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# Markov Processes in Data Center Networks

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**ABSTRACT** A data center network is an important infrastructure in various applications of modern information technologies. Data centers store files with useful information, but the lifetime of these data centers is limited. Once a data center suffers from natural disasters and man-made damages, all the information stored in the files will be lost. Therefore, replicating important files in other data centers is necessary to increase the lifetime of these files in a data center network. In this study, we develop a more general framework for Markov processes in data center networks, that can provide an analysis of file replication processes. A file replication strategy duplicates each important file in at most $d-1$ other data centers such that these files can remain in a data center network under a given data security level in the long term. In this regard, we achieve a sufficient stable condition of data center networks by using a matrix-geometric solution. Then, we provide an expression for the stationary average number of normally operating data center networks. We also develop a computational technique for file lifetime via phase type distribution and the *RG*-factorizations. Lastly, we hope that the methodology and results presented in this study are applicable to research on file lifetime and data security for more general data center networks with a replication mechanism.

**INDEX TERMS** Data center networks, Markovian arrival process, phase type distribution, replication mechanism, *RG*-factorization.

## I. INTRODUCTION

Adata center network is an important infrastructure in various applications of modern information technologies. The lifetime of data centers is limited, and data centers may suffer from natural disasters or man-made damages. File replication has been widely used to reduce the risk of data loss and increase data availability in data center networks [1]–[7]. Therefore, using a Markov process model in large-scale data center networks to study the file replication mechanism, evaluate file lifetime, and assess data security has elicited widespread concern.

Many scholars have studied the mechanism of file replication and the evaluation of file lifetime by using a Markov process. Chun *et al.* [8] presented a Markov chain model for analyzing the expected replica lifetime and designed the Carbonite replication algorithm for keeping data durable at a low cost. Picconi *et al.* [9], [10] used Markov chain repair rates to predict replica durability in distributed hash tables and provided an analytical expression for a system's repair rate. Ramabhadran and Pasquale [11]–[13] developed a model concept of replica loss and repair in distributed storage systems based on a simple Markov chain model and derived an expression for the lifetime of the replicated state. Utard and Vernois [14] constructed a Markov process model peer-to-peer storage system, and analyzed the average lifetime of data in accordance with the availability factors of the system. Lian *et al.* [15] introduced a Markov model for the evolution of a system with brick failures and data repair. Aghajani *et al.* [16] and Sun *et al.* [17] investigated a stochastic model for the evolution of a network. This model converges during distribution to a nonlinear Markov process when the number of nodes goes toward infinity. Feuillet and Robert [18] provided a simple transient Markov process with an absorbing point to investigate the qualitative behavior of a large-scale storage network. Li *et al.* [19] builds a stochastic system of file replication mechanism, in which the replication time of the file follows an exponential distribution, and the joining process of the new data center follows the Poisson process. In terms of data security [20], [21], file replication is one of the effective methods for ensuring the long-term availability of files. However, research on the quantitative relationship between the number of file copies and file lifetime remains lacking.

In using a Markov process to study the mechanism of file replication and evaluate file lifetime, the Markov

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structure of phase type (PH) distribution and the Markov arrival process (MAP) can effectively reduce difficulty in modeling. At present, many scholars have studied demand forecasting [22], inventory management [23], [24], and priority queuing system [25], [26] by using PH distribution and MAP. However, studies on the mechanism of file replication that uses PH distribution and MAP are rare. However, existing research results show that different phases in PH distribution and MAP are highly suitable for describing changes in file lifetime in data center networks.

The major challenges of the current study are threefold. The first challenge is to construct a Markov process model of a large-scale data center network by using PH distribution and MAP. The second challenge is to analyze the joint probability distribution of the number of file copies and number of data centers. The third challenge is to determine the quantitative relationship between the number of file copies and file lifetime.

The contributions of this work are threefold. The first contribution is to develop a more general framework of Markov processes for studying data center networks. Such framework can provide an analysis of the file replication strategy. The file replication strategy replicates important files in at most $d-1$ other data centers such that these files can remain in a data center network under a given data security level in the long term. The second contribution is the use of MAP and PH distribution to establish a quasi-birth-and-death process (QBD) that leads to two effective methods for assessing file lifetime and data security in a data center network. We also develop a computational technique for file lifetime based on PH distribution with finite sizes and the *RG*-factorizations. The third contribution is to popularize the stochastic system of file replication mechanism studied by Li *et al.* [19], and to generalize the replication time of the file from an exponential distribution to PH distribution. The joining process of new data center is generalized from Poisson process to Markov arrival process. In this case, our model is more consistent with the actual situation.

The remainder of this paper is organized as follows. Section 2 describes a file replication mechanism in a data center network. Section 3 establishes a continuous-time Markov process, and expresses the stationary probability vector of a data center network. Section 4 uses MAP and PH distribution to establish a relationship between the number of data centers and number of file copies. A numerical example is used to indicate the effect of the file replication strategy on the average lifetime of files in a data center network. Section 5 develops a computational technique for file lifetime based on PH distribution with finite states and the *RG*-factorizations. Lastly, concluding remarks are provided in Section 6.

## II. MODEL DESCRIPTION

In this section, we describe a large-scale data center network. Some data centers are removed from the network due to failure or malfunction, resulting in the loss of stored data or information. Simultaneously, new data centers are constantly added to this large-scale data center network to enhance its capability to store and protect data. Such a data center network is considered to ensure that stored files are always backed up. This section describes a data storage mechanism in which files are continuously replicated in this data center network. A Markov process is used to provide a more general stochastic model that can analyze the performance evaluation of this system with a data storage mechanism.

To ensure that data are unaffected by natural disasters or emergencies in the data center network, this section assumes that the data center network is based on a wide range of heterogeneous geographic environments. This data center network is connected by a wireless (or wired) communication system; that is, any two data centers in the data center network can achieve fast and efficient connectivity and communication, including business data transmission, acceptance, storage, and processing.

In such situation, when a data center only considers handling large business data, being concerned with the heterogeneous geographical environment of the data center network is unnecessary in the current communication system. Therefore, the data center network is abstracted as a simple undirected network $(V, E)$, where

$$V = \{DC-1, DC-2, DC-3, \ldots, DC-n\},$$
$$E = \{DC-i--DC-j : 1 \le i < j \le n\},$$

$DC-i$ represents the $i$th data center, and $DC-i--DC-j$ represents a service communication (nondirectional) connection between the $i$th and $j$th data centers.

We present the model description and related system parameters of this data center network in the subsequent sections.

