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Symbol Error Rate Minimization Based Constructive Interference Precoding for Multi-User Systems

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ABSTRACT Symbol level precoding based on the concept of constructive interference has been recognized as an advanced version of conventional interference-avoidance precoding for multi-user transmission. With the full knowledge of channel state information (CSI) and symbol information, inter-user interference is adjusted to bring constructive effect to the desired signal. However, the existing schemes are limited within the concept of channel pre-equalization which could be further improved. In this paper, we propose a symbol error rate (SER) minimization based constructive interference precoding scheme for multi-user systems. The constructive interference region is firstly defined based on the analysis of the SER expression, which implies that the proposed scheme is essentially a symbol pre-detection scheme. Then, we prove that the optimal precoded signal for SER minimization shall be the linear combination of the channel vectors. Accordingly, a modified feasible direction algorithm is developed to handle the complex expression of SER, where an extra projection step is proposed to enhance the efficiency of the feasible direction. Finally, the proposed scheme is further extended for transmit power optimization and imperfect CSI cases. Simulation results highlight the efficiency of the modified feasible direction, and the superiority of the proposed scheme in terms of SER and transmit power.

INDEX TERMS Constructive interference precoding, multi-user systems, symbol error rate minimization, modified feasible direction, symbol pre-detection.

I. INTRODUCTION

In multi-user networks, inter-user interference is one of the crucial factors limiting system throughput. Frequency division multiple access (FDMA) and time division multiple access (TDMA) have been widely adopted in current cellular systems to prevent inter-user interference. With larger-scale antennas equipped at the base station (BS) in 5G and future communication systems, the spatial degrees of freedom can be advantageously exploited to further enhance the spectrum efficiency [1]–[3]. Several users are simultaneously served with the same frequency band through spatial division multiple access (SDMA) [4], [5]. To improve the orthogonality of users in spatial domain, precoding technology has been widely investigated to form narrow transmit beams [6]–[9].

Several low-complexity linear precoding schemes have been developed to maintain low-cost mobile units. Matched-filter (MF) precoding contributes to the least complexity, the performance of which depends on the natural orthogonality among the channels [10]. Zero-forcing (ZF) and regularized zero-forcing (RZF) methods can efficiently manage inter-user interference and provide some performance improvements [11]. The low-complexity schemes have been extended in recent works for more practical and complex cases, e.g., secure transmissions [12] and high-speed railway communications [13]. In these schemes, only channel state information (CSI) is exploited to determine the design of precoders. The achievable rates of these schemes are still far from the theoretical upper bounds [14], [15].

A. RELATED WORK

Symbol-level precoding has been investigated as a more advanced technology in most prior studies [16]–[20].

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The general concept of symbol-level precoding is to directly adjust the transmit signal in symbol level to manage inter-user interference. Both CSI and symbol information of all users are exploited in precoding design. The authors in [16] develop an interference neutralization (IN) scheme. A neutralizing signal is sent to generate the opposite signal with respect to the interference through the wireless channel. The summation of received interference signals is forced as zero to achieve interference-free transmission. With the similar approach of IN, the interference steering (IS) scheme employs a steering signal to control the direction of the interference signal [18]. The received interference is adjusted into the orthogonal subspace of the desired signal via interactions among wireless signals. In [19], the concept of symbol-level precoding is adopted in the beamformer design to maximize the signal-to-interference-plus-noise-ratio (SINR) for full-duplex systems. The common feature of these schemes lies in that all interference signals are regarded as troublemakers in communications and should be eliminated with precoding design, which could be classified as interference-avoidance precoding.

Actually, the power of interference could be further utilized since some useful information might be carried by interference signals. Based on this thought, the interference recycling (IR) scheme is proposed in [20]. A recycling signal is generated to interact with interference, which helps to extract useful information from interference. However, only two-user interfering channels are discussed in [20]. A larger scale system will result in much more computational complexity and power consumption. As a breakthrough, the constructive interference precoding scheme is developed in [21]–[23] for multi-user systems. The main idea is to enhance the power of the desired signal with adjusted interference signals. In [22], ZF method is improved with the knowledge of the cross-correlations between interference and desired signal. A correlation rotation (CR) matrix is constructed to align inter-user interference towards the desired signal of each user, which forms constructive interference and achieves better performance than ZF. The authors in [24] provide a more macroscopic scheme, and divert the focus from individual interference to the resultant interference. A constructive interference power minimization precoding (CIPM) scheme is proposed, where the final received symbol is constrained with the exact same angle as the intended symbol. And the CIPM scheme is further extended for maximizing the minimum SINR with the bisection method. To enlarge the degree of freedom in precoding design, the constructive interference region is expanded in [25]–[28]. The phase of the resultant interference is restricted in a more relaxed way with the consideration of the received effective signal-to-noise ratio (SNR). In [26], the minimization of minimum mean square error (MMSE) is considered with constructive interference and power constraint. A semi-closed-form optimal solution is developed with the bisection algorithm to achieve a certain performance gain such as symbol error rate (SER). In [28], the constructive interference region is further relaxed by utilizing the nearest neighbor union bound (UB) on SER probability, which contributes to larger search domains. The concept of constructive interference is further adopted in hybrid precoding design with analog constrains in [29], [30].

It has been shown that the constructive interference precoding scheme is more applicable for multi-user systems than IR, and requires lower transmit power than interference-avoidance schemes. However, the optimization problems considered in existing works, e.g., worst constructive gain maximization, MMSE problem, are not straightforward for symbol detection. The existing constructive interference precoding schemes are still limited within the concept of channel pre-equalization which is consistent with the conventional interference-avoidance precoding. Actually, the SER after symbol detection can be analyzed mathematically at the transmitter side with the full knowledge of CSI and symbol information of all users. Accordingly, optimizing SER is the most direct approach which matches the ultimate goal of signal processing. The authors in [31] provide an analytic bound of average SER in a simplified way, which degenerates the optimization problem into the same form as which in [27]. For imperfect CSI, three convex approximations are obtained based on different conservatism levels to handle the difficulty of the probabilistic constraints [32]. To the best of our knowledge, the problem of designing the SER minimization based constructive interference precoders, which is the main focus of this paper, is not considered in the literature. In a sense, SER minimization based precoding can be further regarded as a symbol pre-detection scheme, but not just for channel pre-equalization.

B. CONTRIBUTIONS

To mitigate the shortfall mentioned above, in this paper, we propose an SER minimization based constructive interference precoding scheme for multi-user systems. The precoder is designed for signal pre-detection to achieve low SER, which matches the ultimate goal of signal processing. The main contributions are summarized as follows:

- The expression of SER is firstly analyzed as the baseline of this work. And the constructive interference region is directly defined by SER, which brings the main difference compared with the existing works [26], [27]. Then, we prove that the optimal precoded signal for SER minimization shall be the linear combination of channel vectors via the analysis of Lagrangian.
- A modified feasible direction algorithm is developed to seek for a near optimal precoded signal. Inspired by the signal space in which the optimal precoded signal shall lie, we propose to enhance the efficiency of the feasible direction via an extra projection step. The complex expression of SER is handled with the calculation of several gradient values which can be obtained off-line.
- The proposed scheme is further extended for transmit power optimization and imperfect CSI cases. The feasible direction requires corresponding adjustment without extra computational complexity.

Since the novel SER-defined constructive interference region provides a large degree of freedom for optimization, the proposed scheme contributes to satisfying performance gains compared with the existing schemes as demonstrated in simulation results.

C. ORGANIZATION

The remainder of this paper is organized as follows. Section II introduces the signal model with constructive interference, and the problem formulation for precoding design. Section III provides the proposed SER minimization based precoding scheme with a modified feasible direction algorithm. Section IV discusses the extension of the proposed scheme. Section V and section VI present simulation results and some conclusions, respectively.

Notations: a, $\mathbf{a}$ and A denote a scalar, vector and matrix, respectively. $\mathbf{A}^T$, $\mathbf{A}^H$, and $\mathbf{A}^\dagger$ denote the transpose, conjugate transpose, and pseudo-inverse of A, respectively. $\mathbf{A}(:, n)$ denotes the n-th column of $\mathbf{A}$. The absolute value and ℓ_2 norm are denoted by $|\cdot|$ and $\|\cdot\|_2$, respectively. I_N denotes the $N \times N$ identity matrix, whereas $\mathbf{0}$ denotes the all-zero matrix. $\angle(\cdot)$, $\mathrm{Re}(\cdot)$, and $\mathrm{Im}(\cdot)$ denotes the angle, real part, and imaginary part of a complex argument, respectively. $\Pr(\cdot)$ and $\mathbb{E}[\cdot]$ denote the probability and statistical expectation, respectively. $\mathrm{diag}(a, b, \ldots)$ denotes the diagonal matrix with entries (a, b, etc.) arranged in order on the diagonal.

