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Electric Vehicle Route Optimization Under Time-of-Use Electricity Pricing

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ABSTRACT This paper addresses an electric vehicle routing problem with time window (E-VRPTW) under time-of-use (TOU) pricing where retail prices vary hour-by-hour to reflect changes in wholesale prices. The proposed solution aims to minimize the electricity-cost as well as traditional objectives: number of used vehicles and total travel distance. In particular, the proposed solution cleverly shifts battery charging to off-peak periods and adjusts the charging duration in order to reduce costs. First, the problem is carefully carved in a mixed integer linear programming model. Second, a constraint programming model is built. Third, a combined model is constructed to exploit the strengths of both models. The computational study based on the well-known benchmarking test instances demonstrates we can reduce the electricity cost by 3.1% on average while not compromising other objectives. We provides benchmarking instances and CPLEX source codes, in order to promote related-research, thus expediting the adoption of energy-efficient scheduling by autonomous taxi company.

INDEX TERMS E-VRPTW, battery, TOU, MIP, CP.

I. INTRODUCTION

Autonomous vehicle (AV) or self-driving is an emerging technology expected to bring fundamental shifts in transportation. In this coming self-driving car era, one conceivable scenario is for people to rely on an autonomous taxi instead of owning a personal vehicle. Some predicted that owning a car will soon be as quaint as owning a horse [26]. There are several reasons. Firstly, solely using autonomous taxis for transportation could cost the same as owning a car [8], although the prediction seems to not be realized yet. Secondly, most accidents are caused by driver error, explaining 94 percent of the crashes [35]. Therefore, more passengers are predicted to depend on autonomous taxis in a form of shared service instead of owning personal vehicles in the coming self-driving car era.

Followed by NuTonomy's debut as a robotaxi (also known as a self-driving taxi or a driverless taxi) service in Singapore, Tesla announced to launch robotaxi as part of broader vision for an autonomous ridesharing network in 2020 during the company's Autonomy Day [34]. Similar efforts have been made virtually by all major auto makers and countless start-ups.

FIGURE 1. Charging stations (Courtesy of TELSA and IONITY).

This inevitable shift creates an urgent need of efficiently orchestrating a fleet of electric AV that are limited by short driving range and slow charging time, once dubbed as an Achilles' heel by Bruglieri *et al.* [21]. To respond to this challenge, researchers have studied a new variant of classical vehicle routing problem (VRP) and termed E-VRP. The E-VRP includes a route with planned detours to charging station (CS) as depicted in Fig. 1, covering arrival times and charging durations. The partial/full and linear/nonlinear charging mechanisms have been extensively studied along with vehicle capacity and time window (TW) constrains.

However, varying electricity prices have not been fully considered, although this would offer an extensive

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opportunity for cost saving. Under time-of-use (TOU) electricity pricing, retail prices vary hour-by-hour to reflect changes in wholesale prices, which are typically announced a day ahead or an hour ahead. Fig. 2 depicts the hourly day-ahead TOU pricing/kWh on January 30, 2019. By simply shifting the charge from 8 pm to 4 am, we could save 68% on electricity costs. The key is to shift the battery charging to off-peak periods in order to minimize the cost in conjunction with other costs: the number of used EVs and the total travel distance.

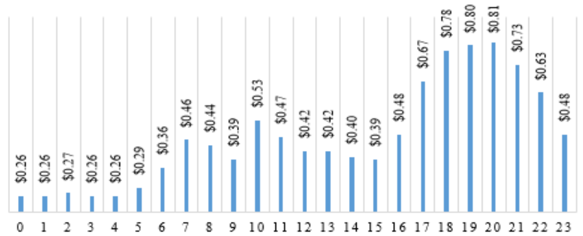

FIGURE 2. Hourly TOU pricing (https://hourlypricing.comed.com/).

The contributions of this paper are threefold. First, we formally formulate E-VRPTW under TOU pricing as an MIP model without explicitly employing a vehicle index for the first time. Second, we devise a constraint programming (CP) model using an intensity function so that energy cost can be effectively calculated. Third, we develop a combined model of MIP-CP to exploit the strengths of both, all for the first time, which performs better than using either of two models alone.

II. LITERATURE REVIEW

A. E-VRP

The most related work has been pioneered by Bard *et al.* [17] where vehicles visit satellite replenishment centers to load depleted items after delivering all preloaded, thus permitting extended tours. They presented a formal MIP model and solved it with the branch and cut method. They introduced a decision variable to track the load of a vehicle when it arrives at customer. A similar approach was later used to track the remaining battery level in E-VRP.

