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Exponential Stability of Neutral-Type Cohen-Grossberg Neural Networks With Multiple Time-Varying Delays

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ABSTRACT This paper deals with the problem for exponential stability of a more general class of neutral-type Cohen-Grossberg neural networks. This class of neutral-type Cohen-Grossberg neural networks involves multiple time-varying delays in the states of neurons and multiple time-varying neutral delays in the time derivatives of the states of neurons. Such neural system cannot be described in the vector-matrix forms due to the existence of the multiple delays. The linear matrix inequality approach cannot be applied to this class of neutral system to determine the stability conditions. This paper provides some sufficient conditions to guarantee the existence, uniqueness and exponential stability of the equilibrium point of the neural system by employing the homeomorphism theory, Lyapunov-Krasovskii functional and inequality techniques. The provided conditions are easy to validate and can also guarantee the global asymptotic stability of the neural system. Two remarks are given to show the provided stability conditions are less conservative than the previous results. Two instructive examples are also given to demonstrate the effectiveness of the theoretical results and compare the provided stability conditions to the previous results.

INDEX TERMS Neutral-type Cohen-Grossberg neural networks; multiple delays; exponential stability; Lyapunov-Krasovskii functional.

I. INTRODUCTION

Since Cohen-Grossberg neural network was proposed [1], it has been extensively investigated by some mathematicians, physicists and computer scientists. These scholars have quickly found that the neural network can be effectively applied in signal processing, pattern recognition, optimization and associative memories so on. These successful applications are dependent largely on the stability of the neural network [2]–[4]. The early neural network model had not the existence of time delay. Now, there is a consensus that time delays always exists because the signal transmission between neurons usually has the phenomenon of limited transmission speed or traffic congestion. Time delay has a great influence on the neural network and it can make the stable network unstable or unstable network stable. In addition, time delay exists not only in the states, but also in the derivatives of states. The time delay existing in the derivatives of states is named neutral delay. In recent years, neutral delay has been introduced into the field of neural network, which produces a kind of neural network called neutral-type neural network. Neutral-type neural network has become a new hot research topic. A number of significant stability results of neutral-type neural networks have been published, see, for example, [5]-[28] and the references therein.

Compared with [5]–[20], the neutral-type neural networks studied in this paper cannot be transformed into the vector-matrix form due to the existence of the multiple delays. As pointed out by [21] and [22], it is not possible to derive stability conditions of the linear matrix inequality forms for the neutral-type neural networks that cannot be expressed in the vector-matrix form. Therefore, we need to construct new Lyapunov-Krasovskii functional and develop new mathematical methods and techniques to obtain stability

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conditions. At the same time, it is noted that the stability results in [5]–[28] only give the sufficient conditions of global asymptotic stability and do not further provide the sufficient conditions of exponential stability, which indicates that the exponential stability of the neutral-type neural networks has not been paid enough attention. These facts have been the main motivations of this paper to focus on the exponential stability of the neutral-type Cohen-Grossberg neural networks with multiple time-varying delays. This paper constructs a moderate Lyapunov-Krasovskii functional and employs inequality techniques to derive new algebraic sufficient conditions to ensure the exponential stability of the neutral-type Cohen-Grossberg neural networks with multiple time-varying delays. The new algebraic conditions are also the sufficient conditions for the global asymptotic stability of the neutral-type Cohen-Grossberg neural networks with multiple time-varying delays. Two instructive examples are provided to indicate that the proposed results reveal new sufficient stability criteria when they are compared with the previously published stability results. Therefore, the proposed stability results enlarge the application domain of neutral-type Cohen-Grossberg neural networks.

II. PRELIMINARIES

Consider the following neutral-type Cohen-Grossberg neural networks with multiple time-varying delays:

$$\begin{aligned}\dot{x}_i(t) = d_i(x_i(t))\Big\{ &- c_i(x_i(t)) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}g_j(x_j(t-\tau_{ij}(t))) + u_i\Big\} \\ &+ \sum_{j=1}^{n} e_{ij}\dot{x}_j(t-\xi_{ij}(t)), \end{aligned} \tag{1}$$

where u_i is external input, e_{ij} are coefficients of the time derivative of the delayed state, a_{ij} and b_{ij} are the strengths of the neuron interconnections. Amplification function $d_i(\cdot)$, behaved function $c_i(\cdot)$, delay functions $\tau_{ij}(\cdot)$ and $\xi_{ij}(\cdot)$, nonlinear activation functions $f_i(\cdot)$ and $g_i(\cdot)$ satisfy the following assumption:

(A_1) There exist some real numbers $\underline{c}_i, \underline{d}_i, \overline{d}_i, l_i, m_i, \xi, \tau, \overline{\xi}$ and $\overline{\tau}$, $i = 1, \ldots, n$, such that for all $x, y \in R$, $x \neq y$,

$$0 < \underline{c}_i \leq \frac{c_i(x) - c_i(y)}{x - y} = \frac{|c_i(x) - c_i(y)|}{|x - y|},$$

$$0 < \underline{d}_i \leq d_i(x) \leq \bar{d}_i, \quad 0 \leq \xi_{ij}(t) \leq \xi, \ 0 \leq \tau_{ij}(t) \leq \tau,$$

$$\dot{\xi}_{ij}(t) \leq \overline{\xi}, \quad \dot{\tau}_{ij}(t) \leq \overline{\tau},$$

$$|f_i(x) - f_i(y)| \leq l_i|x - y|, \quad |g_i(x) - g_i(y)| \leq m_i|x - y|.$$

The initial conditions are $x_i(t) = \varphi_i(t)$ and $\dot{x}_i(t) = \phi_i(t) \in C([-\max\{\tau, \xi\}, 0], R)$, where $C([-\max\{\tau, \xi\}, 0], R)$ is the set of all continuous functions from $[-\max\{\tau, \xi\}, 0]$ to R.

