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Prescribed Finite Time Stabilization of Linear Systems With State Constraints

TINGTING YANG, YICHAO MA, PENGFEI ZHANG, AND JINGYUN XU
School of Engineering, Huzhou University, Huzhou 313000, China

Corresponding author: Pengfei Zhang (pengfei_zhang@zjhu.edu.cn)

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ABSTRACT This paper studies the prescribed finite time stabilization control problem of multi-input linear control systems subject to uncertainties and state constraints. Different from finite-time and fixed-time control methods, a novel switching nonsingular coordinate transformation function based on the block decomposition technique is developed, resulting in a solution that the control action is continuous and bounded based on the transformed system. Moreover, the boundedness of closed-loop system states is ensured with prescribed finite time performance. Furthermore, we incorporate the barrier Lyapunov function into the stabilization control design to restrain the amplitudes of system states. Numerical examples are provided to verify the effectiveness of the proposed prescribed finite time control strategy.

INDEX TERMS Linear system, stabilization control, prescribed finite time, barrier Lyapunov function.

I. INTRODUCTION

Stabilization control is of special interest to linear control systems from the practical and theoretical perspectives. Effective methods have been proposed such as PID control, sliding mode control, model predictive control, backstepping, H_∞ control [1], [2], etc. In particular, sliding mode control is popular due to its effectiveness, the high-order sliding mode control algorithms with finite-time and fixed-time regulations have been applied in the fields of mechanical industries and electronics [3]–[5].

The finite-time and fixed-time stability is widely considered since the attribute of owing a specified time. The finite-time stability for second-order and high-order systems are respectively considered in [6] and [7]. The finite-time stabilization and optimal feedback control for the nonlinear dynamical system are investigated in [8]. The further endeavor made by [10] attains the finite-time stabilization of stochastic high-order nonlinear systems. The literature [11] proposes a constraint stabilization control law with higher-order sliding mode finite-time design, leading that the output of the system remains in some prescribed range. For the above literature, the settling time of finite-time control depends on both the initial conditions and the preselected control parameters. Polyakov [3] studied the nonlinear feedback controller for fixed-time stabilization of linear control systems. In [13], the fixed-time backstepping control design for high-order strict-feedback nonlinear systems via terminal sliding mode is investigated. However, although the fixed-time control method removes the constraint of initial conditions, it can only converge the states before the given settling time, and the predescribed converge time cannot be guaranteed exactly.

Song *et al.* in [1] and [12] consider the time-varying feedback stabilization of normal form nonlinear systems in prescribed finite time by introducing the time-scaling function. In [15], the prescribed-time observers for linear systems in observer canonical form are considered. Different from finite-time and fixed-time control methods, fractional-power terms are not utilized and system states converge to zero at the exact time T. However, the effective time only works for the time $t \in [t_0, t_0 + T)$ and the design parameters should be chosen so that the matrix obtained from the characteristic polynomial is Hurwitz [1]. Moreover, the dynamics considered above are limited to the norm form or the observer canonical form. According to the block control principle in [3], for high-order subsystems transformed by a linear system with multiple inputs, the above algorithms are not applicable.

To overcome the limited time interval of $t \in [t_0, t_0 + T)$, in [2] the prescribed-time consensus and containment control strategies of single integrator multi-agent systems not affected by disturbances are considered. In [14] by introducing a time-varying piecewise function the adaptive fault-tolerant prescribed-time control for nonlinear

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teleoperation systems with position error constraints is considered beyond the limited time interval. Authors in [18] considers the consensus control for wheeled mobile robots by designing the specified-time observer with time scale function under input saturation constraint. However, the continuity of time-varying scaling functions above is not guaranteed. Furthermore, due to the introduction of $\frac{1}{T-t}$, the input boundedness and state constraint issues are necessary to be investigated.

Motivated by the discussions above, we address the nonlinear feedback design for prescribed finite time stabilization of multi-input linear control systems with state constraints. Firstly, in view of the difficulties caused by the distinct subsystem dimensions of multi-input linear systems, a novel switching nonsingular coordinate transformation function based on the block decomposition technique is proposed. Compared with finite time methods [6]–[11] and the fixed time method [3], due to the proposed high-power time scaling function, the convergence time T of closed-loop system states can be explicitly predetermined without the limitation of system initial values. Secondly, the operation time of the control scheme is extended to $t \in [0, \infty)$ compared with the previous works [1], [12] and [16], and the continuity of the switching time-varying scaling function is guaranteed. Moreover, the input boundedness is proven in spite of the existence of term $\frac{1}{T-t}$. Thirdly, the state constraint issue is considered by the barrier Lyapunov function, and the amplitudes of system states can be restrained.

The organization of this article is organized as follows. Section 2 presents some preliminaries and problem statements. Section 3 elaborates the prescribed time stability control design of the linear systems. Section 4 gives the numerical simulation to verify the theoretical analysis. Finally, Section 5 concludes the paper.

Notations : Through this article, $\| x \|$ signifies its standard Euclidean norm. $\| x \|_\infty =$ ess $\sup_{t>p}|x|$ and $\| x \|_p$ is the p norm. Let $\mathbb{R}$ be the set of the real number, $\mathbb{R}^n$ be the set of the n-dimensional vector and $\mathbb{R}^{n\times n}$ be the set of the n-dimensional matrix. The superscript T means the transpose for real matrices and "+" means the pseudo-inverse for real matrices. rown(W) is the row number of the matrix W, ker(W) and range(W) are the null space and the column space of W, and null(W) is the matrix with columns defining the orthonormal basis of ker(W).

II. PROBLEM FORMULATION

Consider the following class of uncertain multi-input linear systems

$$\dot{x}(t) = Ax(t) + Bu(t) + \tau_d(t, x(t)) \tag{1}$$

where $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the control input, matrices $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are the system matrix and control gain matrix. The term $\tau_d(t, x(t)) = B\chi(t, x(t))$ represents the matched uncertainties.

Assumption 1: The matrix pair (A, B) is controllable.

Assumption 2: The term $\| \chi(t, x(t)) \| \le \chi_0(t, x(t))$ for $\forall t > 0$ and $\forall x \in \mathbb{R}^n$, where $\chi_0(t, x(t))$ is a known function.

Remark 1: The observer of the uncertainty with an unknown upper bound has been discussed to improve the system transient performance in our previous work [16]. In fact, the observer design has been discussed warmly, the boundary of the uncertainty is generally unknown, and the norm value of its derivative is assumed to have an unknown upper bound. For instance, in [31] the fixed-time observer is designed to compensate for such uncertainties.

Before moving on, we give the following lemma which will be used in the proof of the main results in the paper.

Lemma 1 [3]: If the pair (A, B) is controllable, then based on the algorithm below,

Initialization: $A_0 = A$, $B_0 = B$, $T_0 = I_n$, $k = 1$.