### A. LIFETIME

Each data center may fail in this network. We assume that the lifetime $X$ of each data center follows a continuous-time PH distribution and its irreducible matrix is expressed as $(\alpha, T)$. $\alpha$ is a substochastic vector of order $m$. $T$ is a transfer rate matrix of order $m$. $T = (T_{i,j})_{m\times m}$, $T_{i,i} < 0$, $T_{i,j} \ge 0$, $i \ne j$, and $Te < 0$. Therefore, its probability distribution function is

$$F(t) = P\{X \le t\} = 1 - \alpha \exp(Tt)e_1, \ t \ge 0. \tag{1}$$

Evidently, $E\left[X^k\right] = (-1)^k \alpha T^{-k} e_1$, where $e_1$ is the column vector with all the elements being one.

### B. JOINING PROCESS OF NEW DATA CENTERS

Each data center may fail, and thus, new data centers should be continually added to the network to enable the data center network to maintain sustainable development through numerous incessant equipment replacements. We assume that the input of a new data center is an MAP. $(D_0, D_1)$ is an irreducible matrix of order $n$, when $D_0 = (d_{i,j}^{(0)})$, $D_1 = (d_{i,j}^{(1)})$, $d_{i,i}^{(0)} < 0$, $d_{i,j}^{(0)} \ge 0$ $(i \ne j)$, $d_{k,l}^{(1)} \ge 0$ and $(D_0 + D_1)e = 0$. $D_1$ is the process of the state transition arrival rate.

### C. FILE REPLICATION STRATEGY

We assume that each file is stored in at most $d \geq 1$ data centers in this network. Once the number of copies of a file is less than $d$ and an available data center exists without storing the file, then the file will be quickly replicated in the data center. We assume that the copy time $Y$ of the file replicated in an available data center follows a continuous-time PH distribution and its irreducible matrix is expressed as $(\beta, S)$. $\beta$ is a substochastic vector of order $m$, and $S$ is a transfer rate matrix of order $m$.

$$F(t) = P\{Y \leq t\} = 1 - \beta \exp(St)e_2, \ t \geq 0. \quad (2)$$

Evidently, $E\left[Y^k\right] = (-1)^k \beta S^{-k} e_2$, where $e_2$ is the column vector with all the elements being one.

### D. FILE LOST PROCESS

Once a data center fails, all the files in this data center will be lost immediately, along with useful information.

We assume that all the random variables involved in the data center network are independent of each other.

## III. THE STEADY-STATE PROBABILITY DISTRIBUTION OF THE DATA CENTER's NUMBER

In this section, we use MAP and PH distribution to study the steady-state probability distribution of the number of normally operating data centers in the data center network and analyze inherent features to ensure the security of data in the files through a file replication strategy.

In this data center network, each data center can fail or malfunction, and its lifetime is subject to PH distribution. Moreover, $(\alpha, T)$ is an irreducible matrix of order $m$. We consider an initial probability vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m)$, and $T$ is a substochastic vector of order $m$ of the infinitesimal generator $Q_{PH}$. The infinitesimal generator $Q_{PH}$ is given by

$$Q_{PH} = \begin{bmatrix} T & T^0 \\ 0 & 0 \end{bmatrix},$$

where $T = (T)_{m \times m}$ and $T^0 = (T_1^0, \ldots, T_m^0)^T = -Te_1$. Meanwhile, new data centers are continually added to the data center network, and the addition process is MAP. The infinitesimal generator $Q_{MAP}$ of MAP is expressed as

$$Q_{MAP} = \begin{bmatrix} D_0 & D_1 & & \\ & D_0 & D_1 & \\ & & \ddots & \ddots \end{bmatrix},$$

where $D_0 = (D_0)_{n \times n}$ represents the transfer rate of the environmental change transfer of the Markov process when a new data center joins the data center network, and $D_1 = (D_1)_{n \times n}$ represents the arrival rate of a new data center when it joins the data center network.

Let $\{N(t), I(t), J(t) : t \geq 0\}$ represent a multidimensional continuous-time Markov process, and $N(t)$ represents the number of available data centers in the data center network at time $t$. $I(t)$ is the phase of the data center lifetime at time $t$, and $J(t)$ is the environmental phase of a new data center joining the data center network at time $t$. Let $\Omega = H_1 \cup H_2$, where $H_1 = \{(0, j) : 1 \leq j \leq n\}$ represents a new data center joining the network, and $j$ is the phase of a joining Markov process of a new data center. $H_2 = \{(N, i, j) : N \geq 0, 1 \leq i \leq m, 1 \leq j \leq n\}$ represents the number of $N$ data centers under normal operation in the data center network. The lifetime of the data center is in phase $i$, and a new data center joining this network is in phase $j$. Then, $\{N(t), I(t), J(t) : t \geq 0\}$ is a QBD process, and its state transition relation is illustrated in Figure 1.

In this figure, let $\oplus^k(\alpha, T)$ denote that the lifetime of $k$ data centers follows the PH distributions $(\alpha, T)$. We then analyze and explain the state transition relationship of the system shown in Figure 1 as follows.

#### 1) LEVEL $n$ TO LEVEL $n$

When $n = 0$, no data center exists in the system at this time, and only a transfer between the phases of MAP occurs in the data center. Then, it can be represented by matrix $D_0$.

When $n \geq 1$, $n$ data centers exist in the system at this time, and a parallel operation of MAP occurs between a new data center and the $n$ data centers' lifetime process. Then, it can be represented by matrix $D_0 \oplus \left(T - \Delta\left((n-1)\, T^0\right)\right)$, where $\Delta(kT^0) = kdiag(t_1^0, t_2^0, \ldots, t_m^0)$.

#### 2) LEVEL $n$ TO LEVEL $n+1$

When $n = 0$ and the system transfers from level 0 to level 1, this transfer indicates that a new data center has joined this system. The matrix is given by $D_1 \otimes \alpha$.

When $n \geq 1$ and the system is from level $n$ to level $n+1$, this transfer indicates that a new data center has joined this system. The matrix is given by $D_1 \otimes I_m$.

#### 3) LEVEL $n$ TO LEVEL $n-1$

When $n = 1$ and the system transfers from level 1 to level 0, this transfer indicates the end of a data center's lifetime and the lost time of service computing capability. The matrix is given by $I_n \otimes T^0$.

When $n \geq 2$ and the system transfers from level $n$ to level $n-1$, this transfer indicates the end of a data center's lifetime and the lost time of service computing capability. The matrix is given by $I_n \otimes \left(nT^0\alpha\right)$.