Remark 1: Compared with [9], [21] and [25], the upper bound of $\frac{c_i(x)-c_i(y)}{x-y}$ is not required, which implies that our conditions are less conservative.

System (1) is a general mathematical expression and includes some neural networks considered in the existed references. For example, system (1) includes the following system studied in [23]

$$\begin{aligned}\dot{x}_i(t) = d_i(x_i(t))\Big\{ &- c_i(x_i(t)) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_{ij}(t))) + u_i\Big\} \\ &+ \sum_{j=1}^{n} e_{ij}\dot{x}_j(t-\xi_j), \end{aligned} \tag{2}$$

the following system studied in [21] and [25]

$$\begin{aligned}\dot{x}_i(t) = d_i(x_i(t))\Big\{ &- c_i(x_i(t)) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}g_j(x_j(t-\tau_{ij})) + u_i\Big\} \\ &+ \sum_{j=1}^{n} e_{ij}\dot{x}_j(t-\xi_{ij}), \end{aligned} \tag{3}$$

and the following system studied in [22]

$$\begin{aligned}\dot{x}_i(t) = &-c_i x_i(t) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_{ij})) + u_i \\ &+ \sum_{j=1}^{n} e_{ij}\dot{x}_j(t-\xi_j). \end{aligned} \tag{4}$$

Before considering the stability, we discuss the existence and uniqueness of the equilibrium point.

Lemma 1 ([22]): Suppose that the map $H(x) \in C^0$ satisfies two properties: $H(x) \neq H(y), x \neq y$ and $||H(x)|| \to \infty$ as $||x|| \to \infty$ with $x, y \in R^n$. Then, $H(x)$ is homeomorphism of R^n.

Lemma 2: Suppose that assumption (A_1) holds and there exist some positive numbers $p_1, \ldots, p_n$ and $\gamma < 1$ such that for every $i = 1, \ldots, n$,

$$\gamma p_i \underline{c}_i - \sum_{j=1}^{n} p_j[\gamma|a_{ji}|l_i + \sum_{j=1}^{n} |b_{ji}|m_i] > 0. \tag{5}$$

Then, system (1) has a unique equilibrium point $x^* = (x_1^*, \ldots, x_n^*)^T$.

Proof. If $x^* = (x_1^*, \ldots, x_n^*)^T$ is one equilibrium point of system (1), then

$$0 = d_i(x_i^*)\Big\{ - c_i(x_i^*) + \sum_{j=1}^{n} a_{ij}f_j(x_j^*) + \sum_{j=1}^{n} b_{ij}g_j(x_j^*) + u_i\Big\},$$

that is,

$$0 = -c_i(x_i^*) + \sum_{j=1}^{n} a_{ij} f_j(x_j^*) + \sum_{j=1}^{n} b_{ij} g_j(x_j^*) + u_i.$$

Define a mapping $H : R^n \to R^n$ by

$$H(x) = (h_1(x), \ldots, h_n(x))^T$$

for $x \in R^n$, where

$$h_i(x) = -c_i(x_i) + \sum_{j=1}^{n} a_{ij} f_j(x_j) + \sum_{j=1}^{n} b_{ij} g_j(x_j) + u_i, i = 1, \ldots, n.$$

For every $x, y \in R^n$ with $x \neq y$, we conclude

$$\begin{aligned}\gamma p_i sgn(x_i - y_i)[h_i(x) - h_i(y)] &\leq -\gamma p_i \underline{c}_i |x_i - y_i| + \sum_{j=1}^{n} \gamma |a_{ij}| p_i l_j |x_j - y_j| \\ &\quad + \sum_{j=1}^{n} |b_{ij}| p_i m_j |x_j - y_j|, \forall i.\end{aligned}$$

In fact, for every $i \in \{i : x_i - y_i = 0\}$,

$$\begin{aligned}0 &= \gamma p_i sgn(x_i - y_i)[h_i(x) - h_i(y)] \\ &\leq -\gamma p_i \underline{c}_i |x_i - y_i| + \sum_{j=1}^{n} \gamma |a_{ij}| p_i l_j |x_j - y_j| \\ &\quad + \sum_{j=1}^{n} |b_{ij}| p_i m_j |x_j - y_j|,\end{aligned}$$

and for every $i \in \{i : x_i - y_i \neq 0\}$,

$$\begin{aligned}&\gamma p_i sgn(x_i - y_i)[h_i(x) - h_i(y)] \\ &= -\gamma p_i sgn(x_i - y_i)[c_i(x_i) - c_i(y_i)] \\ &\quad + \sum_{j=1}^{n} \gamma a_{ij} p_i sgn(x_i - y_i)[f_j(x_j) - f_j(y_j)] \\ &\quad + \sum_{j=1}^{n} \gamma b_{ij} p_i sgn(x_i - y_i)[g_j(x_j) - g_j(y_j)] \\ &\leq -\gamma p_i sgn(x_i - y_i) \frac{c_i(x_i) - c_i(y_i)}{x_i - y_i}(x_i - y_i) \\ &\quad + \sum_{j=1}^{n} \gamma |a_{ij}| p_i l_j |x_j - y_j| \\ &\quad + \sum_{j=1}^{n} |b_{ij}| p_i m_j |x_j - y_j| \\ &\leq -\gamma p_i \underline{c}_i |x_i - y_i| + \sum_{j=1}^{n} \gamma |a_{ij}| p_i l_j |x_j - y_j| \\ &\quad + \sum_{j=1}^{n} |b_{ij}| p_i m_j |x_j - y_j|.\end{aligned}$$

