IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received February 19, 2021, accepted February 27, 2021, date of publication March 3, 2021, date of current version March 11, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3063384

Iterative Learning Consensus Tracking for Multi-Agent Systems With Output Constraints and Data Losses

ZHENGZHENG YU[1], HANWEI ZHANG[1], LIZHI CUI[2], AND JIAQI LIANG[2]

[1]School of Surveying and Land Information Engineering, Henan Polytechnic University, Jiaozuo 454000, China

[2]School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo 454000, China

Corresponding author: Lizhi Cui (cuilizhihpu@163.com)

This work was supported by the National Natural Science Foundation of China under Grant 42074002 and Grant 41931075.

ABSTRACT In this paper, the consensus tracking problem for nonlinear multi-agent systems subjected to the output constraint, data loss and switching topologies is considered. Firstly, a consensus term is defined based on the communication information in present of the nonlinear output saturation and time-varying switching topologies. Since the random data loss is taken into consideration, the consensus term is redesigned by an introducing stochastic variable, which obeys the Bernoulli sequence with known probability. Then, a novel distributed ILC algorithm is designed by using the incomplete communication data of agents, which is more universal than the results without considered nonlinear constraint and random factors simultaneously. Through the contraction mapping method, an obtained convergence condition can also guarantee the asymptotic convergence of the agent along the iteration axis under nonlinear saturation factor and random factors. It is verified that the proposed algorithm can handle more complex situations in the consensus control of multi-agent systems. Simulation examples are further provided to verify the effectiveness of the proposed algorithm.

INDEX TERMS Multi-agent systems (MASs), consensus, saturation, data loss, switching topologies.

I. INTRODUCTION

Consensus control for the multi-agent systems has attracted a growing attention recently. [1], [2]. This is because the multi-agent system has many applications, such as unmanned air vehicles (UAVs), rendezvous and flocking, sensor networks, and formation of robots. Consensus control generally aims at steering the states of nodes in the network to an agreement on certain quantities of interest to perform a joint control task. For the agent model, the existing results cover single integrator model [3]–[6], double integrator model [7]–[9], high-order integrator model [10], [11], linear system [12], [13], and nonlinear system [14]–[17]. Moreover, the information exchange topology, described by a graph, has been thoroughly developed in the existing literature [18], [19]. A series of existing results on the consensus tracking problem were focused on the design and analyses the methods to reach the consensus tracking performance. The consensus algorithm is important to generate complex group level behaviors using simple local coordination rules, which are highly related to practical problems [20]–[22]. In [23], the output feedback protocol based on the event-trigged scheme was designed for the consensus tracking control of the second-order multi-agent systems with unknown inertias.

The iterative learning control strategy has been applied for MASs to achieve learning consensus recently, which is a matured intelligent control technique to achieve high precision tracking performance by the inherent repetition mechanism [24], [25]. Ahn and Chen proposed the first result on formation control using the learning strategy in [26]. Later, the reports on satellite trajectory-keeping [27], mobile robots formation [28], and coordinated train trajectory tracking [29] illustrate successful applications of ILC to MASs. For theoretical research, the contraction mapping method for convergence analysis of affine nonlinear MASs has employed in [30]. The 2D system technique was used to prove the consensus performance in [31] and [32] for linear systems. The Lyapunov function method was introduced in [33] for MASs where agents were of first-order, second order and high-order models, respectively. The data driven iterative learning control protocol was designed for the nonlinear multi-agent systems with completely unknown dynamics in [34].

The associate editor coordinating the review of this manuscript and approving it for publication was Zhiwu Li.

A composite energy function (CEF) based analysis for networked Lagrangian systems was provided in [35]. In [36], the completely distributed iterative learning control scheme was designed for the multi-agent systems with unknown control directions and position constraints based on the barrier composite energy function and multiple piece-wise Nussbaum functions. While various techniques have been developed for the ILC-based MAS consensus, the existing literature mainly focuses on the conventional system setting without any constraint on the system output.

However, when concerning MASs in the real world, it is found that nearly almost all real systems are subject to certain constraints. The constraints arise for the output due to various practical limitations and safety considerations, such as the nonlinear saturation causes by hardware saturation [37]. If ignore such constraints and conduct the conventional control strategy directly, the system output may be beyond the tolerant range and lead to serious problems. In [38], the input saturation problem in the consensus control for the uncertain multi-agent systems by iterative learning control method was considered. On the other hand, the communication among the agents is though the network, however, the network also has its constraints, such as network congestion, bandwidth limitation and network switching, etc. These constraints lead to certain data loss problem and network communication structure changes in the process of communication. To solve the control problem of multi-agent systems with data loss, many results have been obtained. For example, in [39] the consensus problem with data loss was considered as an equivalent asymptotic stability problem. Sufficient condition to solve this asymptotic stability issue was derived by Lyapunov-based methodologies and Linear matrix inequalities (LMIs) techniques. In [40], the consensus method was proposed for linear multi-agent systems such that the multi-agent systems with sampled data and packet losses can reach consensus, where random and deterministic packet losses have considered, respectively. In fact, the communication topology of the agents in the multi-agent systems will change at the same time the data is lost in the network communication [41]. In [42], the mean square consentability problem for multi-agent systems with stochastic switching topology was studied. It has proved that in Markov-switching topologies, the network is mean square consentable under linear consensus protocol if and only if the union of graphs in the switching topology set has globally reachable nodes. In [43], the time-varying formation tracking analysis and design problems for second-order multi-agent systems with switching interaction topologies was studied. Motivated by the existing results, in this paper, we try to propose distributed learning protocol to achieve asymptotical consensus along the iteration axis for the multi-agent systems subjected to output constraint, data loss and switching topologies simultaneously.

To this end, we design a distributed iterative learning control to solve the consensus tracking problem for the multi-agent systems subjected to output constraint, data loss and switching topologies simultaneously. Differing from the existing results on output constraint, data loss and switching topologies respectively, we introduce a general design of distributed learning protocol for the repetitive multi-agent systems. Since the nonlinear saturation factor and random factors take into consideration in this paper, the measured data between the agents are incomplete. In this paper, the data before transmitting to the agents to communicate is suffered with the sensor saturation. Then the data is described by the introduced saturated function, where a relationship exists between the measured output and the agent' actual output. With the limitation of network resource, the agent' saturated data transmitted to its neighbors or the data received from its neighbors maybe loss in the communication process. At the same time, the neighbor agents for a certain agent are also switched in each time instant. Then, we define a stochastic variable to described the data packet of the neighbor agents' whether successfully transmitted or not, where the variable obeys the Bernoulli sequence with known probability. Further, a consensus term is defined based on the incomplete information. On the basis of the consensus term, the distributed learning algorithm is designed. With the incomplete data of the agents, the theoretical analysis is complex than that for the multi-agent systems without consider these problems simultaneously. By using a diagonal matrix, the relationship between the measured saturated data and system output is established firstly. With the use of technology of contraction mapping and a defined α'-norm, the output error of the agents in the compact form is derived to be asymptotically converge to zero under mathematical expectation. Then, convergence of the tracking errors is guaranteed. It is indicated that the proposed algorithm can ensure the multi-agent systems reach the consensus tracking goal although the output of the agents subject to output saturation, data loss and saturation. Illustrative simulations are provided to verify the effectiveness of the proposed algorithm.

