IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received February 20, 2021, accepted February 28, 2021, date of publication March 4, 2021, date of current version March 11, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3063725

Multiobjective Multi-Product Production Planning Problem Using Intuitionistic and Neutrosophic Fuzzy Programming

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This work was supported by the Deanship of Scientific Research, College of Science and Theoretical Studies, Saudi Electronic University, Riyadh, Saudi Arabia.

ABSTRACT In this article, a multiobjective multiproduct production planning (MOMPP) problem discussed for a hardware firm. The hardware firm produces various types of hardware locks and other items in a production run. The firm manager's objectives are minimizing the production cost and inventory holding cost while maximizing the net profit subject to some system constraints. The multiproduct production planning is solved with the last production run information precisely known to the decision-maker, and finally, the model is solved using the intuitionistic and neutrosophic programming approaches, respectively. Also, the multi-product production planning problem is discussed for situations when the product information is vague. The interval-valued trapezoidal neutrosophic numbers used to define this Vagueness. The multiobjective multiproduct production planning problem under fuzziness is solved using the neutrosophic compromise programming. The stepwise solution procedures are discussed using the case study.

INDEX TERMS Interval-valued trapezoidal neutrosophic numbers, intuitionistic fuzzy programming, multiobjective optimization, multiproduct production planning, neutrosophic programming.

I. INTRODUCTION

Production planning is a well-organized strategy under which raw material optimally transformed into finished products while maintaining the quality and costs of the manufactured product. The primary goal of any production planning is to understand the market demands and requirements and keep changing the product design and other up-gradation according to the customer's needs and finally earning profit. For a specific product, it is necessary to evaluate the universal character of machinery, the amount of producing products for the particular period, the number of products types, the demands of the employee's qualification, and the production cycle (process), and the labour character division. Production planning influences the firm's profit and level of the service for customers that are imperative to elasticity in the production planning process and more thoughtfulness to comfort high profit and service level. Many manufacturing enterprises are forced to optimize the production process to win the globalized market's business race. Nowadays, most researchers and industries specialists used optimization techniques on production planning models to ensure the optimum profit with the optimum production of units. Ghosh and Mondal [1] described a production-distribution planning model for a two-echelon supply chain, and the genetic algorithm is used to make the decision. Sohn *et al.* [2] discussed a production planning of L.G. display and presented a mixed-integer programming model. Mosadegh *et al.* [3] considered inventory and shortage, idle time and overtime, workforce level, and currency saving like four criteria in a mid-term planning horizon. Gupta *et al.* [4] described a two-stage transportation problem under a certain and uncertain environment, and for optimal shipment, the fuzzy goal programming was used.

The main aims of the study are to discuss a MOMPP model for a hardware firm. The proposed work is a layout for multiproduct production while achieving the following goals - optimize the production cost, the inventory holding cost, and the net profit. The intuitionistic and neutrosophic fuzzy programming approaches are used to obtain the solution for

The associate editor coordinating the review of this manuscript and approving it for publication was Yizhang Jiang.

MOMPP. The case of interval-valued fuzzy-based information also has been discussed.

II. LITERATURE REVIEW

An essential activity for manufacturing enterprises is production planning. The production plan is usually prepared in many manufacturing companies before the beginning of every financial year. The production plan provides a strategic framework for the production to fulfil each period's demand during the financial year. The perfect production plan executed weekly, monthly, quarterly or even years depends on the products' demand. Production scheduling is allocating the available production resources overtime to satisfy some criteria such as quality, delivery time, demand and supply. A production problem includes machine capacity planning problem, production scheduling problem, storage and freight scheduling problems. Technological advancements, market dynamics and international competitions have significantly impacted the manufacturing industries in the past two decades.

Most of the productions planning problems are multiobjective. Some of the researchers have used the e-constraint method for transforming the multiobjective problem into a single objective, where the objectives functions treated as additional constraints of the problem. Chandra and Fisher [5] described solving the problem for coordinating the production and distribution functions. A single-plant for multi-commodities for multi-periods manufacturing environments where produced items are stored in the plant until products are delivered using a fleet of trucks to the customers. Yan *et al.* [6] described a strategic production-distribution model that included multiple producers, suppliers, distribution centres and customers in which multiple products are manufactured in a single period. The fuzzy multiobjective linear programming model is more efficient in solving real production planning problems, mainly because most companies seek to satisfy more than one objective function to develop a response and flexibility production planning system. Several studies on the application of fuzzy optimization include Zimmermann [7], who first used the concept of the fuzzy set given by Zadeh [8] and studied fuzzy linear programming with several objectives. Some related authors research work on production planning is briefly reported in Table 1.

Ebrahimnejad [29] described the multiobjective linear programming problem where all parameters are represented in terms of fuzzy triangular numbers and solved by the lexicographical approach. Abbaszadeh *et al.* [30] discussed the route planning of robotics research to find the shortest route without colliding from initiation point to destination point so that the amount of energy consumption by a robot would not exceed a predefined amount. The elite artificial bees' colony algorithm is used to solve the robot's fuzzy constrained shortest route problem and compared with the performance of the genetic algorithm and particle swarm optimization algorithm. Bagheri *et al.* [31] discussed a transportation problem with fuzzy costs in the presence of multiple and conflicting objectives is investigated; a fuzzy data envelopment analysis approach was proposed to solve the fuzzy multiobjective transportation problem. Bagheri *et al.* [32] described a fully fuzzy multiobjective transportation problem, and a new fuzzy data envelopment analysis based approach was developed to solve the problems.

Angelov [33] used the concept intuitionistic fuzzy set in optimization problem and converted intuitionistic fuzzy optimization into crisp. Pramanik and Roy [34] proposed the intuitionistic fuzzy goal programming approach to solve vector optimization problems under uncertainty using the concept of an intuitionistic fuzzy set. Pramanik [35] described neutrosophic linear goal programming under fuzzy set theory. Pramanik [36] described neutrosophic multiobjective linear/non-linear programming, neutrosophic goal programming in which the degrees of indeterminacy and falsity (rejection) of objectives and constraints are simultaneously considered together with the degrees of truth membership (satisfaction/acceptance). The author presented the drawbacks of the existing neutrosophic optimization models, and a new framework of multiobjective optimization in the neutrosophic environment was proposed. Abdel-Basset *et al.* [37] describe the technique for solving the linear programming problem in which neutrosophic set theory plays a vital role in a real-life example. Their parameters were presented with the trapezoidal neutrosophic number and presented a neutrosophic linear programming model technique. Abdelfattah [38] represented the entire linear programming coefficient by triangular neutrosophic numbers and used the fuzzy set theory to deal with neutrosophic linear programming models. Bera and Mahapatra [39] used a single-valued trapezoidal neutrosophic number for the linear programming problem with the objective function's coefficient and the constraints' right-hand side. The author also described the simplex algorithm for solving the modified linear programming problem. Das and Edalatpanah [40] build a framework for neutrosophic integer programming with triangular neutrosophic numbers using the aggregate ranking function.

III. PREREQUISITES (SOME BASIC DEFINITIONS)

In this section, we discussed some fundamental definitions regarding the intuitionistic fuzzy number and neutrosophic fuzzy numbers.

Definition 1 (Zadeh [8]): Let X be a fixed set. A fuzzy set $\tilde{A}$ of X is an object having the form $\tilde{A} = \left\{(x, \mu_{\tilde{A}}(x)) : x \in X\right\}$ where $\mu_{\tilde{A}}(x) \in [0, 1]$ represents the degree of membership of the element $x \in X$ being in $\tilde{A}$, and $\mu_{\tilde{A}} : X \to [0, 1]$ is called the membership function.

Definition 2: A fuzzy set $\tilde{A}$ on $\Re$ is convex if and if for every pair of points x_1, x_2 in X, the membership function of $\tilde{A}$ satisfies the inequality

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right\} \quad \forall x_1, \quad x_2 \in X, \ \lambda \in [0, 1]$$

TABLE 1. Research review summary.

Authors	Objective		Techniques used								Remarks
	Single	Multi	GP	FP	LP	SP	GA	BL	IP	NP	
Kanyallar and Adil [9]		✓	✓			✓					Hierarchical Planning Approach, Dynamic Allocation, Goal Programming, Stochastic Programming
Leung and Chang [10]		✓	✓								Goal Programming
Baykasoglu and Gocken [11]		✓		✓							Ranking Method, Tabu Search, Metaheuristic Algorithm
Che and Chiang [12]		✓					✓				Pareto Genetic Algorithm, Modified Pareto Genetic Algorithm
Liu et al.[13]		✓					✓				Aggregate Production Planning, Genetic Algorithm
Sillekens et al. [14]	✓										Mixed Integer Linear Programming, Linear Approximation, Aggregate Production Planning
Ramezanian et al. [15]	✓						✓				Two-phase Aggregate Production Planning, Genetic Algorithm, Tabu search Mixed Integer Linear Programming Model
Mortezaei et al. [16]		✓		✓			✓				Aggregate Production Planning
Liu and Papageorgiou [17]		✓			✓						Mixed-integer Linear Programming, ε-constraint Method and Lexicographic Mini-max Method
Madadi and Wong [18]		✓		✓							Multiobjective Fuzzy Aggregate Production Planning, IBM ILOG CPLEX Optimization Studio Software
Chen and Huang [19]		✓		✓							Aggregate Production Planning, Parametric Programming
Silva and Marins [20]		✓	✓	✓							Fuzzy Goal Programming, Agricultural and Logistics Phase
Singh and Yadav [21]		✓							✓		Interval-valued Intuitionistic, Multiobjective Linear Programming
Gholamian et al. [22]		✓		✓							Mixed-integer Non-linear Programming, GAMS Software, Supply Chain Planning
Lin et al. [23]		✓		✓							Integrated Production Planning, Multiobjective Optimization Evolutionary Algorithm
Mondal et al. [24]	✓									✓	Neutrosophic Geometric Programming, Non-linear Optimization
Meistering and Stadtler [25]		✓						✓			Hierarchical Production Planning, Bi-level Programming
Zhao et al. [26]		✓				✓					Progressive Hedging Algorithm, Multi-stage Stochastic Programming
Goli et al. [27]		✓	✓								Robust Multiobjective Multi-period Aggregate Production Planning, Goal Programming
Hu et al. [28]	✓					✓					Two-stage Stochastic Programming,
Proposed Model		✓							✓	✓	Production Planning Problem, Intuitionistic fuzzy Programming, Neutrosophic Programming, Interval-valued Trapezoidal neutrosophic number

Definition 3 (Mahajan and Gupta [41]): An intuitionistic fuzzy (IF) set $\tilde{A}^I$ in X is a set of ordered triples $\tilde{A}^I = \left\{\left(x, \mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)\right) : x \in X\right\}$, where $\mu_{\tilde{A}^I}(x) : X \to [0,1]$ and $\upsilon_{\tilde{A}^I}(x) : X \to [0,1]$ represent the degree of membership and degree of non-membership of the element $x \in X$ being in $\tilde{A}^I$, respectively, such that $\forall x \in X, 0 \leq \mu_{\tilde{A}^I}(x) + \upsilon_{\tilde{A}^I}(x) \leq 1$.

An IF set $\tilde{A}^I = \left\{\left(x, \mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)\right) : x \in X\right\}$ in X

- ❖ The value of $h_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x) - \upsilon_{\tilde{A}^I}(x)$ is called the degree of non-determinacy (hesitancy) of the element $x \in X$ to $\tilde{A}^I$.
- ❖ is normal if there exists $x_0, x_1 \in X$ such that $\mu_{\tilde{A}^I}(x_0) = 1$ and $\upsilon_{\tilde{A}^I}(x_1) = 1$.
- ❖ is convex if $\forall x_1, x_2 \in X, 0 \leq \lambda \leq 1$,

$$\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left\{\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2)\right\} \text{ and}$$
$$\upsilon_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \leq \max\left\{\upsilon_{\tilde{A}^I}(x_1), \upsilon_{\tilde{A}^I}(x_2)\right\}.$$

Definition 4 (Ebrahimnejad and Verdegay [42]): An IF set $\tilde{A}^I = \left\{\left(x, \mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)\right) : x \in \Re\right\}$ of the real number $\Re$ is called an IF if

- ❖ $\tilde{A}^I$ is normal and convex IF set,
- ❖ $\mu_{\tilde{A}^I}$ is upper semi-continuous and $\upsilon_{\tilde{A}^I}$ is lower semi-continuous and
- ❖ Supp$\tilde{A}^I = \left\{x \in \Re : \upsilon_{\tilde{A}^I}(x) < 1\right\}$ is bounded.

Definition 5 (Smarandache [43]): Let X be a universe of discourse and let $x \in X$. A neutrosophic set A in X is characterized by a truth-membership function $\mu_A^T(x)$, an indeterminacy membership function $\sigma_A^I(x)$, and a falsity-membership function $\upsilon_A^F(x)$, where $\mu_A^T(x), \sigma_A^I(x), \upsilon_A^F(x) \in (0, 1), \forall x \in X$ and $0^+ \leq \sup \mu_A^T(x) + \sup \sigma_A^I(x) + \sup \upsilon_A^F(x) \leq 3^-$.

Definition 6 (Haibin et al. [44]): If X be a universe of discourse and if $x \in X$, a truth-membership function characterizes a single-valued neutrosophic set $\mu_A^T(x)$, an indeterminacy membership function $\sigma_A^I(x)$, and a falsity-membership function $\upsilon_A^F(x)$, where $\mu_A^T(x), \sigma_A^I(x), \upsilon_A^F(x) \in [0, 1], \forall x \in X$ and $0 \leq \sup \mu_A^T(x) + \sup \sigma_A^I(x) + \sup \upsilon_A^F(x) \leq 3$.

Definition 7 (Ishibuchi and Tanak [45]): An interval on $\Re$ is defined as $A = \left[a^L, a^R\right] = \left\{a : a^L \leq a \leq a^R, a \in \Re\right\}$, where a^L is left limit and a^R is the right limit of A.

Definition 8 (Ishibuchi and Tanak [45]): The interval is also defined by

$A = \langle a_c, a_w \rangle = \{a : a_c - a_w \leq a \leq a_c + a_w, a \in \Re\}$, where $a_c = \frac{1}{2}\left(a^R + a^L\right)$ is the centre and $a_w = \frac{1}{2}\left(a^R - a^L\right)$ is the width of A.