Therefore, for every $x, y \in R^n$ with $x \neq y$, we have

$$\begin{aligned}&\sum_{i=1}^{n} \gamma p_i sgn(x_i - y_i)[h_i(x) - h_i(y)] \\ &\leq -\sum_{i=1}^{n} |x_i - y_i| \left\{ \gamma p_i \underline{c}_i - \sum_{j=1}^{n} \gamma |a_{ji}| p_j l_i - \sum_{j=1}^{n} |b_{ji}| p_j m_i \right\} \\ &\leq -\alpha ||x - y||_1,\end{aligned}$$

and

$$\begin{aligned}&\alpha ||x - y||_1 \\ &\leq |\sum_{i=1}^{n} \gamma p_i sgn(x_i - y_i)[h_i(x) - h_i(y)]| \\ &\leq \sum_{i=1}^{n} p_i |h_i(x) - h_i(y)| \\ &\leq \max_{1 \leq i \leq n} \{p_i\} ||H(x) - H(y)||_1, \end{aligned} \tag{6}$$

where

$$\alpha = \min_{1 \leq i \leq n} \left\{ \gamma p_i \underline{c}_i - \sum_{j=1}^{n} p_j [\gamma |a_{ji}| l_i + \sum_{j=1}^{n} |b_{ji}| m_i] \right\}.$$

From (6), we know that if $x \neq y$, then $H(x) \neq H(y)$. In addition, it follows from (6) that

$$\alpha [\max_{1 \leq i \leq n} \{p_i\}]^{-1} ||x||_1 \leq ||H(x)||_1 + ||H(0)||_1.$$

Since $||H(0)||_1$ is bounded, $||H(x)|| \to \infty$ as $||x|| \to \infty$. From Lemma 1, we know that $H(x)$ is homeomorphism of R^n, that is, system (1) has a unique equilibrium point.

III. EXPONENTIAL STABILITY

In this section, we will establish the sufficient conditions for the exponential stability of the equilibrium point of system (1) by constructing a suitable Lyapunov-Krasovskii functional and using inequality techniques. Two examples are provided to demonstrate the effectiveness of the proposed theoretical results and compare the established stability conditions to the previous results.

Theorem 1: Let $\overline{d}_j / \underline{d}_i \geq 1, i, j = 1, \ldots, n$. Suppose that assumption (A_1) holds and there exist some positive numbers $p_1, \ldots, p_n$ and $\gamma < 1$ such that for every $i = 1, 2, \ldots, n$, $\max\{\bar{\xi}, \ \bar{\tau}\} < 1 - \gamma$,

$$\gamma p_i \underline{c}_i \underline{d}_i - \sum_{j=1}^{n} p_j \overline{d}_j (\gamma |a_{ji}| l_i + |b_{ji}| m_i) > 0, \tag{7}$$

$$\gamma p_i - \sum_{j=1}^{n} p_j |e_{ji}| > 0. \tag{8}$$

Then, the equilibrium point of system (1) is exponentially stable.

Proof. From (7), we derive

$$\gamma p_i \underline{c}_i - \sum_{j=1}^{n} p_j \frac{\overline{d}_j}{\underline{d}_i}(\gamma |a_{ji}| l_i + |b_{ji}| m_i) > 0,$$

which implies that inequality (5) holds. Since Lemma 2 shows that system (1) has a unique equilibrium point $x^* = (x_1^*, \dots, x_n^*)^T$, system (1) can be transformed into

$$\begin{aligned} \dot{y}_i(t) = \widetilde{d}_i(y_i(t))\Big\{ &- \widetilde{c}_i(y_i(t)) + \sum_{j=1}^{n} a_{ij}\widetilde{f}_j(y_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}\widetilde{g}_j(y_j(t-\tau_{ij}(t)))\Big\} \\ &+ \sum_{j=1}^{n} e_{ij}\dot{y}_j(t-\xi_{ij}(t)), \ i = 1, \dots, n, \end{aligned} \tag{9}$$

where

$$\begin{aligned} y_i(t) &= x_i(t) - x_i^*, \\ \widetilde{d}_i(y_i(t)) &= d_i(y_i(t) + x_i^*), \\ \widetilde{c}_i(y_i(t)) &= c_i(y_i(t) + x_i^*) - c_i(x_i^*), \\ \widetilde{f}_j(y_j(t)) &= f_j(y_j(t) + x_j^*) - f_j(x_j^*), \\ \widetilde{g}_j(y_j(t-\tau_{ij}(t))) &= g_j(y_j(t-\tau_{ij}(t)) + x_j^*) - g_j(x_j^*). \end{aligned}$$

From (7) and (8), we know that there exists a sufficiently small positive real number λ such that for $i = 1, \dots, n$,

$$e^{\lambda\xi}\gamma + \overline{\xi} - 1 < 0, e^{\lambda\xi}\gamma + \overline{\tau} - 1 < 0, \tag{10}$$

$$\begin{aligned} 2\lambda p_i\gamma - \gamma p_i \underline{c}_i \underline{d}_i + \sum_{j=1}^{n} p_j \overline{d}_j (\gamma |a_{ji}| l_i \\ + e^{\lambda\tau}|b_{ji}| m_i) < 0. \end{aligned} \tag{11}$$

We construct the following Lyapunov-Krasovskii functional

$$\begin{aligned} V(t) = &\sum_{i=1}^{n}\sum_{j=1}^{n} p_i |e_{ij}| \int_{t-\xi_{ij}(t)}^{t} e^{\lambda(s+\xi)}|\dot{y}_j(s)|ds \\ &+ e^{\lambda\xi}\sum_{i=1}^{n}\sum_{j=1}^{n} \overline{d}_i p_i |b_{ij}| m_j \int_{t-\tau_{ij}(t)}^{t} e^{\lambda(s+\tau)}|y_j(s)|ds \\ &+ e^{\lambda(t+\xi)}\sum_{i=1}^{n}[p_i\gamma - \sum_{j=1}^{n} p_j|e_{ji}| sgn(y_i(t)) \\ &\times sgn(\dot{y}_i(t))]|y_i(t)|, \end{aligned} \tag{12}$$

