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Adaptively Robust Square-Root Cubature Kalman Filter Based on Amending

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ABSTRACT To solve the problem of decreased filtering accuracy and even filter divergence for the case that the model errors and measurement outliers exist simultaneously, an adaptively robust square-root cubature Kalman filter (SRCKF) based on amending is proposed in this paper. Based on the error analysis, the judgment criterion and the amending criterion related to the innovation are set. Then the filter could overcome the influence of the measurement outliers with the robust amendment of the measurement noise covariance matrix based on the principle of the innovation covariance matching. To further deal with model errors, the new method of the adaptive amendment of the predicted state based on the innovation is developed and combined. Finally, the proposed algorithm can balance the effect of the prior predicted value and the posterior feedback measurement value on the filtering process and reduce the state estimation error. The simulation results show that the proposed algorithm can effectively suppress the negative impact of the model errors and the measurement outliers and can obtain better estimation performance and obviously decreasing running time compared with the adaptively robust unscented Kalman filter (ARUKF) and the robust multiple fading factors cubature Kalman filter (RMCKF).

INDEX TERMS Adaptively robust filter, adaptive amendment, measurement outliers, model errors, square-root cubature Kalman filter.

I. INTRODUCTION

In recent years, sigma point-based Gaussian approximation filters [1]–[9] have been researched intensively and have an important application in the field of target tracking. Typical ones include the unscented Kalman filter (UKF) [3], [4], the central difference Kalman filter (CDKF) [5], the Gauss-Hermite quadrature filter (GHQF) [6], and the cubature Kalman filter (CKF) [7], [8]. These estimation algorithms have better nonlinear state estimation performance and avoid the calculation of the Jacobian matrix compared with the extended Kalman filter (EKF) [9]. Especially, the CKF has the properties of simplicity, high accuracy, good convergence, etc. Reference [10] which has a popular application in various state estimation areas. The square-root CKF (SRCKF) was proposed to obtain better filtering performance and numerical stability by introducing the square-root of the error covariance matrix into the process of CKF. However, the estimation performance will be degraded and the filtering results will be unreliable in the presence of model errors and measurement outliers.

On the one hand, there are mainly two methods to overcome the model errors [11]. The first is to improve the adaptability and accuracy of the model. The adaptive current statistical (CS) model and the improved interacting multiple model (IMM) were combined with the SRCKF respectively in [11], [12], which effectively improved the maneuvering target tracking accuracy. The second is to develop adaptive filtering algorithms to obtain better filter precision, among which the strong tracking filter (STF) based on the fading memory filtering theory [13]–[16] can effectively resist the bad influence of model errors. On the other hand, it is necessary to use robust techniques for controlling the measurement outliers such as the Huber-based Kalman filter (HKF) [17] and the robust UKF with gain correction [18], [19]. However, the above methods cannot effectively reduce the negative impact of model errors and measurement outliers simultaneously.

In fact, there are several robust filters that can deal with the problem such as the robust M-M filter [20], maximum correntropy Kalman filter [21], and the filters which applied

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heavy-tailed distributions [22], [23] and non-Gaussian distributions [24], [25]. But these estimation methods have limitations of complex theoretical derivation and numerous iterations. Besides, efforts have been devoted to constructing adaptively robust filters in the literature [26]–[29]. Yang [26], Yang *et al.* [27] established the adaptively robust Kalman filter (ARKF) theory which applied the robust estimation methodology to control measurement outliers and introduced the adaptive fading factor to overcome model errors [30]. However, the ARKF may fail when the robust estimation solution of the total state cannot be obtained in some cases [28]. Therefore novel tracking derivative-free filters based on the structure of the ARKF were proposed in [28] and [29] which can overcome the limitations of the ARKF and have fewer iterations. These filters named the adaptively robust unscented Kalman filter (ARUKF) and the robust multiple fading factors cubature Kalman filter (RMCKF) combined the STF with the Huber-based robust estimation. However, the modified measurement covariance matrix and the fading factor were both determined by the innovation in the ARUKF and the RMCKF. If the measurement outliers lead to a big innovation, the system cannot obtain the modified measurement covariance matrix and the fading factor correctly and effectively. Besides, the formation of the fading factor has problems such as the arbitrariness of the introduced position [11], [31], [32] and a large amount of calculation [33], [34].

In this paper, an adaptively robust SRCKF based on amending (ARSRCKF) is proposed to resist the impact of model errors and measurement outliers simultaneously. The main contributions of this paper are summarized as follows. 1) The measurement noise covariance matrix is amended based on the principle of the innovation covariance matching to overcome the measurement outliers influence. 2) The new method that the predicted state is amended directly based on the innovation is combined to deal with model errors, which is proposed as an alternative to the introduction of the fading factor. 3) The judgment criterion and the amending criterion are set based on the error analysis to balance the effect of the prior predicted value and the posterior feedback measurement on the filtering process and to reduce the state estimation error finally. The simulation results are satisfactory and verify the effectiveness of the proposed algorithm.

The paper proceeds as follows; the process of the SRCKF is introduced in Section II. In Section III, the proposed adaptively robust SRCKF is developed based on the given error analysis, the judgment criterion, and the amending criterion. Section IV presents two simulation scenarios that demonstrate the efficiency and performance of the proposed algorithm. Finally, Section V gives the conclusions.