Definition 9 (Interval-Valued Neutrosophic (I.V.N.): set, Broumi and Smarandache [46]) Let X be a non-empty set. Then an interval-valued neutrosophic set $\tilde{A}^{IV}$ of X is defined as:

$$\tilde{A}^{IV} = \left\{ \begin{array}{l} \langle x; \left[\mu_k^{TL}, \mu_k^{TU}\right], \left[\sigma_k^{IL}, \sigma_k^{IU}\right], \left[\upsilon_k^{FL}, \upsilon_k^{FU}\right]\rangle \\ : x \in X \end{array} \right\},$$

where $\left[\mu_k^{TL}, \mu_k^{TU}\right], \left[\sigma_k^{IL}, \sigma_k^{IU}\right]$ and $\left[\upsilon_k^{FL}, \upsilon_k^{FU}\right] \subset [0, 1]$ for each $x \in X$

Definition 10 (Broumi and Smarandache [46]): Let

$$\tilde{A}^{IV} = \left\{ \begin{array}{l} \langle x; \left[\mu_k^{TL}, \mu_k^{TU}\right], \left[\sigma_k^{IL}, \sigma_k^{IU}\right], \left[\upsilon_k^{FL}, \upsilon_k^{FU}\right]\rangle \\ : x \in X \end{array} \right\}$$

be I.V.N. set, then

(i) $\tilde{A}^{IV}$ is empty if $\mu_k^{TL} = \mu_k^{TU} = 0, \sigma_k^{IL} = \sigma_k^{IU} = 1, \upsilon_k^{FL} = \upsilon_k^{FU} = 1$, for all $x \in \tilde{A}$

(ii) Let $\underline{0} = \langle x; 0, 1, 1\rangle$, and $\underline{1} = \langle x; 1, 0, 0\rangle$.

Definition 11 (Interval-Valued Trapezoidal Neutrosophic (IVTN) Number): Let $\mu_{\tilde{a}}, \sigma_{\tilde{a}}, \upsilon_{\tilde{a}} \subset [0, 1]$, and $a_1, a_2, a_3, a_4 \in \Re$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$. Then an interval-valued trapezoidal fuzzy neutrosophic number,

$$\tilde{a} = \left\langle (a_1, a_2, a_3, a_4); \left[\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U\right], \left[\sigma_{\tilde{a}}^L, \sigma_{\tilde{a}}^U\right], \left[\upsilon_{\tilde{a}}^L, \upsilon_{\tilde{a}}^U\right]\right\rangle,$$

Whose degrees of membership, the degrees of indeterminacy and the degrees of non-membership are

$$\mu_{\tilde{a}}(x) = \begin{cases} \mu_{\tilde{a}}^U\left(\dfrac{x - a_1}{a_2 - a_1}\right), & \text{for } a_1 \leq x \leq a_2, \\ \mu_{\tilde{a}}^U, & \text{for } a_2 \leq x \leq a_3, \\ \mu_{\tilde{a}}^U\left(\dfrac{a_4 - x}{a_4 - a_3}\right), & \text{for } a_3 \leq x \leq a_4, \\ 0, & \text{Otherwise} \end{cases}$$

$$\sigma_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x + \sigma_{\tilde{a}}^L(x - a_1)}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2, \\ \sigma_{\tilde{a}}^L, & \text{for } a_2 \leq x \leq a_3, \\ \dfrac{x - a_3 + \sigma_{\tilde{a}}^L(a_4 - x)}{a_4 - x_3}, & \text{for } a_3 \leq x \leq a_4, \\ 1, & \text{Otherwise} \end{cases}$$

$$\upsilon_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x + \upsilon_{\tilde{a}}^L(x - a_1)}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2, \\ \upsilon_{\tilde{a}}^L, & \text{for } a_2 \leq x \leq a_3, \\ \dfrac{x - a_3 + \upsilon_{\tilde{a}}^L(a_4 - x)}{a_4 - x_3}, & \text{for } a_3 \leq x \leq a_4, \\ 1, & \text{Otherwise} \end{cases}$$

Definition 12 (Arithmetic Operations): Let, $\tilde{a} = \left\langle (a_1, a_2, a_3, a_4); \left[\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U\right], \left[\sigma_{\tilde{a}}^L, \sigma_{\tilde{a}}^U\right], \left[\upsilon_{\tilde{a}}^L, \upsilon_{\tilde{a}}^U\right]\right\rangle$, and $\tilde{b} = \left\langle (b_1, b_2, b_3, b_4); \left[\mu_{\tilde{b}}^L, \mu_{\tilde{b}}^U\right], \left[\sigma_{\tilde{b}}^L, \sigma_{\tilde{b}}^U\right], \left[\upsilon_{\tilde{b}}^L, \upsilon_{\tilde{b}}^U\right]\right\rangle$ be two I.V.N. number. Then,

1. $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) : A, B, C\rangle$,

2. $\tilde{a} - \tilde{b} = \langle (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4) : A, B, C\rangle$,

$$\mathbf{3.}\ \tilde{a} * \tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4) : A, B, C\rangle, \\ \text{if } a_4 > 0,\ b_4 > 0 \\ \langle (a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1) : A, B, C\rangle, \\ \text{if } a_4 < 0,\ b_4 > 0 \\ \langle (a_4 b_4, a_3 b_3, a_2 b_2, a_1 b_1) : A, B, C\rangle, \\ \text{if } a_4 < 0,\ b_4 < 0 \end{cases}$$

$$\mathbf{4.}\ \tilde{a}/\tilde{b} = \begin{cases} \langle (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1) : A, B, C\rangle, \\ \text{if } a_4 > 0,\ b_4 > 0 \\ \langle (a_4/b_4, a_3/b_3, a_2/b_2, a_1/b_1) : A, B, C\rangle, \\ \text{if } a_4 < 0,\ b_4 > 0 \\ \langle (a_4/b_1, a_3/b_2, a_2/b_3, a_1/b_4) : A, B, C\rangle, \\ \text{if } a_4 < 0,\ b_4 < 0 \end{cases}$$

$$\mathbf{5.}\ k\tilde{a} = \begin{cases} \left\langle \begin{array}{l} \left(\begin{array}{l} ka_1, ka_2, \\ ka_3, ka_4 \end{array}\right); \left[\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U\right], \\ \left[\sigma_{\tilde{a}}^L, \sigma_{\tilde{a}}^U\right], \left[\upsilon_{\tilde{a}}^L, \upsilon_{\tilde{a}}^U\right] \end{array} \right\rangle, & \text{if } \tilde{a} > 0 \\ \left\langle \begin{array}{l} \left(\begin{array}{l} ka_4, ka_3, \\ ka_2, ka_1 \end{array}\right); \left[\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U\right], \\ \left[\sigma_{\tilde{a}}^L, \sigma_{\tilde{a}}^U\right], \left[\upsilon_{\tilde{a}}^L, \upsilon_{\tilde{a}}^U\right] \end{array} \right\rangle, & \text{if } \tilde{a} < 0 \end{cases}$$

$$\mathbf{6.}\ \tilde{a}^{-1} = \left\langle \begin{array}{l} (1/a_4, 1/a_3, 1/a_2, 1/a_1); \\ \left[\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U\right], \left[\sigma_{\tilde{a}}^L, \sigma_{\tilde{a}}^U\right], \left[\upsilon_{\tilde{a}}^L, \upsilon_{\tilde{a}}^U\right] \end{array} \right\rangle, \quad \tilde{a} \neq 0$$

where,

$$A = \left[\min\left[\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U\right], \quad \min\left[\mu_{\tilde{b}}^L, \mu_{\tilde{b}}^U\right]\right],$$
$$B = \left[\max\left[\sigma_{\tilde{a}}^L, \sigma_{\tilde{a}}^U\right], \quad \max\left[\sigma_{\tilde{b}}^L, \sigma_{\tilde{b}}^U\right]\right] \text{ and}$$
$$C = \left[\max\left[\upsilon_{\tilde{a}}^L, \upsilon_{\tilde{a}}^U\right], \max\left[\upsilon_{\tilde{b}}^L, \upsilon_{\tilde{b}}^U\right]\right].$$

Definition 13 (Score Function, Tharmaraiselvi and Santhi [47]): the score function for the I.V.N. number

$\tilde{a} = \langle (a_1, a_2, a_3, a_4) ; [\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U], [\sigma_{\tilde{a}}^L, \sigma_{\tilde{a}}^U], [\upsilon_{\tilde{a}}^L, \upsilon_{\tilde{a}}^U] \rangle$ is defined as

$$S(\tilde{a}) = \frac{1}{16}(a_1 + a_2 + a_3 + a_4) \times [\mu_{\tilde{a}}(x) + (1 - \sigma_{\tilde{a}}(x)) + (1 - \upsilon_{\tilde{a}}(x))] \quad (1)$$

IV. SOLUTION METHODOLOGIES

Let define the standard form of a multiobjective optimization model, mathematically it can be expressed as follows:

$$\begin{aligned} &Max\ (Min)\, f(x) = \{Z_1(x), Z_2(x), \ldots, Z_k(x)\} \\ &\text{S.t. } g(x) \le 0, \quad x \in X \end{aligned}$$

The multiobjective optimization problem is solved for each objective function separately to obtain the individual optimal solution while ignoring the other objective functions. This procedure repeated for getting individual objective function optimal solution for each objective functions. After that, the upper and lower bounds for each objective functions U_k and $L_k (k = 1, 2, \ldots, K)$ are identified.

In the following sections, we will be discussed the Intuitionistic Fuzzy Programming and Neutrosophic Compromise Programming approaches for solving such MOMPP of a firm.

A. INTUITIONISTIC FUZZY PROGRAMMING

1) MAXIMIZE TYPE OBJECTIVE FUNCTION

Maximize Type Objective Function: The membership and non-membership functions, when the objective functions are maximized type can be defined as follows:

Membership function

$$\mu_k(Z_k(x)) = \begin{cases} 0, & Z_k(x) < L_k \\ \dfrac{Z_k(x) - L_k}{U_k - L_k}, & Z_k(x) \in [L_k, U_k] \\ 1, & Z_k(x) > U_k \end{cases}$$

Non-membership function

$$\upsilon_k(Z_k(x)) = \begin{cases} 1, & Z_k(x) < R_k \\ \dfrac{U_k - Z_k(x)}{U_k - R_k}, & Z_k(x) \in [R_k, U_k] \\ 0, & Z_k(x) > U_k \end{cases}$$

where, $R_k < L_k < U_k$

2) MINIMIZE TYPE OBJECTIVE FUNCTION

Minimize Type Objective Function: Consequently, The membership and non-membership functions for the minimization type objective functions are as follows:

Membership function

$$\mu_k(Z_k(x)) = \begin{cases} 1, & Z_k(x) < L_k \\ \dfrac{U_k - Z_k(x)}{U_k - L_k}, & Z_k(x) \in [L_k, U_k] \\ 0, & Z_k(x) > U_k \end{cases}$$

Non-membership function

$$\upsilon_k(Z_k(x)) = \begin{cases} 0, & Z_k(x) < L_k \\ \dfrac{Z_k(x) - L_k}{W_k - L_k}, & Z_k(x) \in [L_k, W_k] \\ 1, & Z_k(x) > W_k \end{cases}$$

where, $L_k < U_k < W_k$.

The membership and non-membership functions defined above are used to convert the multiobjective optimization problem to a single objective optimization problem is as follows:

$$\begin{aligned} &Max\ \mu_k, \quad Min\ \upsilon_k \\ &\text{Subject to } \mu_k^M = \frac{Z_k(x) - L_k}{U_k - L_k}, \quad \upsilon_k^{NM} = \frac{U_k - Z_k(x)}{U_k - R_k}, \end{aligned}$$

for maximize type objective where $R_k < L_k < U_k$

$$\mu_k^M = \frac{U_k - Z_k(x)}{U_k - L_k}, \quad \upsilon_k^{NM} = \frac{Z_k(x) - L_k}{W_k - L_k},$$

for minimize type objective where $L_k < U_k < W_k$

$$\begin{aligned} &\mu_k \ge \mu_k^M, \quad \upsilon_k \le \upsilon_k^{NM}, \ \mu_k \ge \upsilon_k, \ \mu_k + \upsilon_k \le 1, \\ &\mu_k, \upsilon_k \in [0, 1], \quad k = 1, 2, \ldots, K \\ &g(x) \le 0, \quad x \in X \end{aligned}$$

Or equivalently,

$$\begin{aligned} &Max \sum_{k=1}^{K} (\mu_k - \upsilon_k) \\ &\text{Subject to } \mu_k^M = \frac{Z_k(x) - L_k}{U_k - L_k}, \quad \upsilon_k^{NM} = \frac{U_k - Z_k(x)}{U_k - R_k}, \end{aligned}$$

for maximize type objective, where $R_k < L_k < U_k$

$$\mu_k^M = \frac{U_k - Z_k(x)}{U_k - L_k}, \quad \upsilon_k^{NM} = \frac{Z_k(x) - L_k}{W_k - L_k},$$

for minimize type objective, where $L_k < U_k < W_k$

$$\begin{aligned} &\mu_k \ge \mu_k^M, \quad \upsilon_k \le \upsilon_k^{NM}, \ \mu_k \ge \upsilon_k, \ \mu_k + \upsilon_k \le 1, \\ &\mu_k, \upsilon_k \in [0, 1], \quad k = 1, 2, \ldots, K \\ &g(x) \le 0, \quad x \in X \end{aligned}$$

B. NEUTROSOPHIC COMPROMISE PROGRAMMING

1) MAXIMIZE TYPE OBJECTIVE FUNCTION

Consider the multiobjective optimization problem wherein objective functions are maximization type. In the neutrosophic compromise programming approach first step is to define the lower and upper values, that is,

$$\begin{aligned} &U_k^T = U_k, \quad L_k^T = L_k, \ (\text{for truth membership}) \\ &U_k^I = U_k^T, \quad L_k^I = L_k^T + q_k' \,(U_k^T - L_k^T), \\ &\qquad (\text{for Indeterminacy membership}) \\ &U_k^F = L_k^T + q_k(U_k^T - L_k^T), \quad L_k^F = L_k^T, \\ &\qquad (\text{for falsity membership}, k = 1, 2, \ldots, K) \end{aligned}$$

where q_k and q'_k are tolerance variables, which are chosen for the falsity and indeterminacy Membership function. The membership functions for the neutrosophic environment can be defined as follows:

$$\mu_k^T = \begin{cases} 0, & Z_k \le L_k^T \\ \dfrac{Z_k - L_k^T}{U_k^T - L_k^T}, & L_k^T \le Z_k \le U_k^T \\ 1, & Z_k \ge U_k^T \end{cases}$$

$$\sigma_k^I = \begin{cases} 0, & Z_k \le L_k^I \\ \dfrac{Z_k - L_k^I}{U_k^I - L_k^I}, & L_k^I \le Z_k \le U_k^I \\ 1, & Z_k \ge U_k^I \end{cases}$$

$$\upsilon_k^F = \begin{cases} 1, & Z_k \le L_k^F \\ \dfrac{U_k^F - Z_k}{U_k^F - L_k^F}, & L_k^F \le Z_k \le U_k^F \\ 0, & Z_k \ge U_k^F \end{cases}$$

2) MINIMIZE TYPE OBJECTIVE FUNCTION

Consider the multiobjective optimization problem wherein objective functions are minimization type. In the neutrosophic compromise programming approach first step is to define the lower and upper values, that is,

$$\begin{aligned} U_k^T &= U_k, \quad L_k^T = L_k, \text{ for truth membership} \\ U_k^I &= L_k^T + q'_k(U_k^T - L_k^T), \quad L_k^I = L_k^T, \\ &\quad \text{for Indeterminacy membership} \\ U_k^F &= U_k^T, L_k^F = L_k^T + q_k(U_k^T - L_k^T), \\ &\quad \text{for falsity membership} \quad k = 1, 2, \ldots, K \end{aligned}$$

where q_k and q'_k are tolerance variables, which are chosen for the falsity and indeterminacy membership functions. The membership functions for the neutrosophic environment can be defined as follows:

$$\mu_k^T = \begin{cases} 1, & Z_k \le L_k^T \\ \dfrac{U_k^T - Z_k}{U_k^T - L_k^T}, & L_k^T \le Z_k \le U_k^T \\ 0, & Z_k \ge U_k^T \end{cases}$$

$$\sigma_k^I = \begin{cases} 1, & Z_k \le L_k^I \\ \dfrac{U_k^I - Z_k}{U_k^I - L_k^I}, & L_k^I \le Z_k \le U_k^I \\ 0, & Z_k \ge U_k^I \end{cases}$$

$$\upsilon_k^F = \begin{cases} 0, & Z_k \le L_k^F \\ \dfrac{Z_k - L_k^F}{U_k^F - L_k^F}, & L_k^F \le Z_k \le U_k^F \\ 1, & Z_k \ge U_k^F \end{cases}$$

a: MODEL I

Maximize the Truth (μ_k^T) and Indeterminacy (σ_k^I) membership functions; and minimize the falsity (υ_k^F) membership functions.

