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Optimal Condition-Based Maintenance Strategy via an Availability-Cost Hybrid Factor for a Single-Unit System During a Two-Stage Failure Process

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ABSTRACT This paper develops an optimal condition-based maintenance (CBM) strategy for a single-unit system during two-stage failure, a process that includes a normal stage and delay-time stage. Both stages are divided into the in-control state and out-of-control state. In the in-control state, the items are always produced with 100% quality, whereas in the out-of-control state, the item quality deteriorates, and minor repairs are arranged accordingly to fix this problem. This causes the optimal CBM strategy for system production to exhibit four different scenarios, where different calculations are carried out using renewal reward theory to determine the system profit. Then, the system profit is optimized by an availability-cost hybrid factor that balances the "cost per unit time" and "availability". Finally, to investigate the effects of different decision objectives on the optimization results, a sensitivity study of the cost parameter and availability-cost weight factor is conducted on the optimized results through numerical simulations. According to the simulation results, this availability-cost hybrid factor, as well as the "cost per unit time" and "availability" factors, become less sensitive when it exceeds 0.6.

INDEX TERMS Condition-based maintenance, delay time, hybrid evolution factor, inspection, two-stage failure process.

I. INTRODUCTION

Preventive maintenance (PM) is an efficient and effective maintenance activity used to achieve high product quality and reliability for manufacturing firms [1]–[3]. Depending on the complexity of the product system, PM activities are applied with different modeling policies. For single-unit systems, time-based maintenance (TBM) models are usually established based on age-dependent PM policies or periodic PM policies [4]. These traditional TBM methods are usually implemented by simply replacing the system units at fixed time intervals, therefore causing excessive waste and increasing maintenance burden. To tackle this problem, some extensions are further applied to modify TBM models. For instance, the delay-time method is commonly used to divide the system failure process into two stages, the pristine stage and the stage at which a defect is identified, and afterward, final system failure occurs if the defect is not addressed [5]. With respect to this two-stage process, the system can be inspected periodically, and corrective replacement can be performed upon system failure. In this way, the system is eventually preventively replaced either at every N^{th} inspection or the moment at which an obvious defect is identified, whichever occurs first [6].

However, although modified TBM models exhibit good effectiveness in preventing unexpected failures, their maintenance costs are still too high for a system running long-term. This problem is due to the high frequency of maintenance intervention during periodic inspections [7]. Thus, to reduce the maintenance times and cost, condition-based

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maintenance (CBM) models are gaining popularity due to the fast development of sensing techniques. Compared to TBM models, CBM models can better describe the actual state of systems, especially those using condition monitoring technologies, with low degrees of degradation and minimal repairs [8]–[10]. Currently, there are multifarious CBM models depending on different maintenance restoration degrees, maintenance cost structures, and optimal maintenance policies [4]. Regarding maintenance restoration degrees, three types of maintenance performances are considered: perfect, imperfect, and minimal. For instance, a proactive CBM model for a deteriorating single-unit system was proposed by considering perfect maintenance and imperfect maintenance. With this model, an adaptive maintenance policy is discussed for the determination of an optimal maintenance action and time interval in between two neighboring inspection points [11]. Another example is from an imperfect CBM model developed by the determination of an optimal monitoring interval and degradation level after imperfect preventive repairs. It concluded that the optimal monitoring interval length is directly related to the monitoring cost [12]. An obvious difference between these two models is the optimality criterion, which is determined based on different cost structures, i.e., the total cost and maintenance cost rate. In addition, multilevel CBM models were developed to achieve optimal maintenance strategies for railway infrastructures [13], [14]. Model predictive control (MPC) [15] is used in two models in references [13] and [14] and is different from the CBM strategies mentioned above since it can take control actions according to future events anticipated from dynamic models of the process.

The abovementioned PM models focused on different inspection and maintenance optimization strategies but did not consider product quality. On the other hand, with ever-increasing attention on cost performance from the public, the importance of product quality has become an essential factor in PM activities [16]. Considering both maintenance and quality, the system profit can be determined by the synthetic optimization of several factors, from selling price, quality control, maintenance and reworking costs to production plan, etc., through perfect and imperfect PM polices. For instance, an integrated model was proposed to jointly optimize the imperfect CBM model, quality control and lot sizing by minimizing the total cost per unit time [17]. Another study on the CBM optimization of deteriorating production systems was carried out by jointly considering production, sampling quality control and maintenance factors [18]. Moreover, there are many studies that consider a similar combination of the above multifold factors in different system degradation models, such as the stochastic process model [19] and proportional hazard model [20]. Recently, considering the quality loss of nonconforming products, an economical optimization model was developed by integrating statistical process control and CBM under deteriorating manufacturing system conditions [21]. The lowest expected cost per unit time was produced with superior reliability. Moreover, a dynamic CBM model based on partially observable Markov decision processes was developed to investigate the effect of adjusting condition monitoring quality on the total cost for a stochastic continuous degradation process of a single-unit system [22]. Another example of the joint CBM model considering the production of nonconforming items was proposed for both serial production systems and serial-parallel multistage production systems to reduce the total cost and improve the system output at the same time [23], [24].