II. SYSTEM MODEL

A. SIGNAL MODEL

We consider the downlink transmission of a multi-user multiple-input single-output (MISO) system. The base station (BS) is equipped with N_t transmit antennas to simultaneously serve K users with the same frequency band. The transmit symbol vector at the BS is given by $s = [s_1, s_2 \ldots, s_K]^T \in \mathbb{C}^{K\times 1}$, where s_k denotes the modulated symbol for the k-th user. For simplicity, we assume a block-fading channel $h_k \in \mathbb{C}^{1\times N_t}$ between the BS and the k-th user, where CSI is invariant over the data block. Then, the received signal at all users, $y = [y_1, y_2 \ldots, y_K]^T \in \mathbb{C}^{K\times 1}$, can be given by

$$y = HFs + n = Hx + n, \tag{1}$$

where $F \in \mathbb{C}^{N_t\times K}$ denotes the precoding matrix, $H = [h_1^H, h_2^H, \ldots, h_K^H]^H \in \mathbb{C}^{K\times N_t}$ denotes the channel matrix of the overall system, $n = [n_1, n_2 \ldots, n_K]^T \in \mathbb{C}^{K\times 1}$ denotes complex Gaussian noise vector with each element satisfying $n_k \sim \mathcal{CN}(0, \sigma_k^2)$. The main objective of this paper is to optimize the precoding matrix F and develop the precoded signal vector x to minimize the maximum SER among users with limited transmit power $P_x = \|x\|_2^2$, which will be formulated in subsequent sections.

To characterize the limited scattering feature of the channel, we employ the Saleh-Valenzuela model. The channel vector with normalized power can be formulated by the summation of L_k scattering clusters, i.e.,

$$h_k = \sqrt{\frac{N_t}{L_k L_{c,k}}} \sum_{c=1}^{L_k} \sum_{p=1}^{L_{c,k}} \beta_{c,k}^{(p)} u^H(\theta_{c,k}^{(p)}, \phi_{c,k}^{(p)}), \tag{2}$$

where $L_{c,k}$ denotes the number of propagation paths in the c-th cluster, $\beta_{c,k}^{(p)} \sim \mathcal{CN}(0, 1)$ denotes the complex gain of the p-th path in the c-th cluster, $u(\theta_{c,k}^{(p)}, \phi_{c,k}^{(p)})$ denotes the normalized transmit array response vector corresponding to the azimuth angle $\theta_{c,k}^{(p)}$ and the elevation angle $\phi_{c,k}^{(p)}$ of departure. Without loss of generality, the uniform planar array (UPA) will be considered in further simulations. The $W \times V$-element UPA response vector is variant in two angle domains, which can be expressed as

$$u(\theta, \phi) = \frac{1}{\sqrt{WV}} \Big[1, \ldots, e^{j\frac{2\pi d}{\lambda}(w\sin(\theta)\sin(\phi)+v\cos(\phi))}, \ldots, e^{j\frac{2\pi d}{\lambda}((W-1)\sin(\theta)\sin(\phi)+(V-1)\cos(\phi))}\Big]^T, \tag{3}$$

where $w = 0, 1, \ldots, W-1$, $v = 0, 1, \ldots, V-1$, and $WV = N_t$. In addition, d denotes the distance between adjacent antennas, which is commonly given by the half of the wavelength $d = \lambda/2$.

B. CONSTRUCTIVE INTERFERENCE

Similar to the assumption in [24], [25], [27], M-ary phase-shift-keying (M-PSK) modulation is employed in this paper. The target constellation point for the k-th user can be expressed as $s_k = e^{j\varphi_k}$, where φ_k denotes the angle of the modulation symbol. The definition of constructive interference for the k-th user can be given by

$$\varphi_k - \frac{\pi}{M} \le \angle\,(h_k F(:, t)s_t) \le \varphi_k + \frac{\pi}{M}, \quad t \neq k. \tag{4}$$

When (4) is satisfied, the inter-user interference from the t-th user brings constructive effect to the k-th user, which refers to constructive interference. To satisfy the condition of (4), various constrains are raised to construct different constructive interference regions. In the CR precoding scheme [22], the phase of each inter-user interference is restricted to ensure

$$\angle\,(h_k F(:, t)s_t) = \varphi_k, \quad \forall t \neq k. \tag{5}$$

With the strict phase constraint of each inter-user interference, the phase of the received signal of the k-th user remains at φ_k. In the CIPM precoding scheme [24], a macroscopic constrain is raised to directly restrict the phase of the received signal for the correct detection of the M-PSK symbol, i.e.,

$$\angle\,(h_k F s) = \varphi_k. \tag{6}$$

Essentially, CIPM requires constructive resultant interference, where the constraint is relaxed for each inter-user interference. To further expand the constructive interference region for optimization, [26], [27] contain the received signal within the constructive area with a guard margin away from the decision boundary, which requires

$$\left|\mathrm{Im}(e^{-j\varphi_k} h_k x)\right| \le (\mathrm{Re}(e^{-j\varphi_k} h_k x) - \sqrt{\gamma_k}) \tan\frac{\pi}{M}, \tag{7}$$

where γ_k determines the guard margin. As illustrated in simulation results of [26], the definition of the constructive interference region is of vital importance in precoding performance enhancing. The constructive interference region given by (7) contributes to the best SER performance, whereas (5) leads to the worst. Mathematically, it is clear that an expanded feasible region brings a better optimization result.

C. PROBLEM FORMULATION

Precoding is a part of signal processing in multi-antenna transmission. The ultimate goal of signal processing is confronting the impact of channels and detecting signals at the user side with low SER. Since the constructive interference region given by (7) reserves a guard margin for detection, the SNR at the user side can be guaranteed to a certain extent, which improves SER minimization. Nevertheless, it is still not a direct approach for SER minimization. With the full knowledge of CSI and symbol information, the BS can get a complete analysis of SER for each user once the precoded signal is determined. Accordingly, we consider to develop an SER minimization based constructive interference precoding scheme via solving the following problem

$$\begin{aligned} &\min_{x,\varepsilon}\ \varepsilon \\ &\text{s.t.} \begin{cases} \|x\|_2^2 \le P, \\ S(h_k x, s_k, \sigma_k^2, M) \le \varepsilon, \forall k, \end{cases} \end{aligned} \tag{8}$$

where $S(\cdot)$ is the SER function of the received symbol $h_k x$, the intended symbol s_k, the noise power σ_k^2, and the modulation order M. In (8), the precoded signal x is designed to optimize the worst SER performance ε among users with limited transmit power P. The second constrain determines the constructive interference region with the SER function which has not been considered in existing schemes. The complex expression of SER function brings the main challenge in solving (8). Since a direct consideration is involved at the BS side for SER minimization after symbol detection, the proposed precoding scheme can be further regarded as a symbol pre-detection scheme, which makes a major breakthrough compared with existing pre-equalization precoding schemes.

Remark: Since the precoding matrix F can be obtained via the underdetermined system of equations $(s^T \otimes I_{N_t})$ $\mathbf{vec}(F) = x$, only the precoded signal x is optimized in (8), whereas the precoding matrix F is not considered.

III. PROPOSED SER MINIMIZATION CONSTRUCTIVE INTERFERENCE PRECODING

In this section, we first provide the expression of the SER function based on the analysis of the probability density functions (PDFs). With the optimization problem transformed into the real domain, we prove that the precoded signal shall be the linear combination of the channel vectors for SER minimization. To handle the complexity brought by the SER function, a novel algorithm is developed to design the precoded signal based on the feasible direction method. Particularly, we modify the feasible direction with an extra projection step to improve the efficiency.

A. SER FUNCTION

The correct symbol detection for the k-th user depends on whether the angle of received signal $\angle(y_k)$ falls within the decision region $(\varphi_k - \frac{\pi}{M}, \varphi_k + \frac{\pi}{M})$ with the effect of the Gaussian noise. To avoid the difference among symbols, the decision region can be equivalently rotated onto the positive real axis as follows

$$-\frac{\pi}{M} \le \angle\left(e^{-j\varphi_k} y_k\right) \le \frac{\pi}{M}. \tag{9}$$

Then, the SER of the k-th user can be given by

$$S(h_k x, s_k, \sigma_k^2, M) = 1 - \Pr(|\mathrm{Im}(\tilde{y}_k)| < \mathrm{Re}(\tilde{y}_k)\tan\frac{\pi}{M}), \tag{10}$$

where $\tilde{y}_k \triangleq e^{-j\varphi_k} y_k$. According to (1), the imaginary and real components of $\tilde{y}_k$ can be respectively given by

$$\begin{aligned} \mathrm{Im}(\tilde{y}_k) &= \mathrm{Im}(e^{-j\varphi_k} h_k x) + \mathrm{Im}(e^{-j\varphi_k} n_k), \\ \mathrm{Re}(\tilde{y}_k) &= \mathrm{Re}(e^{-j\varphi_k} h_k x) + \mathrm{Re}(e^{-j\varphi_k} n_k). \end{aligned} \tag{11}$$