and derive

$$\begin{aligned} V(t) \geq\ & e^{\lambda(t+\xi)}\sum_{i=1}^{n}[p_i\gamma - \sum_{j=1}^{n} p_j|e_{ji}| sgn(y_i(t)) \\ &\times sgn(\dot{y}_i(t))]|y_i(t)| \\ \geq\ & \min_{1\leq i\leq n}\{p_i\gamma - \sum_{j=1}^{n} p_j|e_{ji}|\} e^{\lambda(t+\xi)}\|y(t)\|_1 \\ \geq\ & e^{\lambda(t+\xi)}\sum_{i=1}^{n}[p_i\gamma - \sum_{j=1}^{n} p_j|e_{ji}|]|y_i(t)|, \end{aligned} \tag{13}$$

$$\begin{aligned} V(0) \leq\ & e^{\lambda\xi}\max_{1\leq i\leq n}\{p_i\gamma + \sum_{j=1}^{n} p_j|e_{ji}|\}\|y(0)\|_1 \\ &+ e^{\lambda\xi}\sum_{i=1}^{n}\sum_{j=1}^{n} p_i|e_{ij}| \int_{-\xi}^{0} |\dot{y}_j(s)|ds \\ &+ e^{\lambda(\xi+\tau)}\sum_{i=1}^{n}\sum_{j=1}^{n} \overline{d}_i p_i |b_{ij}| m_j \int_{-\tau}^{0} |y_j(s)|ds. \end{aligned} \tag{14}$$

Taking the Dini derivative of the first term and the time derivatives of the all other terms in the Lyapunov-Krasovskii functional $V(t)$ along the trajectories of system (9) and using (8), we have

$$\begin{aligned} \dot{V}(t) =\ & \lambda e^{\lambda(t+\xi)}\sum_{i=1}^{n}[p_i\gamma - \sum_{j=1}^{n} p_j|e_{ji}| sgn(y_i(t)) \\ &\times sgn(\dot{y}_i(t))]|y_i(t)| + e^{\lambda(t+\xi)}\sum_{i=1}^{n}[p_i\gamma \\ &- \sum_{j=1}^{n} p_j|e_{ji}| sgn(y_i(t)) sgn(\dot{y}_i(t))] sgn(y_i(t))\dot{y}_i(t) \\ &+ e^{\lambda(t+\xi)}\sum_{i=1}^{n}\sum_{j=1}^{n} p_i|e_{ij}||\dot{y}_j(t)| - (1-\dot{\xi}_{ij}(t)) \\ &\times e^{\lambda(t-\xi_{ij}(t)+\xi)}\sum_{i=1}^{n}\sum_{j=1}^{n} p_i|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))| \\ &+ e^{\lambda\xi}\sum_{i=1}^{n}\sum_{j=1}^{n} p_i\overline{d}_i|b_{ij}| m_j (e^{\lambda(t+\tau)}|y_j(t)| \\ &- (1-\dot{\tau}_{ij}(t))e^{\lambda(t-\tau_{ij}(t)+\tau)}|y_j(t-\tau_{ij}(t))|) \\ \leq\ & 2\lambda e^{\lambda(t+\xi)}\sum_{i=1}^{n} p_i\gamma|y_i(t)| \\ &+ e^{\lambda t}\sum_{i=1}^{n}\Big\{e^{\lambda\xi} p_i\gamma\, sgn(y_i(t))\dot{y}_i(t) \\ &- e^{\lambda\xi}\sum_{j=1}^{n} p_j|e_{ji}|(sgn(y_i(t)))^2|\dot{y}_i(t)| \\ &+ e^{\lambda\xi}\sum_{j=1}^{n} p_j|e_{ji}||\dot{y}_i(t)| \\ &- (1-\overline{\xi})\sum_{j=1}^{n} p_i|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))|\Big\} \\ &+ e^{\lambda\xi}\sum_{i=1}^{n}\sum_{j=1}^{n} p_i\overline{d}_i|b_{ij}| m_j (e^{\lambda(t+\tau)}|y_j(t)| \\ &- (1-\overline{\tau})e^{\lambda t}|y_j(t-\tau_{ij}(t))|). \end{aligned} \tag{15}$$

For every $y_i(t) \in R$, we conclude

$$\begin{aligned} & e^{\lambda\xi} p_i\gamma\, sgn(y_i(t))\dot{y}_i(t) - e^{\lambda\xi}\sum_{j=1}^{n} p_j|e_{ji}|(sgn(y_i(t)))^2|\dot{y}_i(t)| \\ & + e^{\lambda\xi}\sum_{j=1}^{n} p_j|e_{ji}||\dot{y}_i(t)| - (1-\overline{\xi})\sum_{j=1}^{n} p_i|e_{ij}||\dot{y}_j(t-\xi_{ij})| \end{aligned}$$

$$\leq e^{\lambda\xi}\Big\{-\gamma p_i\underline{c}_i\underline{d}_i|y_i(t)|+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}|l_j|y_j(t)|$$
$$+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}|m_j|y_j(t-\tau_{ij}(t))|\Big\}. \quad (16)$$

