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Finite-Time Projective Synchronization of Stochastic Complex-Valued Neural Networks With Probabilistic Time-Varying Delays

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ABSTRACT This paper addresses finite-time projective synchronization of stochastic complex-valued neural networks (SCVNNs) with probabilistic time-varying delays (PTVs). First, in the complex domain, PTVs are introduced into the studied model and a novel feedback control scheme is constructed. Next, based on inequalities techniques and the Lyapunov stability approach, some novel projective synchronization criteria are established by decomposing SCVNNs into two equivalent real-valued systems. Moreover, a setting time function is created by employing lemma 4. Compared with previous researches, our theory content is an extension and complement to known results. Finally, numerical simulation is presented to validate the effectiveness of theoretical analysis results.

INDEX TERMS Projective synchronization, finite-time, probabilistic time-varying delays, stochastic complex-valued neural networks.

I. INTRODUCTION

In recent decades, the dynamic behaviors study of nonlinear systems have attracted the attention of many researchers [1]–[3]. In particular, the study on neural networks (NNs) has been brought into focus due to the vast potential applications in optimization [4], pattern recognition [5], image processing [6], etc. There are many articles giving the study results, such as dissipativity [7], stability [8]–[10], multistability [11], state estimation [12], and so on.

Synchronization, defined as a problem that two different dynamical signals of NNs coordinating at the same time, is always a significant research topic. Moreover, its extensive potential applications in the field of image encryption [13] and secure communication [14], have attracted more attention. During the latest 10 years, many synchronization types have been studied, namely, complete synchronization [15], anti-synchronization [16], projective synchronization [17]–[21], etc. Among them, projective synchronization is a special kind of phenomenon that shows the synchronization of NNs up to a random scaling factor, which is more general and means that complete synchronization and anti-synchronization can be considered as a special case of projective synchronization. In [19], the authors firstly studied the problem of fixed-time projective synchronization of memristive NNs via a simple controller, and the application of some lemmas greatly simplified the proof process. In [20], for the purpose of realizing projective synchronization of fractional-order system, some criteria were established by means of Gronwall-Bellman integral inequalities. In [21], based on 1-norm and p-norm, some criteria about finite-time projective synchronization were obtained by adopting a suitable Lyapunov function.

In addition, the settling time of synchronization is a hot spot. In real world experiments, finite-time property is more

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powerful, which means better interference suppression performance and fast convergence rate. Hence, for the practical applications, the finite-time (FET) synchronization of NNs is worth studying [21]–[30]. In [27], adding the memristor and reaction diffusion terms to the Cohen-Grossberg neural network, some novel formulas were proposed to ensure FET synchronization. In [28], under FET convergence theory, some FET synchronization criteria of delayed fuzzy neural networks were established. In [29], [30], they discussed the FET problem by employing the same synchronization theorem $\dot{\Im}(\theta(t)) \leq -K_1\theta - K_2\theta^{\mu}$.

It should be noticed that most practical applications of NNs are associated with complex signals, which cannot be solved by real-valued neural networks (RVNNs). However, complex-valued neural networks (CVNNs), as the promotion and generalization of RVNNs, contain complex-valued variables, which can be used to solve such problems, like XOR problem and radar signal processing. Compared with RVNNs, CVNNs is complicated and has richer dynamic characteristics. During the past several years, the dynamical analysis of CVNNs has attracted so much attention of researchers and some interesting results have been published [31]–[38]. For instance, stabilization of CVNNs was investigated via event-triggered control in [36]. In [37], instead of the tradition separation method, the FET synchronization was discussed for CVNNs by defining a new sign function and using conjugate of complex numbers. In [38], considering multiple delays, the complex projection synchronization criteria of CVNNs were derived by separating CVNNs and applying comparison principle of the fractional order systems.

As everyone knows, because of the limited transfer speed, time delays are inevitable in nearly all dynamical systems. Time delay can result in a number of issues, to be specific, instability, oscillation and poor performance of the NNs. In view of this, various time delays have been well studied in theoretical analysis, such as leakage delay [33], discrete delay [39], distributed delay [40], [41], neutral delay [42] and on the like. Probabilistic time varying delay is a special time delay that occurs in a random way owing to probabilistic reasons, which often occurs in real system. In order to better apply CVNNs, it is of practical significance to study the CVNNs with PTVs. In [43], [44], these two studies were concerned with finite-time anti-synchronization and asymptotic anti-synchronization of bidirectional associative memory NNs with PTVs by adopting a suitable Lyapunov function. In [45], by means of linear matrix inequalities, the issue about global stability for SCVNNs with PTVs was studied. Furthermore, besides time delays, stochastic perturbation (SCP) is often existent in real nervous systems. The stochastic perturbation is related to various environmental uncertainties, which is an important impact factor of system behaviour. Considering the stochastic perturbation, many interesting theories are proposed [46]–[49]. In [48], new controllers were designed to realize fixed-time synchronization of the system with SCP. In [49], the stability problem for a new type uncertain SCVNNs was discussed.

Enlightened by the above discussions, the main mission of this study is to explore the problem of FET projective synchronization of SCVNNs with PTVs. The main contents and highlights of this article are listed as follows. (1) By introducing the probabilistic time-varying delays into SCVNNs, one more general NNs is investigated. (2) Considering the stochastic factors, new projective synchronization criteria of the novel SCVNNs are derived by employing synchronization theory and inequality techniques, which casts a new light on the stochastic system. (3) In order to achieve projective synchronization, a new feedback control strategy is designed in this paper and its method applied in the process of proof is more reasonable.

The remaining parts of this article are given as below. In Section 2, the models of SCVNNs with PTVs are given and several preliminaries are provided. In Section3, to assure FET projective synchronization of the SCVNNs, novel criteria are proposed via the new feedback controller. In section 4, the feasibility of the synchronization criteria is verified by numerical simulation. In the end, the summary of this article is given in Section 5.

Notation: $\mathbb{R}$ and $\mathbb{C}$ are the real numbers and complex numbers sets respectively. $\mathbb{R}^{f\times g}$ and $\mathbb{C}^{f\times g}$ are the denotation of any $f \times g$-dimensional real-value and complex-valued matrices. $\mathbb{N}$ denotes $\{1,2,3,\ldots,N\}$. For $\forall\mu \in \mathbb{C}$, let $\mu_m = {\mu_m}^R + j\mu_m^I$, where j stands for the imaginary unite, namely, $j = \sqrt{-1}$. $\mathbb{E}(o(t))$ is the expectation of $o(t)$. $||x_i|| = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}}$. $g_{mn}^n(\cdot) = g_{mn}(t, e_m(t), e_m(t-\tau_n(t)))$.

II. PRELIMINARIES AND MODEL DESCRIPTION

In this article, we pay attention to the SCVNNs with PTVs as below

$$\begin{aligned} du_m(t) = [- c_m u_m(t) &+ \sum_{n=1}^{N} p_{mn} s_n(u_n(t)) \\ &+ \sum_{n=1}^{N} q_{mn} \ell_n(u_n(t-\tau(t)))]dt \\ &+ \sum_{m=1}^{N} g_{mn}(t, u_m(t), u_m(t-\tau(t)))d\omega_m(t). \quad (1) \end{aligned}$$

where $m = 1, 2, \ldots, N$, $u_m(t)$ is the state vector; $c_m > 0$ represents the self-feedback connection weight; p_{mn}, q_{mn} are the connection weight matrices without and with PTVs, respectively; $s(\cdot)$, $\ell(\cdot)$ are the activation function; $0 < \tau(t) < \tau_2$ is the PTVs, where τ_2 is a constant; $g_{mn}(t, u_m(t), u_m(t-\tau(t)))$ stand for the noise intensity function and $\omega_m(t)$ is the scalar standard Brownian motion defined on the complete probability space.

Taking system (1) as drive system, and this is the corresponding response system.

$$dv_m(t) = [-c_m v_m(t) + \sum_{n=1}^{N} p_{mn} s_n(v_n(t))$$

$$+\sum_{n=1}^{N} q_{mn}\ell_n(v_n(t-\tau(t)))+\rho_m(t)]dt$$
$$+\sum_{m=1}^{N} g_{mn}(t,v_m(t),v_m(t-\tau(t)))d\omega_m(t). \quad (2)$$

where $\rho_m(t)$ is control input that will be designed later. The initial conditions of systems (1) and (2) are assumed as below

$$u_m(t)=\varphi^R(t)+j\varphi^I(t), \quad t\in(-\tau,0),\ m\in\mathbb{N}.$$

and

$$v_m(t)=\psi^R(t)+j\psi^I(t), \quad t\in(-\tau,0),\ m\in\mathbb{N}.$$

respectively.