Since $e^{-j\varphi_k} n_k$ is the only random variable in (11) which follows complex Gaussian distribution, $\mathrm{Im}(\tilde{y}_k)$ and $\mathrm{Re}(\tilde{y}_k)$ are independent real Gaussian random variables satisfying

$$\begin{aligned} \mathrm{Im}(\tilde{y}_k) &\sim \mathcal{N}(\mathrm{Im}(\tilde{h}_k x), \frac{\sigma_k^2}{2}), \\ \mathrm{Re}(\tilde{y}_k) &\sim \mathcal{N}(\mathrm{Re}(\tilde{h}_k x), \frac{\sigma_k^2}{2}), \end{aligned} \tag{12}$$

where $\tilde{h}_k \triangleq e^{-j\varphi_k} h_k$. Accordingly, the PDF of $|\mathrm{Im}(\tilde{y}_k)|$ and $-\mathrm{Re}(\tilde{y}_k)\tan\frac{\pi}{M}$ can be respectively given by

$$\begin{aligned} f_{\mathrm{I}}(l) &= \begin{cases} \dfrac{1}{\sqrt{2\pi}\sigma_{\mathrm{I}}}\left(e^{-\frac{(l-\mu_{\mathrm{I}})^2}{2\sigma_{\mathrm{I}}^2}} + e^{-\frac{(l+\mu_{\mathrm{I}})^2}{2\sigma_{\mathrm{I}}^2}}\right) & l \ge 0, \\ 0 & l < 0, \end{cases} \\ f_{\mathrm{R}}(r) &= \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{R}}} e^{-\frac{(r-\mu_{\mathrm{R}})^2}{2\sigma_{\mathrm{R}}^2}}, \end{aligned} \tag{13}$$

where $\mu_{\mathrm{I}} = \mathrm{Im}(\tilde{h}_k x)$, $\mu_{\mathrm{R}} = -\mathrm{Re}(\tilde{h}_k x)\tan\frac{\pi}{M}$, $\sigma_{\mathrm{I}} = \frac{\sigma_k}{\sqrt{2}}$, and $\sigma_{\mathrm{R}} = \frac{\sigma_k}{\sqrt{2}}\tan\frac{\pi}{M}$. Because of the independence between $\mathrm{Im}(\tilde{y}_k)$ and $\mathrm{Re}(\tilde{y}_k)$, the PDF of $G \triangleq |\mathrm{Im}(\tilde{y}_k)| - \mathrm{Re}(\tilde{y}_k)\tan\frac{\pi}{M}$ can be given by the convolution formula, i.e., $f_G(g) = \int_{-\infty}^{+\infty} f_{\mathrm{I}}(l) f_{\mathrm{R}}(g-l)\mathrm{d}l$. Then, the SER function can be expressed as

$$\begin{aligned} &S(h_k x, s_k, \sigma_k^2, M) \\ &= 1 - \Pr(G < 0) = \int_0^{+\infty} f_G(g)\mathrm{d}g \\ &= \int_0^{+\infty} \mathrm{d}g \int_{-\infty}^{+\infty} f_{\mathrm{I}}(l) f_{\mathrm{R}}(g-l)\mathrm{d}l \\ &= \int_{-\infty}^{+\infty} f_{\mathrm{I}}(l)\mathrm{d}l \int_0^{+\infty} f_{\mathrm{R}}(g-l)\mathrm{d}g \end{aligned}$$

$$= \int_{-\infty}^{+\infty} f_{\mathrm{I}}(l) \times \frac{1}{2}\mathrm{erfc}\left(\frac{-l-\mu_{\mathrm{R}}}{\sqrt{2}\sigma_{\mathrm{R}}}\right)\mathrm{d}l$$
$$= \int_{0}^{+\infty} \frac{1}{2\sqrt{2\pi}\sigma_{\mathrm{I}}}\left(e^{-\frac{(l-\mu_{\mathrm{I}})^2}{2\sigma_{\mathrm{I}}^2}} + e^{-\frac{(l+\mu_{\mathrm{I}})^2}{2\sigma_{\mathrm{I}}^2}}\right)\mathrm{erfc}\left(\frac{-l-\mu_{\mathrm{R}}}{\sqrt{2}\sigma_{\mathrm{R}}}\right)\mathrm{d}l, \tag{14}$$

where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{+\infty} e^{-y^2}\mathrm{d}y$. Since the noise power and modulation order are not affected by the precoding design, σ_{I} and σ_{R} can be regarded as constants in (14). And the SER of the k-th user can be determined by μ_{I} and μ_{R}, i.e., $S_k(\mu_{\mathrm{I}}, \mu_{\mathrm{R}}) \triangleq S(h_k x, s_k, \sigma_k^2, M)$. Based on (14), the constructive interference region for SER minimization is shown in Fig. 1. Compared with the region in [24], the proposed region provides a considerable degree of freedom for optimization. Compared with the region in [26], the proposed region shows a different trend especially around the positive real axis, which brings performance gains to the SER minimization based scheme.

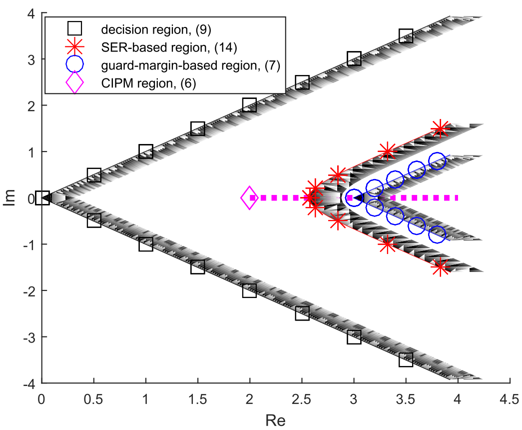

FIGURE 1. Boundaries of decision region and constructive interference regions in different schemes with QPSK, $\varepsilon = 10^{-2}$, $\gamma_k = 9$.

B. MODIFIED FEASIBLE DIRECTION

Since (8) is an optimization problem in complex domain, the gradient of $\mathrm{Im}(\cdot)$ and $\mathrm{Re}(\cdot)$ do not exist. For the convenience of analysing, we consider to equivalently transform (8) into the real domain. Define $\tilde{t} = [\mathrm{Re}(x); -\mathrm{Im}(x)] \in \mathbb{R}^{2N_t \times 1}$ and $t = [\tilde{t}; \varepsilon]$, (8) can be transformed as

$$\begin{aligned} &\min_t \boldsymbol{\beta} t \\ &\text{s.t.} \begin{cases} \|Ct\|_2^2 - P \le 0, \\ S_k(A_k t, -B_k t \tan\dfrac{\pi}{M}) - \boldsymbol{\beta} t \le 0, \forall k, \end{cases} \end{aligned} \tag{15}$$

where

$$\begin{aligned} \boldsymbol{\beta} &= [0_{1\times 2N_t}, 1], C = [I_{2N_t}, 0_{2N_t\times 1}], \\ A_k &= [\mathrm{Im}(\tilde{h}_k), -\mathrm{Re}(\tilde{h}_k), 0], \\ B_k &= [\mathrm{Re}(\tilde{h}_k), \mathrm{Im}(\tilde{h}_k), 0]. \end{aligned} \tag{16}$$

The Lagrangian associated with (15) is given by

$$\begin{aligned} L(t, \gamma, \boldsymbol{\alpha}) &= \boldsymbol{\beta} t + \gamma(\|Ct\|_2^2 - P) \\ &\quad + \sum_{k=1}^{K} \alpha_k (S_k(A_k t, -B_k t \tan\frac{\pi}{M}) - \boldsymbol{\beta} t), \end{aligned} \tag{17}$$

where $\gamma \in \mathbb{R}$ and $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_K] \in \mathbb{R}^{1\times K}$ are Lagrange multipliers associated with the power and constructive interference constraints, respectively. Setting $\partial L(t, \gamma, \boldsymbol{\alpha})/\partial t = 0$, the optimal t can be obtained by

$$\left\{\begin{array}{c} (1 - \sum_{k=1}^{K} \alpha_k)\boldsymbol{\beta}^T + 2\gamma C^T C t + \\ \sum_{k=1}^{K} \alpha_k \left[A_k^T, -B_k^T \tan\dfrac{\pi}{M}\right] \begin{bmatrix} \partial S_k/\partial\mu_{\mathrm{I}} \\ \partial S_k/\partial\mu_{\mathrm{R}} \end{bmatrix} \end{array}\right\} = 0. \tag{18}$$

According to (18), we have the following lemma

Lemma 1: The optimal constructive interference precoded signal x for SER minimization is the linear combination of channel vectors h_k^H.

Proof: Focus on the first $2N_t$ elements of the vectors in (18), we have the following equation

$$\left\{\begin{array}{c} \begin{bmatrix} \mathrm{Re}(x) \\ -\mathrm{Im}(x) \end{bmatrix} + \sum_{k=1}^{K} p_k \begin{bmatrix} \mathrm{Im}(\tilde{h}_k)^T \\ -\mathrm{Re}(\tilde{h}_k)^T \end{bmatrix} \\ + \sum_{k=1}^{K} q_k \begin{bmatrix} \mathrm{Re}(\tilde{h}_k)^T \\ \mathrm{Im}(\tilde{h}_k)^T \end{bmatrix} \end{array}\right\} = 0_{2N_t\times 1}, \tag{19}$$

where $p_k = \frac{\alpha_k}{2\gamma}\frac{\partial S_k}{\partial \mu_{\mathrm{I}}}$ and $q_k = -\frac{\alpha_k}{2\gamma}\frac{\partial S_k}{\partial \mu_{\mathrm{R}}}\tan\frac{\pi}{M}$ are both scalars. Multiply $[I_{N_t}, -jI_{N_t}]$ at both sides of (19), we have

$$x + \sum_{k=1}^{K} j p_k e^{j\varphi_k} h_k^H + \sum_{k=1}^{K} q_k e^{j\varphi_k} h_k^H = 0_{N_t\times 1}. \tag{20}$$

Hence, the optimal x is the linear combination of h_k^H. □

Although Lemma 1 provides a significant guidance for precoding design, it is still extremely difficult to solve the optimization problem (15) based on (18). This is mainly because $\partial S_k/\partial\mu_{\mathrm{I}}$ and $\partial S_k/\partial\mu_{\mathrm{R}}$ are associated with t. The optimal t cannot be expressed by Lagrange multipliers in a closed form. Hence, the Lagrangian method, which has been widely adopted in constructive interference precoding design [24]–[27], [31], is no longer applicable. To handle the complexity brought by the SER function, we propose to seek for a near optimal solution based on the thought of the feasible direction method.