Actually, from (8), (10) and (A_1), we can deduce that for $y_i(t)\neq 0$,

$$e^{\lambda\xi}p_i\gamma\, sgn(y_i(t))\dot{y}_i(t)-e^{\lambda\xi}\sum_{j=1}^{n}p_j|e_{ji}|(sgn(y_i(t)))^2|\dot{y}_i(t)|$$
$$+e^{\lambda\xi}\sum_{j=1}^{n}p_j|e_{ji}||\dot{y}_i(t)|-(1-\overline{\xi})\sum_{j=1}^{n}p_i|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))|$$
$$=e^{\lambda\xi}p_i\gamma\, sgn(y_i(t))\dot{y}_i(t)-(1-\overline{\xi})\sum_{j=1}^{n}p_i|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))|$$
$$=e^{\lambda\xi}\Big\{-\gamma p_i sgn(y_i(t))\widetilde{d}_i(y_i(t))\widetilde{c}_i(y_i(t))$$
$$+\gamma p_i sgn(y_i(t))\widetilde{d}_i(y_i(t))\sum_{j=1}^{n}a_{ij}\widetilde{f}_j(y_j(t))$$
$$+\gamma p_i sgn(y_i(t))\widetilde{d}_i(y_i(t))\sum_{j=1}^{n}b_{ij}\widetilde{g}_j(y_j(t-\tau_{ij}(t)))$$
$$+\gamma p_i sgn(y_i(t))\sum_{j=1}^{n}e_{ij}\dot{y}_j(t-\xi_{ij}(t))\Big\}$$
$$-(1-\overline{\xi})\sum_{j=1}^{n}p_i|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))|$$
$$\leq e^{\lambda\xi}\Big\{-\gamma p_i sgn(y_i(t))\widetilde{d}_i(y_i(t))\frac{\widetilde{c}_i(y_i(t))y_i(t)}{y_i(t)}$$
$$+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}||\widetilde{f}_j(y_j(t))|$$
$$+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}||\widetilde{g}_j(y_j(t-\tau_{ij}(t)))|\Big\}$$
$$+(e^{\lambda\xi}\gamma+\overline{\xi}-1)p_i\sum_{j=1}^{n}|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))|$$
$$\leq e^{\lambda\xi}\Big\{-\gamma p_i\underline{c}_i\underline{d}_i|y_i(t)|+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}|l_j|y_j(t)|$$
$$+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}|m_j|y_j(t-\tau_{ij}(t))|\Big\},$$

and for $y_i(t)=0$,

$$e^{\lambda\xi}p_i\gamma\, sgn(y_i(t))\dot{y}_i(t)-e^{\lambda\xi}\sum_{j=1}^{n}p_j|e_{ji}|(sgn(y_i(t)))^2|\dot{y}_i(t)|$$
$$+e^{\lambda\xi}\sum_{j=1}^{n}p_j|e_{ji}||\dot{y}_i(t)|-(1-\overline{\xi})\sum_{j=1}^{n}p_i|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))|$$
$$=e^{\lambda\xi}\sum_{j=1}^{n}p_j|e_{ji}||\dot{y}_i(t)|-(1-\overline{\xi})\sum_{j=1}^{n}p_i|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))|$$
$$=e^{\lambda\xi}\Big\{-\sum_{j=1}^{n}p_j|e_{ji}|sgn(\dot{y}_i(t))\widetilde{d}_i(y_i(t))\widetilde{c}_i(y_i(t))$$
$$+\sum_{j=1}^{n}p_j|e_{ji}|sgn(\dot{y}_i(t))\widetilde{d}_i(y_i(t))\sum_{j=1}^{n}a_{ij}\widetilde{f}_j(y_j(t))$$
$$+\sum_{j=1}^{n}p_j|e_{ji}|sgn(\dot{y}_i(t))\widetilde{d}_i(y_i(t))\sum_{j=1}^{n}b_{ij}\widetilde{g}_j(y_j(t-\tau_{ij}(t)))$$
$$+\sum_{j=1}^{n}p_j|e_{ji}|sgn(\dot{y}_i(t))\sum_{j=1}^{n}e_{ij}\dot{y}_j(t-\xi_{ij}(t))\Big\}$$
$$-(1-\overline{\xi})\sum_{j=1}^{n}p_i|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))|$$
$$\leq e^{\lambda\xi}\Big\{-\gamma p_i\underline{c}_i\underline{d}_i|y_i(t)|+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}|l_j|y_j(t)|$$
$$+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}|m_j|y_j(t-\tau_{ij}(t))|$$
$$+p_i\gamma\sum_{j=1}^{n}|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))|\Big\}$$
$$-(1-\overline{\xi})\sum_{j=1}^{n}p_i|e_{ij}||\dot{y}_j(t-\xi_{ij}(t))|$$
$$\leq e^{\lambda\xi}\Big\{-\gamma p_i\underline{c}_i\underline{d}_i|y_i(t)|+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}|l_j|y_j(t)|$$
$$+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}|m_j|y_j(t-\tau_{ij}(t))|\Big\},$$

where $\widetilde{c}_i(y_i(t))=|y_i(t)|=\widetilde{f}_i(y_i(t))=0$ when $y_i(t)=0$.

From (10), (11), (15) and (16), we deduce

$$\dot{V}(t)\leq 2\lambda e^{\lambda(t+\xi)}\sum_{i=1}^{n}p_i\gamma|y_i(t)|$$
$$+e^{\lambda t}\sum_{i=1}^{n}e^{\lambda\xi}\Big\{-\gamma p_i\underline{c}_i\underline{d}_i|y_i(t)|$$
$$+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}|l_j|y_j(t)|$$
$$+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}|m_j|y_j(t-\tau_{ij}(t))|\Big\}$$
$$+e^{\lambda\xi}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\overline{d}_i|b_{ij}|m_j(e^{\lambda(t+\tau)}|y_j(t)|$$
$$-(1-\overline{\tau})e^{\lambda t}|y_j(t-\tau_{ij}(t))|)$$
$$\leq e^{\lambda(t+\xi)}\sum_{i=1}^{n}\Big\{2\lambda p_i\gamma|y_i(t)|-\gamma p_i\underline{c}_i\underline{d}_i|y_i(t)|$$

$$+\gamma p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}|l_j|y_j(t)|$$
$$+e^{\lambda\tau}p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}|m_j|y_j(t)|\Big\}$$
$$+(e^{\lambda\xi}\gamma+\overline{\tau}-1)e^{\lambda t}|y_j(t-\tau_{ij}(t))|)$$
$$\leq e^{\lambda(t+\xi)}\sum_{i=1}^{n}\Big\{2\lambda p_i\gamma-\gamma p_i\underline{c}_i\underline{d}_i$$
$$+\sum_{j=1}^{n}p_j\overline{d}_j(\gamma|a_{ji}|l_i+e^{\lambda\tau}|b_{ji}|m_i)\Big\}|y_i(t)|$$
$$\leq 0. \tag{17}$$

Thus, it follows from (13), (14) and (17) that there must exist a real number $M>1$ such that

$$\|x(t)-x^*\|_1\leq Me^{-\lambda t}(\sup_{t\in[-\max\{\tau,\xi\},0]}\|\varphi(t)-x^*\|_1+\sup_{t\in[-\max\{\tau,\xi\},0]}\|\phi(t)\|_1),\quad t\geq 0.$$

Obviously, it is sometimes difficult to find the values of the positive constants $p_1,\ldots,p_n$ satisfying the stability conditions of Theorem 1. Therefore, it is necessary to give a special case of Theorem 1 for $p_1=\cdots=p_n$.