Regarding the degradation process, there exists a failure delay time, which is defined as the time elapsed from the occurrence of a hidden defect to failure. It offers the possibility of performing PM and removing the identified defect before failure [25]. Based on the delay-time concept, studies considering different PM strategies are being carried out. A two-state process was considered to be subject to the delay time model. The process of the system was in the in-control state in the beginning. The system may shift to the out-of-control state later, where the percentage of acceptable good-quality items produced might be low. These PM actions can be conducted by TBM strategies, but this will definitely cause excessive maintenance costs [26]. In contrast, by using the CBM strategy, the current optimization plan is mostly achieved by a single objective of purely minimizing the total cost or maximizing the product profit. However, the product process is affected by multifold aspects, such as cost, availability, and reliability. Therefore, it cannot synchronously visualize all important aspects of the product process in reality. Recently, a review of CBM models was presented, showing that multicriteria methods might be more appropriate and efficient for obtaining the optimal system maintenance policy. Some indicators are in conflict, such as cost, availability, and reliability [27].

Few multicriteria or multi-objective approaches have been investigated for joint models of quality control and preventive maintenance. By considering a multi-objective optimization problem, a multicriteria decision approach was established by using a delay-time model considering the cost and downtime preferences of decision makers, which simultaneously determined inspection intervals and features of maintainability [28]. An integrated model for a single machine was developed by considering schedule, availability, repair time and detection time as constraints to optimize the maintenance and quality plan [29]. Based on previous reviews on the objectives of PM models used in service and manufacturing [30]–[32], cost, reliability and availability were considered the most important factors.

The aim of this study is to propose a CBM strategy via the optimization of an availability-cost hybrid factor and consideration of quality control for a single-unit system during a two-stage failure process. The remainder of this paper is organized as follows. Section II outlines the system and maintenance strategy and shows the PM process of a single-unit system in two-stage production. Section III presents a numerical case study to demonstrate the applicability of the proposed method and a sensitivity analysis of

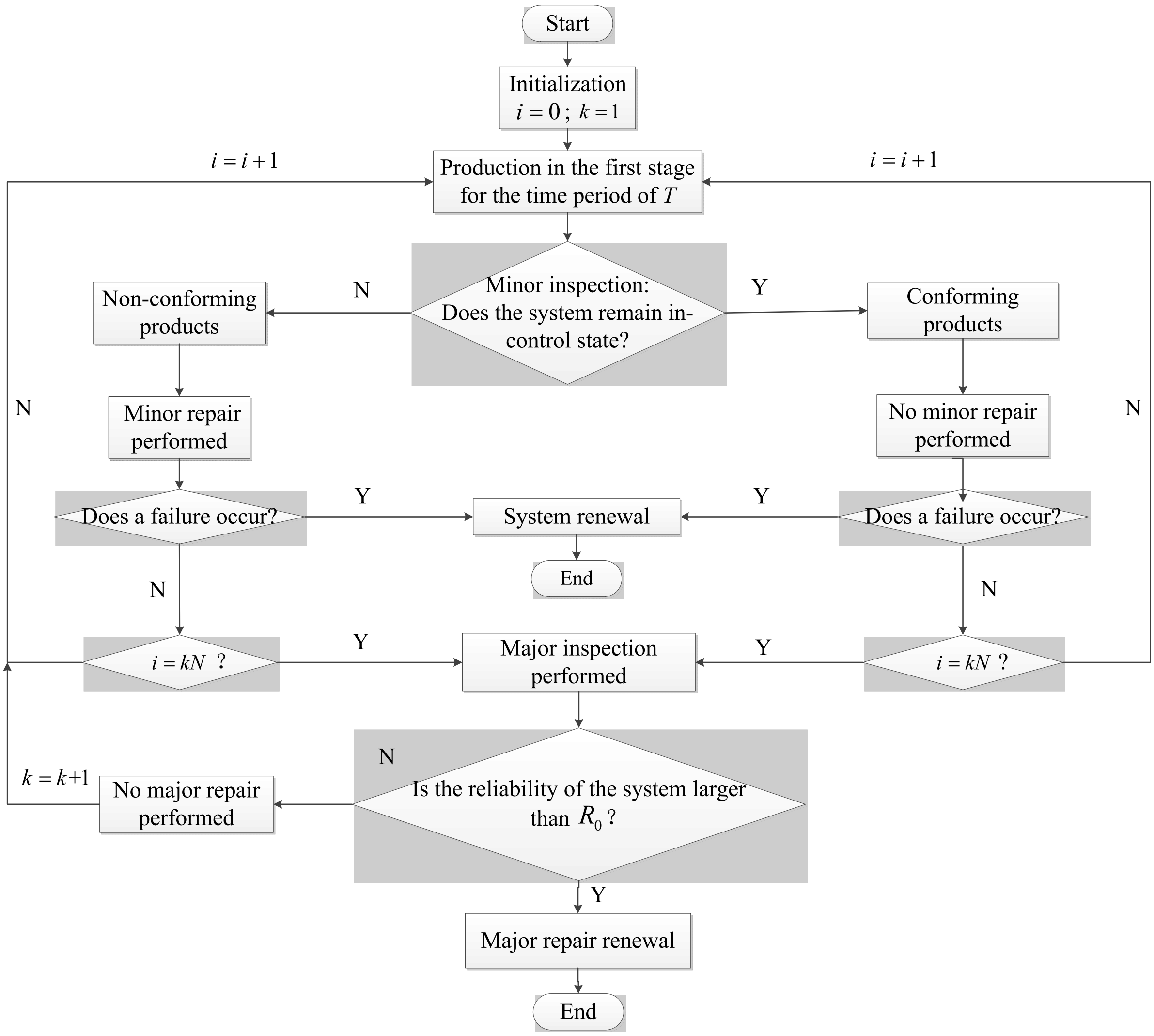

FIGURE 1. **Flow chart of the CBM strategy.**

the cost parameter and availability-cost hybrid factor performed on the optimized results, and Section IV draws major conclusions.