The most significant issue in the proposed scheme is to develop an appropriate descent direction. For the optimization problem (15), a classical approach to obtain a feasible descent direction d is to solve the following linear programming problem [33], [34]

$$\begin{aligned} &\min_{d,z} z \\ &\text{s.t.} \begin{cases} \boldsymbol{\beta} d \le z, \\ 2t^T C^T C d \le z, \text{if } \|Ct\|_2^2 = P, \\ \left[A_k \dfrac{\partial S_k}{\partial \mu_{\mathrm{I}}} - B_k \dfrac{\partial S_k}{\partial \mu_{\mathrm{R}}} \tan\dfrac{\pi}{M} - \boldsymbol{\beta}\right] d \le z, k \in \psi, \\ -1 \le d(n) \le 1, n = 1, \ldots, 2N_t + 1, \end{cases} \end{aligned} \tag{21}$$

where ψ is the index set of active constrains. The first constrain ensures a descent direction for (15), whereas the second and third constrains ensure the feasibility. If the optimal solution satisfies $z < 0$, the corresponding d is the feasible descent direction. Nevertheless, only the feasibility is considered in (21), whereas the efficiency of the direction is ignored. According to (19) in the proof of Lemma 1, an efficient direction for $\tilde{t}$ shall lie in the column space of

$$D_{\mathrm{s}} = \begin{bmatrix} \mathrm{Re}(\tilde{H})^T & \mathrm{Im}(\tilde{H})^T \\ \mathrm{Im}(\tilde{H})^T & -\mathrm{Re}(\tilde{H})^T \end{bmatrix}, \tag{22}$$

where $\tilde{H} \triangleq \mathrm{diag}(e^{-j\varphi_1}, \ldots, e^{-j\varphi_K})H$. Denoting $d \triangleq [d_{\tilde{t}}; d_\varepsilon]$ where $d_{\tilde{t}}$ contains the first $2N_{\mathrm{t}}$ elements of d, we propose to modify the feasible descent direction solved by (21) based on the projection as follows

$$d_{\tilde{t},\perp} = D_{\mathrm{s}} D_{\mathrm{s}}^{\dagger} d_{\tilde{t}}. \tag{23}$$

And we have the following proposition to ensure the feasibility of the modified direction.

Proposition 1: if $\tilde{t}$ lies in the column space of D_s, the modified direction $d^\star \triangleq [d_{\tilde{t},\perp}; d_\varepsilon]$ obtained by (23) maintains feasibility and descent for (15), where $d \triangleq [d_{\tilde{t}}; d_\varepsilon]$ and z are solved by (21) satisfying $z < 0$.

Proof: Please refer to Appendix. □

The remain issue in solving (21) is to determine the gradient value $\frac{\partial S_k}{\partial \mu_{\mathrm{I}}}$ and $\frac{\partial S_k}{\partial \mu_{\mathrm{R}}}$ at a certain point t. Fortunately, the gradient value can be obtained via off-line calculation (or estimation), whereas the detail expressions of $\frac{\partial S_k}{\partial \mu_{\mathrm{I}}}$ and $\frac{\partial S_k}{\partial \mu_{\mathrm{R}}}$ are no longer required. Thus, the proposed scheme handles the complexity brought by the SER function, and is applicable for SER minimization.

C. PROPOSED SER MINIMIZATION BASED CONSTRUCTIVE INTERFERENCE PRECODING ALGORITHM

Based on the thought of the feasible direction method, the proposed SER minimization based constructive interference precoding scheme is summarized in Algorithm 1. Since $\tilde{t}$ is initialized within the column space of D_{s} in step 1, the modified direction $d^{(n)\star}$ in step 9 maintains feasibility and descent according to Proposition 1. Precisely owing to the modification, the prerequisite of Proposition 1 can always be satisfied, which pushes the iteration forward. In step 4, the set of active constrains $\psi^{(n)}$ is determined. The k-th SER constrain is determined as an active constrain when the SER of the k-th user is close to the maximum SER $\varepsilon^{(n)}$ within a certain gap $\rho\varepsilon^{(n)}$. The gap changes correspondingly with revised $\varepsilon^{(n)}$ in each iteration. Especially, if the step size is smaller than the threshold $\Delta\xi$, a narrowed gap will be considered in further iterations with a halved threshold parameter ρ. The definition of the active constrain aims to set a "cordon" before the actual boundary of the constructive interference region. If current $t^{(n)}$ gets close to the "cordon", it will be allowed to move into the region with an appropriate direction. This is helpful to prevent the case that $t^{(n)}$ tends to move from a boundary to another one, which might cause a slight move

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Algorithm 1 Proposed SER Minimization Based Constructive Interference Precoding Algorithm

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Require: Channel matrix H, transmit symbol vector s, noise power σ_k^2, modulation order M, required transmit power P.

Ensure: Precoded signal x.

1: Initialize $\tilde{t}^{(0)} = D_{\mathrm{s}}\boldsymbol{\tau}$ with random $\boldsymbol{\tau} \in \mathbb{R}^{2K\times 1}$, which satisfies $\|x^{(0)}\|_2^2 \le P$.
2: Obtain $\varepsilon^{(0)} = \max(S(h_k x^{(0)}, s_k, \sigma_k^2, M))$ and $t^{(0)} = [\tilde{t}^{(0)}; \varepsilon^{(0)}]$, set $n = 0$.
3: **repeat**
4:    Obtain the index set of active constrains by $\psi^{(n)} = \{k | 0 \le \varepsilon^{(n)} - S(h_k x^{(n)}, s_k, \sigma_k^2, M) \le \rho\varepsilon^{(n)}\}$, where $\rho \in [0, 1)$.
5:    Obtain $d^{(n)} = [d_{\tilde{t}}; d_\varepsilon]$ and $z^{(n)}$ by solving the linear programming problem (21).
6:    **if** $z^{(n)} = 0$ **then**
7:       **break**.
8:    **end if**
9:    Modify the direction by $d^{(n)\star} \triangleq [D_{\mathrm{s}} D_{\mathrm{s}}^{\dagger} d_{\tilde{t}}; d_\varepsilon]$.
10:    Choose the step size ξ via linear search, update $t^{(n+1)} = t^{(n)} + \xi d^{(n)\star}$.
11:    Revise the last term of $t^{(n+1)}$ as the largest element in $\{\varepsilon_k | \varepsilon_k = S(h_k x^{(n+1)}, s_k, \sigma_k^2, M), k = 1, 2, \ldots, K\}$.
12:    **if** $\xi < \Delta\xi$ **then**
13:       $\rho = \rho/2$.
14:    **end if**
15:    $n = n + 1$.
16: **until** $\xi < \Delta\xi$ and $\rho < \Delta\rho$
17: **return** $x = [I_{N_{\mathrm{t}}}, -jI_{N_{\mathrm{t}}}, 0]t^{(n)}$.

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and lead to slow convergence. The iteration from step 3 to step 16 is stopped when $t^{(n)}$ is extremely close to the boundary of the proposed constructive interference region while only a slight move is acceptable.

The computational complexity of Algorithm 1 is mainly caused by solving the linear programming problem (21) in step 5, where $2N_{\mathrm{t}} + 1$ objective variables are involved. Considering the enlarged transmit array scale, the Karmarkar algorithm takes polynomial time to solve the linear programming problem [35]. Assuming that the number of iterations from step 3 to step 16 is N_{iter}, the overall complexity of Algorithm 1 can be given by $O(N_{\mathrm{iter}}(2N_{\mathrm{t}} + 1)^{3.5})$.

IV. EXTENSION OF THE PROPOSED PRECODING SCHEME

In the previous sections, we have developed the scheme for SER minimization with the perfect CSI. The proposed scheme can be extended for transmit power optimization. In addition, imperfect CSI can be handled with appropriate adjustment. The following subsections discuss the details of the extension.

A. TRANSMIT POWER OPTIMIZATION

In this subsection, we extend the proposed scheme to seek for the minimal transmit power with guaranteed SER performance, i.e.,

$$\min_{x} \|x\|_2^2$$
$$\text{s.t. } S(h_k x, s_k, \sigma_k^2, M) \leq \varepsilon_k, \forall k. \quad (24)$$

As analyzed in [24], the minimum power solution can be regarded as a scaled version of the minimized maximum SER solution. And the bisection method can be adopted to provide the solution. However, in the bisection iteration, Algorithm 1 shall be repeated several times, which leads to heavy computational complexity. Actually, the proposed SER minimization scheme can be extended for power optimization without extra computational complexity.