II. SRCKF

The nonlinear discrete-time system with additive noise is shown as follows:

$$\begin{cases} \boldsymbol{x}_{k+1} = f(\boldsymbol{x}_k) + \boldsymbol{w}_k \\ z_{k+1} = h(\boldsymbol{x}_{k+1}) + \boldsymbol{v}_{k+1} \end{cases} \tag{1}$$

where $\boldsymbol{x}_{k+1} \in \mathbf{R}^n$ and $z_{k+1} \in \mathbf{R}^m$ are the state vector and the measurement vector at time $k+1$, respectively. f and h are the nonlinear state transition function and the nonlinear measurement function, respectively. $\boldsymbol{w}_k$ is the process noise. $\boldsymbol{v}_k$ is the measurement noise. $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are mutually independent with $\boldsymbol{w}_k \sim N(0, \boldsymbol{Q}_k)$, $\boldsymbol{v}_k \sim N(0, \boldsymbol{R}_k)$.

To maintain the positive definiteness and the symmetry of the error covariance matrix and thus improve stability and numerical accuracy of the filter, the square-root of the error covariance matrix is introduced into the process of CKF to form the SRCKF algorithm.

Generally, QR decomposition to $\boldsymbol{A}$ is defined as $\boldsymbol{S} = \text{Tria}\ (\boldsymbol{A})$, where $\boldsymbol{S}$ is the lower triangular matrix [11]. The process of the SRCKF can be summarized as follows.

A. TIME UPDATE

The posterior probability density function of the system at time k is assumed to be $p(\boldsymbol{X}_k|\boldsymbol{Z}_k) \sim N(\hat{\boldsymbol{x}_{k|k}}, \boldsymbol{P}_{k|k})$, then:

(1) Factorize

$$\boldsymbol{P}_{k|k} = \boldsymbol{S}_{k|k}(\boldsymbol{S}_{k|k})^T \tag{2}$$

(2) Evaluate the cubature points

$$\boldsymbol{X}_{i,k|k} = \boldsymbol{S}_{k|k}\boldsymbol{\xi}_i + \hat{\boldsymbol{x}}_{k|k},\ i = 1, 2, \ldots m \tag{3}$$

(3) Evaluate the nonlinear propagated cubature points and the predicted state

$$\boldsymbol{X}^*_{i,k+1|k} = f(\boldsymbol{X}_{i,k|k}),\ i = 1, 2, \ldots m \tag{4}$$

$$\hat{\boldsymbol{x}}_{k+1|k} = \frac{1}{m}\sum_{i=1}^{m} \boldsymbol{X}^*_{i,k+1|k} \tag{5}$$

where n and m are the dimensional number of the state vector and the total number of cubature points, respectively, which satisfies $m = 2n$. $\boldsymbol{\xi}_i = \sqrt{m/2}[\mathbf{1}]_i$, $[\mathbf{1}]_i$ is the ith column of the followed points:

$$[\mathbf{1}] = \left\{ \begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}, \begin{pmatrix}0\\1\\\vdots\\0\end{pmatrix}, \ldots \begin{pmatrix}0\\0\\\vdots\\1\end{pmatrix}, \begin{pmatrix}-1\\0\\\vdots\\0\end{pmatrix}, \begin{pmatrix}0\\-1\\\vdots\\0\end{pmatrix}, \ldots \begin{pmatrix}0\\0\\\vdots\\-1\end{pmatrix} \right\} \tag{6}$$

(4) Calculate the square-root of the predicted error covariance

$$\boldsymbol{S}_{k+1|k} = \text{Tria}\left([\boldsymbol{\chi}^*_{k+1|k}, \boldsymbol{S}_Q]\right) \tag{7}$$

where $\boldsymbol{S_Q}$ is the square-root of $\boldsymbol{Q}_k$, which is obtained by Cholesky decomposition($\boldsymbol{S_Q} = \text{Chol}(\boldsymbol{Q}_k)$). $\boldsymbol{\chi}^*_{k+1|k}$ is the $n \times m$ weighted and centered matrix as follows:

$$\boldsymbol{\chi}^*_{k+1|k} = \frac{1}{\sqrt{m}}[\boldsymbol{X}^*_{1,k+1|k} - \hat{\boldsymbol{x}}_{k+1|k}, \boldsymbol{X}^*_{2,k+1|k} - \hat{\boldsymbol{x}}_{k+1|k}, \ldots, \boldsymbol{X}^*_{m,k+1|k} - \hat{\boldsymbol{x}}_{k+1|k}] \tag{8}$$

B. MEASUREMENT UPDATE

(1) Evaluate the cubature points

$$\chi_{i,k+1|k} = S_{k+1|k}\xi_i + \hat{x}_{k+1|k}, i = 1, 2, \ldots m \tag{9}$$

(2) Evaluate the nonlinear propagated cubature points and the predicted measurement

$$Z^*_{i,k+1|k} = h(\chi_{i,k+1|k}), \quad i = 1, 2, \ldots m \tag{10}$$

$$\hat{z}_{k+1|k} = \frac{1}{m}\sum_{i=1}^{m} Z^*_{i,k+1|k} \tag{11}$$

(3) Calculate the innovation and the square-root of innovation covariance matrix

$$e_{k+1} = z_{k+1} - \hat{z}_{k+1|k} \tag{12}$$

$$S_{zz,k+1|k} = \text{Tria}\left([Z_{k+1|k}, S_R]\right) \tag{13}$$

where S_R is the square-root of R_k, which is obtained by Cholesky decomposition($S_R = \text{Chol}(R_k)$). $Z_{k+1|k}$ is the $l \times m$ weighted and centered matrix (l is the dimensional number of the measurement vector) as follows:

$$Z_{k+1|k} = \frac{1}{\sqrt{m}}[Z^*_{1,k+1|k} - \hat{z}_{k+1|k}, Z^*_{2,k+1|k} - \hat{z}_{k+1|k}, \ldots, Z^*_{m,k+1|k} - \hat{z}_{k+1|k}] \tag{14}$$