Notations: The $\|\cdot\|$ denotes the Euclidean norm, $\otimes$ denotes the Kronecker product, $\mathbb{N}$ refers to the set $\{1, \cdots, N\}$, $\mathbf{1}_N$ stands for a column vector filled with ones, I_N denotes the identity matrix of size N. $\mathrm{E}\{\cdot\}$ denotes the mathematical expectation operator.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a group of $N+1$ nodes consisting of N identical followers and one leader indexed by 0. We describe the communication topology among the nodes as $G = (V, E, A)$, where V is the vertex set, E is the edge set and $A = \left[a_{lj}\right] \in \mathbb{R}^{N\times N}$ is the adjacency matrix with elements a_{lj} denoting the connections such that $a_{lj} = 1$ if there is a path from node j to node l, and $a_{lj} = 0$ otherwise. Self-edge (j, j) is not allowed in this paper, i.e., $a_{jj} = 0$.

The dynamics of follower j, $j \in \mathbb{R}$ is described by

$$\begin{cases} x_{i,j}(k+1) = f(t, x_{i,j}(k)) + Bu_{i,j}(k) \\ y_{i,j}(k) = Cx_{i,j}(k) \\ z_{i,j}(k) = sat_{y0j}(y_{i,j}(k)), \end{cases} \tag{1}$$

where $k \in [0, 1, \ldots, T]$ represents the time index ended at T time instant, $j = 1, 2, \ldots, N$ denotes the jth agent, i denotes the operated iteration number, $x_{i,j} \in \mathbb{R}^n$ is the state of each agent, $u_{i,j} \in \mathbb{R}^p$ is the corresponding control input, and $f(.,.)$ is a continuously differentiable unknown nonlinear function. $y_{i,j} \in \mathbb{R}^m$ is the actual output of agent j and $z_{i,j} \in \mathbb{R}^m$ is the measured output of the agent j. $sat_{y0j}(\cdot)$ is the saturation function of the agent j, and $y_{0j} > 0$ represents threshold for each function, where $y_0 = [y_{01}, y_{02}, \ldots, y_{0N}] \in \mathbb{R}^N$.

A desired trajectory $y_d(k)$ is viewed as a virtual leader agent and labeled 0. Assume that there only some of the agents in the system can obtain the desired trajectory. The new graph includes the leader agent is $\bar{G} = (V \cup \{0\}, \bar{E}, \bar{A})$, where $\bar{E}$ is the edges set, $\bar{A}$ represents the adjacency matrix of the graph $\bar{G}$.

Since the multi-agent system is a complex dynamic network system, there is a fast switching in the network sense. Consider the topology of the agents is switched as time varying, where $\bar{G}_{\sigma(k)}$ denotes the switching topology at time k. Define a piecewise function $\sigma(k)$ to describe the randomness of topology switching, which has a mapping relationship with the rules of topology switching over time and records as $\sigma(k) : \{1, 2, \ldots, \infty\} \mapsto \{1, \ldots, M\}$. For a number of known and fixed agents, the number of switchable topologies is bounded and marked as M, i.e., $\bar{G}_{\sigma(k)} \in \left\{\bar{G}_1, \ldots, \bar{G}_M\right\}$. That is, the switching topology at k time is included in the $\bar{G}_{\sigma(k)}$.

For the multi-agent systems with saturated output and switching topologies, considered there is inevitable data loss in the communication between agents. That is, in the communication interaction process, the measured outputs of the agents may exist random data loss such that the received information incomplete.

Thus, the objective of this paper is to design an appropriate algorithm for consensus of the multi-agent systems subject to output saturation, switching topologies and data loss.

To facilitate analysis, the multi-agent systems (1) satisfy the following assumptions

Assumption 1: The nonlinear function $f(\cdot, \cdot)$ satisfies the global Lipschitz condition, that is, there exists a constant $b_f > 0$, for any two different x_1, x_2 such that

$$\|f(k, x_1(k)) - f(k, x_2(k))\| \leq b_f \|x_1(k) - x_2(k)\| \quad (2)$$

Assumption 2: The matrix CB is a full-rank matrix.

Assumption 3: Each switching topology $\bar{G}_{\sigma(k)}$ with respect to k has a spanning tree with the leader agent be the root.

Assumption 4: The initial state of each iteration of the agent is fixed as $x_{i,j}(0) = x_j^0$, where x_j^0 is a constant.

Assumption 5: For a given bounded desired output $y_d(k)$, there are bounded desired state $x_d(k)$ and desired control input $u_d(k)$, satisfying system (1) such that

$$\begin{cases} x_d(k+1) = f(k, x_d(k)) + Bu_d(k) \\ y_d(k) = Cx_d(k) \end{cases} \quad (3)$$

Remark 1: The switching topologies $\bar{G}_{\sigma(k)}$ switches as time varying, each switching topology $\bar{G}_{\sigma(k)}$ satisfies spanning tree assumption. That means that in the time interval $k \in [0, 1, \ldots, T]$, the multi-agent systems do not have isolated agents. The switching topology may remain consistent with or change from the previous time, then the switching is random. The defined piecewise function $\sigma(k)$ described the randomness of the switching topologies along time axis.

Remark 2: The assumption 4 is a typical initialization condition of the ILC method. It is claimed that the initial value of the agent in each iteration need to be identical. Besides, same initial values for any two agents are not a necessary operating condition for the control processing.

III. ALGORITHM DESIGN AND MAIN RESULT

A. ALGORITHM DESIGN

For the multi-agent systems (1) with saturated outputs, we first define a distributed error $\xi_{i,j}(k)$ as follows:

$$\xi_{i,j}(k) = \sum_{l \in N_j} a_{jl}(k)(z_{i,l}(k) - z_{i,j}(k)) + d_j(y_d(k) - z_{i,j}(k)), \quad (4)$$

where $z_{i,j} \in \mathbb{R}^m$ is the measured output of the agent j in ith iteration. $a_{jl}(k)$ is the element of the adjacency matrix $A_{\sigma(k)} = [a_{jl}(k)]$ at time k, where the current topology is $\bar{G}_{\sigma(k)}$. The corresponding Laplacian matrix denotes as

$$L_{\sigma(k)} = diag\left\{\sum_{j=1}^{N} a_{1j}(k), \sum_{j=1}^{N} a_{2j}(k), \cdots, \sum_{j=1}^{N} a_{Nj}(k)\right\} - A_{\sigma(k)},$$

which belongs to the set $\{L_1, \ldots, L_M\}$ with respect to the direct graph $\bar{G}_{\sigma(k)}$. Define matrix $D = diag\{d_1, d_2, \cdots, d_N\}$ to represent the accessibility of the agents to the leader agent, where $D_{\sigma(k)}$ is the matrix at time k.

Remark 3: In each switch, each switching topology is known, but which topology to switch to at different times is random and unknown. Thus, the matrix $L_{\sigma(k)}, D_{\sigma(k)}, A_{\sigma(k)}$ are time-varying, but it is a determined digital matrix independent of time, due to the elements of which only related to the communication topology graph of the current switching.

Consider the multi-agent systems (1) have inevitable data loss in the process of communication between agents. Therefore, we design the following control algorithm for multi-agent systems (1) subject to data loss:

$$u_{i+1,j}(k) = u_{i,j}(k) + \eta_{i,j}(k)\Gamma\xi_{i,j}(k+1), \quad (5)$$

where $\eta_{i,j}(k)$ is defined as a stochastic index to describe the data loss or not at time k. The index is assumed obey the Bernoulli distribution with the probability of $\bar{\eta}$. That is, if the received data for agent j at time k is lost, $\eta_{i,j}(k) = 0$, otherwise, $\eta_{i,j}(k) = 1$.

Remark 4: Since the phenomenon of data loss is completely random in practical, this paper considers that the data is randomly lost in the time domain and iterative domain, which is used to meet the needs of the problem of data loss encountered in the actual system and improve the applicability of the design algorithm.