From the preceding analysis, the infinitesimal generator of the QBD process $\{N(t), I(t), J(t) : t \geq 0\}$ is given by

$$Q = \begin{pmatrix} Q_1^{(0)} & Q_0^{(0)} & & & \\ Q_2^{(1)} & Q_1^{(1)} & Q_0^{(1)} & & \\ & Q_2^{(2)} & Q_1^{(2)} & Q_0^{(2)} & \\ & & Q_2^{(3)} & Q_1^{(3)} & \ddots \\ & & & \ddots & \ddots \end{pmatrix}$$

where

$$Q_1^{(0)} = D_0, Q_0^{(0)} = D_0 \otimes \alpha, Q_2^{(1)} = I_n \otimes T^0,$$
$$Q_1^{(0)} = D_0 \oplus T, Q_0^{(1)} = D_1 \otimes I, Q_2^{(2)} = I_n \otimes (2T^0\alpha),$$

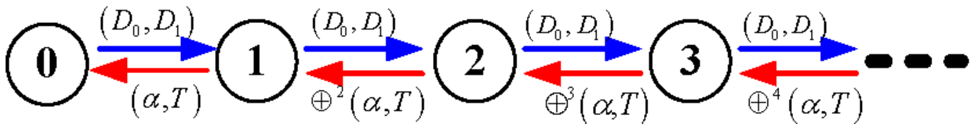


**FIGURE 1.** System state transition relationship.

$$Q_1^{(2)} = D_0 \oplus (T - \Delta(T^0)), Q_0^{(2)} = D_1 \otimes I,$$
$$Q_2^{(3)} = I_n \otimes (3T^0\alpha), Q_1^{(3)} = D_0 \oplus (T - \Delta(2T^0)).$$

Let

$$\begin{aligned} A_k &= Q_0^{(K)} + Q_1^{(K)} + Q_2^{(K)} \\ &= D_1 \otimes I + D_0 \oplus (T - \Delta(k-1)T^0) + I \otimes (kT^0\alpha) \\ &= (D_0 + D_1) \otimes I + I \otimes \left[(T - \Delta((k-1)T^0)) + (kT^0\alpha)\right] \end{aligned}$$

Let $\theta^{(D)}$ indicate the steady-state probability vector of the Markov process $(D_0 + D_1)$. $\theta_k^{(T)}$ indicates the steady-state probability vector of the Markov process $(T-\Delta((k-1)T^0))+(kT^0\alpha)$, and $\theta^{(D)}\otimes\theta_k^{(T)}$ represents the steady-state probability vector of the Markov process $A_k$. Therefore, the drift rate of Markov process $Q$ from levels $K$ to $K-1$ is

$$\begin{aligned} \left(\theta^{(D)} \otimes \theta_k^{(T)}\right) Q_2^{(K)} e \otimes e &= (\theta^{(D)} Ie) \otimes (\theta_k^{(T)}(kT^0\alpha))e \\ &= K\theta_k^{(T)} T^0 \to \infty, \end{aligned} \quad (3)$$

where $\theta_\infty^{(T)} = \lim_{k\to\infty} \theta_k^{(T)}$.

The drift rate of Markov process $Q$ from levels $K$ to $K+1$ is

$$\left(\theta^{(D)} \otimes \theta_k^{(T)}\right) Q_0^{(K)} e = \theta^{(D)} D_1 e \to \infty,$$

Therefore, $K^*$ exists, when $K > K^*$,

$$k\theta_k^{(T)} T^0 > \theta^{(D)} D_1 e.$$

By using the mean draft condition, $k\theta_k^{(T)} T^0 > \theta^{(D)} D_1 e$ indicates that the QBD process $\{N(t), I(t), J(t) : t \geq 0\}$ is irreducible, aperiodic, and positive recurrent.

When the system is in steady state, the steady-state probability vector of the infinitesimal generator $Q$ is $\pi$, where $\pi = (\pi_0, \pi_1, \pi_2, \pi_3, \ldots)$, and $\pi$ satisfies the following conditions:

$$\pi Q = 0, \ \pi e = 1.$$

Li and Cao [28] proved that for the positive recurrent QBD process $Q$, the steady-state probability vector $\pi = (\pi_0, \pi_1, \pi_2, \pi_3, \ldots)$ satisfies the following conditions:

$$\pi_{k+1} = \pi_k R_k = \pi_0 R_0 R_1 ... R_k, \ k \geq 0,$$

where $\pi_0 \left[Q_1^{(0)} + R_0 Q_2^{(1)}\right] = 0$, $\pi_0 \sum_{k=0}^{\infty} \left[\prod_{m=0}^{k-1} R_m\right] e = 1$, $\prod_{m=0}^{-1} R_m = I$.

Under normal conditions, $R_k$ is finite-dimensional and the preceding infinite sum formula should be truncated. Then, we define

$$\{\pi_k(K), 0 \leq k \leq K\}$$

and

$$\pi_0(K) \sum_{k=0}^{K} \left[\prod_{m=0}^{k-1} R_m\right] e = 1.$$

Then

$$\widetilde{Q}(K) = \begin{pmatrix} Q_1^{(0)} & Q_0^{(0)} & & & \\ Q_2^{(1)} & Q_1^{(1)} & Q_0^{(1)} & & \\ & \ddots & \ddots & \ddots & \\ & & Q_2^{(K-1)} & Q_1^{(K-1)} & Q_0^{(K-1)} \\ & & & Q_2^{(K)} & Q_1^{(K)} \end{pmatrix}.$$

By using the algorithms proposed by Bright and Taylor [29], we obtain a steady-state probability distribution $\{\pi_k(K), 0 \leq k \leq K\}$. When the system is in a level state, the mean of the number of normal operating data centers $N$ is

$$E[\mathbf{N}] = \sum_{k=0}^{K} k\pi_k(K)e. \quad (4)$$

On this basis, if $d$ identical files are required to be copied on $d$ different data centers in this data center network, at least $d$ normally operating data centers that can accept the duplicate files. Thus, the probability of successfully copying $d$ identical files on $d$ different data centers is

$$\begin{aligned} Pr\{\mathbf{N} \geq d\} &= \sum_{k=d}^{k} \pi_k(K)e \\ &= \sum_{k=d}^{k} \pi_0(K) \prod_{n=0}^{k-1} R_n e > \pi_0(K) \prod_{n=0}^{d-1} R_n e. \end{aligned} \quad (5)$$

In addition, the probability of successfully copying $m$ identical files on $m(\leq d-1)$ different data centers is

$$\begin{aligned} Pr\{d-1 \geq m \geq \mathbf{N} \geq 1\} &= \sum_{k=1}^{m} \pi_k(K)e \\ &= \sum_{k=1}^{m} \pi_0(K) \prod_{n=0}^{k-1} R_n e. \end{aligned} \quad (6)$$

In particular, the probability of an unsuccessful replication is

$$Pr\{\mathbf{N} = 0\} = \pi_0(K)e. \quad (7)$$

## IV. JOINT PROBABILITY ANALYSIS OF THE NUMBER OF FILE COPIES AND NUMBER OF DATA CENTERS

In this section, we study the random behavior of the number of file copies and describe the relationship between the number of data centers and number of file copies.

