Low Complexity Linear Detectors for Massive MIMO: A Comparative Study

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ABSTRACT Massive multiple-input multiple-output (M-MIMO) is a significant pillar in fifth generation (5G) networks where a large number of antennas is deployed. It provides massive advantages to modern communication systems in data rate, spectral efficiency, number of users serviced simultaneously, energy efficiency, and quality of service (QoS). However, it requires advanced signal processing for data detection. The growing MIMO size leads to complicated scenarios, which makes the detector design a knotty problem. The problem is also becoming more complicated when high-order modulation schemes are exploited and more users are multiplexed. Therefore, it is not practical to employ the maximum likelihood (ML) detector despite the excellent performance. Linear detectors are alternative solutions and relatively simple. Unfortunately, they still need an exact matrix inversion computation, which bears to a significant high complexity. Therefore, several iterative methods are utilized to approximate or evade the matrix inversion rather than computing it. This paper studies the pros and cons of iterative matrix inversion methods where the number of computations and bit-error-rate (BER) are considered to compare between the methods. The comparison is conducted in several scenarios such as different ratio between the number of base station (BS) antennas and user terminal (UT) antennas (β), the number of iterations (n), and the relaxation parameter (ω). This paper also studies the impact of ω in the performance of Richardson (RI) and the successive over-relaxation (SOR) methods. Numerical results show that the conjugate gradient (CG) and optimized coordinate descent (OCD) methods exhibit the lowest complexity with an acceptable performance. In addition, the Gauss-Seidel (GS) method outperforms all other detectors with a trivial complexity increment. It is also noticed that the performance is not improved with every iteration. It is also shown that ω has a great impact and a significant role in achieving a satisfactory performance in both RI and SOR based detectors. From implementation point of view, detectors based on RI, OCD, and CG methods have achieved the highest hardware efficiency (HE) while Jacobi (JA) based detector has obtained the lowest HE. Recent research advances of detection methods are also presented in the open research direction with a potential impact of linear detection methods in initialization and pre-processing.

INDEX TERMS 5G, M-MIMO, detection, relaxation parameter, hardware efficiency, performance, Jacobi, conjugate gradient, Gaus-Seidel, optimized coordinate descent, Richardson, Neumann series.

I. INTRODUCTION

The number of mobile devices and mobile data traffic are tremendously growing year over year. It is anticipated that

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the number of mobile devices will approach 12.3 billion in 2022 while it was 7.6 billion in 2015 [1]. Sequentially, the mobile data rate will grow more than twenty-fold between 2015 and 2023 [17]–[19]. However, mobile carriers are requested to provide higher data rates, better spectral efficiency, and larger network capacity. Fifth generation

(5G) networks are officially implemented by several mobile companies (i.e., Nokia and Erickson) to achieve the user demands resulted from billions of mobile devices [85]. It is also noteworthy that more than 422 million units will be on 5G networks and nearly 12% of an international mobile traffic will be generated by 5G capable devices by 2022 [17]–[19]. By 2024, more than 1.5 billion subscriptions will be exploited on 5G networks [1]. The research is also propelled to beyond 5G (B5G) and sixth generation (6G) communication systems. In 5G networks, several efficient technologies are utilized such as the optical wireless communication (OWC), the internet of things (IoT), the millimeter wave (mmWave), the device-to-device (D2D) technology, the spectrum sharing (SS), ultra dense networks (UDNs), and the massive multiple-input multiple-output (M-MIMO). Since the third generation (3G), the conventional MIMO technology was deployed successfully to improve the performance of transmitters and receivers. It is also a promising candidate to achieve low latency and energy consumption. In [48], it is shown that the M-MIMO technology is a promising technology to support the massive industrial IoT (IIoT). However, multiple interference signals affect the data being transmitted from and to several antennas. Therefore, detection methods are required at the M-MIMO receiver to estimate and extract the transmitted data from received signal vector. Although the optimum bit-error-rate (BER) performance is obtained by the maximum-likelihood (ML), it is not suitable for MIMO system due to a high complexity [4]. In case of M-MIMO networks, the scenario of utilizing the ML detector is becoming more complicated due to a deployment of a large number of antennas. Therefore, the ML detection is prohibited in large-scale MIMO networks. In contrast, linear detectors are simpler than the ML detectors but they involve an unfavourable computation of a matrix inversion and multiplications [77]. It is well known that the matrix inversion complexity of linear detectors is $\mathcal{O}\left(K^3\right)$. However, for zero forcing (ZF) and minimum mean square error (MMSE) detector, the total complexity in terms of complex multiplications is $K^3 + 2(K^2)N$, where K is the number of users and N is the number of antennas at the BS. As for M-MIMO, $N \gg K$, then N is significative for complexity analysis. To this end, several free-matrix-inversion detection methods were proposed for the M-MIMO UL systems. The research aimed to achieve a detector with a low number of computations (low complexity) and a near optimal BER (high performance). The main idea behind proposed methods was either the utilization of iterative methods to approximate the matrix inversion or to evade the computation of exact matrix. This paper presents the pros and cons of each free-matrix-inversion methods: the Neumann series (NS), the successive overrelaxation (SOR), the Gauss-Seidel (GS), the Jacobi (JA), the conjugate gradient (CG), the optimized coordinate descent (OCD), and the Richardson (RI). A comparison between the performance and complexity profile of detectors is also conducted in several scenarios such as different ratio between the number of BS antennas

and user terminals (UTs) antennas (β), and different number of iterations (n). Relaxation parameter (ω) of the RI and SOR methods is also comprehensively discussed. In addition, the normalized resource consumption (NRC) and hardware efficiency (HE) of detectors based on iterative methods are also presented.

Two survey papers related to data detection in M-MIMO systems were published [4], [98]. Although these papers were comprehensive, none of them have presented the numerical results of performance-complexity profile of each detector. In [98], a comprehensive review of fifty years of MIMO detection was provided. Authors have provided an overview and milestones in the development of MIMO detection. The impact of co-channel interference, MIMO channels, and dispersive MIMO channels were comprehensively discussed. However, the article was focused on MIMO networks. In addition, methods to alleviate the matrix inverse computation in data detection were not illustrated. In addition, this paper was also discussed the impact of a comparable number of active users to the number of BS antennas in large-scale MIMO networks. In addition, the applicability of small-scale MIMO detection techniques in large-scale MIMO is also demonstrated. A survey dated on 2019 [4] presented a detailed clarification of the M-MIMO detection fundamentals. It also provides an extensive overview and milestones in the development of optimal detectors. However, it is not presenting the results in terms of performance and complexity to compare the precision-complexity profile of linear iterative methods. In addition, the HE and NRC were not presented. In other words, there is a paucity of numerical results and comparisons. Furthermore, effects of β, n, and ω were not investigated. The impact of precoding and channel estimation techniques were also presented in the context of data detection for M-MIMO. In [111], a comparison between detectors based on the CG, SOR, NS, and RI was provided when the M-MIMO size is 16×128. However, a comparison with JA, GS, OCD based detectors were not presented. In addition, the impact of β, n, and ω were not presented. In addition, the HE and NRC were not demonstrated. In [5], we exploited the SOR, GS, and JA methods to in pre-processing stage to initialize a detector based on an approximate message passing (AMP). In comparison with the traditional AMP, it was shown that the proposed hybrid detectors achieved a significant performance improvement and complexity reduction. However, hardware implementation was not illustrated. In [80], a comparison between data detection methods based on the NS, CG, and JA was demonstrated. In [41], in order to achieve a high convergence rate and a satisfactory performance, a stair matrix is exploited in iterative free-matrix data detection algorithms. Table 1 summarizes the differences between this work and prior relevant articles.

This paper is organized as follows: Section II presents an overview of the M-MIMO detection. Section III presents the fundamentals of linear detectors such as the MMSE and the impact of matrix inversion in data detection. Section IV demonstrates the iterative methods to avoid/approximate

TABLE 1. Prior Relevant Articles.

Reference	NS	RI	SOR	GS	OCD	JA	CG	Several β	n	BER	Complexity	HE
[98]	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
[4]	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗	✗
[111]	✓	✓	✓	✗	✗	✗	✓	✓	✓	✓	✓	✗
[5]	✗	✗	✓	✓	✗	✓	✗	✗	✓	✓	✓	✗
[80]	✓	✗	✗	✓	✗	✓	✓	✓	✓	✓	✓	✗
This work	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

matrix inversion methods. Section V provides a detailed complexity analysis in real multiplications and additions. In addition, the HE, NRC, and implementation comparison are also demonstrated. In Section VI, the results and discussion are presented. Section VII concludes the paper and presents the future research directions.

II. OVERVIEW

Unlike the conventional small scale MIMO, the BS is occupied by a large number of antennas to avail many users in M-MIMO networks. The number of UTs is usually smaller than the number of BS antennas. The growing MIMO size enhances the channel capacity and the link reliability because of a high multiplexing diversity.

The massive number of active antenna elements at the BS and cohere transceiver processing provide tremendous advantages in high data rate, satisfactory throughput, high spectral efficiency, high power efficiency, and a good QoS [10], [62]. The large data volume that could be generated using M-MIMO arrays call for employing advanced optimization algorithms to eliminate the interference effects, the fast fading, and the uncorrelated noise. In addition, the transmit power is notably reduced and the throughput is increased without increasing the transmit power when the number of transmit antennas grows [66]. However, channel estimation is one of the main challenges in M-MIMO systems where pilot contamination effect has a great impact. Across the transmitters, the channel state information (CSI) has to be exchanged very quickly (extremely low latency) [103]. It is also pivotal to address the transmit precoding challenge to focus each signal at destined receiver, especially in a non line of sight (LOS) scenario. In addition, the number of computations associated with the reliable signal detection at the BS is one of the crucial issues in transceivers design. It is noteworthy that a precise and instantaneous CSI is required at the BS to perform uplink (UL) data detection and down-link beamforming [95]. Matched filter (MF) achieves a good performance when the number of active users is small and rich scattering channels are utilized. In vast majority of M-MIMO literature, the propagation channels are assumed uncorrelated to simplify the process. In practical scenario, propagation channels are generally spatially correlated. Therefore, there is a need for advanced data detection algorithms. Due to a high number of computations of the ML detector, sub-optimal linear detection methods are proposed. Table 4 presents the up-to-date detection methods where linear and nonlinear algorithms have been utilized.