In system (1) and (2), the PTVs $\tau(t)$ takes values in $[0,\tau_1]$ and $(\tau_1,\tau_2]$ with certain probability, which can be defined as

$$Prob\{0\le\tau(t)\le\tau_1\}=\gamma_0, Prob\{\tau_1<\tau(t)\le\tau_2\}=1-\gamma_0.$$

where γ_0, τ_1, τ_2 are non-negative constant, and $0\le\gamma_0\le1$, $0\le\tau_1\le\tau_2$. Here τ_1 represents a small time delay.

$$\tau(t)=\begin{cases}\tau_1(t), & \tau(t)\in[0,\tau_1],\\ \tau_2(t), & \tau(t)\in(\tau_1,\tau_2].\end{cases}$$

Then, we state a random variable

$$\gamma(t)=\begin{cases}1, & \tau(t)\in[0,\tau_1],\\ 0, & \tau(t)\in(\tau_1,\tau_2].\end{cases}$$

Hence, we can easily get that $\gamma(t)$ is the Bernoulli distributed sequence, which satisfies the following conditions.

$$\begin{aligned}\text{Prob}\{\gamma(t)=1\}&=\text{Prob}\{\tau(t)\in[0,\tau_1]\}=\mathbb{E}(\tau(t))=\gamma_0,\\ \text{Prob}\{\gamma(t)=0\}&=\text{Prob}\{\tau(t)\in(\tau_1,\tau_2]\}\\ &=\mathbb{E}(1-\tau(t))=1-\gamma_0,\end{aligned}$$

where $\mathbb{E}(\cdot)$ represents the Mathematical expectation.

By adopting the new time delay functions $\tau_1(t)$, $\tau_2(t)$ and the random variable $\gamma(t)$, SCVNNs (1) and (2) can be written as

$$\begin{aligned}du_m(t)=[&-c_mu_m(t)+\sum_{n=1}^{N}p_{mn}s_n(u_n(t))\\ &+\gamma(t)\sum_{n=1}^{N}q_{mn}\ell_n(u_n(t-\tau_1(t)))\\ &+(1-\gamma(t))\sum_{n=1}^{N}q_{mn}\ell_n(u_n(t-\tau_2(t)))]dt\\ &+\sum_{m=1}^{N}g_{mn}(t,u_m(t),u_m(t-\tau_1(t)))d\omega_m(t)\\ &+\sum_{m=1}^{N}g_{mn}(t,u_m(t),u_m(t-\tau_2(t)))d\omega_m(t). \quad (3)\end{aligned}$$

$$\begin{aligned}dv_m(t)=[&-c_mv_m(t)+\sum_{n=1}^{N}p_{mn}s_n(v_n(t))\\ &+\gamma(t)\sum_{n=1}^{N}q_{mn}\ell_n(v_n(t-\tau_1(t)))\\ &+(1-\gamma(t))\sum_{n=1}^{N}q_{mn}\ell_n(v_n(t-\tau_2(t)))+\rho_m(t)]dt\\ &+\sum_{m=1}^{N}g_{mn}(t,v_m(t),v_m(t-\tau_1(t)))d\omega_m(t)\\ &+\sum_{m=1}^{N}g_{mn}(t,v_m(t),v_m(t-\tau_2(t)))d\omega_m(t). \quad (4)\end{aligned}$$

Assumption 1: We consider $s_n(\cdot)$, $\ell_n(\cdot)$ satisfying the following conditions

I.*The activation function $s_n(\cdot)$ and $\ell_n(\cdot)$ can be rewritten as*

$$s_n(\cdot)=s_n^R(\cdot)+js_n^I(\cdot),\ \ell_n(\cdot)=\ell_n^R(\cdot)+j\ell_n^I(\cdot).$$

where j stands for the imaginary unite, namely, $j=\sqrt{-1}$.

II.*The activation functions $s_n(\cdot)$ and $\ell_n(\cdot)$ are Lipschitz continuous. In other words, for any $n\in\mathbb{N}$, there exist positive real numbers $\hbar_n^{RR}$, $\hbar_n^{RI}$, $\hbar_n^{IR}$, $\hbar_n^{II}$ and $\lambda\!\!\!-_n^{RR}$, $\lambda\!\!\!-_n^{RI}$, $\lambda\!\!\!-_n^{IR}$, $\lambda\!\!\!-_n^{II}$ such that*

$$\begin{aligned}\left|s_n^R(v)-s_n^R(u)\right|&\le\hbar_n^{RR}\left|v^R-u^R\right|+\hbar_n^{RI}\left|v^I-u^I\right|,\\ \left|s_n^I(v)-s_n^I(u)\right|&\le\hbar_n^{IR}\left|v^R-u^R\right|+\hbar_n^{II}\left|v^I-u^I\right|,\\ \left|\ell_n^R(v)-\ell_n^R(u)\right|&\le\lambda\!\!\!-_n^{RR}\left|v^R-u^R\right|+\lambda\!\!\!-_n^{RI}\left|v^I-u^I\right|,\\ \left|\ell_n^I(v)-\ell_n^I(u)\right|&\le\lambda\!\!\!-_n^{IR}\left|v^R-u^R\right|+\lambda\!\!\!-_n^{II}\left|v^I-u^I\right|.\end{aligned}$$

III.*The activation functions $s_n(\cdot)$ and $\ell_n(\cdot)$ are bounded. That is to say, for any $n\in\mathbb{N}$, there exist positive constants A_n^1, A_n^2, B_n^1, B_n^2 such that*

$$\left|s_n^R(\cdot)\right|\le A_n^1,\ \left|s_n^I(\cdot)\right|\le A_n^2,\ \left|\ell_n^R(\cdot)\right|\le B_n^1,\ \left|\ell_n^I(\cdot)\right|\le B_n^2.$$

Therefore, system (3) and (4) can be separated as follows

$$\begin{aligned}du_m^R(t)=[&-c_mu_m^R(t)+\sum_{n=1}^{N}p_{mn}^Rs_n^R(u_n(t))\\ &-\sum_{n=1}^{N}p_{mn}^Is_n^I(u_n(t))+\gamma(t)\sum_{n=1}^{N}q_{mn}^R\ell_n^R(u_n(t-\tau_1(t)))\\ &-\gamma(t)\sum_{n=1}^{N}q_{mn}^I\ell_n^I(u_n(t-\tau_1(t)))\\ &+(1-\gamma(t))\sum_{j=1}^{n}q_{mn}^R\ell_n^R(u_n(t-\tau_2(t)))\\ &-(1-\gamma(t))\sum_{j=1}^{n}q_{mn}^I\ell_n^I(u_n(t-\tau_2(t)))]dt\\ &+\sum_{m=1}^{N}g_{mn}^R(t,u_m(t),u_m(t-\tau_1(t)))d\omega_m(t)\\ &+\sum_{m=1}^{N}g_{mn}^R(t,u_m(t),u_m(t-\tau_2(t)))d\omega_m(t). \quad (5)\end{aligned}$$

$$
\begin{aligned}
du_m^I(t) = {} & [-c_m u_m^I(t) + \sum_{n=1}^{N} p_{mn}^I s_n^R(u_n(t)) \\
& + \sum_{n=1}^{N} p_{mn}^R s_n^I(u_n(t)) + \gamma(t) \sum_{n=1}^{N} q_{mn}^I \ell_n^R(u_n(t-\tau_1(t))) \\
& + \gamma(t) \sum_{n=1}^{N} q_{mn}^R \ell_n^I(u_n(t-\tau_1(t))) \\
& + (1-\gamma(t)) \sum_{n=1}^{N} q_{mn}^I \ell_n^R(u_n(t-\tau_2(t))) \\
& + (1-\gamma(t)) \sum_{n=1}^{N} q_{mn}^R \ell_n^I(u_n(t-\tau_2(t)))]dt \\
& + \sum_{m=1}^{N} g_{mn}^I(t, u_m(t), u_m(t-\tau_1(t)))d\omega_m(t) \\
& + \sum_{m=1}^{N} g_{mn}^I(t, u_m(t), u_m(t-\tau_2(t)))d\omega_m(t). \quad (6)
\end{aligned}
$$

$$
\begin{aligned}
dv_m^R(t) = {} & [-c_m v_m^R(t) + \sum_{n=1}^{N} p_{mn}^R s_n^R(v_n(t)) \\
& - \sum_{n=1}^{N} p_{mn}^I s_n^I(v_n(t)) + \gamma(t) \sum_{n=1}^{N} q_{mn}^R \ell_n^R(v_n(t-\tau_1(t))) \\
& - \gamma(t) \sum_{n=1}^{N} q_{mn}^I \ell_n^I(v_n(t-\tau_1(t))) \\
& + (1-\gamma(t)) \sum_{n=1}^{N} q_{mn}^R \ell_n^R(v_n(t-\tau_2(t))) \\
& - (1-\gamma(t)) \sum_{n=1}^{N} q_{mn}^I \ell_n^I(v_n(t-\tau_2(t))) + \rho_m^R(t)]dt \\
& + \sum_{m=1}^{N} g_{mn}^R(t, v_m(t), v_m(t-\tau_1(t)))d\omega_m(t) \\
& + \sum_{m=1}^{N} g_{mn}^R(t, v_m(t), v_m(t-\tau_2(t)))d\omega_m(t). \quad (7)
\end{aligned}
$$

$$
\begin{aligned}
dv_m^I(t) = {} & [-c_m v_m^I(t) + \sum_{n=1}^{N} p_{mn}^I s_n^R(v_n(t)) \\
& + \sum_{n=1}^{N} p_{mn}^R s_n^I(v_n(t)) + \gamma(t) \sum_{n=1}^{N} q_{mn}^I \ell_n^R(v_n(t-\tau_1(t))) \\
& + \gamma(t) \sum_{n=1}^{N} q_{mn}^R \ell_n^I(v_n(t-\tau_1(t))) \\
& + (1-\gamma(t)) \sum_{n=1}^{N} q_{mn}^I \ell_n^R(v_n(t-\tau_2(t))) \\
& + (1-\gamma(t)) \sum_{n=1}^{N} q_{mn}^R \ell_n^I(v_n(t-\tau_2(t))) + \rho_m^I(t)]dt \\
& + \sum_{m=1}^{N} g_{mn}^I(t, v_m(t), v_m(t-\tau_1(t)))d\omega_m(t) \\
& + \sum_{m=1}^{N} g_{mn}^I(t, v_m(t), v_m(t-\tau_2(t)))d\omega_m(t). \quad (8)
\end{aligned}
$$

