IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received February 23, 2021, accepted March 10, 2021, date of publication March 17, 2021, date of current version March 25, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3066402

Switched Fuzzy Sampled-Data Control of Chaotic Systems With Input Constraints

YUNJUN CHEN[1], QIUXIA CAO[2], ZHENYU ZHU[3], (Student Member, IEEE), ZHANGANG WANG[2], AND ZHANSHAN ZHAO[2]

[1]School of Electrical Engineering and Automation, Tiangong University, Tianjin 300387, China
[2]School of Computer Science and Technology, Tiangong University, Tianjin 300387, China
[3]School of Control Science and Engineering, Shandong University, Shandong 250100, China

Corresponding author: Zhangang Wang (wangzhangang@tiangong.com)

This work was supported in part by the Natural Science Foundation of Tianjin under Grant 19JCTPJC48600, Grant 18JCYBJC88100, and Grant 17JCYBJC18500.

ABSTRACT This paper designs a sampled-data controller to exponentially stabilize a kind of chaotic systems (CSs) which are represented by T-S fuzzy model with nonuniform sampling. By employing a switched technology and Lyapunov functional dependent on fuzzy membership-functions (FMFs), a new control criterion for chaotic systems was established. Compared with the existing methods, less conservatism and a large sampling period can be obtained by above approaches. Consequently, the amount of transmitted information is greatly reduced and the efficiency of bandwidth is much improved. Finally, the proposed T-S fuzzy sampled-data (TSFSD) controller is applied to Lorenz system to prove the effectiveness of those methods.

INDEX TERMS Chaotic system (CS), Takagi–Sugeno (T-S) fuzzy model, sampled-data controller, exponential stability.

I. INTRODUCTION

Chaotic phenomenon is a natural nonlinear phenomenon, and it has extensive applications in other areas, which include engineering, biology, physics and so on. Since it has some characteristics, for instance, no periodic, the orderliness of irregularity and the sensitivity is affected by initial values. Thus, chaotic control has become one of the popular topics of nonlinear system during the past decade [1]–[7].The synchronization of the chaotic systems is received widespread attention in some engineering problems such as two-memristor-based chaotic system [8], secure communication [9] image encryption [10], identification of cutting tool wear [11] and so on.Until now, many various approaches have been proposed to analyze and control the CSs, for example in [12] and [13], the authors propose feedback control approach, and continuous sliding mode approach have been proposed in [14] and [15], and fuzzy control approach in [16] and [17] have been advanced.

Recently, the T-S fuzzy model has been widely employed for describing and controlling the behaviors of nonlinear systems. By using a set of fuzzy IF-THEN rules and appropriate membership functions, T-S fuzzy model can describe local linear input and output relations of a nonlinear system. Therefore, the problems of nonlinear systems based on T-S fuzzy models have been researched during the past 20 years, for example, in [18],the authors proposed the problem of non fragile H_∞ filtering for T-S fuzzy system. The authors proposed in [19], investigated problem for nonlinear spatially distributed processes based on T-S fuzzy system and in [17]. The authors proposed robust H_∞ control issue of T-S fuzzy systems with input time-varying delays. Moreover, many famous CSs, for instance Chen system, Rosslers system and Lorenz system, which can be described by T-S fuzzy model. Consequently, chaotic control based on T-S fuzzy system has been studied extensively. In addition, since the convergence rate of exponential asymptotic stability can be described, it is faster than that of asymptotic stability.

Rapid development of modern communication, digital technology and digital hardware technologies make the digital controllers have paid attention to an increasing number of researchers. Digital controllers have many advantages over analog controllers, such as higher reliability, low installation cost and simpler installation. When closed-loop systems are controlled by digital controllers, the systems called sampled-data systems. For the sampled-data systems control, the sampled-data control only needs sampled information of this system at the controller sampling instants.

The associate editor coordinating the review of this manuscript and approving it for publication was Chao-Yang Chen.

Consequently, the amount of transmitted information will be reduced and efficiency bandwidth utilization will be more efficient. Meanwhile, the most important part of the sampling control system is the sampling period, due to it will affect the conservatism and efficiency of the sampling control system. Such as, a small sampling period will limit operating conditions i.e., bandwidth and limited communication capacity. Thus, how to realize the control goal of the sampled-data controller in the sampling period as long as possible is meaningful and important topic. Recently, the TSFSD control is focused on controller problems for CSs [20]–[22] with a larger sampling period by many researchers. In [20], the sampling data control problem with input constraints is investigated, a new time-varying Lyapunov functional is constructed to obtain the criteria on the exponential stability of the chaotic system. However, Jensen inequality is applied to deal with the integral terms generated by the derivative of the Lyapunov function, and the result obtained is still relatively conservative. In view of this problem, in [21], the conservatism of system and sampling period is improved via applying the free-matrix-based time-dependent discontinuous Lyapunov approach of chaotic TSFSD control system which proposed by [22]. It should be pointed out that the approaches mentioned above ignore the main information of the fuzzy membership functions (FMFs), which increases conservatism of the system to a certain extent.

The existing results show that considering a membership-function (MF)-dependent Lyapunov functional can further reduce the conservatism of the system, which is obtained in [24]. In addition, to take full advantage of more information available from the FMFs and the actual sampling pattern, similar to [23], we apply a switched technology in the study of chaotic TSFSD control system, which might obtain a larger sampling period and less conservative results for fuzzy sampled-data control systems.

Inspired by the above discussions, the stabilization problem of chaotic TSFSD control system is investigated in this paper. Firstly, different from [20] and [21], by using FMFs-dependent Lyapunov scheme and a switched approach, the error information of the FMFs between the chaotic system and TSFSD controller is considered, therefore, the less conservatism and larger sampling period than [21] is obtained. That is to say, in the actual sampling process, more available infromation than those are established method in [20]–[22] can be obtained. Furthermore,on the basis of [23], the chaotic system is denoted by the T-S fuzzy system with input constraints, and by constructing a new FMFs-dependent Lyapunov function, exponential stability of the chaotic TSFSD control system is studied via a given attenuation rate. Finally, the proposed controller designed method is applied to Lorenz system to prove the effectiveness of this method.

II. PROBLEM STATEMENT AND PRELIMINARIES

In this section, consider a CS described as follow:

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$

where $f(x(t), u(t))$ is a known nonlinear continuous function which satisfies $f(0, 0) = 0$. $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in \Re^n$ and $u(t) = [u_1(t), u_2(t), \cdots, u_m(t)]^T \in \Re^m$ denote, respectively, state vector and input vector. By using the T-S fuzzy modeling, the CSs (1) can be represented as follows:

Plant Rule i:

$$\begin{aligned} &IF\ \varpi_1(t)\ is\ \aleph_{i1},\ \varpi_2(t)\ is\ \aleph_{i2}, \cdots \varpi_\nu(t)\ is\ \aleph_{i\nu},\ THEN \\ &\dot{x}(t) = A_i x(t) + B_i u(t) \end{aligned} \tag{2}$$

where $\varpi_1(t), \varpi_2(t), \cdots, \varpi_\nu(t)$ are the premise variables, $\aleph_{i1}, \aleph_{i2}, \cdots, \aleph_{i\nu}$ are the fuzzy sets and $i \in \emptyset = \{1, 2 \cdots, r\}$ where r is the number of fuzzy rules. $A_i \in \Re^{n\times n}$ and $B_i \in \Re^{n\times m}$ are the known matrices.