(4) Evaluate the cross-covariance matrix

$$P_{xz,k+1|k} = X_{k+1|k}Z^T_{k+1|k} \tag{15}$$

where $X_{k+1|k}$ is the $n \times m$ weighted and centered matrix as follows:

$$X_{k+1|k} = \frac{1}{\sqrt{m}}[\chi_{1,k+1|k} - \hat{x}_{k+1|k}, \chi_{2,k+1|k} - \hat{x}_{k+1|k}, \ldots, \chi_{m,k+1|k} - \hat{x}_{k+1|k}] \tag{16}$$

(5) Calculate the Kalman gain

$$K_{k+1} = (P_{xz,k+1|k}/S^T_{zz,k+1|k})/S_{zz,k+1|k} \tag{17}$$

(6) Update the estimate state

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(z_{k+1} - \hat{z}_{k+1|k}) \tag{18}$$

(7) Update the square-root of the estimation error covariance matrix

$$S_{k+1|k+1} = \text{Tria}\left([X_{k+1|k} - K_{k+1}Z_{k+1|k}, K_{k+1}S_R]\right) \tag{19}$$

III. ADAPTIVELY ROBUST SRCKF BASED ON AMENDING

A. ERROR ANALYSIS AND AMENDMENT

In nonlinear filtering, the model errors (caused by model mismatch, aircraft manoeuver, abrupt state changes, and some other factors) and the occurrence of measurement outliers are the key reasons that lead to the decline of the filtering precision, the degradation of tracking performance, and even the divergence of the filter.

As described above, if model errors and measurement outliers occur individually, there are effective methods. However, neither the STF nor the robust Kalman filter designed for controlling measurement outliers can overcome the negative impact brought by model errors and measurement outliers simultaneously, which is due to the design concepts of the algorithms: 1) In essence, the STF avoids filter divergence by increasing the weight of current measurement data in the filtering process. Therefore, the performance of the STF will be badly affected by the occurrence of measurement outliers. 2) The reason for the filter divergence caused by model errors is that the previous data play a strong role in the state estimation, so it is necessary to make full use of the information of the current measurement data for correction. However, the robust techniques designed just for controlling measurement outliers will mistake model errors for measurement errors, make an increment in the measurement noise covariance matrix, and further reduce the contributions of measurements. Therefore, it is difficult to overcome model errors effectively.

It can be seen from (18) that the estimated state is composed of the prior predicted state and the feedback of the innovation.

In the event of the model errors at time k, the predicted state and the predicted measurement at the time $k + 1$ will have a deviation, then the accuracy of the state estimation will decline, which is reflected in the large increase of the innovation at the time $k + 1$. The STF adjusts the gain matrix in real-time by introducing the fading factor, which is actually amending the second item of (18). In fact, the first item $\hat{x}_{k+1|k}$, can be directly amended based on the innovation which reflects the trend and the predicted state deviates from the actual state. Then even facing the abrupt state changes, the filter can catch up with the target state in real-time by amending and compensating the predicted state $\hat{x}_{k+1|k}$ according to the actual changes, and finally achieve the purpose of improving the estimation accuracy.

Due to the nonlinear property of measurement, the existence of interference, and the measurement noise, measurement outliers occur from time to time, which will lead to a dramatic increase of the innovation. In this case, still amending the predicted state value will obviously lead to a further increase of the prediction error or even the filter divergence. Then, the measurement noise covariance matrix R_k can be amended to reduce the weight of the measurements in real-time during the filtering process based on the principle of the innovation covariance matching.

Just like the ARUKF and the RMCKF, the proposed algorithm can overcome the negative impact of model errors and measurement outliers simultaneously by combining the above two aspects of the amendment. But if measurement outliers lead to a big innovation, the two amending methods will make an unwanted effect offset. Then the system cannot make the best use of the contributions of the model information and the measurement on the filtering process correctly.

Based on the above analysis, it can be concluded that the changes of the innovation determine whether to amend. And it is necessary to set the thresholds to judge the source of errors, so as to choose how to amend.

