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Parameter Identification of Fractional Order Systems Using a Hybrid of Bernoulli Polynomials and Block Pulse Functions

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ABSTRACT Block pulse functions (BPFs) are piecewise constant and not sufficiently smooth. Therefore, their accuracy is limited when it comes to identifying the parameters of fractional order systems (FOSs). This in turn means that BPFs are incapable of offering highly accurate parameter identification results. However, using a great number of BPFs would significantly increase the dimension of the operational matrix and thereby adds to the computational complexity and burden. To overcome this problem, we present here a hybrid function method for identifying FOSs. The method utilizes a hybrid of Bernoulli polynomials and block pulse functions (HBPBPFs) as the base functions to approximate input and output signals. The fractional integral operational matrix of HBPBPFs is derived and used to convert an FOS to an algebraic system. The parameters of the FOS are successfully identified by minimizing the mean square error between the output of the true system and that of the algebraic representation of the FOS. The simulation experiment verifies that our proposed HBPBPFs method is effective and can generate more accurate identification results than existing BPFs methods.

INDEX TERMS Bernoulli polynomials, block pulse functions, fractional order system, operational matrix, parameter identification.

I. INTRODUCTION

Due to their inherent global property and historical dependence, fractional order models (FOMs) or fractional differential equations (FDEs) exhibit great advantages in describing the dynamic behaviors of real-world systems such as viscoelastic systems [1], the electrical characteristics of a solid oxide fuel cell (SOFC) [2], lithium-ion batteries [3], and anomalous diffusion [4]. Therefore, many researchers have been working on methods for building FOMs for real-world systems by using various identification methods and controlling FOSs [5]–[8].

A number of methods have been proposed for FOS identification. These methods can be roughly divided into two classes: frequency domain methods and time domain methods. For frequency domain methods, D. Valério *et al.* extended the traditional Levy's method to identify an FOS transfer function with commensurate orders [9], and the idea has been adopted to identify arbitrary fractional systems from a frequency response [10]. In [11], a variable damping least squares algorithm was adopted to investigate frequency response based identification of FOSs. In [12], a set membership algorithm was used to identify the coefficients and orders of FOSs with the unknown but bounded noise in frequency domain. In [13], [14], a frequency domain subspace method, which was expressed by a state space model, was proposed to identify FOSs. Several time domain methods have been proposed for FOS identification, including a simplified refined instrumental variable (SRIV) method [15], a recursive error prediction approach [16], and a modulating function method [17]–[19]. In [20], [21], a BPFs-based method was proposed to identify the parameters of FOSs. In this method, the fractional integral operational matrix of BPFs is derived, and the FOS to be identified is converted to an algebraic system. In this way, it avoids the complex and costly computations of the fractional derivatives of input and output signals. While

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the BPFs method is simple and effective, it suffers from at least one important limitation. Specifically, BPFs are piecewise constant and are therefore not sufficiently smooth. This means that using a smaller number of BPFs would compromise the identification accuracy and using a larger number of BPFs would lead to greatly increased dimension of the operational matrix and thus more complex computation.

To overcome the limitation of the BPFs method, a new hybrid functions based identification method is proposed in this paper. The method incorporates a hybrid of Bernoulli polynomials and block pulse functions as the base functions to approximate input and output signals. The fractional integral operational matrix of the hybrid functions is derived and then used to convert the FOS to be identified to an algebraic system. The parameter and order are simultaneously identified by minimizing the mean square error between the output of the true system and that of the algebraic system. Since the hybrid functions are piecewise polynomial, they are more smooth than BPFs. Therefore, the proposed method offers higher identification accuracy than the BPFs method when the same number of base functions are used.

The remainder of the paper is structured as follows. Some basics about fractional calculus are given in Section II. Section III elaborates on the HBPBPFs-based identification method. Section IV gives the error analysis. Numerical simulations are presented in Section V. Finally, conclusions are presented in Section VI.

II. BASIC KNOWLEDGE

A. DEFINITION OF FRACTIONAL INTEGRAL AND DERIVATIVE

Unlike integer calculus, fractional calculus comes with non-unique definitions. Among the most frequently used definitions for fractional calculus are Riemann-Liouville (R-L) definition, Grünwald-Letnikov (G-L) definition, and Caputo definition. In this paper, the R-L fractional integral definition and the Caputo fractional derivative definition are used. The definition details are as follows:

Definition 1: The R-L fractional integral definition of a continuous $f(t)$ is expressed as [22]

$$I^{\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_a^t (t-\tau)^{\alpha-1}f(\tau)\mathrm{d}\tau,$$

where $\alpha > 0$ is called the integral order, and a is the lower limit of the integral.

Definition 2: The Caputo fractional derivative definition of order α ($n-1 < \alpha \leqslant n$) is expressed as [22]

$$D^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}f^{(n)}(\tau)\mathrm{d}\tau,$$

where $\alpha > 0$ is the order of the derivative, and n is the smallest integer greater than α.

The Caputo derivative and R-L fractional integral of a function $f(t)$ are related to each other as follows:

$$I^{\alpha}(D^{\alpha}f(t)) = f(t) - \sum_{k=0}^{n-1} f^{(k)}(0)\frac{t^k}{k!},$$

and under zero initial conditions, in particular, their relation can be expressed as

$$I^{\alpha}(D^{\alpha}f(t)) = f(t).$$

B. HYBRID OF BERNOULLI POLYNOMIALS AND BLOCK PULSE FUNCTIONS

The BPFs are a set of piecewise orthogonal functions defined on the interval [0,T) as

$$\phi_i(t) = \begin{cases} 1, & \frac{i-1}{N}T \leqslant t < \frac{i}{N}T \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, N$, and N is the number of BPFs.