Theorem 2: Let $\overline{d}_j/\underline{d}_i\geq 1, i,j=1,\ldots,n$. Suppose that assumption (A_1) holds and there exists a positive number $\gamma<1$ such that for every $i=1,2,\ldots,n$, $\max\{\overline{\xi},\ \overline{\tau}\}<1-\gamma$,

$$\gamma\underline{c}_i\underline{d}_i-\sum_{j=1}^{n}\overline{d}_j(\gamma|a_{ji}|l_i+|b_{ji}|m_i)>0, \tag{18}$$

$$\gamma-\sum_{j=1}^{n}|e_{ji}|>0. \tag{19}$$

Then, the equilibrium point of system (1) is exponentially stable.

Theorem 1 and Theorem 2 give some stability results for systems (2) and (3).

Corollary 1: Let $\overline{d}_j/\underline{d}_i\geq 1, i,j=1,\ldots,n$. Suppose that assumption (A_1) holds and there exist some positive numbers $p_1,\ldots,p_n$ and $\gamma<1$ such that for every $i=1,2,\ldots,n$, (8) holds and $\overline{\tau}<1-\gamma$,

$$\gamma p_i\underline{c}_i\underline{d}_i-\sum_{j=1}^{n}p_j\overline{d}_jl_i(\gamma|a_{ji}|+|b_{ji}|)>0. \tag{20}$$

Then, the equilibrium point of system (2) is exponentially stable.

Corollary 2: Let $\overline{d}_j/\underline{d}_i\geq 1, i,j=1,\ldots,n$. Suppose that assumption (A_1) holds and there exists a positive number $\gamma<1$ such that for every $i=1,2,\ldots,n$, (19) holds and $\overline{\tau}<1-\gamma$,

$$\gamma\underline{c}_i\underline{d}_i-\sum_{j=1}^{n}\overline{d}_jl_i(\gamma|a_{ji}|+|b_{ji}|)>0. \tag{21}$$

Then, the equilibrium point of system (2) is exponentially stable.

Corollary 3: Let $\overline{d}_j/\underline{d}_i\geq 1, i,j=1,\ldots,n$. Suppose that assumption (A_1) holds and there exist some positive numbers $p_1,\ldots,p_n$ and $\gamma<1$ such that for every $i=1,2,\ldots,n$, (8) and (20) hold. Then, the equilibrium point of system (3) is exponentially stable.

Corollary 4: Let $\overline{d}_j/\underline{d}_i\geq 1, i,j=1,\ldots,n$. Suppose that assumption (A_1) holds and there exists a positive number $\gamma<1$ such that for every $i=1,2,\ldots,n$, (19) and (21) hold. Then, the equilibrium point of system (3) is exponentially stable.

For system (4), Lemma 2 and Corollary 3 give the following result.

Corollary 5: Suppose that there exist some positive numbers $l_i(i=1,\ldots,n)$ such that

$$|f_i(x)-f_i(y)|\leq l_i|x-y|, x,y\in R.$$

(i) If there exist some positive numbers $p_i(i=1,\ldots,n)$ and $\gamma<1$ such that for every $i=1,2,\ldots,n$,

$$\gamma p_ic_i-\sum_{j=1}^{n}p_jl_i(\gamma|a_{ji}|+|b_{ji}|)>0, \tag{22}$$

then system (4) has a unique equilibrium point.

(ii) If there exist some positive numbers $p_i(i=1,\ldots,n)$ and $\gamma<1$ such that for every $i=1,2,\ldots,n$, (8) and (22) hold, then the equilibrium point of system (4) is exponentially stable.

Remark 2: We know that if the equilibrium point of the system is exponentially stable, then it is also globally asymptotically stable. Therefore, our results also provide the sufficient conditions of global asymptotic stability of systems (1)-(4). In [21], [23] and [25], the authors have not discussed the existence and uniqueness of the equilibrium point and only provided the sufficient conditions of the global asymptotic stability. In [22], the author has discussed the existence, uniqueness and global asymptotic stability of the equilibrium point of system (4). However, it is easy to see that the conditions of Corollary 5 can include the criteria of in [22]. Therefore, the results in [22] can be taken as a corollary of our result.

Remark 3: Theorem 1 in [23] gives the following sufficient conditions for global asymptotic stability of system (2):

$$p_i\underline{c}_i\underline{d}_i-\sum_{j=1}^{n}p_j\overline{d}_jl_i(|a_{ji}|+\frac{|b_{ji}|}{1-\overline{\tau}})>0,$$
$$\gamma p_i-\sum_{j=1}^{n}p_j|e_{ji}|>0,\quad i=1,2,\ldots,n. \tag{23}$$

It follows from (20) and $\overline{\tau}<1-\gamma$ that

$$0\leq\sum_{j=1}^{n}p_j\overline{d}_jl_i|b_{ji}|$$
$$<\gamma(p_i\underline{c}_i\underline{d}_i-\sum_{j=1}^{n}p_j\overline{d}_jl_i|a_{ji}|)$$

$$< (1-\bar{\tau})(p_i\underline{c_i}\underline{d}_i - \sum_{j=1}^{n} p_j\bar{d}_j l_i|a_{ji}|),$$

which shows that (23) holds. Therefore, the conditions of Corollary 1 are less conservative than those of Theorem 1 in [23].