The above-discussed membership function is used, and the final problem is defined as follows:

$$\begin{aligned} &Max\ \mu_k, Max\ \sigma_k, Min\ \upsilon_k \\ &\text{Subject to } \mu_k^T = \frac{U_k^T - Z_k}{U_k^T - L_k^T}, \quad \sigma_k^I = \frac{U_k^I - Z_k}{U_k^I - L_k^I}, \\ &\qquad \upsilon_k^F = \frac{Z_k - L_k^F}{U_k^F - L_k^F} \\ &\qquad \mu_k \ge \mu_k^T, \quad \sigma_k \ge \sigma_k^I, \upsilon_k \le \upsilon_k^F \\ &\qquad \mu_k \ge \sigma_k, \quad \mu_k \ge \upsilon_k, \mu_k + \sigma_k + \upsilon_k \le 3, \\ &\qquad \mu_k, \quad \sigma_k, \upsilon_k \in [0, 1], \ k = 1, 2, \ldots, K \\ &\qquad g(x) \le 0 \\ &\qquad x \in X \end{aligned}$$

Or equivalently

$$\begin{aligned} &Max \sum_{k=1}^{K} (\mu_k + \sigma_k - \upsilon_k) \\ &\text{Subject to } \mu_k^T = \frac{U_k^T - Z_k}{U_k^T - L_k^T}, \quad \sigma_k^I = \frac{U_k^I - Z_k}{U_k^I - L_k^I}, \\ &\qquad \upsilon_k^F = \frac{Z_k - L_k^F}{U_k^F - L_k^F} \\ &\qquad \mu_k \ge \mu_k^T, \quad \sigma_k \ge \sigma_k^I, \upsilon_k \le \upsilon_k^F \\ &\qquad \mu_k \ge \sigma_k, \quad \mu_k \ge \upsilon_k, \ \mu_k + \sigma_k + \upsilon_k \le 3, \\ &\qquad \mu_k, \quad \sigma_k, \upsilon_k \in [0, 1], \ k = 1, 2, \ldots, K \\ &\qquad g(x) \le 0 \\ &\qquad x \in X \end{aligned}$$

b: MODEL II

Maximize the Truth (μ_k^T) membership functions, and minimize the Indeterminacy (σ_k^I) and falsity (υ_k^F) membership functions.

The above-discussed membership function is used, and the final problem is defined as follows:

$$\begin{aligned} &Max\ \mu_k, \quad Min\ \sigma_k, \ Min\ \upsilon_k \\ &\text{Subject to } \mu_k^T = \frac{U_k^T - Z_k}{U_k^T - L_k^T}, \quad \sigma_k^I = \frac{U_k^I - Z_k}{U_k^I - L_k^I}, \\ &\qquad \upsilon_k^F = \frac{Z_k - L_k^F}{U_k^F - L_k^F} \\ &\qquad \mu_k \ge \mu_k^T, \quad \sigma_k \le \sigma_k^I, \ \upsilon_k \le \upsilon_k^F \\ &\qquad \mu_k \ge \sigma_k, \quad \mu_k \ge \upsilon_k, \ \mu_k + \sigma_k + \upsilon_k \le 3, \\ &\qquad \mu_k, \quad \sigma_k, \upsilon_k \in [0, 1], \ k = 1, 2, \ldots, K \\ &\qquad g(x) \le 0 \\ &\qquad x \in X \end{aligned}$$

Or equivalently

$$Max \sum_{k=1}^{K} (\mu_k - \sigma_k - \upsilon_k)$$

$$\text{Subject to } \mu_k^T = \frac{U_k^T - Z_k}{U_k^T - L_k^T}, \quad \sigma_k^I = \frac{U_k^I - Z_k}{U_k^I - L_k^I},$$

$$\upsilon_k^F = \frac{Z_k - L_k^F}{U_k^F - L_k^F}$$

$$\mu_k \geq \mu_k^T, \quad \sigma_k \leq \sigma_k^I, \ \upsilon_k \leq \upsilon_k^F$$

$$\mu_k \geq \sigma_k, \quad \mu_k \geq \upsilon_k, \ \mu_k + \sigma_k + \upsilon_k \leq 3,$$

$$\mu_k, \quad \sigma_k, \upsilon_k \in [0, 1], \ k = 1, 2, \ldots, K$$

$$g(x) \leq 0$$

$$x \in X$$

V. MULTIOBJECTIVE MULTIPRODUCT PRODUCTION PLANNING PROBLEM

Every production firms want to maximize profit while maintaining the manufactured product. Quality and reliability are always in priority. Some factors are considered to realize the firm's objectives, and those include the satisfaction of customers demand, timely delivery of goods and services and many others. The firm's manager always wants a perfect balance between total production management and market demands to achieve such objectives. The firms must be formulated a production plan by using scientific methods to provide exact layouts before starting the production process. This case study seeks to establish an efficient production plan that will minimize the production cost and holding cost and maximize a manufacturing firm's profit. The study's primary objective is to determine the optimal number of goods to be produced regarding the products' future expected demands. The production planning and control model for the organization are represented as in fig. 1.

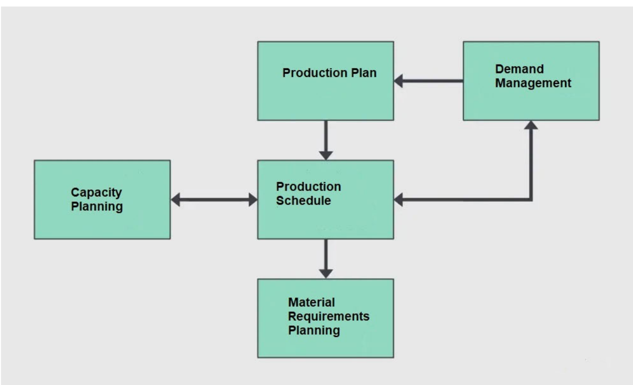

FIGURE 1. **Production planning and control model.**

NOMENCLATURE

INDICES

k - Index for the objective function, $k = 1, 2, \ldots, K$
j - Index for the manufactured item, $j = 1, 2, \ldots, J$
i - Index for machine timing, $i = 1, 2, \ldots, I$
l - Index for the raw-material, $l = 1, 2, \ldots, L$

DECISION VARIABLE

x_j -Manufactured items

PARAMETERS

I_j -Inventory holding cost of the j^{th} unit (in $Rs.$)
R_j -Profit of j^{th} unit (in $Rs.$)
P_j -The production cost of the j^{th} unit (in $Rs.$)
q_{kj} -Quantity of k^{th} Raw material in the j^{th} product
Q_l^* -Economic order quantity of l^{th} raw material
s_j -The volume of the j^{th} unit (in ft^3)
S -The volume of the warehouse (in ft^3)
M_{ij} -Working time (in hours) of the i^{th} machine on j^{th} unit
b_i -Total Working time (in hours) for i^{th} machine
D_{Mj} -Lower (mean) limit of demand
D_{Uj} -Upper (3σ) limit of demand

OBJECTIVE FUNCTION

Z_1 -Minimize the production cost
Z_2 -Minimize the holding cost
Z_3 -Maximize the profit

Haq *et al.* [48] has discussed the multiobjective multiproduct production planning mathematical model is as follows (2), as shown at the bottom of the next page.

PROBLEM OBJECTIVES

The primary aim is to minimize the total production cost of manufacture items which includes different types of cost (raw material cost, labour cost, transportation cost)

$$Min\ Z_1 = \sum_{j=1}^{J} P_j x_j$$

The second objective is to minimize the holding/ carrying cost to reach the lower limit of demand under constraints.

$$Min\ Z_2 = \sum_{j=1}^{J} I_j x_j$$

Every company has one primary goal to maximize its profit which can be achieved by minimizing its overall production cost.

$$Max\ Z_3 = \sum_{j=1}^{J} R_j x_j$$

SYSTEM CONSTRAINTS

The first constraint is related to the availability of the warehouse's space

$$\sum_{j=1}^{J} s_j x_j \leq S$$

TABLE 2. Machine availability.

Machines	No. of Availability
Sarvo voltage stabilizer	3
Generator	3
Drill Machine	4
Lensar	5
Lathe Machine	6
Surface Grinder	3
Safer Machine	2
Milling Machine	3
Power Press	4
Power Putting Plan	4
Forging	3
Polishing	6
Ultrasonic Machine	3
Stripping	3
Skying packing	4
E.T.P. Plant	3

TABLE 3. Product detail.

Products	Zinc (in kg)	Iron (in kg)	Brass (in kg)	Aluminium (in kg)
Hinges	-	0.280	-	-
Door and Window Hardware	-	-	-	0.150
Gate Hardware	-	0.200	-	-
Georgian Scroll Lever Handle	0.550	-	-	-
Home Accessories	-	-	0.450	-
Hooks	0.120	-	-	-
Security Hardware	-	0.210	-	-
Numerals and Alphabets	-	-	0.150	-
Cabinet Hardware	-	-	0.300	-
Handrail Bracket	-	0.350	-	-
Floor Mounted Door Stop	-	-	0.250	-
Reinforcement Hardware	-	0.150	-	-

The second constraint is related to the total machine timing

$$\sum_{j=1}^{J} M_{ij}x_j \leq b_i, \quad i = 1, 2, \ldots, I$$

The third constraint is related to the total availability of the raw materials

$$\sum_{j=1}^{J} q_{lj}x_j \leq Q_l^*, \quad l = 1, 2, \ldots, L$$

moreover, the fourth constraint is about the upper and lower limit of the demands

$$D_{Mj} \leq x_j \leq D_{Uj}, \quad j = 1, 2, \ldots, J$$

VI. CASE STUDY (HARDWARE FIRM)

A case study of a product mix production planning is used to validate the solution procedure. The data is used to formulate the multiproduct production planning for a hardware firm that produces different types of hardware metals items, such as (i) Hinges, (ii) Door and Window Hardware, (iii) Gate Hardware, (iv) Georgian scroll Lever Handle, (v) Home Accessories, (vi) Hooks, (vii) Security Hardware, (viii) Numerals and Alphabets, (ix) Cabinet Hardware, (x) Handrail Bracket, (xi) Floor Mounted Door Stop, (xii) Reinforcement Hardware. The following data are collected from a firm: machine availability, production cost, expected profit, production run time of the machine and the demand for each item. These data are summarized in Tables 2-8.

Table 5 summarizes the average time consumed by each machine during production hours.

$$optimize \begin{cases} Min\ Z_1 = \sum_{j=1}^{J} P_j x_j \quad \text{(Related to production cost)} \\ Min\ Z_2 = \sum_{j=1}^{J} I_j x_j \quad \text{(Relatec to holding/carring cost)} \\ Max\ Z_3 = \sum_{j=1}^{J} R_j x_j \quad \text{(Related to profit)} \\ \text{Subject to set of constraints:} \\ \sum_{j=1}^{J} s_j x_j \leq S \quad \text{(constraint related to warehouse space)} \\ \sum_{j=1}^{J} M_{ij} x_j \leq b_i, \quad i = 1, 2, \ldots, I \ \text{(constraint related to machine timing)} \\ \sum_{j=1}^{J} q_{lj} x_j \leq Q_l^*, \quad l = 1, 2, \ldots, L \ \text{(constraint related to raw material)} \\ D_{Mj} \leq x_j \leq D_{Uj}, \quad j = 1, 2, \ldots, J \ \text{(constraint related to demands)} \end{cases} \tag{2}$$

TABLE 4. Production cost, inventory holding cost, profit and volume of per dozen unit.

Items	Production costs	Inventory Holding cost	Expected Profit	Volume
Hinges	540	12.00	75	0.77
Door and Window Hardware	1250	5.00	215	0.28
Gate Hardware	460	6.50	85	0.54
Georgian Scroll Lever Handle	2530	19.75	535	3.60
Home Accessories	3480	14.25	875	1.80
Hooks	830	8.00	190	0.57
Security Hardware	475	6.00	95	0.60
Numerals and Alphabets	1465	10.50	410	0.64
Cabinet Hardware	3270	13.50	725	1.80
Handrail Bracket	585	9.25	115	1.20
Floor Mounted Door Stop	2315	18.50	450	0.72
Reinforcement Hardware	435	8.25	105	0.40
Total Warehouse volume				1200

Table 6 summarizes the monthly demand for each item (in Dozen)

Before formulating the problem, we have to find out the 3σ limit of the demand (given in Table 6) for the items to be produced by the manufacturer.

Table 7 for the upper and lower limit of the items to be manufactured during the production run. Table 8 summarized the expected annual demand, ordering cost and carrying cost for raw material.

where the economic order quantity Q^* is calculated by using the expression:

$$\text{Economic order quantity } (Q^*) = \sqrt{\frac{2 \times \text{Annual Demand} \times \text{Ordering cost}}{\text{Carrying cost}}}$$

Table 7 for the upper and lower limit of the items which are to be manufactured during a production run.

First, we solved the problem as the single objective optimization problem to set the goals values. Z_1, Z_2, Z_3, as shown at the bottom of the page.

Based on the goal values, a payoff matrix is constructed.

