IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received February 25, 2021, accepted February 27, 2021, date of publication March 2, 2021, date of current version March 11, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3063309

Asynchronous Multi-Sensor State Estimation for Systems Subject to Multiplicative and Cross-Correlated Noises With Measurement Packet Dropping

YIMING MA, MENGJUN ZHANG, BOHAO WANG, AND SHUAI CHEN
School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China
Corresponding author: Shuai Chen (chenshuai@njust.edu.cn)

ABSTRACT This paper is concerned with the optimal state estimation problem for linear discrete-time systems with both multiplicative and cross-correlated noises. The measurement outputs for state estimation are collected from multiple sensors whose sampling rates are different that provide asynchronous data. In addition, the noises that affect the measurement information are correlated among different sensors and also coupled with the process noises as well. The aim of the addressed problem is to propose an optimal state estimation algorithm such that the estimation error is minimized in the mean-square sense with the existence of asynchronous data, possible sensor faults and correlated noises. In order to mitigate the impact of measurement missing, this paper utilizes neural networks to compensate the state estimation when measurement packets are dropping. Then, a fault detection mechanism that utilizes normalized innovation test is adopted to ensure that the abnormal data would be detected and removed. By resorting to the projection theorem and mathematical induction approach, sufficient conditions are derived for the existence of the desired optimal state estimator where the optimized estimation gains are formulated and can be computed iteratively at each time step. The proposed theoretical results are demonstrated via an illustrative numerical example.

INDEX TERMS Correlated noises, multiplicative noises, asynchronous data, multi-sensors, multi-rate systems, neural networks.

I. INTRODUCTION

Sensors have been long playing an essential role in various branches of science research and industrial engineering. With the fast development of electronics technology, nowadays, sensors have been widely applied and implemented in many areas for the purpose of monitoring, surveillance, gathering essential information, some of which even have abilities for data processing [1]. In comparison to the adoption of single sensor, the application where multiple sensors are deployed for specific tasks via collaboration shows particular advantages due to lower cost, better robustness and more flexible configuration. Therefore, to date, systems with multiple sensors have been widely appeared, ranging from military infrastructures such as target tracking, integrated navigation to civilian applications like unmanned automobiles and industrial robots, to name but a few [2], [3]. The merits of multi-sensor application, aside from aforementioned ones, lie mainly on the fact that multiple sensors, as a whole, could provide rich yet complementary information of which we can make full use via collaboration among sensors for better performance.

The filtering or state estimation problem which aims to extract state information from the measurements corrupted by noises or disturbances [4]. So far, because of its practical significance, the state estimation issue has been playing a pivotal role in a variety of areas and quite a lot of techniques have been developed, see [5] for some recent advances. Among the existing approaches, the Kalman filtering technique has attracted special attention, thanks to its capability of providing an elegant yet convenient way to deal with estimation problems for linear systems subject to Gaussian noises [6]

The associate editor coordinating the review of this manuscript and approving it for publication was Bing Li.

and [7]. Note that traditional Kalman filtering technique is only applicable for dealing with additive noises as well as noises of independent distributions. However, in real-world engineering especially in the context of networks, on one hand, noises may probably affect systems in a multiplicative way rather than in an additive form. Examples include but are not limited to, the communications network [8], [9], image process systems [10] and petroleum exploration systems. The existence of multiplicative noises largely hinders the application of Kalman filtering technique and its variants, and there is an urgent demand toward novel paradigms for the corresponding state estimation.

In association with the wide utilization of multi-sensor configuration like wireless sensor networks, the corresponding filteirng/estiamtion/fusion issues have gained increasingly research attention within a multitudes of academic societies such as signal processing and integrated navigation [11], [12]. It should be pointed out that one of the primary challenges resulting from the application of multiple sensors in a system is the phenomenon of asynchronous data due to the different sampling rate of different sensors [13], [14]. The asynchronous data cannot be directly used to estimate the system state by simply employing the existing approaches, which brings substantial difficulties for the state estimation problem for systems with multiple sensors. So far, there have been a rich body of research fruits available in the literature concerning the multi-sensor filtering problems, see e.g., [2], [15]–[18]. The communication transmission delays are also commonly occurred in multi-sensor system and it is not considered in this paper. In [19], [20], the predictive-based estimation mechanism is proposed to handle the time-delays and the mechanism can be adopted in this paper to counter the communication delays.

In particular, a quintessential example that should be mentioned of the application of state estimation via multi-sensor is the integrated navigation system. A typical representative intergraded navigation system usually consists of two (or more) subsystems including, but not limited to, inertial navigation systems (INS) and global navigation satellite systems (GNSS). The advantages of such a combination mainly lie in the better usage of the inherently complementary information provided by different sensors so as to achieve a better performance on the estimation of the states and attitudes of interest [21]. So far, the integrated systems have found extensive applications and the associated multi-sensor-based estimation/fusion issues have stirred considerable attention. Some representative works can be summarized as follows. For instance, [22] has developed a multi-sensor data fusion methodology for INS/GPS/SAR integrated navigation systems. In [23], a new adaptive Kalman filter has studied for multi-sensor integrated navigation system. Recently, a new technique has been designed in [24] to handle unknown noise statistical characteristics of system and outliers in the measurement information.

Note that most of the aforementioned results have been mainly focused on the additive noises or noises with independent distributions. However, in the real-world engineering, on one hand, the noises always appear in a multiplicative form as mentioned before. On the other, in the context of multi-sensors, the noises corrupting different sensors would be cross-correlated due to the fact that the sensors deployed in the similar environment would confront similar, if not the same, external noises or disturbances. Moreover, sometimes, the process and measurement noises would also be correlated with each other. However, to the best of our knowledge, neither multiplicative noises case nor correlated noises case has been thoroughly investigated for the multi-sensor state estimation problem. This gives rise to the first motivation of our current study.

On the other hand, since in many practical engineering such as the aforementioned integrated INS/GNSS navigation, the sensors are always deployed in very harsh sometimes even hostile environments (e.g., high speed, strong disturbance, extreme overload, etc). In such cases, the phenomena of sensor failures are inevitable which might yield abnormal data and degrade the estimation performance. Accordingly, much effort have been devoted to investigation of fault-tolerant (also known as reliable) state estimation algorithms. Up to now, various theories and techniques have been developed with the hope of mitigating the effects on the estimation perforce from possible abnormal data, see, e.g. [25]–[30]. Nevertheless, the relevant issue has not been adequately studied for the case where the multiplicative and cross-correlated noises are taken into simultaneous consideration, which raises the second motivation of our current research.

In response to the above discussions, this paper tackles the optimal state estimation problem for linear discrete-time systems with both multiplicative and cross-correlated noises. The main difficulties of the addressed problem can be identified as follows: i) How to unify the asynchronous measurement data into a framework of identical time-scale and design the corresponding recursive form of the optimal estimate in the existence of both multiplicative and cross-correlated noises? ii) How to design appropriate fault detection scheme as well as missing measurement compensation mechanism, to mitigate the effects from abnormal data on the estimation performance?

To sum up, it is our objective in this paper to provide a systematic framework within which the fault-tolerant and packet dropouts compensation state estimation algorithm can be analyzed and designed for the addressed multi-sensor systems subject to both multiplicative and cross-correlated noises. The main contributions of this paper lie twofold. 1) The estimation issue is first studied for the multi-sensor systems whose measurement outputs are asynchronous and subject to both multiplicative and cross-correlated noises; and 2) a reliable state estimation is proposed where a fault detection mechanism and a packet dropouts compensation method are implemented to prevent the abnormal data from deteriorating the estimation performance and avoid the impact of packet dropouts on the estimate accuracy. The paper is organized as follows. In section II, the problem formulation is presented.

In section III, the sequential estimation algorithm is proposed. In section IV, a numerical example is provided that illustrates the validity of our theoretical algorithm. Section V gives some discussions and conclusions.

II. PROBLEM FORMULATION

Consider a discrete-time linear system of the following form [31]:

$$x(k+1) = \left(\Phi(k) + \sum_{m=1}^{n} A_m(k)\eta_m(k)\right) x(k) + w(k) \tag{1}$$

$$z_i(k_i) = \gamma_i(k_i)\left((C_i(k_i) + H_i(k_i)\zeta_i(k_i))\, x_i(k_i) + v_i(k_i)\right) \tag{2}$$

where $x(k) \in R^n$ is system state; $z_i(k_i) \in R^p$ is the k_i-th measurement collected by sensor i at time $t_i(k_i)$; $t_i(k_i) \in (k-1, k]$ denotes the sampling time of sensor i which obtains the measurement in $(k-1, k]$; $x_i(k_i)$ is the state observed by $z_i(k_i)$; $w(k) \in R^n$ is the process noise and $v_i(k_i) \in R^p$ is the measurement noise; $\gamma_i(k_i)$ is a random sequence obeying the Bernoulli distribution that describes the measurement loss of sensor i; $\Phi(k) \in R^{n\times n}$, $A_m(k) \in R^{n\times n}$, $C_i(k_i) \in R^{p\times n}$ and $H_i(k_i) \in R^{p\times n}$ are known time-varying matrices of compatible dimensions; the initial state x_0 is random with mean of $\bar{x}_0$. $\eta_m(k)$ and $\zeta_i(k_i)$ are zero mean Gaussian white noise sequences.