Erdoğan and Miller-Hooks [29] introduced the green (G)-VRP, where alternative fuel-powered vehicles with a limited fuel capacity visit at fuel stations along the route with a fixed refueling time. Although neither partial refueling, loading capacity, nor time window constraints were considered, this well-defined G-VRP served a precursor of E-VRP where a refueling (battery-charging) becomes the critical restriction.

Finally, Schneider *et al.* [25] introduced E-VRP that incorporates CS within a route in addition to the conventional VRP constraints (capacity restrictions and time windows). Due to the battery capacities, EVs need to visit to CS during delivery tours. The aim was to find feasible tours satisfying battery restriction (the level of battery may never fall below zero).

Table 1 contains the selected articles related to E-VRP. Column 4 shows if the article considered the partial charge since the partial charge is realistic requirement, difficult to model, and computationally expensive. In particular, we have to decide how long to recharge as well as where and when, for each vehicle (Felipe *et al.* [5]). Since recharging at the depot is cheaper than recharging en route, vehicles choose the partial charge en route and making zero battery level when vehicles arrive at the depot. Columns 5-6 mark if the article considered the vehicle load capacity and customer time windows, respectively.

TABLE 1. Selected articles related to E-VRP.

Year	Authors	Approach	Partial	Capa	TW	Note
1998	Bard et al.	Branch and Cut	N	Y	N	Replenish commodity
2012	Erdoğan and Miller-Hooks	MIP/Heuristic	N	N	N	G-VRP
2014	Schneider et al.	MIP/VNS/Tabu	N	Y	Y	E-VRP
2014	Felipe et al.	Heuristic	Y	Y	N	Varying charging technologies
2015	Bruglieri et al.	MIP/VNS	Y	Y	Y	—
2016	Lin et al.	MIP	Y	Y	N	Vehicle load effect on battery consumption
2016	Desaulniers et al.	Branch and Cut	Y	Y	Y	—
2016	Keskin and Çatay	MIP/ALNS	Y	Y	Y	—
2016	Hiermann et al.	Branch-and-Price/ALNS	Y	Y	Y	Heterogeneous vehicles
2017	Montoya et al.	ILS	Y	N	N	Nonlinear charging
2019	Booth and Beck	CP	N	Y	Y	—
2019	Zuo et al.	MIP	Y	Y	Y	Nonlinear charging
2019	Xiao et al.	MIP	N	Y	Y	Varying battery consumption rate
2019	Erdelica et al.	MIP/Heuristic	N	Y	Y	—
2019	Cortés-Murcia et al.	MIP/Heuristic	Y	Y	Y	Satellite customer
2020	Zhu et al.	VNS	N	Y	N	
	This paper	MIP/CP/MIP+CP	Y	Y	Y	Time-Of-Use Electricity Pricing

The partial/full, linear/nonlinear charging, varying charging speed mechanisms, and varying battery consumption rates have been extensively studied along with vehicle capacity and time window constraints as Table 1 presents. For readers interested in E-VRP, see survey papers Pelletier *et al.* [30], Erdelic and Caric [32], and Schiffer *et al.* [24].

B. SCHEDULING UNDER TIME-OF-USE PRICING

Electricity cannot be efficiently stored so it must be generated, transmitted and consumed simultaneously. In addition, electricity demand is imbalanced over time, which causes a difficulty for electricity suppliers to regulate peak load (Mitra *et al.* [31]). As a result, electricity price varies, according to the market demand over time. There are three types of time dependent electricity pricing: time-of-use (TOU) pricing, real-time pricing (RTP) and critical peak pricing (CPP) (Sharma *et al.* [6]). In the TOU pricing (the most widely used among the three), the electricity price schedule is predefined, but it may vary by hour, day, and season. This pricing policy motivates consumers to save cost by shifting their energy consumption from on-peak to off-peak periods. That creates a new research stream called energy-aware production scheduling (Wu *et al.* [28]) where the total energy-cost is minimized in conjunction with other traditional objectives in classical manufacturing environments: single-stage, job-shop, flexible-job-shop, flow-shop, batching, etc.

Dubey and Santoso [1] and Afzalan and Jazizadeh [20] considered TOU pricing and studied the impacts of residential electric vehicle (EV) charging on distribution system.

However, TOU pricing has not been fully studied in E-VRP. To the best of our knowledge, Yang *et al.* [16] and Ferro *et al.* [12] are the only two articles that considered the electricity-price as a time-varying factor. However, Yang *et al.* [16] did not consider the time window constraint and formulated the problem as a nonlinear, nonconvex, and discrete model, which is difficult to solve. Hence, they proposed a learnable partheno-genetic algorithm which combines conventional intelligent algorithms (such as genetic algorithm and tabu search) and a knowledge model. Ferro *et al.* [12] proposed an MIP model which considers several speed levels of EVs and several recharging modes at recharging stations, but it is inefficient due to the explicitly employing a vehicle index in all decision variables. Hence, in this paper, we formally formulate E-VRP with time window constraint under TOU pricing as an MIP model without explicitly employing a vehicle index for the first time.