By using the singleton fuzzifier, product inference engine, and weighted average defuzzifier, the above system(2) can be represented as:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\varpi(t))[A_i x(t) + B_i u(t)] \tag{3}$$

where $\varpi(t) = [\varpi_1(t), \varpi_2(t), \cdots, \varpi_\nu(t)]^T$ and $h_i(\varpi(t))$ represents the normalized membership functions and satisfying:

$$\begin{aligned} h_i(\varpi(t)) &= \frac{\prod_{j=1}^{\nu} \aleph_{ij}(\varpi_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{\nu} \aleph_{ij}(\varpi_j(t))} \\ h_i(\varpi(t)) &\geq 0 \ \sum_{i=1}^{r} h_i(\varpi(t)) = 1 \end{aligned} \tag{4}$$

and $\aleph_{ij}(\varpi_j(t))$ denotes the grade of membership of $\varpi_j(t)$ in $\aleph_{ij}$. In this paper, it is assumed that the control input signal can be obtained by a zero-order holder (ZOH) function with a sequence of hold times:

$$0 = t_0 < t_1 < \cdots < t_k < \cdots < \lim_{k\to+\infty} t_k = +\infty \tag{5}$$

meanwhile, according to the approach of parallel distributed compensation (PDC), the TSFSD controller of system (3) can be designed with the same premise parts and same fuzzy sets with the fuzzy model.

Controller Rule j:

$$\begin{aligned} &IF\ \varpi_1(t)\ is\ \aleph_{j1},\ \varpi_2(t)\ is\ \aleph_{j2}, \cdots \varpi_\nu(t)\ is\ \aleph_{j\nu},\ THEN \\ &u(t) = K_j x(t_k)\ \ t_k \leq t < t_{k+1}\ j \in \emptyset \end{aligned} \tag{6}$$

where $K_j \in \Re^{n\times m}$ is the gain matrix. Hence, the overall fuzzy sampled-data controller is described as:

$$u(t) = \sum_{j=1}^{r} h_j(\varpi(t_k))K_j x(t_k)\ \ t_k \leq t < t_{k+1}\ j \in \emptyset \tag{7}$$

In this paper, sampling time sequence (5) is not required to be periodic. For instance, the interval between t_k and t_{k+1} are variable and bounded and the interval can be described as:

$$0 \leq \hbar_m \leq t_{k+1} - t_k = \hbar_k \leq \hbar_M \tag{8}$$

where $\hbar_m = t - t_k$. For all $t \geq 0$, $\hbar_m$ and $\hbar_M$ represent the upper and the lower bounds of sampling interval, respectively.

Substituting (7) into (3) the following closed-loop system can be represented as:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varpi(t))h_j(\varpi(t_k))[A_i x(t) + B_i K_j x(t_k)]$$
$$t_k \le t < t_{k+1} \tag{9}$$

In practical applications, due to the limited power of the actuator and the control system is always limited by the inputs. Hence, this paper assumes that the input signal is bounded and it satisfies:

$$\|u(t)\| \le \hat{u} \;\; \forall t \ge 0 \tag{10}$$

where $\hat{u}$ represents the maximum input. But, obviously, the globally exponential stabilization of system (9) cannot be achieved under the above situation. In order to realize the globally asymptotically stable of system (9), in this paper, we will design a TSFSD controller (7) and use a LKF depends on the FMFs such that system (9) under such situations is globally asymptotically exponentially stable for any initial condition $x(0) \in \varepsilon(P_i, \rho)$, that is, system (9) is globally asymptotically exponentially stable(GAES) if there exist two constants $\gamma > 0$ and $\delta > 0$ such that

$$\|x(t)\| \le \delta e^{-\gamma t}, \;\; \forall t \ge 0 \tag{11}$$

At the end of this section, some essential lemmas will be introduced.

Lemma 1 ([20]): The inequality

$$\sum_{i=1}^{r} h_i(\varpi(t)) \sum_{j=1}^{r} h_j(\varpi(t_k)) W_{ij} < 0$$

holds with $t_k \le t < t_{k+1}$ and exist symmetric matrices W_{ij}. If $|\varpi(t) - \varpi(t_k)| \le \varrho_i$, for scalars $\varrho_i > 0$ and there exist positive definite matrices $\mathbb{U}_{ij}$, $\mathbb{V}_{ij}$, and any appropriate dimensional matrices $\mathbb{N}_{ij} = \mathbb{N}_{ji}^T$, $\mathbb{N}_{i(j+r)} = \mathbb{N}_{(j+r)i}^T$ for any $i, j \in \emptyset$ such that

$$\mathbb{Z}_{ij} + \mathbb{Z}_{ji} \le \mathbb{N}_{ij} + \mathbb{N}_{ji} \tag{12}$$

$$W_{ij} - 2\mathbb{Z}_{ij} + \sum_{s=1}^{r} \varrho_s(\mathbb{Z}_{is}^{+} + \mathbb{Z}_{sj}^{+}) \le \mathbb{N}_{i(j+r)} + \mathbb{N}_{(j+r)i} \tag{13}$$

$$\begin{bmatrix} \mathbb{Y}_{11} & \mathbb{Y}_{12} \\ * & \mathbb{Y}_{11} \end{bmatrix} < 0 \tag{14}$$

where

$$\mathbb{Z}_{ij} = \mathbb{U}_{ij} - \mathbb{V}_{ij}$$
$$\mathbb{Z}_{ij}^{+} = \mathbb{U}_{ij} + \mathbb{V}_{ij}$$
$$\mathbb{Y}_{11} = \begin{bmatrix} \mathbb{N}_{11} & \cdots & \mathbb{N}_{1r} \\ \vdots & \ddots & \vdots \\ \mathbb{N}_{r1} & \cdots & \mathbb{N}_{rr} \end{bmatrix}, \; \mathbb{Y}_{11} = \begin{bmatrix} \mathbb{N}_{1(r+1)} & \cdots & \mathbb{N}_{1(2r)} \\ \vdots & \ddots & \vdots \\ \mathbb{N}_{r(r+1)} & \cdots & \mathbb{N}_{r(2r)} \end{bmatrix}$$

Lemma 2 ([17]): For $t_k \le t < t_{k+1}$ and system (9), the following inequality holds:

$$\|x(t)\|^2 \le \lambda \|x(t_k)\|^2 \tag{15}$$

where $\lambda = 3(1 + \hbar_M^2 \lambda_1 \lambda_2)e^{3\hbar_M^2 \lambda_3}$ and $\lambda_1 = \max_{i\in\emptyset}\{\|B_i\|\}$, $\lambda_2 = \max_{i\in\emptyset}\{\|K_i\|\}$ and $\lambda_3 = \max_{i\in\emptyset}\{\|A_i\|\}$.

III. MAIN RESULTS

In this section, we will use a new sampling data control method which using an FMF-dependent LKF to exponentially stabilize the fuzzy system with an input saturation constraint. For the sake of simplicity, the following notions are given.

$$\begin{aligned}
\xi(t) &= [x^T(t)\; \dot{x}^T(t)\; x^T(t_k)]^T, \\
e_\iota &= [0_{n,(\iota-1)n}\; I_n\; 0_{n,(3-\iota)n}], \; (\iota = 1, 2, 3), \\
\chi_h &= \sum_{i=1}^{r} h_i(\varpi(t))\chi_i, \; \dot{\chi}_h = \sum_{i=1}^{r} \dot{h}_i(\varpi(t))\chi_i, \\
\tau_k &= (t_{k+1} - t_k), \; \tau(t) = (t - t_k).
\end{aligned}$$

By the following theorem, system (9) under such situations is GAES for any initial condition $x(0) \in \varepsilon(P_i, \rho)$.