Recursion: While rank $(B_k) =$ rown (A_k) do

$$A_{k+1} = B_k^\perp A_k \left(B_k^\perp\right)^T, \quad B_{k+1} = B_k^\perp A_k \widehat{B}_k$$

$$T_{k+1} = \begin{pmatrix} B_k^\perp \\ \widehat{B}_k \end{pmatrix}, \quad k = k+1$$

with $B_k^\perp = \left(\text{null}\left(B_k^T\right)\right)^T$, $\widehat{B}_k = \left(\text{null}\left(B_k^\perp\right)\right)^T$. Then orthogonal matrix G can be given as

$$G = \begin{pmatrix} T_k & 0 \\ 0 & I_{wk} \end{pmatrix} \begin{pmatrix} T_{k-1} & 0 \\ 0 & I_{wk-1} \end{pmatrix} \cdots \begin{pmatrix} T_2 & 0 \\ 0 & I_{w2} \end{pmatrix} T_1 \tag{2}$$

with $w_i = n - rown(T_i)$. According to the coordinate transformation $y = Gx$, system (1) can be reduced to the block form

$$\begin{cases} \dot{y}_1 = A_{1,1}y_1 + A_{1,2}y_2 \\ \dot{y}_2 = A_{2,1}y_1 + A_{2,2}y_2 + A_{2,3}y_3 \\ \cdots \\ \dot{y}_k = A_{k,1}y_1 + \cdots + A_{k,k}y_k + A_{k,k+1}(u+\chi) \end{cases} \tag{3}$$

where $A_{i,j}$ are derived from the matrix GAG^T.

Lemma 2 [3]: Consider the dynamics in (1) and (3), combining with the coordinate transformation $s = \Phi y$ and the inverse transformation $y = \Phi^{-1}s$ below

$$s_i = y_i + \phi_i, \quad i = 1, \ldots, k$$

$$\phi_1 = 0, \ \phi_{i+1} = A_{i,i+1}^+ \Big(\sum_{j=1}^{i} A_{ij}y_j + \sum_{r=1}^{i} \frac{\partial \phi_i}{\partial y_r} \sum_{j=1}^{r+1} A_{rj}y_j\Big) \tag{4}$$

$$y_i = s_i + \psi_i, \ i = 1, \ldots, k; \quad \psi_1 = 0,$$

$$\psi_{i+1} = A_{i,i+1}^+ \Big(\sum_{k=1}^{i} \frac{\partial \psi_i}{\partial s_k} A_{i,i+1}s_{k+1} + \sum_{j=1}^{i} A_{i,j}(s_j + \psi_j)\Big) \tag{5}$$

Then system (1) can be redescripted as the following block subsystem form

$$\begin{cases} \dot{s}_1 = A_{1,2}s_2 \\ \dot{s}_2 = A_{2,3}s_3 \\ \cdots \\ \dot{s}_{k-1} = A_{k-1,k}s_k \\ \dot{s}_k = \tilde{A}_{k,1}s_1 + \tilde{A}_{k,2}s_2 + \cdots + \tilde{A}_{k,k}s_k + A_{k,k+1}(u+\chi) \end{cases} \tag{6}$$

where the matrices $A_{i,j}$ and $\tilde{A}_{i,j}$ can be derived from the matrix ΦG.

Remark 2: According to the algorithm in Lemma 1, it is noted that the dimensions of subsystems $y_1, \ldots, y_k$ in (3) and $s_1, \ldots, s_k$ in (6) may be different. Due to the input matrix $A_{k,k+1}$ with the structure of full row rank, it brings difficulty to design the stabilization controller of multi-input linear systems.

Objective: This paper aims to solve the prescribed finite time stabilization control for multi-input linear systems (1) with both uncertainties and state constraints. We will employ prescribed finite time state feedback controller for systems (1) by a novel switching nonsingular coordinate transformation function based on block decomposition technique.

Remark 3: Compared with the finite time methods [6]–[11] and the fixed time method [3], the convergence time T can be conveniently set by introducing the term $\frac{1}{T-t}$, which can be considered as the role of gain term acting on the control input and adjusting the convergence rate. Besides, different from [1] and [12], the stabilization system of (1) owns the structure of multiple inputs, and the controller proposed in this paper can be obtained without solving certain polynomials like [12]. Moreover, the case of time interval $t > T$ and the control continuity are not investigated in [1], [12] and [16]. Due to the existence of the term $\frac{1}{T-t}$ introduced by the time scale function, it is necessary to analyze the input boundedness and state constraint issues.

III. MAIN RESULTS

This section presents the design and analysis of the prescribed finite time stabilization controller for the multi-input linear system. First, the novel switching coordinate transformation and inverse coordinate transformation are proposed. Second, stabilization control laws are developed and the proof of boundedness is given. Finally, the stabilization control law with state constraints is analyzed via the barrier Lyapunov function.

A. COORDINATE TRANSFORMATION AND INVERSE COORDINATE TRANSFORMATION

To obtain the prescribed-time controller, the switching scaling function is introduced as follows

$$\begin{cases} \mu_1(t,T) := \dfrac{1}{(T-t+\delta)^{\gamma}}, & t \in [0,T) \\ \mu_1(t,T) := 1/\delta^{\gamma}, & t \in [T,\infty] \end{cases} \tag{7}$$

where the parameter T is the prescribed finite time satisfying $0 < T_p \le T$ (T_p is the physically possible time interval which represents the time consuming of signal transmission and processor computing) and is irrelevant to system initial conditions. The parameter δ is a positive value satisfying that $0 < \delta \ll 1$. It is noted that the function $\mu_1(t,T)$ meets that $\mu_1(0,T) = \frac{1}{(T+\delta)^{\gamma}}$, and when $t = T$, $\mu_1(t,T) = 1/\delta^{\gamma}$ with an integer γ greater than 1, which guarantees the continuity of the switching function (7). The function $\mu_1(t,T)$ is a positive non-decreasing function and is continuous everywhere. Moreover, when $\delta = 0$ it becomes the case studied in [12].

Following the above function, we propose a coordinate transformation based on dynamics (6).

Consider the block subsystem (6), design the coordinate transformation $w = Ps$ as follows,

Case 1: $t \in [0, T)$

$$\begin{cases} w_i = \dfrac{s_i}{(T-t+\delta)^{\gamma}} + p_i, & p_1 = 0, \\ p_{i+1} = \sum_{j=1}^{i} \dfrac{a_{i+1,j}}{(T-t+\delta)^{i+\gamma+1-j}} s_j, & 1 \le j \le i \le k \end{cases} \tag{8}$$

Case 2: $t \in [T, \infty)$

$$\begin{cases} w_i = \dfrac{s_i}{\delta^{\gamma}} + p_i, & p_1 = 0, \\ p_{i+1} = \sum_{j=1}^{i} \dfrac{a_{i+1,j}}{\delta^{i+\gamma+1-j}} s_j, & 1 \le j \le i \le k \end{cases} \tag{9}$$

where the coefficients $a_{i,j}$ is a constant designed as

$$a_{i,0} = 0,\ a_{i,i} = 1,\ a_{i,q} = 0,\ q > i \tag{10}$$

$$a_{i+1,j} = A_{i,i+1}^{+}\left(a_{i,j}(i+\gamma-j+k_1) + a_{i,j-1}A_{i-1,j}\right) \tag{11}$$

Remark 4: Compared with [1], [12] and [16], the case of time interval $t > T$ is considered. In terms of equations (8) and (9), the parameter δ is introduced to guarantee the continuity of state w_i for $t \in [0, \infty)$.

Theorem 1: For the coordinate transformation (8-11), the dynamics of w_i, $1 \le i \le k$, can be described as

$$\dot{w}_i = \frac{-k_1}{\sigma(T-t)+\delta} w_i + A_{i,i+1} w_{i+1}, \quad i = 1, \ldots, k-1 \tag{12}$$

where

$$\sigma(T-t) = \begin{cases} T-t & 0 \le t < T \\ 0 & t \ge T \end{cases} \tag{13}$$

and the parameter k_1 is a positive value.

Proof: The proofs are divided into two parts associated with above two cases in (8) and (9).