The Bernoulli polynomials are a special kind of generating function. The Bernoulli polynomials $\delta_m(t)$ of order m are defined as [23]

$$\delta_m(t) = \sum_{k=0}^{m}\binom{m}{k} c_{m-k}t^k,$$

where c_k ($k = 0, 1, \ldots, m$) is called Bernoulli numbers, and it is defined through power series of function $\frac{t}{e^t-1}$ as

$$\frac{t}{e^t-1} = \sum_{k=0}^{\infty} c_k\frac{t^k}{k!} = 1 - \frac{t}{2} + \sum_{k=0}^{\infty} c_{2k}\frac{t^{2k}}{(2k)!} \quad |t| < 2\pi.$$

The term $1-\frac{t}{2}$ is moved to the left side. Since $\frac{t}{e^t-1} - 1 + \frac{t}{2}$ is odd for $t \in \mathbb{R}$, the Bernoulli numbers $c_{2k+1} = 0$ for $k \in \mathbb{N}$. The first five Bernoulli numbers and Bernoulli polynomials are listed as follows:

$$c_0 = 1, \quad c_1 = -\frac{1}{2}, \quad c_2 = \frac{1}{6}, \quad c_4 = -\frac{1}{30}, \quad c_6 = \frac{1}{42},$$
$$\delta_0(t) = 1, \quad \delta_1(t) = t - \frac{1}{2}, \quad \delta_2(t) = t^2 - t + \frac{1}{6},$$
$$\delta_3(t) = t^3 - \frac{3}{2}t^2 + \frac{1}{2}t, \quad \delta_4(t) = t^4 - 2t^3 + t^2 - \frac{1}{30}.$$

Definition 3: The HBPBPFs $\gamma_{nm}(t)$ ($n = 1, 2, \ldots, N$, $m = 0, 1, 2, \ldots, M$) are a set of piecewise functions defined on the interval [0,T) as [24]

$$\gamma_{nm}(t) = \begin{cases} \delta_m(\frac{N}{T}t - n + 1), & \frac{n-1}{N}T \leqslant t < \frac{n}{N}T \\ 0, & \textit{otherwise,} \end{cases} \tag{2}$$

where n is the number of the elementary BPFs and m is the order of Bernoulli polynomials.

C. FUNCTION APPROXIMATION USING HBPBPFS

An absolutely integrable function $f(t) \in [0, T)$ can be expanded onto HBPBPFs as

$$f(t) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} \mu_{nm}\gamma_{nm}(t), \tag{3}$$

where

$$\mu_{nm} = \frac{\langle f(t), \gamma_{nm}(t)\rangle}{\langle \gamma_{nm}(t), \gamma_{nm}(t)\rangle}. \tag{4}$$

The symbol $\langle \cdot, \cdot \rangle$ in (4) is the inner product. Truncating (3) to a finite term, the function $f(t)$ can be approximated as

$$f(t) \cong \sum_{n=1}^{N} \sum_{m=0}^{M} \mu_{nm} \gamma_{nm}(t) = Y^{\mathrm{T}} \Upsilon(t),$$

where

$$Y = [\mu_{10}, \ldots, \mu_{1M}, \mu_{20}, \ldots, \mu_{2M}, \ldots, \mu_{N0}, \ldots, \mu_{NM}]^{\mathrm{T}},$$

and

$$\Upsilon(t) = [\gamma_{10}(t), \ldots, \gamma_{1M}(t), \gamma_{20}(t), \ldots, \gamma_{2M}(t), \ldots, \gamma_{N0}(t), \ldots, \gamma_{NM}(t)]^{\mathrm{T}}.$$

D. FRACTIONAL INTEGRAL OPERATIONAL MATRIX OF HBPBPFS

Definition 4: [20] The fractional integral of the BPFs vector $\Phi(t)$, i.e.,

$$I^{\alpha}\Phi(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \Phi(\tau) \mathrm{d}\tau$$

can be written in a matrix form as

$$I^{\alpha}\Phi(t) = F^{\alpha}_{N\times N} \Upsilon(t), \tag{5}$$

where $F^{\alpha}_{N\times N}$ is an N order square matrix and is called the fractional integral operational matrix of BPFs.

Definition 5: The fractional integral of the HBPBPFs vector $\Upsilon(t)$, i.e.,

$$I^{\alpha}\Upsilon(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \Upsilon(\tau) \mathrm{d}\tau$$

can be written in a matrix form as

$$I^{\alpha}\Upsilon(t) = P^{\alpha}_{s\times s} \Upsilon(t), \tag{6}$$

where $P^{\alpha}_{s\times s}$ is an s order square matrix and is called the fractional integral operational matrix of HBPBPFs.