Further, we need to find a suitable gain matrix under the designed control algorithm (5), such that the multi-agent systems (1) subject to output saturation, switching topologies and data loss can achieve the consensus tracking task for $\forall k \in [1, T]$. Therefore, we need analyze the condition for matrix selection to ensure the consensus of the multi-agent systems (1).

B. MAIN RESULT

Define tracking error $e_{i,j}(k) = y_d(k) - y_{i,j}(k)$. The perfect tracking track of the iterative multi-agents systems (1) in a finite time interval $[1, T]$is achieved if $\lim_{i\to\infty} y_{i,j}(k) = y_d(k)$, $\forall k \in [1, T]\, y_d(k)$, i.e., $e_{i,j}(k) = 0$ as iteration goes to infinite. That is, the asymptotical convergence of tracking error is required to be provide.

Theorem 1: For the multi-agent systems (1) satisfied *Assumptions* 1-5 with algorithm (5), if the gain matrix is selected satisfy

$$\left\| I - (\bar{\eta} \otimes I_p)(L_k + D_k) \otimes \Gamma CB \right\| \le \rho' < 1, \tag{6}$$

such that the tracking error $e_{i,j}(k)$ converges to 0 as iteration goes to infinite.

Proof: Firstly, we given the compact form of the following variable

$$\begin{aligned} \boldsymbol{x}_i(k) &= [x_{i,1}^T(k), x_{i,2}^T(k), x_{i,3}^T(k), \ldots, x_{i,N}^T(k)]^T, \\ \boldsymbol{y}_i(k) &= [y_{i,1}^T(k), y_{i,2}^T(k), y_{i,3}^T(k), \ldots, y_{i,N}^T(k)]^T, \\ \boldsymbol{u}_i(k) &= [u_{i,1}^T(k), u_{i,2}^T(k), u_{i,3}^T(k), \ldots, u_{i,N}^T(k)]^T, \\ \bar{\boldsymbol{\xi}}_i(k) &= [\bar{\xi}_{i,1}^T(k), \ldots, \bar{\xi}_{i,N}^T(k)]^T \end{aligned}$$

Then, the multi-agent systems can be rewritten as

$$\begin{cases} \boldsymbol{x}_i(k+1) = \boldsymbol{f}(k, \boldsymbol{x}_i(k)) + (I_N \otimes B)\boldsymbol{u}_i(k) \\ \boldsymbol{y}_i(k) = (I_N \otimes C)\boldsymbol{x}_i(k) \end{cases} \tag{7}$$

Introduce an index $r_{i,j}(k)$, then we can have the following relationship between actual tracking error and the measure tracking error $e'_{i,j}(k) = r_{i,j}(k)e_{i,j}(k)$, where

$$r_{i,j}(k) = \begin{cases} \dfrac{y_d(k) - y_{0j}}{e_{i,j}(k)}, e_{i,j}(k) < y_d(k) - y_{0j} \\ 1, y_d(k) - y_{0j} \le e_{i,j}(k) \le y_d(k) + y_{0j} \\ \dfrac{y_d(k) + y_{0j}}{e_{i,j}(k)}, y_d(k) + y_{0j} < e_{i,j}(k) \end{cases} \tag{8}$$

Based on (4) and (8), we can have the compact form of the distributed error

$$\xi_{i,j}(k) = \sum_{l \in N_j} a_{j,l}(k)(e'_{i,j}(k) - e'_{i,l}(k)) + d_j(e'_{i,j}(k)) \tag{9}$$

Define the tracking error in the similar way as

$$\begin{aligned} \boldsymbol{e}_i(k) &= [e_{i,1}^T(k), e_{i,2}^T(k), e_{i,3}^T(k), \cdots, e_{i,N}^T(k)], \\ \boldsymbol{e'}_i(k) &= [e'_{i,1}(k)^T, e'_{i,2}(k)^T, e'_{i,3}(k)^T, \cdots, e'_{i,N}(k)^T], \\ r_i(k) &= diag\{r_{i,1}(k), \ldots, r_{i,N}(k)\}. \end{aligned} \tag{10}$$

Then it yields that $\boldsymbol{e'}_i(k) = r_i(k)\boldsymbol{e}_i(k)$.

Then, we can obtain

$$\begin{aligned} \bar{\boldsymbol{\xi}}_i(k) &= ((L_k + D_k) \otimes I_m)\boldsymbol{e'}_i(k) \\ &= ((L_k + D_k)r_i(k) \otimes I_m)\boldsymbol{e}_i(k) \end{aligned} \tag{11}$$

$$\begin{aligned} \boldsymbol{u}_{i+1}(k) = \boldsymbol{u}_i(k) + (\eta_i(k) \otimes I_p)(I_N \otimes \Gamma)\bar{\boldsymbol{\xi}}_i(k+1) = \boldsymbol{u}_i(k) \\ + (\eta_i(k) \otimes I_p)((L_k + D_k)r_i(k+1) \otimes \Gamma)\boldsymbol{e}_i(k+1) \end{aligned} \tag{12}$$

From (12), the control error $\Delta\boldsymbol{u}_{i+1}(k)$ and state error $\Delta\boldsymbol{x}_i(k+1)$ are

$$\begin{aligned} &\Delta\boldsymbol{u}_{i+1}(k) \\ &= \boldsymbol{u}_d(k) - \boldsymbol{u}_{i+1}(k) \\ &= \Delta\boldsymbol{u}_i(k) - (\eta_i(k) \otimes I_p)(I_N \otimes \Gamma)\xi_i(k+1) \\ &= \Delta\boldsymbol{u}_i(k) - (\eta_i(k) \otimes I_p)((L_k + D_k)r_i(k+1) \otimes \Gamma)\boldsymbol{e}_i(k+1) \\ &= \Delta\boldsymbol{u}_i(k) - (\eta_i(k) \otimes I_p)((L_k + D_k))r_i(k+1) \otimes \Gamma C) \\ &\quad \times \Delta\boldsymbol{x}_i(k+1) \\ &= [I - (\eta_i(k) \otimes I_p)\,((L_k + D_k)r_i(k+1) \otimes \Gamma CB)]\Delta\boldsymbol{u}_i(k) \\ &\quad -(\eta_i(k) \otimes I_p)((L_k + D_k)r_i(k+1) \otimes \Gamma C) \\ &\quad \times [\boldsymbol{f}(k, \boldsymbol{x}_d(k)) - \boldsymbol{f}(k, \boldsymbol{x}_i(k))] \end{aligned} \tag{13}$$

$$\begin{aligned} &\Delta\boldsymbol{x}_i(k+1) \\ &= \boldsymbol{x}_d(k+1) - \boldsymbol{x}_i(k+1) \\ &= (\boldsymbol{f}(k, \boldsymbol{x}_d(k)) - \boldsymbol{f}(k, \boldsymbol{x}_i(k))) + (I_N \otimes B)\Delta\boldsymbol{u}_i(k) \end{aligned} \tag{14}$$

where $\boldsymbol{f}(k, \boldsymbol{x}_d(k)) = 1_N \otimes f^T(k, x_d(k))$, $\boldsymbol{f}(k, \boldsymbol{x}_i(k)) = [f^T(k, x_1(k)), \ldots, f^T(k, x_N(k))]^T$.