Step 1: The case of $0 \le t < T$ is firstly considered. According to equation (12), the desired transformed system is given below,

$$\dot{w}_i = \frac{-k_1}{T-t+\delta} w_i + A_{i,i+1} w_{i+1} \tag{14}$$

Then w_{i+1} can be obtained as follows,

$$w_{i+1} = A_{i,i+1}^{+}\left(\dot{w}_i + \frac{k_1}{(T-t+\delta)} w_i\right) + (I - A_{i,i+1}^{+}A_{i,i+1})\frac{s_{i+1}}{(T-t+\delta)^{\gamma}} \tag{15}$$

the second term $(I - A_{i,i+1}^{+}A_{i,i+1})\frac{s_{i+1}}{(T-t+\delta)^{\gamma}}$ is chosen carefully to guarantee the solvability and uniqueness of control input u. With the switching coordinate transformation (8), the derivative of w_i along (14) is obtained as

$$\dot{w}_i = \frac{\gamma s_i}{(T-t+\delta)^{\gamma+1}} + \sum_{j=1}^{i-1} \frac{a_{i,j}(i+\gamma-j)}{(T-t+\delta)^{i+\gamma+1-j}} s_j$$

$$+\frac{A_{i,i+1}s_{i+1}}{(T-t+\delta)^{\gamma}}+\sum_{j=1}^{i-1}\frac{a_{i,j}A_{j,\,j+1}}{(T-t+\delta)^{i+\gamma-j}}s_{j+1} \quad (16)$$

Then one has the solution of (15) as

$$\begin{aligned}w_{i+1}&=A^{+}_{i,i+1}\Big(\frac{\gamma s_i}{(T-t+\delta)^{\gamma+1}}+\sum_{j=1}^{i-1}\frac{a_{i,j}(i+\gamma-j)s_j}{(T-t+\delta)^{i+\gamma+1-j}}\\&+\frac{A_{i,i+1}s_{i+1}}{(T-t+\delta)^{\gamma}}+\sum_{j=1}^{i-1}\frac{a_{i,j}A_{j,\,j+1}}{(T-t+\delta)^{i+\gamma-j}}s_{j+1}\\&+\frac{k_1}{T-t+\delta}\Big(\frac{s_i}{(T-t+\delta)^{\gamma}}+\sum_{j=1}^{i-1}\frac{a_{i,j}s_j}{(T-t+\delta)^{i+\gamma-j}}\Big)\Big)\\&+(I-A^{+}_{i,i+1}A_{i,i+1})\frac{s_{i+1}}{(T-t+\delta)^{\gamma}}\end{aligned}$$

Subsequently, we have

$$\begin{aligned}w_{i+1}&=A^{+}_{i,i+1}\Big(\frac{\gamma s_i+k_1s_i}{(T-t+\delta)^{\gamma+1}}+\frac{A_{i,i+1}s_{i+1}}{(T-t+\delta)^{\gamma}}\\&+\sum_{j=1}^{i-1}\frac{a_{i,j}(i+\gamma-j)}{(T-t+\delta)^{i+\gamma+1-j}}s_j+\sum_{j=1}^{i-1}\frac{a_{i,j}A_{j,\,j+1}s_{j+1}}{(T-t+\delta)^{i+\gamma-j}}\\&+\sum_{j=1}^{i-1}\frac{k_1a_{i,j}s_j}{(T-t+\delta)^{i+\gamma+1-j}}\Big)+\frac{(I-A^{+}_{i,i+1}A_{i,i+1})s_{i+1}}{(T-t+\delta)^{\gamma}}\\&=\frac{s_{i+1}}{(T-t+\delta)^{\gamma}}+A^{+}_{i,i+1}\Big(\sum_{j=1}^{i}\frac{a_{i,j}(i+\gamma-j+k_1)}{(T-t+\delta)^{i+\gamma+1-j}}s_j\\&+\sum_{j=1}^{i-1}\frac{a_{i,j}A_{j,\,j+1}}{(T-t+\delta)^{i+\gamma-j}}s_{j+1}\Big)\end{aligned}$$

With the condition (10), we have $\sum_{j=1}^{i-1}\frac{a_{i,j}A_{j,\,j+1}}{(T-t)^{i+\gamma-j}}s_{j+1}=\sum_{j=1}^{i}\frac{a_{i,j-1}A_{j-1,\,j}}{(T-t)^{i+\gamma+1-j}}s_j$. Rewrite the above equation as follows,

$$\begin{aligned}w_{i+1}&=\frac{s_{i+1}}{(T-t+\delta)^{\gamma}}+A^{+}_{i,i+1}\\&\times\Big(\sum_{j=1}^{i}\frac{a_{i,j}\,(i+\gamma-j+k_1)+a_{i,j-1}A_{j-1,\,j}}{(T-t+\delta)^{i+\gamma+1-j}}s_j\Big).\end{aligned} \quad (17)$$

According to the equation (8), one gets the condition (11).

Step 2: The case of $t\geq T$ is considered. Then w_{i+1} can be obtained as follows,

$$w_{i+1}=A^{+}_{i,i+1}\left(\dot{w}_i+\frac{k_1}{\delta}w_i\right)+(I-A^{+}_{i,i+1}A_{i,i+1})\frac{s_{i+1}}{\delta^{\gamma}} \quad (18)$$

With the dynamics (12), the desired transformed system is

$$\dot{w}_i=\frac{-k_1}{\delta}w_i+A_{i,i+1}w_{i+1} \quad (19)$$

By the similar steps above, we have

$$w_{i+1}=\frac{s_{i+1}}{\delta^{\gamma}}+A^{+}_{i,i+1}\Big(\sum_{j=1}^{i}\frac{a_{i,j}(i+\gamma-j+k_1)}{\delta^{i+\gamma+1-j}}s_j+\sum_{j=1}^{i}\frac{a_{i,j-1}A_{j-1,\,j}}{\delta^{i+\gamma+1-j}}s_j\Big) \quad (20)$$

According to the equation (9), the condition (11) is obtained. The proof is completed.

Remark 5: Different from [16], this paper designs the time-based power function (8) and (9). The continuity of the switching function of (7) is guaranteed and it owns the ability to adjust the transient response performance.

Lemma 3: For the coordinate transformation (8-11), the inverse coordinate transformation $s=P^{-1}w$ can be described as follows,

Case 1: $t\in[0,T)$

$$\begin{cases}s_i=w_i(T-t+\delta)^{\gamma}+l_i,\quad l_1=0,\\ l_{i+1}=\sum_{j=1}^{i}b_{i+1,j}\,(T-t+\delta)^{\gamma-i+j-1}\,w_j,\\ \qquad 1\leq j\leq i\leq k\end{cases} \quad (21)$$

Case 2: $t\in[T,\infty)$

$$\begin{cases}s_i=\delta^{\gamma}w_i+l_i,\quad l_1=0,\\ l_{i+1}=\sum_{j=1}^{i}b_{i+1,j}\delta^{\gamma-i+j-1}w_j,\quad 1\leq j\leq i\leq k\end{cases} \quad (22)$$

where the coefficient $b_{i,j}$ is a constant designed as

$$b_{i,0}=0,\, b_{i,i}=1,\, b_{i,q}=0,\ \ q>i \quad (23)$$

$$b_{i+1,j}=A^{+}_{i,i+1}\left(-b_{i,j}\,(\gamma-i+j+k_1)+b_{i,j-1}A_{j-1,j}\right). \quad (24)$$

Proof: The proofs are divided into two parts associated with the above two cases in (21) and (22).