Nonlinear detectors are not competitive in real applications. For example, expectation propagation detection (EPD) method suffers from a complex low-parallelism iterations. Nonlinear detectors such as the successive interference cancellation (SIC), lattice reduction-aided (LRA) algorithms, and SD are utilized in a small scale MIMO. However, they become non-competitive when utilized in M-MIMO systems because a matrix inversion, QR-decomposition, or Cholesky decomposition are required in which the computational complexity is proportional to the number of antenna elements. In the SIC, the BER performance is usually influenced by the first detected signal [54]. In the SD, radius selection is required for each layer and increased gradually. This process aims to pruning more nodes. Thus, a considerable pruning may need to restart the radius selection algorithm several times, which brings additional computational complexity and energy consumption [21]. For instance, the sphere decoding (SD), require additional hardware in order to compute the sub-optimal solution [83], [96], [107]. Moreover, it is not very hardware friendly because of a variable complexity with various signals and channels which leads to a non-fixed detection throughput which is not competent in real time applications [4]. Detectors based on linear methods are relatively easy to implement and has a simple structure. The MMSE detection is one of the most popular and efficient methods in reducing the number of computations. However, MMSE based detector consists of a matrix inverse and suffers from a remarkable losses in highly loaded and ill-conditioned environments. Therefore, it is not the favourite solution in a real time applications [4]. A pleasant balance between the BER performance and the number of computations of linear M-MIMO UL detector can be achieved by employing two well-known scenarios [87]. The first scenario includes the NS and the Newton iteration (NI) methods where the matrix inversion is approximated, rather than the exact calculation [42], [65], [84]. If the expansion order is greater than 2, complexity of the NS method is almost $\mathcal{O}\left(K^3\right)$. The second scenario aims to solve the linear equations by avoiding the matrix inversion using the iterative methods. The SOR [76], [109], the CG [39], [100], [112], the GS [22], [92], the JA [57], [65], [75], the RI [56], and the OCD [13], [87] methods are examples of such iterative methods. Iterative methods replace matrix-matrix multiplications by matrix-vector products. Therefore, the solution of linear equations using the iterative methods requires a small number of computations compared with the first scenario. In addition, the convergence rate in NS and NI methods is slow when $\beta \approx 1$. The number

TABLE 2. Detection methods for M-MIMO systems.

Method	References	Pros	Cons
Lattice reduction aided algorithms	[114], [53], [72], [81], [52]	• The ill-conditioned channel matrix could be modified to be more orthogonal. • Optimum performance can be achieved.	• Computational complexity is high.
Sparsity based algorithms	[16], [36], [26], [2], [37], [35]	• Optimal performance can be obtained.	• Convergence errors could be happened due to sparse constraints. • stiffness computational complexity is required.
Linear detectors, i.e., ZF and MMSE	[66], [87], [6], [7]	• If columns of the propagation matrix are nearly orthogonal, they work properly. • Relatively simple to implement.	• In the scenario of ill-conditioned environment, poor performance and high computational complexity are obtained. • MMSE obtains a significant performance loss in highly loaded systems.
Detectors based on approximate matrix inversions	[33], [112], [84], [20], [43], [46], [79], [78], [110], [27], [34], [22], [93], [109], [28], [24], [49], [76], [91], [30], [71], [11]	• The optimal performance can be obtained. • If the initial solution is properly selected, the optimal performance can be achieved within few iterations.	• Suffer from a considerable performance loss when the ratio between the BS antenna and the user antennas is close to 1. • The convergence rate is highly affected by the initial solution which could lead to a wrong estimation.
Sphere decoder	[64], [67], [15], [8]	• It can be configured to maintain a satisfactory balance between the computational complexity and the performance.	• High computational complexity when the radius is not properly selected.
Successive interference cancellation	[54], [55]	• It achieves a good performance when the number of BS antennas is greater than the number of user terminals.	• The performance is highly affected by the initial solution. • High computational complexity.
Graph models and belief propagation	[58], [61], [105], [74], [90], [73], [60], [59], [97], [94], [32], [23], [9]	• When the channel correlation is low, the ML performance is obtained.	• Optimal damping factor is not easy to obtain. • Performance is considerably degraded if a bad conditioned factor graph is utilized. • Convergence is not always guaranteed.

of computations in M-MIMO detection methods is crucially affected by the systems' size and the matrix by matrix multiplication. In addition, inverse of the *Gram* matrix is one of the major hindrances in detectors design. Thus, the researcher community attention is attracted to utilize iterative methods in the design of high performance and low complexity detectors.

III. SYSTEM MODEL

In M-MIMO UL system, K users are served by N antennas at the BS, where $K \ll N$. Each UT transmits a data symbol $\mathbf{x} = [x_1, x_2, \ldots, x_K]^T$ to the BS where $x_K \in C$ and C is the modulation alphabet. A vector $\mathbf{y} = [y_1, y_2, \ldots, y_N]^T$ is received at the BS. Entries of the channel matrix (**H**) are feigned to be i.i.d Gaussian random variables with unit variance. The model of M-MIMO based detector is always illustrated in Fig. 1 and presented mathematically as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}, \qquad (1)$$

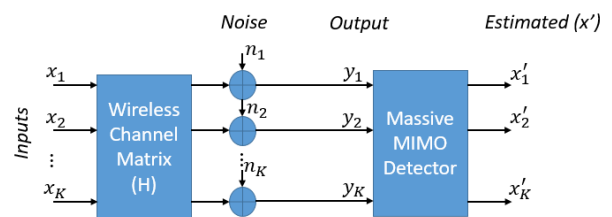**FIGURE 1.** A block diagram of M-MIMO system.

where **n** refers to a noise vector. The ML detector obtains the best performance but the number of computations is extremely high. In the MMSE detector, the signal can be detected by diminishing the mean-square error (MSE) as

$$\mathbf{A}_{MMSE}^H = arg\ \min_{\mathbf{H} \in \mathbb{C}^{N \times K}} \mathbb{E}\|\mathbf{x} - \mathbf{H}^H \mathbf{y}\|^2$$

$$= \left[\mathbf{H}^H \mathbf{H} + \frac{K}{SNR}\mathbf{I}\right]^{-1} \mathbf{H}^H, \qquad (2)$$

where **I** is the identity matrix. The MMSE estimated signal is

$$\hat{\mathbf{x}}_{MMSE} = \mathbf{A}_{MMSE}^H \mathbf{y}. \qquad (3)$$

It is not a trivial task to build a practical detector for M-MIMO systems. However, linear detector would attain a satisfactory performance in UL M-MIMO, but they include unfavourable matrix inversions. The channel matrix in the M-MIMO system will be determined by the number of antennas. More antennas require robust hardware chips in the physical layer. Therefore, matrix inversion is not very hardware friendly where it is the most dominant component in the computational complexity. It also becomes a hard challenge when the system is ill-conditioned and/or the channel matrix is nearly singular. In such scenario, the matrix inversion will not be efficient to equalize the signal. Therefore, linear detectors with iterative methods become substantial to defeat the noise enhancement with a low complexity.

IV. ITERATIVE METHODS

In eighteenth century, iterative JA and GS methods were proposed to avoid the exact matrix computation. However, such methods were rarely used to solve small dimension systems due to a time consumption. In other words, when the system size is small, the required time to compute the exact matrix inverse could be less than the required time when iterative methods are utilized [47]. However, several iterative methods are very useful in storage, area, and computation when the system size is large. In M-MIMO, the problem of matrix inverse increases when the system size is large. The problem is also increased when ill-conditioned system or singular channel matrix are used. In this scenario, the performance of simple detectors is extremely deteriorated. Therefore, advanced signal processing techniques are required to overcome the inherent noise enhancement. In this section, we demonstrate the concepts of several iterative methods to avoid the direct matrix inversion in data detection for M-MIMO. The number of iterations (n), the system's size, and the convergence rate are crucial to obtain a near optimal low-complexity detector [29]. The spectral radius of a channel matrix $\rho(\mathbf{H})$ plays a crucial role in the convergence rate [41]. It is defined as

$$\rho(\mathbf{H}) = max\,|\lambda|, \tag{4}$$

where λ is an eigenvalue of $\mathbf{H}$. However, $\rho(\mathbf{H})$ is closely related to the norm of a matrix. In order to accelerate convergence, a method whose associated matrix has minimal $\rho(\mathbf{H})$ is selected. This section presents several low-complexity detectors based on iterative methods to approximate or avoid the matrix inversion.

A. NEUMANN SERIES

It is a well-known iterative method to approximate the large-scale matrix inversion where the polynomial expansion concept is applied. A sum of infinite number of elements are utilized to express the matrix inversion. It is noteworthy that the matrix inversion is replaced by either matrix-matrix multiplications and \ or matrix-vector multiplications. For MMSE detector, the *Gram* matrix is $\mathbf{G} = \mathbf{H}^H\mathbf{H} + \frac{K}{SNR}\mathbf{I}$. In the NS method, the diagonally dominant *Gram* matrix is

decomposed into the main diagonal matrix ($\mathbf{D}$) and hollow matrix ($\mathbf{E}$), where $\mathbf{G} = \mathbf{D} + \mathbf{E}$ [107]. The elements of $\mathbf{D}$ are considerably higher than the entries of $\mathbf{E}$. The accuracy is improved gradually to approximate the inversion of ($\mathbf{G}$) as

$$\mathbf{G}^{-1} = \sum_{i=0}^{\infty} \left(-\mathbf{D}^{-1}\mathbf{E}\right)^i \mathbf{D}^{-1}, \tag{5}$$

where the condition

$$\lim_{i \to \infty} \left(-\mathbf{D}^{-1}\mathbf{E}\right) = 0, \tag{6}$$

is satisfied. In practical applications, a well-identified n is conducted based on a sum of finite terms (i) as shown in (5). The accuracy of the matrix inversion and the number of computations are highly affected by n. However, the NS method incurs a considerable loss when $\beta \approx 1$. It is also noteworthy that not every iteration could improve the performance. However, every extra iteration could increase the complexity. Therefore, a trade-off between the near optimum performance and the low number of computations is required. In [25], a weighted NS-steepest descent iterative method is proposed to achieve low complexity and satisfactory convergence rate.