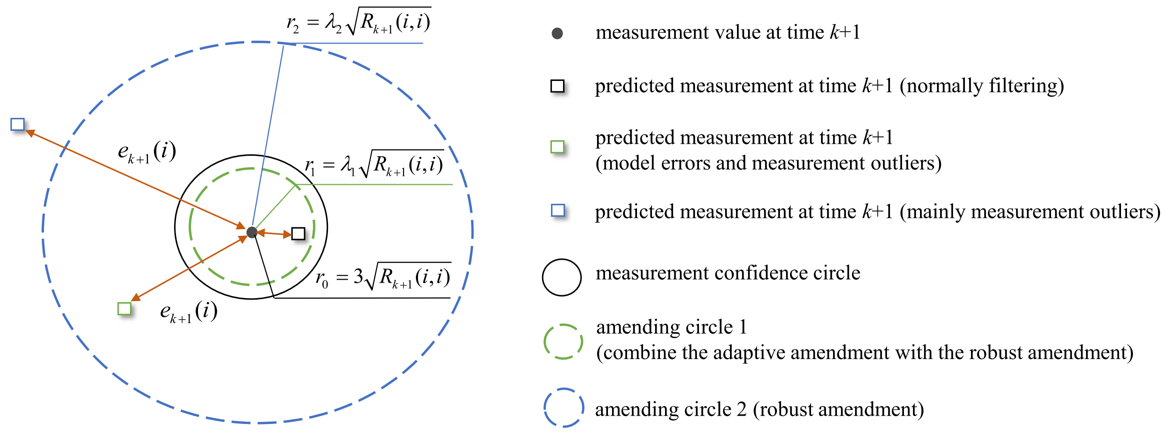

FIGURE 1. **Judgment criteria for amending.**

B. JUDGMENT CRITERION AND AMENDING CRITERION

1) JUDGMENT CRITERION

Draw a circle which is named the measurement confidence circle while the center is the measurement value at time $k+1$ and the radius is $3\sqrt{\boldsymbol{R}_{k+1}}$. Because $z_{k+1} \sim N(\hat{z}_{k+1|k}, \boldsymbol{R}_{k+1})$ and the measurement noise is the Gaussian white noise, when the filter works normally, there is a 99.73% probability that the predicted measurement $\hat{z}_{k+1|k}$ will be within $[z_{k+1} - 3\sqrt{\boldsymbol{R}_{k+1}}, z_{k+1} + 3\sqrt{\boldsymbol{R}_{k+1}}]$ based on the probability theory (3-σ rule). Equivalently, $\hat{z}_{k+1|k}$ will be in the measurement confidence circle with a 99.73% probability. The non-representational measurement circle is just used to visually help illustrate the judgment criterion.

Similarly, the amending threshold can be equivalent to the radius of another circle which is named the amending circle. When the model errors or the measurement outliers occur, the absolute value of the innovation will increase and exceed the threshold which means that the predicted measurement $\hat{z}_{k+1|k}$ will appear outside the amending circle.

Fig. 1 is the schematic diagram of judgment criteria for amending. The black solid circle is the measurement confidence circle, and the two dotted circles are the amending circles. The parameters λ_1 and λ_2 are adjustment factors. $\boldsymbol{R}_{k+1}(i, i)$ is the ith diagonal element of $\boldsymbol{R}_{k+1}$, which reflects the measurement error of variables in the corresponding dimensions.

According to the previous analysis, it is necessary to set the thresholds to judge the source of errors and determine whether to amend. The two thresholds correspond to r_1 and r_2 respectively in Fig. 1. If the ith component of the absolute value of the innovation $|\boldsymbol{e}_{k+1}(i)| < r_1$, there is no need to amend. If $r_1 < |\boldsymbol{e}_{k+1}(i)| < r_2$, the adaptive amendment of the predicted state and the robust amendment of the measurement noise covariance matrix will be combined. If $r_2 < |\boldsymbol{e}_{k+1}(i)|$, the increase of the innovation is mainly due to the occurrence of measurement outliers. In this case, only the measurement noise covariance matrix is amended.

In the event of measurement outliers especially the mutant measurements, there is a larger deviation between the measurement value and the predicted measurement. Therefore the radius of the amending circle corresponding to r_2 is larger. By contrast, the changes of the innovation caused by the model errors are smaller because the process that the changes of acceleration variable transmit to the measurable position dimension and velocity variables is not instantaneous. If the predicted measurement appears outside the amending circle corresponding to r_2, the deviation of the innovation will be mainly due to measurement outliers. Then the adaptive amendment of the predicted state based on the innovation will make an unwanted offset for the robust amendment of the measurement noise covariance matrix and even make the filter not robust. Therefore, only the measurement noise covariance matrix is amended in this case.

The parameters λ_1 and λ_2 are used to balance the effect of the measurement value and the predicted value on the amending area. λ_1 can be set in the interval of [2,3]. r_1 is a bit smaller than r_0 to reduce the state estimation deviation caused by the error accumulation during the filtering process and enhance the sensitivity to the target state changes. λ_2 is related to the system measurement accuracy and is generally set larger to overcome the negative impact of the mutant measurements. The setting interval is based on probability theory and experience, in which the filtering algorithm can achieve the optimal performance. And the basic principle of avoiding excessive amendment on the filtering process needs to be followed when setting λ_1 and λ_2. The main purpose of this principle is to set a reasonable amending area by setting λ_1 and λ_2, to balance the effect of the prior predicted value and the posterior feedback measurement value on the filtering process, and thus reduce the state estimation error.

2) AMENDING CRITERION

If the predicted measurement value is outside the area of r_2, the robust amendment is applied directly to the measurement noise covariance matrix $\boldsymbol{R}_{k+1}$ to reduce the weight

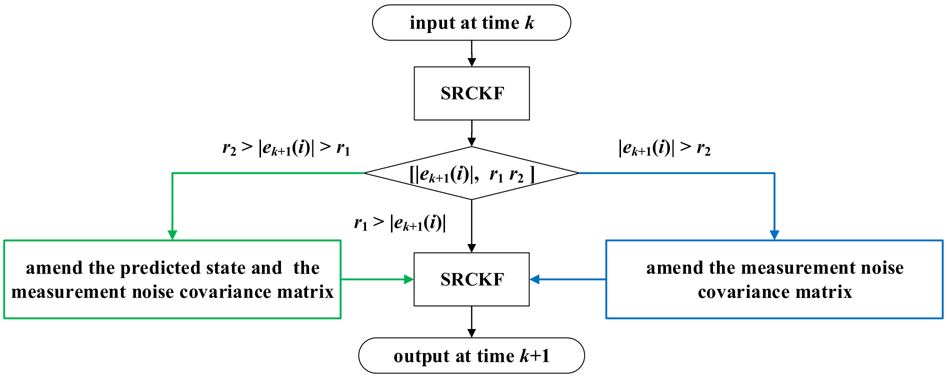

FIGURE 2. Structure diagram of the algorithm.