Theorem 1: For given positive scalars α and ρ, if there exist positive symmetric matrices P_i, $M_{11,i}$, Z, symmetric matrices

$$F_i, U_i = \begin{bmatrix} F_{11,i} & F_{12,i} \\ * & F_{22,i} \end{bmatrix}, \quad M_i = \begin{bmatrix} M_{11,i} & M_{12,i} \\ * & M_{22,i} \end{bmatrix},$$

and any appropriate dimensional matrices W, H, N_ι, $(\iota = 1, 2, 3)$, $i, j \in \emptyset$, satisfying the following LMIs:

$$\dot{P}_h \le 0, \quad \dot{U}_h \le 0, \quad \dot{F}_h \le 0, \quad \dot{M}_h \le 0 \tag{16}$$

$$\begin{bmatrix} \Omega(\tau_k, 0) & \frac{\tau_k}{2}\widetilde{F} \\ * & -Z \end{bmatrix} < 0 \tag{17}$$

$$\begin{bmatrix} \Omega(\tau_k, \tau_k) & e^{-\alpha\hbar_M}\sqrt{\tau}_k W & e^{-\alpha\hbar_M}\sqrt{\tau}_k H & \frac{\tau_k}{2}\widetilde{F} \\ * & -U_i & 0 & 0 \\ * & * & -M_{11,i} & 0 \\ * & * & * & -Z \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \hat{u}^2 I & K_j \\ * & \rho^{-1} P_i \end{bmatrix} \ge 0 \tag{19}$$

where

$$\begin{aligned}
\Omega(\tau_k, 0) = {} & Sym\{e_1 P_i e_2^T + e^{-2\alpha\hbar_M} W^T (e_1 - e_3)^T \\
& + e^{-2\alpha\hbar_M} H^T (e_1 - e_3)^T - e^{-2\alpha\hbar_M} (e_1 - e_3) \\
& \times M_{12,i} e_3^T\} + 2\alpha e_1 P_i e_1^T + \tau_k e_2 U_i e_2^T \\
& + \frac{\tau_k^2}{4} e_2 Z e_2^T + \tau_k\{[e_1\; e_3] F_i [e_1\; e_3]^T\} \\
& + \tau_k\{[e_2\; e_3] M_i [e_2\; e_3]^T\} + Sym\{(e_1 N_1 + e_2 N_2 \\
& + e_3 N_3)(A_i e_1^T + B_i K_{\theta,j} e_3^T - e_2^T)\} \\
\Omega(\tau_k, \tau_k) = {} & Sym\{e_1 P_i e_2^T + e^{-2\alpha\hbar_M} W^T (e_1 - e_3)^T \\
& + e^{-2\alpha\hbar_M} H^T (e_1 - e_3)^T - e^{-2\alpha\hbar_M} (e_1 - e_3) \\
& \times M_{12,i} e_3^T\} + 2\alpha e_1 P_i e_1^T + \frac{\tau_k^2}{4} e_2 Z e_2^T - \tau_k \\
& \times e^{-2\alpha\hbar_M} e_3 M_{22,i} e_3^T + (2\alpha\hbar_M - 1)\tau_k\{[e_1\; e_3] \\
& \times F_i [e_1\; e_3]^T\} + Sym\{(e_1 N_1 + e_2 N_2 + e_3 N_3) \\
& \times (A_i e_1^T + B_i K_{\theta,j} e_3^T - e_2^T)\} \\
\widetilde{F} = {} & [F_{11,i}\; 0\; F_{12,i}]^T
\end{aligned}$$

then the system (9) will be GAES for under initial condition $x(0) \in \varepsilon(P_i, \rho)$.

Proof: Construct the following FMF-dependent LKF:

$$V(t) = \sum_{l=1}^{4} V_l(t), \quad (l = 1, 2, 3, 4) \tag{20}$$

where

$$\begin{aligned}
V_1(t) &= \sum_{i=1}^{r} h_i(\varpi(t))x^T(t)P_i x(t) \\
V_2(t) &= (\tau_k - \tau(t))\tau(t)\sum_{i=1}^{r} h_i(\varpi(t)) \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}^T F_i \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix} \\
V_3(t) &= (\tau_k - \tau(t))\sum_{i=1}^{r} h_i(\varpi(t)) \\
&\quad \times \int_{t_k}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)U_i\dot{x}(s)ds \\
V_4(t) &= (\tau_k - \tau(t))\sum_{i=1}^{r} h_i(\varpi(t)) \\
&\quad \times \int_{t_k}^{t} e^{2\alpha(s-t)} \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix}^T M_i \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix} ds.
\end{aligned}$$

Then we have:

$$\lim_{t\to t_k} V_l(t) = V_l(t_k), \quad l = 1 \tag{21}$$

and

$$\lim_{t\to t_{k+}} V_l(t) = \lim_{t\to t_{k-}} V_l(t) = \lim_{t\to t_k} V_l(t) = 0, \; l = 2, 3, 4. \tag{22}$$

Since, $\lim_{t\to t_k} V(t) = V(t_k)$, thus, LKF (20) is continuous.

Letting the time derivative of $V(t)$ along the trajectory of the system (9) yields:

$$\dot{V}(t) + 2\alpha V(t) = \sum_{l=1}^{4} (\dot{V}_l(t) + 2\alpha V_l(t)), \; (l = 1, 2, 3, 4) \tag{23}$$

where

$$\begin{aligned}
\dot{V}_1(t) + 2\alpha V_1(t) &= 2x^T(t)P_h\dot{x}(t) + x^T(t)\dot{P}_h x(t) \\
&\quad + 2\alpha x^T(t)P_h x(t) \\
\dot{V}_2(t) + 2\alpha V_2(t) &= -\tau(t) \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}^T F_h \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix} \\
&\quad + (\tau_k - \tau(t)) \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}^T F_h \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix} \\
&\quad + (\tau_k - \tau(t))\tau(t) \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}^T \dot{F}_h \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix} \\
&\quad + 2(\tau_k - \tau(t))\tau(t)\xi^T(t)\widetilde{F}\dot{x}(t) \\
&\quad + 2(\tau_k - \tau(t))\tau(t) \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}^T F_h \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix} \\
\dot{V}_3(t) + 2\alpha V_3(t) &= (\tau_k - \tau(t)) \int_{t_k}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)\dot{U}_h\dot{x}(s)ds \\
&\quad + (\tau_k - \tau(t))(\dot{x}^T(t)U_h\dot{x}(t)) \\
&\quad - \int_{t_k}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)U_h\dot{x}(s)ds \\
\dot{V}_4(t) + 2\alpha V_4(t) &= -\int_{t_k}^{t} e^{2\alpha(s-t)} \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix}^T M_h \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix} ds \\
&\quad + (\tau_k - \tau(t)) \\
&\quad \times \int_{t_k}^{t} e^{2\alpha(s-t)} \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix}^T \dot{M}_h \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix} ds \\
&\quad + (\tau_k - \tau(t)) \begin{bmatrix} \dot{x}(t) \\ x(t_k) \end{bmatrix}^T M_h \begin{bmatrix} \dot{x}(t) \\ x(t_k) \end{bmatrix}.
\end{aligned} \tag{24}$$

For matrix $Z > 0$, we have:

$$\begin{aligned}
&2(\tau_k - \tau(t))\tau(t)\xi^T(t)\widetilde{F}\dot{x}(t) \\
&\quad \leq \frac{\tau_k^2}{4}(\xi^T(t)\widetilde{F}Z^{-1}\widetilde{F}^T\xi(t) + \dot{x}^T(t)Z\dot{x}(t))
\end{aligned} \tag{25}$$

and due to $M_{11,i} > 0$ according to the Schur complement, we can obtain:

$$\begin{bmatrix} M_{11,i} & I_n \\ * & M_{11,i}^{-1} \end{bmatrix} \geq 0. \tag{26}$$