Step 1: The case of $0\leq t<T$ is firstly considered. The derivative of dynamic (21) with $b_{i,q}=0$ and $b_{i,0}=0$ is given as

$$\begin{aligned}&A_{i,i+1}s_{i+1}\\&=\sum_{j=1}^{i-1}-b_{i,j}(\gamma-i+j)\,(T-t+\delta)^{\gamma-i+j-1}\,w_j\\&+\sum_{j=1}^{i-1}b_{i,j}\,(T-t+\delta)^{\gamma-i+j}\Big(-\frac{k_1w_j}{T-t+\delta}+A_{j,j+1}w_{j+1}\Big)\\&-(k_1+\gamma)\,w_i(T-t+\delta)^{\gamma-1}+A_{i,i+1}w_{i+1}(T-t+\delta)^{\gamma}\\&=\sum_{j=1}^{i-1}-b_{i,j}(\gamma-i+j+k_1)\,(T-t+\delta)^{\gamma-i+j+1}\,w_j\\&+\sum_{j=1}^{i-1}b_{i,j}\,(T-t+\delta)^{\gamma-i+j}A_{j,j+1}w_{j+1}\\&-(k_1+\gamma)\,w_i(T-t+\delta)^{\gamma-1}+A_{i,i+1}w_{i+1}(T-t+\delta)^{\gamma}\end{aligned}$$

then the solution of s_{i+1} gives

$$\begin{aligned}s_{i+1}&=A^{+}_{i,i+1}\sum_{j=1}^{i}(-b_{i,j}\,(\gamma-i+j+k_1))\\&\times(T-t+\delta)^{\gamma-i+j+1}\,w_j+A^{+}_{i,i+1}A_{i,i+1}w_{i+1}\\&\times(T-t+\delta)^{\gamma}\\&+\left(I-A^{+}_{i,i+1}A_{i,i+1}\right)w_{i+1}(T-t+\delta)^{\gamma}\end{aligned}$$

$$+\sum_{j=1}^{i-1} A_{i,i+1}^{+} b_{i,j}(T-t+\delta)^{\gamma-i+j} A_{j,j+1} w_{j+1}$$
$$= A_{i,i+1}^{+} \sum_{j=1}^{i} (-b_{i,j}(\gamma - i + j + k_1)) \times (T-t+\delta)^{\gamma-i+j+1} w_j$$
$$+\sum_{j=1}^{i} A_{i,i+1}^{+} b_{i,j}(T-t+\delta)^{\gamma-i+j} A_{j,j+1} w_{j+1}$$
$$= A_{i,i+1}^{+} \sum_{j=1}^{i} \left(-b_{i,j}(\gamma - i + j + k_1) + b_{i,j-1} A_{j-1,j}\right) \times (T-t+\delta)^{\gamma-i+j+1} w_j + (T-t+\delta)^{\gamma} w_{i+1}$$

where the term $\left(I - A_{i,i+1}^{+} A_{i,i+1}\right) w_{i+1}(T-t+\delta)^{\gamma}$ is chosen to eliminate the term $A_{i,i+1}^{+} A_{i,i+1} w_{i+1}(T-t+\delta)^{\gamma}$. By the coordinate transformation (21), it gives

$$s_{i+1} = \sum_{j=1}^{i} b_{i+1,j}(T-t+\delta)^{\gamma-i+j+1} w_j + (T-t+\delta)^{\gamma} w_{i+1}$$

Then the equation (24) is obtained.

Step 2: The case of $t \geq T$ is considered. Similar to the above equations, one gets

$$s_{i+1} = A_{i,i+1}^{+} \sum_{j=1}^{i+1} \left(-b_{i,j}(\gamma - i + j + k_1) + b_{i,j-1} A_{j-1,j}\right) \times \delta^{\gamma-i+j+1} w_j + \delta^{\gamma} w_{i+1}$$

By the coordinate transformation (22), the condition (24) is obtained. The proof is completed.

Remark 6: According to the block decomposition technique in Lemma 1 and Lemma 2, the original system (1) is transformed into the system (6), which owns the structure of different dimensions of subsystems. The coordinate transformation proposed in (8-11) is utilized to obtain the transformed system (12) from (6). With the transformed system (12), if the state $w_i, i = 1, \ldots, k-1$ is bounded, then the state w_{i-1} can be bounded in prescribed time T. With the nonsingular coordinate transformations introduced in Lemma 1 and 2, the state x of the system (1) will be bounded in prescribed time T.

Remark 7: As we all know, there are countless solutions of w_{i+1} and s_{i+1} due to $A_{i,,i+1}(I - A_{i,i+1}^{\dagger} A_{i,i+1}) = 0$, the term $(I - A_{i,i+1}^{+} A_{i,i+1}) \frac{s_{i+1}}{(T-t+\delta)^{\gamma}}$ and $\left(I - A_{i,i+1}^{+} A_{i,i+1}\right) w_{i+1}(T - t + \delta)^{\gamma}$ are selected specially for coordinate transformation and controller design in the next subsection.

B. PRESCRIBED-TIME STABILIZATION WITHOUT STATE CONSTRAINTS

Recalling the above coordinate transformations, we give the control strategy as below

$$u = -\sum_{p=1}^{k} \widetilde{A}_{k,p} s_p + (\phi(t))^{\gamma} A_{k,k+1}^{+} \times \Big(-\frac{\gamma s_k}{(\phi(t))^{\gamma+1}} - \sum_{j=1}^{k-1} \frac{a_{k,j}(k+\gamma-j)}{(\phi(t))^{k+\gamma+1-j}} s_j - \sum_{j=1}^{k-1} \frac{a_{k,j} A_{j,\,j+1}}{(\phi(t))^{k+\gamma-j}} s_{j+1} - \frac{k_1}{(\phi(t))^{\gamma}} w_k\Big) \tag{25}$$

with $\phi(t) = \sigma(T-t) + \delta$. Then the block subsystems w can be given as

$$\begin{cases} dotw_i = -\dfrac{k_1}{\phi(t)} w_i + A_{i,i+i} w_{i+1}, \ i = 1, \ldots, k-1 \\ \dot{w}_k = -\dfrac{k_1}{(\phi(t))^{\gamma}} w_k + \dfrac{A_{k,k+1}}{(\phi(t))^{\gamma}} \chi(t, w) \end{cases} \tag{26}$$

Theorem 2: Consider the dynamics (1) with transformations (3-12) and control law (25), the transformed system states w and s, the original system state x and control input u are prescribed finite time bounded.

Proof: Applying the coordinate transformations (8) and (9), we have

$$w_k = \frac{s_k}{(\sigma(T-t)+\delta)^{\gamma}} + \sum_{j=1}^{k-1} \frac{a_{k,j}}{(\sigma(T-t)+\delta)^{k+\gamma-j}} s_j \tag{27}$$

Then the derivative of w_k along systems (6) is obtained as

$$\dot{w}_k = \frac{\gamma s_k}{(\phi(t))^{\gamma+1}} + \frac{A_{k,k+1}(\sum_{p=1}^{k} \widetilde{A}_{k,p} s_p + u + \chi(t,w))}{(\phi(t))^{\gamma}} + \sum_{j=1}^{k-1} \frac{a_{k,j}(k+\gamma-j)}{(\phi(t))^{k+1+\gamma-j}} s_j + \sum_{j=1}^{k-1} \frac{a_{k,j} A_{j,\,j+1}}{(\phi(t))^{k+\gamma-j}} s_{j+1}$$

By combining the controller (25) with the above equation, the dynamics (26) is obtained.