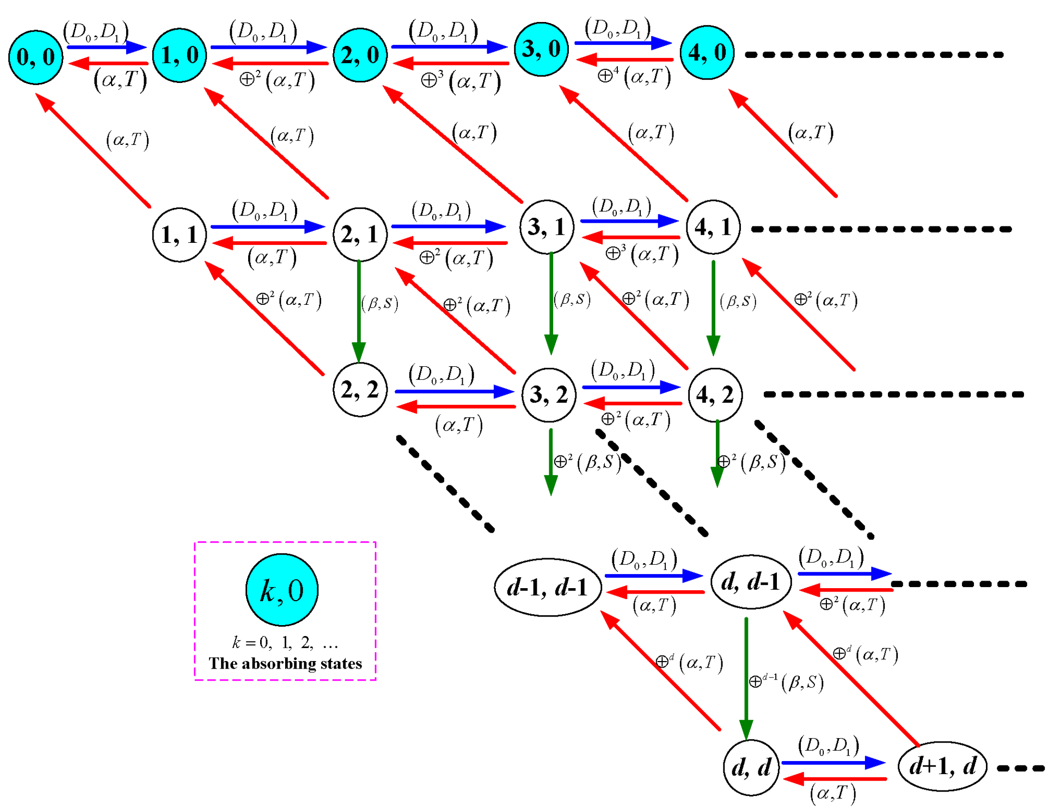


**FIGURE 2.** System state transition relationship.

In the data center network, let $N(t)$ represent the number of data centers operating normally at time $t$, and $M(t)$ represents the number of successful copies of the same file in different data centers at time $t$. Evidently, $N(t) \in \{0, 1, 2, \ldots\}$ and $M(t) \in \{0, 1, 2, \ldots, d\}$, and $I(t)$ is the phase of the data center's PH lifetime, $J(t)$ is the environmental phase of MAP joining the new data center, and $Z(t)$ is the phase of PH copying time of the file data. $\{N(t), M(t), I(t), J(t), Z(t) : t \geq 0\}$ is a multidimensional Markov chain, which is also a QBD process. The state transition relationship is illustrated in Figure 2.

In this figure, $\oplus^k(\alpha, T)$ indicates that the lifetime of $k$ data centers follows the PH distribution $(\alpha, T)$, and $\oplus^k(\beta, S)$ indicates that the copy time of the $k$ file follows the PH distribution $(\beta, S)$. As shown in Figure 2, the state space of the QBD process $\{N(t), M(t), I(t), J(t), Z(t) : t \geq 0\}$ is expressed as

$$\Theta = \Delta \cup \Theta_1 \cup \Theta_2 \cup \Theta_3 \cup \cdots = \Delta \cup \left( \bigcup_{k=1}^{\infty} \Theta_k \right),$$

where $\Delta = \{(k, 0) : k = 0, 1, 2 \ldots\}$ is a set of all the absorption states. Observing the columns in Figure 2, we write level $k$ as follows:

$$k \in \{1, 2, 3, \ldots, d-1\} : \Theta_k = \{(k, 1), (k, 2), \ldots, (k, k)\};$$

and we write level $l$ as follow:

$$l \in \{d, d+1, d+2, \ldots\} : \Theta_l = \{(l, 1), (l, 2), \ldots, (l, d)\};$$

From these levels, the infinitesimal generator of the QBD process $\{N(t), M(t), I(t), J(t), Z(t) : t \geq 0\}$ in sub-state space $\bigcup_{k=1}^{\infty} \Theta_k$ is given by

$$T = \begin{pmatrix} A_{1,1} & A_{1,2} & & & \\ A_{2,1} & A_{2,2} & A_{2,3} & & \\ & A_{3,2} & A_{3,3} & A_{3,4} & \\ & & \ddots & \ddots & \ddots \end{pmatrix}$$

$$A_{1,1} = (C \oplus T),$$
$$A_{1,2} = (D \otimes I \otimes \beta, 0);$$

For $2 \leq k \leq d-1$,

$$A_{k,k-1} = \begin{pmatrix} \Phi^{(1)} & & & & & & \\ R^{(2)} & \Phi^{(2)} & & & & & \\ & R^{(3)} & \Phi^{(3)} & & & & \\ & & \ddots & \ddots & & & \\ & & & R^{(i)} & \Phi^{(i)} & & \\ & & & & \ddots & \ddots & \\ & & & & & R^{(k-1)} & \Phi^{(k-1)} \\ & & & & & & R^{(k)} \end{pmatrix},$$

$$\Phi^{(i)} = I \otimes ((k-i)T^0\alpha) \otimes I, i = 1, 2, \ldots, k-2;$$
$$\Phi^{(k-1)} = I \otimes (T^0\alpha)e;$$
$$R^{(i)} = I \otimes (iT^0\alpha) \otimes I, i = 1, 2, \ldots, k-1;$$
$$R^{(k)} = I \otimes (kT^0\alpha);$$

$$A_{k,k} = \begin{pmatrix} Z^{(1)} & N^{(1)} & & & & & \\ & Z^{(2)} & N^{(2)} & & & & \\ & & \ddots & \ddots & & & \\ & & & Z^{(i)} & N^{(i)} & & \\ & & & & \ddots & \ddots & \\ & & & & & Z^{(k-1)} & N^{(k-1)} \\ & & & & & & Z^{(k)} \end{pmatrix},$$

$$\begin{aligned} Z^{(i)} &= C \oplus (T - \triangle(k-1)T^0) \oplus (S - (\triangle(i-1)S^0)), \\ & \quad i = 1, 2, \ldots, k-1; \\ Z^{(k)} &= C \oplus (T - \triangle(k-1)T^0); \\ N^{(i)} &= I \otimes I \otimes (iS^0\beta), i = 1, 2, \ldots, k-2; \\ N^{(k-1)} &= I \otimes I \otimes ((k-1)S^0); \end{aligned}$$

$$A_{k,k+1} = \begin{pmatrix} D\otimes I\otimes I & & & & \\ & D\otimes I\otimes I & & & \\ & & D\otimes I\otimes I & & \\ & & & \ddots & \\ & & & & D\otimes I\otimes I \\ & & & & D\otimes I\otimes \beta \quad 0 \end{pmatrix},$$