The payoff matrix is used to define the aspiration levels for the objective functions. The bounds for all the three objective functions are determined as: $1427045 \leq Z_1(x) \leq 1634160$, $10770.25 \leq Z_2(x) \leq 11994.50$ and $309315 \leq Z_3(x) \leq 355510$. The memberships and non-membership functions for the three objective functions for using intuitionistic fuzzy

$$Z_1 = 540x_1 + 1250x_2 + 460x_3 + 2530x_4 + 3480x_5 + 830x_6 + 475x_7 + 1465x_8 + 3270x_9 + 585x_{10} + 2315x_{11} + 435x_{12}$$
$$Z_2 = 12x_1 + 5x_2 + 6.5x_3 + 19.75x_4 + 14.25x_5 + 8x_6 + 6x_7 + 10.5x_8 + 13.5x_9 + 9.25x_{10} + 18.5x_{11} + 8.25x_{12}$$
$$Z_3 = 75x_1 + 215x_2 + 85x_3 + 535x_4 + 875x_5 + 190x_6 + 95x_7 + 410x_8 + 725x_9 + 115x_{10} + 450x_{11} + 105x_{12}$$

Subject to constraints

$$\begin{pmatrix} 0.77 & 0.28 & 0.54 & 3.6 & 1.8 & 0.57 & 0.6 & 0.64 & 1.8 & 1.2 & 0.72 & 0.4 \\ 1.5 & 0.1 & 0.3 & 0.5 & 1.5 & 0.1 & 1.1 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.2 & 0.4 & 1.2 & 0.5 & 0.2 & 1.1 & 0.1 & 1.2 & 1.3 & 0.1 & 0.1 & 0.1 \\ 0.3 & 1.5 & 0.4 & 1.1 & 0.2 & 1.5 & 0.5 & 0.2 & 0.1 & 0.1 & 0.3 & 0.1 \\ 1.5 & 0.5 & 1.2 & 1.4 & 0.6 & 1.0 & 0.1 & 1.1 & 0.1 & 0.2 & 1.1 & 1.1 \\ 1.5 & 1.5 & 1.3 & 1.3 & 0.5 & 0.6 & 1.3 & 1.1 & 1.2 & 1.0 & 1.2 & 0.8 \\ 1.1 & 0.5 & 0.2 & 1.5 & 0.3 & 0.4 & 0.2 & 0.5 & 0.2 & 1.0 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 & 1.1 & 0.3 & 0.1 & 0.3 & 0.1 & 0.1 & 0.3 & 0.6 & 0.2 \\ 0.3 & 1.0 & 0.3 & 0.4 & 0.3 & 0.5 & 0.6 & 1.2 & 0.4 & 0.5 & 0.2 & 1.0 \\ 0.2 & 0.2 & 0.3 & 1.0 & 0.3 & 1.5 & 0.4 & 0.5 & 1.0 & 0.7 & 1.0 & 0.1 \\ 0.1 & 0.2 & 1.0 & 0.3 & 1.5 & 0.7 & 1.0 & 0.4 & 0.5 & 1.2 & 0.6 & 0.3 \\ 1.5 & 1.5 & 1.1 & 1.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.5 & 1.2 & 1.5 & 1.3 & 0.3 & 1.2 & 0.6 & 0.5 & 1.5 & 0.8 & 1.1 & 1.5 \\ 0.2 & 0.6 & 1.5 & 0.5 & 0.7 & 0.1 & 0.3 & 0.4 & 0.1 & 1.0 & 0.2 & 0.5 \\ 0.3 & 0.4 & 1.3 & 0.1 & 1.0 & 0.5 & 0.2 & 0.1 & 1.0 & 1.2 & 0.1 & 0.5 \\ 1.0 & 0.6 & 0.2 & 0.1 & 0.5 & 0.6 & 0.3 & 1.1 & 0.7 & 0.3 & 1.0 & 0.6 \\ 1.1 & 1.1 & 0.5 & 0.7 & 0.4 & 0.1 & 0.2 & 0.2 & 0.1 & 1.0 & 0.2 & 0.1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{pmatrix} \leq \begin{pmatrix} 1200 \\ 624 \\ 624 \\ 832 \\ 1040 \\ 1248 \\ 624 \\ 416 \\ 624 \\ 832 \\ 832 \\ 624 \\ 1248 \\ 624 \\ 624 \\ 832 \\ 624 \end{pmatrix}$$

$$110 \leq x_1 \leq 140,\quad 108 \leq x_2 \leq 135,\ 104 \leq x_3 \leq 128,\ 96 \leq x_4 \leq 126,\ 84 \leq x_5 \leq 99,\ 86 \leq x_6 \leq 104,$$
$$76 \leq x_7 \leq 88,\quad 60 \leq x_8 \leq 75,\ 70 \leq x_9 \leq 79,\ 67 \leq x_{10} \leq 88,\ 68 \leq x_{11} \leq 80,\ 66 \leq x_{12} \leq 81,$$
$$x_j \in \text{integer} \quad j = 1, 2, \ldots, 12$$

TABLE 5. Average time consumed.

Machine/ Item	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
M_1	1.5	0.1	0.3	0.5	1.5	0.1	1.1	0.5	-	-	-	--
M_2	0.2	0.4	1.2	0.5	0.2	1.1	0.1	1.2	1.3	0.1	0.1	0.1
M_3	0.3	1.5	0.4	1.1	0.2	1.5	0.5	0.2	0.1	0.1	0.3	0.1
M_4	1.5	0.5	1.2	1.4	0.6	1.0	0.1	1.1	0.1	0.2	1.1	1.1
M_5	1.5	1.5	1.3	1.3	0.5	0.6	1.3	1.1	1.2	1.0	1.2	0.8
M_6	1.1	0.5	0.2	1.5	0.3	0.4	0.2	0.5	0.2	1.0	0.1	0.3
M_7	0.2	0.1	0.2	1.1	0.3	0.1	0.3	0.1	0.1	0.3	0.6	0.2
M_8	0.3	1.0	0.3	0.4	0.3	0.5	0.6	1.2	0.4	0.5	0.2	1.0
M_9	0.2	0.2	0.3	1.0	0.3	1.5	0.4	0.5	1.0	0.7	1.0	0.1
M_{10}	0.1	0.2	1.0	0.3	1.5	0.7	1.0	0.4	0.5	1.2	0.6	0.3
M_{11}	1.5	1.5	1.1	1.1	-	-	-	-	-	-	-	-
M_{12}	1.5	1.2	1.5	1.3	0.3	1.2	0.6	0.5	1.5	0.8	1.1	1.5
M_{13}	0.2	0.6	1.5	0.5	0.7	0.1	0.3	0.4	0.1	1.0	0.2	0.5
M_{14}	0.3	0.4	1.3	0.1	1.0	0.5	0.2	0.1	1.0	1.2	0.1	0.5
M_{15}	1.0	0.6	0.2	0.1	0.5	0.6	0.3	1.1	0.7	0.3	1.0	0.6
M_{16}	1.1	1.1	0.5	0.7	0.4	0.1	0.2	0.2	0.1	1.0	0.2	0.1

TABLE 6. The demand for each item (in Dozen).

Month/Items	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
JAN	125	118	112	102	82	93	75	65	69	73	71	69
FEB	105	101	107	98	85	85	72	61	70	69	68	65
MAR	112	109	103	94	81	88	77	58	66	64	65	60
APR	108	105	105	101	83	81	75	53	67	61	61	67
MAY	98	97	93	97	87	78	71	57	68	63	63	67
JUNE	110	115	108	105	89	89	78	63	72	71	70	69
JULY	130	127	118	110	95	97	82	69	75	76	74	79
AUG	95	98	89	88	88	78	78	61	65	61	70	66
SEP	107	101	99	76	81	81	73	58	69	55	64	62
OCT	105	103	102	83	76	82	75	53	70	59	66	59
NOV	111	107	106	89	78	85	69	58	72	72	68	64
DEC	119	114	101	103	81	89	72	62	74	75	71	66

TABLE 7. 3σ limit for the demand.

Items	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Mean	110	108	104	96	84	86	76	60	70	67	68	66
S.D.	10	9	8	10	5	6	4	5	3	7	4	5
$\overline{X}+3\sigma$	140	135	128	126	99	104	88	75	79	88	80	81

TABLE 8. Annual demand, ordering cost and carrying cost of raw material.

Raw Material	Annual Demand (in kg)	Ordering cost (per order)	Carrying cost (per kg per year)	Economic Order Quantity (Q^*) (in kg)
Zinc	7000	4000	23.50	1543.69
Iron	14000	1100	8	1962.14
Brass	12000	8000	72	1632.99
Aluminium	2400	1200	18	565.68

programming are constructed as follows:

$$\mu_1(Z_1(x)) = \begin{cases} 1 & \text{if } Z_1(x) < 1427045 \\ \dfrac{1634160 - Z_1(x)}{1634160 - 1427045} & \text{if } Z_1(x) \in [1427045, 1634160] \\ 0 & \text{if } Z_1(x) > 1634160 \end{cases}$$

$$\upsilon_1(Z_1(x)) = \begin{cases} 0 & \text{if } Z_1(x) < 1447756.5 \\ \dfrac{Z_1(x) - 1447756.5}{1634160 - 1447756.5} & \text{if } Z_1(x) \\ & \in [1447756.5, 1634160] \\ 1 & \text{if } Z_1(x) > 1634160 \end{cases}$$

$$\mu_2(Z_2(x)) = \begin{cases} 1 & \text{if } Z_2(x) < 10770.25 \\ \dfrac{11994.5 - Z_2(x)}{11994.5 - 10770.25} & \text{if } Z_2(x) \in [10770.25, 11994.5] \\ 0 & \text{if } Z_2(x) > 11994.5 \end{cases}$$

$$\upsilon_2(Z_2(x)) = \begin{cases} 0 & \text{if } Z_2(x) < 10892.675 \\ \dfrac{Z_2(x) - 10892.675}{11994.5 - 10892.675} & \text{if } Z_2(x) \\ & \in [10892.675, 11994.5] \\ 1 & \text{if } Z_2(x) > 11994.5 \end{cases}$$

$$\mu_3(Z_3(x)) = \begin{cases} 0 & \text{if } Z_3(x) < 309315 \\ \dfrac{Z_3(x) - 309315}{355510 - 309315} & \text{if } Z_3(x) \in [309315, 355510] \\ 1 & \text{if } Z_3(x) > 355510 \end{cases}$$

$$\upsilon_3(Z_3(x)) = \begin{cases} 1 & \text{if } Z_3(x) < 309315 \\ \dfrac{350890.5 - Z_3(x)}{350890.5 - 309315} & \text{if } Z_3(x) \in [309315,\ 350890.5] \\ 0 & \text{if } Z_3(x) > 350890.5 \end{cases}$$

The intuitionistic fuzzy programming model for the discussed problem of Eqn. 2 is as follows $Max \sum_{i=1}^{3} (\mu_i - \upsilon_i)$, as shown at the bottom of the page. The compromise solution for the intuitionistic model is calculated and summarized in Table 9.

TABLE 9. Intuitionistic compromise solution.

Objective Values	The optimal number of quantities to be produced by the firm.
Z_1=1515320 Z_2=11173 Z_3=331205	x_1=110, x_2=108, x_3=104, x_4=96, x_5=99, x_6=86, x_7=76, x_8=69, x_9=77, x_{10}=67, x_{11}=68, x_{12}=66

The Truth, Indeterminacy and falsity membership functions for using neutrosophic programming are constructed as follows:

$$\mu_1^T(Z_1(x)) = \begin{cases} 1 & \text{if } Z_1(x) < 1427045 \\ \dfrac{1634160 - Z_1(x)}{1634160 - 1427045} & \text{if } Z_1(x) \in [1427045, 1634160] \\ 0 & \text{if } Z_1(x) > 1634160 \end{cases}$$

$$Max \sum_{i=1}^{3} (\mu_i - \upsilon_i)$$

Subject to the constraints

$$\mu_1 \geq \frac{1634160 - Z_1(x)}{1634160 - 1427045}, \quad \upsilon_1 \leq \frac{Z_1(x) - 1447756.5}{1634160 - 1447756.5}$$

$$\mu_2 \geq \frac{11994.5 - Z_2(x)}{11994.5 - 10770.25}, \quad \upsilon_2 \leq \frac{Z_2(x) - 10892.675}{11994.5 - 10892.675}$$

$$\mu_3 \geq \frac{Z_3(x) - 309315}{355510 - 309315}, \quad \upsilon_3 \leq \frac{350890.5 - Z_3(x)}{350890.5 - 309315}$$

$$\begin{pmatrix} 0.77 & 0.28 & 0.54 & 3.6 & 1.8 & 0.57 & 0.6 & 0.64 & 1.8 & 1.2 & 0.72 & 0.4 \\ 1.5 & 0.1 & 0.3 & 0.5 & 1.5 & 0.1 & 1.1 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.2 & 0.4 & 1.2 & 0.5 & 0.2 & 1.1 & 0.1 & 1.2 & 1.3 & 0.1 & 0.1 & 0.1 \\ 0.3 & 1.5 & 0.4 & 1.1 & 0.2 & 1.5 & 0.5 & 0.2 & 0.1 & 0.1 & 0.3 & 0.1 \\ 1.5 & 0.5 & 1.2 & 1.4 & 0.6 & 1.0 & 0.1 & 1.1 & 0.1 & 0.2 & 1.1 & 1.1 \\ 1.5 & 1.5 & 1.3 & 1.3 & 0.5 & 0.6 & 1.3 & 1.1 & 1.2 & 1.0 & 1.2 & 0.8 \\ 1.1 & 0.5 & 0.2 & 1.5 & 0.3 & 0.4 & 0.2 & 0.5 & 0.2 & 1.0 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 & 1.1 & 0.3 & 0.1 & 0.3 & 0.1 & 0.1 & 0.3 & 0.6 & 0.2 \\ 0.3 & 1.0 & 0.3 & 0.4 & 0.3 & 0.5 & 0.6 & 1.2 & 0.4 & 0.5 & 0.2 & 1.0 \\ 0.2 & 0.2 & 0.3 & 1.0 & 0.3 & 1.5 & 0.4 & 0.5 & 1.0 & 0.7 & 1.0 & 0.1 \\ 0.1 & 0.2 & 1.0 & 0.3 & 1.5 & 0.7 & 1.0 & 0.4 & 0.5 & 1.2 & 0.6 & 0.3 \\ 1.5 & 1.5 & 1.1 & 1.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.5 & 1.2 & 1.5 & 1.3 & 0.3 & 1.2 & 0.6 & 0.5 & 1.5 & 0.8 & 1.1 & 1.5 \\ 0.2 & 0.6 & 1.5 & 0.5 & 0.7 & 0.1 & 0.3 & 0.4 & 0.1 & 1.0 & 0.2 & 0.5 \\ 0.3 & 0.4 & 1.3 & 0.1 & 1.0 & 0.5 & 0.2 & 0.1 & 1.0 & 1.2 & 0.1 & 0.5 \\ 1.0 & 0.6 & 0.2 & 0.1 & 0.5 & 0.6 & 0.3 & 1.1 & 0.7 & 0.3 & 1.0 & 0.6 \\ 1.1 & 1.1 & 0.5 & 0.7 & 0.4 & 0.1 & 0.2 & 0.2 & 0.1 & 1.0 & 0.2 & 0.1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{pmatrix} \leq \begin{pmatrix} 1200 \\ 624 \\ 624 \\ 832 \\ 1040 \\ 1248 \\ 624 \\ 416 \\ 624 \\ 832 \\ 832 \\ 624 \\ 1248 \\ 624 \\ 624 \\ 832 \\ 624 \end{pmatrix}$$