Taking the norm of both sides of (14)

$$\begin{aligned} \|\Delta\boldsymbol{x}_i(k+1)\| &\le \|\boldsymbol{f}(k, \boldsymbol{x}_d(k)) - \boldsymbol{f}(k, \boldsymbol{x}_i(k))\| \\ &\quad + b_B \|\Delta\boldsymbol{u}_i(k)\|, \\ b_B &= \|I_N \otimes B\| \end{aligned} \tag{15}$$

According to *Assumption* 1, it yields that

$$\|\boldsymbol{f}(k, \boldsymbol{x}_d(k)) - \boldsymbol{f}(k, \boldsymbol{x}_i(k))\| \le b_f \|\Delta\boldsymbol{x}_i(k)\| \tag{16}$$

Subsisting (16) into (15), we have

$$\|\Delta\boldsymbol{x}_i(k+1)\| \le b_f \|\Delta\boldsymbol{x}_i(k)\| + b_B \|\Delta\boldsymbol{u}_i(k)\| \tag{17}$$

From the inequality relationship described in formula (17), it can be recursively obtained

$$\begin{aligned} \|\Delta\boldsymbol{x}_i(k)\| &\le b_f \|\Delta\boldsymbol{x}_i(k-1)\| + b_B \|\Delta\boldsymbol{u}_i(k-1)\|, \\ \|\Delta\boldsymbol{x}_i(k-1)\| &\le b_f \|\Delta\boldsymbol{x}_i(k-2)\| + b_B \|\Delta\boldsymbol{u}_i(k-2)\|, \\ &\vdots \\ \|\Delta\boldsymbol{x}_i(1)\| &\le b_f \|\Delta\boldsymbol{x}_i(0)\| + b_B \|\Delta\boldsymbol{u}_i(0)\|, \end{aligned} \tag{18}$$

That further equals to

$$\|\Delta\boldsymbol{x}_i(k)\| \le \sum_{l=0}^{k-1} b_f^{k-1-l} b_B \|\Delta\boldsymbol{u}_i(l)\| + b_f^k \|\Delta\boldsymbol{x}_i(0)\| \tag{19}$$

Taking norm of the both sides of (13), it yields that

$$\begin{aligned}&\|\Delta \boldsymbol{u}_{i+1}(k)\|\\&\le \left\|I-(\eta_i(k)\otimes I_p)\left((L_k+D_k)r_i(k+1)\otimes\Gamma CB\right)\right\|\|\Delta\boldsymbol{u}_i(k)\|\\&\quad+b_f\left\|(\eta_i(k)\otimes I_p)((L_k+D_k)r_i(k+1)\otimes\Gamma C)\right\|\|\Delta\boldsymbol{x}_i(k)\|\end{aligned}\tag{20}$$

Subsisting (19) into (20), we have

$$\begin{aligned}&\|\Delta \boldsymbol{u}_{i+1}(k)\|\\&\le \left\|I-(\eta_i(k)\otimes I_p)(L_k+D_k)r_i(k+1)\otimes\Gamma CB\right\|\|\Delta\boldsymbol{u}_i(k)\|\\&\quad+b_f\left\|(\eta_i(k)\otimes I_p)((L_k+D_k)r_i(k+1)\otimes\Gamma C)\right\|\\&\quad\times\sum_{l=0}^{k-1}{b_f}^{k-1-l}b_B\|\Delta\boldsymbol{u}_i(l)\|\\&\quad+b_f\left\|(\eta_i(k)\otimes I_p)((L_k+D_k)r_i(k+1)\otimes\Gamma C)\right\|b_f^k\|\Delta\boldsymbol{x}_i(0)\|\end{aligned}\tag{21}$$

Taking α'-norm of the both sides of (21), where $\alpha' > \max[1, b_f]$

$$\begin{aligned}&\|\Delta u_{i+1}(k)\|(1/\alpha')^k\\&\le\tilde{\rho}_i(k)\|\Delta u_i(k)\|(1/\alpha')^k+\frac{b_1(k)}{\alpha'}\\&\quad\times\sum_{l=0}^{k-1}\left(\frac{b_f}{\alpha'}\right)^{k-1-l}b_B\|\Delta u_i(l)\|(1/\alpha')^l+b_1(k)b_f^k\|\Delta x_i(0)\|\\&\le\tilde{\rho}_i(k)\|\Delta u_i(k)\|(1/\alpha')^k+\frac{b_1(k)}{\alpha'}\\&\quad\times\sum_{l=0}^{k-1}\left(\frac{b_f}{\alpha'}\right)^{k-1-l}b_B\sup_{1\le k\le T}\{\|\Delta u_i(l)\|(1/\alpha')^l\}\\&\quad+b_1(k)b_f^k\|\Delta x_i(0)\|,\end{aligned}\tag{22}$$

where

$$\begin{aligned}\tilde{\rho}_i(k)&=\left\|I-(\eta_i(k)\otimes I_p)(L_k+D_k)r_i(k+1)\otimes\Gamma CB\right\|,\\b_1(k)&=b_f\left\|(\eta_i(k)\otimes I_p)((L_k+D_k)r_i(k+1)\otimes\Gamma C)\right\|.\end{aligned}$$

Further we can obtain that from (22)

$$\begin{aligned}\|\Delta\boldsymbol{u}_{i+1}\|_{\alpha'}\le\left(\tilde{\rho}_i(k)+\frac{b_1(k)b_B(1-(\frac{b_f}{\alpha'})^k)}{\alpha'-b_f}\right)\|\Delta\boldsymbol{u}_i\|_{\alpha'}\\+b_1(k)b_f^k\|\Delta\boldsymbol{x}_i(0)\|.\end{aligned}\tag{23}$$

Because of the introduction of random variable $\eta_i(k)$, the multi-agent iterative system (1) can become a random system meanwhile, then we need to further consider the convergence of the system in the meaning of mathematical expectation.

Taking the mathematical expectation operation $E\{\cdot\}$ of both sides of (23), it yields that

$$\begin{aligned}&\mathrm{E}\left\{\|\Delta\boldsymbol{u}_{i+1}(k)\|_{\alpha'}\right\}\\&\le E\left\{\left(\|\tilde{\rho}_i(k)\|+\frac{\|b_1(k)\|\,b_B(1-(b_f/\alpha')^k)}{\alpha'-b_f}\right)\right\}E\{\|\Delta\boldsymbol{u}_i(k)\|_{\alpha'}\}\\&\quad+\mathrm{E}\left\{b_1(k)b_f^k\|\Delta\boldsymbol{x}_i(0)\|\right\}\\&\le\left(\rho+\frac{\rho_1b_B(1-(b_f/\alpha')^k)}{\alpha'-b_f}\right)\mathrm{E}\{\|\Delta\boldsymbol{u}_i(k)\|_{\alpha'}\}+\varepsilon\\&\le\tilde{\rho}\mathrm{E}\{\|\Delta\boldsymbol{u}_i(k)\|_{\alpha'}\}+\varepsilon\end{aligned}\tag{24}$$

where

$$\begin{aligned}\tilde{\rho}&=\rho+\frac{\rho_1b_B\left(1-(b_f/\alpha')^k\right)}{\alpha'-b_f}\\\rho&=\mathrm{E}\{\|\tilde{\rho}_i(k)\|\}=\left\|I-(\bar{\eta}\otimes I_p)(L_k+D_k)\otimes\Gamma CB\right\|\\\rho_1&=\mathrm{E}\{\|b_1(k)\|\}=b_f\left\|(\bar{\eta}\otimes I_p)(L_k+D_k)\otimes\Gamma CB\right\|,\\\mathrm{E}\left\{\eta_{i,j}(k)\right\}&=\bar{\eta}_j,\\\bar{\eta}&=diag\{\bar{\eta}_1,\bar{\eta}_2,\ldots,\bar{\eta}_N\},\\\varepsilon&=\rho_1b_f^k\|\Delta\boldsymbol{x}_i(0)\|=\rho_1b_f^kb_0.\end{aligned}$$

Then, if $\tilde{\rho} < 1$, $\lim_{i\to\infty}\sup\mathrm{E}\{\|\Delta\boldsymbol{u}_i\|_\alpha\}\le\frac{\varepsilon}{1-\tilde{\rho}}$ holds.