The synchronization errors are defined as $e_m(t) = v_m(t) - \lambda u_m(t)$, $m \in \mathbb{N}$, where constant λ is the real scaling factor. Then we can obtain as below

$$
\begin{aligned}
de_m(t) = {} & [-c_m e_m(t) + \sum_{n=1}^{N} p_{mn} s_n(v_n(t)) \\
& -\lambda \sum_{n=1}^{N} p_{mn} s_n(u_n(t)) + \gamma(t) \sum_{n=1}^{N} q_{mn} \ell_n(v_n(t-\tau_1(t))) \\
& -\lambda\gamma(t) \sum_{n=1}^{N} q_{mn} \ell_n(u_n(t-\tau_1(t))) \\
& + (1-\gamma(t)) \sum_{n=1}^{N} q_{mn} \ell_n(v_n(t-\tau_2(t))) \\
& -\lambda(1-\gamma(t)) \sum_{n=1}^{N} q_{mn} \ell_n(u_n(t-\tau_2(t))) + \rho_m(t)]dt \\
& + \sum_{m=1}^{N} g_{mn}(t, e_m(t), e_m(t-\tau_1(t)))d\omega_m(t) \\
& + \sum_{m=1}^{N} g_{mn}(t, e_m(t), e_m(t-\tau_2(t)))d\omega_m(t). \quad (9)
\end{aligned}
$$

The error system (9) can be rewritten as the real and imaginary parts, then one gets

$$
\begin{aligned}
de_m^R(t) = {} & [-c_m e_m^R(t) \\
& + \sum_{n=1}^{N} p_{mn}^R s_n^R(v_n(t)) - \sum_{n=1}^{N} p_{mn}^I s_n^I(v_n(t)) \\
& -\lambda(\sum_{n=1}^{N} p_{mn}^R s_n^R(u_n(t)) - \sum_{n=1}^{N} p_{mn}^I s_n^I(u_n(t))) \\
& +\gamma(t)(\sum_{n=1}^{N} q_{mn}^R \ell_n^R(v_n(t-\tau_1(t))) \\
& - \sum_{n=1}^{N} q_{mn}^I \ell_n^I(v_n(t-\tau_1(t)))) \\
& -\lambda\gamma(t)(\sum_{n=1}^{N} q_{mn}^R \ell_n^R(u_n(t-\tau_1(t))) \\
& - \sum_{n=1}^{N} q_{mn}^I \ell_n^I(u_n(t-\tau_1(t)))) \\
& +(1-\gamma(t))(\sum_{n=1}^{N} q_{mn}^R \ell_n^R(v_n(t-\tau_2(t)))
\end{aligned}
$$

$$-\sum_{n=1}^{N} q_{mn}^{I}\ell_{n}^{I}(v_n(t-\tau_2(t))))$$
$$-\lambda(1-\gamma(t))(\sum_{n=1}^{N} q_{mn}^{R}\ell_{n}^{R}(u_n(t-\tau_2(t)))$$
$$-\sum_{n=1}^{N} q_{mn}^{I}\ell_{n}^{I}(u_n(t-\tau_2(t)))) + \rho_m^R(t)]dt$$
$$+\sum_{m=1}^{N} g_{mn}^{R}(t, e_m(t), e_m(t-\tau_1(t)))d\omega_m(t)$$
$$+\sum_{m=1}^{N} g_{mn}^{R}(t, e_m(t), e_m(t-\tau_2(t)))d\omega_m(t) \quad (10)$$
$$de_m^I(t) = [-c_m e_m^I(t)$$
$$+\sum_{n=1}^{N} p_{mn}^{I}s_n^R(v_n(t)) + \sum_{n=1}^{N} p_{mn}^{R}s_n^I(v_n(t))$$
$$-\lambda(\sum_{n=1}^{N} p_{mn}^{I}s_n^R(u_n(t)) + \sum_{n=1}^{N} p_{mn}^{R}s_n^I(u_n(t)))$$
$$+\gamma(t)(\sum_{n=1}^{N} q_{mn}^{I}\ell_{n}^{R}(v_n(t-\tau_1(t)))$$
$$+\sum_{n=1}^{N} q_{mn}^{R}\ell_{n}^{I}(v_n(t-\tau_1(t))))$$
$$-\lambda\gamma(t)(\sum_{n=1}^{N} q_{mn}^{I}\ell_{n}^{R}(u_n(t-\tau_1(t)))$$
$$+\sum_{n=1}^{N} q_{mn}^{R}\ell_{n}^{I}(u_n(t-\tau_1(t))))$$
$$+(1-\gamma(t))(\sum_{n=1}^{N} q_{mn}^{I}\ell_{n}^{R}(v_n(t-\tau_2(t)))$$
$$+\sum_{n=1}^{N} q_{mn}^{R}\ell_{n}^{I}(v_n(t-\tau_2(t))))$$
$$-\lambda(1-\gamma(t))(\sum_{n=1}^{N} q_{mn}^{I}\ell_{n}^{R}(u_n(t-\tau_2(t)))$$
$$+\sum_{n=1}^{N} q_{mn}^{R}\ell_{n}^{I}(u_n(t-\tau_2(t)))) + \rho_m^I(t)]dt$$
$$+\sum_{m=1}^{N} g_{mn}^{I}(t, e_m(t), e_m(t-\tau_1(t)))d\omega_m(t)$$
$$+\sum_{m=1}^{N} g_{mn}^{I}(t, e_m(t), e_m(t-\tau_2(t)))d\omega_m(t). \quad (11)$$

For the stochastic system:

$$du(t) = h(t, u(t))dt + g(t, u(t))d\omega(t).$$

where $\omega(t)$ is an m-dimensional Brownian motion defined on the complete probability space and it is clearly $\mathbb{E}\omega(t) = 0$; $g(\cdot)$ is the noise intensity function; The first hitting time is denoted as $T(u_0, \omega) = \inf\{T \geq 0 | u_0 = 0, t \geq T\}$, which is the settling time function.

To further study, we have the following preparations.

Definition 1: SCVNNs (3) can achieve FET projective synchronization with system (4), if there exists a time $T^* > 0$, such that $\lim_{t\to T^*} ||e(t)|| = 0$ and $||e(t)|| \equiv 0$ for $\forall t > T^*$, where T^* is called the settling time.

Assumption 2 [43]: There exist nonnegative constants D_1 and D_2, such that

$$Trace[g^T(t, x_1, x_2)g(t, x_1, x_2)] \leq x_1 D_1 x_1 + x_2 D_2 x_2.$$

Lemma 1: Inequality of arithmetic and geometric means

$$a^2 + b^2 \geq 2ab.$$

Lemma 2 [47]: If $x_1, x_2, \ldots, x_n > 0$ and $0 < a < 1$, we obtain that

$$\sum_{i=1}^{n} X_i^a \geq (\sum_{i=1}^{n} X_i)^a.$$

Lemma 3 [43]: Let $u_1, u_2, \ldots, u_n \in R^n$ are arbitrary vectors,then the following condition satisfying

$$||u_1||^b + ||u_2||^b + \cdots + ||u_n||^b \geq (||u_1||^2 + ||u_2||^2 + \cdots + ||u_n||^2)^{b/2},$$

where $0 < b < 2$ is a real number.

Lemma 4 [29]: Suppose $\Im(\theta(t)) : R^n \to R$ is C-regular, and that $\theta(t) : [0, +\infty] \to R^n$ is absolutely continuous on any compact subinterval of $[0, +\infty]$. If $\Im(v(t))$ satisfies

$$\dot{\Im}(\theta(t)) \leq -K_1\theta - K_2\theta^{\mu},$$

where $\mu \in (0, 1)$ and $K_1, K_2 > 0$, then

$$T^* = \frac{1}{K_1(1-\mu)} \ln \frac{K_1 V^{1-\mu}(0) + K_2}{K_2}.$$

III. MAIN RESULTS

In this subsection, we obtain certain novel conditions to assure FET projective synchronization of consider SCVNNs (3) and (4) by employing lemmas in Section II.