of measurement outliers based on the principle the innovation covariance matching. When the filter works normally, the actual and the theoretical innovation covariance matrix match as follows:

$$\boldsymbol{e}_{k+1}\boldsymbol{e}_{k+1}^T = \boldsymbol{P}_{zz,k+1|k} = \boldsymbol{P}_{z,k+1|k} + \boldsymbol{R}_{k+1} \tag{20}$$

where $\boldsymbol{P}_{zz,k+1|k}$ and $\boldsymbol{P}_{z,k+1|k}$ are the innovation covariance matrix and the observation covariance matrix respectively which are determined by:

$$\boldsymbol{P}_{zz,k+1|k} = \boldsymbol{S}_{zz,k+1|k}\boldsymbol{S}_{zz,k+1|k}^T \tag{21}$$

$$\boldsymbol{P}_{z,k+1|k} = \boldsymbol{P}_{zz,k+1|k} - \boldsymbol{R}_{k+1} \tag{22}$$

However, when measurement outliers occur, the innovation covariance matrix needs adaptation:

$$\boldsymbol{P}_{zz,k+1|k} = \boldsymbol{P}_{z,k+1|k} + \mu_{k+1}\boldsymbol{R}_{k+1} \tag{23}$$

where μ_{k+1} is the scale factor to make (20) satisfied again. Then the square-root of the innovation covariance matrix in (13) will become:

$$\boldsymbol{S}_{zz,k+1|k} = \text{Tria}\left([\boldsymbol{Z}_{k+1|k}, \sqrt{\mu_{k+1}}\boldsymbol{S}_{\boldsymbol{R}}]\right) \tag{24}$$

And the square-root of the estimation error covariance matrix in (19) will become:

$$\boldsymbol{S}_{k+1|k+1} = \text{Tria}\left([\boldsymbol{X}_{k+1|k} - \boldsymbol{K}_{k+1}\boldsymbol{Z}_{k+1|k}, \sqrt{\mu_{k+1}}\boldsymbol{K}_{k+1}\boldsymbol{S}_{\boldsymbol{R}}]\right) \tag{25}$$

Then (24) and (25) will be used in the measurement update process. According to [18], the adaptive scale factor is calculated by

$$\mu_{k+1} = \left(\boldsymbol{e}_{k+1}\boldsymbol{e}_{k+1}^T - tr[\boldsymbol{P}_{z,k+1|k}]\right)/tr(\boldsymbol{R}_{k+1}) \tag{26}$$

where tr is an operator for getting the trace of a matrix. It can be seen from (23) that the scale factor will increase in the event of measurement outliers. Then the measurement noise covariance matrix increases and the filtering gain decreases, which reduces the effect of measurement outliers on the state update process.

If the measurement systems can measure the position and velocity variables, the innovation at time $k+1$ can used to directly compensate and amend the predicted states of position and velocity variables at time $k+1$. As $k+1$. As for the predicted states of acceleration variables, the changes of the amended velocity variables can be used. Therefore, when the predicted measurement value is in the area between r_1 and r_2, the criteria are combined with the amended robust measurement noise matrix:

$$\begin{cases} \widehat{\boldsymbol{x}}_{k+1|k}(1) = \hat{\boldsymbol{x}}_{k+1|k}(1) + \boldsymbol{e}_{k+1}(1) \cdot k_1 \\ \widehat{\boldsymbol{x}}_{k+1|k}(2) = \hat{\boldsymbol{x}}_{k+1|k}(2) + \boldsymbol{e}_{k+1}(2) \cdot k_2 \\ \widehat{\boldsymbol{x}}_{k+1|k}(3) = \hat{\boldsymbol{x}}_{k+1|k}(3) + \left(\hat{\boldsymbol{x}}_{k+1|k}(2) - \widehat{\boldsymbol{x}}_{k+1|k}(2)\right) \cdot k_3 \end{cases} \tag{27}$$

where $\hat{\boldsymbol{x}}_{k+1|k}(i)$ is the ith component of the predicted state, corresponding to the position, velocity and acceleration dimension respectively. $\widehat{\boldsymbol{x}}_{k+1|k}(i)$ is the ith component of the amended predicted state. k_1, k_2 and k_3 are the corresponding amending factors of the position, velocity and acceleration dimension.

To avoid the excessive or inadequate amending effect, three aspects can be considered in the actual setting of k_1, k_2 and k_3: Firstly, the amending factors can be set based on experience or obtained offline by various intelligent algorithms. Secondly, the factors satisfy the formula of $x = vt + 0.5at^2$. After setting any two parameters in k_1, k_2, and k_3, the third parameter can be calculated by solving this formula. Thirdly, the proportion of information provided by the innovation should be considered. For example, if the innovation obtained by the measurement system only includes the deviation of the position dimension and velocity dimension, k_1 and k_2 can be set slightly larger.

C. IMPLEMENTATION OF THE ADAPTIVELY ROBUST SRCKF

The structure diagram of the algorithm is shown in Fig. 2. The involved steps are as follows:

1) Obtain the innovation at time $k+1$ using (2) to (12).
2) Determine whether to amend and distinguish which amending circle the predicted measurement is in based on the judgment criterion. If no amendment is needed, continue the filtering process according to (13) to (19).