$$Z_1 = 540x_1 + 1250x_2 + 460x_3 + 2530x_4 + 3480x_5 + 830x_6 + 475x_7 + 1465x_8 + 3270x_9 + 585x_{10} + 2315x_{11} + 435x_{12}$$

$$Z_2 = 12x_1 + 5x_2 + 6.5x_3 + 19.75x_4 + 14.25x_5 + 8x_6 + 6x_7 + 10.5x_8 + 13.5x_9 + 9.25x_{10} + 18.5x_{11} + 8.25x_{12}$$

$$Z_3 = 75x_1 + 215x_2 + 85x_3 + 535x_4 + 875x_5 + 190x_6 + 95x_7 + 410x_8 + 725x_9 + 115x_{10} + 450x_{11} + 105x_{12}$$

$$110 \leq x_1 \leq 140, \quad 108 \leq x_2 \leq 135,\ 104 \leq x_3 \leq 128,\ 96 \leq x_4 \leq 126,\ 84 \leq x_5 \leq 99,\ 86 \leq x_6 \leq 104,$$

$$76 \leq x_7 \leq 88, \quad 60 \leq x_8 \leq 75,\ 70 \leq x_9 \leq 79,\ 67 \leq x_{10} \leq 88,\ 68 \leq x_{11} \leq 80,\ 66 \leq x_{12} \leq 81,$$

$$x_j \in \text{integer}\ j = 1, 2, \ldots, 12, \quad \mu_i \geq \upsilon_i,\ \mu_i + \upsilon_i \leq 1,\ \forall \mu_i \,\&\, \upsilon_i \in [0, 1],\ i = 1, 2, 3.$$

$$\sigma_1^I(Z_1(x)) = \begin{cases} 1 & \text{if } Z_1(x) < 1427045 \\ \dfrac{1613448.5 - Z_1(x)}{1613448.5 - 1427045} & \text{if } Z_1(x) \in [1427045, 1613448.5] \\ 0 & \text{if } Z_1(x) > 1613448.5 \end{cases}$$

$$\upsilon_1^F(Z_1(x)) = \begin{cases} 0 & \text{if } Z_1(x) < 1447756.5 \\ \dfrac{Z_1(x) - 1447756.5}{1634160 - 1447756.5} & \text{if } Z_1(x) \in [1447756.5,\ 1634160] \\ 1 & \text{if } Z_1(x) > 1634160 \end{cases}$$

$$\mu_2^T(Z_2(x)) = \begin{cases} 1 & \text{if } Z_2(x) < 10770.25 \\ \dfrac{11994.5 - Z_2(x)}{11994.5 - 10770.25} & \text{if } Z_2(x) \in [10770.25, 11994.5] \\ 0 & \text{if } Z_2(x) > 11994.5 \end{cases}$$

$$\sigma_2^I(Z_2(x)) = \begin{cases} 1 & \text{if } Z_2(x) < 10770.25 \\ \dfrac{11872.075 - Z_2(x)}{11872.075 - 10770.25} & \text{if } Z_2(x) \in [10770.25, 11872.075] \\ 0 & \text{if } Z_2(x) > 11872.075 \end{cases}$$

$$\upsilon_2^F(Z_2(x)) = \begin{cases} 0 & \text{if } Z_2(x) < 10892.675 \\ \dfrac{Z_2(x) - 10892.675}{11994.5 - 10892.675} & \text{if } Z_2(x) \in [10892.675,\ 11994.5] \\ 1 & \text{if} Z_2(x) > 11994.5 \end{cases}$$

$$\mu_3^T(Z_3(x)) = \begin{cases} 0 & \text{if } Z_3(x) < 309315 \\ \dfrac{Z_3(x) - 309315}{355510 - 309315} & \text{if } Z_3(x) \in [309315,\ 355510] \\ 1 & \text{if } Z_3(x) > 355510 \end{cases}$$

$$\sigma_3^I(Z_3(x)) = \begin{cases} 0 & \text{if } Z_3(x) < 313934.5 \\ \dfrac{Z_3(x) - 313934.5}{355510 - 313934.5} & \text{if } Z_3(x) \in [313934.5,\ 355510] \\ 1 & \text{if } Z_3(x) > 355510 \end{cases}$$

$$\upsilon_3^F(Z_3(x)) = \begin{cases} 1 & \text{if } Z_3(x) < 309315 \\ \dfrac{350890.5 - Z_3(x)}{350890.5 - 309315} & \text{if } Z_3(x) \in [309315,\ 350890.5] \\ 0 & \text{if } Z_3(x) > 350890.5 \end{cases}$$

The neutrosophic programming model I for the Eqn. 2 is as shown at the bottom of the next page. The neutrosophic programming model II for Eqn. 2 is as shown at the bottom of the page 37479. The compromise solution for both neutrosophic models is calculated and summarized in table 10.

TABLE 10. Neutrosophic compromise solution.

Model	Objective Values	The optimal number of quantities to be produced by the firm.
I	Z_1=1524255 Z_2=11343.75 Z_3=333815	x_1=110, x_2=108, x_3=104, x_4=96, x_5=99, x_6=91, x_7=76, x_8=75, x_9=74, x_{10}=68, x_{11}=68, x_{12}=78
II	Z_1=1524385 Z_2=11345.50 Z_3=333805	x_1=110, x_2=108, x_3=104, x_4=96, x_5=99, x_6=103,x_7=77, x_8=75, x_9=72,x_{10}=69, x_{11}=68, x_{12}=68

TABLE 11. Optimal compromise solution.

Methods	Objectives	Compromise value of Decision Variables
Intuitionistic fuzzy Programming	Z_1=1515320 Z_2=11173 Z_3=331205	x_1=110, x_2=108, x_3=104, x_4=96, x_5=99, x_6=86, x_7=76, x_8=69,x_9=77, x_{10}=67, x_{11}=68, x_{12}=66
Neutrosophic Programming **(Model I)**	Z_1=1524255 Z_2=11343.75 Z_3=333815	x_1=110, x_2=108, x_3=104, x_4=96, x_5=99, x_6=91, x_7=76, x_8=75,x_9=74, x_{10}=68, x_{11}=68, x_{12}=78
Neutrosophic Programming **(Model II)**	Z_1=1524385 Z_2=11345.50 Z_3=333805	x_1=110, x_2=108, x_3=104, x_4=96, x_5=99, x_6=103, x_7=77, x_8=75,x_9=72, x_{10}=69, x_{11}=68, x_{12}=68

It can be observed that from the solutions in Table 10, maximizing indeterminacy (Model I) improve the maximizing-type objective function as well as the other objective functions while minimizing the indeterminacy (Model II) reduced the maximizing-type objective function & increase the value of other minimize type objectives function. Therefore, it can be concluded that maximizing indeterminacy (Model I) can be slightly better than minimizing it (Model II) for exact data.

The intuitionistic and neutrosophic compromise solution is given in table 11.

The graphical presentation of the compromise optimal number of units of each item is shown in fig. 2.

VII. PRODUCTION PLANNING MODEL UNDER INTERVAL-VALUED NEUTROSOPHIC NUMBERS

The earlier discussed production planning problem is formulated under interval-valued trapezoidal neutrosophic numbers. Since most of the production planning cases, the data availability is not a specific value in general. To illustrate the case, let suppose the multiobjective production planning model's objective functions are in the form of interval-valued trapezoidal neutrosophic numbers. The stepwise solution procedure is given in the next section.

A. SOLUTION PROCEDURE

The step of the solution procedure to solve the interval-valued trapezoidal neutrosophic numbers production planning problem is as follows:

Step 1: State the interval-valued trapezoidal neutrosophic numbers problem.

Step 2: Convert the interval-valued trapezoidal neutrosophic numbers problem using the score function of Eqn. 1 into the interval-valued problem.

Step 3: Convert interval-valued problem into the crisp form using $\alpha-$ cut method. As for $[a, b]$

$$\alpha a + (1-\alpha)b$$

Step 4: Estimate the ideal points for each objective function of the interval-valued problem subject to given constraints.

Step 5: Determine the maximum and minimum values for each objective function subject to given constraints.

Step 6: Applied Neutrosophic Compromise Programming Approach and use LINGO software to obtain the optimum compromise solution.

The resulting mathematical model of interval-valued trapezoidal neutrosophic numbers is as follows:

$$Min\ Z_1 = \sum_{j=1}^{J} \left\langle \begin{matrix} \left(P_j^1, P_j^2, P_j^3, P_j^4\right)\left[\mu_{\tilde{P}_j}^L, \mu_{\tilde{P}_j}^U\right], \\ \left[\sigma_{\tilde{P}_j}^L, \sigma_{\tilde{P}_j}^U\right], \left[\upsilon_{\tilde{P}_j}^L, \upsilon_{\tilde{P}_j}^U\right] \end{matrix} \right\rangle x_j$$

$$Min\ Z_2 = \sum_{j=1}^{J} \left\langle \begin{matrix} \left(I_j^1, I_j^2, I_j^3, I_j^4\right)\left[\mu_{\tilde{I}_j}^L, \mu_{\tilde{I}_j}^U\right], \\ \left[\sigma_{\tilde{I}_j}^L, \sigma_{\tilde{I}_j}^U\right], \left[\upsilon_{\tilde{I}_j}^L, \upsilon_{\tilde{I}_j}^U\right] \end{matrix} \right\rangle x_j$$

$$Max\ Z_3 = \sum_{j=1}^{J} \left\langle \begin{matrix} \left(R_j^1, R_j^2, R_j^3, R_j^4\right)\left[\mu_{\tilde{R}_j}^L, \mu_{\tilde{R}_j}^U\right], \\ \left[\sigma_{\tilde{R}_j}^L, \sigma_{\tilde{R}_j}^U\right], \left[\upsilon_{\tilde{R}_j}^L, \upsilon_{\tilde{R}_j}^U\right] \end{matrix} \right\rangle x_j$$

Model I

$$\text{Maximize} = \sum_{\mathrm{i}=1}^{3} (\mu_{\mathrm{i}} + \sigma_i - \upsilon_i)$$

Subject to constraints

$$\mu_1 \geq \mu_1^T(Z_1), \quad \sigma_1 \geq \sigma_1^I(Z_1),\ \upsilon_1 \leq \upsilon_1^F(Z_1)$$
$$\mu_2 \geq \mu_2^T(Z_2), \quad \sigma_2 \geq \sigma_2^I(Z_2),\ \upsilon_2 \leq \upsilon_1^F(Z_2)$$
$$\mu_3 \geq \mu_3^T(Z_3), \quad \sigma_3 \geq \sigma_3^I(Z_3),\ \upsilon_3 \leq \upsilon_1^F(Z_3)$$
$$Z_1 = 540x_1 + 1250x_2 + 460x_3 + 2530x_4 + 3480x_5 + 830x_6 + 475x_7 + 1465x_8 + 3270x_9 + 585x_{10} + 2315x_{11} + 435x_{12}$$
$$Z_2 = 12x_1 + 5x_2 + 6.5x_3 + 19.75x_4 + 14.25x_5 + 8x_6 + 6x_7 + 10.5x_8 + 13.5x_9 + 9.25x_{10} + 18.5x_{11} + 8.25x_{12}$$
$$Z_3 = 75x_1 + 215x_2 + 85x_3 + 535x_4 + 875x_5 + 190x_6 + 95x_7 + 410x_8 + 725x_9 + 115x_{10} + 450x_{11} + 105x_{12}$$

$$\begin{pmatrix} 0.77 & 0.28 & 0.54 & 3.6 & 1.8 & 0.57 & 0.6 & 0.64 & 1.8 & 1.2 & 0.72 & 0.4 \\ 1.5 & 0.1 & 0.3 & 0.5 & 1.5 & 0.1 & 1.1 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.2 & 0.4 & 1.2 & 0.5 & 0.2 & 1.1 & 0.1 & 1.2 & 1.3 & 0.1 & 0.1 & 0.1 \\ 0.3 & 1.5 & 0.4 & 1.1 & 0.2 & 1.5 & 0.5 & 0.2 & 0.1 & 0.1 & 0.3 & 0.1 \\ 1.5 & 0.5 & 1.2 & 1.4 & 0.6 & 1.0 & 0.1 & 1.1 & 0.1 & 0.2 & 1.1 & 1.1 \\ 1.5 & 1.5 & 1.3 & 1.3 & 0.5 & 0.6 & 1.3 & 1.1 & 1.2 & 1.0 & 1.2 & 0.8 \\ 1.1 & 0.5 & 0.2 & 1.5 & 0.3 & 0.4 & 0.2 & 0.5 & 0.2 & 1.0 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 & 1.1 & 0.3 & 0.1 & 0.3 & 0.1 & 0.1 & 0.3 & 0.6 & 0.2 \\ 0.3 & 1.0 & 0.3 & 0.4 & 0.3 & 0.5 & 0.6 & 1.2 & 0.4 & 0.5 & 0.2 & 1.0 \\ 0.2 & 0.2 & 0.3 & 1.0 & 0.3 & 1.5 & 0.4 & 0.5 & 1.0 & 0.7 & 1.0 & 0.1 \\ 0.1 & 0.2 & 1.0 & 0.3 & 1.5 & 0.7 & 1.0 & 0.4 & 0.5 & 1.2 & 0.6 & 0.3 \\ 1.5 & 1.5 & 1.1 & 1.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.5 & 1.2 & 1.5 & 1.3 & 0.3 & 1.2 & 0.6 & 0.5 & 1.5 & 0.8 & 1.1 & 1.5 \\ 0.2 & 0.6 & 1.5 & 0.5 & 0.7 & 0.1 & 0.3 & 0.4 & 0.1 & 1.0 & 0.2 & 0.5 \\ 0.3 & 0.4 & 1.3 & 0.1 & 1.0 & 0.5 & 0.2 & 0.1 & 1.0 & 1.2 & 0.1 & 0.5 \\ 1.0 & 0.6 & 0.2 & 0.1 & 0.5 & 0.6 & 0.3 & 1.1 & 0.7 & 0.3 & 1.0 & 0.6 \\ 1.1 & 1.1 & 0.5 & 0.7 & 0.4 & 0.1 & 0.2 & 0.2 & 0.1 & 1.0 & 0.2 & 0.1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{pmatrix} \leq \begin{pmatrix} 1200 \\ 624 \\ 624 \\ 832 \\ 1040 \\ 1248 \\ 624 \\ 416 \\ 624 \\ 832 \\ 832 \\ 624 \\ 1248 \\ 624 \\ 624 \\ 832 \\ 624 \end{pmatrix}$$

$$110 \leq x_1 \leq 140, \quad 108 \leq x_2 \leq 135,\ 104 \leq x_3 \leq 128,\ 96 \leq x_4 \leq 126,\ 84 \leq x_5 \leq 99,$$
$$86 \leq x_6 \leq 104, \quad 76 \leq x_7 \leq 88,\ 60 \leq x_8 \leq 75,\ 70 \leq x_9 \leq 79,\ 67 \leq x_{10} \leq 88,$$
$$68 \leq x_{11} \leq 80, \quad 66 \leq x_{12} \leq 81,\ x_j \in \text{integer}\ j = 1, 2, \ldots, 12,\ \mu_{\mathrm{i}} \geq \sigma_i,\ \mu_{\mathrm{i}} \geq \upsilon_i,$$
$$\mu_{\mathrm{i}} + \sigma_i + \upsilon_i \leq 3, \quad \forall \mu_{\mathrm{i}},\ \sigma_i,\ \upsilon_i \in [0, 1],\ i = 1, 2, 3.$$