Next, the stabilization property of system (26) is analyzed. Denote the candidate Lyapunov function $V_i = \frac{w_i^T w_i}{2}$, $i = 1, \ldots, k$.

Step 1: The case of $i = k$. The derivative of V_k along (26) is $\dot{V}_k = -\frac{k_1}{(\phi(t))^{\gamma}} w_k^T w_k + \frac{1}{(\phi(t))^{\gamma}} w_k^T A_{k,k+1} \chi(t,w)$. By applying Young's inequality with constant parameter $0 < \lambda < k_1$, we have

$$\dot{V}_k \leq -\frac{2\left(k_1 - \lambda \parallel A_{k,k+1} \parallel\right)}{(\phi(t))^{\gamma}} V_k(t) + \frac{1}{4\lambda(\phi(t))^{\gamma}} \chi^2(t,w) \tag{28}$$

Then

$$V_k \leq \exp^{-2(k_1 - \lambda \|A_{k,k+1}\|) \int_0^t \frac{1}{\phi^{\gamma}(t)} d\tau} V(0) + \frac{\chi_0^2}{4\lambda} \int_0^t exp^{-2(k_1-\lambda\|A_{k,k+1}\|)\int_{\tau}^t \frac{1}{\phi^{\gamma}(s)} ds} \frac{1}{\phi^{\gamma}(\tau)} d\tau \tag{29}$$

When $t < T$, we have

$$V_k(t) \leq exp^{-2(k_1-\lambda\|A_{k,k+1}\|)\int_0^t \frac{1}{\phi^{\gamma}(\tau)} d\tau} V(0) + \frac{\chi_0^2}{4\lambda} exp^{-2(k_1-\lambda\|A_{k,k+1}\|)\int_0^t \frac{1}{\phi^{\gamma}(s)} ds} \times \int_0^t exp^{2(k_1-\lambda\|A_{k,k+1}\|)\int_0^{\tau} \frac{1}{\phi^{\gamma}(s)} ds} d\int_0^{\tau} \frac{1}{\phi^{\gamma}(s)} ds$$

$$= exp^{-2(k_1-\lambda\|A_{k,k+1}\|)\int_0^t \frac{1}{\phi^{\gamma}(\tau)}d\tau} V(0) + \frac{\chi_0^2}{4\lambda}\frac{1}{2(k_1-\lambda \parallel A_{k,k+1} \parallel)} \times(1-exp^{-2(k_1-\lambda\|A_{k,k+1}\|)\int_0^t \frac{1}{\phi^{\gamma}(s)}ds})$$

It is known that the first term above is monotonically decreasing. Define $\Theta_1 = exp^{\frac{-2(k_1-\lambda\|A_{k,k+1}\|)(\frac{1}{\phi^{\gamma-1}}-\frac{1}{(T+\delta)^{\gamma-1}})}{\gamma-1}}$, then $V_k(T_-) \leq \Theta_1 V(0) + \frac{\chi_0^2}{8\lambda(k_1-\lambda\|A_{k,k+1}\|)}(1-\Theta_1)$. Due to the fact that $V_k(t) = \frac{w_k^T w_k}{2}$, and one has

$$\parallel w_k(T_-) \parallel_\infty \leq \sqrt{2V_k(T_-)} = \epsilon_{k,1} \tag{30}$$

As t goes to T, we have $\lim_{t\to T}\phi^{\gamma-1} = \frac{1}{\delta^{\gamma-1}}$. The parameter δ is chosen as $0 < \delta \ll 1$, we know that if δ is close to zero, $\lim_{t\to T}\phi^{\gamma-1} = +\infty$ and $\lim_{t\to T}\Theta_1 = 1$. Specially, if $\chi(t) \equiv 0$, $\tau_d \equiv 0$ and $\delta = 0$, we have $\lim_{t\to T} V(t) = 0$. When $t \geq T$, it gives

$$\begin{aligned} V_k(t) &\leq exp^{-2(k_1-\lambda\|A_{k,k+1}\|)(t-T)/\delta^{\gamma}} V(T) \\ &\quad + \frac{\chi_0^2}{8\lambda(k_1-\lambda \parallel A_{k,k+1} \parallel)} \\ &\quad \times exp^{-2(k_1-\lambda\|A_{k,k+1}\|)(t-T)/\delta^{\gamma}} \\ &\quad \times(exp^{2(k_1-\lambda\|A_{k,k+1}\|)(t-T)/\delta^{\gamma}}-1) \\ &= exp^{-2(k_1-\lambda\|A_{k,k+1}\|)(t-T)/\delta^{\gamma}} V(T) \\ &\quad + \frac{\chi_0^2}{4\lambda}\frac{1}{2(k_1-\lambda \parallel A_{k,k+1} \parallel)} \\ &\quad \times(1-exp^{-2(k_1-\lambda\|A_{k,k+1}\|)(t-T)/\delta^{\gamma}}) \end{aligned} \tag{31}$$

Subsequently, the state w_k is bounded as

$$\parallel w_k \parallel_\infty \leq \sqrt{\frac{\chi_0^2}{4\delta\lambda(k_1-\lambda \parallel A_{k,k+1} \parallel)}} = \epsilon'_{k,1} \tag{32}$$

Step 2: The case of $i = k-1$. Due to the fact that

$$\dot{w}_{k-1} = -\frac{k_1}{\sigma(T-t)+\delta}w_{k-1} + A_{k-1,k}w_k \tag{33}$$

Then we have

$$\frac{d\left(e^{\int_0^t \frac{k_1}{\sigma(T-s)+\delta}ds}w_{k-1}\right)}{dt} = e^{\int_0^t \frac{k_1}{\sigma(T-s)+\delta}ds}A_{k-1,k}w_k \tag{34}$$

When $t < T$, we have

$$e^{\int_0^t \frac{k_1}{T-s+\delta}ds} = e^{k_1(-\ln(T-s+\delta)\ |_0^t)} = \left(\frac{T+\delta}{T-t+\delta}\right)^{k_1} \tag{35}$$

Recalling equation (34), we have

$$\left(\frac{T+\delta}{T-t+\delta}\right)^{k_1} w_{k-1} = w_{k-1}(0) + \int_0^t \left(\frac{T+\delta}{T+\delta-s}\right)^{k_1} A_{k-1,k}w_k ds \tag{36}$$

then it leads to

$$w_{k-1} = \left(\frac{T-t+\delta}{T+\delta}\right)^{k_1} w_{k-1}(0) + \int_0^t \left(\frac{T-t+\delta}{T+\delta-s}\right)^{k_1} A_{k-1,k}w_k ds \tag{37}$$

Then one holds

$$\begin{aligned} &\parallel w_{k-1}(T_-) \parallel_\infty \\ &\leq (T+\delta-t)(\parallel w_{k-1}(0)\frac{(T+\delta-t)^{k_1-1}}{(T+\delta)^{k_1}} \parallel \\ &\quad + \frac{\parallel A_{k-1,k}w_k \parallel}{k_1-1}(1-\frac{(T-t+\delta)^{k_1-1}}{(T+\delta)^{k_1-1}})) \\ &= (T+\delta-t)\,\Upsilon_{k-1} = \epsilon_{k,2} \end{aligned} \tag{38}$$