The following assumptions are needed in the establishment of our main results.

Assumption 1: It is assumed that the initial state x_0 is independent of $w(k)$, $v_i(k_i)$, $\eta_m(k)$ and $\zeta_i(k_i)$, whereas the multiplicative noises $\eta_m(k)$ and $\zeta_i(k_i)$ are independent of the process noise $w(k)$ and measurement noise $v_i(k_i)$. The signal $\gamma_i(k_i)$ is independent with other parameters.

Assumption 2: The statistical properties of the noise sequences $w(k)$, $v_i(k_i)$, $\eta_m(k)$, $\zeta_i(k_i)$ and x_0 are as follows:

$$\begin{cases} E\{w(k)\} = 0 \\ E\{w(k)w^{\mathrm{T}}(l)\} = Q(k)\delta(k-l) \end{cases}$$

$$\begin{cases} E\{v_i(k_i)\} = 0 \\ E\{w(k-1)v_i^{\mathrm{T}}(k_i)\} = \Gamma_i \end{cases}$$

$$\begin{cases} E\{v_i(k_i)v_j^{\mathrm{T}}(k_j)\} = R_{ij}, & \text{if} \quad t_i(k_i), t_j(k_j) \in (k-1, k] \\ E\{v_i(k_i)v_j^{\mathrm{T}}(k_j)\} = 0, & \text{otherwise} \end{cases}$$

$$\begin{cases} E\{x_0\} = \bar{x}_0 \\ E\{[x_0 - \bar{x}_0][x_0 - \bar{x}_0]^{\mathrm{T}}\} = P_0 \end{cases}$$

$$\begin{cases} E\{\eta_m(k)\} = 0 \\ E\{\eta_m(k)\eta_l^{\mathrm{T}}(b)\} = \sigma_m\delta(m-l)\delta(k-b) \end{cases}$$

$$\begin{cases} E\{\zeta_i(k_i)\} = 0 \\ E\{\zeta_i(k_i)\zeta_j^{\mathrm{T}}(l_j)\} = \rho_i\delta(i-j)\delta(k_i-l_j). \end{cases}$$

Here, the notation $E\{\}$ denotes the mathematical exception while $\delta(a-b)$ represents the Kronecker function

$$\delta(a-b) = \begin{cases} 1, & \text{if } a = b \\ 0, & \text{if } a \neq b \end{cases}$$

From Assumption 2 we know that previous step process noise $w(k-1)$ is correlated with the measurement noise $v_i(k_i)$, and the measurement noises from different sensors are also cross-correlated at the same time step.

For simplicity, denote $R_i = R_{ii} > 0, i = 1, 2, \ldots, N$. We assume the sampling of sensor 1 is uniform which has the highest rate of all sensors and the sampling time of $z_1(k_1)$ is k. The relationship among different sampling rates is described as follows:

$$n_i = n_1/l_i, \quad i = 1, 2, \ldots, N \tag{3}$$

where n_i is the sampling rate of sensor i; the signal l_i is a known positive integer; the total number of sensors is N.

Remark 1: In this paper, we assume that the sampling of sensor 1 is uniform while the sampling of the rest of the sensors are un-uniform but at specific time instants within the pre-specified time intervals. The signal $z_1(k)$ is the k-th measurement of sensor 1 which is gathered at time step k. For sensor 2 to N, $z_i(k_i)$ is the k_i-th measurement of sensor i in the interval $(l_i(k_i-1), l_ik_i]$. The sampling time of sensor 2 to N are also known in this paper and it is in the time interval $(l_i(k_i-1)+j_i-1, l_i(k_i-1)+j_i]$. The signal j_i is used to determine the specific unit time interval of sampling and $1 < j_i \leq l_i$. Therefore, we can establish the relationship between k and k_i and it is $k = l_i(k_i-1)+j_i$.

III. OPTIMAL FUSION ALGORITHM

A. ASYNCHRONOUS MULTI-RATE MODEL

In order to unify the asynchronous measurement data into a framework of identical time-scale before state estimation, the measurement equation is firstly reconstructed by employing the similar technique from [32], where the state observed by $z_i(k_i)$ (i.e., $x_i(k_i)$) is over the time interval $(l_i(k_i-1)+j_i-1, l_i(k_i-1)+j_i]$, namely,

$$x_i(k_i) = \left(a_i(k_i)I + b_i(k_i)\Phi^{-1}(l_i(k_i-1)+j_i-1)\right) \cdot x(l_i(k_i-1)+j_i) \tag{4}$$

and

$$x_i(k_i) = (b_i(k_i)I + a_i(k_i)\Phi(l_i(k_i-1)+j_i-1)) \cdot x(l_i(k_i-1)+j_i-1) \tag{5}$$

where

$$a_i(k_i) = t_i(k_i) - l_i(k_i-1) - j_i + 1$$
$$b_i(k_i) = l_i(k_i-1) + j_i - t_i(k_i).$$

The following lemma provides a method to reconstruct the system state $x(k)$ in the measurement equation with the purpose of transforming the asynchronous measurement data into those with identical time scale, for the subsequent analysis and design.

Lemma 1: By means of (4), the original system model (1) – (2) can be reconstructed as follows:

$$x(k+1) = \left(\Phi(k) + \sum_{m=1}^{n} A_m(k)\eta_m(k)\right) x(k) + w(k) \tag{6}$$

$$z_i\left(k_i\right) = \gamma_i(k_i)\left(\left(\bar{C}_i(k) + \bar{H}_i(k)\zeta_i(k_i)\right)x(k) + v_i\left(k_i\right)\right). \quad (7)$$

In above equations, we have

$$\bar{C}_i(k) = C_i(k_i)\Phi_i\left(k_i\right) \quad (8)$$
$$\bar{H}_i(k) = H_i(k_i)\Phi_i\left(k_i\right) \quad (9)$$

where

$$\Phi_i\left(k_i\right) = \left(a_i(k_i)I + b_i(k_i)\Phi^{-1}\left(l_i\left(k_i-1\right)+j_i-1\right)\right). \quad (10)$$

Proof: Substituting (4) into measurement equation (2) yields

$$\begin{aligned} z_i\left(k_i\right) &= \gamma_i\left(k_i\right)\left(\left(C_i(k_i) + H_i(k_i)\zeta_i(k_i)\right)\right. \\ &\quad \cdot\Phi_i\left(k_i\right)x\left(l_i\left(k_i-1\right)+j_i\right) + v_i\left(k_i\right)) \\ &= \gamma_i\left(k_i\right)\left(\left(C_i(k_i)\Phi_i\left(k_i\right) + H_i(k_i)\Phi_i\left(k_i\right)\zeta_i(k_i)\right)\right. \\ &\quad \cdot x\left(l_i\left(k_i-1\right)+j_i\right) + v_i\left(k_i\right)) \end{aligned} \quad (11)$$

where $j_i = k - l_i(k_i - 1)$.

The equation (11) can be rewritten as follows:

$$z_i\left(k_i\right) = \gamma_i(k_i)\left(\left(\bar{C}_i(k) + \bar{H}_i(k)\zeta_i(k_i)\right)x(k) + v_i\left(k_i\right)\right). \quad (12)$$

The proof of the lemma is complete now.

B. OPTIMAL ESTIMATION ALGORITHM

Lemma 2: (Orthogonal Projection Theorem [34]): Given three random vectors X, Z_1, Z_2 with second moments. Defining $Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}$, the following equality holds:

$$\begin{aligned} &\hat{E}[X/Z] \\ &\quad = \hat{E}\left[X/Z_1\right] + \hat{E}\left[\widetilde{X}/\widetilde{Z}_2\right] \\ &\quad = \hat{E}\left[X/Z_1\right] + E\left[\widetilde{X}\widetilde{Z}_2^{\mathrm{T}}\right]\left[E\left[\widetilde{Z}_2\widetilde{Z}_2^{\mathrm{T}}\right]\right]^{-1}\widetilde{Z}_2 \end{aligned} \quad (13)$$

where $\hat{E}[\ ]$ denotes orthogonal projection and

$$\widetilde{X} = X - \hat{E}\left[X/Z_1\right], \quad \widetilde{Z}_2 = Z_2 - \hat{E}\left[Z_2/Z_1\right]. \quad (14)$$

If the stochastic processes above are gaussian, then orthogonal projection is actually identical with conditional expectation [35].