Subject to set of constraints:

$$\sum_{j=1}^{J} s_j x_j \leq S, \quad \sum_{j=1}^{J} M_{ij} x_j \leq b_i, \; i = 1, 2, \ldots, I$$

$$\sum_{j=1}^{J} q_{lj} x_j \leq Q_l^*, \quad l = 1, 2, \ldots, L,$$

$$D_{Mj} \leq x_j \leq D_{Uj}, \quad j = 1, 2, \ldots, J$$

B. NUMERICAL EXAMPLE

The objective functions of the multiobjective production planning model are in the form of interval-valued trapezoidal neutrosophic numbers. Table 12 summarizes the Production Cost, Inventory Holding Cost and Profit, which all are defined in the form of interval-valued trapezoidal neutrosophic numbers.

The score function is used to convert the trapezoidal interval-valued neutrosophic number into interval number then Table 12 will be changed into interval-valued form. Table 13 summarizes the Production Cost, Inventory Holding Cost and Profit in the form of interval-valued.

The mathematical formulation of the problem under neutrosophic environment (Vagueness) of the Eqn 2 is as follows Minimize Z_1, Minimize Z_2, and Minimize Z_3, shown at the bottom of the page 37481.

Model II

$$\text{Maximize} = \sum_{\mathrm{i}=1}^{3} (\mu_{\mathrm{i}} - \sigma_i - \upsilon_i)$$

Subject to constraints

$$\mu_1 \geq \mu_1^T(Z_1), \quad \sigma_1 \leq \sigma_1^I(Z_1), \; \upsilon_1 \leq \upsilon_1^F(Z_1)$$

$$\mu_2 \geq \mu_2^T(Z_2), \quad \sigma_2 \leq \sigma_2^I(Z_2), \; \upsilon_2 \leq \upsilon_1^F(Z_2)$$

$$\mu_3 \geq \mu_3^T(Z_3), \quad \sigma_3 \leq \sigma_3^I(Z_3), \; \upsilon_3 \leq \upsilon_1^F(Z_3)$$

$$Z_1 = 540x_1 + 1250x_2 + 460x_3 + 2530x_4 + 3480x_5 + 830x_6 + 475x_7 + 1465x_8 + 3270x_9 + 585x_{10} + 2315x_{11} + 435x_{12}$$

$$Z_2 = 12x_1 + 5x_2 + 6.5x_3 + 19.75x_4 + 14.25x_5 + 8x_6 + 6x_7 + 10.5x_8 + 13.5x_9 + 9.25x_{10} + 18.5x_{11} + 8.25x_{12}$$

$$Z_3 = 75x_1 + 215x_2 + 85x_3 + 535x_4 + 875x_5 + 190x_6 + 95x_7 + 410x_8 + 725x_9 + 115x_{10} + 450x_{11} + 105x_{12}$$

$$\begin{pmatrix} 0.77 & 0.28 & 0.54 & 3.6 & 1.8 & 0.57 & 0.6 & 0.64 & 1.8 & 1.2 & 0.72 & 0.4 \\ 1.5 & 0.1 & 0.3 & 0.5 & 1.5 & 0.1 & 1.1 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.2 & 0.4 & 1.2 & 0.5 & 0.2 & 1.1 & 0.1 & 1.2 & 1.3 & 0.1 & 0.1 & 0.1 \\ 0.3 & 1.5 & 0.4 & 1.1 & 0.2 & 1.5 & 0.5 & 0.2 & 0.1 & 0.1 & 0.3 & 0.1 \\ 1.5 & 0.5 & 1.2 & 1.4 & 0.6 & 1.0 & 0.1 & 1.1 & 0.1 & 0.2 & 1.1 & 1.1 \\ 1.5 & 1.5 & 1.3 & 1.3 & 0.5 & 0.6 & 1.3 & 1.1 & 1.2 & 1.0 & 1.2 & 0.8 \\ 1.1 & 0.5 & 0.2 & 1.5 & 0.3 & 0.4 & 0.2 & 0.5 & 0.2 & 1.0 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 & 1.1 & 0.3 & 0.1 & 0.3 & 0.1 & 0.1 & 0.3 & 0.6 & 0.2 \\ 0.3 & 1.0 & 0.3 & 0.4 & 0.3 & 0.5 & 0.6 & 1.2 & 0.4 & 0.5 & 0.2 & 1.0 \\ 0.2 & 0.2 & 0.3 & 1.0 & 0.3 & 1.5 & 0.4 & 0.5 & 1.0 & 0.7 & 1.0 & 0.1 \\ 0.1 & 0.2 & 1.0 & 0.3 & 1.5 & 0.7 & 1.0 & 0.4 & 0.5 & 1.2 & 0.6 & 0.3 \\ 1.5 & 1.5 & 1.1 & 1.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.5 & 1.2 & 1.5 & 1.3 & 0.3 & 1.2 & 0.6 & 0.5 & 1.5 & 0.8 & 1.1 & 1.5 \\ 0.2 & 0.6 & 1.5 & 0.5 & 0.7 & 0.1 & 0.3 & 0.4 & 0.1 & 1.0 & 0.2 & 0.5 \\ 0.3 & 0.4 & 1.3 & 0.1 & 1.0 & 0.5 & 0.2 & 0.1 & 1.0 & 1.2 & 0.1 & 0.5 \\ 1.0 & 0.6 & 0.2 & 0.1 & 0.5 & 0.6 & 0.3 & 1.1 & 0.7 & 0.3 & 1.0 & 0.6 \\ 1.1 & 1.1 & 0.5 & 0.7 & 0.4 & 0.1 & 0.2 & 0.2 & 0.1 & 1.0 & 0.2 & 0.1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{pmatrix} \leq \begin{pmatrix} 1200 \\ 624 \\ 624 \\ 832 \\ 1040 \\ 1248 \\ 624 \\ 416 \\ 624 \\ 832 \\ 832 \\ 624 \\ 1248 \\ 624 \\ 624 \\ 832 \\ 624 \end{pmatrix}$$

$$110 \leq x_1 \leq 140, \quad 108 \leq x_2 \leq 135, \; 104 \leq x_3 \leq 128, \; 96 \leq x_4 \leq 126, \; 84 \leq x_5 \leq 99,$$

$$86 \leq x_6 \leq 104, \quad 76 \leq x_7 \leq 88, \; 60 \leq x_8 \leq 75, \; 70 \leq x_9 \leq 79, \; 67 \leq x_{10} \leq 88,$$

$$68 \leq x_{11} \leq 80, \quad 66 \leq x_{12} \leq 81, \; x_j \in \text{integer}\, j = 1, 2, \ldots, 12, \; \mu_{\mathrm{i}} \geq \sigma_i, \; \mu_{\mathrm{i}} \geq \upsilon_i,$$

$$\mu_{\mathrm{i}} + \sigma_i + \upsilon_i \leq 3, \quad \forall \mu_{\mathrm{i}}, \sigma_i, \upsilon_i \in [0, 1], \; i = 1, 2, 3.$$

TABLE 12. The IVTN form of the production cost, inventory holding cost, profit.

Production costs	Inventory Holding cost	Expected Profit
530, 540, 550, 560; [0.7,0.9],[0.1,0.3],[0.5,0.7]	10, 11, 12, 13; [0.7,0.9],[0.1,0.3],[0.5,0.7]	70, 75, 80, 85; [0.7,0.9],[0.1,0.3],[0.5,0.7]
1220, 1235, 1250, 1265; [0.5,0.7],[0.3,0.5],[0.4,0.6]	4, 4.5, 5, 5.5; [0.5,0.7],[0.3,0.5],[0.4,0.6]	200, 215, 225, 235; [0.5,0.7],[.3,0.5],[.4,0.6]
450, 458, 466, 472; [0.8,1.0],[0.2,0.4],[0.1,0.3]	6, 6.25, 6.5, 6.75; [0.8,1.0],[0.2,0.4],[0.1,0.3]	75, 85, 95, 100; [0.8,1.0],[0.2,0.4],[0.1,0.3]
2510, 2522, 2536, 2545; [0.7,1.0],[0.2,0.3],[0.2,0.5]	18.25, 19, 19.75, 20.5; 0.7,1.0],[0.2,0.3],[0.2,0.5]	510, 525, 540, 550; [0.7,1.0],[.2,0.3],[.2,0.5]
3450, 3465, 3475, 3495; [0.7,1.0],[0.2,0.3],[0.2,0.5]	13.8, 14.1, 14.3, 14.45; [0.7,1.0],[.2,0.3],[.2,0.5]	825, 855, 880, 895; [0.7,1.0],[.2,0.3],[.2,0.5]
815, 822, 832, 845; [0.7,0.9],[0.1,0.4],[0.3,0.5]	7.25, 7.75, 8.25, 8.75; [0.7,0.9],[0.1,0.4],[0.3,0.5]	168, 180, 195, 205; [0.7,0.9],[.1,0.4],[.3,0.5]
450, 460, 470, 485; [0.3,0.7],[0.1,0.4],[0.3,0.7]	4, 5, 6, 7; [0.3,0.7],[0.1,0.4],[0.3,0.7]	80, 85, 95, 100; [0.3,0.7],[0.1,0.4],[0.3,0.7]
1440, 1455, 1470, 1480; [0.2,0.7],[0.2,0.5],[0.3,0.6]	9, 10, 11, 12; [0.2,0.7],[0.2,0.5],[0.3,0.6]	390, 405, 415, 422; [0.2,0.7],[.2,0.5],[.3,0.6]
3240, 3252, 3265, 3280; [0.6,0.8],[0.3,0.6],[0.2,0.4]	11.5, 12.25, 13.50, 15; [0.6,0.8],[.3,0.6],[0.2,0.4]	695, 715, 730, 740; [0.6,0.8],[.3,0.6],[.2,0.4]
550, 566, 578, 590; [0.4,0.7],[0.2,0.6],[0.2,0.5]	8, 8.5, 9.25, 10; [0.4,0.7],[0.2,0.6],[0.2,0.5]	98, 110, 118, 125; [0.4,0.7],[.2,0.6],[0.2,0.5]
2280, 2298, 2310, 2328; [0.5,0.9],[0.1,0.3],[0.3,0.6]	17, 17.75, 18.5, 19.25; [0.5,0.9],[.1,0.3],[0.3,0.6]	415, 435, 455, 475; [0.5,0.9],[.1,0.3],[.3,0.6]
425, 432, 442, 446; [0.6,0.8],[0.2,0.4],[0.4,0.6]	7.5, 8.25, 9, 9.3; [0.6,0.8],[0.2,0.4],[0.4,0.6]	85, 98, 112, 122; [0.6,0.8],[0.2,0.4],[0.4,0.6]

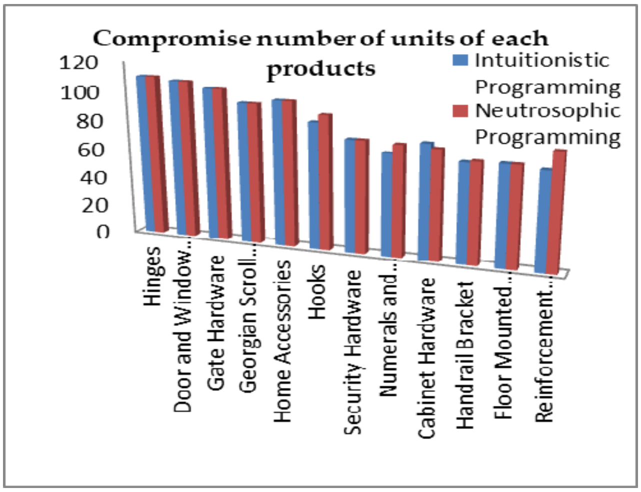

FIGURE 2. Compromise number of units.

TABLE 13. The interval-valued form of the production cost, inventory holding cost, profit.