B. GAUSS-SEIDEL

The GS is exploited to settle a group of linear equations by computing the solution in an iterative behavior where the Hermitian positive semi-definite matrix ($\mathbf{A}$) of the regularized $\mathbf{G}$ is decomposed into strictly lower triangular entries ($\mathbf{L}$), strictly upper triangular elements ($\mathbf{U}$), and the diagonal entries ($\mathbf{D}$) [31]. In other words, the matrix $\mathbf{A}$ is presented as

$$\mathbf{A} = \mathbf{D} + \mathbf{L} + \mathbf{U}. \tag{7}$$

Therefore, $\mathbf{x}$ is estimated based on the output of matched filter ($\hat{\mathbf{x}}_{MF}$) as

$$\hat{\mathbf{x}}^{(n)} = [\mathbf{D} + \mathbf{L}]^{-1}\left[\hat{\mathbf{x}}_{MF} - \mathbf{U}\hat{\mathbf{x}}^{(n-1)}\right], \quad n = 1, 2, \cdots. \tag{8}$$

where $\hat{\mathbf{x}}^{(n)}$ is the estimated signal and is refined repeatedly through each iteration. However, if a prior knowledge of the initial solution ($\hat{\mathbf{x}}^{(0)}$) is not available, a zero vector can be utilized [3]. The initial solution is refined during the iterations [92]. Unfortunately, internal sequential iterations structure is utilized in the GS method and it is not fitting the parallel computation. However, the performance of the GS method is better than the NS method [104]. In [12], a detector using the GS method with acceleration and preconditioning refinement is proposed. In addition, JA method is exploited in the matrix initialization.

C. SUCCESSIVE OVER-RELAXATION

The SOR is an iterative method to avert the large-dimension matrix inversion. It improves the accuracy of GS method by utilizing a relaxation parameter (ω) [63], [99]. The signal is estimated as

$$\hat{\mathbf{x}}^{(n)} = \left[\frac{1}{\omega}\mathbf{D} + \mathbf{L}\right]^{-1}\left[\hat{\mathbf{x}}_{MF} + \left[\left[\frac{1}{\omega} - 1\right]\mathbf{D} - \mathbf{U}\right]\hat{\mathbf{x}}^{[n-1]}\right]. \tag{9}$$

Convergence of the SOR method was demonstrated in [28] and it is greatly affected by the relaxation parameter (ω). In the SOR, **G** is pre-determined and utilized as an input which increases the number of computations.

In the MIMO system, the convergence rate of the SOR method is usually satisfactory when $0 < \omega < 2$ [28]. The uncertain or improper selection of ω could lead to a performance deterioration or a high complexity of SOR based detector. It is also noteworthy that the pre-computation of **G** could lead to a high computational complexity. In [102], non-adaptive SOR (NA-SOR) detector with fixed ω and adaptive SOR (A-SOR) with iteratively changed ω are proposed. In highly correlated channels, it is shown that the A-SOR detector obtains better performance. However, the NA-SOR detector is more firm to ω. Compared with existing methods, the implementation results show that the A-SOR and NA-SOR achieve better throughput.

D. JACOBI
It is another iterative methods where the solution is estimated as

$$\hat{\mathbf{x}}^{(n)} = \mathbf{D}^{-1} \left[\hat{\mathbf{x}}_{MF} + [\mathbf{D} - \mathbf{A}] \hat{\mathbf{x}}^{(n-1)} \right], \qquad (10)$$

which holds if:

$$\lim_{n \to \infty} \left(\mathbf{I} - \mathbf{D}^{-1}\mathbf{A} \right)^n = 0. \qquad (11)$$

The initial values can be selected as

$$\hat{\mathbf{x}}^{(0)} = \mathbf{D}^{-1}\hat{\mathbf{x}}_{MF}. \qquad (12)$$

The JA based detector attains an acceptable performance when β is small. However, it suffers from slow convergence rate [40], and thus, implying high latency. In addition, it is not obtaining a quasi-optimal accuracy when $\beta \approx 1$. In general, computational complexity of the JA, SOR, and GS is very close. It is also noteworthy that the first iteration in JA method is a multiplication free which decreases the number of computations. However, the SOR and GS methods can achieve higher hardware efficiency.

E. CONJUGATE GRADIENT
The CG method is another method to solve linear equations such as (2 and is advocated by [39] where the signal can be estimated as

$$\hat{\mathbf{x}}^{(n+1)} = \hat{\mathbf{x}}^{(n)} + \alpha^{(n)}\mathbf{p}^{(n)}, \qquad (13)$$

where $\mathbf{p}^{(n)}$ is the conjugate direction taking into account the matrix **A**, i.e.,

$$\left(\mathbf{p}^{(n)} \right)^H \mathbf{A}\mathbf{p}^{(j)} = 0, \quad for \; n \neq j, \qquad (14)$$

where $\mathbf{p}^{(n)}$ and other scalar parameters were comprehensively demonstrated in [39]. In the CG method, the solution is refined iteratively where the search is performed in the conjugate direction with a movement towards the best solution. However, the step size and the direction are identified first

and the solution is updated by moving a step in the search direction next. The CG method obtains better performance and lower number of computations in comparison with the NS based detector [100]. The convergence rate of CG method reduces rabidly when $\beta \approx 1$. It also could achieve a good performance with low complexity as compared with the SOR, GS, JA, and RI methods. In [51], a modified CG method is proposed to avoid the strong data dependency between iterations and elements.

F. RICHARDSON
In RI based method, symmetric matrices are utilized. The positive semi-definite property of regularized **G** is exploited to perform iterative iterations where the signal can be detected accordingly. The convergence rate is very sensitive to a selection of relaxation parameter (ω) where $0 < \omega \leq \frac{2}{\lambda}$ and λ is the largest eigenvalue of the symmetric positive definite matrix **H** [44]. The estimated signal is obtained as

$$\mathbf{x}^{(n+1)} = \mathbf{x}^{(n)} + \omega \left[\mathbf{y} - \mathbf{H}\mathbf{x}^{(n)} \right] \quad n = 0, 1, 2, \cdots. \qquad (15)$$

If a prior knowledge of $\mathbf{x}^{(0)}$ is missing, a zero vector can be considered without loss of generality. It can also be selected as $\hat{\mathbf{x}}^{(0)} = \mathbf{D}^{-1}\hat{\mathbf{x}}_{MF}$ and iteratively refined. The accuracy and the number of computations are highly affected by the value of ω. In [45], a CG method is exploited to enhance the performance of second-order RI method. Moreover, ω is selected based on eigenvalues to speed up the convergence rate.

G. OPTIMIZED COORDINATE DESCENT
In the CD method, the line search step size is determined by the knowledge of channel gains values. It can be used to solve over-determined linear systems where a series of coordinate-wise updates are utilized to achieve the near optimal solution of a convex optimization problem [50]. The estimated signal obtained as

$$\hat{\mathbf{x}}_k = \left[\|\mathbf{h}_k\|^2 + N_0 \right]^{-1} \mathbf{h}_k^H \left[\mathbf{y} - \sum_{j \neq k} \mathbf{h}_j\mathbf{x}_j \right], \qquad (16)$$

where N_o is the noise variance.

In the OCD method, a pre-processing and restructuring are needed to reduce the operations during each iteration. The CD based detector is a high throughput estimation system. A low complexity OCD detector can be implemented in a high-throughput FPGA design for M-MIMO systems [87]. This detector refines the estimated signals for each user in a sequential pattern. Therefore, it affords a high latency [70].

V. COMPLEXITY ANALYSIS AND HARDWARE EFFICIENCY
In this section, the complexity analysis is presented based on the number of real multiplications and additions required to estimate the signal. In addition, a comparison on system level deployment is also provided. The computational complexity

TABLE 3. Computational complexity.

Method	Multiplications	Addition
NS [88]	$4K^3(n-2)+(2K+1)K^2+(4N-1)K$	$4K^3+2NK^2+4KN$
RI [29]	$(4N+4n)K^2+2KN$	$(4N+4n-2)K^2+2NK$
SOR [28]	$(4N+4n-2)K^2+2(N-n+1)K$	$(2N+4n-3)K^3+(2N-3n)K+2n-2$
GS [22]	$(4N+4n-2)K^2+2(N-2n+1)K$	$(4N+4n-4)K^2+(2N-4n)K+2n-2$
JA [113]	$(4N+4n+1)K^2+2NK$	$(4K+4n-2)K^2+2(N-n)K$
CG [106]	$NK^2+n(K^2+6K)$	$NK^2+K^2-2K-1+N(K^2+NK+5K+N+2)$

TABLE 4. Implementation comparison On XILINX VIRTEX-7 FPGA WITH 128 BS antennas and 64QAM.

| Detection Method	NS [89]	RI [82]	SOR [102]	OCD [86]	GS [108]	CG [101]	JA [65]
Number of Users	8	8	8	8	8	8	16
LUT Slices	168,125	29,470	6,394	23,914	18,976	3,324	6,330
FF slices	193,451	25,261	26,448	43,008	15,864	3,878	28010
DSP48 units	1,059	396	232	774	232	33	1,312
Frequency [MHz]	317	213	270	258	309	412	200
Latency [cycle]	1,356 [66]	154	672	795	311	375	121
Throughput [Mbps]	603	246	67.5	376	48	20	158
Throughput/LUTs	3,587	8,348	10,557	15,723	2,530	6,017	24,961
Throughput/FFs	3,118	9,739	2,553	8,743	3,026	5,158	5,641
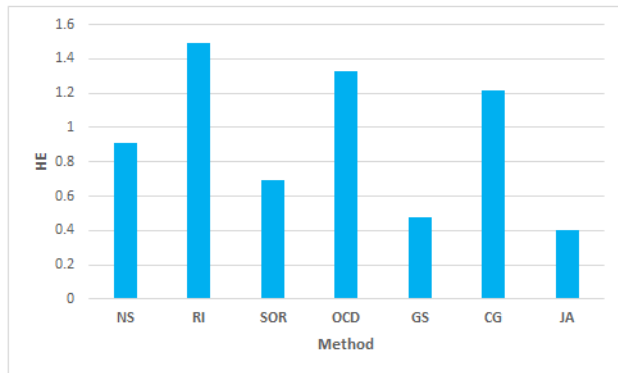
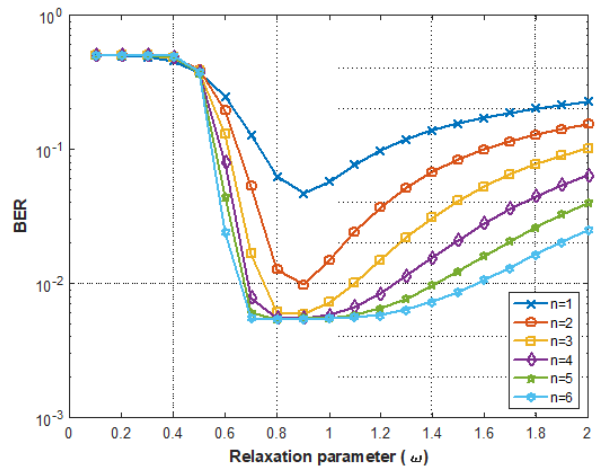**FIGURE 2.** Hardware efficiency (HE) of data detection methods.