C. CONSTRAINT PROGRAMMING

Researchers often formally present scheduling problems in an MIP in order to exactly capture the problems (exact method) and prove optimality of small-sized test instances. Then, heuristic models that are equipped with the same objectives and constraints are proposed for a rapid calculation of large-sized instances.

Recently, the CP (IBM CPLEX) has received lots of attentions from both practitioners and researchers as an alternative and/or complementary of MIP and heuristics, owning to its following strengths: conciseness, natural-language like, flexibility, and performance and successfully applied to VRP problems (Shaw [27]; Backer *et al.* [7]; Ham [2], Ham 2020).

However, CP has not been seen in E-VRP literature. Recently, Booth and Beck [19] applied CP to E-VRP. However, they did not consider the partial charge. We propose a full CP formulation for E-VRP under TOU.

D. HOME ENERGY MANAGEMENT SYSTEM

This study is primarily for the emerging robotaxi service providers that employ a fleet of EVs. However, the home energy management system (HEMS) connected to microgrid can contribute to reduction of energy cost and improvement of voltage stability (Ganjehlou *et al.* [14]) by efficiently orchestrating energy consumptions of EVs, heating and air conditioning, and home appliances. We can further seek a global optimization and maximize the benefits by connecting neighborhood HEMSs (Gholinejad *et al.* [15]).

E. COMBINED MODEL OF MIP-CP

The two-index MIP model for homogeneous vehicles (Schneider *et al.* [25]) does not explicitly employ a vehicle index, but it successfully determines the number of used vehicles. This efficient modelling technique with valid inequalities successfully solves quite large-sized instances. However, the MIP model for E-VRP under TOU relies on lots of binary variables to determine the energy-cost at the time of charging, making the model intractable even for small-sized instances.

We develop a combined model to use the complementary strengths of MIP and CP for solving problems that are otherwise intractable using either of these two methods alone. For these problems, both pure MIP- and pure CP-based approaches may not perform well (Jain and Grossmann [39]). The combined model inherits most of the efficient MIP equations and replaces TOU related variables/equations with CP.

III. PROBLEM DESCRIPTION AND SOLUTIONS

This paper considers E-VRPTW under TOU which finds the optimal delivery tour, delivery time, and recharging time for a set of homogeneous EVs with a loading capacity. We assume the following situations. All customers with known demand must be serviced within a desired delivery time window by exactly one vehicle. Every EV starts delivery from a depot at time zero and comes back to the same depot after delivery. An EV can serve multiple customers whose total demand does not exceed EV's load capacity. The battery level decreases in proportion to the travel distance, and electricity is consumed only during travel. EV's battery must always be above zero. EV can visit the CS to recharge its battery within maximum capacity. In addition to the standard E-VRPTW, a total energy cost is considered in this paper. In particular, the prices of electricity vary over discrete time intervals. The key is to shift the recharging time to off-peak periods in order to minimize the energy cost.

Fig. 3 contrasts the conventional E-VRPTW with E-VRPTW under TOU with a small benchmark instance (C103-5) having 1-EV, 2-CS, and 5-customer. Fig. 3(a) shows a tour when TOU electricity-price was not considered. The proposed model found the minimum traveling distance of 177 Km with a cost of \$2.7930 for recharging. On the other hand, Fig. 3(b) represents a tour when TOU was considered. The proposed model reduced the energy-cost by 8.7% by adjusting the recharging amount and duration at CSs according to TOU pricing. Note the total amounts of recharging (98 MW) in both scenarios are equal, but the costs are different. The EV starts from the depot (D) at time 0, serves customer 65 at 67 (note the time windows are successfully met), and visits the CS (S) for 35 time units for partial charging. Then, the EV visits customer 98 at time 236 and returns to the CS for full charging, spending 268 time units. The EV resumes the tour serving customers 20, 24, and 57 in order and visits the CS at 1090 for 39 time units. Finally, the EV arrives at the deport at 1169.