Production costs	Inventory Holding cost	Expected Profit
[530, 548.9]	[11.9, 13]	[70, 75.56]
[1220, 1241.43]	[4, 4.79]	[220.05, 220.71]
[453.71, 465.33]	[6.28, 6.67]	[75, 94.44]
[2510, 2533.75]	[18.25, 19.56]	[510, 537.5]
[3450, 3468.75]	[13.8, 14.26]	[825, 862.5]
[817.82, 830.56]	[7.25, 7.81]	[168, 193.89]
[450, 461.11]	[4, 5.89]	[80, 85.56]
[1440, 1467.5]	[9, 10.75]	[390, 413..25]
[3255, 3271.88]	[13.13, 14.19]	[705.62, 720]
[550, 575]	[8, 9.06]	[98, 116.25]
[2280, 2308.33]	[17, 17.83]	[415, 437.22]
[425, 441]	[7.5, 8.44]	[85, 109.5]

The solution thus obtained is the idle solution—the pay-off matrix constructed by using the idle solutions. Finally, the constructed payoff matrix helped to define the aspiration level for each objective functions. The bounds for the three objective functions are determined as: $1419949 \leq Z_1(x) \leq 1626021$, $10559.17 \leq Z_2(x) \leq 11744.98$ and $303770 \leq Z_3(x) \leq 349245.9$. The Truth, Indeterminacy and falsity membership functions for using neutrosophic programming are constructed as follows:

$$\mu_1^T(Z_1(x)) = \begin{cases} 1 & \text{if } Z_1(x) < 1419949 \\ \dfrac{1626021 - Z_1(x)}{1626021 - 1419949} & \text{if } Z_1(x) \in [1419949, 1626021] \\ 0 & \text{if } Z_1(x) > 1626021 \end{cases}$$

$$\sigma_1^I(Z_1(x)) = \begin{cases} 1 & \text{if } Z_1(x) < 1419949 \\ \dfrac{1606021 - Z_1(x)}{1606021 - 1419949} & \text{if } Z_1(x) \in [1419949, 1606021] \\ 0 & \text{if } Z_1(x) > 1606021 \end{cases}$$

$$\upsilon_1^F(Z_1(x)) = \begin{cases} 0 & \text{if } Z_1(x) < 1489949 \\ \dfrac{Z_1(x) - 1489949}{1626021 - 1489949} & \text{if } Z_1(x) \in [1489949, 1626021] \\ 1 & \text{if } Z_1(x) > 1626021 \end{cases}$$

$$\mu_2^T(Z_2(x)) = \begin{cases} 1 & \text{if } Z_2(x) < 10559.17 \\ \dfrac{11744.98 - Z_2(x)}{11744.98 - 10559.17} & \text{if } Z_2(x) \in [10559.17, 11744.98] \\ 0 & \text{if } Z_2(x) > 11744.98 \end{cases}$$

$$\sigma_2^I(Z_2(x)) = \begin{cases} 1 & \text{if } Z_2(x) < 10559.17 \\ \dfrac{11244.98 - Z_2(x)}{11244.98 - 10559.17} & \text{if } Z_2(x) \in [10559.17, 11244.98] \\ 0 & \text{if } Z_2(x) > 11244.98 \end{cases}$$

$$\upsilon_2^F(Z_2(x)) = \begin{cases} 0 & \text{if } Z_2(x) < 10759.17 \\ \dfrac{Z_2(x) - 10759.17}{11744.98 - 10759.17} & \text{if } Z_2(x) \in [10759.17, 11744.98] \\ 1 & \text{if } Z_2(x) > 11744.98 \end{cases}$$

$$\mu_3^T(Z_3(x)) = \begin{cases} 0 & \text{if } Z_3(x) < 303770 \\ \dfrac{Z_3(x) - 303770}{349245.9 - 303770} & \text{if } Z_3(x) \in [303770, 349245.9] \\ 1 & \text{if } Z_3(x) > 349245.9 \end{cases}$$

$$\sigma_3^I(Z_3(x)) = \begin{cases} 0 & \text{if } Z_3(x) < 313770 \\ \dfrac{Z_3(x) - 313770}{349245.9 - 313770} & \text{if } Z_3(x) \in [313770, 349245.9] \\ 1 & \text{if } Z_3(x) > 349245.9 \end{cases}$$

$$\upsilon_3^F(Z_3(x)) = \begin{cases} 1 & \text{if } Z_3(x) < 303770 \\ \dfrac{329245.9 - Z_3(x)}{329245.9 - 303770} & \text{if } Z_3(x) \in [303770, \ 329245.9] \\ 0 & \text{if } Z_3(x) > 329245.9 \end{cases}$$

Then, the neutrosophic programming model I is as shown at the bottom of the next page. The neutrosophic programming model II is as shown at the bottom of the page 37483. The compromise solution for the interval-valued trapezoidal neutrosophic model is calculated using the neutrosophic programming approach. The Compromise Solution at distinct α values is given in tables 14 and 15 for both models I and II.

$$\begin{aligned} \text{Minimize } Z_1 = {} & [530, 548.9]x_1 + [1220, 1241.43]x_2 + [453.71, 465.33]x_3 + [2510, 2533.75]x_4 + [3450, 3468.75]x_5 \\ & + [817.82, 830.56]x_6 + [450, 461.11]x_7 + [1440, 1467.5]x_8 + [3255, 3271.88]x_9 + [550, 575]x_{10} \\ & + [2280, 2308.33]x_{11} + [425, 441]x_{12} \\ \text{Minimize } Z_2 = {} & [11.9, 13]x_1 + [4, 4.79]x_2 + [6.28, 6.67]x_3 + [18.25, 19.56]x_4 + [13.8, 14.26]x_5 + [7.25, 7.81]x_6 \\ & + [4, 5.89]x_7 + [9, 10.75]x_8 + [13.13, 14.19]x_9 + [8, 9.06]x_{10} + [17, 17.83]x_{11} + [7.5, 8.44]x_{12} \\ \text{Maximize } Z_3 = {} & [70, 75.56]x_1 + [220.05, 220.71]x_2 + [75, 94.44]x_3 + [510, 537.5]x_4 + [825, 862.5]x_5 \\ & + [168, 193.89]x_6 + [80, 85.56]x_7 + [390, 413.25]x_8 + [705.62, 720]x_9 + [98, 116.25]x_{10} \\ & + [415, 437.22]x_{11} + [85, 109.5]x_{12} \end{aligned}$$

Subject to constraints

$$\begin{pmatrix} 0.77 & 0.28 & 0.54 & 3.6 & 1.8 & 0.57 & 0.6 & 0.64 & 1.8 & 1.2 & 0.72 & 0.4 \\ 1.5 & 0.1 & 0.3 & 0.5 & 1.5 & 0.1 & 1.1 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.2 & 0.4 & 1.2 & 0.5 & 0.2 & 1.1 & 0.1 & 1.2 & 1.3 & 0.1 & 0.1 & 0.1 \\ 0.3 & 1.5 & 0.4 & 1.1 & 0.2 & 1.5 & 0.5 & 0.2 & 0.1 & 0.1 & 0.3 & 0.1 \\ 1.5 & 0.5 & 1.2 & 1.4 & 0.6 & 1.0 & 0.1 & 1.1 & 0.1 & 0.2 & 1.1 & 1.1 \\ 1.5 & 1.5 & 1.3 & 1.3 & 0.5 & 0.6 & 1.3 & 1.1 & 1.2 & 1.0 & 1.2 & 0.8 \\ 1.1 & 0.5 & 0.2 & 1.5 & 0.3 & 0.4 & 0.2 & 0.5 & 0.2 & 1.0 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 & 1.1 & 0.3 & 0.1 & 0.3 & 0.1 & 0.1 & 0.3 & 0.6 & 0.2 \\ 0.3 & 1.0 & 0.3 & 0.4 & 0.3 & 0.5 & 0.6 & 1.2 & 0.4 & 0.5 & 0.2 & 1.0 \\ 0.2 & 0.2 & 0.3 & 1.0 & 0.3 & 1.5 & 0.4 & 0.5 & 1.0 & 0.7 & 1.0 & 0.1 \\ 0.1 & 0.2 & 1.0 & 0.3 & 1.5 & 0.7 & 1.0 & 0.4 & 0.5 & 1.2 & 0.6 & 0.3 \\ 1.5 & 1.5 & 1.1 & 1.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.5 & 1.2 & 1.5 & 1.3 & 0.3 & 1.2 & 0.6 & 0.5 & 1.5 & 0.8 & 1.1 & 1.5 \\ 0.2 & 0.6 & 1.5 & 0.5 & 0.7 & 0.1 & 0.3 & 0.4 & 0.1 & 1.0 & 0.2 & 0.5 \\ 0.3 & 0.4 & 1.3 & 0.1 & 1.0 & 0.5 & 0.2 & 0.1 & 1.0 & 1.2 & 0.1 & 0.5 \\ 1.0 & 0.6 & 0.2 & 0.1 & 0.5 & 0.6 & 0.3 & 1.1 & 0.7 & 0.3 & 1.0 & 0.6 \\ 1.1 & 1.1 & 0.5 & 0.7 & 0.4 & 0.1 & 0.2 & 0.2 & 0.1 & 1.0 & 0.2 & 0.1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{pmatrix} \leq \begin{pmatrix} 1200 \\ 624 \\ 624 \\ 832 \\ 1040 \\ 1248 \\ 624 \\ 416 \\ 624 \\ 832 \\ 832 \\ 624 \\ 1248 \\ 624 \\ 624 \\ 832 \\ 624 \end{pmatrix}$$

$$\begin{aligned} & 110 \leq x_1 \leq 140, \quad 108 \leq x_2 \leq 135, \ 104 \leq x_3 \leq 128, \ 96 \leq x_4 \leq 126, \ 84 \leq x_5 \leq 99, \ 86 \leq x_6 \leq 104, \\ & 76 \leq x_7 \leq 88, \quad 60 \leq x_8 \leq 75, \ 70 \leq x_9 \leq 79, \ 67 \leq x_{10} \leq 88, \ 68 \leq x_{11} \leq 80, \ 66 \leq x_{12} \leq 81, \\ & x_j \in \text{integer } j = 1, 2, \ldots, 12 \end{aligned}$$

It can be observed that from the solutions in Table 14 and 15, maximizing indeterminacy (Model I) increases the maximization-type objective function slightly, at the same time, increases the minimization-type objective functions slightly. Simultaneously, minimizing the indeterminacy (Model II) decreases the minimization-type and maximization-type objective functions slightly. Therefore, it can be concluded that minimizing indeterminacy (Model II) can be slightly better in the interval-valued trapezoidal neutrosophic case than maximizing it (Model I).

VIII. MOTIVATION AND CONTRIBUTION OF THE STUDY

This study is motivated by Neutrosophic programming being a new research area with the potential to capture the decision-makers truth and Indeterminacy goals. The following are the contributions of the study:

Model I

$$\text{Maximize} = \sum_{\mathrm{i}=1}^{3} (\mu_\mathrm{i} + \sigma_i - \upsilon_i)$$

Subject to constraints

$$\mu_1 \geq \mu_1^T(Z_1), \quad \sigma_1 \geq \sigma_1^I(Z_1), \ \upsilon_1 \leq \upsilon_1^F(Z_1)$$
$$\mu_2 \geq \mu_2^T(Z_2), \quad \sigma_2 \geq \sigma_2^I(Z_2), \ \upsilon_2 \leq \upsilon_1^F(Z_2)$$
$$\mu_3 \geq \mu_3^T(Z_3), \quad \sigma_3 \geq \sigma_3^I(Z_3), \ \upsilon_3 \leq \upsilon_1^F(Z_3)$$

$$\begin{pmatrix} 0.77 & 0.28 & 0.54 & 3.6 & 1.8 & 0.57 & 0.6 & 0.64 & 1.8 & 1.2 & 0.72 & 0.4 \\ 1.5 & 0.1 & 0.3 & 0.5 & 1.5 & 0.1 & 1.1 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.2 & 0.4 & 1.2 & 0.5 & 0.2 & 1.1 & 0.1 & 1.2 & 1.3 & 0.1 & 0.1 & 0.1 \\ 0.3 & 1.5 & 0.4 & 1.1 & 0.2 & 1.5 & 0.5 & 0.2 & 0.1 & 0.1 & 0.3 & 0.1 \\ 1.5 & 0.5 & 1.2 & 1.4 & 0.6 & 1.0 & 0.1 & 1.1 & 0.1 & 0.2 & 1.1 & 1.1 \\ 1.5 & 1.5 & 1.3 & 1.3 & 0.5 & 0.6 & 1.3 & 1.1 & 1.2 & 1.0 & 1.2 & 0.8 \\ 1.1 & 0.5 & 0.2 & 1.5 & 0.3 & 0.4 & 0.2 & 0.5 & 0.2 & 1.0 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 & 1.1 & 0.3 & 0.1 & 0.3 & 0.1 & 0.1 & 0.3 & 0.6 & 0.2 \\ 0.3 & 1.0 & 0.3 & 0.4 & 0.3 & 0.5 & 0.6 & 1.2 & 0.4 & 0.5 & 0.2 & 1.0 \\ 0.2 & 0.2 & 0.3 & 1.0 & 0.3 & 1.5 & 0.4 & 0.5 & 1.0 & 0.7 & 1.0 & 0.1 \\ 0.1 & 0.2 & 1.0 & 0.3 & 1.5 & 0.7 & 1.0 & 0.4 & 0.5 & 1.2 & 0.6 & 0.3 \\ 1.5 & 1.5 & 1.1 & 1.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.5 & 1.2 & 1.5 & 1.3 & 0.3 & 1.2 & 0.6 & 0.5 & 1.5 & 0.8 & 1.1 & 1.5 \\ 0.2 & 0.6 & 1.5 & 0.5 & 0.7 & 0.1 & 0.3 & 0.4 & 0.1 & 1.0 & 0.2 & 0.5 \\ 0.3 & 0.4 & 1.3 & 0.1 & 1.0 & 0.5 & 0.2 & 0.1 & 1.0 & 1.2 & 0.1 & 0.5 \\ 1.0 & 0.6 & 0.2 & 0.1 & 0.5 & 0.6 & 0.3 & 1.1 & 0.7 & 0.3 & 1.0 & 0.6 \\ 1.1 & 1.1 & 0.5 & 0.7 & 0.4 & 0.1 & 0.2 & 0.2 & 0.1 & 1.0 & 0.2 & 0.1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{pmatrix} \leq \begin{pmatrix} 1200 \\ 624 \\ 624 \\ 832 \\ 1040 \\ 1248 \\ 624 \\ 416 \\ 624 \\ 832 \\ 832 \\ 624 \\ 1248 \\ 624 \\ 624 \\ 832 \\ 624 \end{pmatrix}$$

$$Z_1 = [530, 548.9]x_1 + [1220, 1241.43]x_2 + [453.71, 465.33]x_3 + [2510, 2533.75]x_4 + [3450, 3468.75]x_5 + [817.82, 830.56]x_6 + [450, 461.11]x_7 + [1440, 1467.5]x_8 + [3255, 3271.88]x_9 + [550, 575]x_{10} + [2280, 2308.33]x_{11} + [425, 441]x_{12}$$

$$Z_2 = [11.9, 13]x_1 + [4, 4.79]x_2 + [6.28, 6.67]x_3 + [18.25, 19.56]x_4 + [13.8, 14.26]x_5 + [7.25, 7.81]x_6 + [4, 5.89]x_7 + [9, 10.75]x_8 + [13.13, 14.19]x_9 + [8, 9.06]x_{10} + [17, 17.83]x_{11} + [7.5, 8.44]x_{12}$$

$$Z_3 = [70, 75.56]x_1 + [220.05, 220.71]x_2 + [75, 94.44]x_3 + [510, 537.5]x_4 + [825, 862.5]x_5 + [168, 193.89]x_6 + [80, 85.56]x_7 + [390, 413.25]x_8 + [705.62, 720]x_9 + [98, 116.25]x_{10} + [415, 437.22]x_{11} + [85, 109.5]x_{12}$$

$$110 \leq x_1 \leq 140, \quad 108 \leq x_2 \leq 135, \ 104 \leq x_3 \leq 128, \ 96 \leq x_4 \leq 126, \ 84 \leq x_5 \leq 99, \ 86 \leq x_6 \leq 104,$$
$$76 \leq x_7 \leq 88, \quad 60 \leq x_8 \leq 75, \ 70 \leq x_9 \leq 79, \ 67 \leq x_{10} \leq 88, \ 68 \leq x_{11} \leq 80, \ 66 \leq x_{12} \leq 81,$$
$$x_j \in \text{integer}\, j = 1, 2, \ldots, 12, \quad \mu_\mathrm{i} \geq \sigma_i, \ \mu_\mathrm{i} \geq \upsilon_i, \ \mu_\mathrm{i} + \sigma_i + \upsilon_i \leq 3, \ \forall \mu_\mathrm{i}, \ \sigma_i, \ \upsilon_i \in [0, 1], \ i = 1, 2, 3.$$