$P^{\alpha}_{s\times s}$ can be obtained by expanding HBPBPFs onto the following BPFs:

$$\Upsilon(t) = \Psi_{s\times s}\Phi(t) \quad (s = N(M+1)), \tag{7}$$

where $\Phi(t) = [\phi_1(t), \phi_2(t), \cdots, \phi_s(t)]^{\mathrm{T}}$ is the BPFs vector, and the matrix $\Psi_{s\times s}$ is called the transition matrix from HBPBPFs to BPFs. In general, for arbitrary N and M, the matrix $\Psi_{s\times s}$ is a block-diagonal matrix and can be expressed as

$$\Psi_{s\times s} = \begin{pmatrix} A & 0 & 0 & \cdots & 0 \\ 0 & A & 0 & \cdots & 0 \\ 0 & 0 & A & \cdots & 0 \\ 0 & 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & 0 & A \end{pmatrix},$$

where $A = (\omega^{(m)}_{ni})_{(M+1)\times(M+1)}$ is an $M+1$ order square matrix, and its element $\omega^{(m)}_{ni}$ is

$$\omega^{(m)}_{ni} = \sum_{k=0}^{m} \binom{m}{k} \frac{c_{m-k}[i^{k+1} - (i-1)^{k+1}]}{(k+1)(M+1)^k}$$

with $1 \leqslant i \leqslant M$.

Using the transition matrix $\Psi_{s\times s}$, one can obtain

$$\begin{aligned} I^{\alpha}\Upsilon(t) &= I^{\alpha}(\Psi_{s\times s}\Phi(t)) = \Psi_{s\times s}(I^{\alpha}\Phi(t)) \\ &= \Psi_{s\times s}F^{\alpha}\Phi(t). \end{aligned} \tag{8}$$

From (7), it can be known that $\Phi(t) = \Psi^{-1}_{s\times s}\Upsilon(t)$, and by substituting this equation into (8), one has

$$I^{\alpha}\Upsilon(t) = \Psi_{s\times s}F^{\alpha}\Psi^{-1}_{s\times s}\Upsilon(t).$$

Therefore, the fractional integral operational matrix of the HBPBPFs can be expressed as

$$P^{\alpha}_{s\times s} = \Psi_{s\times s}F^{\alpha}\Psi^{-1}_{s\times s}.$$

III. PARAMETER IDENTIFICATION OF FOSS USING AN HBPBPFS OPERATIONAL MATRIX

Consider an FOS described by the fractional differential equation

$$\sum_{i=1}^{n} a_i D^{\alpha_i} y(t) = \sum_{j=1}^{m} b_j D^{\beta_j} u(t), \tag{9}$$

where $u(t)$ and $y(t)$ represent the input and output signals of the system, $(a_i, b_j) \in \mathbb{R}$, α_i and β_j are arbitrary real positive numbers that satisfy $\alpha_1 \leqslant \alpha_2 \leqslant \cdots \leqslant \alpha_n$, $\beta_1 \leqslant \beta_2 \leqslant \cdots \leqslant \beta_m$, and $\beta_m < \alpha_n$.

Performing α_n order fractional integration on both sides of system (9), one can get

$$\sum_{i=1}^{n-1} a_i I^{\alpha_n-\alpha_i} y(t) + a_n y(t) = \sum_{j=1}^{m} b_j I^{\alpha_n-\beta_j} u(t). \tag{10}$$

$y(t)$ and $u(t)$ are expanded onto HBPBPFs as follows:

$$y(t) = Y^{\mathrm{T}}\Upsilon(t), \quad u(t) = U^{\mathrm{T}}\Upsilon(t).$$

Then, the fractional integral of $y(t)$ and $u(t)$ can be expressed as

$$I^{\alpha_n-\alpha_i} y(t) = Y^{\mathrm{T}}(I^{\alpha_n-\alpha_i}\Upsilon(t)) = Y^{\mathrm{T}}P^{\alpha_n-\alpha_i}\Upsilon(t), \tag{11}$$

and

$$I^{\alpha_n-\beta_j} u(t) = U^{\mathrm{T}}(I^{\alpha_n-\beta_j}\Upsilon(t)) = U^{\mathrm{T}}P^{\alpha_n-\beta_j}\Upsilon(t). \tag{12}$$

(11) and (12) are substituted into system (10), which can be written as

$$\begin{aligned} & Y^{\mathrm{T}}(a_n I + a_{n-1}P^{\alpha_n-\alpha_{n-1}} + \cdots + a_1 P^{\alpha_n-\alpha_1})\Upsilon(t) \\ &\quad = U^{\mathrm{T}}(b_m P^{\alpha_n-\beta_m} + b_{m-1}P^{\alpha_n-\beta_{m-1}} + \cdots \\ &\qquad + b_1 P^{\alpha_n-\beta_1})\Upsilon(t). \end{aligned} \tag{13}$$

Let

$$\begin{aligned} H &= a_n I + a_{n-1}P^{\alpha_n-\alpha_{n-1}} + \cdots + a_1 P^{\alpha_n-\alpha_1}, \\ E &= b_m P^{\alpha_n-\beta_m} + b_{m-1}P^{\alpha_n-\beta_{m-1}} + \cdots + b_1 P^{\alpha_n-\beta_1}. \end{aligned}$$

Eliminating the $\Upsilon(t)$ in (13), one can get

$$Y^{\mathrm{T}} = U^{\mathrm{T}}EH^{-1}.$$

Accordingly, the output $y(t)$ can be written as

$$y(t) = U^{\mathrm{T}} E H^{-1} \Upsilon(t). \tag{14}$$

Remark 1: Equation (14) is the HBPBPFs operational matrix representation of FOS (9). It is an algebraic representation. An advantage of this representation is that it eliminates the need for fractional integration of input and output signals. Instead, it only involves algebraic computation, which means much less computational complexity. Furthermore, the matrices H^{-1}, E contain the system parameters a_i, b_j and the fractional orders α_i, β_j, which makes it possible to identify both coefficients and orders.