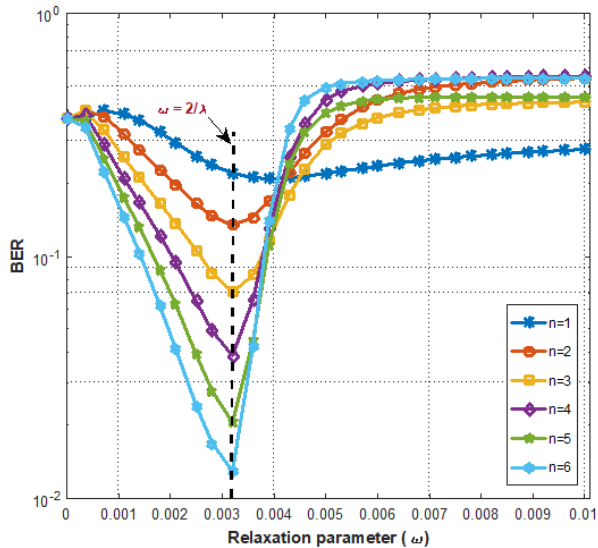**FIGURE 3.** BER performances vs. relaxation parameter (ω) for 64 × 256 MIMO when SNR = 16dB, RI method.

FIGURE 4. BER performances vs. relaxation parameter (ω) for 64 × 256 MIMO when SNR = 16dB, SOR method.

multiplications and additions required by several detection methods. The NS based detector requires a large *n* to achieve a satisfactory performance, hence, the computational complexity of the NS based detector is presented when $n \geq 3$.

Table 4 demonstrates the key implementation results of data detection methods on Xilinx Virtex-7 FPGA platform. Detectors consist of pre-processing and computations of the *Gram* matrix. Therefore, the normalized resource consumption (*NRC*), or normalized area, is presented in [14] as

$$NRC = LUTs + FFs + DSPs \times 280, \qquad (17)$$

where *LUTs*, *FFs*, and *DSPs* are look-up table, flip-flops, and digital signal processing units, respectively. Detectors based on the RI and JA have obtained the lowest latency while NS based architecture has the highest latency. The hardware efficiency (HE) is illustrated as [82]

$$HE = \frac{Throughput}{NRC}. \qquad (18)$$

of the NS method is $\mathcal{O}\left(K^3\right)$. For the RI, SOR, GS, CG, and JA methods, the computational complexity is $\mathcal{O}\left(NK^2\right)$. However, the complexity analysis based on the $\mathcal{O}$ does not show the differences. Therefore, Table 3 presents the number of real
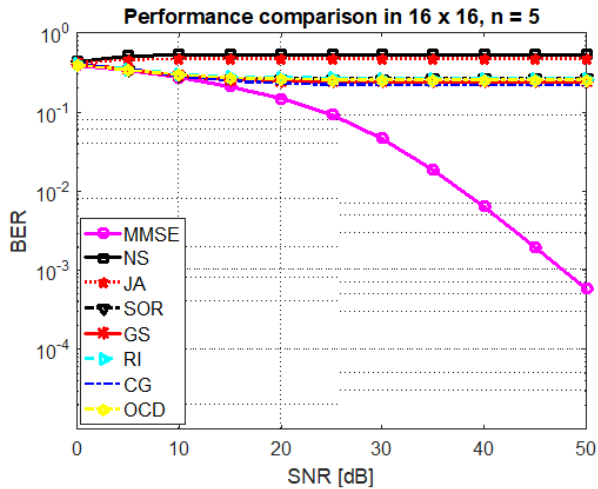
FIGURE 5. BER performance vs. SNR for 16 × 16 M-MIMO where *n* = 5.

Figure 2 shows that the RI based detector has achieved the highest HE, followed by OCD and CG based detectors. Detectors based on the GS and JA have obtained the lowest HE. It is also noteworthy that the architecture based on the NS method has achieved higher HE than the SOR based architecture.

VI. SIMULATION AND DISCUSSION

In this section, the BER performance and the computational complexity are presented for different detection techniques

(the NS, GS, SOR, JA, RI, OCD, and CG). Comparison among iterative methods is presented in BER performance and number of real multiplications. In order to draw useful insights, we consider i.i.d complex Gaussian random variables with zeo mean and unitary variance, various β and M-MIMO environments, i.e., 16 × 16, 16 × 256, 32 × 256, 64 × 256, and 64QAM. The simulations are carried out in MATLAB to present the relationship between the BER and the average SNR.

Selection of ω is crucial to attain a near optimum BER performance of the RI and SOR detectors. Figure 3 shows the BER performance of the MMSE signal detection using the RI method versus ω at SNR = 16dB. The performance improves when ω increases and the best performance is achieved when the value of $\omega = \frac{2}{\lambda}$ where λ is the largest eigenvalue of **H**. Then, the performance is decreased when $\omega > \frac{2}{\lambda}$. Figure 4 illustrates the BER performance of the MMSE signal detection utilizing the SOR method versus ω at SNR = 16dB. A satisfactory BER performance is achieved when $0 < \omega < 2$. At $\omega = 0.8$, the best BER performance is achieved. In the research, the best value of ω is extracted from Figs. 3 and 4 and utilized in the following RI and SOR simulations.

Figure 5 demonstrates the BER performance of the MMSE using several iterative matrix inversion methods in 16 × 16 M-MIMO system. The performance is deteriorated even when *n* is large because the ratio between number of BS antennas and UTs antennas is $\beta = 1$. In other words, iterative matrix inversion methods are not suitable to achieve
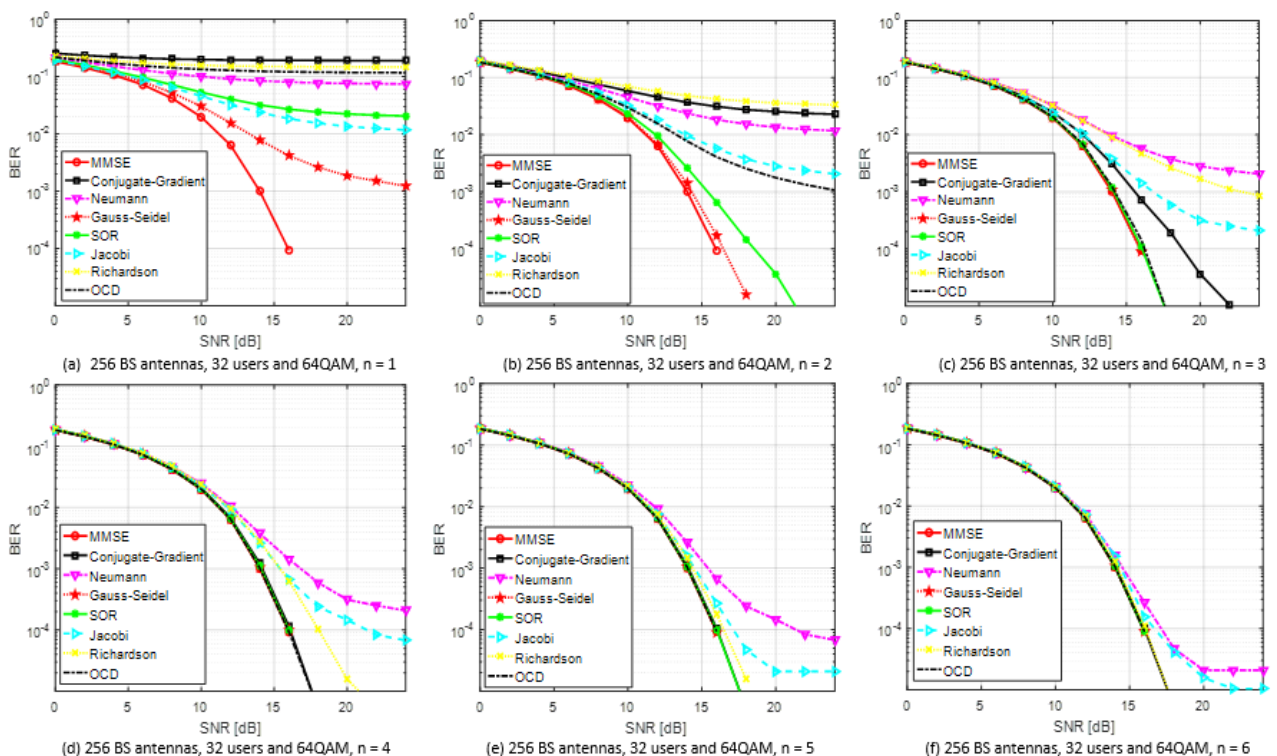

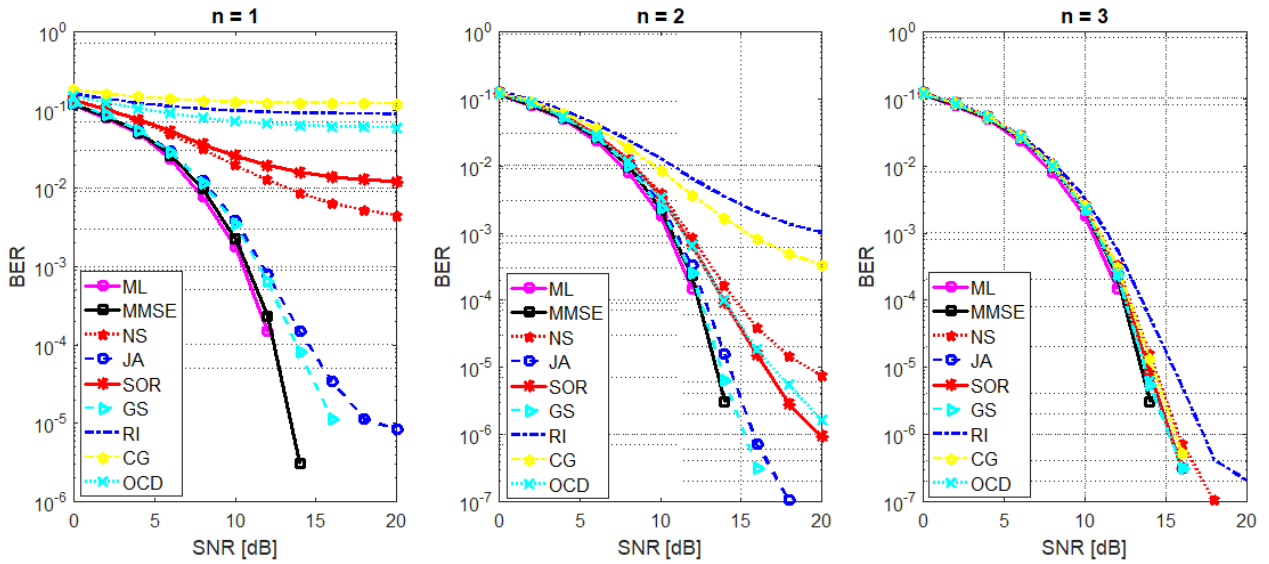**FIGURE 6.** BER performance vs. SNR for 32 × 256 M-MIMO and 64QAM.