1. It serves as an additional contribution to the literature on production planning.
2. A case study is provided in which a solution procedures for multiobjective multiproduct problem formulation is reported.
3. In this study, a new approach based on neutrosophic has been applied.
4. The approach is compared with Intuitionistic fuzzy programming, and the result proves to be better.
5. The applicability of Interval-valued Neutrosophic numbers has also been discussed and reported.
6. Both certainty and Vagueness are considered. Certainties are considered in the numerical example 1, whereas interval-valued trapezoidal neutrosophic number (Vagueness) for the coefficients of objective functions are considered in example 2. In this study, both certainty and Vagueness are considered,

Model II

$$\text{Maximize } = \sum_{\mathrm{i}=1}^{3} (\mu_{\mathrm{i}} - \sigma_i - \upsilon_i)$$

Subject to constraints

$$\mu_1 \geq \mu_1^T(Z_1), \quad \sigma_1 \leq \sigma_1^I(Z_1), \ \upsilon_1 \leq \upsilon_1^F(Z_1)$$

$$\mu_2 \geq \mu_2^T(Z_2), \quad \sigma_2 \leq \sigma_2^I(Z_2), \ \upsilon_2 \leq \upsilon_1^F(Z_2)$$

$$\mu_3 \geq \mu_3^T(Z_3), \quad \sigma_3 \leq \sigma_3^I(Z_3), \ \upsilon_3 \leq \upsilon_1^F(Z_3)$$

$$\begin{pmatrix}
0.77 & 0.28 & 0.54 & 3.6 & 1.8 & 0.57 & 0.6 & 0.64 & 1.8 & 1.2 & 0.72 & 0.4 \\
1.5 & 0.1 & 0.3 & 0.5 & 1.5 & 0.1 & 1.1 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.2 & 0.4 & 1.2 & 0.5 & 0.2 & 1.1 & 0.1 & 1.2 & 1.3 & 0.1 & 0.1 & 0.1 \\
0.3 & 1.5 & 0.4 & 1.1 & 0.2 & 1.5 & 0.5 & 0.2 & 0.1 & 0.1 & 0.3 & 0.1 \\
1.5 & 0.5 & 1.2 & 1.4 & 0.6 & 1.0 & 0.1 & 1.1 & 0.1 & 0.2 & 1.1 & 1.1 \\
1.5 & 1.5 & 1.3 & 1.3 & 0.5 & 0.6 & 1.3 & 1.1 & 1.2 & 1.0 & 1.2 & 0.8 \\
1.1 & 0.5 & 0.2 & 1.5 & 0.3 & 0.4 & 0.2 & 0.5 & 0.2 & 1.0 & 0.1 & 0.3 \\
0.2 & 0.1 & 0.2 & 1.1 & 0.3 & 0.1 & 0.3 & 0.1 & 0.1 & 0.3 & 0.6 & 0.2 \\
0.3 & 1.0 & 0.3 & 0.4 & 0.3 & 0.5 & 0.6 & 1.2 & 0.4 & 0.5 & 0.2 & 1.0 \\
0.2 & 0.2 & 0.3 & 1.0 & 0.3 & 1.5 & 0.4 & 0.5 & 1.0 & 0.7 & 1.0 & 0.1 \\
0.1 & 0.2 & 1.0 & 0.3 & 1.5 & 0.7 & 1.0 & 0.4 & 0.5 & 1.2 & 0.6 & 0.3 \\
1.5 & 1.5 & 1.1 & 1.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.5 & 1.2 & 1.5 & 1.3 & 0.3 & 1.2 & 0.6 & 0.5 & 1.5 & 0.8 & 1.1 & 1.5 \\
0.2 & 0.6 & 1.5 & 0.5 & 0.7 & 0.1 & 0.3 & 0.4 & 0.1 & 1.0 & 0.2 & 0.5 \\
0.3 & 0.4 & 1.3 & 0.1 & 1.0 & 0.5 & 0.2 & 0.1 & 1.0 & 1.2 & 0.1 & 0.5 \\
1.0 & 0.6 & 0.2 & 0.1 & 0.5 & 0.6 & 0.3 & 1.1 & 0.7 & 0.3 & 1.0 & 0.6 \\
1.1 & 1.1 & 0.5 & 0.7 & 0.4 & 0.1 & 0.2 & 0.2 & 0.1 & 1.0 & 0.2 & 0.1
\end{pmatrix}
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{pmatrix}
\leq
\begin{pmatrix} 1200 \\ 624 \\ 624 \\ 832 \\ 1040 \\ 1248 \\ 624 \\ 416 \\ 624 \\ 832 \\ 832 \\ 624 \\ 1248 \\ 624 \\ 624 \\ 832 \\ 624 \end{pmatrix}$$

$$\begin{aligned}
Z_1 = {} & [530, 548.9]x_1 + [1220, 1241.43]x_2 + [453.71, 465.33]x_3 + [2510, 2533.75]x_4 + [3450, 3468.75]x_5 \\
& + [817.82, 830.56]x_6 + [450, 461.11]x_7 + [1440, 1467.5]x_8 + [3255, 3271.88]x_9 + [550, 575]x_{10} \\
& + [2280, 2308.33]x_{11} + [425, 441]x_{12}
\end{aligned}$$

$$\begin{aligned}
Z_2 = {} & [11.9, 13]x_1 + [4, 4.79]x_2 + [6.28, 6.67]x_3 + [18.25, 19.56]x_4 + [13.8, 14.26]x_5 + [7.25, 7.81]x_6 + [4, 5.89]x_7 \\
& + [9, 10.75]x_8 + [13.13, 14.19]x_9 + [8, 9.06]x_{10} + [17, 17.83]x_{11} + [7.5, 8.44]x_{12}
\end{aligned}$$

$$\begin{aligned}
Z_3 = {} & [70, 75.56]x_1 + [220.05, 220.71]x_2 + [75, 94.44]x_3 + [510, 537.5]x_4 + [825, 862.5]x_5 + [168, 193.89]x_6 \\
& + [80, 85.56]x_7 + [390, 413.25]x_8 + [705.62, 720]x_9 + [98, 116.25]x_{10} + [415, 437.22]x_{11} + [85, 109.5]x_{12}
\end{aligned}$$

$$110 \leq x_1 \leq 140, \quad 108 \leq x_2 \leq 135, \ 104 \leq x_3 \leq 128, \ 96 \leq x_4 \leq 126, \ 84 \leq x_5 \leq 99, \ 86 \leq x_6 \leq 104,$$

$$76 \leq x_7 \leq 88, \quad 60 \leq x_8 \leq 75, \ 70 \leq x_9 \leq 79, \ 67 \leq x_{10} \leq 88, \ 68 \leq x_{11} \leq 80, \ 66 \leq x_{12} \leq 81,$$

$$x_j \in \text{integer} j = 1, 2, \ldots, 12, \quad \mu_{\mathrm{i}} \geq \sigma_i, \ \mu_{\mathrm{i}} \geq \upsilon_i, \ \mu_{\mathrm{i}} + \sigma_i + \upsilon_i \leq 3, \ \forall \mu_{\mathrm{i}}, \ \sigma_i, \ \upsilon_i \in [0, 1], \ i = 1, 2, 3.$$

TABLE 14. Compromise solution (Model I).

α	Objective Values	μ_k^T	σ_k^I	υ_k^F	$\mu_k,\ \sigma_k,\ \upsilon_k$
0.30	$Z_1 = 1520719$	$\mu_1^T = 0.5109973$	$\sigma_1^I = 0.4584367$	$\upsilon_1^F = 0.2261285$	$\mu^T = 0.5109973$
	$Z_2 = 10986.83$	$\mu_2^T = 0.6393545$	$\sigma_2^I = 0.3764206$	$\upsilon_2^F = 0.2309340$	$\sigma^I = 0.3737944$
	$Z_3 = 327030.7$	$\mu_3^T = 0.5114949$	$\sigma_3^I = 0.3737944$	$\upsilon_3^F = 0.08695312$	$\upsilon^F = 0.2309340$
0.3696931	$Z_1 = 1519371$	$\mu_1^T = 0.5175397$	$\sigma_1^I = 0.4656824$	$\upsilon_1^F = 0.2162205$	$\mu^T = 0.5157448$
	$Z_2 = 10972.32$	$\mu_2^T = 0.6515864$	$\sigma_2^I = 0.3975703$	$\upsilon_2^F = 0.2162205$	$\sigma^I = 0.3792423$
	$Z_3 = 327224$	$\mu_3^T = 0.5157448$	$\sigma_3^I = 0.3792423$	$\upsilon_3^F = 0.07936675$	$\upsilon^F = 0.2162205$
0.50	$Z_1 = 1523753$	$\mu_1^T = 0.4962743$	$\sigma_1^I = 0.4421312$	$\upsilon_1^F = 0.2484255$	$\mu^T = 0.4958508$
	$Z_2 = 10992.23$	$\mu_2^T = 0.6348024$	$\sigma_2^I = 0.3685496$	$\upsilon_2^F = 0.2364097$	$\sigma^I = 0.3537404$
	$Z_3 = 326319.3$	$\mu_3^T = 0.4958508$	$\sigma_3^I = 0.3537404$	$\upsilon_3^F = 0.1148788$	$\upsilon^F = 0.2484255$
0.70	$Z_1 = 1530702$	$\mu_1^T = 0.4625519$	$\sigma_1^I = 0.4047842$	$\upsilon_1^F = 0.2994958$	$\mu^T = 0.4624475$
	$Z_2 = 10825.47$	$\mu_2^T = 0.7754303$	$\sigma_2^I = 0.6117044$	$\upsilon_2^F = 0.0672513$	$\sigma^I = 0.3109214$
	$Z_3 = 324800.2$	$\mu_3^T = 0.4624475$	$\sigma_3^I = 0.3109214$	$\upsilon_3^F = 0.1745054$	$\upsilon^F = 0.2994958$

TABLE 15. Compromise solution (Model II).

α	Objective Values	μ_k^T	σ_k^I	υ_k^F	$\mu_k,\ \sigma_k,\ \upsilon_k$
0.30	$Z_1 = 1511177$	$\mu_1^T = 0.5573015$	$\sigma_1^I = 0.4397180$	$\upsilon_1^F = 0.1560039$	$\mu^T = 0.4612125$
	$Z_2 = 10932.27$	$\mu_2^T = 0.6853644$	$\sigma_2^I = 0.4559747$	$\upsilon_2^F = 0.1755896$	$\sigma^I = 0.4559747$
	$Z_3 = 324744.1$	$\mu_3^T = 0.4612125$	$\sigma_3^I = 0.3093383$	$\upsilon_3^F = 0.1767099$	$\upsilon^F = 0.1767099$
0.3905304	$Z_1 = 1511603$	$\mu_1^T = 0.5552315$	$\sigma_1^I = 0.4174255$	$\upsilon_1^F = 0.1591387$	$\mu^T = 0.4716033$
	$Z_2 = 10916.05$	$\mu_2^T = 0.6990407$	$\sigma_2^I = 0.4596218$	$\upsilon_2^F = 0.1591387$	$\sigma^I = 0.4596218$
	$Z_3 = 325316.6$	$\mu_3^T = 0.4716033$	$\sigma_3^I = 0.3226580$	$\upsilon_3^F = 0.1581619$	$\upsilon^F = 0.1591387$
0.50	$Z_1 = 1515419$	$\mu_1^T = 0.5367166$	$\sigma_1^I = 0.4269204$	$\upsilon_1^F = 0.1871784$	$\mu^T = 0.4554086$
	$Z_2 = 10908.16$	$\mu_2^T = 0.7056991$	$\sigma_2^I = 0.4411346$	$\upsilon_2^F = 0.1511295$	$\sigma^I = 0.4411346$
	$Z_3 = 324480.1$	$\mu_3^T = 0.4554086$	$\sigma_3^I = 0.3018983$	$\upsilon_3^F = 0.1870703$	$\upsilon^F = 0.1871784$
0.70	$Z_1 = 1522855$	$\mu_1^T = 0.5006332$	$\sigma_1^I = 0.3669587$	$\upsilon_1^F = 0.2418242$	$\mu^T = 0.4249364$
	$Z_2 = 10767.08$	$\mu_2^T = 0.8246692$	$\sigma_2^I = 0.3968417$	$\upsilon_2^F = 0.008022844$	$\sigma^I = 0.3968417$
	$Z_3 = 323094.4$	$\mu_3^T = 0.4249364$	$\sigma_3^I = 0.2628366$	$\upsilon_3^F = 0.2414648$	$\upsilon^F = 0.2418242$

unlike other studies where only one is considered, e.g. see Table 1.

7. The proposed Neutrosophic fuzzy programming considers the independent indeterminacy/neutral degree, which is the area of incognizance of a proposition's value. The proposed technique's selection is quite adequate, explanatory, and a good representative of real-life situations.

IX. CONCLUSION

The present study discussed the formulation and solution procedure of a hardware firm's multiobjective multiproduct production planning problem. The formulated model is solved using intuitionistic and neutrosophic programming. The study revealed that the neutrosophic programming approach provided a better solution than intuitionistic fuzzy programming. Further, the same problem is discussed under the interval-valued trapezoidal neutrosophic fuzzy environment, and finally, it is solved using the neutrosophic programming approach. This study helps to understand the application of intuitionistic and neutrosophic programming approaches in solving production-related problems. The more complex and other production-related problem can be solved using the intuitionistic and neutrosophic programming approaches. Also, the interval-valued neutrosophic fuzzy environment production problems can be solved using the neutrosophic programming approach. This article presented a profound study on intuitionistic and neutrosophic programming approaches to solve multiobjective optimization problems under certain and neutrosophic fuzzy environments. In future, some more complex production planning problem will be considered with some new solution approaches under the different kinds of fuzzy logic, such as Pythagorean fuzzy interactive Hamacher power aggregation operators; Pythagorean fuzzy interaction power Bonferroni means aggregation operators in multiple attribute decision making and many others.

ACKNOWLEDGMENT

The authors are grateful to the editor and anonymous reviewers who helped to improve the work's quality and presentation with their numerous comments and suggestions.

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