For $l \geq d$,

$$A_{l,l-1} = \begin{pmatrix} \Phi^{(1)} & & & & & & \\ R^{(2)} & \Phi^{(2)} & & & & & \\ & R^{(3)} & \Phi^{(3)} & & & & \\ & & \ddots & \ddots & & & \\ & & & R^{(i)} & \Phi^{(i)} & & \\ & & & & \ddots & \ddots & \\ & & & & & R^{(d-1)} & \Phi^{(d-1)} \\ & & & & & & R^{(d)} \quad \Phi^{(d)} \end{pmatrix},$$

$$\begin{aligned} \Phi^{(i)} &= I \otimes ((l-i)T^0\alpha) \otimes I, i = 1, 2, \ldots, d-1, \\ \Phi^{(d)} &= I \otimes ((l-d)T^0\alpha) \otimes \beta; \\ R^{(i)} &= I \otimes (iT^0\alpha) \otimes I, i = 1, 2, \ldots, d-1, \\ R^{(d)} &= I \otimes (dT^0\alpha) \otimes \beta; \end{aligned}$$

$$A_{l,l} = \begin{pmatrix} Z^{(1)} & N^{(1)} & & & & & \\ & Z^{(2)} & N^{(2)} & & & & \\ & & \ddots & \ddots & & & \\ & & & Z^{(i)} & N^{(i)} & & \\ & & & & \ddots & \ddots & \\ & & & & & Z^{(d-1)} & N^{(d-1)} \\ & & & & & & Z^{(d)} \end{pmatrix},$$

$$\begin{aligned} Z^{(i)} &= C \oplus (T - \triangle(l-1)T^0) \oplus (S - (\triangle(i-1)S^0)), \\ & \quad i = 1, 2, \ldots, d-1; \\ Z^{(d)} &= C \oplus (T - \triangle(l-1)T^0); \\ N^{(i)} &= I \otimes I \otimes (iS^0\beta), i = 1, 2, \ldots, d-2; \\ N^{(d-1)} &= I \otimes I \otimes ((d-1)S^0); \end{aligned}$$

$$A_{l,l+1} = \begin{pmatrix} D\otimes I\otimes I & & & & \\ & D\otimes I\otimes I & & & \\ & & D\otimes I\otimes I & & \\ & & & \ddots & \\ & & & & D\otimes I\otimes I \\ & & & & D\otimes I\otimes \beta \quad 0 \end{pmatrix}.$$

Notably, the QBD process

$$\{N(t), M(t), I(t), J(t), Z(t) : t \geq 0\}$$

contains a set of absorption states

$$\Delta = \{(k, 0) : k = 0, 1, 2 \ldots\}.$$

For simplicity, we compress this set of absorption states into absorption state $\Delta^*$. In such situation, the infinitesimal generator of the QBD process $\{N(t), M(t), I(t), J(t), Z(t) : t \geq 0\}$ in the modified state space $\Delta^* \cup \left(\bigcup_{k=1}^{\infty} \Theta_k\right)$ is

$$\mathbf{Q} = \begin{pmatrix} T_1 & T_1^0 \\ 0 & 0 \end{pmatrix},$$

where

$$T_1^0 = -T_1 e = (\lambda; \lambda, 0; \lambda, 0, 0; \lambda, 0, 0, 0; \lambda, 0, 0, 0, 0; \ldots)',$$

And $a'$ represents the transpose of row vector $a$.

Let

$$\chi = \inf\{t \geq 0 : M(t) = 0, N(t) \in \{1, 2, 3, \ldots\}\}.$$

Then, the random variable $\chi$ is the length of time required to reach the absorption state set $\Delta$ or the absorption state $\Delta^*$ for the first time; that is, the random variable $\chi$ is a first arrival time, indicating that the file can-not be successfully replicated in a different data center network for the first time, along with the lifetime of file data in the data center network.

Theorem 1 shows that the first arrival time $\chi$ is the PH distribution with infinite dimension, where the initial probability vector $\alpha = (\alpha_{\Delta^*}, \alpha_1, \alpha_2, \alpha_3, \ldots)$ and $\alpha_{\Delta^*} \in [0, 1]$. For $1 \leq k \leq d$,

$$\alpha_k = \left(\alpha_{k,1}, \alpha_{k,2}, \cdots, \alpha_{k,k-1}, \alpha_{k,k}\right),$$

and for $l \geq d+1$,

$$\alpha_l = \left(\alpha_{l,1}, \alpha_{l,2}, \cdots, \alpha_{l,d-1}, \alpha_{l,d}\right).$$

*Theorem 1:* In the data center network, the first arrival time $\chi$ is the PH distribution with infinite dimension, and its irreduciblility is expressed as $(\widetilde{\alpha}, T_1)$, where $\widetilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \cdots)$, $\widetilde{\alpha} e = 1 - \alpha_{\Delta^*}$. In addition, Markov process

$T_1 + T_1^0\widetilde{\alpha}$ is irreducible. Moreover the $k$th moment of the first arrival time $\chi$ is

$$E\left[\chi^k\right] = (-1)^k\, k!\widetilde{\alpha}T_1^{-k}e,\ \ k = 1, 2, 3, \ldots \tag{8}$$