Lemma 3: If sensor i does not obtain the measurement in time interval $(k-1, k]$, we have

$$\begin{cases} \hat{x}_i(k|k) = \hat{x}_{i-1}(k|k) \\ P_i(k|k) = P_{i-1}(k|k). \end{cases} \quad (15)$$

If the measurement of sensor i is lost, we use the neural network to obtain $\hat{x}_i(k|k) - \hat{x}_{i-1}(k|k)$ and let $P_i(k|k) = P_{i-1}(k|k)$.

Theorem 1: For the reconstructed system (6)-(7), we assume that the optimal state estimate $\hat{x}_{i-1}(k|k)$ and its estimation error covariance $P_{i-1}(k|k)$ are known. Given $\gamma_i(k_i) \equiv 1$ and the initial values x_0, $P(0)$, $S(0)$.

For $i = 0$, $\hat{x}_0(k|k)$ denotes the one-step prediction of $\hat{x}(k-1|k-1)$ and $P_0(k|k)$ denotes the prediction error covariance $P(k-1|k-1)$. The state $\hat{x}_0(k|k)$ and its error covariance $P_0(k|k)$ are given by

$$\begin{cases} \hat{x}_0(k|k) = \Phi(k-1)\hat{x}(k-1|k-1) \\ \begin{cases} P_0(k|k) = \Phi(k-1)P(k-1|k-1)\Phi^{\mathrm{T}}(k-1) \\ \qquad + \sum_{m=1}^{n}\sigma_m A_m(k-1)S(k-1)A_m^{\mathrm{T}}(k-1) \\ \qquad + Q(k-1) \end{cases} \end{cases} \quad (16)$$

When $z_i(k_i)$ is obtained in $(k-1, k]$, the recurrence formulas of sequential data fusion are given by

$$\begin{cases} B_{ji}(k) = \left(I - K_j(k)\bar{C}_j(k)\right)B_{(j-1)i}(k) \\ \qquad + K_j(k)\bar{C}_j(k)\Gamma_i + K_j(k)R_{ji} \\ \tilde{z}_i\left(k_i\right) = z_i\left(k_i\right) - \bar{C}_i(k)\hat{x}_{i-1}(k|k) \\ S(k) = \Phi(k-1)S(k-1)\Phi^{\mathrm{T}}(k-1) \\ \qquad + \sum_{m=1}^{n}\sigma_m A_m(k-1)S(k-1)A_m^{\mathrm{T}}(k-1) \\ \qquad + Q(k-1) \\ K_i(k) = [P_{i-1}(k|k)\bar{C}_i^{\mathrm{T}}(k) + \Gamma_i - B_{(i-1)i}(k)]\left[\bar{C}_i(k)\right. \\ \qquad \cdot P_{i-1}(k|k)\bar{C}_i^{\mathrm{T}}(k) + \rho_i\bar{H}_i(k)S(k)\bar{H}_i^{\mathrm{T}}(k) \\ \qquad + R_i + \bar{C}_i(k)\left(\Gamma_i - B_{(i-1)i}(k)\right) \\ \qquad \left. + \left(\Gamma_i^{\mathrm{T}} - B_{(i-1)i}^{\mathrm{T}}(k)\right)\bar{C}_i^{\mathrm{T}}(k)\right]^{-1} \\ \hat{x}_i(k|k) = \hat{x}_{i-1}(k|k) + K_i(k)\tilde{z}_i\left(k_i\right) \\ P_i(k|k) = P_{i-1}(k|k) - K_i(k) \\ \qquad \cdot[\bar{C}_i(k)P_{i-1}^{\mathrm{T}}(k|k) + \Gamma_i^{\mathrm{T}} - B_{(i-1)i}^{\mathrm{T}}(k)] \end{cases} \quad (17)$$

where the optimal estimate $\hat{x}_i(k|k)$ is obtained by using the measurements of sensor $1-i$ at time k, $P_i(k|k)$ is the covariance of $\tilde{x}_i(k|k)$, $K_i(k)$ is the gain of sensor i, $S(k)$ is the covariance of $x(k)$, and $\tilde{z}(k_i)$ is the innovation.

The optimal state estimate $\hat{x}(k|k) = \hat{x}_N(k|k)$ and the estimation error covariance $P(k|k) = P_N(k|k)$.

Proof: By using the orthogonal projection theorem (Lemma 2) in combination with the mathematical induction approach, the proof of the theorem is given as follows. For convenience of derivation, we first give the following notations.

$$\begin{aligned} \hat{x}(k|k-1) &= E\left(x(k)|\mathbb{Z}^{k-1,N}\right) \\ \tilde{x}(k|k-1) &= x(k) - \hat{x}(k|k-1) \\ P(k|k-1) &= E\left[\tilde{x}(k|k-1)\tilde{x}^{\mathrm{T}}(k|k-1)\right] \\ \hat{x}_i(k|k) &= E\left[x(k)|\mathbb{Z}^{k-1,N}, \mathbb{Z}_{k-1}^{k}(i)\right] \\ \tilde{x}_i(k|k) &= x(k) - \hat{x}_i(k|k) \\ P_i(k|k) &= E\left[\tilde{x}_i(k|k)\tilde{x}_i^{\mathrm{T}}(k|k)\right] \\ \hat{x}(k|k) &= E\left[x(k)|\mathbb{Z}^{k,N}\right] \\ P(k|k) &= E\left\{[x(k) - \hat{x}(k|k)][x(k) - \hat{x}(k|k)]^{\mathrm{T}}\right\} \\ S(k) &= E\left[x(k)x^{\mathrm{T}}(k)\right] \\ B_{ji}(k) &= E[\hat{x}_j(k|k)v_i^{\mathrm{T}}(k_i)] \end{aligned}$$

For $i = 1, 2, \ldots, N$, we further denote

$$\begin{aligned}\mathbb{Z}_i^k &= \{z_i(k_i), 0 < t_i(k_i) \leq k\}\\ \mathbb{Z}^{k,i} &= \{z_g(k_g), 0 < t_g(k_g) \leq k; g = 1, 2, \cdots, i\}\\ \mathbb{Z}_{k-1}^k(i) &= \{z_g(k_g), k-1 < t_g(k_g) \leq k; g = 1, 2, \cdots, i\}\end{aligned}$$

where $\mathbb{Z}_i^k$ represents the measurements of sensor i at time k and before. $\mathbb{Z}^{k,i}$ represents the measurements of sensor $1-i$ at time k and before. $\mathbb{Z}_{k-1}^k(i)$ represents the measurements of sensor $1-i$ which are in time interval $(k-1, k]$.

Firstly, the optimal one-step prediction is computed by

$$\begin{aligned}&\hat{x}(k|k-1)\\ &\quad= E[x(k)|\mathbb{Z}^{k-1,N}]\\ &\quad= E\left[\Phi(k-1)x(k-1)|\mathbb{Z}^{k-1,N}\right]\\ &\qquad+E\left[\sum_{m=1}^{n} A_m(k-1)\eta_m(k-1)x(k-1)|\mathbb{Z}^{k-1,N}\right]\\ &\qquad+E[w(k-1)|\mathbb{Z}^{k-1,N}]\\ &\quad= \Phi(k-1)\hat{x}(k-1|k-1).\end{aligned} \tag{18}$$

Since $\tilde{x}(k-1|k-1)$ is independent of $w(k-1)$, the covariance of $\tilde{x}(k|k-1)$ and $x(k)$ are calculated as follows:

$$\begin{aligned}&P(k|k-1)\\ &\quad= E\{[\tilde{x}(k|k-1)\tilde{x}^{\mathrm{T}}(k|k-1)]\}\\ &\quad= E\Bigg\{\Bigg[(\Phi(k-1)+\sum_{m=1}^{n} A_m(k-1)\eta_m(k-1))\\ &\qquad\cdot x(k-1)+w(k-1)-\Phi(k-1)\hat{x}(k-1|k-1)\Bigg]\\ &\qquad\cdot\Bigg[(\Phi(k-1)+\sum_{m=1}^{n} A_m(k-1)\eta_m(k-1))x(k-1)\\ &\qquad+w(k-1)-\Phi(k-1)\hat{x}(k-1|k-1)\Bigg]^{\mathrm{T}}\Bigg\}\\ &\quad= \Phi(k-1)P(k-1|k-1)\Phi^{\mathrm{T}}(k-1)\\ &\qquad+\sum_{m=1}^{n}\sigma_m A_m(k-1)S(k-1)A_m^{\mathrm{T}}(k-1)\\ &\qquad+Q(k-1)+\Phi(k-1)E\left\{\tilde{x}(k-1\mid k-1)w(k-1)^{\mathrm{T}}\right\}\\ &\qquad+E\left\{w(k-1)\tilde{x}^{\mathrm{T}}(k-1\mid k-1)\right\}\Phi^{\mathrm{T}}(k-1)\\ &\quad= \Phi(k-1)P(k-1|k-1)\Phi^{\mathrm{T}}(k-1)\\ &\qquad+\sum_{m=1}^{n}\sigma_m A_m(k-1)S(k-1)A_m^{\mathrm{T}}(k-1)\\ &\qquad+Q(k-1)\end{aligned} \tag{19}$$

and

$$\begin{aligned}S(k) &= E\left[x(k)x^{\mathrm{T}}(k)\right]\\ &= E\Bigg\{\Bigg[(\Phi(k-1)+\sum_{m=1}^{n} A_m(k-1)\eta_m(k-1))\\ &\quad\cdot x(k-1)+w(k-1)\Bigg]\Bigg[(\Phi(k-1)+\sum_{m=1}^{n} A_m(k-1)\\ &\quad\cdot\eta_m(k-1))x(k-1)+w(k-1)\Bigg]^{\mathrm{T}}\Bigg\}\\ &= \Phi(k-1)S(k-1)\Phi^{\mathrm{T}}(k-1)\\ &\quad+\sum_{m=1}^{n}\sigma_m A_m(k-1)S(k-1)A_m^{\mathrm{T}}(k-1)\\ &\quad+Q(k-1).\end{aligned} \tag{20}$$