For any appropriate dimensional matrices H, it is calculated that:

$$\int_{t_k}^{t} \begin{bmatrix} \dot{x}(s) \\ H\xi(t) \end{bmatrix}^T \begin{bmatrix} M_{11,i} & I_n \\ * & M_{11,i}^{-1} \end{bmatrix} \begin{bmatrix} \dot{x}(s) \\ H\xi(t) \end{bmatrix} ds \geq 0 \tag{27}$$

thus, we have:

$$\begin{aligned}
-\int_{t_k}^{t} \dot{x}^T(s)M_{11,i}\dot{x}(s)ds &\leq \tau(t)(H\xi(t))^T M_{11,i}^{-1}H\xi(t) \\
&\quad + 2(H\xi(t))^T(x(t) - x(t_k)).
\end{aligned} \tag{28}$$

Similarly, for any appropriate dimensional matrices W, we have:

$$\begin{aligned}
-\int_{t_k}^{t} \dot{x}^T(s)U_i\dot{x}(s)ds &\leq \tau(t)(W\xi(t))^T U_i^{-1}W\xi(t) \\
&\quad + 2(W\xi(t))^T(x(t) - x(t_k)).
\end{aligned} \tag{29}$$

based on the above inequalities and $\tau_k \in (0, \hbar_M]$, we obtain:

$$\begin{aligned}
\dot{V}_2(t) + 2\alpha V_2(t) &\leq -\tau(t) \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}^T F_h \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix} \\
&\quad + (\tau_k - \tau(t)) \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}^T F_h \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix} \\
&\quad + (\tau_k - \tau(t))\tau(t) \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}^T \dot{F}_h \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix} \\
&\quad + 2(\tau_k - \tau(t))\tau(t) \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}^T F_h \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix} \\
&\quad + \frac{\tau_k^2}{4}(\xi^T(t)\widetilde{F}Z^{-1}\widetilde{F}^T\xi(t) + \dot{x}^T(t)Z\dot{x}(t))
\end{aligned} \tag{30}$$

$$\dot{V}_3(t)+2\alpha V_3(t) \leq (\tau_k-\tau(t))\int_{t_k}^{t} e^{-2\alpha\hbar_M}\dot{x}^T(s)\dot{U}_h\dot{x}(s)ds$$
$$+(\tau_k-\tau(t))(\dot{x}^T(t)U_h\dot{x}(t))$$
$$+e^{-2\alpha\hbar_M}\tau(t)(W\xi(t))^T U_i^{-1}W\xi(t)$$
$$+2(W\xi(t))^T(x(t)-x(t_k)) \quad (31)$$
$$\dot{V}_4(t)+2\alpha V_4(t) \leq -\int_{t_k}^{t} e^{2\alpha(s-t)}\begin{bmatrix}\dot{x}(s)\\x(t_k)\end{bmatrix}^T M_h\begin{bmatrix}\dot{x}(s)\\x(t_k)\end{bmatrix}ds$$
$$+(\tau_k-\tau(t))\begin{bmatrix}\dot{x}(t)\\x(t_k)\end{bmatrix}^T M_h\begin{bmatrix}\dot{x}(t)\\x(t_k)\end{bmatrix}$$
$$+e^{-2\alpha\hbar_M}\tau(t)(H\xi(t))^T M_{11,i}^{-1}H\xi(t)$$
$$+2(H\xi(t))^T(x(t)-x(t_k))$$
$$-2e^{-2\alpha\hbar_M}(x(t)-x(t_k))^T M_{12,h}x(t_k)$$
$$-\tau(t)e^{-2\alpha\hbar_M}x^T(t_k)M_{22,h}x(t_k). \quad (32)$$

Now,we will consider how to ensure $\dot{P}_h \leq 0, \dot{U}_h \leq 0, \dot{F}_h \leq 0, \dot{M}_h \leq 0$, similar with [17] and due to $\sum_{i=1}^{r} h_i(\varpi(t))=1$, we have that:

$$\dot{P}_h = \sum_{i=1}^{r}\dot{h}_i(\varpi(t))P_i = \sum_{q=1}^{r-1}\dot{h}_q(\varpi(t))(P_q-P_r) \quad (33)$$
$$\dot{U}_h = \sum_{i=1}^{r}\dot{h}_i(\varpi(t))U_i = \sum_{q=1}^{r-1}\dot{h}_q(\varpi(t))(U_q-U_r) \quad (34)$$
$$\dot{F}_h = \sum_{i=1}^{r}\dot{h}_i(\varpi(t))F_i = \sum_{q=1}^{r-1}\dot{h}_q(\varpi(t))(F_q-F_r) \quad (35)$$
$$\dot{M}_h = \sum_{i=1}^{r}\dot{h}_i(\varpi(t))M_i = \sum_{q=1}^{r-1}\dot{h}_q(\varpi(t))(M_q-M_r) \quad (36)$$

where $\dot{h}_q(\varpi(t))$ is the time derivative for FMFs of the qth rule, and it is positive or negative over time.

In order to ensure $\dot{P}_h \leq 0, \dot{U}_h \leq 0, \dot{F}_h \leq 0, \dot{M}_h \leq 0$, based on switching approach, we have:

$$\begin{cases} if \quad \dot{h}_q(\varpi(t))<0 \\ \Rightarrow (P_q-P_r)>0, \quad (U_q-U_r)>0, \\ (F_q-F_r)>0, \quad (M_q-M_r)>0 \\ if \quad \dot{h}_q(\varpi(t))\geq 0 \\ \Rightarrow (P_q-P_r)\leq 0, \quad (U_q-U_r)\leq 0, \\ (F_q-F_r)\leq 0, \quad (M_q-M_r)\leq 0 \end{cases} \quad (37)$$

where, there are 2^{r-1} possible cases in (37) and suppose that:

$$R_\theta = \{\theta : \textit{The possible permutations of } \dot{\varpi}(t) \;\; \textit{for } \theta \in \hat{\emptyset}\} \quad (38)$$
$$S_\theta = \{\theta : \textit{The possible constraints of } P_i, U_i, F_i, M_i \;\; \textit{for } \theta \in \hat{\emptyset}\} \quad (39)$$

where, $\theta \in \hat{\emptyset} = \{1, 2\cdots, 2^{r-1}\}$ and (37) can be presented as:

$$\textit{if } R_\theta, \textit{then } S_\theta. \quad (40)$$

On the other hand, if the FMF-dependent matrices P_h, U_h, F_h and M_h satisfy the rule (40), than we can obtain $\dot{P}_h \leq 0, \dot{U}_h \leq 0, \dot{F}_h \leq 0, \dot{M}_h \leq 0$.

Based on the rule (40), a new TSFSD controller with the switched technology can be presented as:

$$u_\theta(t) = \sum_{j=1}^{r} h_j(\varpi(t_k))K_{\theta,j}x(t_k) \quad (41)$$

then the final TSFSD controller (7) can be shown:

$$u(t): \begin{cases} u_1(t) = K_{1,h}x(t_k) \\ u_2(t) = K_{2,h}x(t_k) \\ \vdots \\ u_{2^{r-1}}(t) = K_{2^{r-1},h}x(t_k) \end{cases} \quad (42)$$

For any appropriately dimensioned matrices N_ι, ($\iota = 1, 2, 3$) we have:

$$0 = 2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varpi(t))h_j(\varpi(t_k))[x^T(t)N_1+\dot{x}^T(t)N_2$$
$$+x^T(t_k)N_3][-\dot{x}(t)+A_ix(t)+B_iK_{\theta,j}x(t_k)]. \quad (43)$$