*Proof:* The infinitesimal generator of the QBD process $\{N(t), M(t), I(t), J(t), Z(t) : t \geq 0\}$ in modified state space $\Delta^* \cup \left(\bigcup_{k=1}^{\infty} \Theta_k\right)$ is given by $\mathbf{Q}$

$$\mathbf{Q} = \begin{pmatrix} T_1 & T_1^0 \\ 0 & 0 \end{pmatrix}.$$

Moreover, it is irreducible in sub-state space $\bigcup_{k=1}^{\infty} \Theta_k$. Thus, the absorption state $\Delta^*$ shows that the first arrival time $\chi$ is the length of time required for the first arrival to the absorption state $\Delta^*$. Hence, the first arrival time $\chi$ is the PH distribution with infinite dimension, and its irreducibility is expressed as $(\widetilde{\alpha}, T_1)$. From Li [27], let the transfer rate matrix $T_1$ be nonsingular, with the probability density function $f(t) = \alpha\exp(T_1 t)T_1^0$. The Laplace-Stieltjes transformation of PH$(T_1, T_1^0)$ can be calculated, and $f^*(s) = 1 - \alpha e + \alpha(sI - T_1)^{-1}T_1^0$, for $s$ differential $k$ times, and $s = 0$. The formula for obtaining $k$ moment is $E[\chi^k] = (-1)^k k!\widetilde{\alpha}T_1^{-k}e, k = 1, 2, 3\ldots$. The $k$th moment $E\left[\chi^k\right]$ of the first arrival time $\chi$ can be derived. This scenario completes the proof. ■

To calculate the $k$th moment $E\left[\chi^k\right]$ of the first arrival time $\chi$, the inverse matrix of matrix $T_1$ should be derived first. From Section 2.1 of Li and Cao [28], we define the $U-$measure as

$$\begin{aligned} \mathbf{U}_0 &= A_{1,1}, \\ \mathbf{U}_k &= A_{k+1,k+1} + A_{k+1,k}(-U_{k-1})^{-1}A_{k,k+1},\ \ k = 1, 2, 3, \ldots; \end{aligned}$$

Define the $R-$measure as

$$\mathbf{R}_k = A_{k+1,k}(-\mathbf{U}_{k-1})^{-1},\ \ k = 1, 2, 3, \ldots;$$

Define the $G-$measure as

$$\mathbf{G}_l = (-\mathbf{U}_l)^{-1}A_{l+1,l+2},\ \ l = 0, 1, 2, \ldots;$$

Based on the $U-$measure, $R-$measure and $G-$measure, using the Theorem 1 in Li and Cao [28], the $RG-$decomposition of matrix $T_1$:

$$T_1 = (I - \mathbf{R}_L)\, \mathbf{U}_D\, (I - \mathbf{G}_U),$$

where

$$\mathbf{U}_D = \mathrm{diag}(\mathbf{U}_0, \mathbf{U}_1, \mathbf{U}_2, \ldots),$$

$$\mathbf{R}_L = \begin{pmatrix} 0 & & & & \\ \mathbf{R}_1 & 0 & & & \\ & \mathbf{R}_2 & 0 & & \\ & & \mathbf{R}_3 & 0 & \\ & & & \ddots & \ddots \end{pmatrix},$$

$$\mathbf{G}_U = \begin{pmatrix} 0 & G_0 & & & \\ & 0 & G_1 & & \\ & & 0 & G_2 & \\ & & & 0 & \ddots \\ & & & & \ddots \end{pmatrix}.$$

So, we have obtained

$$T_1^{-1} = (I - \mathbf{G}_U)^{-1}\, \mathbf{U}_D^{-1}\, (I - \mathbf{R}_L)^{-1}.$$

Let

$$\begin{aligned} X_k^{(l)} &= \mathbf{R}_l\mathbf{R}_{l-1}\cdots\mathbf{R}_{l-k+1}, 1 \leq k \leq l, \\ Y_k^{(l)} &= \mathbf{G}_l\mathbf{G}_{l+1}\cdots\mathbf{G}_{l+k-1}, k \geq l, l \geq 0, \end{aligned}$$

Then

$$U_D^{-1} = \mathrm{diag}(\mathbf{U}_0^{-1}, \mathbf{U}_1^{-1}, \mathbf{U}_2^{-1}, \ldots),$$

$$(I - \mathbf{G}_U)^{-1} = \begin{pmatrix} I & Y_1^{(0)} & Y_2^{(0)} & Y_3^{(0)} & \cdots \\ & I & Y_1^{(1)} & Y_2^{(1)} & \cdots \\ & & I & Y_1^{(2)} & \cdots \\ & & & I & \cdots \\ & & & & \ddots \end{pmatrix},$$

$$(I - \mathbf{R}_L)^{-1} = \begin{pmatrix} I & & & & \\ X_1^{(1)} & I & & & \\ X_2^{(2)} & X_1^{(2)} & I & & \\ X_3^{(3)} & X_2^{(3)} & X_1^{(3)} & I & \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Let

$$T_1^{-1} = \begin{pmatrix} t_{0,0} & t_{0,1} & t_{0,2} & \cdots \\ t_{1,0} & t_{1,1} & t_{1,2} & \cdots \\ t_{2,0} & t_{2,1} & t_{2,2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

From $T_1^{-1} = (I - \mathbf{G}_L)^{-1}\, \mathbf{U}_D^{-1}\, (I - \mathbf{R}_U)^{-1}$, we can get

$$t_{m,n} = \begin{cases} \mathbf{U}_m^{-1}X_{m-n}^{(m)} + \sum_{i=1}^{\infty} Y_i^{(m)}\mathbf{U}_{i+m}^{-1}X_{i+m-n}^{(i+m)}, & 0 \leq n \leq m-1 \\ \mathbf{U}_m^{-1} + \sum_{i=1}^{\infty} Y_i^{(m)}\mathbf{U}_{i+m}^{-1}X_i^{(i+m)}, & n = m \\ Y_{n-m}^{(m)}\mathbf{U}_n^{-1} + \sum_{i=n-m+1}^{\infty} Y_i^{(m)}\mathbf{U}_{i+m}^{-1}X_{i-n+m}^{(i+m)}, & n \geq m+1 \end{cases}$$

Then the mean of the first arrival time $\chi$ is given by

$$\begin{aligned} E[\chi] &= -\widetilde{\alpha}T_1^{-1}e = -\widetilde{\alpha}\,(I - \mathbf{G}_L)^{-1}\,\mathbf{U}_D^{-1}\,(I - \mathbf{R}_U)^{-1}e \\ &= -\sum_{i=1}^{\infty} \alpha_i t_{i-1,0} - \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} \alpha_i t_{i-1,j}e \end{aligned} \tag{9}$$

Finally, given the complexity of the calculation process of MAP and PH distribution, the exponential distribution is briefly explained as a special case. We present a simple example to illustrate how file lifetime $\chi$ depends on the maximum number of identical backups: $d \in (2, 70)$. Let $\lambda = 1$ and $\beta = 6$. As shown in Figure 3, the mean $E[\chi]$ increases, as $d$ increases. In addition, when $d$ increases to a certain value $\Theta$, the mean $E[\chi]$ will no longer change significantly. The number of file copies is controlled within $\Theta$ to minimize the cost of file replication on the basis of

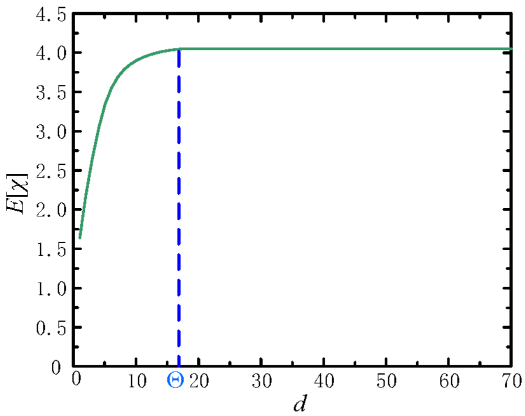


**FIGURE 3.** File lifetime depends on the key parameter *d*.

keeping file lifetime relatively long. Such phenomenon will be valuable in designing and optimizing a data center network with a file replication mechanism.