3) If the predicted measurement value is in the area between r_1 and r_2, calculate the scale factor μ_{k+1} with (13), (21), (22) and (26). Then amend the predicted state based on the amending criterion with (27). Finally, complete the filtering process using (7) – (12), (24), (14) – (18) and (25).
4) If the predicted measurement value is outside the area of r_2, complete the measurement update process using (24), (14) – (18) and (25) after calculating the scale factor μ_{k+1} and the predicted state needs no amendment.
5) Repeat the above steps.

IV. SIMULATIONS AND DISCUSSION

In this section, the proposed algorithm is compared with the conventional SRCKF, the ARUKF [28], and the RMCKF [29] to illustrate the filter efficiency in the scenarios with the model errors or the measurement outliers. In the simulations, the state vector is $\boldsymbol{X}_k = [x, \dot{x}, y, \dot{y}]^T$ and the measurement vector is $\boldsymbol{Z}_k = [x, y]^T$, where x and y are the position variables, with $\dot{x}$ and $\dot{y}$ being the corresponding velocity variables. So there are only two amending factors in the simulations. Then the innovation can be used to compensate and amend the predicted states of position variables. As for the predicted states of velocity variables, the changes of the amended position variables can be used. The constant velocity motion and coordinated turning motion are set and modeled as $\boldsymbol{F}_{CV}$ and $\boldsymbol{F}_{CT}$ respectively by:

$$\boldsymbol{F}_{CV} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{28}$$

$$\boldsymbol{F}_{CT} = \begin{bmatrix} 1 & \frac{sin(wT)}{w} & 0 & \frac{cos(wT)-1}{w} \\ 0 & cos(wT) & 0 & -sin(wT) \\ 0 & \frac{1-cos(wT)}{w} & 1 & \frac{sin(wT)}{w} \\ 0 & sin(wT) & 0 & cos(wT) \end{bmatrix} \tag{29}$$

where w is the turning rate of 4 rad/s and T is the sampling time with $T = 1$ s. The initial target state $\boldsymbol{x}_0 = [1000\,m, 200\,m/s, 1000\,m, 10\,m/s]^T$, initial covariance matrix is $\boldsymbol{P}_0 = diag(15, 15, 15, 5)$. The covariance matrices of the process noise and the measurement noise are taken as $\boldsymbol{Q}_k = diag(1, 1, 1, 1)$ and $\boldsymbol{R}_k = diag(100, 100)$ respectively.

The Root Mean Square Error (RMSE) of the position are used to quantify and evaluate the performance of various filtering algorithms, and the 200-run Monte Carlo simulation is conducted to obtain the statistically average results. The RMSE is defined by:

$$RMSE = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(\boldsymbol{X}_i(k) - \hat{\boldsymbol{x}}_i^j(k|k)\right)^2} \tag{30}$$

Where N is the total number of Monte Carlo simulations. $\boldsymbol{X}_i(k)$ and $\hat{\boldsymbol{x}}_i^j(k|k)$ are the true value and the estimated state value of the ith component of the target state at time k in the jth simulation. In each simulation, the entire process is realized for 225 s and the initial estimated value is consistent with the initial target state value.

A. MODEL ERRORS

Model errors caused by the model mismatch and the state mutations are taken into consideration in the first scenario. The target is set for a hybrid maneuver with constant velocity motion and coordinated turning motion, but all four algorithms only use the model $\boldsymbol{F}_{CV}$ when tracking, so as to introduce the model errors caused by the model mismatch. The coordinated turning motion is during 50 s ∼ 100 s and 140 s ∼ 190 s, and the target maintains the constant velocity motion during the other time. At the same time, the state mutations $[0, 30\ m/s, 0, 30\ m/s]^T$ and $[0, 20\ m/s, 0, 20\ m/s]^T$ are introduced at epochs of 30 s and 120 s, so as to introduce the model errors caused by state mutations.

Fig. 3 presents the true and estimated target trajectories, which shows that all four algorithms can track the trajectory of the target. However, during the two turning motion periods when there is a model mismatch, the tracking performance of the SRCKF decreases and the estimated target trajectory cannot fit the true target trajectory well.

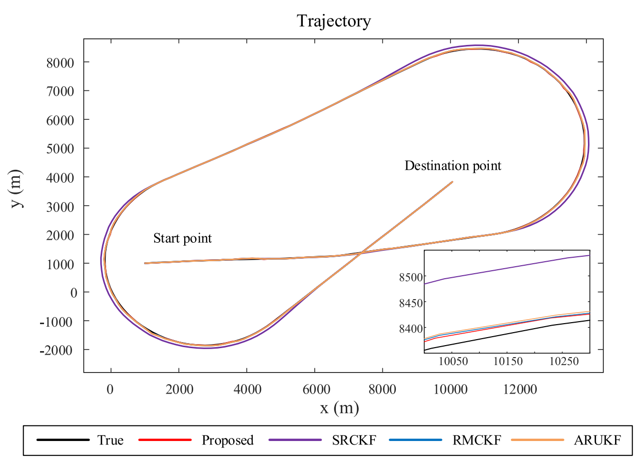

FIGURE 3. The true and estimated target trajectories.

Fig. 4 and Fig. 5 present the comparison of RMSE position curves among the four algorithms. The corresponding mean RMSEs and running time are listed in Table 1.