II. PREVENTIVE MAINTENANCE STRATEGY FOR A SINGLE-UNIT SYSTEM

A. SINGLE-UNIT SYSTEM

The single-unit system investigated in this study is a continuous production line that follows a two-stage failure process, including a normal stage and delay-time stage. In the normal stage, the system remains perfect until a major defect is initiated. The time of defect occurrence is a random variable that follows a Weibull distribution. In the delay-time stage, this defect gradually grows until failure. As soon as failure occurs, the system shuts down immediately. Only deterioration failure is considered in this model, which is relevant to the major defect.

In addition, two states (i.e., the in-control state and out-of-control state) related to product quality are considered during each stage of the failure process. At the start of the production line, the system is in the in-control state, which means that the product items are high quality. Once the production process enters the out-of-control state, the system still runs, but non-conforming items are produced, yielding a lower profit. Note that the time elapsed between these two states is a random variable in both normal and delay-time stages and follows exponential distributions.

B. MAINTENANCE STRATEGY

Figure 1 shows the flow chart of the CBM strategy proposed for the single-unit system during a two-stage failure process. Minor inspections are carried out periodically to promptly identify the state of the system. Once an out-of-control state is detected, a minor repair is carried out immediately to bring the system back to its in-control state. Furthermore, the single-unit system is monitored every N minor inspections by performing a perfect major inspection to determine if the state of the system is beyond a predetermined reliability threshold, R_0. If the system reliability drops below R_0, a major repair will be performed to replace the system; otherwise, no major repair is performed. In addition, whenever a failure occurs, a failure repair will be performed to renew the system. This That is, the system will be renewed either by a major repair or by a failure repair. In summary, the number of minor inspections N and minor inspection interval T are decision variables. In addition, minor repairs, major repairs and failure repairs will usually take some time to rectify the system. However, for the sake of simplicity, the duration of both minor inspections and major inspections are ignored, as they are short compared to the whole production process.

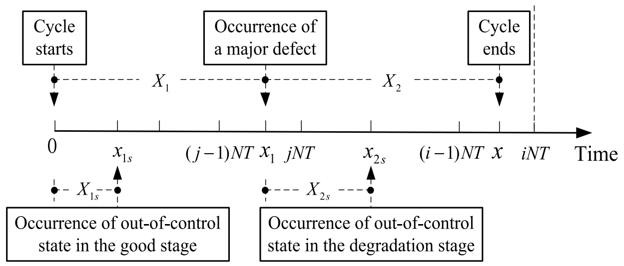

FIGURE 2. Illustration of system renewal by failure repair.

C. SYSTEM RENEWAL DUE TO FAILURE REPAIRS

This part provides the calculation approach used in the case that the single-unit system is renewed until a failure occurs, as illustrated in Figure 2. The occurrence of failure x is in the range from $(i-1)NT$ to iNT, where $i = 1, 2, 3\ldots$ and N and T are the number of periodic minor inspections and the time interval between two neighboring minor inspections, respectively. In this study, we let X_1 denote the random period of a new cycle in which a major defect is detected, and x_1 denotes the occurrence time of the major defect, where $x_1 \in [(j-1)NT, jNT]$ and $j = 1, 2, \ldots, i$. Similarly, we let X_2 denote the random period of the major defect in which a physical failure is detected. Further, X_{1s} and X_{2s} are denoted the random periods of the in-control state in the normal stage and delay-time stage, respectively.

Then, the system reliability at arbitrary time x_t can be formulated as

$$R(x_t) = \frac{1 - F_2(x_t + x_0 - x_1)}{1 - F_2(x_0 - x_1)},$$
$$x_0 = jNT, (j+1)NT, \ldots, (i-1)NT \qquad (1)$$

where x_t denotes the time point before which the system does not fail and $F_2(x_t) = P(X_2 < x_t)$ denotes the cumulative distribution function of X_2. The reliability calculated by Eq. (1) is only determined by the data collected from the delay-time stage of the system. This is because no failure occurred in the normal stage. Therefore, the reliability of the system in that stage is much higher than the predetermined reliability threshold of condition PM (i.e., R_0).

Next, we define L_1 as the total cycle length, which consists of X_1 and X_2, as shown in Figure 2, together with the repair time and corrective maintenance time. Then, the expectation of L_1 is calculated:

$$E(L_1; N, T) = \sum_{i=1}^{\infty}\sum_{j=1}^{i}[\int_{(i-1)NT}^{iNT}\int_{(j-1)NT}^{jNT}(x + T_{rf})$$
$$\times f_1(x_1)f_2(x - x_1)1(\min(R(jNT), \ldots,$$
$$\times R(i-1)NT) \geq R_0)dx_1dx] \qquad (2)$$

where $f_1(x_1)$ and $f_2(x - x_1)$ denote the probability density functions of x_1 and the remaining time to failure in the delay-time stage, respectively, and $1(*)$ denotes the indicator function, which is given by

$$1(\min(R(jNT), \ldots, R(i-1)NT) \geq R_0)$$
$$= \begin{cases} 1, & \text{if } \min R(jNT), \ldots, R(i-1)NT) \geq R_0 \text{ is true} \\ 0, & \text{otherwise} \end{cases}$$

Furthermore, the total cost of system renewal is jointly affected by the cost of minor inspections and profit, both of which are also influenced by product quality. In this way, the expected total cost of failure repair renewal can be calculated:

$$E(C_1; N, T) = \sum_{i=1}^{\infty}\sum_{j=1}^{i}\{\int_{(i-1)NT}^{iNT}\int_{(j-1)NT}^{jNT}[(i-1)(C_c + NC_i)$$
$$+ \text{int}[\frac{x - (i-1)NT}{T}]C_i + C_{rf} + E(C_{ijf})]$$
$$\times f_1(x_1)f_2(x - x_1)1(\min(R(jNT), \ldots,$$
$$\times R(i-1)NT) \geq R_0)dx_1dx\} \qquad (3)$$

where C_1 denotes the total cost of system renewal due to a failure repair, C_c denotes the average cost of system condition monitoring, C_i denotes the average cost of a minor inspection, C_{rf} denotes the average cost of a failure repair, C_{ijf} denotes the modified system profit of system renewal due to a failure repair, which refers to the expected minor repair cost minus the expected profit of system renewal due to a failure repair, and int $[*]$ denotes the integer function.