Let $\hat{\theta} = [\hat{a}_1, \cdots, \hat{a}_n, \hat{b}_1, \cdots, \hat{b}_m; \hat{\alpha}_1, \cdots, \hat{\alpha}_n, \hat{\beta}_1, \cdots, \hat{\beta}_m]$ be the estimation of the true parameters $\theta^* = [a_1, \cdots, a_n, b_1, \cdots, b_m; \alpha_1, \cdots, \alpha_n, \beta_1, \cdots, \beta_m]$. For a specific estimation of system parameters, the corresponding output is given by

$$\hat{y}(t) = U^{\mathrm{T}} \hat{E} \hat{H}^{-1} \Upsilon(t).$$

If $\hat{\theta}$ is as close as θ^*, then $\hat{y}(t)$ will also be as close as $y(t)$. More specifically, if $\hat{\theta} \to \theta^*$, then $\hat{y}(t) \to y(t)$. Observing this, one can get the optimal estimation $\hat{\theta}^*$ by minimizing the mean square error between the true output $y(t)$ and the estimated output $\hat{y}(t)$, i.e.,

$$\hat{\theta}^* = \arg\min \frac{1}{L} \sum_{k=1}^{L} [y(kh) - \hat{y}(kh)]^2, \tag{15}$$

where h is the sampling interval, and L is the number of the data. The optimization problem (15) can be solved using a well-developed optimization tool, for example, the interpoint method. For the sake of simplification, the `fmincon` function in MATLAB optimization toolbox is adopted. The principle of parameter identification of the fractional system using the proposed method is shown in Fig. 1.

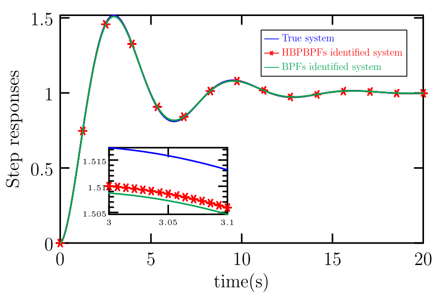

FIGURE 1. Step responses for Example 1 without noise.

IV. ERROR BOUNDS

In this section, we obtain the error bounds of best approximation for the HBPBPFs. The following theorem is established in terms of a Sobolev norm. Assuming that $y^{(k)}(t)$ is continuous and bounded on $[0, T]$, one has

$$\exists L' > 0, \quad \forall t \in [0, T], \quad \left| y^{(k)}(t) \right| \leqslant L',$$

where k is a nonnegative integer.

Definition 6: [26] The Sobolev norm is defined on the interval (a, b) for $\mu \geqslant 0$ by

$$\| f \|_{H^{\mu}(a,b)} = \left(\sum_{k=0}^{\mu} \int_a^b \left| f^{(k)}(x) \right|^2 \mathrm{d}x \right)^{\frac{1}{2}} = \left(\sum_{k=0}^{\mu} \| f^{(k)}(x) \|_{L^2(a,b)}^2 \right)^{\frac{1}{2}},$$

where $f^{(k)}(x)$ denotes the kth derivative of $f(x)$.

The symbol $|f|_{H^{\mu;M}(0,T)}$, which was introduced in [26], is expanded by

$$|f|_{H^{\mu;M}(0,T)} = \left(\sum_{k=\min(\mu, M+1)}^{\mu} \| f^{(k)} \|_{L^2(0,T)}^2 \right)^{\frac{1}{2}}.$$

Lemma 1: [26] Assuming that $f \in H^{\mu}(0, 1)$, if $P_M f = \sum_{m=0}^{M} c_m \beta_m$ is the best approximation of f, then

$$\| f - P_M f \|_{L^2(0,1)} \leqslant c M^{-\mu} |f|_{H^{\mu;M}(0,1)},$$

where c depends on μ, and $\mu \geqslant 0$.

Theorem 1: Suppose that $y_n : (\frac{n-1}{N}T, \frac{n}{N}T) \to \mathbb{R}$ $(n = 1, 2, \ldots, N)$ is a function in $H^{\mu}(\frac{n-1}{N}T, \frac{n}{N}T)$. For $t \in (0, T)$, consider the function $F_n y_n : (0, T) \to \mathbb{R}$ such that $(F_n y_n)(t) = y_n(\frac{1}{N}(t + (n-1)T))$, then for $0 \leqslant l \leqslant \mu$, one has

$$\left\| (F_n y_n)^{(l)} \right\|_{L^2(0,T)} = N^{1-2l} \left\| y_n^{(l)} \right\|^2_{L^2(\frac{n-1}{N}T, \frac{n}{N}T)}.$$

Proof: For $0 \leqslant l \leqslant \mu$, one has

$$\begin{aligned} &\left\| (F_n y_n)^{(l)} \right\|^2_{L^2(0,T)} \\ &= \int_0^T \left| (F_n y_n)^{(l)}(t) \right|^2 \mathrm{d}t \\ &= \int_0^T \left| y_n^{(l)}(\frac{1}{N}(t + (n-1)T)) \right|^2 \mathrm{d}t \\ &\overset{u = \frac{1}{N}(t + (n-1)T)}{=\!=\!=\!=\!=} \int_{\frac{n-1}{N}T}^{\frac{n}{N}T} N^{-2l} \left| y_n^{(l)}(u) \right|^2 N \mathrm{d}u \\ &= N^{1-2l} \left\| y_n^{(l)}(u) \right\|^2_{L^2(\frac{n-1}{N}T, \frac{n}{N}T)}. \end{aligned}$$

□

Theorem 2: Assuming that function $y(t) \in H^{\mu}(0, T)$ is expanded onto HBPBPFs as $y(t) = Y^{\mathrm{T}} \Upsilon(t)$, and $Y^{\mathrm{T}} \Upsilon(t)$ is the best approximation of $y(t)$, then the error is bounded as follows:

$$\left\| y(t) - Y^{\mathrm{T}} \Upsilon(t) \right\|_{L^2(0,T)} \leqslant \frac{c L' \sqrt{T}}{(MN)^{\mu}},$$

where N is the number of BPFs, M is the order of Bernoulli polynomials, and $\mu \geqslant 0$, $M \geqslant \mu - 1$.