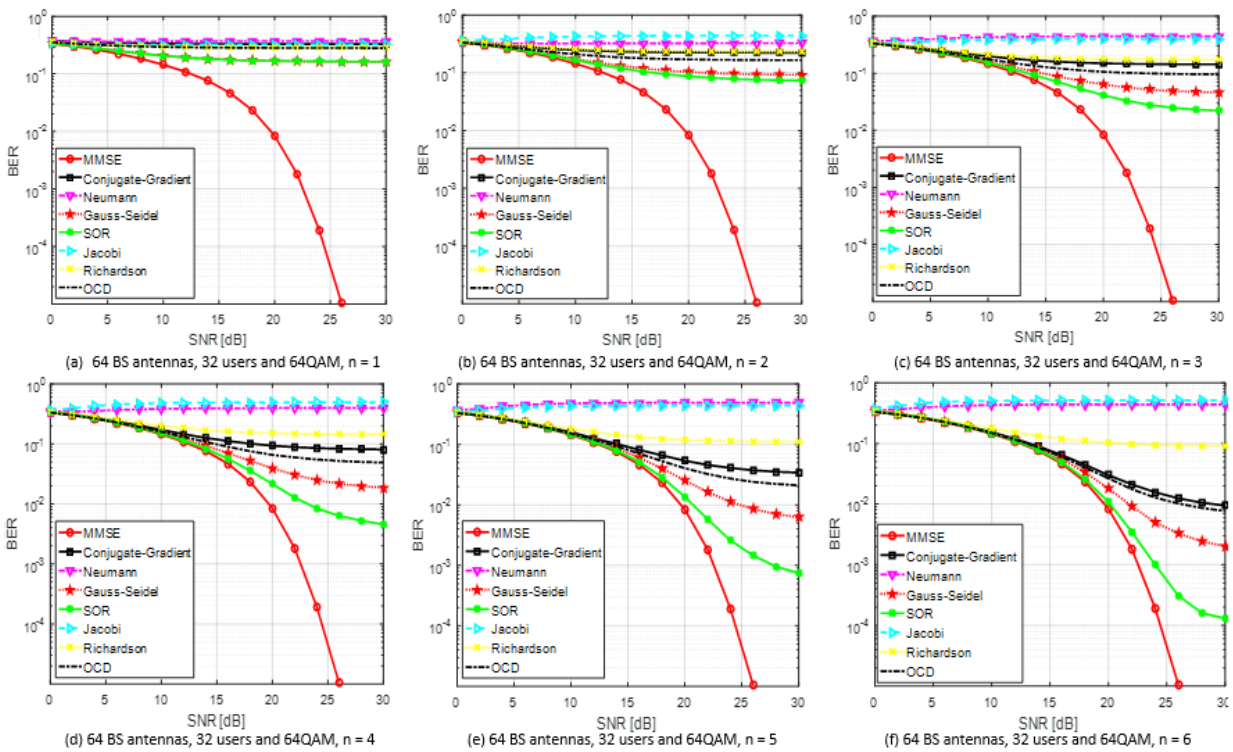**FIGURE 7.** BER performance vs. SNR for 16 × 256 M-MIMO and 64QAM.

FIGURE 8. BER performance vs. SNR for 32 × 64 M-MIMO and 64QAM.

ML performance when the general M-MIMO assumption ($N \gg K$) is not applied. When $N = K$, the classical MMSE detector is preferred.

Figure 6 till Fig. 9 show the performance of detectors based on several approximate matrix inversion methods at different MIMO sizes. It is clear that the performance profile of detectors based on approximate matrix inversion methods

converges to the MMSE performance when β is smaller than 1. The detector based on the GS method achieves the best BER performance when $n = 1$ and 2. When β is large (close to 1), detectors based on the NS and JA methods suffer from a considerable performance loss and the performance does not improve through iterations. It is also clear that a detector based on CG, NS, and JA methods requires a high number of
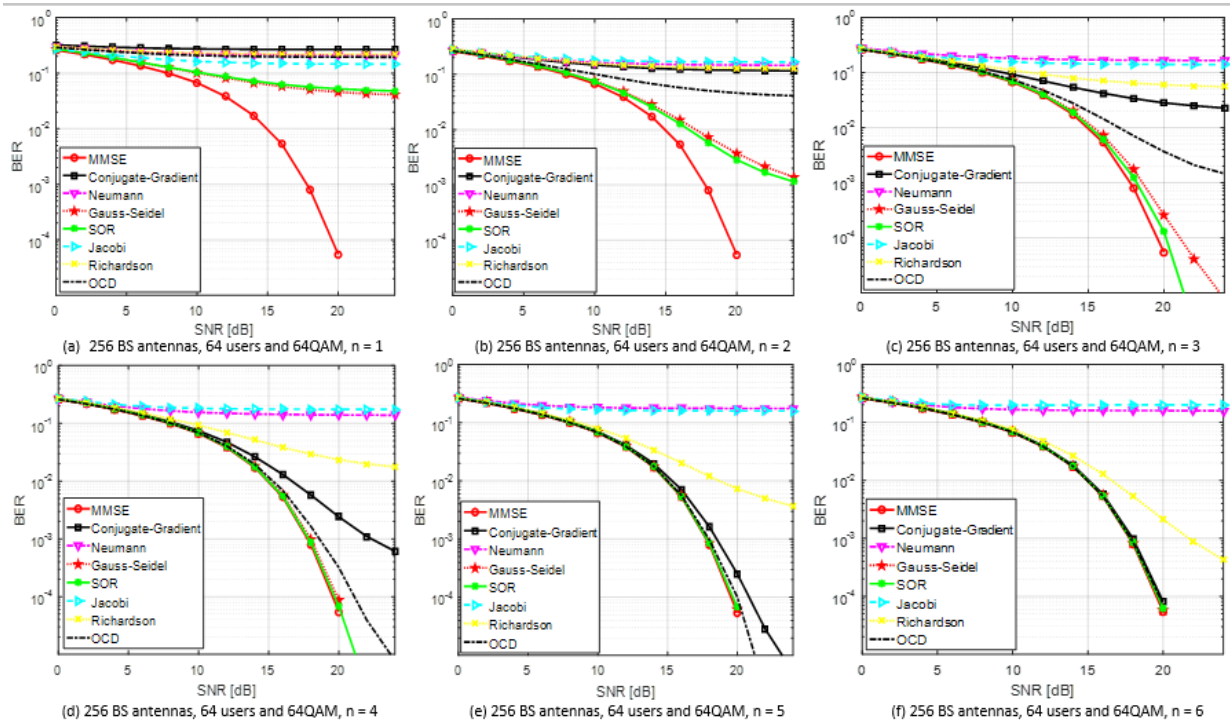
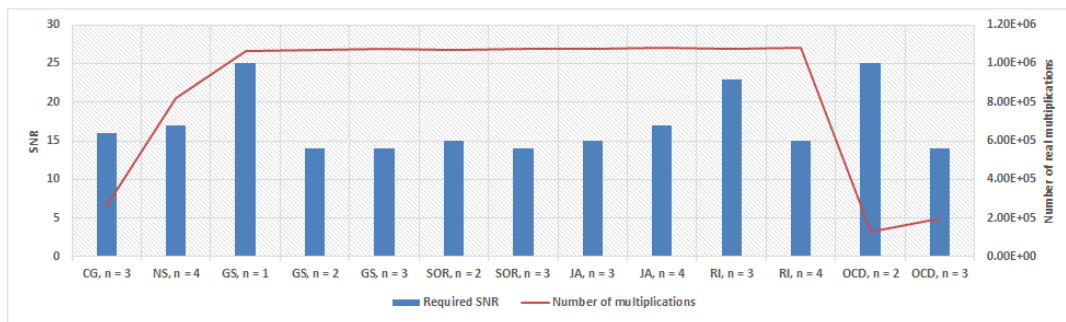**FIGURE 9.** BER performance vs. SNR for 64 × 256 M-MIMO and 64QAM.

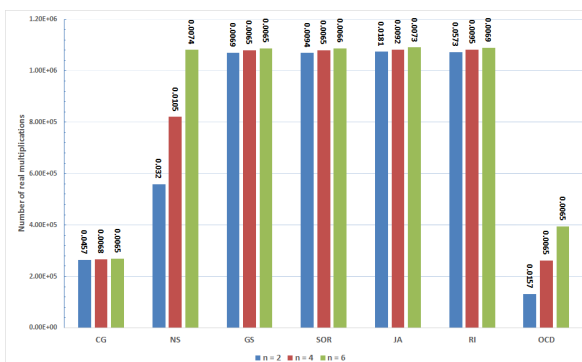**FIGURE 10.** Performance vs. complexity to attain BER = 10^{-2} in 32 × 256 MIMO.