Assuming that the major defect occurred between the n-1th and nth minor inspections, the range of x_1 (i.e., the initial point of the major defect) is then narrowed down to $[(j-1)NT + (n-1)T, (j-1)NT + nT]$. Likewise, the range of x (i.e. point of failure) is narrowed down to $[(i-1)NT + (m-1)T, (i-1)NT + mT]$, where $m > n$. To simplify the expression of the following formulas, we adopt the following abbreviations:

$$\begin{cases} a_1 = (j-1)NT + (n-1)T, & a_2 = (j-1)NT + nT \\ b_1 = (i-1)NT + (m-1)T, & b_2 = (i-1)NT + mT \end{cases}$$

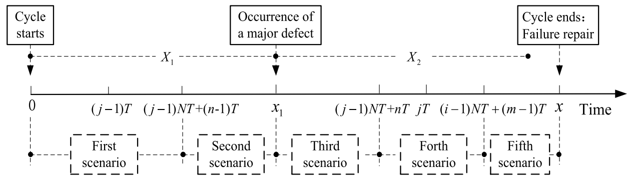

FIGURE 3. **Five scenarios of system renewal due to failure repairs, with different minor inspection intervals.**

Then, as shown in Figure 3, the entire production process of system renewal due to a failure repair is naturally divided into five scenarios, in which the C_{ijf} parameter should be calculated in different ways.

In the first scenario, calculations of the profit depend on whether or not a shift from the in-control state to the out-of-control state happens. In the case that the shift does not occur, the system profit is calculated as

$$P_{oc} = P_{in}T[1 - F_{1s}(T)] \tag{4}$$

where P_{oc} denotes the system profit, P_{in} denotes the profit of the process in the in-control state per unit time, and $[1 - F_{1s}(T)]$ denotes the probability that no shift occurs. On the contrary, as soon as the shift occurs, the system profit is then calculated as

$$P_{oc} = P_{in}\int_0^T x_{1s}f_{1s}(x_{1s})dx_{1s} + P_{out}(T - \frac{\int_0^T x_{1s}f_{1s}(x_{1s})dx_{1s}}{F_{1s}(T)})]F_{1s}(T) \tag{5}$$

where P_{out} denotes the profit of the process in the out-of-control state per unit time, and the first and second terms on the right side of Eq. (5) denote the system profit in the in-control state and out-of-control state, respectively. Furthermore, the expected total cost of minor repairs in the first scenario (denoted E_1 (C)) is given by

$$E_1(C) = [(j-1)N + n - 1]F_{1s}(T)C_{mr} \tag{6}$$

To perform a summation of Equations (4)-(6), the expected modified system profits C_{ijf} for a given interval, Ψ, can be derived as

$$\begin{aligned} &E(C_{ijf}|\psi \in (0, a_1]) \\ &\quad = [(j-1)N + n - 1] \\ &\qquad \times [F_{1s}(T)C_{mr} - \int_0^T x_{1s}f_{1s}(x_{1s})dx_{1s}(P_{in} - P_{out}) \\ &\qquad - F_{1s}(T)(P_{out} - P_{in})T - P_{in}T] \end{aligned} \tag{7}$$

Similarly, the expectations of the modified system profits for the second to fifth scenarios are sequentially calculated, as shown in the following equations.

$$E(C_{ijf}) = \begin{cases} F_{1s}(x_1 - a_1)C_{mr} - F_{1s}(x_1 - a_1)P_{out}(x_1 - a_1) \\ -(1 - F_{1s}(x_1 - a_1)) \\ \times P_{in}(x_1 - a_1) - \int_0^{x_1 - a_1} x_{1s}f_{1s}(x_{1s})dx_{1s} \\ \times (P_{in} - P_{out}), \\ \qquad \psi \in (a_1, x_1] \\ [1 - F_{2s}(a_2 - x_1)][P_{in}(a_2 - x_1) \\ +(P_{in} - P_{out})\int_0^{a_2 - x_1} x_{2s}f_{2s}(x_{2s})dx_{2s} \\ +F_{2s}(a_2 - x_1)(a_2 - x_1)(P_{out} - P_{in})], \\ \qquad \psi \in (x_1, a_2] \\ [(i-j)N + (m - n - 1)][F_{2s}(T)C_{mr} - F_{2s}(T) \\ \times (P_{out} - P_{in})T - P_{in}T \\ - \int_0^T x_{2s}f_{2s}(x_{2s})dx_{2s}(P_{in} - P_{out})], \\ \qquad \psi \in (a_2, b_1] \\ F_{2s}(x - b_1)C_{mr} - \int_0^{x - b_1} x_{2s}f_{2s}(x_{2s})dx_{2s} \\ \times (P_{in} - P_{out}) - P_{in}(x - b_1) \\ -F_{2s}(x - b_1)(P_{out} - P_{in})(x - b_1), \\ \qquad \psi \in (b_1, x] \end{cases} \tag{8}$$

D. SYSTEM RENEWAL DUE TO MAJOR REPAIRS

This part provides the calculation approach of system renewal due to major repairs when the single-unit system is renewed by condition-based preventive replacement at time point iNT, as illustrated in Figure 4.