It follows from (23)−(43) that:

$$\dot{V}(t)+2\alpha V(t)$$
$$\leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varpi(t))h_j(\varpi(t_k))\xi^T(t)\Xi_{(\tau_k,\tau(t))}\xi(t). \quad (44)$$

where

$$\Xi_{(\tau_k,\tau(t))} = \frac{(\tau_k-\tau(t))}{\tau_k}\Xi_{(\tau_k,0)} + \frac{\tau(t)}{\tau_k}\Xi_{(\tau_k,\tau_k)}$$
$$\Xi_{(\tau_k,0)} = \Omega(\tau_k,0) + \frac{\tau_k^2}{4}\widetilde{F}Z^{-1}\widetilde{F}^T$$
$$\Xi_{(\tau_k,\tau_k)} = \Omega(\tau_k,\tau_k) + \frac{\tau_k^2}{4}\widetilde{F}Z^{-1}\widetilde{F}^T$$
$$+e^{-2\alpha\hbar_M}\tau(t)W^TU_i^{-1}W$$
$$+e^{-2\alpha\hbar_M}\tau(t)H^TM_{11,i}^{-1}H$$

due to $\frac{(\tau_k-\tau(t))}{\tau_k} + \frac{\tau(t)}{\tau_k} = 1$, according to the definition of convex combination, $\Xi_{(\tau_k-\tau(t))} < 0$ if and only if $\Xi_{(\tau_k,0)} < 0$ and $\Xi_{(\tau_k,\tau_k)} < 0$ hold. Applying Schur complement, we have:

$$\dot{V}(t)+2\alpha V(t) \leq 0, \quad t_k \leq t < t_{k+1} \quad (45)$$

which implies for natural numbers $k \geq 0$, we have:

$$V(t) > V(t_{k+1}) > 0, \quad t_k \leq t < t_{k+1} \quad (46)$$

Integrating the preceding inequality (45) for $t \in (t, t_k]$, It is clear that we have:

$$V(t) \leq e^{-2\alpha(t-t_k)}V(t_k)$$
$$\leq e^{-2\alpha(t-t_{k-1})}V(t_{k-1})$$
$$\leq \cdots$$
$$\leq e^{-2\alpha t}V(0). \quad (47)$$

By using Lemma 2 and (46), we have that for $t \in (t, t_k]$:

$$\begin{aligned}\|x(t)\|^2 &\le \lambda\|x(t_k)\|^2 \\ &= \frac{\lambda}{\eta_{\min}(P_i)}\eta_{\min}(P_i)\|x(t_k)\|^2 \\ &\le \frac{\lambda}{\eta_{\min}(P_i)}x^T(t_k)P_ix(t_k) \\ &= \frac{\lambda}{\eta_{\min}(P_i)}V(t_k) \\ &\le \frac{\lambda}{\eta_{\min}(P_i)}e^{-2\alpha t_k}V(0) \\ &= \frac{\lambda}{\eta_{\min}(P_i)}e^{-2\alpha t}e^{-2\alpha(t-t_k)}V(0) \\ &\le e^{2\alpha\hbar_M}\frac{\lambda}{\eta_{\min}(P_i)}e^{-2\alpha t}V(0) \end{aligned} \tag{48}$$

it means whatever initial condition $x(0) \in \varepsilon(P_i, \rho)$, we have:

$$\|x(t)\| \le e^{\alpha\hbar_M}\sqrt{\frac{\lambda}{\eta_{\min}(P_i)}}e^{-\alpha t}. \tag{49}$$

For any natural numbers t and k satisfy $t_k \le t < t_{k+1}$. Hence, we can see that:

$$u^T(t)u(t) \le \sum_{j=1}^{r} h_j(\varpi(t_k))x^T(t_k)K_{\theta,j}^TK_{\theta,j}x(t_k) \quad t_k \le t < t_{k+1}\ j \in \emptyset. \tag{50}$$

Moreover, based on (20) and (47), we obtain:

$$x^T(t_k)P_ix(t_k) = V(t_k) \le V(0) = x^T(0)P_ix(0). \tag{51}$$

By using Schur complement, (19) can be expressed as follow:

$$\hat{u}^{-2}K_{\theta,j}^TK_{\theta,j} \le \rho^{-1}P_i. \tag{52}$$

Thus, for any initial condition $x(0) \in \varepsilon(P_i, \rho)$,

$$u^T(t)u(t) \le \hat{u}^2. \tag{53}$$

It is clear that input constraint condition (10) is satisfied. This completes the proof.

Theorem 2: For given positive scalars α, ρ, ϵ_1 and ϵ_2, if there exist positive symmetric matrices $\bar{P}_i, \bar{M}_{11,i}, \bar{Z}$, symmetric matrices

$$\bar{F}_i, \bar{U}_i = \begin{bmatrix}\bar{F}_{11,i} & \bar{F}_{12,i}\\ * & \bar{F}_{22,i}\end{bmatrix}, \quad \bar{M}_i = \begin{bmatrix}\bar{M}_{11,i} & \bar{M}_{12,i}\\ * & \bar{M}_{22,i}\end{bmatrix},$$

and any appropriate dimensional matrices $\bar{W}, \bar{H}, \bar{Y}_{\theta,j}, i, j \in \emptyset$, and appropriate dimensional nonsingular matrix N, satisfying the following LMIs:

$$\dot{\bar{P}}_i \le 0, \quad \dot{\bar{U}}_h \le 0, \quad \dot{\bar{F}}_h \le 0, \quad \dot{\bar{M}}_h \le 0 \tag{54}$$

$$\begin{bmatrix}\bar{\Omega}(\tau_k, 0) & \frac{\tau_k}{2}\bar{F}\\ * & -\bar{Z}\end{bmatrix} < 0 \tag{55}$$

$$\begin{bmatrix}\bar{\Omega}(\tau_k, \tau_k) & e^{-\alpha\hbar_M}\sqrt{\tau_k}\bar{W} & e^{-\alpha\hbar_M}\sqrt{\tau_k}\bar{H} & \frac{\tau_k}{2}\bar{F}\\ * & -\bar{U}_i & 0 & 0\\ * & * & -\bar{M}_{11,i} & 0\\ * & * & * & -\bar{Z}\end{bmatrix} < 0 \tag{56}$$

$$\begin{bmatrix}\hat{u}^2I & \bar{Y}_{\theta,j}\\ * & \rho^{-1}\bar{P}_i\end{bmatrix} \ge 0 \tag{57}$$

where

$$\begin{aligned}\bar{\Omega}(\tau_k, 0) &= Sym\{e_1\bar{P}_ie_2^T + e^{-2\alpha\hbar_M}\bar{W}^T(e_1-e_3)^T \\ &\quad + e^{-2\alpha\hbar_M}\bar{H}^T(e_1-e_3)^T - e^{-2\alpha\hbar_M}(e_1-e_3) \\ &\quad \times \bar{M}_{12,i}e_3^T\} + 2\alpha e_1\bar{P}_ie_1^T + \tau_ke_2\bar{U}_ie_2^T + \frac{\tau_k^2}{4}e_2\bar{Z}e_2^T \\ &\quad + \tau_k\{[e_1\ e_3]\bar{F}_i[e_1\ e_3]^T\} + \tau_k\{[e_2\ e_3]\bar{M}_i[e_2\ e_3]^T\} \\ &\quad + Sym\{(e_1+\epsilon_1e_2+\epsilon_2e_3)(A_iNe_1^T + B_i\bar{Y}_{\theta,j}e_3^T \\ &\quad - Ne_2^T)\} \\ \bar{\Omega}(\tau_k, \tau_k) &= Sym\{e_1\bar{P}_ie_2^T + e^{-2\alpha\hbar_M}\bar{W}^T(e_1-e_3)^T \\ &\quad + e^{-2\alpha\hbar_M}\bar{H}^T(e_1-e_3)^T - e^{-2\alpha\hbar_M}(e_1-e_3)\bar{M}_{12,i}e_3^T\} \\ &\quad + 2\alpha e_1\bar{P}_ie_1^T + \frac{\tau_k^2}{4}e_2\bar{Z}e_2^T - \tau_ke^{-2\alpha\hbar_M}e_3\bar{M}_{22,i}e_3^T \\ &\quad + (2\alpha\hbar_M - 1)\tau_k\{[e_1\ e_3]\bar{F}_i[e_1\ e_3]^T\} \\ &\quad + Sym\{(e_1+\epsilon_1e_2+\epsilon_2e_3)(A_iNe_1^T + B_i\bar{Y}_{\theta,j}e_3^T \\ &\quad - Ne_2^T)\} \\ \bar{F} &= [N^TF_{11,i}N \quad 0 \quad N^TF_{12,i}N]^T \\ K_{\theta,j} &= \bar{Y}_{\theta,j}N^{-1}\end{aligned}$$

then the system (9) will be GAES for any initial condition $x(0) \in \varepsilon(\bar{P}_i, \rho)$.