## V. STOCHASTIC BEHAVIOR ANALYSIS OF THE NUMBER OF FILE COPIES

In this section, a new birth-and-death process of finite state space is established by modifying the copy rate of files in the data center network. The random behavior of the number of file data copies is studied. Moreover the time of the first loss of file data backup is examined.

Note that $\{N(t), I(t), J(t) : t \geq 0\}$ is QBD process, the steady state probability distribution of the normal operating data center is

$$\theta_0 = \pi_0 e;\ \theta_k = \pi_{k+1} e = \pi_k R_k e = \pi_0 R_0 R_1 \dots R_k e,\ \ k \geq 0$$

As shown in Figure 2, when this data center network is in steady state, the following results can be obtained.

**(a)** If a file has only one copy in the data center network, then the file adopts the copying rate of another data center, given by

$$S_1^0 = S^0 \pi_0 \sum_{n=2}^{\infty} \theta_n. \tag{10}$$

In the steady state of the system, the copying time of this file is subject to a PH distribution and its irreducible matrix is expressed as $(\beta, S_1)$, where

$$S_1 = S\pi_0 \sum_{n=2}^{\infty} \theta_n. \tag{11}$$

Evidently,

$$S_1^0 = S^0 \pi_0 \sum_{n=2}^{\infty} \theta_n. \tag{12}$$

**(b)** If a file has $k$ identical copies(i.e., a file has been duplicated in $k$ different data centers) in the data center network for $2 \leq k \leq d-1$, then the $k$ identical files adopt the copying rate of another data center, given by

$$S_k^0 = kS^0 \pi_0 \sum_{n=k+1}^{\infty} \theta_n. \tag{13}$$

In the steady state of the system, the copying time of this file is subject to a PH distribution and its irreducible matrix is expressed as $(\beta, S_k)$, where

$$S_k = \left[S - \Delta((k-1)S^0)\right] \sum_{n=k+1}^{\infty} \theta_n. \tag{14}$$

Evidently,

$$S_k^0 = kS^0 \sum_{n=k+1}^{\infty} \theta_n. \tag{15}$$

In the data center network, let $\oplus^k(a, T)$ denote that $k$ data centers with PH lifetime distribution $(a, T)$ fail simultaneously and are removed from the data center network. We denote $M(t)$ as the number of identical copies of one file at time $t$, then $\{M(t) : t \geq 0\}$ is a QBD process in a finite state space $\mathbf{E} = \{0, 1, 2, \dots, d-1, d\}$, the state transition relation of which is depicted in Figure 4.

Let

$$\eta = \inf\{t \geq 0 : M(t) = 0\}.$$

Then, $\eta$ is the lifetime of a file that remains in the data center network. It is also the first lost time of the file that will possibly disappear in the data center network.

Let

$$\varphi = \begin{pmatrix} \Upsilon^1 & P^1 & & & & & \\ \Gamma^2 & \Upsilon^2 & P^2 & & & & \\ & \ddots & \ddots & \ddots & & & \\ & & \Gamma^i & \Upsilon^i & P^i & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & \Gamma^{d-1} & \Upsilon^{d-1} & P^{d-1} \\ & & & & & \Gamma^d & \Upsilon^d \end{pmatrix},$$

$$\varphi^0 = \begin{pmatrix} e_2 \otimes T^0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

When $i = 1$,

$$\Upsilon^1 = (S_1 \oplus T), P^1 = (S_1^0 \beta \otimes I);$$

When $i = 2, 3, \dots, d-1$,

$$\Gamma^i = I \otimes (iT^0 \alpha), \Upsilon^i = S_i \oplus (T - \Delta(i-1)T^0), P^i = S_i^0 \beta \otimes I;$$

When $i = d$,

$$\Gamma^i = I \otimes (iT^0 \alpha), \Upsilon^d = (T - \Delta(d-1)T^0).$$

*Theorem 2:* In this data center network, the lifetime $\eta$ of a file follows a PH distribution with an irreducibility representation of $(\widetilde{\gamma}, \varphi)$. The initial probability vector is $(\widetilde{\gamma}, \gamma_0)$, where $\widetilde{\gamma} = (\gamma_1, \gamma_2, \dots, \gamma_d)$, $\gamma_0 \in [0, 1]$, and $\widetilde{\gamma} e = 1 - \gamma_0$. The

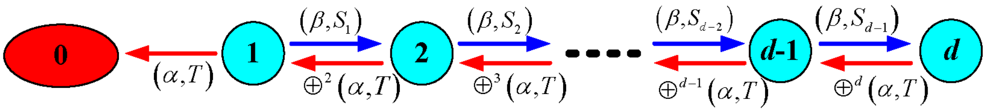


**FIGURE 4.** State transition of the file replication process.

Markov process $\left(\varphi+\varphi^0\widetilde{\gamma}\right)$ is also irreducible. Furthermore, the $k$th moment of the lifetime $\eta$ of a file is given by

$$E\left[\eta^k\right]=(-1)^k k!\widetilde{\gamma}\varphi^{-k}e,\ k=1,2,3,\ldots \tag{16}$$

*Proof:* Checking that the infinitesimal generator of the birth-death process $\{M(t):t\geq 0\}$ is in state space is easy.

$$\mathbf{E}=\{\text{level }0,\text{level}1,\text{level}2,\ldots,\text{level}d-1,\text{level}d\}$$

is given by

$$\mathbf{Q}=\begin{pmatrix}\varphi & \varphi^0\\ 0 & 0\end{pmatrix}.$$

Evidently, the lifetime $\eta$ of a file follows a PH distribution with size $d$ and an irreducibility representation of $(\widetilde{\gamma},\varphi)$. Moreover the Markov process $\left(\varphi+\varphi^0\widetilde{\gamma}\right)$ is irreducible. In addition, some simple computations can lead to the $k$th moment of lifetime $\eta$. This scenario completes the proof. ■