Firstly, (24) can be equivalently transformed into the real domain as

$$\min_{\tilde{t}} \|\tilde{t}\|_2^2$$
$$\text{s.t. } S_k(\tilde{A}_k\tilde{t}, -\tilde{B}_k\tilde{t}\tan\frac{\pi}{M}) - \varepsilon_k \leq 0, \quad \forall k, \quad (25)$$

where $\tilde{A}_k = [\text{Im}(\tilde{h}_k), -\text{Re}(\tilde{h}_k)]$, $\tilde{B}_k = [\text{Re}(\tilde{h}_k), \text{Im}(\tilde{h}_k)]$. The Lagrangian associated with (25) is given by

$$\tilde{L}(\tilde{t}, \tilde{\boldsymbol{\alpha}}) = \|\tilde{t}\|_2^2 + \sum_{k=1}^{K} \tilde{\alpha}_k (S_k(\tilde{A}_k\tilde{t}, -\tilde{B}_k\tilde{t}\tan\frac{\pi}{M}) - \varepsilon_k), \quad (26)$$

where $\tilde{\boldsymbol{\alpha}} = [\tilde{\alpha}_1, \ldots, \tilde{\alpha}_K] \in \mathbb{R}^{1\times K}$ contains Lagrange multipliers. Note that (17) and (26) are in a similar form, Lemma 1 can be extended as follows

Lemma 2: The optimal constructive interference precoded signal x for transmit power optimization is the linear combination of channel vectors h_k^H.

Proof: Setting $\partial\tilde{L}(\tilde{t}, \tilde{\boldsymbol{\alpha}})/\partial\tilde{t} = 0$, the optimal $\tilde{t}$ can be obtained by

$$\tilde{t} = \sum_{k=1}^{K} \frac{\tilde{\alpha}_k}{2}\left[-\tilde{A}_k^T, \tilde{B}_k^T\tan\frac{\pi}{M}\right]\begin{bmatrix}\partial S_k/\partial\mu_{\text{I}} \\ \partial S_k/\partial\mu_{\text{R}}\end{bmatrix}. \quad (27)$$

Multiply $[I_{N_t}, -jI_{N_t}]$ at both sides of (27), we have

$$x = \sum_{k=1}^{K} \frac{\tilde{\alpha}_k e^{j\varphi_k}}{2}(-j\frac{\partial S_k}{\partial\mu_{\text{I}}} + \frac{\partial S_k}{\partial\mu_{\text{R}}}\tan\frac{\pi}{M})h_k^H. \quad (28)$$

Hence, the optimal x is the linear combination of h_k^H. □

To extend the proposed scheme, the key part is to revise the feasible descent direction. Specifically, the feasible descent direction $d_{\tilde{t}}$ for (25) can be obtained based on the following linear programming problem

$$\min_{d_{\tilde{t}}, z} z$$
$$\text{s.t.} \begin{cases} 2\tilde{t}^T d_{\tilde{t}} \leq z, \\ \left[\tilde{A}_k\dfrac{\partial S_k}{\partial\mu_{\text{I}}} - \tilde{B}_k\dfrac{\partial S_k}{\partial\mu_{\text{R}}}\tan\frac{\pi}{M}\right]d_{\tilde{t}} \leq z, k \in \tilde{\psi}, \\ -1 \leq d_{\tilde{t}}(n) \leq 1, n = 1, \ldots, 2N_t, \end{cases} \quad (29)$$

where $\tilde{\psi}$ is the index set of active constrains. According to Lemma 2, the projection $d_{\tilde{t},\perp} = D_s D_s^{\dagger} d_{\tilde{t}}$ is also necessary to enhance the efficiency of the direction. And it is worth mentioning that $d_{\tilde{t},\perp}$ is still a feasible descent direction for (25) according to the following proposition.

Proposition 2: if $\tilde{t}$ lies in the column space of D_s, the modified direction $d_{\tilde{t},\perp} = D_s D_s^{\dagger} d_{\tilde{t}}$ maintains feasibility and descent for (25)*, where $d_{\tilde{t}}$ and z are solved by* (29) *satisfying* $z < 0$.

Proof: Please refer to Appendix. □

According to the analysis above, Algorithm 1 can be extended for power optimization with the revised feasible direction. And none extra computational task is involved.

B. IMPERFECT CSI

Considering imperfect CSI, the actual channel vector can be given by

$$\hat{h}_k = h_k + \Delta h_k, \quad (30)$$

where h_k denotes the estimated CSI, and Δh_k denotes the estimation error. And the overall estimation error matrix $\Delta H = [\Delta h_1^H, \ldots, \Delta h_K^H]^H$ is assumed to be Gaussian-distributed with $\mathbb{E}[\Delta H \Delta H^H] = \sigma_H^2 I_K$. With imperfect CSI, the imaginary and real components of $\tilde{y}_k$ in (11) can be rewritten as

$$\begin{aligned}\text{Im}(\tilde{y}_k) &= \text{Im}(e^{-j\varphi_k}h_k x) + \text{Im}(e^{-j\varphi_k}n_k) \\ &\quad + \text{Im}(e^{-j\varphi_k}\Delta h_k x), \\ \text{Re}(\tilde{y}_k) &= \text{Re}(e^{-j\varphi_k}h_k x) + \text{Re}(e^{-j\varphi_k}n_k) \\ &\quad + \text{Re}(e^{-j\varphi_k}\Delta h_k x).\end{aligned} \quad (31)$$

Since $\mathbb{E}[x^H \Delta h_k^H \Delta h_k x] = \|x\|_2^2\sigma_H^2/N_t$, we have

$$\text{Im}(e^{-j\varphi_k}\Delta h_k x), \text{Re}(e^{-j\varphi_k}\Delta h_k x) \sim \mathcal{N}(0, \frac{\|x\|_2^2\sigma_H^2}{2N_t}). \quad (32)$$

Because of the independence between noise and estimation error, $\text{Im}(\tilde{y}_k)$ and $\text{Re}(\tilde{y}_k)$ are independent Gaussian random variables satisfying

$$\begin{aligned}\text{Im}(\tilde{y}_k) &\sim \mathcal{N}(\text{Im}(\tilde{h}_k x), \frac{\sigma_k^2}{2} + \frac{\|x\|_2^2\sigma_H^2}{2N_t}), \\ \text{Re}(\tilde{y}_k) &\sim \mathcal{N}(\text{Re}(\tilde{h}_k x), \frac{\sigma_k^2}{2} + \frac{\|x\|_2^2\sigma_H^2}{2N_t}).\end{aligned} \quad (33)$$

Hence, the SER function with imperfect CSI can be expressed as

$$\begin{aligned}&\hat{S}(h_k x, s_k, \sigma_k^2, M, \sigma_H^2) \\ &= \int_0^{+\infty} \frac{1}{2\sqrt{2\pi}\sigma_{\text{I}}}\left(e^{-\frac{(l-\mu_{\text{I}})^2}{2\hat{\sigma}_{\text{I}}^2}} + e^{-\frac{(l+\mu_{\text{I}})^2}{2\hat{\sigma}_{\text{I}}^2}}\right) \\ &\quad \times \text{erfc}\left(\frac{-l-\mu_{\text{R}}}{\sqrt{2}\hat{\sigma}_{\text{R}}}\right)\text{d}l,\end{aligned} \quad (34)$$

where $\hat{\sigma}_{\text{I}}^2 = \frac{\sigma_k^2}{2} + \frac{\|x\|_2^2\sigma_H^2}{2N_t}$ and $\hat{\sigma}_{\text{R}} = \hat{\sigma}_{\text{I}}\tan\frac{\pi}{M}$. Compared with (14), $\hat{\sigma}_{\text{I}}$ and $\hat{\sigma}_{\text{R}}$ are no longer constants but change with the transmit power, whereas μ_{I} and μ_{R} are determined by estimated CSI. Assume that σ_H remains constant within a symbol period, the SER of the k-th user can be determined by μ_{I}, μ_{R}, and $P_x = \|x\|_2^2$, i.e., $\hat{S}_k(\mu_{\text{I}}, \mu_{\text{R}}, P_x) \triangleq \hat{S}(h_k x, s_k, \sigma_k^2, M, \sigma_H^2)$. According to the novel SER function

(34), the optimal constructive precoded signal can be given based on the following lemma.

Lemma 3: In the case of imperfect CSI with Gaussian-distributed estimation error, the optimal constructive interference precoded signal x for SER minimization is the linear combination of the estimated channel vectors h_k^H.