FIGURE 11. Number of real multiplications and BER in 32 × 256 MIMO, SNR = 12dB.

iterations to obtain a satisfactory BER performance. The RI detector outperforms the NS and JA based detectors. However, RI detector requires a large n to achieve the optimum

performance. In Fig. 7, the ML concept for data detection is compared with other data detection methods. It is clear that the OCD based detector has achieved a quasi-optimum performance when $n = 1$ while other methods require extra iterations.

Figure 10 presents the number of iterations (n) and the required SNR to obtain a BER = 10^{-2} in 32 × 256 MIMO. Obviously, the required performance is achieved at SNR = 14dB using detectors based on the GS, SOR and OCD methods. However, the detector based on GS and SOR suffer from a high computational complexity while the detector based on OCD method has the lowest computational complexity. Figure 11 shows the comparison between several detection methods in terms of the number of real multiplications in $n = 2$, 4, and 6 as mentioned in Table 3. It also shows the BER above each bar at SNR = 12dB. It is clear that the CG and OCD based detectors obtain the lowest number

of real multiplications and a satisfactory BER performance. However, they need a high number of iterations to attain a good BER performance. Complexity of the detector based on NS method is lower than the complexity of the detector based on GS, JA, RI and SOR methods.

VII. CONCLUSION AND OPEN RESEARCH

In this paper, crucial issues of data detection in M-MIMO UL systems are presented. This paper studies the performance-complexity profile of various detectors based on several iterative methods at different MIMO sizes and different iteration levels. It is shown that the GS detector perform better than other detectors when β is small. In addition, all detectors have obtained a considerable performance loss when $\beta = 1$. Optimum values of ω are presented for both the RI and SOR methods. Unlike the practical scenario, a plethora of M-MIMO literature assumes that the propagation channels are spatially uncorrelated which could lead to misleading conclusions. The detector based on RI, OCD, and CG methods can achieve a high HE in correlated channels. Furthermore, this paper illustrates that the OCD detector has the lowest complexity.

Since 2017, there is a substantial trend in a research community to exploit the machine learning in data detection. For instance, deep networks in M-MIMO detector's design based on a projected gradient descent method is utilized in [68]. A modified DetNet is proposed where a relatively small number of parameters is required to optimize [69]. Unfortunately, the training is unstable for realistic and correlated channels. In addition, scalability of the DetNet algorithm is poor because of a relatively large number of training parameters. In 2018-2020, there is a notable trend in a research community to exploit the DL to build a robust M-MIMO detector. A model-driven DL network is proposed based on the orthogonal approximate message passing network (OAMP-Net) [38]. Although the existing detection methods achieve a near optimal performance, there is still an open room for further research work. For instance, the utilization of iterative free matrix inversion methods with deep learning in detector's design should be investigated where efficient initialization could impact greatly the performance and the computational complexity.

REFERENCES

[1] Ericsson—Press Release, "5G estimated to reach 1.5 billion subscriptions in 2024—Ericsson mobility report—Ericsson," Ericsson, Stockholm, Sweden, Tech. Rep., Nov. 2018.

[2] A. Kumar, A. K. Sah, and A. K. Chaturvedi, "Improved sparsity behaviour and error localization in detectors for large MIMO systems," in *Proc. IEEE Globecom Workshops (GC Wkshps)*, Dec. 2016, pp. 1–6.

[3] M. Albreem, M. Juntti, and S. Shahabuddin, "Efficient initialisation of iterative linear massive MIMO detectors using a stair matrix," *Electron. Lett.*, vol. 56, no. 1, pp. 50–52, Jan. 2020.

[4] M. A. Albreem, M. Juntti, and S. Shahabuddin, "Massive MIMO detection techniques: A survey," *IEEE Commun. Surveys Tuts.*, vol. 21, no. 4, pp. 3109–3132, Nov. 2019.

[5] M. A. Albreem, A. Kumar, M. H. Alsharif, I. Khan, and B. J. Choi, "Comparative analysis of data detection techniques for 5G massive MIMO systems," *Sustainability*, vol. 12, no. 21, p. 9281, Nov. 2020.

[6] C. D. Altamirano, J. Minango, H. C. Mora, and C. De Almeida, "BER evaluation of linear detectors in massive MIMO systems under imperfect channel estimation effects," *IEEE Access*, vol. 7, pp. 174482–174494, 2019.

[7] C. D. Altamirano, J. Minango, C. D. Almeida, and N. Orozco, "On the asymptotic BER of MMSE detector in massive MIMO systems," in *Proc. Int. Conf. Appl. Technol.* Ecuador: Springer, 2019, pp. 57–68.

[8] M. Arfaoui, H. Ltaief, Z. Rezki, M. Alouini, and D. Keyes, "Efficient sphere detector algorithm for massive MIMO using GPU hardware accelerator," *Procedia Comput. Sci.*, vol. 80, p. 2169–2180, Jun. 2016.

[9] V. M. Baeza and A. G. Armada, "Noncoherent massive MIMO," in *Wiley 5G Ref: The Essential 5G Reference Online*. Hoboken, NJ, USA: Wiley, 2019, pp. 1–28.

[10] E. Björnson, E. G. Larsson, and M. Debbah, "Massive MIMO for maximal spectral efficiency: How many users and pilots should be allocated?" *IEEE Trans. Wireless Commun.*, vol. 15, no. 2, pp. 1293–1308, Feb. 2016. R.

[11] R. Chataut, R. Akl, and U. K. Dey, "Least square regressor selection based detection for uplink 5G massive MIMO systems," in *Proc. IEEE 20th Wireless Microw. Technol. Conf. (WAMICON)*, Apr. 2019, pp. 1–6.

[12] R. Chataut, R. Akl, and M. Robaei, "Accelerated and preconditioned refinement of Gauss-Seidel method for uplink signal detection in 5G massive MIMO systems," in *Proc. 10th Annu. Comput. Commun. Workshop Conf. (CCWC)*, Jan. 2020, pp. 0083–0089.

[13] J.-C. Chen, "A low complexity data detection algorithm for uplink multiuser massive MIMO systems," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 8, pp. 1701–1714, Aug. 2017.

[14] J. Chen, Z. Zhang, H. Lu, J. Hu, and G. E. Sobelman, "An intra-iterative interference cancellation detector for large-scale MIMO communications based on convex optimization," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 63, no. 11, pp. 2062–2072, Nov. 2016.

[15] J.-C. Chi, Y.-C. Yeh, I.-W. Lai, and Y.-H. Huang, "Sphere decoding for spatial permutation modulation MIMO systems," in *Proc. IEEE Int. Conf. Commun. (ICC)*, May 2017, pp. 1–8.

[16] J. W. Choi and B. Shim, "New approach for massive MIMO detection using sparse error recovery," in *Proc. IEEE Global Commun. Conf.*, Dec. 2014, pp. 3754–3759.

[17] Cisco, "Cisco visual networking index: Global mobile data traffic forecast update, 2015–2020 white paper," Cisco, White Paper, Feb. 2016, pp. 1–39.

[18] Cisco, "Cisco visual networking index: Global mobile data traffic forecast update, 2016-2021," Cisco, White Paper, Feb. 2017, pp. 1–35.

[19] Cisco, "Cisco visual networking index: Forecast and trends, 2017-2022," Cisco, White Paper, Nov. 2018, pp. 1–38.

[20] H. Costa and V. Roda, "A scalable soft Richardson method for detection in a massive MIMO system," *Przeglad Elektrotechniczny*, vol. 92, no. 5, pp. 199–203, Aug. 2016.

[21] T. Cui, S. Han, and C. Tellambura, "Probability-distribution-based node pruning for sphere decoding," *IEEE Trans. Veh. Technol.*, vol. 62, no. 4, pp. 1586–1596, May 2013.

[22] L. Dai, X. Gao, X. Su, S. Han, C.-L. I, and Z. Wang, "Low-complexity soft-output signal detection based on Gauss–Seidel method for uplink multiuser large-scale MIMO systems," *IEEE Trans. Veh. Technol.*, vol. 64, no. 10, pp. 4839–4845, Oct. 2015.

[23] T. Datta, N. Srinidhi, A. Chockalingam, and B. S. Rajan, "A hybrid RTS-BP algorithm for improved detection of large-MIMO M-QAM signals," in *Proc. Nat. Conf. Commun. (NCC)*, Jan. 2011, pp. 1–5.

[24] Q. Deng, L. Guo, C. Dong, J. Lin, D. Meng, and X. Chen, "High-throughput signal detection based on fast matrix inversion updates for uplink massive multiuser multiple-input multi-output systems," *IET Commun.*, vol. 11, no. 14, pp. 2228–2235, Sep. 2017.

[25] Q. Deng, X. Liang, X. Wang, M. Huang, C. Dong, and Y. Zhang, "Fast converging iterative precoding for massive MIMO systems: An accelerated weighted Neumann series-steepest descent approach," *IEEE Access*, vol. 8, pp. 50244–50255, 2020.

[26] Y. Fadlallah, A. Aissa-El-Bey, K. Amis, and D. Pastor, "Low-complexity detector for very large and massive MIMO transmission," in *Proc. IEEE 16th Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Jun. 2015, pp. 251–255.

[27] L. Fang, L. Xu, and D. D. Huang, "Low complexity iterative MMSE-PIC detection for medium-size massive MIMO," *IEEE Wireless Commun. Lett.*, vol. 5, no. 1, pp. 108–111, Feb. 2016.

[28] X. Gao, L. Dai, Y. Hu, Z. Wang, and Z. Wang, "Matrix inversion-less signal detection using SOR method for uplink large-scale MIMO systems," in *Proc. IEEE Global Commun. Conf.*, Dec. 2014, pp. 3291–3295.

[29] X. Gao, L. Dai, Y. Ma, and Z. Wang, "Low-complexity near-optimal signal detection for uplink large-scale MIMO systems," *Electron. Lett.*, vol. 50, no. 18, pp. 1326–1328, Aug. 2014.

[30] X. Gao, L. Dai, C. Yuen, and Y. Zhang, "Low-complexity MMSE signal detection based on Richardson method for large-scale MIMO systems," in *Proc. IEEE 80th Veh. Technol. Conf. (VTC-Fall)*, Sep. 2014, pp. 1–5.

[31] X. Gao, Z. Lu, Y. Han, and J. Ning, "Near-optimal signal detection with low complexity based on Gauss-Seidel method for uplink large-scale MIMO systems," in *Proc. IEEE Int. Symp. Broadband Multimedia Syst. Broadcast.*, Jun. 2014, pp. 1–4.