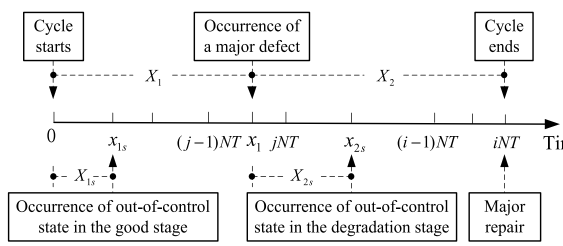

FIGURE 4. **Illustration of system renewal by a major repair.**

By defining L_2 as the cycle length of major repair renewal, the expectation of L_2 can be then calculated as

$$\begin{aligned} E(L_2; N, T) = &\sum_{i=1}^{\infty}\sum_{j=1}^{i}\int_{(j-1)NT}^{jNT}(iNT + T_{rp})f_1(x_1) \\ &\times [1 - F_2(iNT - x_1)]dx_1 1(\min(R(jNT), \ldots, \\ &\times R(i-1)NT) \geq R_0)1(R(iNT) < R_0) \end{aligned} \tag{9}$$

where $[1 - F_2(iNT - x_1)]$ denotes the survival probability of no major failure during the period $[x_1, iNT]$. The indicator function $1(*)$ shows that the reliability of the system is larger than R_0 during the period $[x_1,\ iNT)$ but smaller than R_0 at point iNT, which means that the system is renewed at iNT.

The calculation of the expected total cost formula derived for major repair renewal is similar to that derived for failure repair renewal, as given by

$$\begin{aligned} &E(C_2; N, T) \\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{i}\{\int_{(j-1)NT}^{jNT}[i(C_c+NC_i)+C_{rp}+E(C_{ijp})] \\ &\times f_1(x_1)(1-F_2(iNT-x_1))1(\min(R(jNT),\ldots, \\ &\times R(i-1)NT)\geq R_0)1(R(iNT)<R_0)dx_1\} \end{aligned} \quad (10)$$

where C_{rp} denotes the average costs of a major repair and C_{ijp} denotes the modified system profit of major repair renewal, which refers to the expected minor repair cost minus the expected profit of major repair renewal.

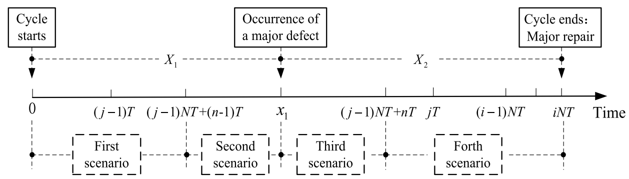

FIGURE 5. Four scenarios of system renewal by a major repair with different minor inspection intervals.

As shown in Figure 5, the entire production process of major repair renewal is divided into four scenarios for the system. The calculations of the expectations of C_{ijp} for to these four scenarios are similar to those derived in the preceding subsection, as given by

$$E(C_{ijp}) = \begin{cases} [(j-1)N+n-1][F_{1s}(T)C_{mr}-(P_{in}-P_{out}) \\ \times\int_0^T x_{1s}f_{1s}(x_{1s})dx_{1s} \\ -F_{1s}(T)(P_{out}-P_{in})T-P_{in}T], \\ \qquad \psi\in(0,a_1] \\ F_{1s}(x_1-a_1)C_{mr}-(P_{in}-P_{out})\int_0^{x_1-a_1} \\ \times x_{1s}f_{1s}(x_{1s})dx_{1s}-P_{in}(x_1-a_1) \\ -F_{1s}(x_1-a_1)(P_{out}-P_{in})(x_1-a_1), \\ \qquad \psi\in(a_1,x_1] \\ F_{2s}(a_2-x_1)C_{mr}-(P_{in}-P_{out})\int_0^{a_2-x_1} \\ \times x_{2s}f_{2s}(x_{2s})dx_{2s}-P_{in}(a_2-x_1) \\ -F_{2s}(a_2-x_1)(P_{out}-P_{in})(a_2-x_1), \\ \qquad \psi\in(x_1,a_2] \\ [(i-j+1)N-n][F_{2s}(T)C_{mr}-(P_{in}-P_{out}) \\ \times\int_0^T x_{2s}f_{2s}(x_{2s})dx_{2s} \\ -F_{2s}(T)(P_{out}-P_{in})T-P_{in}T], \\ \qquad \psi\in(a_2,iNT] \end{cases} \quad (11)$$

E. EXPECTED TOTAL COST

Based on expectation calculations with respect to system renewal due to failure repairs and major repairs, the expected cost per unit time can be further calculated according to the renewal reward theorem [33], as given by

$$E(C_\Lambda) = \frac{E(C)}{E(L)} \quad (12)$$

where C_Λ denotes the cost per unit time and C and L are the total cost and cycle length respectively. The expectation of total cycle length L is given by

$$E(L) = E(L_1; N, T) + E(L_2; N, T) \quad (13)$$

The expectation of total cost C is given by

$$E(C) = E(C_1; N, T) + E(C_2; N, T) \quad (14)$$

F. EXPECTED SYSTEM AVAILABILITY

System availability is calculated by the ratio between mean up time (MUT) and the sum of the MUT and mean down time (MDT) [34]. By denoting the length of the MUT as L_U, the expected system availability is written as

$$E(A) = \frac{E(L_U)}{E(L)} \quad (15)$$

where $E(L_U)$ is given by

$$\begin{aligned} E(L_U) &= E(L_U|\text{ renewal of failure repairs} \\ &\quad \cup \text{renewal of major repairs}) \\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{i}\{[\int_{(i-1)NT}^{iNT}\int_{(j-1)NT}^{jNT} xf_1(x_1)f_2(x-x_1) \\ &\quad \times 1(\min(R(jNT),\ldots,R(i-1)NT)\geq R_0)dx_1dx] \\ &\quad + \int_{(j-1)NT}^{jNT}(iNT)f_1(x_1)[1-F_2(iNT-x_1)]dx_1 \\ &\quad \times 1(\min(R(jNT),\ldots, \\ &\quad \times R(i-1)NT)\geq R_0)1(R(iNT)<R_0)\} \end{aligned} \quad (16)$$

