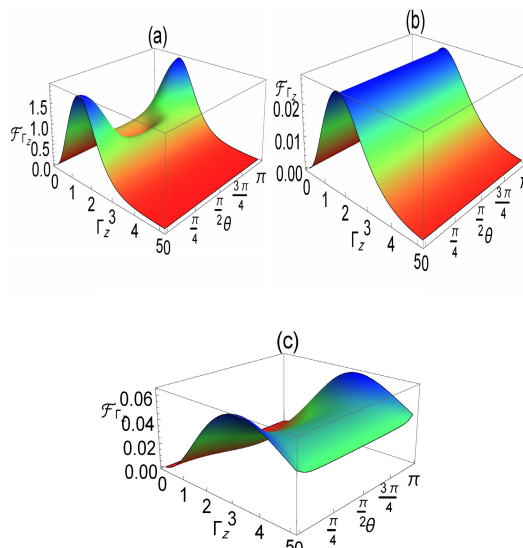

FIGURE 7. The behavior of $\mathcal{F}_{\Gamma_z}$ in Z- direction, with $J_x = 0.2$, $J_y = 0.5$ and $J_z = 0.9$. (a) $T = 1$, $M_z = 0$, (b) $T = 1$, $M_z = 3$, (c) $T = 3$, $M_z = 3$.

external interactions may be slowing down the decoherence and increasing the entanglement.

The numerical behaviour of $\mathcal{F}_T$ is displayed in Fig.(6). It shows that, the general effects of the temperature is an appearance at $T > 0$. In the absence of Γ_z and M_z, the behaviour of $\mathcal{F}_T$ is similar to that displayed in Fig.(3.a). Comparing with Fig.(6.b and c), the effect of T in Fig.(3.b and c) has a slight dependence on θ. But in the presence of the coupling strength of the two interactions, we can improve the generated decoherence areas by temperature.

Finally, Fig.(7) shows the estimation degree of the coupling strength of KSEWA-interaction in Z-direction. For small temperature $T = 1$, the function $\mathcal{F}_{\Gamma_z}$ starts from the minimum bounds and gradually increases with growing Γ_z, then it is decayed. The coupling strength of magnetic field $M_z = 3$ may play a control role to keep the survival of $\mathcal{F}_{\Gamma_z}$, however,

FIGURE 5. The behavior of $\mathcal{F}_\theta$ in Z- direction,, with $J_x = 0.2$, $J_y = 0.5$ and $J_z = 0.9$. (a) $\Gamma_z = 0 = M_z$, (b) $\Gamma_z = 4$, $M_z = 0$, (c) $\Gamma_z = 4$, $M_z = 4$.

it decreases the maximum bounds. According to the increasing in T with $M_z = 3$, the function $\mathcal{F}_{\Gamma_z}$ increasing and the decoherence areas disappeared.

IV. CONCLUSION

In this contribution, We discussed how the two copies of the thermal XYZ chain as a channel affected the quantum teleportation. The teleportation channel consists of a two-qubit system of XYZ chain model influenced by an external magnetic field and KSEWA-interaction in X and Z direction. The density operator of the channel system has been obtained analytically and the unknown state has been forced to pass inside the channel. The final output teleported state in X and Z direction have been obtained on the computational basis. The QFI has been used as an estimator of some parameters as the weight function of the initial state, the temperature and the coupling strength parameter of the KSEWA-interaction.

Our results show that, the weight function plays a central role in the behaviour of the QFI and the estimation degree. It was shown that, the possibility of estimating in Z direction is much better than that displayed in X direction. The degree of estimation decrease as the temperature increases, where the QFI in X direction suffers sudden death at $\theta = \pi/2$ with magnetic and KSEWA-interactions. However, the teleported state has an enormous estimation degree at zero temperature in X and Z direction. The effect of temperature on the degree of estimation in the presence of coupling strength of the magnetic and the KSEWA-interaction is discussed. The general behaviour of the QFI is shown that, the temperature has greatly estimation degree after zero temperature, it is suddenly decreased with growing the temperature. As well the strength of external interactions decreases the upper bounds of the temperature in X and Z directions, specially in X direction the QFI is limited around $\theta = \pi/2$. A similar effect is shown in the coupling strength of the KSEWA-interaction, where the QFI of the teleported system decreases as the coupling strength increases. At $\Gamma > 0$ the evolution of QFI in Z direction is better than that exhibited in X direction and the amplitudes of the upper bounds in X less than that displayed in Z direction.

In summary, to teleport the maximum entangled state via a channel of the XYZ thermal state, one has to increase the coupling strength of KSEWA-interaction and magnetic field in the Z-direction, as well setting fewer values of the temperature. By estimating the temperature, we found that it reduces the information. The KSEWA-interaction is very benefited to increase the information according to the weight function, it also slowed the decoherence.

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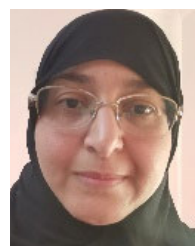
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