For the purpose of achieving FET projective synchronization of SCVNNs with PTVs, we propose the following feedback controller.

$$\rho_m^R(t) = -sign(e_m^R(t))(k_1^R + k_2^R \left|e_m^R(t-\tau_1(t))\right| + k_3^R \left|e_m^R(t-\tau_2(t))\right| + k_4^R \left|e_m^R(t)\right|^{\alpha}),$$
$$\rho_m^I(t) = -sign(e_m^I(t))(k_1^I + k_2^I \left|e_m^I(t-\tau_1(t))\right| + k_3^I \left|e_m^I(t-\tau_2(t))\right| + k_4^I \left|e_m^I(t)\right|^{\alpha}). \quad (12)$$

where $0 < \alpha < 1$, $k_1^R, k_2^R, k_3^R, k_4^R, k_1^I, k_2^I, k_3^I, k_4^I$ are non-negative constants to be defined in the theorem, and $m \in \mathbb{N}$.

Theorem 1: Under Assumptions 1, the stochastic CVNNs (3) and (4) with PTVs will achieve projective synchronization in finite time if

$$
\begin{aligned}
&\zeta_1, \zeta_2, \zeta_3, \zeta_4 \le 0,\\
&\quad \phi_1, \phi_2 > 0,\\
&\quad\quad k_1^R \ge (3-\lambda)\sum_{n=1}^{N}(p_{mn}^{R*}A_n^1 + p_{mn}^{I*}A_n^2 + q_{mn}^{R*}B_n^1\\
&\quad\quad\quad + q_{mn}^{I*}B_n^2),\\
&\quad\quad k_1^I \ge (3-\lambda)\sum_{j=1}^{n}(p_{mn}^{I*}A_n^1 + p_{mn}^{R*}A_n^2 + q_{mn}^{I*}B_n^1\\
&\quad\quad\quad + q_{mn}^{R*}B_n^2).
\end{aligned}
$$

where

$$
\begin{aligned}
\zeta_1 &= (-k_2^R + D_2)/\gamma_0(t)\\
&\quad + \sum_{n=1}^{N}(q_{mn}^{R*}\bar{\lambda}_n^{RR} + q_{mn}^{I*}\bar{\lambda}_n^{IR} + q_{mn}^{I*}\bar{\lambda}_n^{RR} + q_{mn}^{R*}\bar{\lambda}_n^{IR}),\\
\zeta_2 &= (-k_2^I + E_2)/\gamma_0(t)\\
&\quad + \sum_{n=1}^{N}(q_{mn}^{R*}\bar{\lambda}_n^{RI} + q_{mn}^{I*}\bar{\lambda}_n^{II} + q_{mn}^{I*}\bar{\lambda}_n^{RI} + q_{mn}^{R*}\bar{\lambda}_n^{II}),\\
\zeta_3 &= (-k_3^R + D_4)/(1-\gamma_0(t))\\
&\quad + \sum_{n=1}^{N}(q_{mn}^{R*}\bar{\lambda}_n^{RR} + q_{mn}^{I*}\bar{\lambda}_n^{IR} + q_{mn}^{I*}\bar{\lambda}_n^{RR} + q_{mn}^{R*}\bar{\lambda}_n^{IR}),\\
\zeta_4 &= (-k_3^I + E_4)/(1-\gamma_0(t))\\
&\quad + \sum_{n=1}^{N}(q_{mn}^{R*}\bar{\lambda}_n^{RI} + q_{mn}^{I*}\bar{\lambda}_n^{II} + q_{mn}^{I*}\bar{\lambda}_n^{RI} + q_{mn}^{R*}\bar{\lambda}_n^{II}),\\
\phi_1 &= -(-2c_m - k_2^R/\gamma_0(t) - k_3^R/(1-\gamma_0(t)) + D_1 + D_3\\
&\quad + \sum_{n=1}^{N}(2p_{mn}^{R*}\hbar_n^{RR} + 2p_{mn}^{I*}\hbar_n^{IR} + p_{mn}^{R*}\hbar_n^{RI} + p_{mn}^{I*}\hbar_n^{II}\\
&\quad + p_{mn}^{I*}\hbar_n^{RR} + p_{mn}^{R*}\hbar_n^{IR} + q_{mn}^{R*}\bar{\lambda}_n^{RR} + q_{mn}^{I*}\bar{\lambda}_n^{IR}\\
&\quad + q_{mn}^{R*}\bar{\lambda}_n^{RI} + q_{mn}^{I*}\bar{\lambda}_n^{II}),\\
\phi_2 &= -(-2c_m - k_2^I/\gamma_0(t) - k_3^I/(1-\gamma_0(t)) + E_1 + E_3\\
&\quad + \sum_{n=1}^{N}(2p_{mn}^{I*}\hbar_n^{RI} + 2p_{mn}^{R*}\hbar_n^{II} + p_{mn}^{R*}\hbar_n^{RI} + p_{mn}^{I*}\hbar_n^{II}\\
&\quad + p_{mn}^{I*}\hbar_n^{RR} + p_{mn}^{R*}\hbar_n^{IR} + q_{mn}^{I*}\bar{\lambda}_n^{RR} + q_{mn}^{R*}\bar{\lambda}_n^{IR}\\
&\quad + q_{mn}^{I*}\bar{\lambda}_n^{RI} + q_{mn}^{R*}\bar{\lambda}_n^{II}).
\end{aligned}
$$

Proof: For obtaining the main results, we choose the following Lyapunov function

$$V(t) = V_1(t) + V_2(t).$$

where

$$V_1(t) = \sum_{m=1}^{N}[e_m^R(t)]^2, \quad V_2(t) = \sum_{m=1}^{N}[e_m^I(t)]^2.$$

Along with the error trajectory (10), from $V_1(t)$, we can derive

$$
\begin{aligned}
dV_1(t) &= \mathcal{L}V_1(t) + \mathcal{H}V_1(t)\\
&= \mathcal{L}V(t)dt\\
&\quad + 2\sum_{m=1}^{N} e_m^R g_{mn}^R(t, e_m(t), e_m(t-\tau(t)))d\omega_m(t).\\
\mathcal{L}V_1(t) &= 2\sum_{m=1}^{N} e_m^R(t)[-c_m e_m^R(t)\\
&\quad + \sum_{n=1}^{N} p_{mn}^R s_n^R(v_n(t)) - \lambda\sum_{n=1}^{N} p_{mn}^R s_n^R(u_n(t))\\
&\quad - \sum_{n=1}^{N} p_{mn}^I s_n^I(v_n(t)) + \lambda\sum_{n=1}^{N} p_{mn}^I s_n^I(u_n(t))\\
&\quad + \gamma(t)(\sum_{n=1}^{N} q_{mn}^R \ell_n^R(v_n(t-\tau_1(t)))\\
&\quad - \lambda\sum_{n=1}^{N} q_{mn}^R \ell_n^R(u_n(t-\tau_1(t))))\\
&\quad - \gamma(t)(\sum_{n=1}^{N} q_{mn}^I \ell_n^I(v_n(t-\tau_1(t)))\\
&\quad - \lambda\sum_{n=1}^{N} q_{mn}^I \ell_n^I(u_n(t-\tau_1(t))))\\
&\quad + (1-\gamma(t))(\sum_{n=1}^{N} q_{mn}^R \ell_n^R(v_n(t-\tau_2(t)))\\
&\quad - \lambda\sum_{n=1}^{N} q_{mn}^R \ell_n^R(u_n(t-\tau_2(t))))\\
&\quad - (1-\gamma(t))(\sum_{n=1}^{N} q_{mn}^I \ell_n^I(v_n(t-\tau_2(t)))\\
&\quad - \lambda\sum_{n=1}^{N} q_{mn}^I \ell_n^I(u_n(t-\tau_2(t)))\\
&\quad - sign(e_m^R(t))(k_1^R + k_2^R\left|e_m^R(t-\tau_1(t))\right|\\
&\quad + k_3^R\left|e_m^R(t-\tau_2(t))\right| + k_4^R\left|e_m^R(t)\right|^{\alpha})]dt\\
&\quad + \sum_{m=1}^{N} Trace((g_{mn}^{1R}(\cdot))^T g_{mn}^{1R}(\cdot))\\
&\quad + \sum_{m=1}^{N} Trace((g_{mn}^{2R}(\cdot))^T g_{mn}^{2R}(\cdot)).
\end{aligned}
$$

According to the Assumption 1, we can conclude that

$$\sum_{n=1}^{N} p_{mn}^R s_n^R(v_n(t)) - \lambda\sum_{n=1}^{N} p_{mn}^R s_n^R(u_n(t))$$

$$
\begin{aligned}
&= \sum_{n=1}^{N} p_{mn}^{R} s_{n}^{R}(v_n(t)) - \sum_{n=1}^{N} p_{mn}^{R} s_{n}^{R}(\lambda u_n(t)) \\
&\quad + \sum_{n=1}^{N} p_{mn}^{R} s_{n}^{R}(\lambda u_n(t)) - \sum_{n=1}^{N} p_{mn}^{R} s_{n}^{R}(u_n(t)) \\
&\quad + \sum_{n=1}^{N} p_{mn}^{R} s_{n}^{R}(u_n(t)) - \lambda \sum_{n=1}^{N} p_{mn}^{R} s_{n}^{R}(u_n(t)) \\
&\leq \sum_{n=1}^{N} p_{mn}^{R*}(\hbar_n^{RR} \left|e_n^R(t)\right| + \hbar_n^{RI} \left|e_n^I(t)\right|) \\
&\quad +2 \sum_{n=1}^{N} p_{mn}^{R*} A_n^1 + (1-\lambda) \sum_{n=1}^{N} p_{mn}^{R*} A_n^1.
\end{aligned}
$$