On the basis of the *Lemma* 1 in [44]

$$\begin{aligned}\rho&=\left\|I-(\bar{\eta}\otimes I_p)(L_k+D_k)r_i(k+1)\otimes\Gamma CB\right\|\\&=\left\|I-\left((\bar{\eta}\otimes I_p)(L_k+D_k)\otimes\Gamma CB\right)(r_i(k+1)\otimes I_N)\right\|\end{aligned}$$

That is if $\left\|I-(\bar{\eta}\otimes I_p)(L_k+D_k)\otimes\Gamma CB\right\| < 1$ holds, then $\rho < 1$. From (24), it is indicated that there exists a big enough α' such that $\tilde{\rho} < 1$ if $\rho < 1$ holds. Then, the control input error satisfies

$$\lim_{i\to\infty}\sup\mathrm{E}\{\|\Delta\boldsymbol{u}_i\|_\alpha\}\le\frac{\varepsilon}{1-\tilde{\rho}}$$

It is worth pointing out that $\lim_{i\to\infty}\mathrm{E}\{\|\Delta\boldsymbol{u}_i(k)\|_{\alpha'}\}=0$ holds if the initial values of the agents satisfied $x_{i,j}(0)=x_j^0=x_d$.

Taking α'-norm of the both sides of (19), it yields,

$$\begin{aligned}&\|\Delta\boldsymbol{x}_i(k+1)\|_{\alpha'}\\&\le\sup\left\{\sum_{l=0}^{k-1}b_f^{k-1-l}b_B\|\Delta\boldsymbol{u}_i(l)\|\right\}\\&\quad+b_f^k\|\Delta\boldsymbol{x}_i(0)\|_{\alpha'}\\&\le\frac{b_f(1-\alpha'^{-(\lambda-1)T_d})}{\alpha'^{\lambda-1}-1}\|\Delta\boldsymbol{u}_i(k)\|_{\alpha'}+b_f^k\|\Delta\boldsymbol{x}_i(0)\|_{\alpha'}\end{aligned}\tag{25}$$

That is, if $\Delta\boldsymbol{x}(0)=0$, $\mathrm{E}\{\|\Delta\boldsymbol{x}_i(k)\|_{\alpha'}\}=0$ as $\lim_{i\to\infty}\mathrm{E}\{\|\Delta\boldsymbol{u}_i(k)\|_{\alpha'}\}=0$.

Further, $\lim_{i\to\infty}\mathrm{E}\{\|\Delta e_i(k)\|_{\alpha'}\}=0$, that means that when the appropriate gain matrix is selected to satisfy $\tilde{\rho} < 1$, the consensus tracking task of the agents is fulfill by the algorithm (5), i.e., $\lim_{i\to\infty}y_{i,j}(k)=y_d(k)$, $\forall k\in[1,T]$. The proof is complete. ■

Remark 5: Although the same initial value assumption is strong in the assumption 4, the proposed ILC design is also applicable to the control problem under the initialization condition with initial offset. In this case, the system cannot be guaranteed to converge asymptotically to 0, but converges to the bound of a convex hull related to the initial value of an agent. For relaxing the assumption of initial condition, there have been methods to remove this limitation by adding an initial value learning, such as [45], [46].

Remark 6: The proposed protocol is proved to be effective to solve the consensus tracking problem for the nonlinear multi-agent systems with output constraint, data loss and switching topologies. Since the controller design is only data-based, then the ILC protocol is suitable for the more general nonlinear dynamics, as shown in [24], [25], [34], and the protocol proposed in this paper also can be extended to the handle with the consensus for the nonlinear multi-agent systems with heterogeneous dynamics.

Remark 7: The main result of Theorem 1 shows that, it is necessary to make this condition satisfy all times in theory. However, this condition is essentially only related to the number of topological structures, and does not need to be satisfied at all times, because it has been assumed that the switching topology of agents switches randomly between $\bar{G}_1$ and $\bar{G}_M$.

IV. SIMULATION

In this section, we verify the consensus tracking of a multi-agent system contains 5 agents, where the dynamics are shown as follows:

$$\begin{cases} x_{i,j}(k+1) = \begin{bmatrix} \cos(0.67x_{i,j_1}(k)) + 1.2x_{i,j_1}(k) \\ -1.2\sin(0.8x_{i,j_2}(k)) + 1.1x_{i,j_2}(k) \end{bmatrix} \\ + \begin{bmatrix} 2 & -1.1 \\ -0.98 & 1.5 \end{bmatrix} u_{i,j}(k) \\ y_{i,j}(k) = \begin{bmatrix} 1 & 2 \end{bmatrix} x_{i,j}(k) \end{cases} \tag{26}$$

and given the desired trajectory

$$y_d(k) = [1.1\sin(0.27k) + \cos(0.1\pi k)],$$

$k \in [1, 50]$.

Consider the threshold of each agent is

$$[y_{01}, y_{02}, y_{03}, y_{04}, y_{05}] = [1.7, 1.5, 1.6, 1.4, 1.8].$$

It can be seen that the desired trajectory is within the measurable range of the system.

In the simulation, we assume the switching graphs have 4 kinds of topologies as shown in Fig. 1, where $\bar{G}_{\sigma(k)} \in [\bar{G}_1, \ldots, \bar{G}_4]$.

Define that the switching signal function $\sigma(k)$ changes randomly on the interval [0,1]. The switching rules are set as follows

$$\begin{cases} \sigma_i(k) \in [0, 0.25), \sigma_i(k) = 1, \bar{G}_{\sigma_i(k)} = \bar{G}_1 \\ \sigma_i(k) \in [0.25, 0.5), \bar{G}_{\sigma_i(k)} = \bar{G}_2 \\ \sigma_i(k) \in [0.5, 0.75), \bar{G}_{\sigma_i(k)} = \bar{G}_3 \\ \sigma_i(k) \in [0.75, 1), \bar{G}_{\sigma_i(k)} = \bar{G}_4 \end{cases}$$

The switching signal function is random. Fig. 2 shows the switching of multi-agent communication topology when the system changes with time in one simulation. The 1,2,3,4 in the diagram refers to the switchable topology diagram $\bar{G}_1, \bar{G}_2, \bar{G}_3, \bar{G}_4$, respectively.

In order to verify the effectiveness of the proposed method, we set two cases of data loss rates by $\bar{\eta} = 0.9$ and $\bar{\eta} = 0.5$. The multi-agent system example is simulated in two cases.

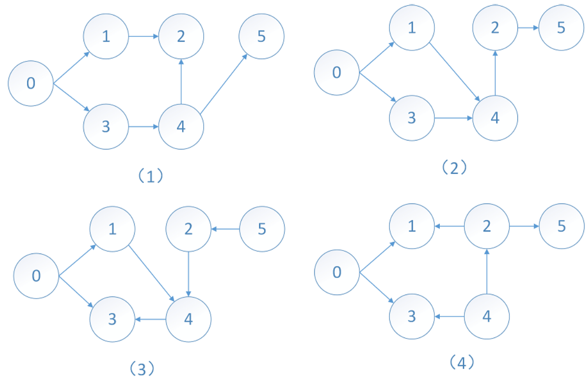

FIGURE 1. Switching interaction graphs.