G. AVAILABILITY-COST HYBRID FACTOR

In practical applications, a dual-attribute criterion that considers effects by both periodic minor inspection intervals and the number N^* is certainly more available than single-attribute criteria of CBM optimizations for a single-unit system during a two-stage failure process. Therefore, hybrid factor V, which combines the contributions of both cost and availability, is employed simultaneously to determine the minor and major inspection intervals [35], as given by

$$V = w_1\frac{C_\Lambda}{C_\Lambda^*} - (1-w_1)\frac{A}{A^*} \quad (17)$$

where A^* is the maximum system availability, C_Λ^* is the minimum cost, and w_1 is the availability-cost weight factor that reflects the preferences of the decision maker. Then, the optimized periodic minor inspection interval T^* and number N^* can be determined by the minimization of hybrid evolution factor V with preassigned w_1.

III. NUMERICAL CASE STUDY

To show the applicability of the proposed CBM strategy, a two-stage production process was simulated for a single-unit system based on the parameters provided in Wang's study [26]. Among these input parameters, X_1 and X_2 follow two different Weibull distributions, X_{1s} and X_{2s} follow two exponential distributions, and the weight factor w_1 is set

TABLE 1. Distributions for X_1 and X_2.

Distributions	Parameters
$f_1(x_1)=\alpha_1\beta_1(\alpha_1 x_1)^{\beta_1-1}e^{-(\alpha_1 x_1)^{\beta_1}}$	α_1=0.01 β_1=2.4
$f_2(x_2)=\alpha_2\beta_2(\alpha_2 x_2)^{\beta_2-1}e^{-(\alpha_2 x_2)^{\beta_2}}$	α_2=0.02 β_2=2.4
$f_{1s}(x_{1s})=\alpha_{1s}e^{-\alpha_{1s}x_{1s}}$	α_{1s}=0.1
$f_{2s}(x_{2s})=\alpha_{2s}e^{-\alpha_{2s}x_{2s}}$	α_{2s}=0.2

TABLE 2. Parameters for the CBM strategy and costs.

T_d	T_{rf}	T_{rp}	R_0	P_{in}	P_{out}
3000	16	4	0.65	120	60
C_{mr}	C_{rf}	C_{rp}	C_i	C_c	
50	1600	300	12	60	

as 0.65. The relevant parameters of these distributions are presented in Table 1. The other parameters related to the CBM strategy and costs are shown in Table 2.

The proposed CBM strategy was carried out with a desktop PC equipped with an Intel i7 8565U CPU and 8GB memory. It approximately takes 228.4 seconds to complete the optimization of CBM. Optimization of CBM for the numerical case is implemented through MATLAB scripts with the following steps.

Step 1: Assign initial values for all variables in Eq. (1) to Eq. (14) by using the parameters in Table 1 and Table 2.

Step 2: Calculate cycle lengths L_1 and L_2 according to Eq. (2) and Eq. (9), the probability of exceeding the pre-determined threshold and the survival probability of the single-unit system to determine whether the system is renewed by failure repairs or major repairs.

Step 3: Calculate the total costs according to system renewal due to failure repairs and major repairs according to Eq. (3) and Eq. (10) and determine the expected total cost per unit time by using Eq. (12).

Step 4: Calculate $E(L_U)$ by using Eq. (16) and further calculate the expected system availability by using Eq. (15).

Step 5: Calculate the maximum availability of the system A^* and the minimum cost C^*_Λ to determine availability-cost hybrid factor V. Then, the optimality of V (indicated by V^*) is obtained from the optimized T and N values (namely, T^* and N^*, respectively) for a given w_1.

By following the steps above, the expected cost per unit time, system availability and availability-cost hybrid factor are obtained and plotted in Figs. 6, 7 and 8, respectively.

As shown in Fig. 6, the minimum expected cost per unit time is observed for the situation where $N = 5$ and $T = 2$.

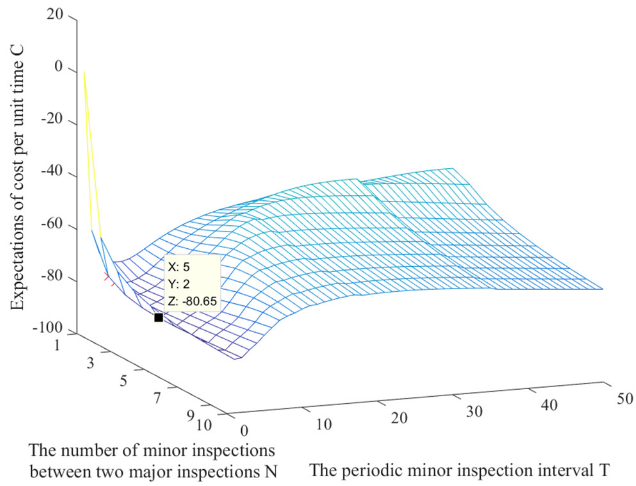

FIGURE 6. Three-dimensional plot of the expected cost per unit time.