Since state w_k and parameters T, k_1 are bounded, Υ_{k-1} is bounded. Similarly, we have

$$\begin{aligned} &\parallel w_{k-2}(T_-) \parallel_\infty \\ &\leq (T+\delta-t)^2(\parallel w_{k-2}(0)\frac{(T+\delta-t)^{k_1-2}}{T^{k_1}} \parallel \\ &\quad + \parallel A_{k-2,k-1} \parallel \Upsilon_{k-1}\frac{1}{k_1-1}(1-\frac{(T+\delta-t)^{k_1-1}}{(T+\delta)^{k_1-1}})) \\ &= (T+\delta-t)^2\,\Upsilon_{k-2} \end{aligned} \tag{39}$$

By the same steps, we have $w_{k-q} \leq (T+\delta-t)^q\,\Upsilon_{k-q}$, $q = 3, \ldots, k-1$. When $t > T$, we have

$$\parallel w_{i-1} \parallel_\infty \leq \epsilon_{k,1}\Upsilon'_{k-1} \tag{40}$$

with $\Upsilon'_{k-1} = (1+ \parallel A_{k,k+1} \parallel exp(\frac{-k_1}{\delta}+1))$. By the same steps, we have w_{k-q}, $q = 3, \ldots, k-1$ are bounded. With the inverse coordinate transformation (21) and (22), then

$$s_{j+1} = \sum_{q=1}^{j+1} b_{j+1,q}(T+\delta-t)^{\gamma-j+q+1}w_q \tag{41}$$

and the control input (25) is rewritten as

$$\begin{aligned} u = &-\sum_{p=1}^{k}\widetilde{A}_{k,p}s_p - \frac{A^+_{k,k+1}\gamma s_k}{\phi} \\ &-\sum_{j=1}^{k-1}\frac{A^+_{k,k+1}a_{k,j}(k+\gamma-j)}{\phi^{k+1-j}}s_j \\ &-\sum_{j=1}^{k-1}\frac{A^+_{k,k+1}a_{k,j}A_{j,\,j+1}}{\phi^{k-j}}s_{j+1} - k_1A^+_{k,k+1}w_k \end{aligned} \tag{42}$$

According to the coordinate transformations (8) and (9), one has w_k is bounded, then $-\frac{s_k}{\phi}$, $\sum_{j=1}^{k-1}\frac{a_{k,j}}{(\phi)^{k+\gamma-j}}s_j$ and $-\sum_{j=1}^{k-1}\frac{A^+_{k,k+1}a_{k,j}A_{j,\,j+1}}{\phi^{k-j}}s_{j+1}$ are bounded. Then the control input is bounded. The proof is completed.

Remark 8: The boundary value $\epsilon_{k,1}$ of the state w_k in (30) is related to the magnitude of the uncertainties χ_0, and it can be adjusted by choosing the parameters k_1. Indeed, the system uncertainty has a significant influence on the control performance. The advanced nonlinear control methods in [24] and [25] can provide a good solution for dealing with various uncertainties.

C. PRESCRIBED-TIME STABILIZATION WITH STATE CONSTRAINTS

In this section, the barrier Lyapunov function is proposed for restraining the amplitudes of system states. Before moving on, we give the following lemma.

Lemma 4: [22] For any positive constant k_w, the following inequality holds for any vector $x \in \mathbb{R}^n$ in the interval $\| x \| < k_w$,

$$\log \frac{k_w^2}{k_w^2 - x^T x} \le \frac{x^T x}{k_w^2 - x^T x} \tag{43}$$

where log(•) denotes the natural logarithm of •.

Then consider system (1) by coordinate transformations in Theorem 1 and Lemma 3, we have

$$\begin{cases} \dot{w}_i = -\frac{k_1}{\phi(t)} w_i + A_{i,i+i} w_{i+1}, \ i = 1, \ldots, k-1 \\ \dot{w}_k = \bar{\Xi} + \frac{A_{k,k+1}}{(\phi(t))^\gamma}(u + \sum_{q=1}^k \widetilde{A}_{k,q} s_q + \gamma(t, w)) \end{cases} \tag{44}$$

with $\bar{\Xi} = \frac{\gamma s_k}{(\sigma(T-t)+\delta)^{\gamma+1}} + \sum_{j=1}^{k-1} \frac{a_{k,j}(k+\gamma-j)}{(\sigma(T-t)+\delta)^{k+1+\gamma-j}} s_j + \sum_{j=1}^{k-1} \frac{a_{k,j} A_{j,\,j+1}}{(\sigma(T-t)+\delta)^{k+\gamma-j}} s_{j+1}$. Then we design the controller u below,

$$\begin{aligned} u = &-\sum_{p=1}^{k} \widetilde{A}_{k,p} s_p + (\phi(t))^\gamma A_{k,k+1}^+ \\ &\times \Big(-\frac{\gamma s_k}{(\phi(t))^{\gamma+1}} - \sum_{j=1}^{k-1} \frac{a_{k,j}(k+\gamma-j)}{(\phi(t))^{k+\gamma+1-j}} s_j \\ &- \sum_{j=1}^{k-1} \frac{a_{k,j} A_{j,\,j+1}}{(\phi(t))^{k+\gamma-j}} s_{j+1} - \frac{k_1}{(\phi(t))^r} w_k \\ &- \frac{\gamma_1}{2} \frac{w_k}{(\phi(t))^r (k_{wk}^2 - w_k^T w_k)}\Big) \end{aligned} \tag{45}$$

Theorem 3: Consider the dynamics (1) with transformations (3-12) and control law (45), the transformed system states w and s, the original system state x and control input u are prescribed finite time bounded with amplitude constraints.

Proof: Define the BLF Lyapunov function candidate as

$$V_k = \log \frac{k_{wk}^2}{k_{wk}^2 - w_k^T w_k} \tag{46}$$

Hence, the states w_k in (44) with the boundary $\| w_k \| < k_{wk}$ is guaranteed if V_k is made bounded. Due to equations (38) and (39), states $w_q, \ q = 1, \ldots, k-1$ are also bounded. The derivative of V_k along (44) is given as

$$\begin{aligned} \dot{V}_k &= \frac{w_k^T \dot{w}_k}{k_{wk}^2 - w_k^T w_k} \\ &= \frac{w_k^T}{(k_{wk}^2 - w_k^T w_k)} (\bar{\Xi} + \frac{A_{k,k+1}}{(\phi(t))^\gamma} (u \\ &\quad + \sum_{q=1}^{k} \widetilde{A}_{k,q} s_q + \gamma(t, w))) \end{aligned} \tag{47}$$