Secondly, the optimal state estimation of sensor i at time k can be deduced by Lemma 2:

$$\begin{aligned}\hat{x}_i(k|k) &= E\left[x(k)|\mathbb{Z}^{k-1,N}, \mathbb{Z}_{k-1}^k(i-1), z_i(k_i)\right]\\ &= E\left[x(k)|\mathbb{Z}^{k-1,N}, \mathbb{Z}_{k-1}^k(i-1)\right]+K_i(k)\\ &\quad\cdot\left(z_i(k_i)-E\left[z_i(k_i)|\mathbb{Z}^{k-1,N}, \mathbb{Z}_{k-1}^k(i-1)\right)\right]\\ &= \hat{x}_{i-1}(k|k)+K_i(k)\tilde{z}_i(k_i)\end{aligned} \tag{21}$$

where

$$\begin{aligned}K_i(k) &= E\{\left[x(k)-\hat{x}_{i-1}(k|k)\right]\\ &\quad\cdot\left[z_i(k_i)-\bar{C}_i(k)\hat{x}_{i-1}(k|k)\right]^{\mathrm{T}}\}\\ &\quad\cdot(E\{[z_i(k_i)-\bar{C}_i(k)\hat{x}_{i-1}(k|k)]\\ &\quad\cdot[z_i(k_i)-\bar{C}_i(k)\hat{x}_{i-1}(k|k)]^{\mathrm{T}}\})^{-1}\\ &= E\left[\tilde{x}_{i-1}(k|k)\tilde{z}_i^{\mathrm{T}}(k_i)\right](E\left[\tilde{z}_i(k_i)\tilde{z}_i^{\mathrm{T}}(k_i)\right])^{-1}.\end{aligned} \tag{22}$$

From (17), we can deduce

$$\begin{aligned}\tilde{z}_i(k_i) &= z_i(k_i)-\bar{C}_i(k)\hat{x}_{i-1}(k|k)\\ &= \left(\bar{C}_i(k)+\bar{H}_i(k)\zeta_i(k_i)\right)x(k)+v_i(k_i)\\ &\quad-\bar{C}_i(k)\hat{x}_{i-1}(k|k)\\ &= \bar{C}_i(k)\tilde{x}_{i-1}(k|k)+\bar{H}_i(k)\zeta_i(k_i)x(k)+v_i(k_i).\end{aligned} \tag{23}$$

Therefore,

$$\begin{aligned}&E\left[\tilde{z}_i(k_i)\tilde{z}_i^{\mathrm{T}}(k_i)\right]\\ &\quad= E\{[\bar{C}_i(k)\tilde{x}_{i-1}(k|k)+\bar{H}_i(k)\zeta_i(k_i)x(k)+v_i(k_i)]\\ &\qquad\cdot[\bar{C}_i(k)\tilde{x}_{i-1}(k|k)+\bar{H}_i(k)\zeta_i(k_i)x(k)+v_i(k_i)]^{\mathrm{T}}\}\\ &\quad= \bar{C}_i(k)P_{i-1}(k|k)\bar{C}_i^{\mathrm{T}}(k)+\rho_i\bar{H}_i(k)S(k)\bar{H}_i^{\mathrm{T}}(k)\\ &\qquad+R_i+E\left[\bar{C}_i(k)\tilde{x}_{i-1}(k|k)v_i^{\mathrm{T}}(k_i)\right]\\ &\qquad+E\left[v_i(k_i)\tilde{x}_{i-1}^{\mathrm{T}}(k|k)\bar{C}_i(k)^{\mathrm{T}}\right]\\ &\quad= \bar{C}_i(k)P_{i-1}(k|k)\bar{C}_i^{\mathrm{T}}(k)+\rho_i\bar{H}_i(k)S(k)\bar{H}_i^{\mathrm{T}}(k)\\ &\qquad+R_i+\bar{C}_i(k)(\Gamma_i-B_{(i-1)i}(k))\\ &\qquad+(\Gamma_i^{\mathrm{T}}-B_{(i-1)i}^{\mathrm{T}}(k))\bar{C}_i^{\mathrm{T}}(k)\end{aligned} \tag{24}$$

where

$$\begin{aligned}&E\left[\bar{C}_i(k)\tilde{x}_{i-1}(k|k)v_i^{\mathrm{T}}(k_i)\right]\\ &\quad= E\left[\bar{C}_i(k)\left(x(k)-\hat{x}_{i-1}(k|k)\right)v_i^{\mathrm{T}}(k_i)\right]\\ &\quad= \bar{C}_i(k)\left(E\left[x(k)v_i^{\mathrm{T}}(k_i)\right]-E\left[\hat{x}_{i-1}(k|k)v_i^{\mathrm{T}}(k_i)\right]\right)\end{aligned}$$

$$
\begin{aligned}
&= \bar{C}_i(k)\Big(E\Big[\Big(\big(\Phi(k-1)+\sum_{m=1}^{n}A_m(k-1)\eta_m(k-1)\big)\\
&\quad \cdot x(k-1)+w(k-1)\Big)v_i^{\mathrm{T}}(k_i)\Big]\\
&\quad -E\left[\hat{x}_{i-1}(k|k)v_i^{\mathrm{T}}(k_i)\right]\Big)\\
&= \bar{C}_i(k)(\Gamma_i - B_{(i-1)i}(k)). \qquad (25)
\end{aligned}
$$

From the notation $B_{ji}(k) = E[\hat{x}_j(k|k)v_i^{\mathrm{T}}(k_i)]$, $B_{ji}(k)$ can be deduced,where $j < i$.

$$
\begin{aligned}
&E\left[\hat{x}_j(k|k)v_i^{\mathrm{T}}(k_i)\right]\\
&\quad = E[(\hat{x}_{j-1}(k|k)+K_j(k)(z_j(k_j)\\
&\qquad -\bar{C}_j(k)\hat{x}_{j-1}(k|k)))v_i^{\mathrm{T}}(k_i)]\\
&\quad = (I-K_j(k)\bar{C}_j(k))E[\hat{x}_{j-1}(k|k)v_i^{\mathrm{T}}(k_i)]\\
&\qquad +K_j(k)\bar{C}_j(k)\Gamma_i + K_j(k)R_{ji}. \qquad (26)
\end{aligned}
$$

For $j = 0$,

$$
\begin{aligned}
B_{0i}(k) &= E\left[\hat{x}(k|k-1)v_i^{\mathrm{T}}(k_i)\right]\\
&= E\left[\Phi(k-1)\hat{x}(k-1|k-1)v_i^{\mathrm{T}}(k_i)\right]\\
&= 0. \qquad (27)
\end{aligned}
$$

For $j > 0$,

$$
\begin{aligned}
B_{ji}(k) = \big(I-K_j(k)\bar{C}_j(k)\big)B_{(j-1)i}(k)&\\
+K_j(k)\bar{C}_j(k)\Gamma_i + K_j(k)R_{ji}.& \qquad (28)
\end{aligned}
$$

For $E\left[\tilde{x}_{i-1}(k|k)\tilde{z}_i^{\mathrm{T}}(k_i)\right]$, we get

$$
\begin{aligned}
&E\left[\tilde{x}_{i-1}(k|k)\tilde{z}_i^{\mathrm{T}}(k_i)\right]\\
&\quad = E\{\tilde{x}_{i-1}(k|k)[z_i(k_i)-\bar{C}_i(k)\hat{x}_{i-1}(k|k)]^{\mathrm{T}}\}\\
&\quad = E\{\tilde{x}_{i-1}(k|k)[(\bar{C}_i(k)+\bar{H}_i(k)\zeta_i(k_i))x(k)+v_i(k_i)\\
&\qquad -\bar{C}_i(k)\hat{x}_{i-1}(k|k)]^{\mathrm{T}}\}\\
&\quad = E\{\tilde{x}_{i-1}(k|k)[\bar{C}_i(k)\tilde{x}_{i-1}(k|k)+\bar{H}_i(k)\zeta_i(k_i)x(k)\\
&\qquad +v_i(k_i)]^{\mathrm{T}}\}\\
&\quad = P_{i-1}(k|k)\bar{C}_i^{\mathrm{T}}(k)+\Gamma_i - B_{(i-1)i}(k) \qquad (29)
\end{aligned}
$$

where

$$
\begin{aligned}
&E\left[\tilde{x}_{i-1}(k|k)v_i^{\mathrm{T}}(k_i)\right]\\
&\quad = E\left\{\left[x(k)-\hat{x}_{i-1}(k|k)\right]v_i^{\mathrm{T}}(k_i)\right\}\\
&\quad = E\left[x(k)v_i^{\mathrm{T}}(k_i)\right]-E\left[\hat{x}_{i-1}(k|k)v_i^{\mathrm{T}}(k_i)\right]\\
&\quad = \Gamma_i - B_{(i-1)i}(k). \qquad (30)
\end{aligned}
$$