Using the same analysis method, we can get the following inequality

$$
\begin{aligned}
&\mathcal{L}V_1(t) \\
&\leq 2 \sum_{m=1}^{N} \left|e_m^R(t)\right| \{ \left|e_m^R(t)\right| (-c_m + \sum_{n=1}^{N} p_{mn}^{R*} \hbar_n^{RR} \\
&\quad + \sum_{n=1}^{N} p_{mn}^{I*} \hbar_n^{IR}) + \left|e_m^I(t)\right| (\sum_{n=1}^{N} p_{mn}^{R*} \hbar_n^{RI} + \sum_{n=1}^{N} p_{mn}^{I*} \hbar_n^{II}) \\
&\quad + \gamma(t) \left|e_n^R(t-\tau_1(t))\right| (\sum_{n=1}^{N} q_{mn}^{R*} \lambda_n^{RR} + \sum_{n=1}^{N} q_{mn}^{I*} \lambda_n^{IR}) \\
&\quad + \gamma(t) \left|e_n^I(t-\tau_1(t))\right| (\sum_{n=1}^{N} q_{mn}^{R*} \lambda_n^{RI} + \sum_{n=1}^{N} q_{mn}^{I*} \lambda_n^{II}) \\
&\quad + (1-\gamma(t)) \left|e_n^R(t-\tau_2(t))\right| (\sum_{n=1}^{N} q_{mn}^{R*} \lambda_n^{RR} + \sum_{n=1}^{N} q_{mn}^{I*} \lambda_n^{IR}) \\
&\quad + (1-\gamma(t)) \left|e_n^I(t-\tau_2(t))\right| (\sum_{n=1}^{N} q_{mn}^{R*} \lambda_n^{RI} + \sum_{n=1}^{N} q_{mn}^{I*} \lambda_n^{II}) \\
&\quad + (3-\lambda) \sum_{n=1}^{N} (p_{mn}^{R*} A_n^1 + p_{mn}^{I*} A_n^2 + q_{mn}^{R*} B_n^1 + q_{mn}^{I*} B_n^2) \\
&\quad - k_1^R - k_2^R \left|e_m^R(t-\tau_1(t))\right| - k_3^R \left|e_m^R(t-\tau_2(t))\right| \\
&\quad - k_4^R \left|e_m^R(t)\right|^{\alpha} \} + e_m^R(t) D_1 e_m^R(t) \\
&\quad + e_m^R(t-\tau_1(t)) D_2 e_m^R(t-\tau_1(t)) + e_m^R(t) D_3 e_m^R(t) \\
&\quad + e_m^R(t-\tau_2(t)) D_4 e_m^R(t-\tau_2(t)). \qquad (13)
\end{aligned}
$$

Similarly, imaginary part $V_2(t)$ along with the error system (11) is analyzed by

$$
\begin{aligned}
&dV_2(t) \\
&= \mathcal{L}V_2(t) + \mathcal{H}V_2(t) \\
&= \mathcal{L}V_2(t)dt + 2 \sum_{m=1}^{N} e_m^I g_{mn}^I(t, e_m(t), e_m(t-\tau(t))) d\omega_m(t).
\end{aligned}
$$

Then we have

$$
\begin{aligned}
&\mathcal{L}V_2(t) \\
&\leq 2 \sum_{i=1}^{n} \left|e_m^I(t)\right| \{ \left|e_m^R(t)\right| (\sum_{n=1}^{N} p_{mn}^{I*} \hbar_n^{RR} + \sum_{n=1}^{N} p_{mn}^{R*} \hbar_n^{IR}) \\
&\quad + \left|e_m^I(t)\right| (-c_m + \sum_{n=1}^{N} p_{mn}^{I*} \hbar_n^{RI} + \sum_{n=1}^{N} p_{mn}^{R*} \hbar_n^{II}) \\
&\quad + \gamma(t) \left|e_n^R(t-\tau_1(t))\right| (\sum_{n=1}^{N} q_{mn}^{I*} \lambda_n^{RR} + \sum_{n=1}^{N} q_{mn}^{R*} \lambda_n^{IR}) \\
&\quad + \gamma(t) \left|e_n^I(t-\tau_1(t))\right| (\sum_{n=1}^{N} q_{mn}^{I*} \lambda_n^{RI} + \sum_{n=1}^{N} q_{mn}^{R*} \lambda_n^{II}) \\
&\quad + (1-\gamma(t)) \left|e_n^R(t-\tau_2(t))\right| (\sum_{n=1}^{N} q_{mn}^{I*} \lambda_n^{RR} + \sum_{n=1}^{N} q_{mn}^{R*} \lambda_n^{IR}) \\
&\quad + (1-\gamma(t)) \left|e_n^R(t-\tau_2(t))\right| (\sum_{n=1}^{N} q_{mn}^{I*} \lambda_n^{RI} + \sum_{n=1}^{N} q_{mn}^{R*} \lambda_n^{II}) \\
&\quad + (3-\lambda) \sum_{j=1}^{n} (p_{mn}^{I*} A_n^1 + p_{mn}^{R*} A_n^2 + q_{mn}^{I*} B_n^1 + q_{mn}^{R*} B_n^2) \\
&\quad - k_1^I - k_2^I \left|e_m^I(t-\tau_1(t))\right| - k_3^I \left|e_m^I(t-\tau_2(t))\right| \\
&\quad - k_4^I \left|e_m^I(t)\right|^{\alpha} \} + e_m^I(t) E_1 e_m^I(t) \\
&\quad + e_m^I(t-\tau_1(t)) E_2 e_m^I(t-\tau_1(t)) + e_m^I(t) E_3 e_m^I(t) \\
&\quad + e_m^I(t-\tau_2(t)) E_4 e_m^I(t-\tau_2(t)). \qquad (14)
\end{aligned}
$$

From **Theorem** 1, equations (13) and (14), it is clear that we can derive the following inequality.

$$
\begin{aligned}
&\mathbb{E}(\mathcal{L}V_1(t) + \mathcal{L}V_2(t)) \\
&\leq 2 \sum_{m=1}^{N} \left|e_m^R(t)\right|^2 (-c_m + \sum_{n=1}^{N} p_{mn}^{R*} \hbar_n^{RR} + \sum_{n=1}^{N} p_{mn}^{I*} \hbar_n^{IR}) \\
&\quad + 2 \sum_{i=1}^{n} \left|e_i^I(t)\right|^2 (-c_m + \sum_{n=1}^{N} p_{mn}^{I*} \hbar_n^{RI} + \sum_{n=1}^{N} p_{mn}^{R*} \hbar_n^{II}) \\
&\quad + 2 \sum_{m=1}^{N} \left|e_m^R(t)\right| \left|e_n^I(t)\right| (\sum_{n=1}^{N} p_{mn}^{R*} \hbar_n^{RI} + \sum_{n=1}^{N} p_{mn}^{I*} \hbar_n^{II} \\
&\quad + \sum_{n=1}^{N} p_{mn}^{I*} \hbar_n^{RR} + \sum_{n=1}^{N} p_{mn}^{R*} \hbar_n^{IR}) \\
&\quad + 2\gamma_0(t) \sum_{m=1}^{N} \left|e_m^R(t)\right| \left|e_n^R(t-\tau_1(t))\right| (\sum_{n=1}^{N} q_{mn}^{R*} \lambda_n^{RR} \\
&\quad + \sum_{n=1}^{N} q_{mn}^{I*} \lambda_n^{IR} - k_2^R/\gamma_0(t)) \\
&\quad + 2\gamma_0(t) \sum_{m=1}^{N} \left|e_m^R(t)\right| \left|e_n^I(t-\tau_1(t))\right| (\sum_{n=1}^{N} q_{mn}^{R*} \lambda_n^{RI} \\
&\quad + \sum_{n=1}^{N} q_{mn}^{I*} \lambda_n^{II})
\end{aligned}
$$