Remark 4: Theorem 1 in [21] provides the following sufficient conditions for the global asymptotic stability of system (3):

$$\begin{aligned}\sigma_i = 2\underline{c_i}\underline{d_i} &- \sum_{j=1}^{n}(\bar{d}_i l_j|a_{ij}| + \bar{d}_j l_i|a_{ji}|)\\ &- \sum_{j=1}^{n}(\bar{d}_i l_j|b_{ij}| + \bar{d}_j l_i|b_{ji}|)\\ &- \sum_{j=1}^{n}(\bar{d}_i\bar{c}_i|e_{ij}| + \bar{d}_j\bar{c}_j|e_{ji}|)\\ &- \sum_{j=1}^{n}\sum_{k=1}^{n}(\bar{d}_i l_i|a_{ki}||e_{kj}| + \bar{d}_k l_i|b_{ki}||e_{kj}|)\\ &- \sum_{j=1}^{n}\sum_{k=1}^{n}(\bar{d}_j l_k|a_{jk}||e_{ji}| + \bar{d}_j l_k|b_{jk}||e_{ji}|) > 0,\end{aligned}$$

and $1-\sum_{j=1}^{n}|e_{ji}| > 0, i = 1,\ldots,n$, where $0 < \underline{c}_i \le \frac{c_i(x)-c_i(y)}{x-y} \le \bar{c}_i$. Example 2 indicates the above conditions $\sigma_i > 0 (i = 1,\ldots,n)$ cannot be satisfied while the conditions of Corollary 3 can be satisfied.

Remark 5: Theorem 1 in [25] provides the following sufficient conditions for the global asymptotic stability of system (3):

$$\begin{aligned}\epsilon_i = \frac{\underline{c}_i^2}{l_i^2} &- \sum_{j=1}^{n}|\sum_{k=1}^{n}a_{ki}a_{kj}| - \sum_{j=1}^{n}\sum_{k=1}^{n}(|a_{ji}||b_{jk}|\\ &+|a_{ji}||e_{jk}| + |a_{jk}||b_{ji}| + |b_{ji}||e_{jk}| + |b_{ji}||b_{jk}|) > 0,\\ \varepsilon_{ij} = \frac{1}{n\underline{d}_i^2} &- \frac{1}{\bar{d}_i^2}\sum_{k=1}^{n}(|e_{ji}||e_{jk}| + |a_{jk}||e_{ji}| + |b_{jk}||e_{ji}|) > 0,\end{aligned}$$

where $i,j = 1,\ldots,n$. Example 2 indicates the above conditions $\varepsilon_{ij} > 0 (i,j = 1,\ldots,n)$ cannot be satisfied while the conditions of Corollary 3 can be satisfied.

Remark 6: In [22], the author has stated that it is not possible to derive stability conditions of the linear matrix inequality forms for the neutral-type neural network that cannot be expressed in the vector-matrix form. In [21], the author has also stated since the neutral-type neural networks cannot be expressed in the vector-matrix form, the linear matrix inequality approach cannot be applied to this class of neutral system to determine the stability conditions. Therefore, it is impossible to derive the linear matrix inequality criteria for system (1). In this case, although the criteria in this paper ignore the symbol of network parameters, the criteria are easier to verify and can guarantee the existence, uniqueness and exponential stability of the equilibrium point.

Example 1: Consider system (1) with the following system matrices and the network functions:

$$A\begin{pmatrix}1 & -1 & 1 & -1\\ -1 & 1 & 1 & -1\\ 1 & 1 & 1 & 1\\ 1 & -1 & -1 & -1\end{pmatrix},$$

$$B=\begin{pmatrix}1 & 1 & -1 & 1\\ -1 & -1 & 1 & -1\\ 1 & 1 & 1 & 1\\ -1 & -1 & -1 & 1\end{pmatrix},$$

$$E=\begin{pmatrix}0.1 & -0.1 & -0.1 & -0.1\\ -0.1 & 0.1 & 0.1 & 0.1\\ 0.1 & -0.1 & 0.1 & -0.1\\ -0.1 & -0.1 & -0.1 & 0.1\end{pmatrix},$$

$d_1(x) = 1.5 + 0.5sinx, d_2(x) = 1.5 - 0.5cosx, d_3(x) = 1.5 - 0.5sinx, d_4(x) = 1.5 + 0.5cosx, c_1(x) = c_2(x) = c_3(x) = 9x, c_4(x) = 8\ x, f_i(x) = 0.5tanh(x), g_i(x) = 0.25tanh(x), u_i = 0.5, \xi_{ij}(t) = \tau_{ij}(t) = 0.05cost + 0.05, i = j; \xi_{ij}(t) = \tau_{ij}(t) = 0.05sint + 0.05, i \neq j; i,j = 1,2,3,4.$

It is easy to calculate $\underline{c_1} = \underline{c_2} = \underline{c_3} = 9, \underline{c_4} = 8, \underline{d}_i = 1, \bar{d}_i = 2, \bar{\xi} = \bar{\tau} = 0.05, l_i = 0.5, m_i = 0.25, \sum_{j=1}^{4}|e_{ji}| = 0.4$ and

$$\gamma\underline{c_i}\underline{d}_i - \sum_{j=1}^{4}\bar{d}_j(\gamma|a_{ji}|l_i + |b_{ji}|m_i) = \begin{cases}5\gamma - 2, & i = 1,2,3;\\ 4\gamma - 2, & i = 4.\end{cases}$$

Therefore, for every $\gamma \in (0.5, 0.95)$, all conditions of Theorem 2 are satisfied.