Substituting (24) and (29) to (22) results in

$$
\begin{aligned}
K_i(k) &= E\left[\tilde{x}_{i-1}(k|k)\tilde{z}_i^{\mathrm{T}}(k_i)\right]\left(E\left[\tilde{z}_i(k_i)\tilde{z}_i^{\mathrm{T}}(k_i)\right]\right)^{-1}\\
&= [P_{i-1}(k|k)\bar{C}_i^{\mathrm{T}}(k)+\Gamma_i - B_{(i-1)i}(k)][\bar{C}_i(k)\\
&\quad \cdot P_{i-1}(k|k)\bar{C}_i^{\mathrm{T}}(k)+\rho_i\bar{H}_i(k)S(k)\bar{H}_i^{\mathrm{T}}(k)\\
&\quad +R_i+\bar{C}_i(k)\cdot(\Gamma_i - B_{(i-1)i}(k))\\
&\quad +(\Gamma_i^{\mathrm{T}} - B_{(i-1)i}^{\mathrm{T}}(k))\bar{C}_i^{\mathrm{T}}(k)]^{-1}. \qquad (31)
\end{aligned}
$$

Finally, we can get the covariance of estimation error $\tilde{x}_i(k|k)$ as follows:

$$
\begin{aligned}
&P_i(k|k)\\
&\quad = E\left[\tilde{x}_i(k|k)\tilde{x}_i^{\mathrm{T}}(k|k)\right]\\
&\quad = E\{\left[x(k)-\hat{x}_i(k|k)\right]\left[x(k)-\hat{x}_i(k|k)\right]^{\mathrm{T}}\}\\
&\quad = E\{\left[\tilde{x}_{i-1}(k|k)-K_i(k)\tilde{z}_i(k_i(k))\right]\\
&\qquad \cdot\left[\tilde{x}_{i-1}(k|k)-K_i(k)\tilde{z}_i(k_i)\right]^{\mathrm{T}}\}\\
&\quad = P_{i-1}(k|k)-E\left[\tilde{x}_{i-1}(k|k)\tilde{z}_i^{\mathrm{T}}(k_i(k))\right]K_i^{\mathrm{T}}(k)-K_i(k)\\
&\qquad \cdot E\left[\tilde{z}_i(k_i)\tilde{x}_{i-1}^{\mathrm{T}}(k|k)\right]+K_i(k)E\left[\tilde{z}_i(k_i)\tilde{z}_i^{\mathrm{T}}(k_i)\right]K_i^{T}(k)\\
&\quad = P_{i-1}(k|k)-E\left[\tilde{x}_{i-1}(k|k)\tilde{z}_i^{\mathrm{T}}(k_i)\right]\\
&\qquad \cdot\left(E\left[\tilde{z}_i(k_i)\tilde{z}_i^{\mathrm{T}}(k_i)\right]\right)^{-1}E\left[\tilde{z}_i(k_i)\tilde{x}_{i-1}^{\mathrm{T}}(k|k)\right]\\
&\quad = P_{i-1}(k|k)-K_i(k)[\bar{C}_i(k)P_{i-1}^{\mathrm{T}}(k|k)+\Gamma_i^{\mathrm{T}}-B_{(i-1)i}^{\mathrm{T}}(k)]. \qquad (32)
\end{aligned}
$$

Therefore, the proof is completed now.

Remark 2: In the estimation algorithm proposed in theorem 1, since $z_1(k_1)$ will be obtained at any time k, the measurement $z_1(k_1)$ is used for state estimation at first and the measurement matrix of sensor 1 meets $C_1(k_1) = \bar{C}_1(k)$. Then, state estimation is performed by using the measurements with sampling rates from high to low at time k. If sensor i does not obtain the measurement in time interval $(k-1, k]$, $\hat{x}_i(k|k)$ and $P_i(k|k)$ are given by (15). The equations (15) mean the estimate will not be updated when $z_i(k_i)$ is not observed in $(k-1, k]$. On the other hand, if $z_i(k_i)$ is available but $z_{i-1}(k_{i-1})$ is not available in $(k-1, k]$, we use $B_{(i-2)i}(k)$ instead of $B_{(i-1)i}(k)$ in (17).

C. PACKET DROPOUTS COMPENSATION

The loss of measurement often occurs in practical engineering, which will inevitably affect the accuracy of state estimation. In order to reduce the influence of measurement loss in the system, here, we use a neural network to compensate the estimate by borrowing its strong capability in approximating nonlinear relationships [33]. The neural network will be trained when the measurement is normal, and when the measurement is missing, it will be working in prediction mode to generate desired output for compensation.

Specifically, a generalized regression neural network (GRNN) is used for packet dropouts compensation in this paper. GRNN is an improved version of radial basis function neural network(RBFNN), which has good nonlinear mapping ability and fast learning speed. In our proposed framework, the GRNN has a structure shown in Fig. 1.

In the proposed structure of GRNN, the number of neurons in the input layer is equal to the dimension of the input vector and the number of neurons in the output layer is equal to the dimension of the output vector. The number of neurons in the pattern layer is equal to the number of samples. In this paper, the input vector of GRNN is $\hat{x}_{i-1}(k|k)$ and the output vector

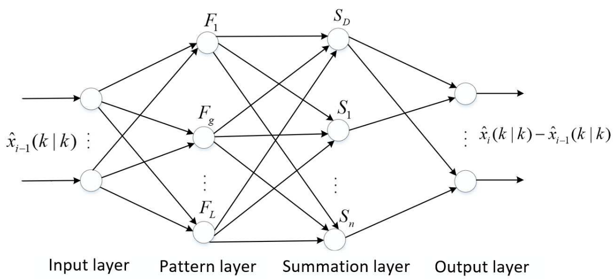


FIGURE 1. **Generalized regression neural network topology architecture.**

is $\hat{x}_i(k|k) - \hat{x}_{i-1}(k|k)$. Therefore, the number of neurons in input layer and output layer is n. The activation function of pattern layer neuron is Gaussian function as follows:

$$F_g = \exp(-\frac{(X-X_g)^{\mathrm{T}}(X-X_g)}{2\varrho^2}) \quad g = 1, 2 \ldots, L \tag{33}$$

where X is the input vector; X_g is the input vector of sample g; ϱ is a scale and L is the number of samples.

The summation layer neuron is a weighted sum of the output of the pattern layer as follows:

$$S_j = \sum_{g=1}^{L} Y_{jg} F_g \quad j = 1, 2, \ldots, n \tag{34}$$

$$S_D = \sum_{g=1}^{L} F_g \tag{35}$$

where Y_{jg} is the j-th element in output vector of sample g and n is the dimension of output vector.

Finally, the value of neuron in the output layer is

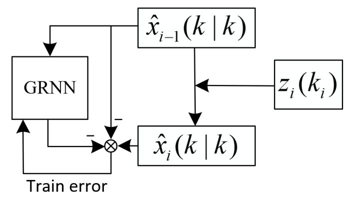
$$y_j = \frac{S_j}{S_D} = \frac{\sum_{g=1}^{L} Y_{jg} F_g}{\sum_{g=1}^{L} F_g} \quad j = 1, 2, \ldots, n. \tag{36}$$

The Fig 2(a) and Fig 2(b) show the process of packet dropouts compensation.

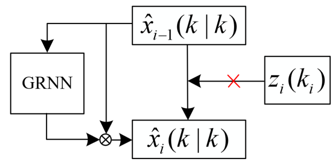
Remark 3: It is worth mentioning that, most of the existing literature concerning the estimation issue subject to measurement missing phenomenon are mainly applying the stochastic analysis and design methods. That is, based on the stochastic assumption on the measurement dropping, algorithms are developed to achieve the desired specifications that are raised in a probabilistic sense. Consequently, the obtained approaches can only satisfy the requirements in a statistical meaning, and sometimes show poor performance in practice. In view of this, we here utilize the GRNN, by virtual of its capability of modeling complex nonlinear models, to compensate the missing measurements, thereby mitigating the effects from random loss of data. It will be shown in the simulation part that such a scheme is of help to improve the estimation performance.