Proof: For $n = 1, 2, \ldots, N$, we consider the function $y_n : (\frac{n-1}{N}T, \frac{n}{N}T) \to \mathbb{R}$ such that $y_n(t) = y(t)$ for all

$t \in (\frac{n-1}{N}T, \frac{n}{N}T)$. For simplicity, we denote that $P_M y_n = \sum_{m=0}^{M} \mu_{nm}\gamma_{nm}(t)$. By using Theorem 1, one has

$$\begin{aligned}
&\left\|y(t) - Y^{\mathrm{T}}\Upsilon(t)\right\|^2_{L^2(0,T)} \\
&\quad = \sum_{n=1}^{N}\left\|y_n - \sum_{m=0}^{M}\mu_{nm}\gamma_{nm}(t)\right\|^2_{L^2(\frac{n-1}{N}T,\frac{n}{N}T)} \\
&\quad = N^{-1}\sum_{n=1}^{N}\|F_n y_n - P_M(F_n y_n)\|^2_{L^2(0,T)} \\
&\quad \leqslant c^2N^{-1}M^{-2\mu}\sum_{k=min(\mu,M+1)}^{\mu}\sum_{n=1}^{N}\left\|(F_n y_n)^{(k)}\right\|^2_{L^2(0,T)} \\
&\quad = c^2M^{-2\mu}\sum_{k=min(\mu,M+1)}^{\mu}\sum_{n=1}^{N}N^{-2k}\left\|y_n^{(k)}\right\|^2_{L^2(\frac{n-1}{N}T,\frac{n}{N}T)} \\
&\quad = c^2M^{-2\mu}\sum_{k=min(\mu,M+1)}^{\mu}N^{-2k}\left\|y^{(k)}\right\|^2_{L^2(0,T)} \\
&\quad = c^2M^{-2\mu}N^{-2\mu}\left\|y^{(\mu)}\right\|^2_{L^2(0,T)} \\
&\quad \leqslant c^2M^{-2\mu}N^{-2\mu}L'^2T.
\end{aligned}$$

Therefore,

$$\left\|y(t) - Y^{\mathrm{T}}\Upsilon(t)\right\|_{L^2(0,T)} \leqslant \frac{cL'\sqrt{T}}{(MN)^{\mu}}. \tag{16}$$

□

Theorem 3: Suppose that function $y(t) \in H^{\mu}(0,T)$ is expanded onto BPFs as $y(t) = K^{\mathrm{T}}\Phi(t)$. Then the error is bounded as follows:

$$\left\|y(t) - K^{\mathrm{T}}\Phi(t)\right\|_{L^2(0,T)} \leqslant \frac{cL'\sqrt{T}}{N^{\mu}},$$

where $\mu \geqslant 0$, and $N \geqslant \mu - 1$.

Proof: Suppose that the function $y(t)$ is expanded onto BPFs as

$$y(t) = \sum_{i=1}^{N} k_i'\phi_i(t) = K^{\mathrm{T}}\Phi(t) = P_N y(t).$$

Then, from Lemma 1, one has

$$\begin{aligned}
&\|y(t) - P_N y(t)\|^2_{L^2(0,T)} \\
&\quad \leqslant c^2N^{-2\mu}\left(\sum_{k=min\{\mu,N+1\}}^{\mu}\|y^{(k)}\|^2_{L^2(0,T)}\right) \\
&\quad \leqslant c^2N^{-2\mu}\|y^{(\mu)}\|^2_{L^2(0,T)} \\
&\quad \leqslant c^2N^{-2\mu}L'^2T.
\end{aligned}$$

Therefore,

$$\|y(t) - P_N y(t)\|_{L^2(0,T)} \leqslant \frac{cL'\sqrt{T}}{N^{\mu}}. \tag{17}$$

□

Theorem 4: Suppose $y(t) \in H^{\mu}(0,T)$. Then

$$\|I^{\alpha}y(t) - I^{\alpha}(Y^{\mathrm{T}}\Upsilon(t))\|_{L^2(0,T)} \leqslant \frac{1}{\Gamma(\alpha)}\frac{cL'\sqrt{T}}{(MN)^{\mu}}$$

where $\mu \geqslant 0$ and $\alpha > 1$.

Proof:

$$\begin{aligned}
&\|I^{\alpha}y(t) - I^{\alpha}(Y^{\mathrm{T}}\Upsilon(t))\|_{L^2(0,T)} \\
&\quad \leqslant \left\|\frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}(y(\tau) - Y^{\mathrm{T}}\Upsilon(\tau))d\tau\right\|_{L^2(0,T)} \\
&\quad \leqslant \left\|\frac{1}{\Gamma(\alpha)}\int_0^T (T-\tau)^{\alpha-1}(y(\tau) - Y^{\mathrm{T}}\Upsilon(\tau))d\tau\right\|_{L^2(0,T)} \\
&\quad \leqslant \frac{1}{\Gamma(\alpha)}\int_0^T \left\|y(\tau) - Y^{\mathrm{T}}\Upsilon(\tau)\right\|_{L^2(0,T)} d\tau \\
&\quad \leqslant \frac{1}{\Gamma(\alpha)}cM^{-\mu}N^{-\mu}\left\|y^{(\mu)}\right\|_{L^2(0,T)} \\
&\quad = \frac{1}{\Gamma(\alpha)}\frac{cL'T}{(MN)^{\mu}}.
\end{aligned}$$

□

Theorem 5: Suppose $y(t) \in H^{\mu}(0,T)$. Then

$$\|I^{\alpha}y(t) - I^{\alpha}(K^{\mathrm{T}}\Phi(t))\|_{L^2(0,T)} \leqslant \frac{1}{\Gamma(\alpha)}\frac{cL'\sqrt{T}}{N^{\mu}}$$

where $\mu \geqslant 0$ and $\alpha > 1$.