If we choose $\gamma = 0.45, p_1 = p_2 = p_3 = 0.0762, p_4 = 0.0857$, then

$$\begin{aligned}0 &< \gamma p_i\underline{c_i}\underline{d}_i - \sum_{j=1}^{4}p_j\bar{d}_j(\gamma|a_{ji}|l_i + |b_{ji}|m_i)\\ &= 0.45\ p_i\underline{c}_i - 0.95\sum_{j=1}^{4}p_j = 0.01, \quad i = 1,2,3,4,\\ 0 &< p_i\gamma - \sum_{j=1}^{4}p_j|e_{ji}| = 0.45\ p_i - 0.1\sum_{j=1}^{4}p_j\\ &= \begin{cases}0.00286, & i = 1,2,3;\\ 0.007135, & i = 4.\end{cases}\end{aligned}$$

Thus, all conditions of Theorem 1 are satisfied.

Example 2: Consider system (3) with the following system matrices and the network functions:

$$A\begin{pmatrix}-1 & 1 & -1 & 1 & -1\\ -1 & -1 & 1 & 1 & -1\\ 1 & -1 & 1 & 1 & 1\\ -1 & 1 & -1 & -1 & -1\\ 1 & 1 & 1 & 1 & 1\end{pmatrix},$$

$$B=\begin{pmatrix}1 & -1 & 1 & -1 & 1\\ -1 & 1 & -1 & 1 & -1\\ -1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & 1\\ -1 & -1 & 1 & -1 & 1\end{pmatrix},$$

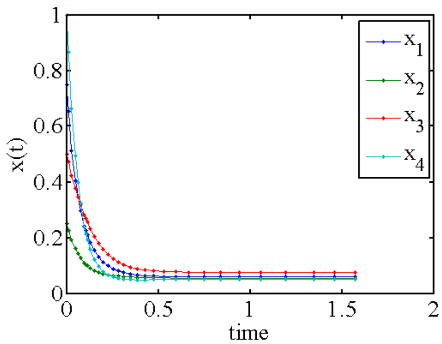

FIGURE 1. The solution trajectory of system (1) .

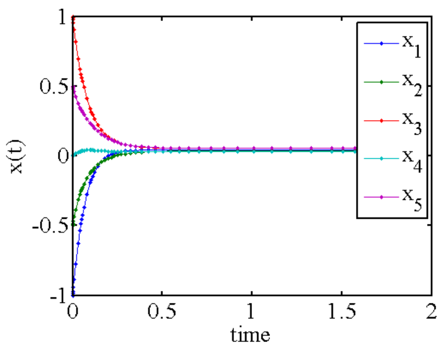

FIGURE 2. The solution trajectory of system (3) .

$$E=\begin{pmatrix} 0.1 & -0.1 & -0.1 & -0.1 & -0.1\\ -0.1 & 0.1 & 0.1 & 0.1 & 0.1\\ 0.1 & -0.1 & -0.1 & 0.1 & -0.1\\ -0.1 & -0.1 & -0.1 & 0.1 & -0.1\\ 0.1 & 0.1 & -0.1 & 0.1 & -0.1 \end{pmatrix},$$

$d_1(x) = 1.5 + 0.5sinx, d_2(x) = 1.5 - 0.5cosx, d_3(x) = d_5(x) = 1.5 - 0.5sinx, d_4(x) = 1.5 + 0.5cosx, c_1(x) = c_2(x) = c_3(x) = c_4(x) = 12x, c_5(x) = 11\ x, f_i(x) = 0.5tanh(x), u_i = 0.5, \xi_{ij} = \tau_{ij} = 0.1, i = j; \xi_{ij}(t) = \tau_{ij}(t) = 0.05, i \neq j; i, j = 1, 2, 3, 4, 5.$

In this case, we calculate $\underline{c_1} = \underline{c_2} = \underline{c_3} = \underline{c_4} = 12, \underline{c_5} = 11, \underline{d}_i = 1, \overline{d}_i = 2, l_i = 0.5, \sum_{j=1}^{5} |e_{ji}| = 0.5,$

$$\gamma\underline{c}_i\underline{d}_i - \sum_{j=1}^{5}\overline{d}_j l_i(\gamma|a_{ji}| + |b_{ji}|) = \begin{cases} 7\gamma - 5, & i = 1, 2, 3, 4;\\ 6\gamma - 5, & i = 5.\end{cases}$$

Therefore, for every $\gamma \in (5/6, 1)$, all conditions of Corollary 4 are satisfied.

If we choose $\gamma = 0.8, p_1 = p_2 = p_3 = p_4 = 0.0229, p_5 = 0.0250$, then

$$0 < \gamma p_i\underline{c}_i\underline{d}_i - \sum_{j=1}^{5} p_j\overline{d}_j l_i(\gamma|a_{ji}| + |b_{ji}|)$$
$$= 0.8\, p_i\underline{c}_i - 1.8\sum_{j=1}^{5} p_j = 0.01, i = 1, 2, 3, 4, 5,$$
$$0 < p_i\gamma - \sum_{j=1}^{5} p_j|e_{ji}| = \begin{cases} 0.00666, & i = 1, 2, 3, 4;\\ 0.00834, & i = 5.\end{cases}$$

Thus, all conditions of Corollary 3 are satisfied.

On the other hand, we calculate

$$\sigma_1 = 2\times 12 - 2\times 5 - 2\times 5 - 0.2\sum_{j=1}^{5}(\bar{c}_i + \bar{c}_j)$$
$$-0.2\times 5^2 - 0.2\times 5^2$$
$$= -0.2\sum_{j=1}^{5}(\bar{c}_i + \bar{c}_j) - 6 < 0,$$
$$\varepsilon_{ij} = \frac{1}{5} - \frac{0.21\times 5}{4} = -0.0625 < 0,$$

where $\bar{c}_1 = \bar{c}_2 = \bar{c}_3 = \bar{c}_4 = 12, \bar{c}_5 = 11$. Therefore, the conditions of in [21] and [25] cannot be satisfied.

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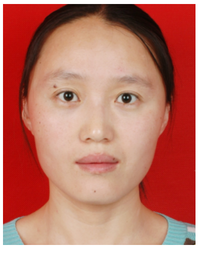

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