$$+2\gamma_0(t)\sum_{m=1}^{N}\left|e_m^I(t)\right|\left|e_n^R(t-\tau_1(t))\right|(\sum_{n=1}^{N}q_{mn}^{I*}\lambda_n^{RR}$$
$$+\sum_{n=1}^{N}q_{mn}^{R*}\lambda_n^{IR})$$
$$+2\gamma_0(t)\sum_{m=1}^{N}\left|e_m^I(t)\right|\left|e_n^I(t-\tau_1(t))\right|(\sum_{n=1}^{N}q_{mn}^{I*}\lambda_n^{RI}$$
$$+\sum_{n=1}^{N}q_{mn}^{R*}\lambda_n^{II}-k_2^I/\gamma_0(t))$$
$$+2(1-\gamma_0(t))\sum_{m=1}^{N}\left|e_m^R(t)\right|\left|e_n^R(t-\tau_2(t))\right|(\sum_{n=1}^{N}q_{mn}^{R*}\lambda_n^{RR}$$
$$+\sum_{n=1}^{N}q_{mn}^{I*}\lambda_n^{IR}-k_3^R/(1-\gamma_0(t)))$$
$$+2(1-\gamma_0(t))\sum_{m=1}^{N}\left|e_m^R(t)\right|\left|e_n^I(t-\tau_2(t))\right|(\sum_{n=1}^{N}q_{mn}^{R*}\lambda_n^{RI}$$
$$+\sum_{n=1}^{N}q_{mn}^{I*}\lambda_n^{II})$$
$$+2(1-\gamma_0(t))\sum_{m=1}^{N}\left|e_m^I(t)\right|\left|e_n^R(t-\tau_2(t))\right|(\sum_{n=1}^{N}q_{mn}^{I*}\lambda_n^{RR}$$
$$+\sum_{n=1}^{N}q_{mn}^{R*}\lambda_n^{IR})$$
$$+2(1-\gamma_0(t))\sum_{m=1}^{N}\left|e_m^I(t)\right|\left|e_n^I(t-\tau_2(t))\right|(\sum_{n=1}^{N}q_{mn}^{I*}\lambda_n^{RI}$$
$$+\sum_{n=1}^{N}q_{mn}^{R*}\lambda_n^{II}-k_3^I/(1-\gamma_0(t)))$$
$$-2\sum_{m=1}^{N}k_4^R\left|e_m^R(t)\right|^{1+\alpha}-2\sum_{m=1}^{N}k_4^I\left|e_m^I(t)\right|^{1+\alpha}$$
$$+e_m^R(t)D_1e_m^R(t)+e_m^R(t-\tau_1(t))D_2e_m^R(t-\tau_1(t))$$
$$+e_m^R(t)D_3e_m^R(t)+e_m^R(t-\tau_2(t))D_4e_m^R(t-\tau_2(t))$$
$$+e_m^I(t)E_1e_m^I(t)+e_m^I(t-\tau_1(t))E_2e_m^I(t-\tau_1(t))$$
$$+e_m^I(t)E_3e_m^I(t)+e_m^I(t-\tau_2(t))E_4e_m^I(t-\tau_2(t)).$$

By utilizing **Lemma** 1, it can be obtained

$$2\sum_{m=1}^{N}\left|e_m^R(t)\right|\left|e_m^R(t-\tau_\iota(t))\right|$$
$$\leq\sum_{m=1}^{N}\left|e_m^R(t)\right|^2+\sum_{m=1}^{N}\left|e_m^R(t-\tau_\iota(t))\right|^2,$$
$$2\sum_{m=1}^{N}\left|e_m^R(t)\right|\left|e_m^I(t-\tau_\iota(t))\right|$$
$$\leq\sum_{m=1}^{N}\left|e_m^R(t)\right|^2+\sum_{m=1}^{N}\left|e_m^I(t-\tau_\iota(t))\right|^2,$$
$$2\sum_{m=1}^{N}\left|e_m^I(t)\right|\left|e_m^R(t-\tau_\iota(t))\right|$$
$$\leq\sum_{m=1}^{N}\left|e_m^I(t)\right|^2+\sum_{m=1}^{N}\left|e_m^R(t-\tau_\iota(t))\right|^2,$$
$$2\sum_{m=1}^{N}\left|e_m^I(t)\right|\left|e_m^I(t-\tau_\iota(t))\right|$$
$$\leq\sum_{m=1}^{N}\left|e_m^I(t)\right|^2+\sum_{m=1}^{N}\left|e_m^I(t-\tau_\iota(t))\right|^2.$$

where $\iota=1,2$.

Then we get

$$\mathbb{E}(\mathcal{L}V_1(t)+\mathcal{L}V_2(t))$$
$$\leq\sum_{m=1}^{N}\left|e_m^R(t)\right|^2(-2c_m-k_2^R/\gamma_0(t)-k_3^R/(1-\gamma_0(t))$$
$$+D_1+D_3+\sum_{n=1}^{N}(2p_{mn}^{R*}\hbar_n^{RR}+2p_{mn}^{I*}\hbar_n^{IR}$$
$$+p_{mn}^{R*}\hbar_n^{RI}+p_{mn}^{I*}\hbar_n^{II}+p_{mn}^{I*}\hbar_n^{RR}+p_{mn}^{R*}\hbar_n^{IR}+q_{mn}^{R*}\lambda_n^{RR}$$
$$+q_{mn}^{I*}\lambda_n^{IR}+q_{mn}^{R*}\lambda_n^{RI}+q_{mn}^{I*}\lambda_n^{II})$$
$$+\sum_{i=1}^{n}\left|e_i^I(t)\right|^2(-2c_m-k_2^I/\gamma_0(t)-k_3^I/(1-\gamma_0(t))$$
$$+E_1+E_3+\sum_{n=1}^{N}(2p_{mn}^{I*}\hbar_n^{RI}+2p_{mn}^{R*}\hbar_n^{II}$$
$$+p_{mn}^{R*}\hbar_n^{RI}+p_{mn}^{I*}\hbar_n^{II}+p_{mn}^{I*}\hbar_n^{RR}+p_{mn}^{R*}\hbar_n^{IR}+q_{mn}^{I*}\lambda_n^{RR}$$
$$+q_{mn}^{R*}\lambda_n^{IR}+q_{mn}^{I*}\lambda_n^{RI}+q_{mn}^{R*}\lambda_n^{II})$$
$$+\gamma_0(t)\sum_{m=1}^{N}\left|e_n^R(t-\tau_1(t))\right|^2((-k_2^R+D_2)/\gamma_0(t)$$
$$+\sum_{n=1}^{N}(q_{mn}^{R*}\lambda_n^{RR}+q_{mn}^{I*}\lambda_n^{IR}+q_{mn}^{I*}\lambda_n^{RR}+q_{mn}^{R*}\lambda_n^{IR}))$$
$$+\gamma_0(t)\sum_{m=1}^{N}\left|e_n^I(t-\tau_1(t))\right|^2((-k_2^I+E_2)/\gamma_0(t)$$
$$+\sum_{n=1}^{N}(q_{mn}^{R*}\lambda_n^{RI}+q_{mn}^{I*}\lambda_n^{II}+q_{mn}^{I*}\lambda_n^{RI}+q_{mn}^{R*}\lambda_n^{II}))$$
$$+(1-\gamma_0(t))\sum_{m=1}^{N}\left|e_n^R(t-\tau_1(t))\right|^2((-k_3^R+D_4)/(1-\gamma_0(t))$$
$$+\sum_{n=1}^{N}(q_{mn}^{R*}\lambda_n^{RR}+q_{mn}^{I*}\lambda_n^{IR}+q_{mn}^{I*}\lambda_n^{RR}+q_{mn}^{R*}\lambda_n^{IR}))$$
$$+(1-\gamma_0(t))\sum_{m=1}^{N}\left|e_n^R(t-\tau_1(t))\right|^2((-k_3^I+E_4)/(1-\gamma_0(t))$$

$$+\sum_{n=1}^{N}(q_{mn}^{R*}\lambda_n^{RI}+q_{mn}^{I*}\lambda_n^{II}+q_{mn}^{I*}\lambda_n^{RI}+q_{mn}^{R*}\lambda_n^{II}))$$
$$-2\sum_{m=1}^{N}k_4^R\left|e_m^R(t)\right|^{1+\alpha}-2\sum_{m=1}^{N}k_4^I\left|e_m^I(t)\right|^{1+\alpha}.$$

Employing **Lemma** 2, we have

$$-k_4^R\sum_{m=1}^{N}\left|e_m^R(t)\right|^{1+\alpha}=-k_4^R\sum_{m=1}^{N}(\left|e_m^R(t)\right|^2)^{\frac{1+\alpha}{2}}$$
$$\leq -k_4^R(\sum_{m=1}^{N}\left|e_m^R(t)\right|^2)^{\frac{1+\alpha}{2}},$$
$$-k_4^I\sum_{m=1}^{N}\left|e_m^I(t)\right|^{1+\alpha}=-k_4^I\sum_{m=1}^{N}(\left|e_m^I(t)\right|^2)^{\frac{1+\alpha}{2}}$$
$$\leq -k_4^I(\sum_{m=1}^{N}\left|e_m^I(t)\right|^2)^{\frac{1+\alpha}{2}}.$$