Proof:

$$\begin{aligned}
&\|I^{\alpha}y(t) - I^{\alpha}(K^{\mathrm{T}}\Phi(t))\|_{L^2(0,T)} \\
&\quad \leqslant \left\|\frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}(y(\tau) - K^{\mathrm{T}}\Phi(\tau))d\tau\right\|_{L^2(0,T)} \\
&\quad \leqslant \left\|\frac{1}{\Gamma(\alpha)}\int_0^T (T-\tau)^{\alpha-1}(y(\tau) - K^{\mathrm{T}}\Phi(\tau))d\tau\right\|_{L^2(0,T)} \\
&\quad \leqslant \frac{1}{\Gamma(\alpha)}\int_0^T \left\|y(\tau) - K^{\mathrm{T}}\Phi(\tau)\right\|_{L^2(0,T)} d\tau \\
&\quad \leqslant \frac{1}{\Gamma(\alpha)}cN^{-\mu}\left\|y^{(\mu)}\right\|_{L^2(0,T)} \\
&\quad = \frac{1}{\Gamma(\alpha)}\frac{cL'T}{N^{\mu}}.
\end{aligned}$$

□

Remark 2: From Theorem 2 and Theorem 4, we can see that $\|y(t) - Y^{\mathrm{T}}\Upsilon(t)\| \to 0$ and $\|I^{\alpha}y(t) - I^{\alpha}(Y^{\mathrm{T}}\Upsilon(t))\| \to 0$ when $NM \to \infty$. The larger the value of NM, the more accurate the approximation for $Y^{\mathrm{T}}\Upsilon(t)$ to $y(t)$ and $I^{\alpha}(Y^{\mathrm{T}}\Upsilon(t))$ to $I^{\alpha}y(t)$. Accordingly, the identified output $\hat{y}(t)$ is closer to the real output $y^*(t)$.

Remark 3: From Theorem 3 and Theorem 5, we can see that the error of BPFs depends on N. Because $(NM)^{\mu}$ is larger than N^{μ}, the approximation error caused by HBPBPFs is smaller than that by BPFs when the number of BPFs is the same. Accordingly, the identification results obtained by HBPBPFs are more accurate than those by BPFs.

V. EXPERIMENT RESULTS

In this section, we use the derived HBPBPFs operational matrix of the fractional order integration to identify FOSs in order to demonstrate the better effectiveness and accuracy of the proposed method than the BPFs method. We also provide two types of error criteria to represent the good performance. One is the relative error ε_1 and the other is the mean square error ε_2. They are defined as

$$\epsilon_1 = \frac{\| \hat{\theta} - \theta^* \|}{\| \theta^* \|},$$

$$\epsilon_2 = \frac{1}{L}\sum_{k=1}^{L}[y(kh) - \hat{y}(kh)]^2,$$

where $y(t)$ is the output of the true system, $\hat{y}(t)$ is the output of the identified system, θ^* is the vector of true parameters, and $\hat{\theta}$ is the vector of identified parameters.

A. EXAMPLE 1

Consider an FOS as follows:

$$a_1 D^{\alpha_1} y(t) + a_2 D^{\alpha_2} y(t) = b_1 D^{\beta_1} u(t). \quad (18)$$

The parameter vector of system (18) is $\theta^* = [a_1, a_2, b_1; \alpha_1, \alpha_2, \beta_1]$, where $a_1 = 1, a_2 = 1, b_1 = 1, \alpha_1 = 0, \alpha_2 = 1.7$, and $\beta_1 = 0$. All these parameters are supposed to be unknown and need to be identified.

First, the case that the system output is not corrupted by noise is tested. A unit step signal as the input signal is selected to excite the system, and then the output signal is recorded. In the experiment, the number of elementary BPFs N is set to 40 and 100, respectively, the order of Bernoulli polynomials $M = 1$, T is set to 20, and the sampling interval is $h = 0.01$s. In the optimization process, the optimization initial condition is $[1 \ \ 1 \ \ \cdots \ \ 1]$, where the dimension of optimization initial condition vector is the same as the number of identified parameters. Table 1 lists the identification results obtained using our proposed method and the BPFs method. Obviously, with the increase in the N value, our proposed method generates decreased relative error and mean square error, and when the N value stays the same, offers more accurate identification results than the BPFs method. A more straightforward illustration of this finding is given by Fig. 1 and Fig. 2, showing the step responses and the Bode diagrams of the true system and those of the identified systems. It can be seen that the identified output of our proposed method $\hat{y}(t)$ is closer to the real output $y(t)$ than that of the BPFs method.