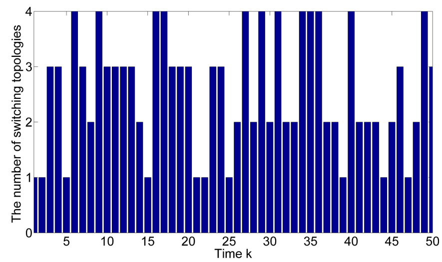

FIGURE 2. The switching topology as time increases in one iteration.

Select the gain matrix as $\Gamma = \begin{bmatrix} 0.13 & 0.14 \end{bmatrix}^T$. Through verification, both data loss rates and four switching structures meet the convergence condition. With the given control gain, the control effect of the proposed algorithm on the system is shown in Figs 3 to Fig. 7.

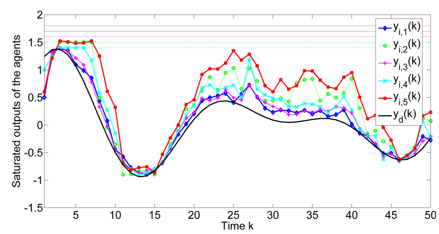

FIGURE 3. The measured outputs of agents at 20th iteration with switching topology and 10% data drops.

In the presence of data loss and saturation constraint, we present the simulation results of multi-agent systems. The output curve of the system at the 20th iteration is shown in Fig. 3 and Fig. 4. In Fig. 3 and Fig. 4, different colors with marked lines represent different agents, and the corresponding solid lines represent the saturation threshold of each agent. We can see in Fig. 3 that when the system has 20 iterations, some of the output at case1 $\bar{\eta} = 0.9$ (10% data loss) is out of the threshold range. However, in Fig. 4 there are more saturation constraints in the case of the data loss rate at case 2 $\bar{\eta} = 0.5$ (50% data loss) with the same system gain and saturation. The output of the system is more dispersed, which indicating that more data is lost. It shows that the amount of data loss has a direct impact on the output of the system under the same configuration conditions.

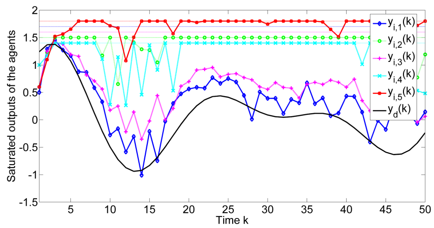

FIGURE 4. The measured outputs of agents at 20th iteration with switching topology and 50%data drops.

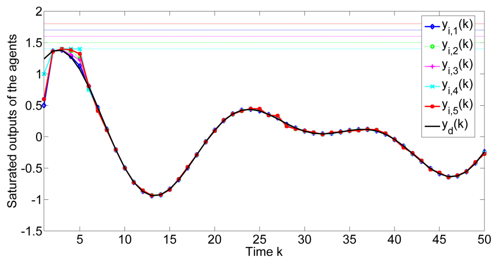

FIGURE 5. The measured outputs of agents at 80th iteration with switching topology and 10% data drops.

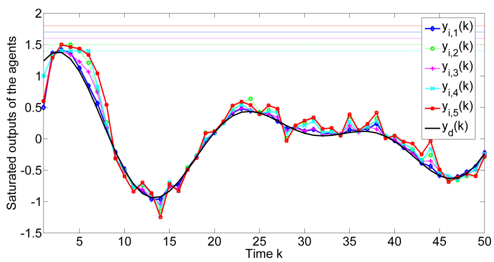

FIGURE 6. The measured outputs of agents at 80th iteration with switching topology and 50% data drops.

With the increase of iteration numbers, although the topology of the system is switched randomly, the agents asymptotically converge to the desired trajectory under the proposed algorithm. Compared with the output in Fig. 3 and Fig. 4, it can be seen from Fig. 5 and Fig. 6 that systems at 20 iterations, the output of the system in 80 iterations has obviously tended to the desired trajectory. But the amount of data loss still affects the output of the system. As shown in Fig. 5, when the system loses 10% data, only a small part of the time value is not fully tracked at 80 iterations. However, when the data is lost by 50%, as shown in Fig. 6, although the output of the system has approached the desired trajectory, there are still large tracking errors at each time.

The system can finally achieve complete tracking of the desired trajectory as iteration number increases. As can be seen from Fig. 3 and Fig. 5, with the increase of iteration numbers, the output of the system is asymptotically converge to the desired trajectory. Therefore, even at different data loss rates, the multi-agent system subject to saturation constraints and switching topologies can finally achieve complete tracking of the same desired trajectory in a finite time interval, and the results are shown in Fig. 7.

It can be seen from the above results that the multi-agent system subject to switching topologies, data loss rate and

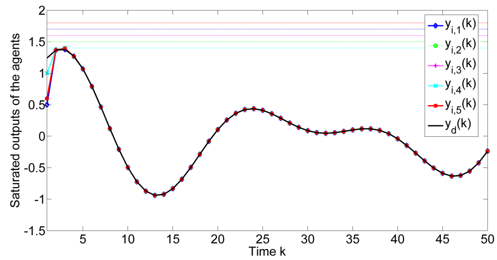

FIGURE 7. The measured outputs of agents at 200th iteration with switching topologies and data dropouts.

saturation constraints can achieve complete tracking of the desired trajectory under the proposed algorithm. However, because the different data loss has a direct impact on the convergence rate of the system, the tracking error of the system is not the same under the two data loss rates, as shown in Fig. 8 and Fig. 9. It shows that the less data loss, the faster the system converges.

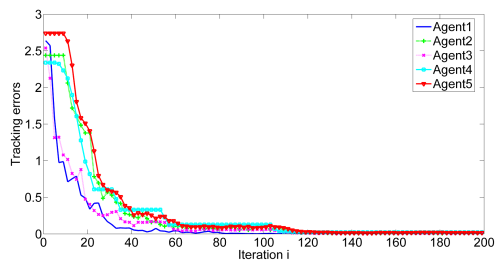

FIGURE 8. The tracking errors of agents with switching topology, output saturation, 10% data dropouts as iteration increases.

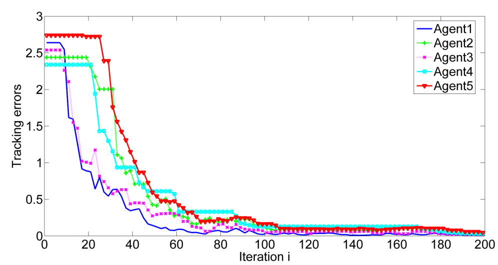

FIGURE 9. The tracking errors of agents with switching topology, output saturation, 50% data dropouts as iteration increases.

Further, a multi-agent system composed of 5 brush DC permanent magnet linear motor is given next to verify the effectiveness of the algorithm, in which the nonlinear model of each motor is shown as

$$\begin{cases} \dot{x}(t) = \upsilon(t), \\ \upsilon(t) \ = \frac{u(t) - f_{friction}(t) - f_{ripple}(t)}{m}, \end{cases} \tag{27}$$

where $f_{friction}(t)$ and $f_{ripple}(t)$ are friction (N) and thrust pulse (N), $u(t)$ is thrust (N), m is mass (kg), $x(t)$ denotes position (m), $\upsilon(t)$ is velocity (m/s), t is continuous time (s).

The model of friction shown as follows

$$f_{friction}(t) = (f_c + (f_s - f_c)e^{-(\dot{x}/\dot{x}_\delta)^\delta} + f_\upsilon \dot{x}) sgn\dot{x}, \tag{28}$$

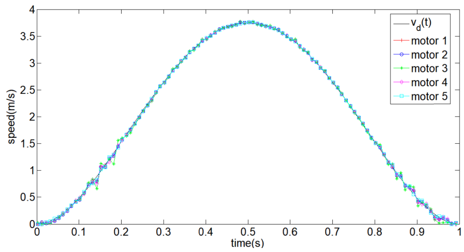

FIGURE 10. The speeds of motors under fixed topology without data drops and output saturation at 50th iteration.