Then from **Theorem** 1, we get

$$\mathbb{E}(\mathcal{L}V_1(t)+\mathcal{L}V_2(t))$$
$$\leq\sum_{m=1}^{N}\left|e_m^R(t)\right|^2(-2c_m-k_2^R/\gamma_0(t)-k_3^R/(1-\gamma_0(t))$$
$$+D_1+D_3+\sum_{n=1}^{N}(2p_{mn}^{R*}\hbar_n^{RR}+2p_{mn}^{I*}\hbar_n^{IR}$$
$$+p_{mn}^{R*}\hbar_n^{RI}+p_{mn}^{I*}\hbar_n^{II}+p_{mn}^{I*}\hbar_n^{RR}+p_{mn}^{R*}\hbar_n^{IR}+q_{mn}^{R*}\lambda_n^{RR}$$
$$+q_{mn}^{I*}\lambda_n^{IR}+q_{mn}^{R*}\lambda_n^{RI}+q_{mn}^{I*}\lambda_n^{II})$$
$$+\sum_{i=1}^{n}\left|e_i^I(t)\right|^2(-2c_m-k_2^I/\gamma_0(t)-k_3^I/(1-\gamma_0(t))$$
$$+E_1+E_3+\sum_{n=1}^{N}(2p_{mn}^{I*}\hbar_n^{RI}+2p_{mn}^{R*}\hbar_n^{II}$$
$$+p_{mn}^{R*}\hbar_n^{RI}+p_{mn}^{I*}\hbar_n^{II}+p_{mn}^{I*}\hbar_n^{RR}+p_{mn}^{R*}\hbar_n^{IR}+q_{mn}^{I*}\lambda_n^{RR}$$
$$+q_{mn}^{R*}\lambda_n^{IR}+q_{mn}^{I*}\lambda_n^{RI}+q_{mn}^{R*}\lambda_n^{II})$$
$$-2k_4^R(\sum_{m=1}^{N}\left|e_m^R(t)\right|^2)^{\frac{1+\alpha}{2}}-2k_4^I(\sum_{m=1}^{N}\left|e_m^I(t)\right|^2)^{\frac{1+\alpha}{2}}$$
$$=-\phi_1\sum_{m=1}^{N}\left|e_m^R(t)\right|^2-\phi_2\sum_{m=1}^{N}\left|e_m^I(t)\right|^2$$
$$-2k_4^R(\sum_{m=1}^{N}\left|e_m^R(t)\right|^2)^{\frac{1+\alpha}{2}}-2k_4^I(\sum_{m=1}^{N}\left|e_m^I(t)\right|^2)^{\frac{1+\alpha}{2}}$$
$$\leq-\phi\mathbb{E}(V(t))-2k_4\mathbb{E}(V(t)^{\frac{1+\alpha}{2}}).$$

where **Lemma** 3 have been utilized, and $\phi=\min\{\phi_1,\phi_2\}$, $k_4=\min\{k_4^R,k_4^I\}$.

Based on **Lemma** 4, it can yield that SCVNNS (3) and (4) can achieve FET projective synchronization. Additionally, the setting time can be obtained as below

$$T^*=\frac{2}{\phi(1-\alpha)}\ln(1+\frac{\phi*[V(0)]^{\frac{1-\alpha}{2}}}{2K_4})$$
$$\leq\frac{2}{\phi(1-\alpha)}\ln(1+\frac{\phi*||e(0)||^{1-\alpha}}{2K_4}). \tag{15}$$

The proof is accomplished. □

Remark 1: If λ = 1 or λ = −1, then the SCVNNs (3) and (4) will realize the complete synchronization or anti-synchronization, respectively.

Remark 2: In this study, we focus on PTVs. In particular, when $\gamma_0=1$, the system will become the SCVNNs with time varying delay $\tau_1(t)$ as a special case for system (3).

Remark 3: Up to now, to our knowledge, the study on FET projective synchronization of the SCVNNs is very few. Therefore, regard some models of other literature as special cases of our considered system [46]–[48], the theoretical results that we obtained are an extension and complement to the study of CVNNs. In addition, considering the similar system (3) and (4), but without stochastic perturbations, we can use the same method to analyze. The proof is omitted here.

Remark 4: In references [43], authors investigated the finite-time anti-synchronization for memristive bidirectional associative memory neural networks. Compared with [43], we use the novel SCVNNs, which is rarely researched. And we consider probabilistic time-varying delays in our model. In addition, we are concerned with the finite-time projective synchronization problem. This can be regarded as an extension and complement to the research content of SCVNNs. Furthermore, different from other literature about stochastic systems [45] and [49], we applied the Lyapunov stability approach instead of traditional linear matrix inequalities, which casts a new light on the stochastic system.

IV. NUMERICAL SIMULATION

In this subsection, for the purpose of displaying the correctness of the control scheme and theoretical content, some simulation results are given as follows.

Consider the following drive system and response system of SCVNNs:

$$du_m(t)=[-c_mu_m(t)+\sum_{n=1}^{N}p_{mn}s_n(u_n(t))$$
$$+\gamma_0\sum_{n=1}^{N}q_{mn}\ell_n(u_n(t-\tau_1(t)))$$
$$+(1-\gamma_0)\sum_{n=1}^{N}q_{mn}\ell_n(u_n(t-\tau_2(t)))]dt$$
$$+\sum_{m=1}^{N}g_{mn}(t,u_m(t),u_m(t-\tau_1(t)))d\omega_m(t)$$
$$+\sum_{m=1}^{N}g_{mn}(t,u_m(t),u_m(t-\tau_2(t)))d\omega_m(t), \tag{16}$$

$$
\begin{aligned}
dv_m(t) = {} & [-c_m v_m(t) + \sum_{n=1}^{N} p_{mn} s_n(v_n(t)) \\
& + \gamma_0 \sum_{n=1}^{N} q_{mn} \ell_n(v_n(t-\tau_1(t))) \\
& + (1-\gamma_0) \sum_{n=1}^{N} q_{mn} \ell_n(v_n(t-\tau_2(t))) + \rho_m(t)]dt \\
& + \sum_{m=1}^{N} g_{mn}(t, v_m(t), v_m(t-\tau_1(t)))d\omega_m(t) \\
& + \sum_{m=1}^{N} g_{mn}(t, v_m(t), v_m(t-\tau_2(t)))d\omega_m(t). \quad (17)
\end{aligned}
$$

Select the activation function as $s(u(t)) = \ell(u(t)) = \tanh(u^R(t)) + j\tanh(u^I(t))$, $\tau_{11}(t) = \tau_{21}(t) = 0.75 - 0.25 * \cos(t)$, $\tau_{12}(t) = \tau_{22}(t) = 0.75 - 0.25 * \sin(t)$, $c_1 = 1$, $c_2 = 2$, $\gamma_0 = 0.2$,

$$
P = \begin{pmatrix} 1.4 - 0.4j & 0.6 + j \\ 1.2 + 1.1j & -0.4 + 0.6j \end{pmatrix},
$$
$$
Q = \begin{pmatrix} 0.1 + 1.2j & -0.5 + 0.8j \\ -1.6 + j & 0.2 + 0.6j \end{pmatrix}.
$$

Pick an initial state arbitrarily, for example

$$
u(t) = (2.2 - 2j, 1.2 - 3.6j),
$$
$$
v(t) = (-4.5 + 5.8j, -7.4 - 1.5j).
$$

Based on **Theorem** 1, we choose the follow control scheme.

$$
\begin{aligned}
\rho_1^R &= -sign(e_1^R(t))\{8.5 + 9.75 * |e_1^R(t-\tau_1(t))| \\
&\quad +2.5 * |e_1^R(t-\tau_2(t))| + 0.5 * |e_1^R(t)|^{0.4}\}, \\
\rho_1^I &= -sign(e_1^I(t))\{6.5 + 10.25 * |e_1^I(t-\tau_1(t))| \\
&\quad +2.5 * |e_1^I(t-\tau_2(t))| + 0.5 * |e_1^I(t)|^{0.4}\}, \\
\rho_2^R &= -sign(e_2^R(t))\{8.5 + 11.25 * |e_2^R(t-\tau_1(t))| \\
&\quad +2.5 * |e_2^R(t-\tau_2(t))| + 0.5 * |e_2^R(t)|^{0.4}\}, \\
\rho_2^I &= -sign(e_2^I(t))\{6.5 + 10.35 * |e_2^I(t-\tau_1(t))| \\
&\quad +2.5 * |e_2^I(t-\tau_2(t))| + 0.5 * |e_2^I(t)|^{0.4}\}. \quad (18)
\end{aligned}
$$

The Brownian motion satisfies

$$
\begin{aligned}
& g_{mn}^R(t, u_m(t), u_m(t-\tau_1(t))) \\
&\quad = diag\{-0.4u_1^R(t) \\
&\qquad +0.3u_1^R(t-\tau_1(t)), -0.5u_2^R(t) + 0.2u_2^R(t-\tau_1(t))\}, \\
& g_{mn}^R(t, u_m(t), u_m(t-\tau_2(t))) \\
&\quad = diag\{-0.4u_1^R(t) \\
&\qquad +0.4u_1^R(t-\tau_1(t)), -0.5u_2^R(t) + 0.2u_2^R(t-\tau_1(t))\}, \\
& g_{mn}^I(t, u_m(t), u_m(t-\tau_1(t))) \\
&\quad = diag\{-0.4u_1^I(t) \\
&\qquad +0.3u_1^I(t-\tau_1(t)), -0.5u_2^I(t) + 0.2u_2^I(t-\tau_1(t))\}, \\
& g_{mn}^I(t, u_m(t), u_m(t-\tau_2(t))) \\
&\quad = diag\{-0.4u_1^I(t) \\
&\qquad +0.4u_1^I(t-\tau_1(t)), -0.5u_2^I(t) + 0.2u_2^I(t-\tau_1(t))\}.
\end{aligned}
$$

In response system (17), $g_{mn}(t, v_m(t), v_m(t-\tau(t)))$ is similar to $g_{mn}(t, u_m(t), u_m(t-\tau(t)))$, the corresponding equations are omitted.