Second, the case that the system output is corrupted by noise is tested. Two different white noises, one with an SNR of 5 and the other with an SNR of 10, are considered. The value of N is set to 40, and $M = 1$. A Monte Carlo simulation is completed by executing 50 identifications independently. The mean value of the 50 identification results is regarded as the final identification result, as listed in Table 2. It can be seen that the identification results are satisfactory even though the output is corrupted by noise. Furthermore, our proposed method offers more accurate results than the BPFs method. Again, a more straightforward illustration of this finding is given by Fig. 3 and Fig. 4, showing the step responses and Bode diagrams of the true system and those of the identified systems with SNR = 5dB. We can see that the identified output of our proposed method is closer to the real output than that of the BPFs method.

TABLE 1. Identification results for Example 1 without noise.

Parameter	True	$N = 40$		$N = 100$	
		HBPBPFs	BPFs	HBPBPFs	BPFs
a_1	1	1.0012	1.0054	1.0000	1.0005
a_2	1	0.9987	0.9913	1.0013	1.0016
b_1	1	1.0005	1.0035	0.9992	0.9984
α_1	0	0.0141	0.0156	0.0137	0.0139
α_2	1.7	1.7078	1.7052	1.7086	1.7087
β_1	0	0.0139	0.0151	0.0135	0.0132
ϵ_1	-	8.78E-03	1.02E-02	8.68E-03	8.70E-03
ϵ_2	-	5.65E-06	1.22E-03	2.16E-07	1.96E-03
Time(s)	-	852	0.27	1798	0.72

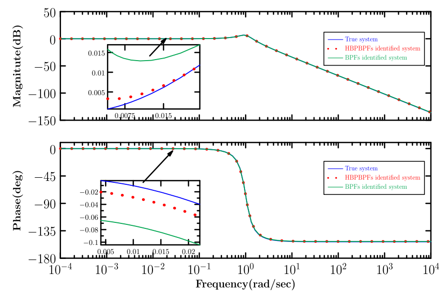

FIGURE 2. Bode diagrams for Example 1 without noise.

TABLE 2. Identification results for Example 1 with noise.

Parameter	True	SNR=5dB(Mean)		SNR=10dB(Mean)	
		HBPBPFs	BPFs	HBPBPFs	BPFs
a_1	1	0.9996	1.0031	1.0004	1.0046
a_2	1	1.0028	0.9931	1.0010	0.9935
b_1	1	0.9974	1.0033	0.9987	1.0020
α_1	0	0.0153	0.0179	0.0146	0.0164
α_2	1.7	1.7060	1.7001	1.7067	1.7042
β_1	0	0.0147	0.0179	0.0141	0.0156
ϵ_1	-	0.0092	0.0110	0.0088	0.0101
ϵ_2	-	3.67E-05	1.11E-04	2.47E-05	6.93E-05
Time	-	10.8(h)	12.74(s)	10.5(h)	11.98(s)

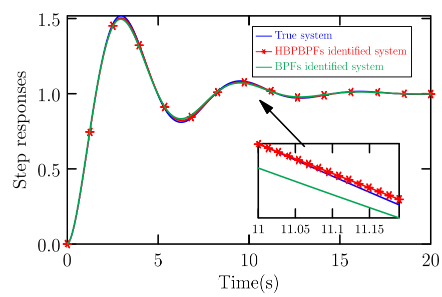

FIGURE 3. Step responses for Example 1 with SNR = 5dB.

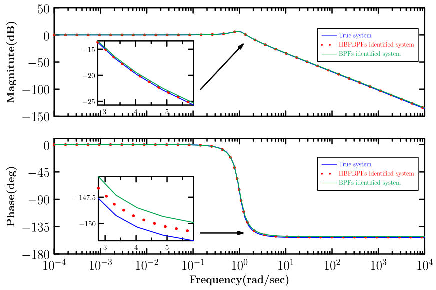

FIGURE 4. Bode diagrams for Example 1 with SNR = 5dB.

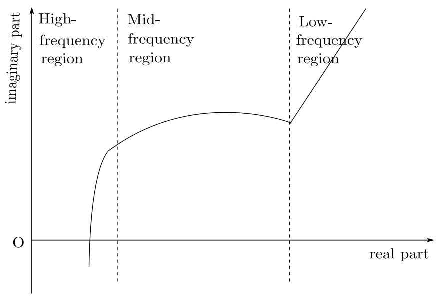

FIGURE 5. Impedance spectra of the lithium-ion battery [3].

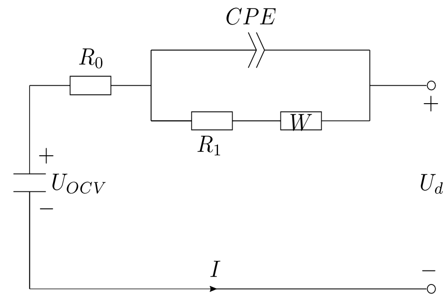

FIGURE 6. Schematic of fractional equivalent circuit model.

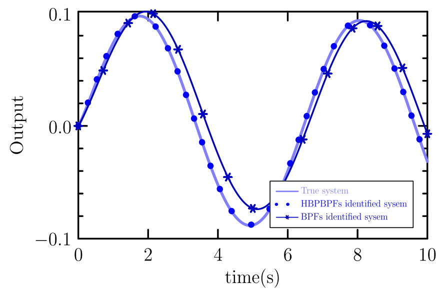

FIGURE 7. The output of the true system, HBPBPFs identified system and BPFs identified system for Example 2.