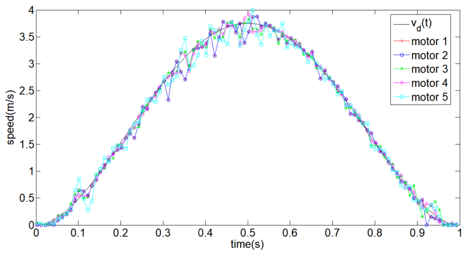

FIGURE 11. The speeds of motors with switching topology, output saturation, 15% data dropouts at 50th iteration.

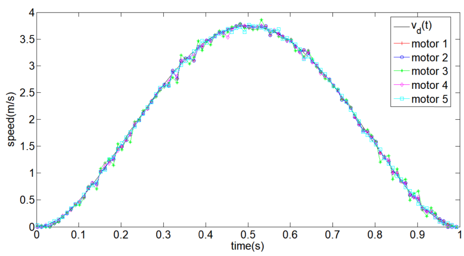

FIGURE 12. The speeds of motors at 300th iteration.

where f_s is static friction, f_c is the minimum value of Coulomb friction, $\dot{x}_\delta$ is lubrication parameters, f_υ is load parameters and δ is empirical parameters.

According to the benefit of slot in motor structure, the pulse dynamic model produced by magneto resistive is

$$f_{ripple}(t) = b_1 \sin(\omega_0 x(t)), \tag{29}$$

where b_1 is amplitude, ω_0 is angular velocity.

The desired speed of the linear motor is designed as follows

$$\upsilon_d(\tau) = (x_0 - x_f)(60\tau^3 - 30\tau^4 - 30\tau^2), \tag{30}$$

where $\tau = t/(t_f - t_0)$, x_0 is initial position, x_f is terminal position. $x_0 = 0$, $x_f = 2m$, $t_f = 1s$. Sampling period during simulation set as $0.001s$.

The nonlinear model parameters are as follows $m = 0.59kg$, $\dot{x}_\delta = 0.1$, $\delta = 1$, $f_c = 10N$, $f_s = 20N$, $f_\upsilon = 10N \cdot s \cdot m^{-1}$, $b_1 = 8.5N$, $\omega_0 = 314s^{-1}$. Set $\boldsymbol{u}(t) = 0$ and initial motor speed $\upsilon(t) = 0$.

Without considering the data loss and saturation constraints factors, and assuming that the motors communicate under the first topology of Fig. 1, the speed simulation results of 5 DC motors are shown in Fig. 10. Fig. 10 shows the convergence tracking effect of five DC motors in the system at the 50th iteration.

Taking switching topologies, data loss and saturation constraint factors into account, the effectiveness of the proposed algorithm is also verified. Set 15% data loss and saturation threshold to 4. The topology of the system is randomly switched as shown in Fig. 1. The simulation results are shown in Fig. 11 and Fig. 12. Fig. 11 shows the output results of motor speed considering the random switching of system topology and the existence of data loss and saturation constraint. The output results when the motor speed in the system reaches the consistent state are shown in Fig. 12.

V. CONCLUSION

For a class of repetitive multi-agent systems, its consensus tracking problem subjective to switching topologies, data loss and saturation constraints is considered. The distributed ILC algorithm is established by the incomplete interaction data. With the use of graph theory and contraction mapping technical, the convergence of the control input error is firstly analyzed and further the convergence of the tracking error is derived. The obtained sufficient condition provides the selection criterion of the control gain, under which the proposed ILC algorithm can ensure the agents reach consensus tracking objective even the output saturation, data loss and time-varying switching topologies factors exist. Finally, the effectiveness is verified by simulation examples.

REFERENCES

[1] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[2] W. Ren, "On consensus algorithms for double-integrator dynamics," *IEEE Trans. Autom. Control*, vol. 53, no. 6, pp. 1503–1509, Jul. 2008.

[3] W. Jia and J. Wang, "Observer-based distributed fault detection for heterogeneous multi-agent systems," *Appl. Sci.*, vol. 10, no. 21, p. 7466, Oct. 2020.

[4] B. Ning, Q.-L. Han, and Z. Zuo, "Practical fixed-time consensus for integrator-type multi-agent systems: A time base generator approach," *Automatica*, vol. 105, pp. 406–414, Jul. 2019.

[5] Z. Wang, J. Xu, X. Song, and H. Zhang, "Consensus conditions for multi-agent systems under delayed information," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 65, no. 11, pp. 1773–1777, Nov. 2018.

[6] D. Poulsen, M. Defoort, and M. Djemai, "Mean square consensus of double-integrator multi-agent systems under intermittent control: A stochastic time scale approach," *J. Franklin Inst.*, vol. 356, no. 16, pp. 9076–9094, Nov. 2019.

[7] Y. Feng and W. X. Zheng, "Group consensus control for discrete-time heterogeneous first-and second-order multi-agent systems," *IET Control Theory Appl.*, vol. 12, no. 6, pp. 753–760, Apr. 2018.

[8] Y. Zheng, Q. Zhao, J. Ma, and L. Wang, "Second-order consensus of hybrid multi-agent systems," *Syst. Control Lett.*, vol. 125, pp. 51–58, Mar. 2019.

[9] C. Sun, G. Hu, L. Xie, and M. Egerstedt, "Robust finite-time connectivity preserving coordination of second-order multi-agent systems," *Automatica*, vol. 89, pp. 21–27, Mar. 2018.

[10] Y. Xie and Z. Lin, "Global optimal consensus for higher-order multi-agent systems with bounded controls," *Automatica*, vol. 99, pp. 301–307, Jan. 2019.

[11] A. Abdessameud and A. Tayebi, "Distributed consensus algorithms for a class of high-order multi-agent systems on directed graphs," *IEEE Trans. Autom. Control*, vol. 63, no. 10, pp. 3464–3470, Oct. 2018.

[12] Y.-Y. Qian, L. Liu, and G. Feng, "Output consensus of heterogeneous linear multi-agent systems with adaptive event-triggered control," *IEEE Trans. Autom. Control*, vol. 64, no. 6, pp. 2606–2613, Jun. 2019.

[13] Y. Yang, H. Xu, and D. Yue, "Observer-based distributed secure consensus control of a class of linear multi-agent systems subject to random attacks," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 66, no. 8, pp. 3089–3099, Aug. 2019.