[32] Y. Gao, H. Niu, and T. Kaiser, "Massive MIMO detection based on belief propagation in spatially correlated channels," in *Proc. 11th Int. ITG Conf. Syst., Commun. Coding (SCC)*, Feb. 2017, pp. 1–6.

[33] S. Ghacham, M. Benjillali, and Z. Guennoun, "Low-complexity detection for massive MIMO systems over correlated Rician fading," in *Proc. 13th Int. Wireless Commun. Mobile Comput. Conf. (IWCMC)*, Jun. 2017, pp. 1677–1682.

[34] V. Gupta, A. K. Sah, and A. K. Chaturvedi, "Iterative matrix inversion based low complexity detection in large/massive MIMO systems," in *Proc. IEEE Int. Conf. Commun. Workshops (ICC)*, May 2016, pp. 712–717.

[35] Z. Hajji, K. Amis, and A. Aissa-El-Bey, "Turbo detection based on sparse decomposition for massive MIMO transmission," in *Proc. 9th Int. Symp. Turbo Codes Iterative Inf. Process. (ISTC)*, Sep. 2016, pp. 290–294.

[36] Z. Hajji, K. Amis, A. Aissa-El-Bey, and F. Abdelkefi, "Low-complexity half-sparse decomposition-based detection for massive MIMO transmission," in *Proc. 5th Int. Conf. Commun. Netw. (COMNET)*, Nov. 2015, pp. 1–6.

[37] R. Hayakawa and K. Hayashi, "Error recovery with relaxed MAP estimation for massive MIMO signal detection," in *Proc. IEEE Int. Symp. Inform. Theory Appl.*, Oct. 2016, pp. 478–482.

[38] H. He, C.-K. Wen, S. Jin, and G. Y. Li, "A model-driven deep learning network for MIMO detection," in *Proc. IEEE Global Conf. Signal Inf. Process. (GlobalSIP)*, Nov. 2018, pp. 584–588.

[39] Y. Hu, Z. Wang, X. Gaol, and J. Ning, "Low-complexity signal detection using CG method for uplink large-scale MIMO systems," in *Proc. IEEE Int. Conf. Commun. Syst.*, Nov. 2014, pp. 477–481.

[40] F. Jiang, C. Li, and Z. Gong, "A low complexity soft-output data detection scheme based on Jacobi method for massive MIMO uplink transmission," in *Proc. IEEE Int. Conf. Commun. (ICC)*, May 2017, pp. 1–5.

[41] F. Jiang, C. Li, Z. Gong, and R. Su, "Stair matrix and its applications to massive MIMO uplink data detection," *IEEE Trans. Commun.*, vol. 66, no. 6, pp. 2437–2455, Jun. 2018.

[42] B. Kang, J.-H. Yoon, and J. Park, "Low complexity massive MIMO detection architecture based on Neumann method," in *Proc. Int. SoC Design Conf. (ISOCC)*, Nov. 2015, pp. 293–294.

[43] B. Kang, J.-H. Yoon, and J. Park, "Low-complexity massive MIMO detectors based on Richardson method," *ETRI J.*, vol. 39, no. 3, pp. 326–335, Jun. 2017.

[44] I. A. Khoso, X. Dai, M. N. Irshad, A. Khan, and X. Wang, "A low-complexity data detection algorithm for massive MIMO systems," *IEEE Access*, vol. 7, pp. 39341–39351, 2019.

[45] I. A. Khoso, X. Zhang, and A. H. Shaikh, "Low-complexity signal detection for large-scale MIMO systems with second-order Richardson method," *Electron. Lett.*, vol. 56, no. 9, pp. 467–469, Apr. 2020.

[46] B. Y. Kong and I.-C. Park, "Low-complexity symbol detection for massive MIMO uplink based on Jacobi method," in *Proc. IEEE 27th Annu. Int. Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC)*, Sep. 2016, pp. 1–5.

[47] E. G. Larsson, "MIMO detection methods: How they work [lecture notes]," *IEEE Signal Process. Mag.*, vol. 26, no. 3, pp. 91–95, May 2009.

[48] B. M. Lee and H. Yang, "Massive MIMO with massive connectivity for industrial Internet of Things," *IEEE Trans. Ind. Electron.*, vol. 67, no. 6, pp. 5187–5196, Jun. 2020.

[49] Y. Lee, "Decision-aided Jacobi iteration for signal detection in massive MIMO systems," *Electron. Lett.*, vol. 53, no. 23, pp. 1552–1554, Nov. 2017.

[50] Y. Lee, "Hybrid Kaczmarz and coordinate-descent iterations for signal detection in massive MIMO systems," *Electron. Lett.*, vol. 55, no. 11, pp. 665–667, May 2019.

[51] L. Liu, G. Peng, P. Wang, S. Zhou, Q. Wei, S. Yin, and S. Wei, "Energy- and area-efficient recursive-conjugate-gradient-based MMSE detector for massive MIMO systems," *IEEE Trans. Signal Process.*, vol. 68, pp. 573–588, 2020.

[52] M. M. Lwin and M. F. M. Salleh, "Feasibility of lattice reduction aided detection techniques in massive MIMO," in *Proc. Int. Conf. Signals Syst. (ICSigSys)*, May 2018, pp. 131–135.

[53] S. Lyu and C. Ling, "Sequential lattice reduction," in *Proc. 8th Int. Conf. Wireless Commun. Signal Process. (WCSP)*, Oct. 2016, pp. 1–5.

[54] M. Mandloi, M. A. Hussain, and V. Bhatia, "An improved multiple feedback successive interference cancellation algorithm for MIMO detection," in *Proc. 8th Int. Conf. Commun. Syst. Netw. (COMSNETS)*, Jan. 2016, pp. 1–6.

[55] M. Mandloi, M. A. Hussain, and V. Bhatia, "Adaptive multiple stage K-best successive interference cancellation algorithm for MIMO detection," *Telecommun. Syst.*, vol. 66, no. 1, pp. 1–16, Sep. 2017.

[56] J. Minango and C. Almeida, "Optimum and quasi-optimum relaxation parameters for low-complexity massive MIMO detector based on Richardson method," *Electron. Lett.*, vol. 53, no. 16, pp. 1114–1115, Aug. 2017.

[57] J. Minango and A. C. Flores, "Low-complexity MMSE detector based on refinement Jacobi method for massive MIMO uplink," *Phys. Commun.*, vol. 26, pp. 128–133, Feb. 2018.

[58] T. L. Narasimhan and A. Chockalingam, "Channel hardening-exploiting message passing (CHEMP) receiver in large MIMO systems," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Apr. 2014, pp. 815–820.

[59] T. L. Narasimhan and A. Chockalingam, "Detection and decoding in large-scale MIMO systems: A non-binary belief propagation approach," in *Proc. IEEE 79th Veh. Technol. Conf. (VTC Spring)*, May 2014, pp. 1–5.

[60] T. L. Narasimhan and A. Chockalingam, "EXIT chart based design of irregular LDPC codes for large-MIMO systems," *IEEE Commun. Lett.*, vol. 17, no. 1, pp. 115–118, Jan. 2013.

[61] T. L. Narasimhan and A. Chockalingam, "Channel hardening-exploiting message passing (CHEMP) receiver in large-scale MIMO systems," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 847–860, Oct. 2014.

[62] H. Quoc Ngo, E. G. Larsson, and T. L. Marzetta, "Energy and spectral efficiency of very large multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1436–1449, Apr. 2013.

[63] C. N. Cuong, T. T. Hong, and L. D. Khai, "Hardware implementation of the efficient SOR-based massive MIMO detection for uplink," in *Proc. IEEE-RIVF Int. Conf. Comput. Commun. Technol. (RIVF)*, Mar. 2019, pp. 1–6.

[64] K. Nikitopoulos, D. Chatzipanagiotis, C. Jayawardena, and R. Tafazolli, "MultiSphere: Massively parallel tree search for large sphere decoders," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2016, pp. 1–6.

[65] X. Qin, Z. Yan, and G. He, "A near-optimal detection scheme based on joint steepest descent and Jacobi method for uplink massive MIMO systems," *IEEE Commun. Lett.*, vol. 20, no. 2, pp. 276–279, Feb. 2016.

[66] F. Rusek, D. Persson, B. Kiong Lau, E. G. Larsson, T. L. Marzetta, and F. Tufvesson, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.

[67] A. K. Sah and A. K. Chaturvedi, "An MMP-based approach for detection in large MIMO systems using sphere decoding," *IEEE Wireless Commun. Lett.*, vol. 6, no. 2, pp. 158–161, Apr. 2017.

[68] N. Samuel, T. Diskin, and A. Wiesel, "Deep MIMO detection," in *Proc. IEEE 18th Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Jul. 2017, pp. 1–5.

[69] N. Samuel, T. Diskin, and A. Wiesel, "Learning to detect," *IEEE Trans. Signal Process.*, vol. 67, no. 10, pp. 2554–2564, May 2019.

[70] P. Seidel, S. Paul, and J. Rust, "Low-complexity 2-coordinates descent for near-optimal MMSE soft-output massive MIMO uplink data detection," in *Proc. 27th Eur. Signal Process. Conf. (EUSIPCO)*, Sep. 2019, pp. 1–5.

[71] L. Shao and Y. Zu, "Joint Newton iteration and Neumann series method of convergence-accelerating matrix inversion approximation in linear precoding for massive MIMO systems," *Math. Problems Eng.*, vol. 2016, pp. 1–5, May 2016.

[72] K. A. Singhal, T. Datta, and A. Chockalingam, "Lattice reduction aided detection in large-MIMO systems," in *Proc. IEEE 14th Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Jun. 2013, pp. 594–598.

[73] P. Som, T. Datta, A. Chockalingam, and B. S. Rajan, "Improved large-MIMO detection based on damped belief propagation," in *Proc. IEEE Inf. Theory Workshop (ITW)*, Jan. 2010, pp. 1–5.

[74] P. Som, T. Datta, N. Srinidhi, A. Chockalingam, and B. S. Rajan, "Low-complexity detection in large-dimension MIMO-ISI channels using graphical models," *IEEE J. Sel. Topics Signal Process.*, vol. 5, no. 8, pp. 1497–1511, Dec. 2011.