With Lemma 4, we have $\log \frac{k_{wk}^2}{k_{wk}^2 - w_k^T w_k} \le \frac{w_k^T w_k}{k_{wk}^2 - w_k^T w_k}$. Then according to controller (45) and Young's inequality, the derivative of V_k can be rewritten as

$$\begin{aligned} \dot{V}_k &= \frac{-k_1 w_k^T w_k}{(\phi(t))^r (k_{wk}^2 - w_k^T w_k)} \\ &\quad - \frac{\gamma_1}{2} \frac{w_k^T w_k}{(\phi(t))^r (k_{wk}^2 - w_k^T w_k)^2} \\ &\quad + \frac{w_k^T A_{k,k+1} \gamma(t, w)}{(\phi(t))^r (k_{wk}^2 - w_k^T w_k)} \\ &\le \frac{-k_1}{(\phi(t))^r} V_k - \frac{\gamma_1}{2} \frac{w_k^T w_k}{(\phi(t))^r (k_{wk}^2 - w_k^T w_k)^2} \\ &\quad + \frac{w_k^T w_k}{2(\phi(t))^r (k_{wk}^2 - w_k^T w_k)^2} \\ &\quad + \frac{1}{2(\phi(t))^r} \| A_{k,k+1} \gamma(t, w) \|^2 \\ &= \frac{-\bar{k}}{(\phi(t))^r} V_k + \frac{1}{2(\phi(t))^r} \| A_{k,k+1} \|^2 \gamma^2(t, w) \end{aligned} \tag{48}$$

with $\bar{k} = \frac{2k_1 + \gamma_1 - 1}{2} > 0$. Similar to (28), we have that the variable w_k is bounded and remains in the sets $\| w_i \| < k_{wk}$, if the initial value of $w_i(0)$ is in the set $\| w_i(0) \| < k_{wk}$. Then recalling (44), states $w_i, \ i = 1, \ldots, k-1$ are also bounded. With $s = \Phi G x$, $s = P^{-1} w$, Φ and G are nonsingular transformation, we can know that the states of x and s are prescribed-time bounded and limited by $\| x \|_\infty < \| P^{-1} \Phi G \|_2^2 \max(k_{wi})$. The proof is completed.

Remark 9: With the proposed controller (45) the state w_k is designed to be bounded in prescribed time by (44), and then states $w_i, \ i = 1, \ldots, k-1$ are bounded in prescribed time. In fact, the barrier Lyapunov function considered in (46) can only ensure the constant constraint, the time-varying barrier Lyapunov function which is close to practical applications can be referred to [23]. Moreover, the high-order full-state constraints can be solved based on [19]–[21], which will be our future work.

IV. SIMULATION

In this section, we verify the effectiveness of the proposed stabilization control methods in (25) and (45). We consider the coefficients of the dynamics in (1) as $A = [1.5, -3.5, 2.5; -3, 0, 5; 0, -2, 5]$ and $B = [2, 0; -2, 2; 0, -5]$. The matched uncertainties are assumed to be $\tau_d = [2; 0; -5] * sin(t)$, the desired prescribed convergence time is chosen as $T = 1s$ and the positive value δ is chosen as $\delta = 0.05$. The parameters $\gamma_1 = 1$. Then according to the block principle (2) and coordinate transformation (6), we have

$$A_{11} = -0.9259, \quad A_{12} = \begin{bmatrix} -2.1376 & 6.7792 \end{bmatrix},$$

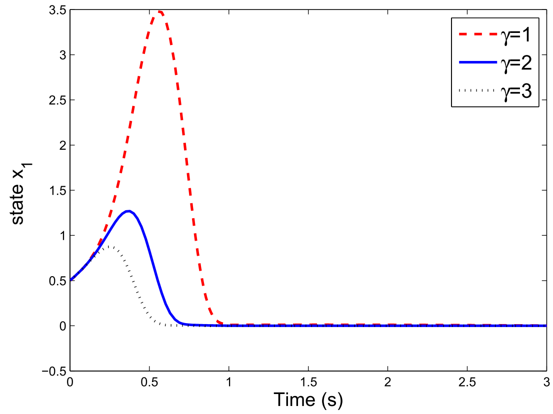

FIGURE 1. The state x_1 with controller (25).

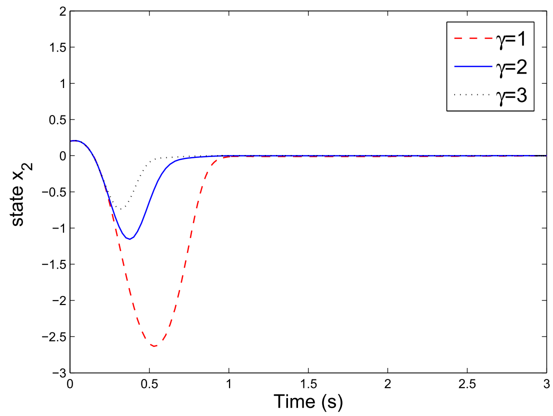

FIGURE 2. The state x_2 with controller (25).

$$A_{23} = \begin{bmatrix} -2.8098 & 2.0000 \\ -0.3239 & -5.0000 \end{bmatrix},$$
$$\tilde{A}_{21} = \begin{bmatrix} 0.0521 \\ 0.7378 \end{bmatrix}, \quad \tilde{A}_{22} = \begin{bmatrix} 3.8236 & 2.1582 \\ -0.7676 & 2.6764 \end{bmatrix},$$
$$G = \begin{bmatrix} 0.6804 & 0.6804 & 0.2722 \\ -0.6804 & 0.7245 & -0.1102 \\ -0.2722 & -0.1102 & 0.9559 \end{bmatrix},$$
$$\Phi = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0.0392 & 1.0000 & 0 \\ -0.1242 & 0 & 1.0000 \end{bmatrix}$$

With Theorem 1, the proposed non-singular coordinate transformation can be obtained as

$$w = \begin{bmatrix} \frac{1}{(\sigma(T-t)+\delta)^{\gamma}} & \mathbb{O} \\ \frac{A_{12}^{\dagger}(\gamma+k_1)}{(\sigma(T-t)+\delta)^{(\gamma+1)}} & \frac{1}{(\sigma(T-t)+\delta)^{\gamma}} \end{bmatrix} \tag{49}$$

To get fair comparison with [16], we use the same initial values as $x_0 = (0.5, 0.2, 0.3)^T$. The cases $\gamma = 1$ to $\gamma = 3$ are chosen in (8) and (9), respectively. Compared with case $\gamma = 1$ in [16], Figures 1-3 show the evolutions of states for system (1). It is clearly shown that states $x_i,\ i = 1, 2, 3$ can converge to the neighbor of the origin in the desired prescribed time $T = 1s$, which proofs the proposed

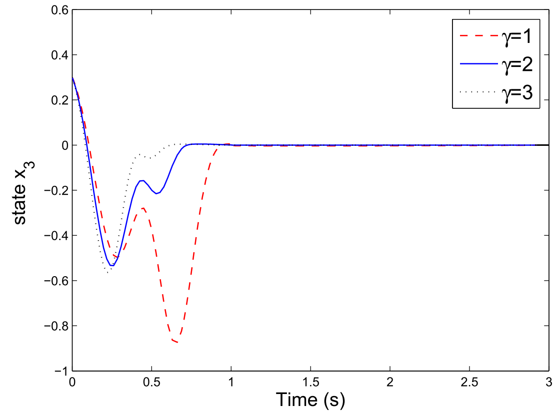

FIGURE 3. The state x_3 with controller (25).

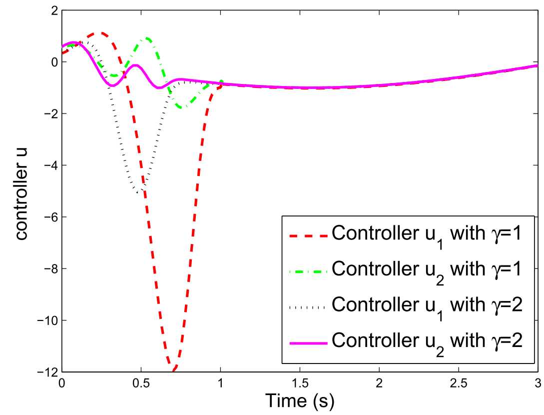

FIGURE 4. Comparison of controller u with $\gamma = 1$ and $\gamma = 2$.