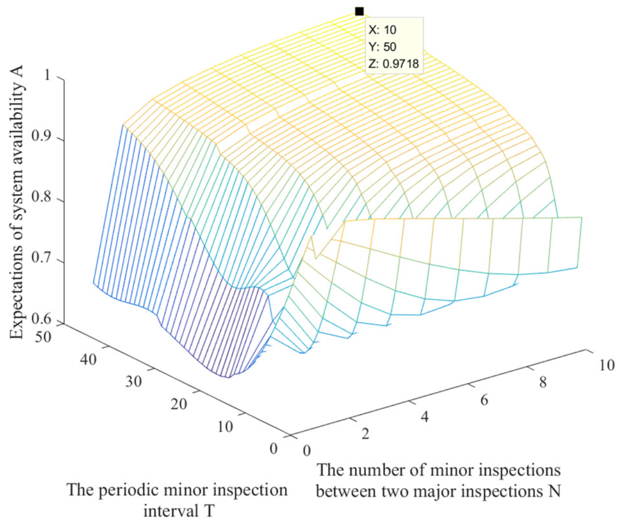

FIGURE 7. Three-dimensional plot of the expected system availability.

A major inspection should be performed after every 5 minor inspections, with a minor inspection time interval of 2.

Likewise, according to the calculation results shown in Fig. 7, the minimum expected system availability is observed for the situation where $N = 10$ and $T = 50$. In this case, a major inspection should be performed after every 10 minor inspections, with a minor inspection time interval of 50. The selection of either "cost per unit time" or "availability" as the only decision objective will result in completely different optimized CBM strategies for the single-unit system.

To reach a trade-off solution, an availability-cost hybrid factor is created by Eq. (17), where $w_1 = 0.65$ is selected as the decision objective. According to the calculation results shown in Fig. 8, the minimum expected hybrid factor is observed for the situation where $N = 4$ and $T = 2$. This optimized CBM strategy is quite similar to the one obtained from Fig. 6 where the "cost per unit time" is considered as

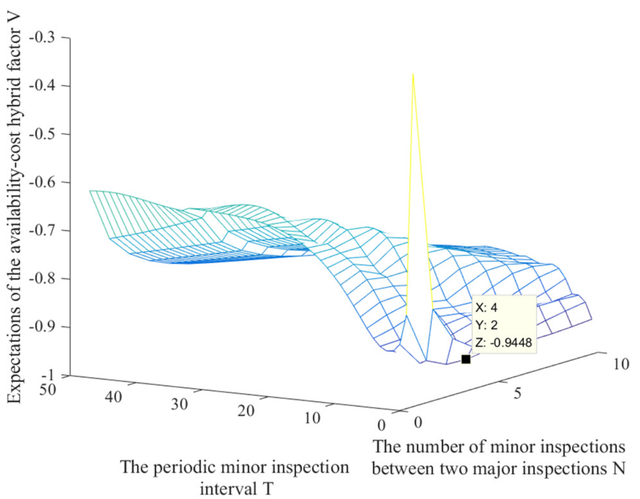

FIGURE 8. Three-dimensional plot of the expected availability-cost hybrid factor.

TABLE 3. Sensitivity analysis of costs.

C_{rf}	400	800	1200	1600	2000	2400	2800	3200
C_Λ^*	-83.7	-82.5	-81.5	-80.6	-79.8	-79.0	-78.2	-77.6
$N*$	8	7	6	5	5	5	4	4

the only decision objective. This implies that ''cost per unit time'' plays an important role in the optimization of the CBM strategy, especially when the availability-cost weight factor is chosen to be 0.65.

The influence of the predetermined cost parameters on cost optimization is first discussed by performing a sensitivity analysis by changing the average cost of a failure repair C_{rf}, which is apparently larger than other costs. Table 3 shows the changing optimal CBM strategies in terms of C_{rf}. We chose 8 different values in a board range spanning from 400 to 3200. According to Table 3, the optimum C_Λ^* and N^* values show a positive relationship and negative relationship with C_{rf}, respectively. This is because the C_{rf} parameter is directly related to failure loss. A high failure loss will result in a high cost value per unit time (i.e., an increase in C_Λ^*) and short period of major repair (i.e., a decrease in N^*).

Furthermore, the effects of the availability-cost weight factor on the optimized CBM strategy are investigated through a sensitivity analysis by increasi ng the w_1 parameter from 0 to 1 with an increment of 0.2. With reference w_1 of 0.65, the sensitivity factor $\Delta_{w_1}^V$ calculated by Eq. (18) is employed to quantitatively characterize the sensitivity of the w_1 parameter according to Ref. [36]. A high absolute value of $\Delta_{w_1}^V$ indicates that the V^* calculation is sensitive to the input w_1 parameter.

$$\Delta_{w_1}^V = \frac{V_{w_1}^* - V_{w_1=0.65}^*}{V_{w_1=0.65}^*} \tag{18}$$

where $V_{w_1}^*$ and $V_{w_1=0.65}^*$ are the optimum V value for an arbitrary given w_1 and $w_1 = 0.65$, respectively. Likewise, sensitivity factors $\Delta_{w_1}^{C_\Lambda}$ and $\Delta_{w_1}^A$ for the total cost per unit time and availability of the single-unit system can be obtained in the same way, as shown in Eq. (19) and Eq. (20), respectively.

$$\Delta_{w_1}^C = \frac{C_{w_1}^* - C_{w_1=0.65}^*}{C_{w_1=0.65}^*} \tag{19}$$

$$\Delta_{w_1}^A = \frac{A_{w_1}^* - A_{w_1=0.65}^*}{A_{w_1=0.65}^*} \tag{20}$$

where $C_{w_1}^*$ and $C_{w_1=0.65}^*$ are the minimum cost for an arbitrary given w_1 and $w_1 = 0.65$, respectively, and $A_{w_1}^*$ and $A_{w_1=0.65}^*$ are the maximum system availability values for an arbitrary given w_1 and $w_1 = 0.65$, respectively.