[75] W. Song, X. Chen, L. Wang, and X. Lu, "Joint conjugate gradient and Jacobi iteration based low complexity precoding for massive MIMO systems," in *Proc. IEEE/CIC Int. Conf. Commun. China (ICCC)*, Jul. 2016, pp. 1–5.

[76] Y. Sun, Z. Li, C. Zhang, R. Zhang, F. Yan, and L. Shen, "Low complexity signal detector based on SSOR iteration for large-scale MIMO systems," in *Proc. 9th Int. Conf. Wireless Commun. Signal Process. (WCSP)*, Oct. 2017, pp. 1–6.

[77] X. Tan, J. Jin, K. Sun, Y. Xu, M. Li, Y. Zhang, Z. Zhang, X. You, and C. Zhang, "Enhanced linear iterative detector for massive multiuser MIMO uplink," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 67, no. 2, pp. 540–552, Feb. 2020.

[78] C. Tang, C. Liu, L. Yuan, and Z. Xing, "Approximate iteration detection with iterative refinement in massive MIMO systems," *IET Commun.*, vol. 11, no. 7, pp. 1152–1157, May 2017.

[79] C. Tang, C. Liu, L. Yuan, and Z. Xing, "High precision low complexity matrix inversion based on Newton iteration for data detection in the massive MIMO," *IEEE Commun. Lett.*, vol. 20, no. 3, pp. 490–493, Mar. 2016.

[80] C. Tang, Y. Tao, Y. Chen, C. Liu, L. Yuan, and Z. Xing, "Approximate iteration detection and precoding in massive MIMO," *China Commun.*, vol. 15, no. 5, pp. 183–196, May 2018.

[81] O. H. Toma and M. El-Hajjar, "Element-based lattice reduction aided K-best detector for large-scale MIMO systems," in *Proc. IEEE 17th Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Jul. 2016, pp. 1–5.

[82] J. Tu, M. Lou, J. Jiang, D. Shu, and G. He, "An efficient massive MIMO detector based on second-order Richardson iteration: From algorithm to flexible architecture," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 67, no. 11, pp. 4015–4028, Nov. 2020.

[83] D. Vordonis and V. Paliouras, "Sphere decoder for massive MIMO systems," in *Proc. IEEE Nordic Circuits Syst. Conf. (NORCAS), NORCHIP Int. Symp. Syst.-Chip (SoC)*, Oct. 2019, pp. 1–6.

[84] F. Wang, C. Zhang, X. Liang, Z. Wu, S. Xu, and X. You, "Efficient iterative soft detection based on polynomial approximation for massive MIMO," in *Proc. Int. Conf. Wireless Commun. Signal Process. (WCSP)*, Oct. 2015, pp. 1–5.

[85] A. Weissberger, "5 nordic countries aim to be 1st interconnected 5G region in the world," in *Proc. Int. Conf. Wireless Commun. Signal Process.*, China, Jun. 2018.

[86] M. Wu, C. Dick, J. R. Cavallaro, and C. Studer, "High-throughput data detection for massive MU-MIMO-OFDM using coordinate descent," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 63, no. 12, pp. 2357–2367, Dec. 2016.

[87] M. Wu, C. Dick, J. R. Cavallaro, and C. Studer, "FPGA design of a coordinate descent data detector for large-scale MU-MIMO," in *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS)*, May 2016, pp. 1894–1897.

[88] M. Wu, B. Yin, A. Vosoughi, C. Studer, J. R. Cavallaro, and C. Dick, "Approximate matrix inversion for high-throughput data detection in the large-scale MIMO uplink," in *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS)*, May 2013, pp. 2155–2158.

[89] M. Wu, B. Yin, G. Wang, C. Dick, J. R. Cavallaro, and C. Studer, "Large-scale MIMO detection for 3GPP LTE: Algorithms and FPGA implementations," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 916–929, Oct. 2014.

[90] S. Wu, L. Kuang, Z. Ni, J. Lu, D. Huang, and Q. Guo, "Low-complexity iterative detection for large-scale multiuser MIMO-OFDM systems using approximate message passing," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 902–915, Oct. 2014.

[91] Z. Wu, L. Ge, X. You, and C. Zhang, "Efficient near-MMSE detector for large-scale MIMO systems," in *Proc. IEEE Int. Workshop Signal Process. Syst. (SiPS)*, Oct. 2017, pp. 1–6.

[92] Z. Wu, Y. Xue, X. You, and C. Zhang, "Hardware efficient detection for massive MIMO uplink with parallel Gauss-Seidel method," in *Proc. 22nd Int. Conf. Digit. Signal Process. (DSP)*, Aug. 2017, pp. 1–5.

[93] Z. Wu, C. Zhang, Y. Xue, S. Xu, and X. You, "Efficient architecture for soft-output massive MIMO detection with Gauss-Seidel method," in *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS)*, May 2016, pp. 1886–1889.

[94] Y. Xiong, N. Wei, and Z. Zhang, "A low-complexity iterative GAMP-based detection for massive MIMO with low-resolution ADCs," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Mar. 2017, pp. 1–6.

[95] W. Xu, H. A. Alshamary, T. Al-Naffouri, and A. Zaib, "Optimal joint channel estimation and data detection for massive SIMO wireless systems: A polynomial complexity solution," *IEEE Trans. Inf. Theory*, vol. 66, no. 3, pp. 1822–1844, Mar. 2020.

[96] Y. Xue, L. Liu, G. Peng, H. Zhang, S. Yin, J. Wei, and S. Wei, "Hardware efficient signal detector based on Lanczos method for massive MIMO systems," in *Proc. IEEE 9th Int. Conf. Commun. Softw. Netw. (ICCSN)*, May 2017, pp. 523–527.

[97] J. Yang, C. Zhang, S. Xu, and X. You, "Efficient stochastic detector for large-scale MIMO," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP)*, Mar. 2016, pp. 6550–6554.

[98] S. Yang and L. Hanzo, "Fifty years of MIMO detection: The road to large-scale MIMOs," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 4, pp. 1941–1988, 4th Quart., 2015.

[99] X. Yaohua and W. Jian, "Signal detection algorithm based on SOR algorithm in massive MIMO system," in *Proc. 14th IEEE Int. Conf. Signal Process. (ICSP)*, Aug. 2018, pp. 662–667.

[100] B. Yin, M. Wu, J. R. Cavallaro, and C. Studer, "Conjugate gradient-based soft-output detection and precoding in massive MIMO systems," in *Proc. IEEE Global Commun. Conf.*, Dec. 2014, pp. 3696–3701.

[101] B. Yin, M. Wu, J. R. Cavallaro, and C. Studer, "VLSI design of large-scale soft-output MIMO detection using conjugate gradients," in *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS)*, May 2015, pp. 1498–1501.

[102] A. Yu, S. Jing, X. Tan, Z. Wu, Z. Yan, Z. Zhang, X. You, and C. Zhang, "Efficient successive over relaxation detectors for massive MIMO," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 67, no. 6, pp. 2128–2139, Jun. 2020.

[103] A. Zaib, M. Masood, A. Ali, W. Xu, and T. Y. Al-Naffouri, "Distributed channel estimation and pilot contamination analysis for massive MIMO-OFDM systems," *IEEE Trans. Commun.*, vol. 64, no. 11, pp. 4607–4621, Nov. 2016.

[104] J. Zeng, J. Lin, and Z. Wang, "An improved Gauss-Seidel algorithm and its efficient architecture for massive MIMO systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 65, no. 9, pp. 1194–1198, Sep. 2018.

[105] J. Zeng, J. Lin, and Z. Wang, "Low complexity message passing detection algorithm for large-scale MIMO systems," *IEEE Wireless Commun. Lett.*, vol. 7, no. 5, pp. 708–711, Oct. 2018.

[106] J. Zeng, J. Lin, Z. Wang, and Y. Chen, "Hybrid preconditioned CG detection with sequential update for massive MIMO systems," in *Proc. IEEE Int. Workshop Signal Process. Syst. (SiPS)*, Oct. 2019, pp. 207–212.

[107] C. Zhang, X. Liang, Z. Wu, F. Wang, S. Zhang, Z. Zhang, and X. You, "On the low-complexity, hardware-friendly tridiagonal matrix inversion for correlated massive MIMO systems," *IEEE Trans. Veh. Technol.*, vol. 68, no. 7, pp. 6272–6285, Jul. 2019.

[108] C. Zhang, Z. Wu, C. Studer, Z. Zhang, and X. You, "Efficient soft-output Gauss-Seidel data detector for massive MIMO systems," *IEEE Trans. Circuits Syst. I, Reg. Papers*, early access, Oct. 26, 2018, doi: 10.1109/TCSI.2018.2875741.

[109] P. Zhang, L. Liu, G. Peng, and S. Wei, "Large-scale MIMO detection design and FPGA implementations using SOR method," in *Proc. 8th IEEE Int. Conf. Commun. Softw. Netw. (ICCSN)*, Jun. 2016, pp. 206–210.

[110] Z. Zhang, J. Wu, X. Ma, Y. Dong, Y. Wang, S. Chen, and X. Dai, "Reviews of recent progress on low-complexity linear detection via iterative algorithms for massive MIMO systems," in *Proc. IEEE/CIC Int. Conf. Commun. China (ICCC Workshops)*, Jul. 2016, pp. 1–6.

[111] S. Zhao, B. Shen, and Q. Hua, "A comparative study of low-complexity MMSE signal detection for massive MIMO systems," *KSII Trans. Internet Inf. Syst.*, vol. 12, no. 4, pp. 1504–1526, Apr. 2018.

[112] J. Zhou, J. Hu, J. Chen, and S. He, "Biased MMSE soft-output detection based on conjugate gradient in massive MIMO," in *Proc. IEEE 11th Int. Conf. ASIC (ASICON)*, Nov. 2015, pp. 1–4.

[113] J. Zhou, Y. Ye, and J. Hu, "Biased MMSE soft-output detection based on Jacobi method in massive MIMO," in *Proc. IEEE Int. Conf. Communication Problem-Solving*, Dec. 2014, pp. 442–445.

[114] Q. Zhou and X. Ma, "Element-based lattice reduction algorithms for large MIMO detection," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 274–286, Feb. 2013.

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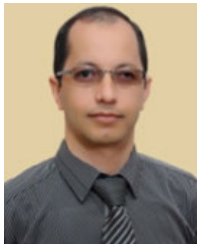
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