Proof: Define

$$\begin{aligned}\bar{P}_i &= N^TP_iN, \bar{U}_i = N^TU_iN, \bar{Z}_i = N^TZ_iN \\ \bar{F}_i &= diag\{N^T, N^T\}F_idiag\{N, N\} \\ \bar{M}_i &= diag\{N^T, N^T\}M_idiag\{N, N\} \\ \bar{H} &= N^THdiag\{N, N, N\} \\ \bar{W} &= N^TWdiag\{N, N, N\} \\ N_1 &= N^{-1}, N_2 = \epsilon_1N^{-1}, N_3 = \epsilon_2N^{-1}.\end{aligned}$$

Pre- and post-multiplying (44) by $diag\{N^T, N^T, N^T\}$ and $diag\{N, N, N\}$, respectively, we have (55) and (56). Pre- and post- multiply (19) by $diag\{I, N^T\}$ and $diag\{I, N\}$, respectively, we obtain (57). This completes the proof.

Corollary 1: Assume the $|\varpi(t) - \varpi(t_k)| \le \varrho_i$, for given positive scalars α, ρ, ϵ_1 and ϵ_2, if there exist positive symmetric matrices $\bar{P}_i, \bar{M}_{11,i}, \bar{Z}, \mathbb{U}_{1ij}, \mathbb{V}_{1ij}, \mathbb{U}_{2ij}, \mathbb{V}_{2ij}$, symmetric matrices

$$\bar{F}_i, \bar{U}_i = \begin{bmatrix}\bar{F}_{11,i} & \bar{F}_{12,i}\\ * & \bar{F}_{22,i}\end{bmatrix}, \quad \bar{M}_i = \begin{bmatrix}\bar{M}_{11,i} & \bar{M}_{12,i}\\ * & \bar{M}_{22,i}\end{bmatrix},$$

and appropriate dimensional nonsingular matrix N, and any appropriate dimensional matrices $\bar{W}, \bar{H}, \bar{Y}_{\theta,j}, \mathbb{N}_{ij} = \mathbb{N}_{ji}^T, \mathbb{N}_{i(j+r)} = \mathbb{N}_{(j+r)i}^T, i, j \in \emptyset$, satisfying the following conditions:

$$\dot{\bar{P}}_i \le 0, \quad \dot{\bar{U}}_h \le 0, \ \dot{\bar{F}}_h \le 0, \ \dot{\bar{M}}_h \le 0 \tag{58}$$

$$\mathbb{Z}_{\ell ij} + \mathbb{Z}_{\ell ji} \le \mathbb{N}_{\ell ij} + \mathbb{N}_{\ell ji} \tag{59}$$

$$\Xi_{\ell ij} - 2\mathbb{Z}_{\ell ij} + \sum_{s=1}^{r}\varrho_s(\mathbb{Z}_{\ell is}^{+} + \mathbb{Z}_{\ell sj}^{+}) \le \mathbb{N}_{\ell i(j+r)} + \mathbb{N}_{\ell(j+r)i} \tag{60}$$

$$\begin{bmatrix} \mathbb{Y}_{11} & \mathbb{Y}_{12} \\ * & \mathbb{Y}_{11} \end{bmatrix} < 0 \tag{61}$$

$$\begin{bmatrix} \hat{u}^2 I & \bar{Y}_{\theta,j} \\ * & \rho^{-1}\bar{P}_i \end{bmatrix} \geq 0 \tag{62}$$

where

$$\mathbb{Z}_{\ell ij} = \mathbb{U}_{\ell ij} - \mathbb{V}_{\ell ij}, \quad \mathbb{Z}^{+}_{\ell ij} = \mathbb{U}_{\ell ij} + \mathbb{V}_{\ell ij}, \quad (\ell = 1, 2)$$

$$\mathbb{Y}_{11} = \begin{bmatrix} \mathbb{N}_{11} & \cdots & \mathbb{N}_{1r} \\ \vdots & \ddots & \vdots \\ \mathbb{N}_{r1} & \cdots & \mathbb{N}_{rr} \end{bmatrix},$$

$$\mathbb{Y}_{11} = \begin{bmatrix} \mathbb{N}_{1(r+1)} & \cdots & \mathbb{N}_{1(2r)} \\ \vdots & \ddots & \vdots \\ \mathbb{N}_{r(r+1)} & \cdots & \mathbb{N}_{r(2r)} \end{bmatrix}$$

$$\Xi_{1ij} = \begin{bmatrix} \bar{\Omega}(\tau_k, 0) & \frac{\tau_k}{2}\bar{F} \\ * & -\bar{Z} \end{bmatrix} < 0$$

$$\Xi_{2ij} = \begin{bmatrix} \bar{\Omega}(\tau_k, \tau_k) & e^{-\alpha\hbar_M}\sqrt{\tau_k}\bar{W} & e^{-\alpha\hbar_M}\sqrt{\tau_k}\bar{H} & \frac{\tau_k}{2}\bar{F} \\ * & -\bar{U}_i & 0 & 0 \\ * & * & -\bar{M}_{11,i} & 0 \\ * & * & * & -\bar{Z} \end{bmatrix} < 0$$

Then the system (9) will be GAES for any initial condition $x(0) \in \varepsilon(\bar{P}_i, \rho)$.

Remark 1: The above controller gain matrix $K_{\theta,j} = \bar{Y}_{\theta,j} N^{-1}$,

$$\Xi_{1ij} = \begin{bmatrix} \bar{\Omega}(\tau_k, 0) & \frac{\tau_k}{2}\bar{F} \\ * & -\bar{Z} \end{bmatrix}$$

and

$$\Xi_{2ij} = \begin{bmatrix} \bar{\Omega}(\tau_k, \tau_k) & e^{-\alpha\hbar_M}\sqrt{\tau_k}\bar{W} & e^{-\alpha\hbar_M}\sqrt{\tau_k}\bar{H} & \frac{\tau_k}{2}\bar{F} \\ * & -\bar{U}_i & 0 & 0 \\ * & * & -\bar{M}_{11,i} & 0 \\ * & * & * & -\bar{Z} \end{bmatrix},$$

are the same as Theorem 2.