Table 4 compares the N^*, T^*, $\Delta_{w_1}^V$, $\Delta_{w_1}^{C_\Lambda}$ and $\Delta_{w_1}^A$ parameters calculated for all w_1 parameters spanning from 0 to 1. Both N^* and T^* parameters are significantly reduced with increasing w_1 from 0.2 to 0.4, probably because of the ever-increasing concerns of cost instead of system availability. In addition, note that $w_1 = 0.6$ also depicts a clear threshold line for the $\Delta_{w_1}^V$, $\Delta_{w_1}^{C_\Lambda}$ and $\Delta_{w_1}^A$ calculations. In circumstances where w_1 is no less than 0.6, the ''availability-cost hybrid'', ''cost per unit time'' and ''availability'' factors are not very sensitive to the selection of w_1.

TABLE 4. Sensitivity analysis of weighting factors.

W_1	$N*$	$T*$	$\Delta_{w_1}^V$	$\Delta_{w_1}^{C_\Lambda}$	$\Delta_{w_1}^A$
0	10	50	7.90%	19.49%	17.21%
0.2	10	50	1.62%	19.49%	17.21%
0.4	5	1	2.44%	5.14%	6.37%
0.6	3	2	0.70%	1.81%	3.30%
0.8	4	2	0.23%	0.00%	0.00%
1	5	2	-0.58%	-0.59%	-3.12%

Table 4 shows that the $\Delta_{w_1}^A$ parameter reaches its highest value of 17.21% when $w_1 = 0$ and $w_1 = 0.2$, both of which indicate that the availability of the system achieves its maximum. The corresponding values of $\Delta_{w_1}^{C_\Lambda}$ also achieve the highest values, which indicates that the expected costs per unit time are the highest. Then, continuously increasing w_1 results in an obvious reduction in both $\Delta_{w_1}^{C_\Lambda}$ and $\Delta_{w_1}^A$, which reflects a significant decrease in both the expected cost per unit time and the availability of the system. Therefore, the expected cost per unit time achieves its minimum when $w_1 = 1$. In the case when $w_1 = 1$, hybrid factor V is only related to C_Λ. As w_1 decreases from 1 to 0, cost factor C_Λ becomes less important while availability factor A becomes more significant for hybrid factor V. In reality, the determination of value w_1 depends on the preferences of manufacturing firms. A proper value of w_1 will be decided by the decision makers when the decision objectives of a manufacturing firm focus on both cost and system availability.

IV. CONCLUSION

This paper proposes a complete CBM strategy for a single-unit system during a two-stage (i.e., the normal stage and delay-time stage) process. In both stages, two states, namely, the in-control state when production is in a good state and the out-of-control state when low product quality and productivity result, are addressed successively for the description of product quality. The proposed CBM strategy is composed of a combination of minor inspections and condition-based major inspections. Once an out-of-control state is detected by the minor inspection, a minor repair is carried out immediately to bring the system back to its in-control state. A major inspection is conducted after every N minor inspections to calculate the system reliability. When the system reliability reaches threshold R_0, a major repair is carried out with unit replacement. In our work, the system profit is calculated by fully considering the possible scenarios in the minor and major inspections. A hybrid evolution factor that synthetically combines system "availability" with "cost per unit time" is employed for profit optimization with the proposed CBM strategy. The two key parameters, which are the total number of minor inspections between two adjacent major inspections (i.e., N) and the time interval between two adjacent minor inspections (i.e., T) in the PM policy are optimized. By assuming weight factor w_1 to be 0.65, the optimized N and T values are 4 and 2, respectively. In the end, the sensitivity study was performed on the availability-cost hybrid, cost per unit time and availability factors with an increase in w_1. w_1 becomes less sensitive when it exceeds 0.6.

APPENDIX

NOTATIONS

- X_1 the random time to the initial major defect with probability distribution function $f_1(x_1)$ and cumulative distribution function $F_1(x_1) = P(X_1 < x)$
- X_2 the random time from the initial major defect to failure with probability distribution function $f_2(x_2)$ and cumulative distribution function $F_2(x_2) = P(X_2 < x)$
- X_{1s} the random time to the shift to the out-of-control state from the beginning of the in-control state when the process is in X_1 with probability distribution function $f_{1s}(x_{1s})$ and cumulative distribution function $F_{1s}(x_{1s}) = P(X_{1s} < x)$
- X_{2s} the random time to the shift to the out-of-control state from the beginning of the in-control state when the process is in X_2 with probability distribution function $f_{2s}(x_{2s})$ and cumulative distribution function $F_{2s}(x_{2s}) = P(X_{2s} < x)$
- R_0 the predetermined threshold of reliability over which the system needs to be repaired
- T decision variable; the time interval between two neighboring minor inspections
- N decision variable; the condition-based inspection (major inspection) performed after every N minor inspections
- T_d the predesigned operational lifetime of the system
- T_{rf} time for a failure repair
- T_{rp} time for a major repair
- C_i average cost of a minor inspection
- C_c average cost of the system condition monitoring per time
- C_{mr} average cost of a minor repair
- C_{rf} average cost of a failure repair
- C_{rp} average cost of PM replacement
- P_{in} profit during an in-control state process per unit time
- P_{out} profit during an out-of-control state process per unit time

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