As seen from Fig. 4 and Fig. 5, although the filtering accuracy of the SRCKF is higher when the target model is accurate, it will be affected badly by the model mismatch and the state mutations which lead to the large jumps of the RMSE curves. By contrast, the proposed algorithm, the RMCKF, and the ARUKF can obtain the faster convergence rate and higher filtering accuracy. Besides, the proposed algorithm and the RMCKF outperform than the ARUKF facing the model mismatch and the state mutations.

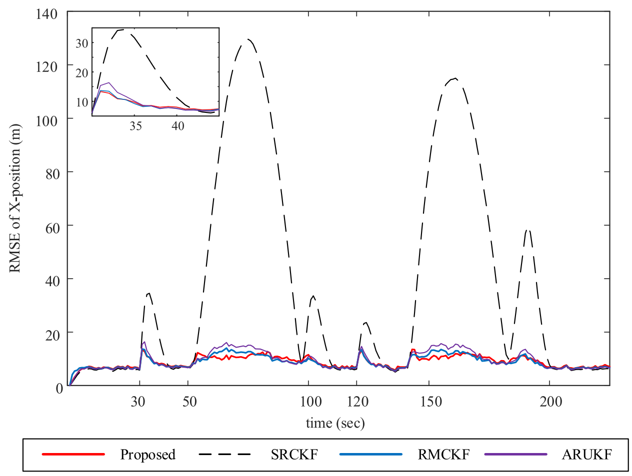

FIGURE 4. RMSEs of X-position in the first scenario.

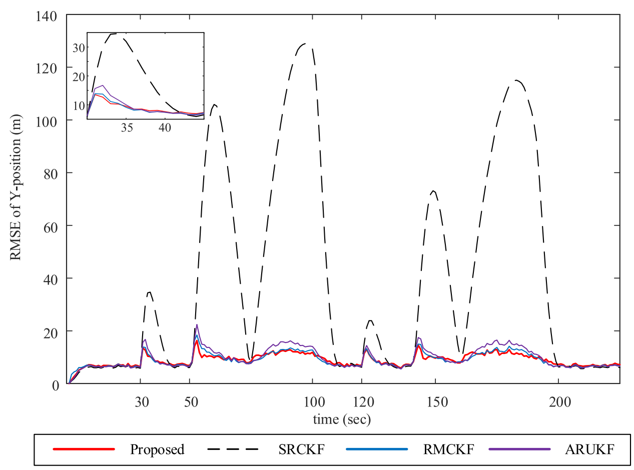

FIGURE 5. RMSEs of Y-position in the first scenario.

TABLE 1. Comparison of the position tracking performances and running time in the first scenario.

Algorithm	Mean RMSE of X-position (m)	Mean RMSE of Y-position (m)	Running time (s)
Proposed	19.8013	20.1521	6.4533
SRCKF	87.2316	91.3877	5.2257
RMCKF	19.8024	20.2238	12.2284
ARUKF	21.0655	21.6242	12.0186

As shown in Table 1, the estimation accuracy of the proposed algorithm is similar to that of the RMCKF and higher than that of the ARUKF. Additionally, compared with the SRCKF, the running time of the RMCKF and the ARUKF increases by 134.01% and 129.99% respectively, while the proposed algorithm only increases by 22.51%. The proposed algorithm has advantages in running time and computational load due to two reasons: 1) The proposed algorithm amends the predicted state directly and avoids the calculation of the fading factor. 2) The proposed algorithm amends the measurement noise covariance matrix based on the principle of the innovation covariance matching. However, the robust correction in RMCKF and ARUKF involves a lot of matrix inversion operations. The advantages will become more pronounced as the dimensions of state vector and measurement vector increase.

The scenario verifies the effectiveness of the proposed algorithm in the presence of model errors (caused by the model mismatch or the target state mutations). By setting the reasonable amending threshold and amending criterion, the proposed algorithm directly amends the predicted state. Finally, the proposed algorithm not only obtains high filtering accuracy but also has obvious advantages in running time and computational load.

B. MEASUREMENT OUTLIERS

In the second scenario, the model errors (the same as those in the first scenario) and the measurement outliers are introduced simultaneously, i.e., $z_{30} = z_{30} + [300\ m,\ 300\ m/s,\ 300\ m,\ 300\ m/s]^T$, $z_{70} = z_{70} + [400\ m,\ 400\ m/s,\ 400\ m,\ 400\ m/s]^T$, $z_{120} = z_{120} + [500\ m,\ 500\ m/s,\ 500\ m,\ 500\ m/s]^T$. In addition, the measurements during 200 s $\sim$ 202 s are set to zero to introduce the continuous measurement outliers.

The comparison of RMSE position curves among the four algorithms are depicted in Fig. 6 and Fig. 7. And the corresponding mean RMSEs are listed in Table 2.