D. FAULT DETECTION

The innovation vector $\tilde{z}_i(k_i)$ is defined in (17) and $\tilde{z}_i(k_i)$ obeys zero-mean Gaussian distribution when sensors work in good



(a) The neural network train stage



(b) The neural network prediction stage

FIGURE 2. **The process of packet dropouts compensation.**

condition. However, in practical engineering, due to various reasons, the senors might confront many kinds of faults, which probably makes the mean of $\tilde{z}_i(k_i)$ non-zero. In these cases, the mean of $\tilde{z}_i(k_i)$ can be used to determine the working status of sensors.

Since the innovation vector $\tilde{z}_i(k_i)$ obeys zero-mean Gaussian distribution, we can obtain from (24) that

$$\begin{aligned}\Sigma_i(k) = &\bar{C}_i(k)P_{i-1}(k|k)\bar{C}_i^{\mathrm{T}}(k) + \rho_i\bar{H}_i(k)S(k)\bar{H}_i^{\mathrm{T}}(k)\\ &+R_i + \bar{C}_i(k)\big(\Gamma_i - B_{(i-1)i}(k)\big)\\ &+\big(\Gamma_i^{\mathrm{T}} - B_{(i-1)i}^{\mathrm{T}}(k)\big)\bar{C}_i^{\mathrm{T}}(k).\end{aligned} \tag{37}$$

Since R_i is a real symmetric matrix, it is not difficult to observe that $\Sigma_i(k) = \Sigma_i^{\mathrm{T}}(k)$, which indicates that $\Sigma_i(k)$ is a normal matrix. Consequently, there exists a unitary matrix $U_i(k)$ such that

$$U_i^{-1}(k)\Sigma_i(k)U_i(k) = \Lambda_i(k) \tag{38}$$

where $\Lambda_i(k)$ is a diagonal matrix consisting of the eigenvalues of $\Sigma_i(k)$.

By using Mahalanobis transform, we have

$$\begin{aligned}\psi_i(k) &= \Lambda_i^{-\frac{1}{2}}(k)U_i^{-1}(k)\tilde{z}_i(k_i)\\ \psi_i(k) &= [d_{1,i}(k), d_{2,i}(k), \cdots, d_{p,i}(k)]^{\mathrm{T}}\end{aligned} \tag{39}$$

where $\psi_i(k)$ is normalized innovation and $d_{\kappa,i}(k)$ ($\kappa = 1, 2, \ldots, p$) is a random variable that obeys standard normal distribution.

The test statistic is $d_{\kappa,i}(k)$ in this Hypothesis Testing.The null hypothesis is $H_0 : E\{d_{\kappa,i}(k)\} = 0$ and the alternative hypothesis is $H_1 : E\{d_{\kappa,i}(k)\} \neq 0$ which will be accepted when the null hypothesis is rejected. Therefore, we have

$$H_0 : E\{d_{\kappa,i}(k)\} = 0 \quad \text{versus} \quad H_1 : E\{d_{\kappa,i}(k)\} \neq 0.$$

The significance of Hypothesis Testing is τ and the rejection region of null hypothesis H_0 is $|d_{\kappa,i}(k)| > \theta_\tau$. In addition, the rejection region of alternative hypothesis H_1 is

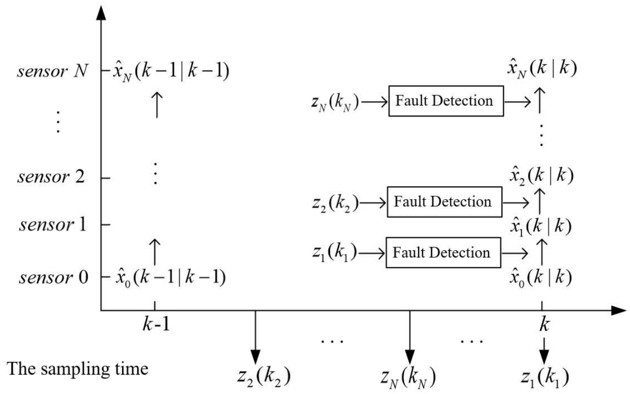


FIGURE 3. Algorithm flow diagram.

$|d_{\kappa,i}(k)| < \theta_\tau$. When alternative hypothesis is accepted for any $d_{\kappa,i}(k)$ ($\kappa = 1, 2, \ldots, p$), we believe that sensor i is abnormal. Otherwise, we believe that sensor i works well. Therefore, we define the fault detection function as follows:

$$\Omega_i(k) = \begin{cases} 0, & \text{for } \kappa, \forall \left|d_{\kappa,i}(k)\right| \le \theta_\tau \\ 1, & \text{for } \kappa, \exists \left|d_{\kappa,i}(k)\right| > \theta_\tau \end{cases} \tag{40}$$

where θ_τ is the threshold of fault detection regulating the sensitivity of fault detection. When any $d_{\kappa,i}(k)$ exceeds threshold θ_τ, system determines that the sensor i is faulty, and then the measurement of faulty sensor will be removed from the data fusion at time k. In such cases, state estimate and estimation error covariance are given by (15). The value of θ_τ can be found in standard normal distribution table and it meets the equation

$$\int_{\theta_\tau}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt = \frac{\tau}{2} \tag{41}$$

which indicates the null hypothesis will be rejected with a certain probability τ.

The fault detection procedure is shown in Fig 3.

Remark 4: In this paper, fault detection algorithm is performed in order from sensor 1 to N. If sensor $z_i(k_i)$ is obtained in $(k-1, k]$, $\tilde{z}_i(k_i)$ is used for fault detection of sensor i.

IV. NUMERICAL SIMULATION

A numerical example is given to illustrate the performance of the fusion algorithm.

Given a system containing two sensors, whose dynamics are described by (1) and (2). The sampling rate of sensor 1 is higher than sensor 2, and the sampling of sensor 1 is uniform while the sampling of sensor 2 is non-uniform. The sampling rate of sensor 2 is n_2 and meets the equation $n_1 = 2n_2$. The system parameters are as follows [36]:

$$\Phi(k) = \begin{bmatrix} 0 & -0.5 \\ 1 & 1 \end{bmatrix},$$
$$A_1(k) = A_2(k) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$C_i(k_i) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad H_i(k_i) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

TABLE 1. Average of mean square error.

State	Case 1	Case 2	Case 3	Case 4
Dimension 1	0.6625	0.5936	0.6290	0.6083
Dimension 2	1.3814	1.2055	1.2783	1.2273

The noises $w(k)$, $\eta_m(k)$, $\zeta_1(k_1)$ and $\zeta_2(k_2)$ are zero mean Gaussian white noise sequences with the following coefficients.

$$Q(k) = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$
$$\sigma_1 = 2 \quad, \sigma_2 = 2 \quad, \rho_1 = 4 \quad, \rho_2 = 4.$$

Assume zero mean cross-correlated noises $m_1(k)$, $m_2(k)$ and $n(k)$ are selected as follows:

$$v_1(k) = 0.8w(k-1) + m_1(k) + n(k),$$
$$v_2(k) = 0.6w(k-1) + m_2(k) + 0.6n(k),$$
$$E[m_1(k)m_1^{\mathrm{T}}(k)] = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix},$$
$$E[m_2(k)m_2^{\mathrm{T}}(k)] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$E[n(k)n^{\mathrm{T}}(k)] = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}.$$

The initial values are chosen as

$$x_0 = [1, 0]^{\mathrm{T}}, P(0) = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, S(0) = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}.$$

The numerical simulation are performed in the following four cases: 1) Case 1, the algorithm in this paper is performed only with sensor 1; 2) Case 2, the algorithm in this paper is performed with sensors 1 and 2; 3) Case 3, the multi-rate multi-sensor data fusion algorithm in [37] is combined with asynchronous model (4) and the algorithm is performed with sensors 1 and 2; 4) Case 4, the algorithm in [38] is performed with sensors 1 and 2.

The average of mean square error of different algorithms are shown in the Table 1.

Remark 5: We took 300 sampling points. Results are obtained from a 100 Monte Carlo simulation.

In Table 1, we can know the performance of the algorithm proposed in this paper is better than other algorithms. From Cases 1 and 2, we see that state estimation error will decrease when the number of sensors increases. From Cases 2,3,4, we learn that the algorithm proposed in this article shows better performance in dealing with cross-correlated noises and multiplicative noise than [37] and [38].