B. EXAMPLE 2: MODELING OF LITHIUM-ION BATTERIES

As a new secondary clean and renewable energy, lithium-ion batteries have been used widely in electric vehicles due to their high specific energy, long service life, and low self-discharge rate [3]. The state of charge (SOC) indicates how much energy is remaining with a battery. The accuracy of SOC estimation is a key problem in battery management systems (BMSs). Therefore, proper estimation of battery power can effectively predict and control the driving distance of electric vehicles and helps to extend the service life of batteries. Accurate estimation of SOC is the first consideration to make when developing power batteries. However, SOC cannot be measured directly, but indirectly by measured variables such as current and terminal voltage. Here, we build a fractional order model between current and terminal voltage so that our proposed method can be used to estimate the SOC of lithium-ion batteries.

Fig. 5 shows a common lithium-ion electrochemical impedance spectrogram [3]. From this spectrogram, a fractional order SOC model can be easily obtained, which can be expressed as [3]

$$\frac{U_d(s) - U_{OCV}(s)}{I(s)} = \frac{R_1}{1 + R_1 C_1 s^{\alpha}} + \frac{R_2}{1 + R_2 C_2 s^{\beta}} + \frac{1}{W s^{\gamma}} + R_0.$$

According to the electrochemical principle and Fig. 5, we propose a fractional order model for lithium-ion batteries based on Randles circuit [27], where the fractional differential orders are within (0, 1). Fig.6 shows the fractional equivalent impedance circuit of Randles circuit. The electronic component *CPE* in Fig.6 represents a fractional order impedance component such as the constant phase element. Based on what we know from the electrochemical principle, the impedance of *CPE* can be expressed in the following transfer function equation:

$$Z_{CPE}(s) = \frac{1}{C s^{\alpha_{CPE}}},$$

where C is the parameter of model element, α_{CPE} is the order, CPE represents the ideal capacitor if $\alpha_{CPE} = 1$ and the resistor if $\alpha_{CPE} = 0$. Then, the Warburg element's transfer function is given as

$$Z_W(s) = \frac{1}{W_D s^{\alpha_D}},$$

where Z_W is the impedance of the Warburg element, W_D is the coefficient of the element, and α_D is the order. The transfer function of the fractional impedance model of lithium-ion batteries can be expressed as

$$\frac{U_d(s) - U_{OCV}(s)}{I(s)} \tag{19}$$

$$= \frac{R_1 W_D s^{\alpha_D} + 1}{W_D s^{\alpha_D} + R_1 C W_D s^{\alpha_D + \alpha_{CPE}} + C s^{\alpha_{CPE}}} + R_0, \tag{20}$$

where R_0 is the pure resistance, and R_1 is the charge transfer resistance. Letting $U(s) = I(s)$ and $Y(s) = U_d(s) - U_{OCV}(s)$, equation (19) can be written as a fractional differential equation:

$$\begin{aligned} &W_D D^{\alpha_D} y(t) + C D^{\alpha_{CPE}} y(t) + R_1 C W_D D^{\alpha_D + \alpha_{CPE}} y(t) \\ &= u(t) + (R_0 + R_1) W_D D^{\alpha_D} u(t) \\ &\quad + R_0 C D^{\alpha_{CPE}} u(t) + R_0 R_1 C W_D D^{\alpha_D + \alpha_{CPE}} u(t), \end{aligned} \tag{21}$$

where the parameter to be estimated is as follows:

$$\theta = [R_0 \quad R_1 \quad C \quad W_D \quad \alpha_{CPE} \quad \alpha_D].$$

TABLE 3. Identification results of Example 2.

Method	R_0	R_1	C	W_D	α_{CPE}	α_D	ϵ_2
HBPBPFs	0.0199	0.9768	36.9961	23.1733	0.2270	0.4933	7.24E-08
BPFs	0.0211	0.9654	40.3271	25.2628	0.4952	0.2165	3.56E-06

The proposed method is used to identify model (21). The number of elementary BPFs to be used N is set to 50, the order of Bernoulli polynomials M is set to 1, T is equal to 10, and $h = 0.05$s. In the optimization process, the optimization initial condition is $[1\ 1\ \cdots\ 1]$, where the dimension of the optimization initial condition vector is the same as the number of identified parameters. The sinusoidal input signal $u(t) = \sin(t)$ is applied to excite the system, and the parameter θ is estimated using the proposed method and the BPFs method. Table 3 shows the identification results. We can see that the mean square error generated by the HBPBPFs method is smaller than that by the BPFs method. This suggests that the system (21) modeled by the HBPBPFs method is more accurate than that by the BPFs method. Fig. 7 gives the sinusoidal response of the true system and the HBPBPFs identified system.

VI. CONCLUSION

In this paper, we propose a method based on the fractional integral operational matrix of HBPBPFs to investigate the FOS identification problem, in which the orders and parameters are unknown. This HBPBPFs-based method is different from existing BPFs methods in that it comprises of polynomial functions that are piecewise smooth on each subinterval. This uniqueness enables the proposed method to approximate signals more accurately than BPFs-based methods if the same number of elemental BPFs are used. The proposed method utilizes the fractional integral operational matrix of HBPBPFs to convert an FOS to an algebraic system, which simplifies the computation of fractional derivatives of input and output signals and allows for estimating both the coefficients and orders of the FOS. Therefore, we conclude that the proposed method is more accurate than BPFs-based methods. This conclusion is experimentally verified by several simulation examples. However, the fractional integral operational matrix of HBPBPFs involves the complex operation of matrix inversion, the proposed method's computation time is too high, so the proposed method cannot be employed in online system identification studies. For future research works, we will use the proposed method to identify FOSs with time delays.

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