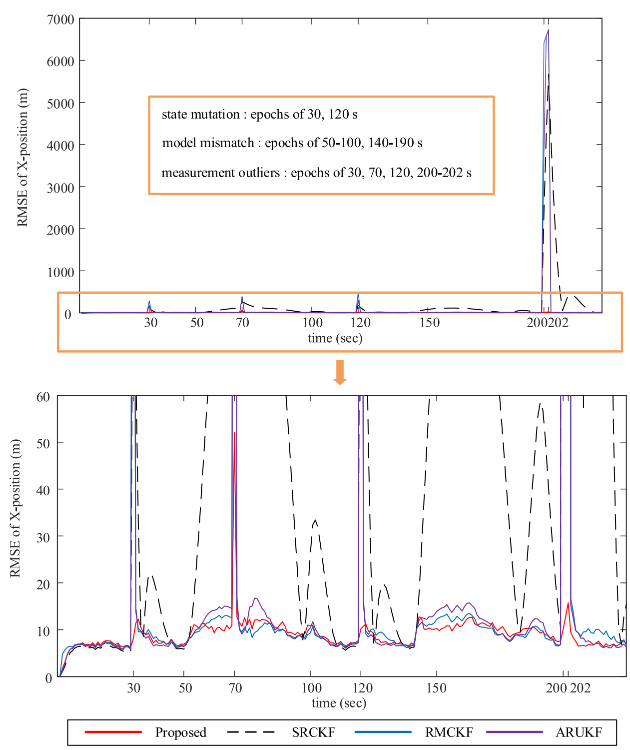

FIGURE 6. RMSEs of X-position in the third scenario.

As clearly seen from Fig. 6, Fig. 7 and Table 2, the curves of the RMCKF, the ARUKF, and the SRCKF exhibit large jumps at the moment when the measurement outliers are introduced. And the RMCKF and the ARUKF can recover

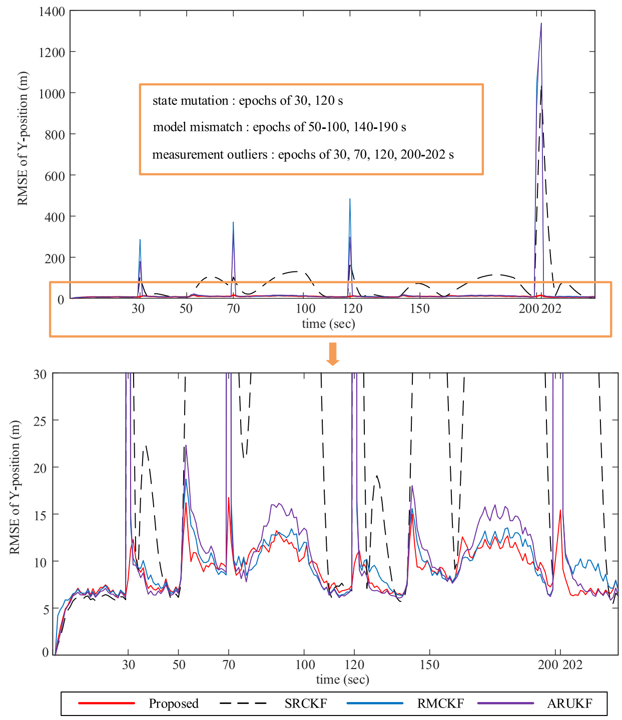

FIGURE 7. RMSEs of Y-position in the third scenario.

TABLE 2. Comparison of the position tracking performances in the third scenario.

Algorithm	Mean RMSE of X-position (m)	Mean RMSE of Y-position (m)
Proposed	20.4821	20.3054
SRCKF	353.8406	143.3784
RMCKF	228.7952	67.7731
ARUKF	210.1556	63.5655

more quickly and have higher accuracy than the SRKCF. However, the RMSE curves of the RMCKF and the ARUKF have even higher peaks than SRCKF at the moment when the model errors and measurement outliers occur simultaneously. This simulation phenomenon also appeared in [28] and [29] which verifies the previous analysis; that is, the effect of the fading factor will make an unwanted offset for the effect of the modified measurement covariance matrix.

By contrast, the proposed algorithm can not only converge rapidly but also maintain higher filtering accuracy and stability. And there are no large jumps of the curves like the other algorithms because the proposed algorithm can make the best use of the contributions of the model information and the measurements correctly based on the amending threshold and amending criterion. As seen from the mean RMSEs in Table 2, the proposed algorithm has obviously better estimation performance. It can be further found that the proposed algorithm is affected by the simultaneous occurrence of the model mismatch and measurement outliers at 70 s. The simultaneous occurrence of the state mutations and measurement outliers at 30 s and 120 s have little influence on the filtering process. And the proposed algorithm stays robust to the continuous measurement outliers during 200 $\sim$ 202 s.

V. CONCLUSION

In this paper, a novel adaptively robust SRCKF algorithm is proposed aiming to obtain good estimation performance in the presence of model errors and measurement outliers simultaneously. Based on the error analysis, the judgment criterion and the amending criterion related to the innovation are set. Then the measurement noise covariance matrix is amended based on the principle of the innovation covariance matching to suppress the measurement outliers influence, and the new method that the predicted state is amended directly based on the innovation is combined to deal with model errors. Finally, the proposed algorithm can balance the effect of the prior predicted value and the posterior feedback measurement value on the filtering process and reduce the state estimation error.

The proposed algorithm is evaluated in two scenarios and the simulation results demonstrate that: 1) The estimation accuracy of the proposed algorithm is similar to that of the RMCKF and higher than that of the ARUKF and the SRCKF in the presence of model errors. Besides, the proposed algorithm obtains obviously decreasing running time. Therefore, the adaptive amendment of the predicted state based on the innovation is effective and can be an alternative to the introduction of the fading factor. 2) The proposed algorithm can not only converge rapidly but also maintain higher filtering accuracy and stability than the other algorithms even when the model errors and measurement outliers occur simultaneously.

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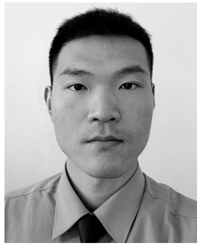

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