In order to show the effect of packet dropouts compensation, we set packet dropouts rate as 30% and the mean of $\gamma_i(k_i)$ is set as follows:

$$E\{\gamma_1(k_1)\} = E\{\gamma_2(k_2)\} = 0.7.$$

In order to ensure adequate training of the neural networks, we set the measurement not to be lost when $k <= 40$ and the number of samples in train set is 40. When the training

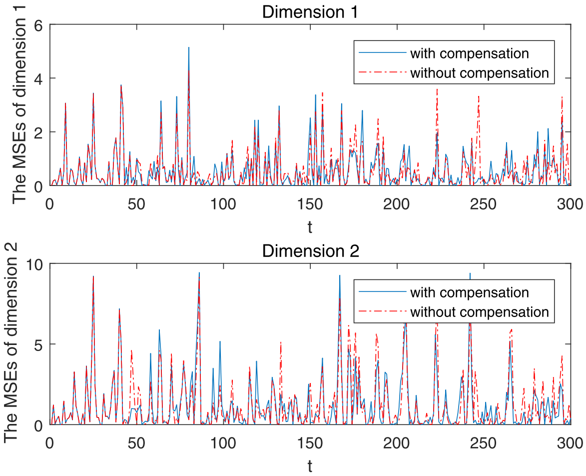


FIGURE 4. Compensation for with and without packet dropouts compensation.

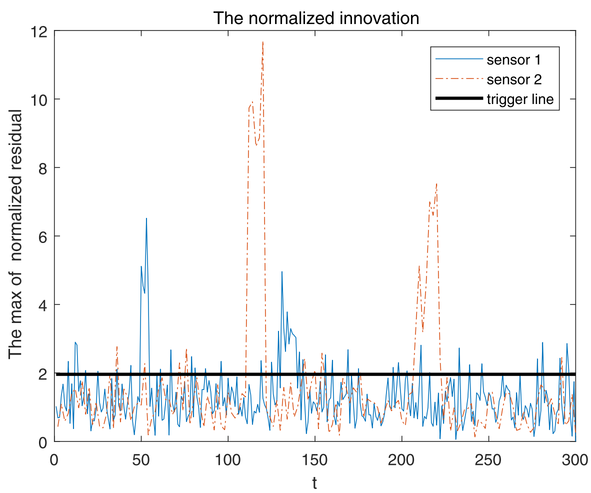


FIGURE 5. The value of normalized innovation and trigger line.

TABLE 2. Average of mean square error.

State	With compensation	Without compensation
Dimension 1	0.6564	0.6609
Dimension 2	1.3829	1.4184

set is full, the old data are removed and the new data are added in training set. The training steps of neural networks are set to 200; the goal error is set to 0.01; spread of radial basis functions is set to 5. By comparing the average of mean square error of the algorithm with and without neural networks compensation shown in Table 2, we see that the proposed neural network compensation algorithm is effective.

Next, we are going to verify the effectiveness of fault detection algorithm. For this purpose, we add abrupt faults and ramp faults manually in the measurement data.

The abrupt fault is added to sensor 1 and sensor 2 in time interval $[50s, 55s]$ and time interval $[110s, 120s]$, respectively. The ramp fault is added to sensor 1 and sensor 2 in

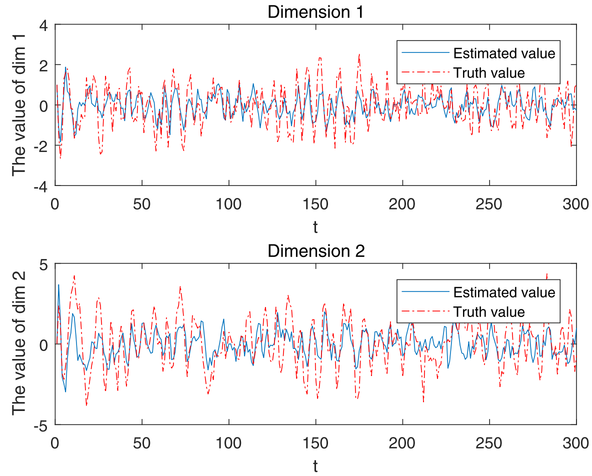


FIGURE 6. The filter result of algorithm with Ma7 detection.

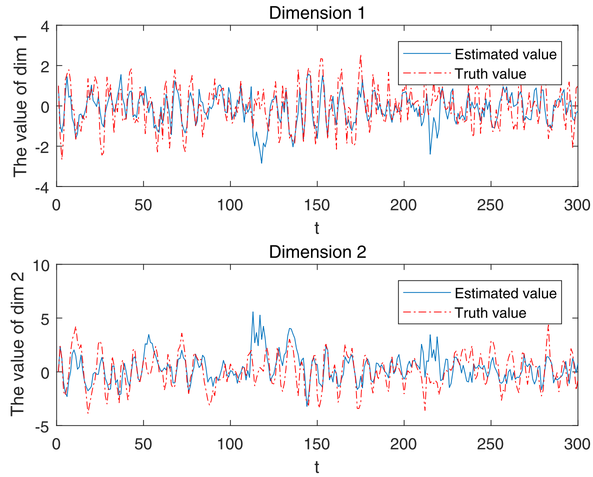


FIGURE 7. The filter result of algorithm without fault detection.

TABLE 3. Average of mean square error.

State	With fault detection	Without fault detection
Dimension 1	0.6656	0.6946
Dimension 2	1.3038	2.0640

time interval $[120s, 140s]$ and time interval $[200s, 220s]$, respectively. The fixed vector $[14, 10]^{\mathrm{T}}$ is added to the measurements of sensors to describe the abrupt fault. The linear time-varying value is set to $0.5t$ to simulate ramp fault where t is duration of sensor failure. The significant of Hypothesis Testing in fault detection is set to 2.5%. We can get the figure of maximum value in normalized innovation vector $\psi_i(k)$ and the trigger line of fault detection in Fig. 5.

The comparison between algorithms with and without fault detection is shown in Table 3.

The figures(Fig 5, 6 and 7) and Table 3 illustrate that the fault detection algorithm can effectively isolate faulty sensors and avoid the corruption of the state estimation.

V. CONCLUSION

In this paper, the optimal estimation algorithm has been proposed for asynchronous multi-rate multi-sensor discrete linear dynamic systems with multiplicative and cross-correlation noises. Packet dropouts compensation based on neural networks have been designed to effectively reduce the influence of packet dropouts on state estimation. The fault detection algorithm has been proposed to avoid the degradation of estimation accuracy caused by sensor fault. Numerical examples have been given showing the effectiveness of the developed algorithms. The presented algorithms could be applied to deal with practical problems in many fields such as industrial robot and integrated navigation.

REFERENCES

[1] R. Caballero-Águila, A. Hermoso-Carazo, J. Linares-Pérez, and Z. Wang, "A new approach to distributed fusion filtering for networked systems with random parameter matrices and correlated noises," *Inf. Fusion*, vol. 45, pp. 324–332, Jan. 2019.

[2] L. Yan, B. Xiao, Y. Xia, and M. Fu, "The modeling and estimation of asynchronous multirate multisensor dynamic systems," in *Proc. Chin. Control Conf.*, Xi'an, China, 2013, pp. 4676-4681.

[3] L. Ma, Z. Wang, Q.-L. Han, and H.-K. Lam, "Variance-constrained distributed filtering for time-varying systems with multiplicative noises and deception attacks over sensor networks," *IEEE Sensors J.*, vol. 17, no. 7, pp. 2279–2288, Apr. 2017.

[4] H. Geng, Z. Wang, Y. Liang, Y. Cheng, and F. E. Alsaadi, "State estimation for asynchronous sensor systems with Markov jumps and multiplicative noises," *Inf. Sci.*, vol. 417, pp. 1–19, Nov. 2017.

[5] L. Zou, Z. Wang, H. Dong, and Q. Han, "Moving horizon estimation with multirate measurements and correlated noises," *Int. J. Robust Nonlinear Control*, vol. 30, no. 17, pp. 7429–7445, Sep. 2020.

[6] Y. Liu, X. Fan, C. Lv, J. Wu, L. Li, and D. Ding, "An innovative information fusion method with adaptive Kalman filter for integrated INS/GPS navigation of autonomous vehicles," *Mech. Syst. Signal Process.*, vol. 100, pp. 605–616, Feb. 2018.

[7] Z. Duan and X. R. Li, "Lossless linear transformation of sensor data for distributed estimation fusion," *IEEE Trans. Signal Process.*, vol. 59, no. 1, pp. 362–372, Jan. 2011.

[8] Y. Li, S. Liu, M. Zhong, and S. X. Ding, "State estimation for stochastic discrete-time systems with multiplicative noises and unknown inputs over fading channels," *Appl. Math. Comput.*, vol. 320, pp. 116–130, Mar. 2018.

[9] L. Ma, Z. Wang, Y. Liu, and F. E. Alsaadi, "Distributed filtering for nonlinear time-delay systems over sensor networks subject to multiplicative link noises and switching topology," *Int. J. Robust Nonlinear Control*, vol. 29, no. 10, pp. 2941–2959, Jul. 2019.

[10] W.-Q. Liu, X.-M. Wang, and Z.-L. Deng, "Robust weighted fusion Kalman estimators for multisensor systems with multiplicative noises and uncertain-covariances linearly correlated white noises," *Int. J. Robust Nonlinear Control*, vol. 27, no. 12, pp. 2019–2052, Sep. 2016.