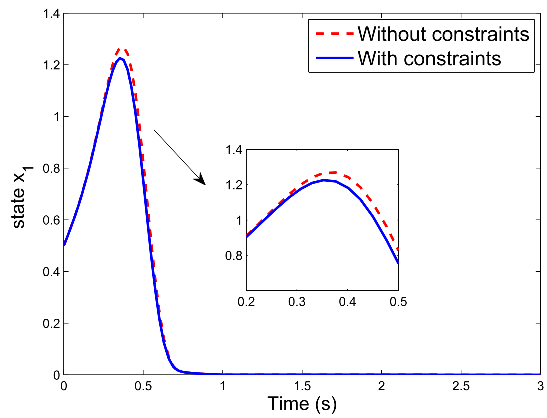

FIGURE 5. Comparison of the state x_1 with controller (25) and (42).

controller (25) is effective when $0 \leq t < T$ and $t \geq T$. Moreover, the transient response performance can be effectively improved by choosing a larger γ, which confirm that the high power nonsingular coordinate transformations (8)-(9) and (21)-(22) are significant and necessary. Figure 4 presents the plots of controller (25) and the boundedness of the controller is proved. In Figures 5-7, it can been seen that the amplitude of state x can be limited. To further show the effectiveness of the controller (45), we assume the constraint bound of w_k as $k_{wk} = 0.18$. In Figures 8-9, the value γ

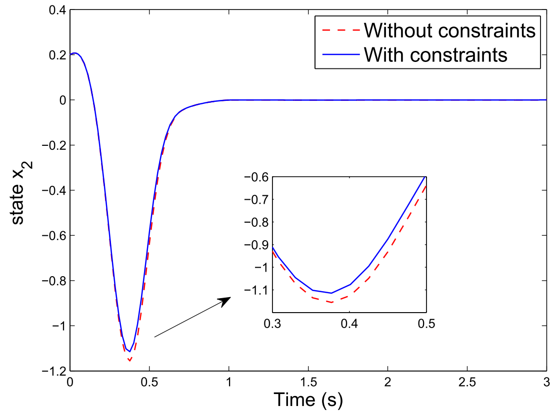

FIGURE 6. Comparison of the state x_2 with controller (25) and (42).

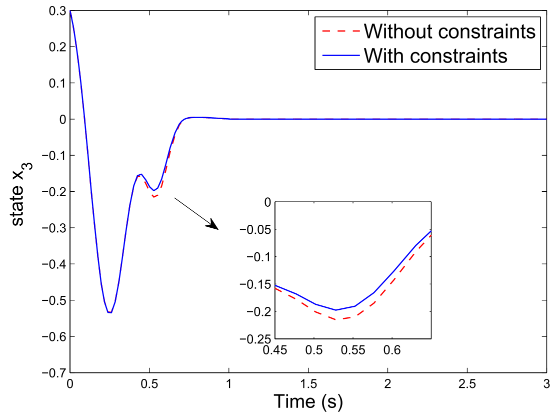

FIGURE 7. Comparison of the state x_3 with controller (25) and (42).

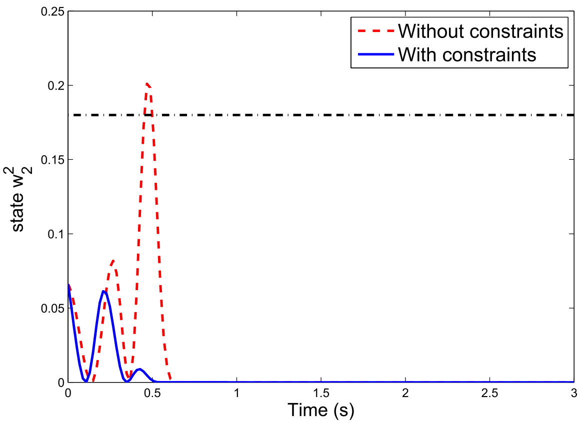

FIGURE 8. State w_2^2 without the influence of the uncertainty τ_d.

is given as $\gamma = 2$. In Figure 8, we assume the system uncertainty $\tau_d = 0$, and the comparison of state response w_2^2 with and without the constraint bound of $k_{wk} = 0.18$ is shown. With equation (49), the dimension of state w_2 is 2. From Figure 8, it illustrates that the state w_2^2 will converge to zero without the existence of the uncertainty τ_d. Moreover, the controller proposed in (45) can bring the state w_2^2 into the specified constraint range $k_{wk} = 0.18$, while the state w_2^2 with controller (42) will be out of limitation range. In Figure 9, we assume the uncertainty τ_d as $\tau_d = [2; 0; -5] * sin(t)$,

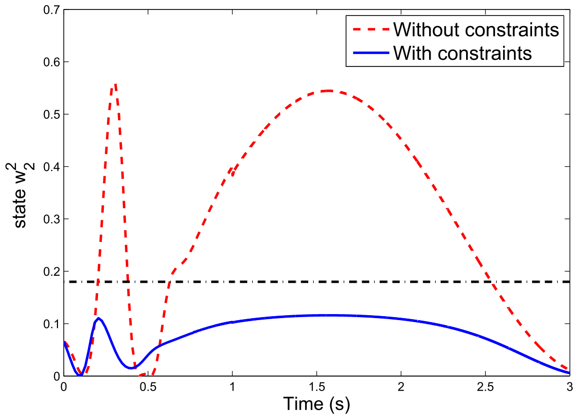

FIGURE 9. State w_2^2 with the influence of the uncertainty τ_d.

and the comparison of state response w_2^2 with and without the constraint bound of $k_{wk} = 0.18$ is shown. From Figure 9, it shows that the state w_2^2 can converge into the specified constraint range $k_{wk} = 0.18$ with the controller (45). It is proven that the proposed controllers are effective.

V. CONCLUSION AND FUTURE WORK

In this paper, based on the block decomposition technique and novel switching nonsingular coordinate transformation methods, the prescribed finite time stabilization control laws for multi-input linear control systems are presented, which fills the gap of the control time interval in the case of $t > T$ while guaranteeing continuity and boundedness of the controller. Furthermore, the amplitudes of system states are restrained with constant boundary value by introducing the barrier Lyapunov function. Future research work will focus on the extension of the results to the prescribed-time consensus problem of multiple linear systems with time-varying communication delay inspired by [27]–[30]. Furthermore, the event trigger prescribed time control of linear systems which can reduce the burden of actuators will be another future work.

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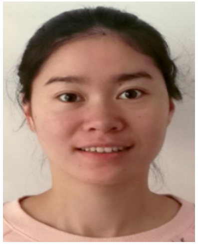

TINGTING YANG was born in Heilongjiang, China. She received the B.S. and Ph.D. degrees from Dalian Maritime University, in 2012 and 2020, respectively. Her current research interests include formation control of underwater vehicles, disturbance observers, and fault tolerant control.

YICHAO MA was born in Zhejiang, China. He received the B.S. degree from the Keyi College of Zhejiang Sci-Tech University, in 2019. His current research interests include stabilization control, formation control, and fault tolerant control.

PENGFEI ZHANG was born in Heilongjiang, China. He received the B.S. and Ph.D. degrees from Dalian Maritime University, in 2012 and 2020, respectively. His current research interests include stabilization/tracking control of underactuated surface/underwater vehicles, nonlinear systems, and disturbance observers.

JINGYUN XU was born in Jiangxi, China, in 1980. He received the Ph.D. degree in control science and engineering from Tongji University, China, in 2017. His research interests include unmanned vehicle control, signal processing, state and life estimation, health management of lithium-ion batteries in electrical vehicles, and energy storage.