Proof: Based on the novel SER function (34), the Lagrangian associated with (15) can be rewritten as

$$\hat{L}(t,\hat{\gamma},\hat{\boldsymbol{\alpha}}) = \boldsymbol{\beta}t + \hat{\gamma}(\|Ct\|_2^2 - P) + \sum_{k=1}^{K}\hat{\alpha}_k(\hat{S}_k(A_k t, -B_k t\tan\frac{\pi}{M}, \|Ct\|_2^2) - \boldsymbol{\beta}t), \quad (35)$$

where $\hat{\gamma} \in \mathbb{R}$ and $\hat{\boldsymbol{\alpha}} = [\hat{\alpha}_1, \ldots, \hat{\alpha}_K] \in \mathbb{R}^{1\times K}$ are Lagrange multipliers. Setting $\partial\hat{L}(t,\hat{\gamma},\hat{\boldsymbol{\alpha}})/\partial t = 0_{(2N_t+1)\times 1}$, the optimal t can be obtained by

$$0 = (1 - \sum_{k=1}^{K}\hat{\alpha}_k)\boldsymbol{\beta}^T + 2\hat{\gamma}C^T Ct + \sum_{k=1}^{K}\hat{\alpha}_k(A_k^T\frac{\partial\hat{S}_k}{\partial\mu_I} - B_k^T\frac{\partial\hat{S}_k}{\partial\mu_R}\tan\frac{\pi}{M} + 2C^T Ct\frac{\partial\hat{S}_k}{\partial P_x}). \quad (36)$$

Focus on the first $2N_t$ elements of the vectors in (36), we have the following equation

$$\left\{\begin{bmatrix}\mathrm{Re}(x)\\ -\mathrm{Im}(x)\end{bmatrix} + \sum_{k=1}^{K}\hat{p}_k\begin{bmatrix}\mathrm{Im}(\tilde{h}_k)^T\\ -\mathrm{Re}(\tilde{h}_k)^T\end{bmatrix} + \sum_{k=1}^{K}\hat{q}_k\begin{bmatrix}\mathrm{Re}(\tilde{h}_k)^T\\ \mathrm{Im}(\tilde{h}_k)^T\end{bmatrix}\right\} = 0_{2N_t\times 1}, \quad (37)$$

where

$$\hat{p}_k = \frac{\hat{\alpha}_k(\partial\hat{S}_k/\partial\mu_I)}{2\hat{\gamma} + \sum_{k=1}^{K}2\hat{\alpha}_k(\partial\hat{S}_k/\partial P_x)},$$
$$\hat{q}_k = \frac{-\hat{\alpha}_k\tan(\frac{\pi}{M})(\partial\hat{S}_k/\partial\mu_R)}{2\hat{\gamma} + \sum_{k=1}^{K}2\hat{\alpha}_k(\partial\hat{S}_k/\partial P_x)}, \quad (38)$$

are both scalars. Multiply $[I_{N_t}, -jI_{N_t}]$ at both sides of (38), we have

$$x + \sum_{k=1}^{K}j\hat{p}_k e^{j\varphi_k}h_k^H + \sum_{k=1}^{K}\hat{q}_k e^{j\varphi_k}h_k^H = 0_{N_t\times 1}. \quad (39)$$

Hence, the optimal x is the linear combination of the estimated channel vectors h_k^H. □

Despite the SER function gets more complex for imperfect CSI cases, the proposed scheme is still feasible with the adjustment of the feasible descent direction. Specifically, the third constrain in (21) shall be rewritten as

$$\left[A_k\frac{\partial\hat{S}_k}{\partial\mu_I} - B_k\frac{\partial\hat{S}_k}{\partial\mu_R}\tan\frac{\pi}{M} + \frac{\partial\hat{S}_k}{\partial P_x}2t^T C^T C - \boldsymbol{\beta}\right]d \le z. \quad (40)$$

According to Lemma 3, the projection step (23) is also necessary to enhance the efficiency of the direction. And it is worth mentioning that $d_{\tilde{t},\perp}$ is still a feasible descent direction for (15) with imperfect CSI according to the following proposition.

Proposition 3: if $\tilde{t}$ lies in the column space of D_s obtained by the estimated channel matrix, the modified direction $d^\star \triangleq [d_{\tilde{t},\perp}; d_\varepsilon]$ obtained by (23) *maintains feasibility and descent for* (15) *under imperfect CSI cases, where $d \triangleq [d_{\tilde{t}}; d_\varepsilon]$ and $z < 0$ are solved by* (21)*, the third constrain of which is rewritten as* (40).

Proof: Please refer to Appendix. □

According to the analysis above, imperfect CSI can be handled with revised feasible direction. The major difference is that the SER function for imperfect CSI contains one more variable, i.e., the transmit power. Nevertheless, the gradient value can still be obtained off-line, which means the proposed schemes for perfect and imperfect CSI have the similar computational complexity.

Remark: Lemma 1 and 2 imply that x shall lie in the column space of H^H. Exactly, if x contains the component which lies in the null space of H, this component will make no sense for adjusting the received symbols, which is simply a waste of the transmit power. For imperfect CSI cases, the estimation error component of complete CSI is transformed as an independent Gaussian-distributed random variable by the linear combination of x. Hence, only the estimated CSI is involved to form x as illustrated in Lemma 3.

V. SIMULATION RESULTS

In this section, numerical results based on Monte Carlo simulations are presented to evaluate the performance of the proposed constructive interference precoding scheme. In the default simulation system, the BS is equipped with $N_t = 4\times 4$ antenna array to serve $K = 12$ users with quadrature phase shift keying (QPSK) modulation. The transmit power is set as $P = K$. Without loss of generality, the noise power of each user is assumed as $\sigma^2 = \sigma_k^2, \forall k$. In the following simulations, SNR is defined by the ratio of the transmit power to the noise power, i.e., SNR $= P/\sigma^2$. The multi-antenna channel between each pair of transceivers consists of $L_k = 3$ scattering clusters, each of which contains $L_{c,k} = 5$ propagation paths. The azimuth and elevation angles of departure are uniformly distributed in $[0, 2\pi)$ with a 10-degree angular spread. All simulation results are calculated over 100000 channel realizations.

A. PERFECT CSI

In this subsection, we investigate the performance of precoding schemes in terms of SER and transmit power with perfect CSI under several cases of system parameters.

Fig. 2 provides the comparison between the proposed SER minimization scheme and the existing constructive interference precoding schemes in terms of the maximum SER among users. The CR scheme in [22], the constructive interference maximized minimum SINR (CIMM) scheme in [24], and the MMSE scheme in [26] are presented as benchmarks.

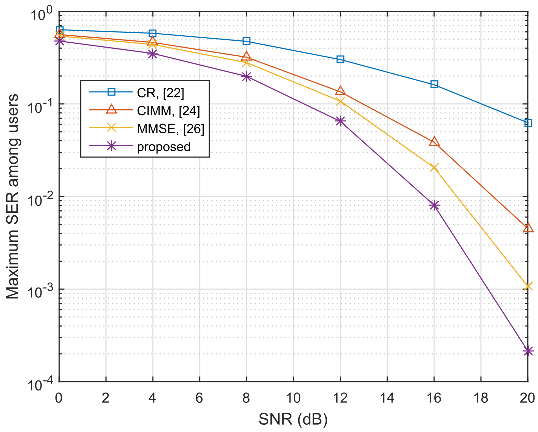

FIGURE 2. The maximum SER performance among $K = 12$ users with different constructive interference precoding schemes under perfect CSI with QPSK, $M = 4$.

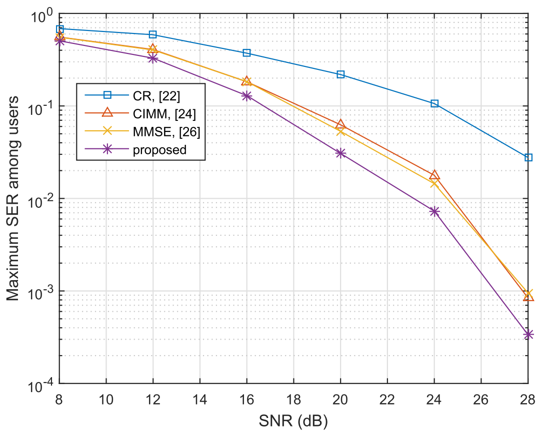

FIGURE 3. The maximum SER performance among $K = 12$ users with different constructive interference precoding schemes under perfect CSI with 8-PSK, $M = 8$.

Since each inter-user interference is restricted to provide the same phase as the intended symbol in the CR scheme, the strict constrain leads to the worst SER performance. Owing to the macroscopic consideration of the resultant interference, the other schemes contribute to lower SER. Since the MMSE scheme considers an expanded constructive interference region with a guard margin for detection, it outperforms the CIMM scheme with an SNR gain of 2 dB at the level of SER $= 10^{-2}$, which recovers the results in [26]. The proposed scheme provides the best SER performance, since the constructive interference region is directly determined by the SER function. Compared with the MMSE scheme, an SNR gain of 1.8 dB can be obtained at the level of SER $= 10^{-2}$. And a larger SNR gain can be obtained at a lower SER level. Fig. 3 further provides the SER performance with 8-PSK modulation. Lower performance gaps can be observed among different schemes, since the high modulation order makes different constructive interference regions tend to be the same. Nevertheless, the proposed scheme still provides a significant performance gain compared with the existing schemes.