[11] Q. Liu, Z. Wang, Q.-L. Han, and C. Jiang, "Quadratic estimation for discrete time-varying non-Gaussian systems with multiplicative noises and quantization effects," *Automatica*, vol. 113, Mar. 2020, Art. no. 108714, doi: 10.1016/j.automatica.2019.108714.

[12] J. Hu, Z. Wang, and H. Gao, "Recursive filtering with random parameter matrices, multiple fading measurements and correlated noises," *Automatica*, vol. 49, no. 11, pp. 3440–3448, Nov. 2013.

[13] Y. Chen, X. Sheng, L. Guo, P. Li, H. Hao, and J. Yin, "Asynchronous multisonar data integration approach used to detect sparse targets," *IEEE Access*, vol. 7, pp. 37908–37917, 2019.

[14] K. Zhang, Z. Wang, L. Guo, Y. Peng, and Z. Zheng, "An asynchronous data fusion algorithm for target detection based on multi-sensor networks," *IEEE Access*, vol. 8, pp. 59511–59523, Mar. 2020.

[15] L. P. Yan, B. S. Liu, and D. H. Zhou, "The modeling and estimation of asynchronous multirate multisensor dynamic systems," *Aerosp. Sci. Technol.*, vol. 10, no. 1, pp. 63–71, Jan. 2006.

[16] X. R. Li, "Optimal linear estimation fusion—Part VII: Dynamic systems," in *Proc. 6th Int. Conf. Inf. Fusion*, 2003, pp. 455–462.

[17] H. Lin and S. Sun, "Distributed fusion estimator for multisensor multirate systems with correlated noises," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 7, pp. 1131–1139, Jul. 2018.

[18] L. Yan, C. Di, Q. M. J. Wu, Y. Xia, and S. Liu, "Distributed fusion estimation for multisensor systems with non-Gaussian but heavy-tailed noises," *ISA Trans.*, vol. 101, pp. 160–169, Jun. 2020.

[19] J. Hu, Z. Wang, and G.-P. Liu, "Delay compensation-based state estimation for time-varying complex networks with incomplete observations and dynamical bias," *IEEE Trans. Cybern.*, early access, Jan. 15, 2021, doi: 10.1109/TCYB.2020.3043283.

[20] J. Hu, G.-P. Liu, H. Zhang, and H. Liu, "On state estimation for nonlinear dynamical networks with random sensor delays and coupling strength under event-based communication mechanism," *Inf. Sci.*, vol. 511, pp. 265–283, Feb. 2020.

[21] L. Zou, Z. Wang, J. Hu, and Q.-L. Han, "Moving horizon estimation meets multi-sensor information fusion: Development, opportunities and challenges," *Inf. Fusion*, vol. 60, pp. 1–10, Aug. 2020.

[22] S. Gao, Y. Zhong, X. Zhang, and B. Shirinzadeh, "Multi-sensor optimal data fusion for INS/GPS/SAR integrated navigation system," *Aerosp. Sci. Technol.*, vol. 13, nos. 4–5, pp. 232–237, Jun. 2009.

[23] B. Fu, J. Liu, and Q. Wang, "Multi-sensor integrated navigation system for ships based on adaptive Kalman filter," in *Proc. IEEE Int. Conf. Mechatronics Autom. (ICMA)*, Aug. 2019, pp. 186–191.

[24] H.-Q. Zhai and L.-H. Wang, "The robust residual-based adaptive estimation Kalman filter method for strap-down inertial and geomagnetic tightly integrated navigation system," *Rev. Sci. Instrum.*, vol. 91, no. 10, Oct. 2020, Art. no. 104501, doi: 10.1063/5.0019305.

[25] X. He, Z. Wang, Y. Liu, and D. H. Zhou, "Least-squares fault detection and diagnosis for networked sensing systems using a direct state estimation approach," *IEEE Trans. Ind. Informat.*, vol. 9, no. 3, pp. 1670–1679, Aug. 2013.

[26] S. Wang, X. Zhan, Y. Zhai, and B. Liu, "Fault detection and exclusion for tightly coupled GNSS/INS system considering fault in state prediction," *Sensors*, vol. 20, no. 3, p. 590, Jan. 2020.

[27] H. Dong, Z. Wang, and H. Gao, "Fault detection for Markovian jump systems with sensor saturations and randomly varying nonlinearities," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 59, no. 10, pp. 2354–2362, Oct. 2012.

[28] H. Dong, Z. Wang, J. Lam, and H. Gao, "Fuzzy-model-based robust fault detection with stochastic mixed time delays and successive packet dropouts," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 42, no. 2, pp. 365–376, Apr. 2012.

[29] F. Nemati, S. M. Safavi Hamami, and A. Zemouche, "A nonlinear observer-based approach to fault detection, isolation and estimation for satellite formation flight application," *Automatica*, vol. 107, pp. 474–482, Sep. 2019.

[30] R. Iqbal, T. Maniak, F. Doctor, and C. Karyotis, "Fault detection and isolation in industrial processes using deep learning approaches," *IEEE Trans. Ind. Informat.*, vol. 15, no. 5, pp. 3077–3084, May 2019.

[31] Z. Wang and H. Qiao, "Robust filtering for bilinear uncertain stochastic discrete-time systems," *IEEE Trans. Signal Process.*, vol. 50, no. 3, pp. 560–567, Mar. 2002.

[32] H. Lin and S. Sun, "Distributed fusion estimation for multi-sensor asynchronous sampling systems with correlated noises," *Int. J. Syst. Sci.*, vol. 48, no. 5, pp. 952–960, Apr. 2017.

[33] D. F. Specht, "A general regression neural network," *IEEE Trans. Neural Netw.*, vol. 2, no. 6, pp. 568–576, Nov. 1991.

[34] L. Zhang and X.-D. Zhang, "An optimal filtering algorithm for systems with multiplicative/additive noises," *IEEE Signal Process. Lett.*, vol. 14, no. 7, pp. 469–472, Jul. 2007.

[35] R. E. Kalman, "A new approach to linear filtering and prediction problems," *J. Basic Eng.*, vol. 82, no. 1, pp. 35–45, Mar. 1960.

[36] F. Yang, Z. Wang, and Y. S. Hung, "Robust Kalman filtering for discrete time-varying uncertain systems with multiplicative noises," *IEEE Trans. Autom. Control*, vol. 47, no. 7, pp. 1179–1183, Jul. 2002.

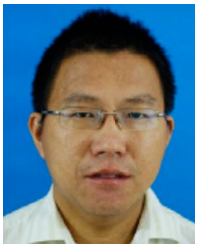
[37] S. Hang, L. Yan, B. Liu, and J. Zhu, "A sequential asynchronous multirate multisensor data fusion algorithm for state estimation," *Chin. J. Electron.*, vol. 17, no. 4, pp. 630–632, Oct. 2008.

[38] Y. Liu, L. Yan, B. Xiao, Y. Xia, and M. Fu, "Multirate multisensor data fusion algorithm for state estimation with cross-correlated noises," in *Knowledge Engineering and Management* (Advances in Intelligent Systems and Computing). Beijing, China: Springer, 2014, pp. 19–29.

YIMING MA was born in Henan, China, in 1997. He received the B.E. degree from the Department of Automation, North China University of Water Resources and Electric Power, Zhengzhou, China, in 2015. He is currently pursuing the M.E. degree with the School of Automation, Nanjing University of Science and Technology, Nanjing, China. His current research interests include state estimation, information fusion filtering, and integrated navigation. He is a very Active Reviewer for many international journals.

MENGJUN ZHANG was born in Anhui, China, in 1995. He received the bachelor's degree from the School of Mechanical and Electrical Engineering, Daqing Normal University, Daqing, China. He is currently pursuing the master's degree in control theory and control engineering with the School of Automation, Nanjing University of Science and Technology, Nanjing, China. His current research interests include satellite navigation and integrated navigation.

BOHAO WANG received the B.E. degree in automation from the Nanjing University of Science and Technology, Nanjing, China, in 2018, where he is currently pursuing the master's degree in control engineering. In 2019, he joined the Micro-PNT Team for integrated navigation system research. His main research interests include guidance, navigation, and control of aerospace vehicles.



SHUAI CHEN received the B.Sc. degree in computer science and technology and the Ph.D. degree in navigation guidance and control specialty from the Nanjing University of Science and Technology, Nanjing, China, in 2003 and 2008, respectively.

Since June 2015, he has been a Postdoctoral Research of combined Navigation Research and Intelligent Navigation Research with the Nanjing University of Science and Technology. He is currently a Professor with the School of Automation, Nanjing University of Science and Technology, and a Visiting Scholar with the University of New South Wales, Australia. His research interests include navigation systems, signal processing, artificial intelligence, and control theory and applications. He is an Active Reviewer for many international journals.