Under control scheme (18), the state trajectories of system (16) and (17) are presented in Fig.1 and Fig.2. Obviously, we can get that SCVNNs (16) and (17) can realize projective synchronization, where the projective coefficient $\lambda = 2$. Based on the evolution of the errors curve in Fig.3, it is clear that the state errors are quickly converging to stable.

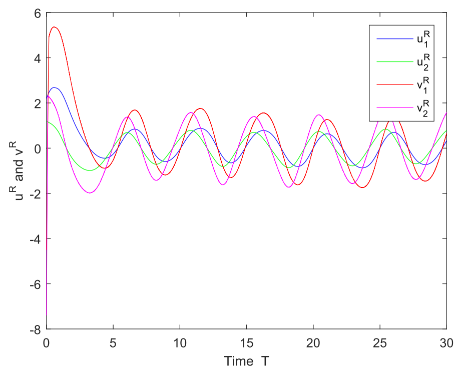

FIGURE 1. Real parts of system (16) and (17) under control (18), where the coefficient $\lambda = 2$.

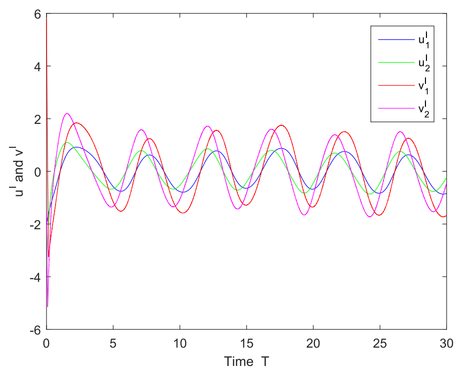

FIGURE 2. Imaginary parts of system (16) and (17) under control (18), where the coefficient $\lambda = 2$.

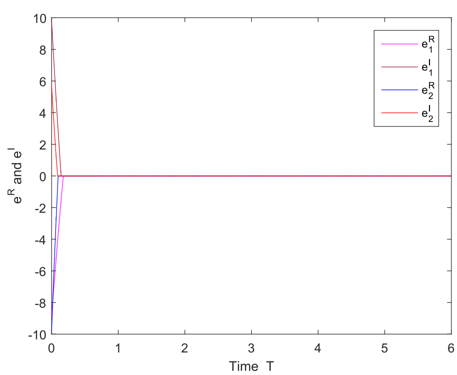

FIGURE 3. Time evolution of the error between the drive-response systems, where the coefficient $\lambda = 2$.

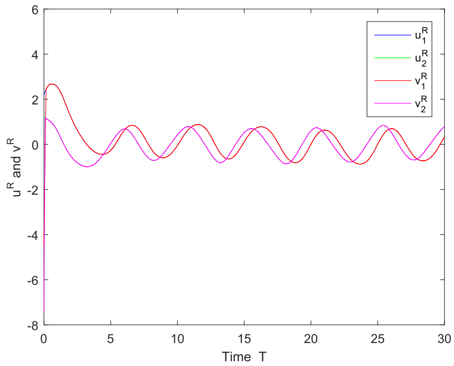

FIGURE 4. Real parts of system (16) and (17) under control (18), where the coefficient $\lambda = 1$.

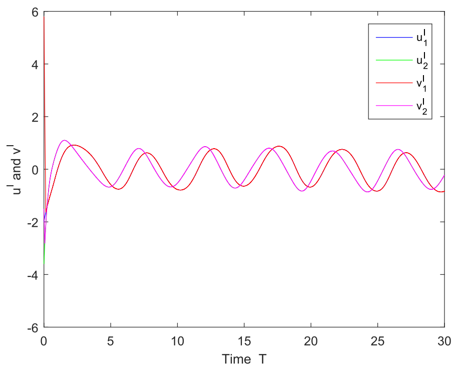

FIGURE 5. Imaginary parts of system (16) and (17) under control (18), where the coefficient $\lambda = 1$.

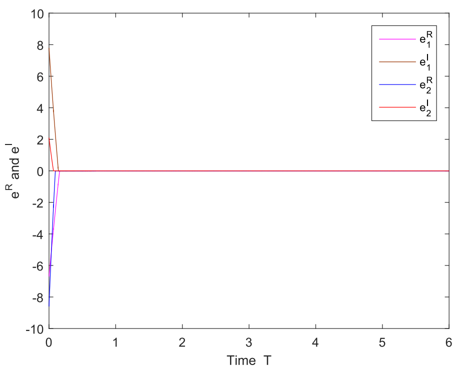

FIGURE 6. Time evolution of the error between the drive-response systems, where the coefficient $\lambda = 1$.

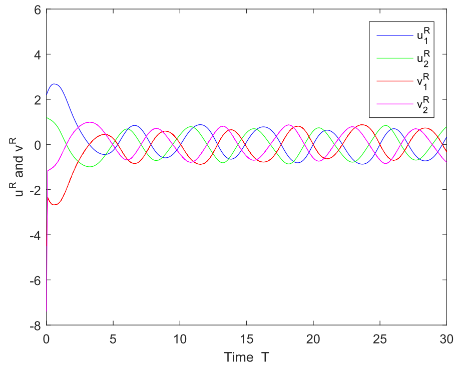

FIGURE 7. Real parts of system (16) and (17) under control (18), where the coefficient $\lambda = -1$.

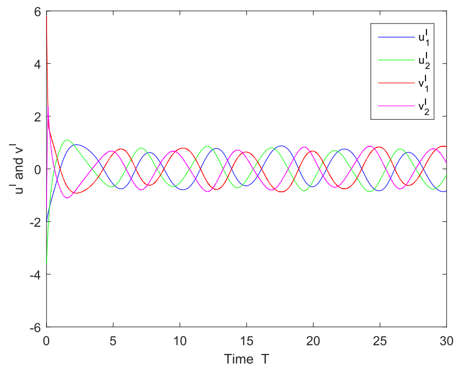

FIGURE 8. Imaginary parts of system (16) and (17) under control (18), where the coefficient $\lambda = -1$.

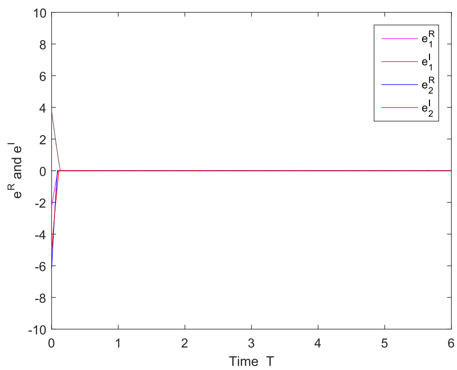

FIGURE 9. Time evolution of the error between the drive-response systems, where the coefficient $\lambda = -1$.

By calculation with the mentioned parameters, it is light to check all the algebraic criteria. Hence, as Fig.3 illustrates, we deduce from **Theorem** 1 that the considered system (3) and (4) can realize FET projective synchronization.

Based on the discussion of the article and the summary of **Remark** 1, we know that SCVNNs (3) and (4) can realize the complete synchronization and anti-synchronization when the coefficient changes.

If $\lambda = 1$, as Fig.6 shows, SCVNNs (3) and (4) can realize complete synchronization, and the state trajectories are shown in Fig.4 and Fig.5. If $\lambda = -1$, Fig.7-9 demonstrate that the system (3) and (4) can achieve anti-synchronization. Since the system error stabilization time is independent of the coefficient, the errors all converge to zero in finite time.

V. CONCLUSION

In this paper, we are concerned with FET projective synchronization of the SCVNNs with PTVs by separating SCVNNs. Instead of linear matrix inequalities, we apply Lyapunov stability approach to obtain several novel criteria for ensuring FET projective synchronization via some inequalities techniques, which can cast a new light on stochastic system and provide a reference for the application of large time delay intervals. Finally, the obtained criteria are demonstrated by numerical examples, and the setting time is estimated. In the future, stability of SCVNNs is the issue for our research work.

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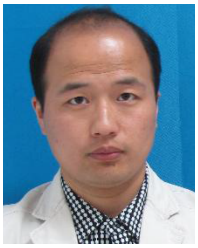

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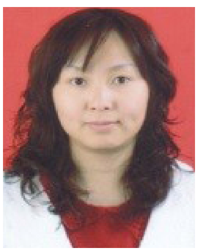

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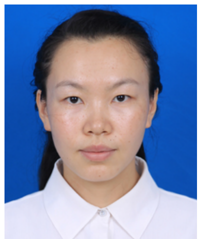

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