IV. SIMULATION

In this section, gives a numerical examples to demonstrate that the proposed method effectiveness. Consider the following chaotic Lorenz system with an input term:

$$\begin{cases} \dot{x}_1(t) = -ax_1(t) + ax_2(t) + u(t) \\ \dot{x}_2(t) = cx_1(t) - x_2(t) - x_1(t)x_3(t) \\ \dot{x}_3(t) = x_1(t)x_2(t) - bx_3(t) \end{cases} \tag{63}$$

Note that $x_1(t) \in [-d, d]$, and the Lorenz system (63) can be expressed by using the T-S fuzzy modeling method as follow:

$$Rule1 : IF x_1(t) \text{ is } \zeta_1, THEN \dot{x}(t) = A_1 x(t) + B_1 u(t)$$
$$Rule2 : IF x_1(t) \text{ is } \zeta_2, THEN \dot{x}(t) = A_2 x(t) + B_2 u(t)$$

with

$$A_1 = \begin{bmatrix} -a & a & 0 \\ c & -1 & -d \\ 0 & d & -b \end{bmatrix}, A_2 = \begin{bmatrix} -a & a & 0 \\ c & -1 & d \\ 0 & -d & -b \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

where $a = 10, b = \frac{8}{3}, c = 28, d = 25$, and the FMFs are $h_1(\varpi(t)) = \frac{1}{2}(1 + \frac{x_1(t)}{d})$ and $h_2(\varpi(t)) = 1 - h_1(\varpi(t))$. The given initial condition is $x(0) = (-0.3, 0.1, 0.2)^T$. For given $\varrho_1 = 0.12$ and $\varrho_2 = 1$, by using Corollary 1 when $\epsilon_1 = 0.15$ and $\epsilon_2 = 1.3$ under the constraint $S_1 : \{P_1 > P_2, U_1 > U_2, F_1 > F_2, M_1 > M_2\}$ we have $h = 0.0697, 0.0696, 0.0694, 0.0692, 0.0690$ under $\alpha = 0.1, 0.2, 0.3, 0.4, 0.5$ respectively. Meanwhile, for the constraint $S_2 : \{P_1 \leq P_2, U_1 \leq U_2, F_1 \leq F_2, M_1 \leq M_2\}$ when $\epsilon_1 = 0.15$ and $\epsilon_2 = 1.25$ we can get $h = 0.0717, 0.0716, 0.0714, 0.0712, 0.0711$ under $\alpha = 0.1, 0.2, 0.3, 0.4, 0.5$ respectively. Similar to Algorithm 1 of Ref.23, we can obtain the maximum sampling period $h = 0.0697, 0.0696, 0.0694, 0.0692, 0.0690$ for different α. In Table 1, a detailed comparison the maximum sampling period between Corollary 1 and the results of [20] and [21], can be shown. Meanwhile, under the possible permutations of $\dot{h}_1(\varpi(t))$, which include $\dot{h}_1(\varpi(t)) < 0$ and $\dot{h}_1(\varpi(t)) \geq 0$ and correspond to S_1 and S_2 respectively, we can get corresponding control gains as follow:

$$K_{1,1} = \begin{bmatrix} -7.1648 & -26.2767 & -10.4339 \end{bmatrix}$$
$$K_{1,2} = \begin{bmatrix} 5.3898 & 2.3505 & -14.5889 \end{bmatrix}$$
$$K_{2,1} = \begin{bmatrix} -5.3049 & -25.0191 & -9.3138 \end{bmatrix}$$
$$K_{2,2} = \begin{bmatrix} 7.3954 & 2.8378 & -13.0551 \end{bmatrix}$$

TABLE 1. Maximum sampling period h under constant sample for different α.

α	0.1	0.2	0.3	0.4	0.5
[20]	0.0339	0.0330	0.0322	0.0313	0.0302
[21]	0.0556	0.0548	0.0540	0.0532	0.0522
Corollary 1	0.0697	0.0696	0.0694	0.0692	0.0690

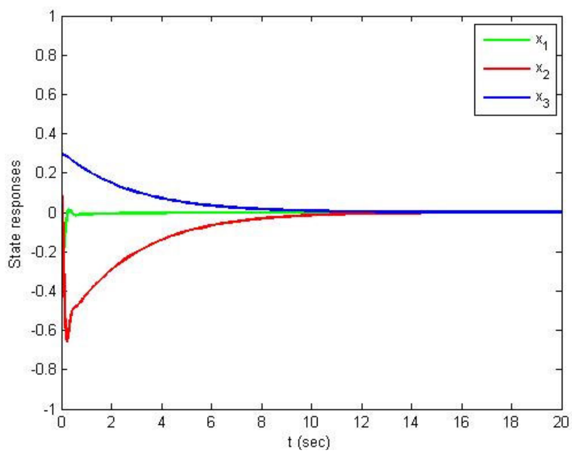

FIGURE 1. State response of system (63).

The Figure 1 shows state response curves of system(63), which indicates the sampled-data controller designed in this

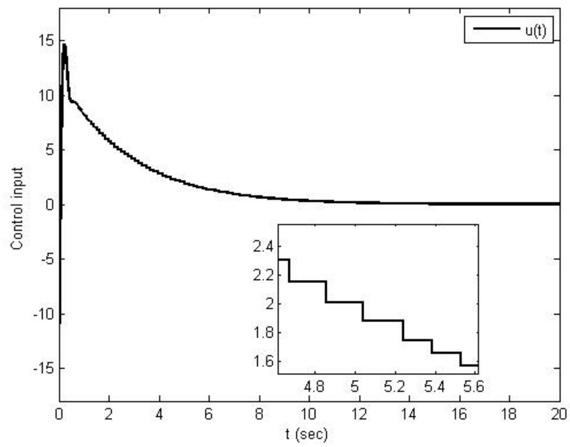

FIGURE 2. Input response of system (63).

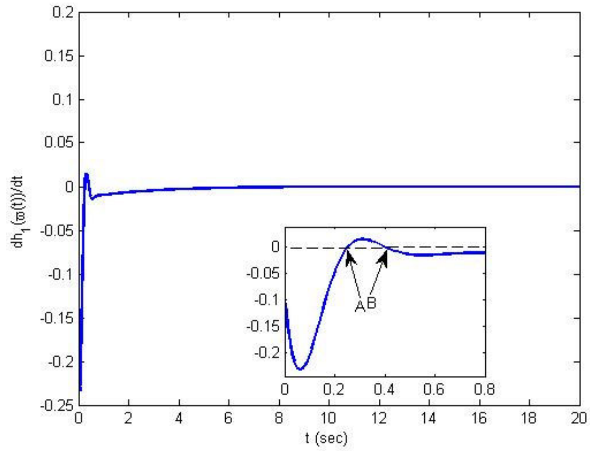

FIGURE 3. State response of $dh_1(\varpi(t))/dt$.

paper can effectively stabilize chaotic systems. The curve of control input $u(t)$ is shown in Figure 2, and input saturation constraint $\hat{u} = 20$ can be satisfied in Figure 2. In the figure 3,we can get that the controller is $u_1(t)$ during the simulation time interval [0,A]s,and switch to $u_2(t)$ in the simulation time interval [A,B]s.Then, it switch to $u_1(t)$ again in [B,0.8]s.

V. CONCLUSION

In this paper, a TSFSD controller has been investigated to get the exponential stability of a class of CSs which are represented by T-S fuzzy scheme with input constraints via a switched technology. By employing a switched approach and FMFs-dependent Lyapunov functional for stabilization of Lorenz system, we can get less conservatism and a large sampling period to prove the merits and effectiveness of proposed scheme. Nevertheless, the results proposed in this paper are still somewhat conservative, since the type-2 fuzzy set can solve the problem that the membership function of the classical fuzzy set is too accurate or difficult to determine, the fuzzy system in this paper can be replaced with the type-2 fuzzy system in the future work to model more uncertainties and further reduce the conservatism of the system. On the other hand, due to the problem of time-delay exists widely in nonlinear systems, and it is one of the reasons which leading to the unstability of nonlinear systems. Thus, it is great practical significance to study the stability of nonlinear time-delay(NTD) systems. In the future, we will use this approach to study on stability of NTD Systems. Moreover, input saturation is a common constraint behavior in the practical chaotic systems, which will cause instability of the system. Inspired by [25], chaotic synchronization control under input saturation will be another research direction of our future work.

REFERENCES

[1] H. Wang and Q. X. Zhu, "Adaptive output feedback control of stochastic nonholonomic systems with nonlinear parameterization," *Automatica*, vol. 98, pp. 247–255, Dec. 2018.