[14] T. Ma, Z. Zhang, and B. Cui, "Variable impulsive consensus of nonlinear multi-agent systems," *Nonlinear Anal., Hybrid Syst.*, vol. 31, pp. 1–18, Feb. 2019.
[15] Y. Zhang, H. Liang, H. Ma, Q. Zhou, and Z. Yu, "Distributed adaptive consensus tracking control for nonlinear multi-agent systems with state constraints," *Appl. Math. Comput.*, vol. 326, pp. 16–32, Jun. 2018.
[16] S. J. Yoo, "Distributed consensus tracking of a class of asynchronously switched nonlinear multi-agent systems," *Automatica*, vol. 87, pp. 421–427, Jan. 2018.
[17] P. Duan, Z. Duan, G. Chen, and L. Shi, "Distributed state estimation for uncertain linear systems: A regularized least-squares approach," *Automatica*, vol. 117, Jul. 2020, Art. no. 109007.
[18] L. Fang and P. J. Antsaklis, "On communication requirements for multi-agent consensus seeking," in *Networked Embedded Sensing and Control*. Berlin, Germany: Springer, 2006, pp. 53–67.
[19] A. Tahbaz-Salehi and A. Jadbabaie, "A necessary and sufficient condition for consensus over random networks," *IEEE Trans. Autom. Control*, vol. 53, no. 3, pp. 791–795, Apr. 2008.
[20] S. Khoo, L. Xie, and Z. Man, "Robust finite-time consensus tracking algorithm for multirobot systems," *IEEE/ASME Trans. Mechatronics*, vol. 14, no. 2, pp. 219–228, Apr. 2009.
[21] W. Ren and R. W. Beard, *Distributed Consensus in Multi-Vehicle Cooperative Control*. London, U.K.: Springer, 2008.
[22] S. Yang, S. Tan, and J.-X. Xu, "Consensus based approach for economic dispatch problem in a smart grid," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4416–4426, Nov. 2013.
[23] P. Duan, K. Liu, N. Huang, and Z. Duan, "Event-based distributed tracking control for second-order multiagent systems with switching networks," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 50, no. 9, pp. 3220–3230, Sep. 2020.
[24] A. Tayebi and C.-J. Chien, "A unified adaptive iterative learning control framework for uncertain nonlinear systems," *IEEE Trans. Autom. Control*, vol. 52, no. 10, pp. 1907–1913, Oct. 2007.
[25] C. Yin, J.-X. Xu, and Z. Hou, "A high-order internal model based iterative learning control scheme for nonlinear systems with time-iteration-varying parameters," *IEEE Trans. Autom. Control*, vol. 55, no. 11, pp. 2665–2670, Nov. 2010.
[26] H. S. Ahn and Y. Q. Chen, "Iterative learning control for multi-agent formation," in *Proc. ICCAS-SICE*, Aug. 2009, pp. 3111–3116.
[27] H.-S. Ahn, K. L. Moore, and Y. Chen, "Trajectory-keeping in satellite formation flying via robust periodic learning control," *Int. J. Robust Nonlinear Control*, vol. 20, no. 14, pp. 1655–1666, Sep. 2010.
[28] X. Chen and Y. Jia, "Stereo vision-based formation control of mobile robots using iterative learning," in *Proc. Int. Conf. Humanized Syst.*, Sep. 2010, pp. 62–67.
[29] H. Sun, Z. Hou, and D. Li, "Coordinated iterative learning control schemes for train trajectory tracking with overspeed protection," *IEEE Trans. Autom. Sci. Eng.*, vol. 10, no. 2, pp. 323–333, Apr. 2013.
[30] S. Yang, J.-X. Xu, D. Huang, and Y. Tan, "Optimal iterative learning control design for multi-agent systems consensus tracking," *Syst. Control Lett.*, vol. 69, pp. 80–89, Jul. 2014.
[31] D. Meng, Y. Jia, and J. Du, "Consensus seeking via iterative learning for multi-agent systems with switching topologies and commu-nication time-delays," *Int. J. Robust Nonlinear Control*, vol. 26, no. 17, pp. 3772–3790, 2016.
[32] D. Meng and K. L. Moore, "Learning to cooperate: Networks of formation agents with switching topologies," *Automatica*, vol. 64, pp. 278–293, Feb. 2016.
[33] J. Li and J. Li, "Distributed adaptive fuzzy iterative learning control of coordination problems for higher order multi-agent systems," *Int. J. Syst. Sci.*, vol. 47, no. 10, pp. 2318–2329, Jul. 2016.
[34] X. Bu, Z. Hou, Q. Yu, and Y. Yang, "Quantized data driven iterative learning control for a class of nonlinear systems with sensor saturation," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 50, no. 12, pp. 5119–5129, Dec. 2020.
[35] S. Yang and J.-X. Xu, "Leader–follower synchronisation for networked lagrangian systems with uncertainties: A learning approach," *Int. J. Syst. Sci.*, vol. 47, no. 4, pp. 956–965, Mar. 2016.
[36] N. Yang, J. Li, and C. He, "Completely distributed neuro-learning consensus with position constraints and partially unknown control directions," *Neurocomputing*, vol. 418, pp. 251–262, Dec. 2020.
[37] J. M. Morse, "The significance of saturation," *Qualitative Health Res.*, vol. 5, no. 2, pp. 147–149, May 1995.
[38] N. Yang and J. Li, "Distributed iterative learning coordination control for leader–follower uncertain non-linear multi-agent systems with input saturation," *IET Control Theory Appl.*, vol. 13, no. 14, pp. 2252–2260, Sep. 2019.
[39] X. Gong, Y.-J. Pan, and A. Pawar, "A novel leader following consensus approach for multi-agent systems with data loss," *Int. J. Control, Autom. Syst.*, vol. 15, no. 2, pp. 763–775, Apr. 2017.
[40] W. Zhang, Y. Tang, T. Huang, and J. Kurths, "Sampled-data consensus of linear multi-agent systems with packet losses," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 11, pp. 2516–2527, Nov. 2017.
[41] D. Fan, H. Shen, and L. Dong, "Stability analysis of multi-agent tracking systems with quasi-cyclic switching topologies," *Appl. Sci.*, vol. 10, no. 24, p. 8889, Dec. 2020.
[42] Y. Zhang and Y.-P. Tian, "Consentability and protocol design of multi-agent systems with stochastic switching topology," *Automatica*, vol. 45, no. 5, pp. 1195–1201, May 2009.
[43] J. Liu, J.-A. Fang, Z. Li, and G. He, "Time-varying formation tracking for second-order multi-agent systems subjected to switching topology and input saturation," *Int. J. Control, Autom. Syst.*, vol. 18, no. 4, pp. 991–1001, Apr. 2020.
[44] Y.-D. Zhang and Y.-C. Fang, "Learning control for systems with saturated output," *Acta Automatica Sinica*, vol. 37, no. 1, pp. 92–98, May 2011.
[45] J. Li and J. Li, "Adaptive fuzzy iterative learning control with initial-state learning for coordination control of leader-following multi-agent systems," *Fuzzy Sets Syst.*, vol. 248, pp. 122–137, Aug. 2014.
[46] R. Hou, L. Cui, X. Bu, and J. Yang, "Distributed formation control for multiple non-holonomic wheeled mobile robots with velocity constraint by using improved data-driven iterative learning," *Appl. Math. Comput.*, vol. 395, Apr. 2021, Art. no. 125829.

ZHENGZHENG YU received the B.S. and M.S. degrees from Henan Polytechnic University, Jiaozuo, China, in 2004 and 2007, respectively, where he is currently pursuing the Ph.D. degree. His research interests include mathematical modeling, data mining, information processing, and so on.

HANWEI ZHANG received the B.S. degree from Yunnan University, Kunming, China, in 1990, the M.S. degree from Nanjing University, Nanjing, China, in 1993, and the Ph.D. degree from the University of Science and Technology of China, in 2004. He is currently a Professor with Henan Polytechnic University, Jiaozuo, China. His research interests include applied mathematics and mathematical modeling.

LIZHI CUI received the Ph.D. degree in control theory and control engineering from the East China University of Science and Technology, Shanghai, China, in 2015. He is currently a Lecturer with Henan Polytechnic University. His research interests include intelligent signal processing, matrix decomposition, and control theory.

JIAQI LIANG received the B.S. degree in computer science and technology from the Henan University of Science and Technology, Luoyang, China, in 2010, and the M.S. degree in automation from the School of Electric Engineering and Automation, Jiaozuo, China, in 2018. She is currently pursuing the Ph.D. degree with Henan Polytechnic University. Her research interests include iterative learning control and multiagent systems.