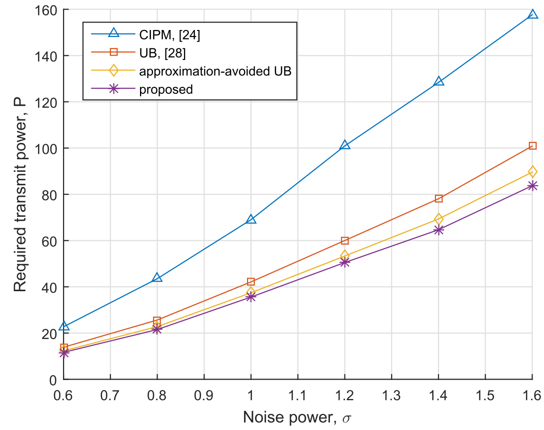

FIGURE 4. The required transmit power versus the noise power to achieve $\varepsilon_k = 10^{-2}, \forall k$ with $N_t = 4 \times 4$.

Fig. 4 shows the required transmit power versus the noise power to achieve $\varepsilon_k = 10^{-2}, \forall k$ in (24). The CIPM scheme in [24] and the UB scheme in [28] are presented as benchmarks. Owing to the expanded constructive interference region, the UB scheme requires lower transmit power than the CIPM scheme. In [28], the upper bound of SER is adopted to generate the parameter γ_k of the guard-margin-based region (7). To avoid the approximation, we further provide the power performance with the parameter determined by the exact SER expression (14). With the same shape of the constructive interference region, the approximation-avoided scheme outperforms the original scheme in [28]. The proposed scheme achieves the best performance with the novel shape of the SER-defined region. Since the SER-defined region shows a similar trend as the guard-margin-based region around the area away from the real axis, the similarity leads to the slight performance gain between the proposed and UB schemes. Practically, the similarity results from that SER is mainly determined by an individual component of the Gaussian noise when the received constellation point gets away from the real axis. Fig. 5 provides the power performance with $N_t = 4 \times 8$

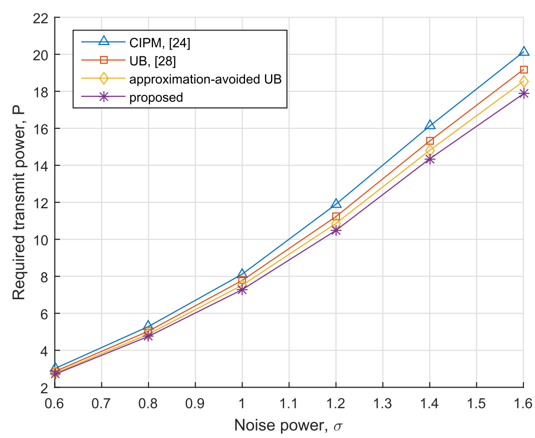

FIGURE 5. The required transmit power versus the noise power to achieve $\varepsilon_k = 10^{-2}, \forall k$ with $N_t = 4 \times 8$.

antenna array. Owing to the enhanced orthogonality among channels brought by the enlarged antenna scale, much lower transmit power is required in Fig. 5, and the performance gap becomes even smaller. Nevertheless, the proposed scheme still reduces transmit power by 12% compared with the CIPM scheme and by 7% compared with the UB scheme. Hence, the proposed scheme is suitable for large-scale antenna arrays in 5G and future communication systems.

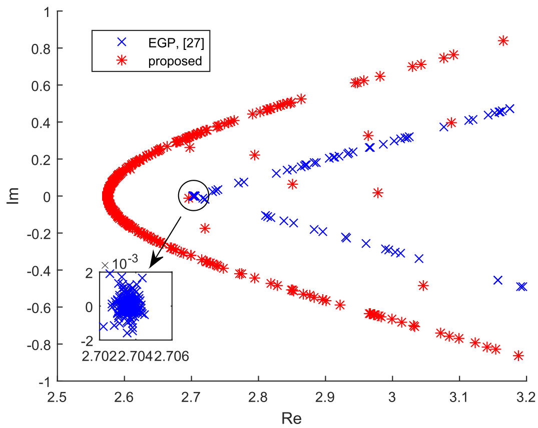

FIGURE 6. The distribution of the received signals without noise to achieve $\varepsilon_k = 10^{-2}$, $\forall k$ with different constructive interference precoding schemes under $\sigma^2 = 1$.

Fig. 6 depicts the distribution of the received signals without noise on the modulation constellation to achieve $\varepsilon_k = 10^{-2}$, $\forall k$ in (24) at $\sigma^2 = 1$. The comparison of the proposed scheme with the EGP scheme in [27] is presented. With the aim of minimizing the transmit power, the received signals tend to get close to the boundary brought by the SER requirement, which reflects Fig. 1. Since the slope of the boundary of the guard-margin-based region contains a discrete point at the real axis, abundant received signals are gathered at the discrete point with the EGP scheme. Hence, the performance of the EGP scheme is mainly dependent upon the definition of γ_k in (7), which is consistent with the scheme in [24]. With a higher modulation order, the angle between the upper and lower boundaries becomes further smaller, which results in more conspicuous aggregation of received signals. Hence, the performance of different schemes tends to be the same, which reflects the results in Fig. 3. Owing to the improved definition of the constructive interference region, the proposed scheme better center the received signals into the required SER region with a more moderate trend around the real axis compared with the EGP scheme. Mathematically, the feasible region for optimization is fully utilized by the proposed scheme to make the received signals close to the origin of coordinates, which contributes to lower power consumption.

Fig. 7 shows the average SER performance of $K = 12$ users with different constructive interference precoding schemes. Although the proposed scheme mainly focuses on minimizing the maximum SER among users, it still achieves the best average SER performance. Compared with the curves

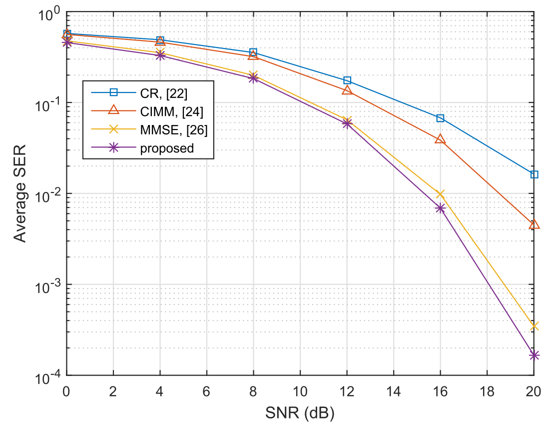

FIGURE 7. The average SER performance of $K = 12$ users with different constructive interference precoding schemes with QPSK, $M = 4$.

in Fig. 2, the CR scheme in [22] and the MMSE scheme in [26] provide significant improvements in terms of average SER, since the fairness among users is broken with the definition of the constructive interference region. The proposed SER-defined region enlarges available search domains for optimization while ensuring the fairness, which contributes to stable and the best SER performance for each user.

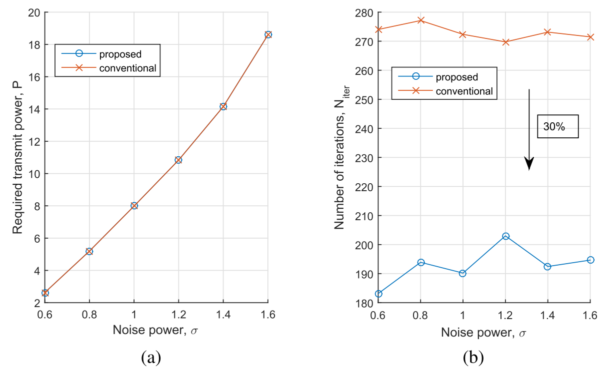

FIGURE 8. The comparison between the proposed and conventional feasible directions in terms of transmit power (a) and the number of iterations (b) to achieve $\Delta\xi = 10^{-5}$ and $\Delta\rho = 10^{-10}$.

To show the advance of the proposed feasible direction modified by an extra projection step, Fig. 8 presents the comparison between the proposed scheme and the conventional feasible direction method in terms of the transmit power performance and the number of iterations. The feasible direction given by the conventional method is directly obtained by (29). As shown in Fig. 8(a), the proposed scheme and the conventional scheme provide similar transmit power performance. In Fig. 8(b), however, the proposed scheme requires 30% fewer iterations for convergence than the conventional feasible direction method. It demonstrates that the proposed projection step provides a more effective feasible direction, and contributes to less convergence time than the conventional scheme.

Fig. 9 shows the SER performance with different numbers of users involved in the system. The proposed scheme and

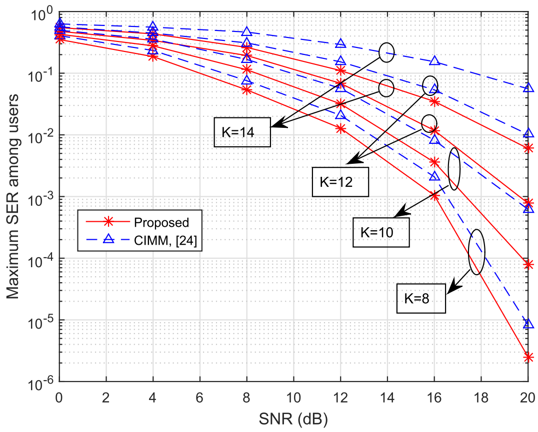

FIGURE 9. The maximum SER performance among users versus SNR with different numbers of users involved in the system, $K = 8, 10, 12, 14$.

the CIMM scheme are presented for comparison. For both schemes, a conspicuous upward trend of SER can be observed with an increasing number of users. It implies the BS prefers serving fewer users simultaneously which refers to the femtocell network. The proposed scheme shows a slight advantage in the 8-user systems compared with the CIMM scheme. Nevertheless, the performance gap dramatically increases when more users are involved. Accordingly, the proposed scheme is more effective for handling the multi-user interference.