To calculate the mean $E[\eta]$ of the first arrival time $\eta$, we need to derive the inverse matrix of matrix $\varphi$ first. Using Section 2.1 of Li and Cao [28], we define the $U-$measure as

$$\begin{aligned}\mathbf{U}_0&=\Upsilon^1,\\ \mathbf{U}_i&=\Upsilon^i+\Gamma^i(-\mathbf{U}_{i-1})^{-1}P^{i-1},\ 1\leq i\leq d.\end{aligned}$$

Define the $R-$measure as

$$\mathbf{R}_i=\Gamma^i(-\mathbf{U}_{i-1})^{-1},\ 1\leq i\leq d.$$

Define the $G-$measure as

$$\mathbf{G}_i=(-\mathbf{U}_i)^{-1}P^i,\ 1\leq i\leq d-1.$$

Based on the $U-$measure, $R-$measure and $G-$measure, using the Theorem 1 in Li and Cao [28], the $RG-$decomposition of matrix $\varphi$ is

$$\varphi=(I-\mathbf{R}_L)\,\mathbf{U}_D\,(I-\mathbf{G}_U)\,,$$

where

$$\begin{aligned}\mathbf{U}_D&=\text{diag}(\mathbf{U}_0,\mathbf{U}_1,\mathbf{U}_2,\ldots,\mathbf{U}_{d-1},\mathbf{U}_d),\\ \mathbf{R}_L&=\begin{pmatrix}0 &&&&\\ \mathbf{R}_1 & 0 &&&\\ & \ddots & \ddots &&\\ && \mathbf{R}_{d-1} & 0 &\\ &&& \mathbf{R}_d & 0\end{pmatrix},\\ \mathbf{G}_U&=\begin{pmatrix}0 & G_0 &&&\\ & 0 & G_1 &&\\ && \ddots & \ddots &\\ &&& 0 & G_d\\ &&&& 0\end{pmatrix}.\end{aligned}$$

So, we can obtain

$$\varphi^{-1}=(I-\mathbf{G}_U)^{-1}\,\mathbf{U}_D^{-1}\,(I-\mathbf{R}_L)^{-1}.$$

Let

$$\begin{aligned}X_k^{(l)}&=\mathbf{R}_l\mathbf{R}_{l-1}\cdots\mathbf{R}_{l-k+1},\,1\leq k\leq l\leq d,\\ Y_k^{(l)}&=\mathbf{G}_l\mathbf{G}_{l+1}\cdots\mathbf{G}_{l+k-1},\,0\leq l\leq k\leq d-1.\end{aligned}$$

Then

$$\begin{aligned}U_D^{-1}&=\text{diag}(\mathbf{U}_0^{-1},\mathbf{U}_1^{-1},\mathbf{U}_2^{-1},\ldots,\mathbf{U}_d^{-1}),\\ (I-\mathbf{G}_U)^{-1}&=\begin{pmatrix}I & Y_1^{(0)} & Y_2^{(0)} & \cdots & Y_{d-1}^{(0)}\\ & I & Y_1^{(1)} & \cdots & Y_{d-2}^{(0)}\\ && \ddots & \ddots & \vdots\\ &&& \ddots & Y_1^{(d-1)}\\ &&&& I\end{pmatrix},\\ (I-\mathbf{R}_L)^{-1}&=\begin{pmatrix}I &&&&\\ X_1^{(1)} & I &&&\\ \vdots & \vdots & \ddots &&\\ X_{d-1}^{(d-1)} & X_{d-2}^{(d-2)} & X_{d-3}^{(d-3)} & I &\\ X_d^{(d)} & X_{d-1}^{(d)} & X_{d-2}^{(d)} & X_{d-3}^{(d)} & I\end{pmatrix}.\end{aligned}$$

Let

$$\varphi^{-1}=\begin{pmatrix}\varphi_{0,0} & \varphi_{0,1} & \varphi_{0,2} & \cdots & \varphi_{0,d}\\ \varphi_{1,0} & \varphi_{11} & \varphi_{1,2} & \cdots & \varphi_{1,d}\\ \varphi_{2,0} & \varphi_{2,1} & \varphi_{2,2} & \cdots & \varphi_{2,d}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \varphi_{d,0} & \varphi_{d,1} & \varphi_{d,2} & \cdots & \varphi_{d,d}\end{pmatrix}.$$

By $\varphi^{-1}=(I-\mathbf{G}_U)^{-1}\,\mathbf{U}_D^{-1}\,(I-\mathbf{R}_L)^{-1}$, So we can obtain

$$\varphi_{m,n}=\begin{cases}\mathbf{U}_mX_{m-n}^{(m)}+\sum\limits_{i=1}^{d-m}Y_i^{(m)}\mathbf{U}_{i+m}^{-1}X_{i+m-n}^{(i+m)},\\ 0\leq n\leq m-1,0\leq m\leq d-1,\\ \\ \mathbf{U}_m^{-1}+\sum\limits_{i=1}^{d-m}Y_i^{(m)}\mathbf{U}_{i+m}^{-1}X_i^{(i+m)},\\ n=m,\ 0\leq m\leq d-1,\\ \\ Y_{n-m}^{(m)}\mathbf{U}_n^{-1}+\sum\limits_{i=n-m+1}^{d-m}Y_i^{(m)}\mathbf{U}_{i+m}^{-1}X_{i-n+m}^{(i+m)},\\ m+1\leq n\leq d,0\leq m\leq d-1,\\ \\ \mathbf{U}_d^{-1}X_{d-n}^{(d)},\,m=c,\ 0\leq n\leq d-1,\\ \mathbf{U}_d^{-1},\,m=n=c\end{cases}$$

Then the mean of the first arrival time $\eta$ is given by

$$E[\eta] = -\widetilde{\alpha}\varphi^{-1}e = -\widetilde{\alpha}(I - \mathbf{G}_U)^{-1}\mathbf{U}_D^{-1}(I - \mathbf{R}_L)^{-1}e$$
$$= -\sum_{i=1}^{\infty}\alpha_i\varphi_{i-1,0} - \sum_{j=1}^{\infty}\sum_{i=1}^{\infty}\alpha_i\varphi_{i-1,j}e. \quad (17)$$

## VI. CONCLUSION

First, to ensure the security of the file in the data center network, a Markov process is used to provide a more general stochastic model that can evaluate the system performance of the data storage mechanism. Second, three types of Markov process are constructed to express the random behavior of some important performance indicators of the data center network. Finally, we propose a file replication strategy and its feasibility is verified through numerical computations. The results and methods presented in this study provide an important development path for research on the data security mechanism in data center networks. This study also presents a significant Markov process calculation theory and method.

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