[2] R. Rakkiyappan, L. V. Preethi, Q. X. Zhu, and Z. S. Yao, "Exponential synchronization of Markovian jumping chaotic neural networks with sampled-data and saturating actuators," *Nonlinear Anal., Hybrid Syst.*, vol. 24, pp. 28–44, May 2017.

[3] Z. Zhu, Z. Zhao, H. Cui, and F. Shi, "Improved T-S fuzzy control for uncertain time-delay coronary artery system," *Complexity*, vol. 2019, May 2019, Art. no. 3864843.

[4] Y. Wang and H. Yu, "Fuzzy synchronization of chaotic systems via intermittent control," *Chaos, Solitons Fractals*, vol. 106, pp. 154–160, Jan. 2018.

[5] B. Wang and Q. X. Zhu, "Stability analysis of semi-Markov switched stochastic systems," *Automatica*, vol. 94, pp. 72–80, Aug. 2018.

[6] S. Mobayen, "Chaos synchronization of uncertain chaotic systems using composite nonlinear feedback based integral sliding mode control," *ISA Trans.*, vol. 77, pp. 100–111, Jun. 2018.

[7] Z. Z. Zhao, J. Zhang, G. Ding, and D. K. Zhang, "Chaos synchronization of coronary artery system based on higher order sliding mode adaptive control," *Acta Phys. Sinica*, vol. 64, no. 21, pp. 264–271, 2015.

[8] Q. Lai, Z. Wan, L. K. Kengne, P. D. K. Kuate, and C. Chen, "Two-memristor-based chaotic system with infinite coexisting attractors," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, early access, Dec. 11, 2020, doi: 10.1109/TCSII.2020.3044096.

[9] H. Jahanshahi, A. Yousefpour, J. M. Munoz-Pacheco, I. Moroz, Z. Wei, and O. Castillo, "A new multi-stable fractional-order four-dimensional system with self-excited and hidden chaotic attractors: Dynamic analysis and adaptive synchronization using a novel fuzzy adaptive sliding mode control method," *Appl. Soft Comput.*, vol. 87, Feb. 2020, Art. no. 105943.

[10] K. Shahna and A. Mohamed, "A novel image encryption scheme using both pixel level and bit level permutation with chaotic map," *Appl. Soft Comput.*, vol. 90, May 2020 , Art. no. 106162.

[11] H. T. Yau, C. C. Wang, J. Y. Chang, and X. Y. Su, "A study on the application of synchronized chaotic systems of different fractional orders for cutting tool wear diagnosis and identification," *IEEE Access*, vol. 7, pp. 15903–15911, 2019.

[12] Y. Xu, M. Fang, Y. Pan, K. Shi, and Z. Wu, "Event-triggered output synchronization for nonhomogeneous agent systems with periodic denial-of-service attacks," *Int. J. Robust Nonlinear Control*, Sep. 2020, doi: 10.1002/rnc.5223.

[13] S. Dong, G. Chen, M. Liu, and Z. Wu, "Cooperative neural-adaptive fault-tolerant output regulation for heterogeneous nonlinear uncertain multiagent systems with disturbance," *Sci. China Inf. Sci.*, doi: 10.1007/s11432-020-3122-6,2020.

[14] F. Z. Nian, X. M. Liu, and Y. Q. Zhang, "Sliding mode synchronization of fractional-order complex chaotic system with parametric and external disturbances," *Chaos, Solitons Fractals*, vol. 116, pp. 22–28, Nov. 2018.

[15] X. Y. Chen, J. H. Park, J. D. Cao, and J. L. Qiu, "Sliding mode synchronization of multiple chaotic systems with uncertainties and disturbances," *Appl. Math. Comput.*, vol. 308, pp. 161–173, Nov. 2018.

[16] Y. J. Liu, J. H. Park, B. Z. Guo, and Y. J. Shu, "Further results on stabilization of chaotic systems based on fuzzy memory sampled-data control," *IEEE Trans. Fuzzy Syst.*, vol. 26, pp. 1040–1045, Nov. 2017.

[17] L. B. Wu, G. H. Yang, H. Wang, and F. L. Wang, "Adaptive fuzzy asymptotic tracking control of uncertain nonaffine nonlinear systems with non-symmetric dead-zone nonlinearities," *Inf. Sci.*, vol. 348, pp. 1–14, Oct. 2016.
[18] X. H. Chang and G. H. Yang, "Nonfragile H_∞ filtering of continuous-time fuzzy systems," *IEEE Trans. Signal Process.*, vol. 59, no. 4, pp. 1528–1538, Dec. 2011.
[19] H.-N. Wu, J.-W. Wang, and H.-X. Li, "Exponential stabilization for a class of nonlinear parabolic pde systems via fuzzy control approach," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 2, pp. 318–329, Oct. 2012.
[20] Z. G. Wu, P. Shi, H. Su, and J. Chu, "Sampled-data fuzzy control of chaotic systems based on a T–S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 1, pp. 153–163, Mar. 2014.
[21] H. B. Zeng, K. T. Teo, Y. He, and W. Wang, "Sampled-data stabilization of chaotic systems based on a TS fuzzy model," *Inf. Sci.*, vol. 483, pp. 262–272, May 2019.
[22] T. H. Lee and J. H. Park, "New methods of fuzzy sampled-data control for stabilization of chaotic systems," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 12, pp. 2026–2034, Dec. 2017.
[23] X. Wang, J. Park, K. She, S. Zhong, and L. Shi, "Stabilization of chaotic systems with T–S fuzzy model and nonuniform sampling: A switched fuzzy control approach," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 6, pp. 1263–1271, Oct. 2019.
[24] Z. P. Wang and H. N. Wu, "On fuzzy sampled-data control of chaotic systems via a time-dependent Lyapunov functional approach," *IEEE Trans. Cybern.*, vol. 45, no. 4, pp. 819–829, Aug. 2015.
[25] Q. Wang and Z. G. Wu, "Robust output feedback control for input-saturated systems based on a sliding mode observer," *Circuits Syst. Signal Process.*, vol. 1, pp. 1–15, Nov. 2020.

YUNJUN CHEN was born in 1976. He received the M.S. degree in 2005 and the Ph.D. degree from Tiangong University, in 2016. He is currently a Lecturer with the School of Electrical Engineering and Automation, Tiangong University. His current research interests include intelligent control systems, time-delay systems, and intelligent manufacturing.

QIUXIA CAO was born in 1995. She received the B.S. degree from the Sanquan College of Xinxiang Medical University, in 2018. She is currently pursuing the M.S. degree with the School of Computer Science and Technology, Tiangong University. Her current research interest includes the synchronization of coronary artery time-delay systems.

ZHENYU ZHU (Student Member, IEEE) was born in 1993. He received the B.S. and M.S. degrees from the School of Computer Science and Technology, Tiangong University, in 2016 and 2020, respectively. He is currently pursuing the Ph.D. degree with the School of Control Science and Engineering, Shandong University. His research interests include fuzzy control, robust control, and their applications.

ZHANGANG WANG was born in 1975. He received the M.S. degree in 2005 and the Ph.D. degree from Tiangong University, in 2016. He is currently an Associate Professor with the School of Computer Science and Technology, Tiangong University. His current research interests include software engineering, block chain, and big data.

ZHANSHAN ZHAO was born in 1980. He received the M.S. degree from the Harbin Institute of Technology, in 2006, and the Ph.D. degree from Tianjin University, in 2010. He was a Visiting Scholar with RMIT University, Australia, in 2018. He is currently a Professor with the School of Computer Science and Technology, Tiangong University. His current research interests include time-delay systems, sliding mode control, and chaotic systems.