B. IMPERFECT CSI

Fig. 10 provides the comparison between the proposed scheme and the existing schemes under imperfect CSI with $\sigma_H^2 = 0.2$ and $K = 8$. The CR scheme in [22] can hardly achieve the strict requirement for each inter-user interference with imperfect CSI, which leads to the worst performance. And a dramatic performance gap can be observed between the CR scheme and the schemes which consider the resultant interference. At the level of SER $= 10^{-3}$, the proposed scheme achieves an SNR gain of 1 dB compared with the CIMM scheme in [24], and an SNR gain of 0.5 dB compared with the MMSE scheme in [26]. With the consideration of SER affected by the CSI estimation error, the transmit power becomes one of the parameters for the definition of the constructive interference region in the proposed scheme and the safe approximation (SA) scheme in [32]. Both schemes outperform other existing schemes [22], [24], [26] in dealing with the issue of robust symbol level precoding. Furthermore, since none approximation is involved in the proposed scheme, a slight SNR gain of 0.25 dB can be obtained by the proposed scheme compared with the SA scheme. Comparing slopes of the curves in Fig. 2 and Fig. 10, all considered precoding schemes in Fig. 10 suffer slower declines of SER because of imperfect CSI.

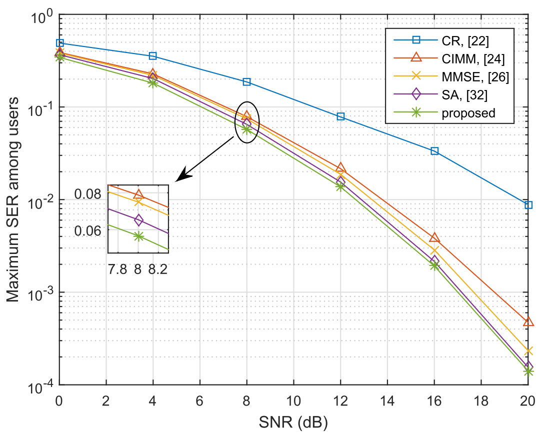

FIGURE 10. The maximum SER performance among $K = 8$ users with different constructive interference precoding schemes under imperfect CSI with $\sigma_H^2 = 0.2$.

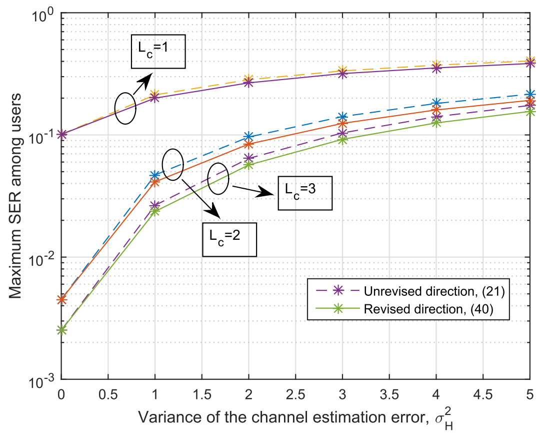

FIGURE 11. The maximum SER performance among $K = 8$ users versus the variance of the channel estimation error with different feasible directions under $L_c = 1, 2, 3$.

Fig. 11 shows the SER performance versus the variance of the estimation error with $K = 8$ and SNR $= 15$ dB. The comparison between the feasible direction given by (21) and its revised version based on (40) is presented to illustrate the effectiveness of the proposed scheme for imperfect CSI. A larger performance gain can be obtained by the revised feasible direction with increasing σ_H^2. Moreover, we investigate the impact of the multi-path channel. With constant transmit power, the maximum SER among users decreases with increasing L_c, since more spatial diversity can be achieved by more paths in multi-user systems. The performance gap brought by the multi-path channel gets slight with increasing L_c, which implies that inter-user interference can be naturally separated when L_c is large enough.

VI. CONCLUSION

In this paper, we have proposed an SER minimization based constructive interference precoding scheme for multi-user systems. With the full knowledge of CSI and symbol information, the constructive interference region is directly defined based on the analysis of the SER expression. To handle the complex expression of SER, we have developed a modified feasible direction algorithm, where an extra projection step is proposed to enhance the efficiency of the feasible direction.

Furthermore, the proposed scheme has been extended for transmit power optimization and imperfect CSI cases.

Owing to the direct consideration of the SER after detection, the proposed scheme is essentially a symbol pre-detection scheme, which makes a major breakthrough compared with existing pre-equalization precoding schemes. The proposed SER-defined constructive interference region provides a large degree of freedom for optimization. Simulation results have illustrated that the proposed scheme achieves superior performance in terms of SER and transmit power compared with existing precoding schemes, and the modified feasible direction contributes to quicker convergence than the conventional method.

This article also leads to several open problems. Firstly, the SER function is analyzed with M-PSK modulation. Further investigations are required for other modulation schemes. Secondly, the complexity of the feasible direction method is positively related to the number of antennas. It will lead to high computational complexity especially for large scale antenna cases in future communication systems.

APPENDIX
PROOF OF THE PROPOSITION 1, 2 AND 3

We first provide the proof of Proposition 1. Define the feasible solution obtained by (21) as z and d, which satisfies $z < 0$. Then, we have

$$\boldsymbol{\beta} d = \boldsymbol{\beta} d^{\star} = d_{\varepsilon} \leq z < 0. \tag{41}$$

Hence, $d^{\star}$ is a descent direction for (15).

For the power constrain, we have

$$\begin{aligned} 2t^T C^T C d^{\star} &= 2\tilde{t}^T d_{\tilde{t},\perp} \\ &= 2\tilde{t}^T D_{\mathrm{s}} D_{\mathrm{s}}^{\dagger} d_{\tilde{t}} \\ &\overset{(a)}{=} 2\tilde{t}^T d_{\tilde{t}} \\ &= 2t^T C^T C d \leq z < 0, \end{aligned} \tag{42}$$

where the equation (a) results from the prerequisite that $\tilde{t}$ lies in the column space of D_{s}. For the SER constrain, we have

$$\begin{aligned} &[\frac{\partial S_k}{\partial \mu_{\mathrm{I}}} A_k - \frac{\partial S_k}{\partial \mu_{\mathrm{R}}} \tan(\frac{\pi}{M}) B_k - \boldsymbol{\beta}] d^{\star} \\ &\quad = \frac{\partial S_k}{\partial \mu_{\mathrm{I}}} \tilde{A}_k d_{\tilde{t},\perp} - \frac{\partial S_k}{\partial \mu_{\mathrm{R}}} \tan(\frac{\pi}{M}) \tilde{B}_k d_{\tilde{t},\perp} - d_{\varepsilon} \\ &\quad = \frac{\partial S_k}{\partial \mu_{\mathrm{I}}} \tilde{A}_k D_{\mathrm{s}} D_{\mathrm{s}}^{\dagger} d_{\tilde{t}} - \frac{\partial S_k}{\partial \mu_{\mathrm{R}}} \tan(\frac{\pi}{M}) \tilde{B}_k D_{\mathrm{s}} D_{\mathrm{s}}^{\dagger} d_{\tilde{t}} - d_{\varepsilon} \\ &\quad \overset{(a)}{=} \frac{\partial S_k}{\partial \mu_{\mathrm{I}}} \tilde{A}_k d_{\tilde{t}} - \frac{\partial S_k}{\partial \mu_{\mathrm{R}}} \tan(\frac{\pi}{M}) \tilde{B}_k d_{\tilde{t}} - d_{\varepsilon} \\ &\quad = [\frac{\partial S_k}{\partial \mu_{\mathrm{I}}} A_k - \frac{\partial S_k}{\partial \mu_{\mathrm{R}}} \tan(\frac{\pi}{M}) B_k - \boldsymbol{\beta}] d \leq z < 0, \end{aligned} \tag{43}$$

where $\tilde{A}_k = [\mathrm{Im}(\tilde{h}_k), -\mathrm{Re}(\tilde{h}_k)]$, $\tilde{B}_k = [\mathrm{Re}(\tilde{h}_k), \mathrm{Im}(\tilde{h}_k)]$. The equation (a) in (43) results from that $\tilde{A}_k^T$ and $\tilde{B}_k^T$ are the column components of D_{s}. According to (42) and (43), $d^{\star}$ is a feasible direction for (15).

The proof of Proposition 2 and 3 can be similarly given with that of Proposition 1, and are omitted.

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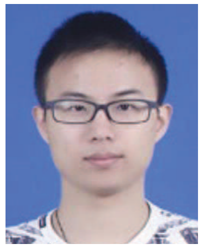

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