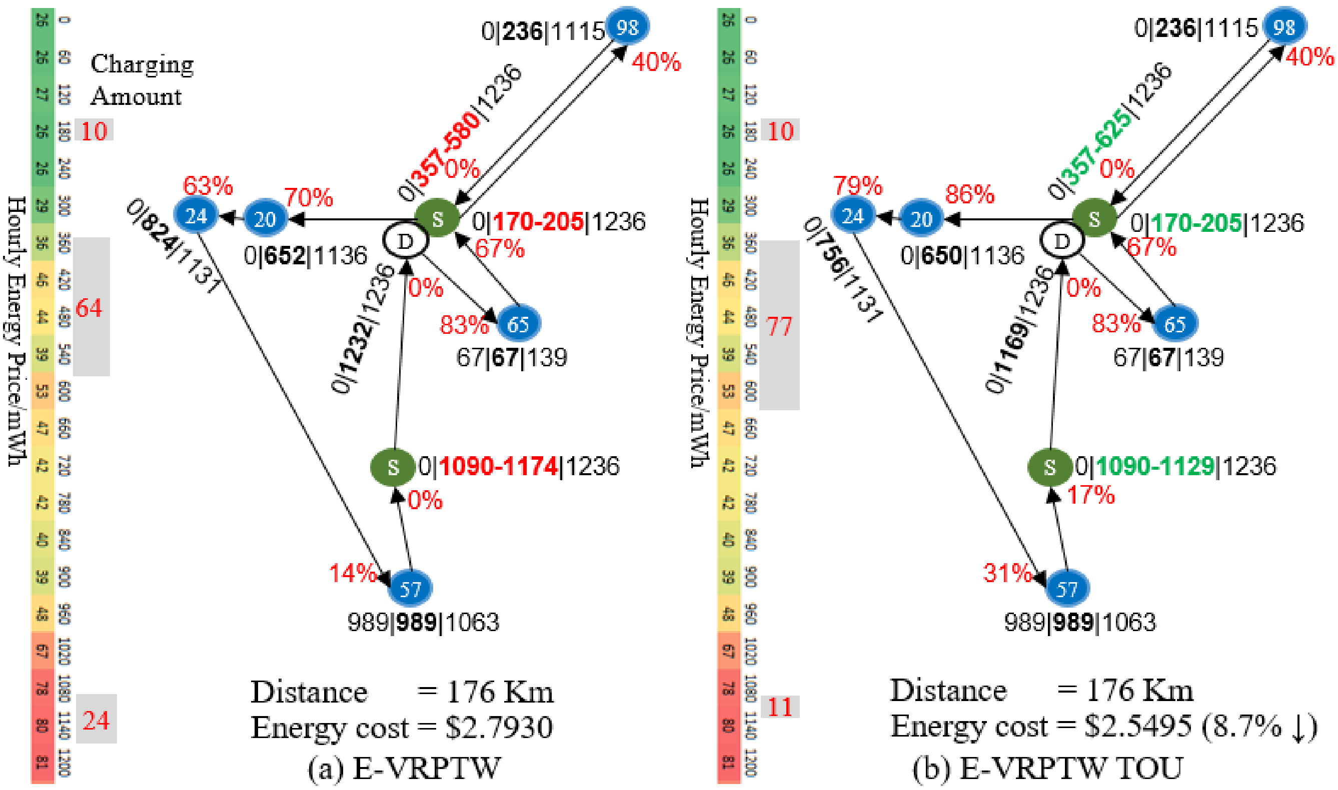

FIGURE 3. Energy-aware scheduling: Energy cost minimization (C103-5 instance).

In this paper, we have three objectives. The first is to minimize the number of used EVs. The second is to minimize total travel distance of EVs. The third is to minimize total energy cost for recharging. This paper proposes a lexicographic optimization approach using mixed integer linear programming, constraint programming and combined models.

A. MIXED INTEGER PROGRAMING MODELS

Table 2 summarizes parameters and decision variables of MIP model.

The mathematical model of E-VRPTW under TOU pricing is formulated as a mixed-integer program as follows:

$$\text{Minimize } M_1 \cdot TV + M_2 \cdot TD + TC \tag{A1}$$

$$\text{s.t. } TV = \sum_{oj \in E} X_{0j} \tag{A2}$$

$$TD = \sum_{ij \in E} d_{ij} X_{ij} \tag{A3}$$

$$TC = \sum_{i \in F'} \sum_{t \in T} c_t L_{it} \tag{A4}$$

$$\sum_{ij \in E} X_{ij} = 1 \quad \forall i \in V \tag{A5}$$

$$\sum_{ij \in E} X_{ij} \le 1 \quad \forall i \in F' \tag{A6}$$

$$\sum_{ij \in E} X_{ij} = \sum_{jk \in E} X_{jk} \quad \forall j \in V' \tag{A7}$$

$$s_i + A_i + t_{ij} X_{ij} + s\left(X_{ij} - 1\right) \le A_j \quad \forall ij \in E_v \tag{A8}$$

$$D_i + t_{ij} X_{ij} + s\left(X_{ij} - 1\right) \le A_j \quad \forall ij \in E_f \tag{A9}$$

$$e_i \le A_i \le l_i \quad \forall i \in V \tag{A10}$$

$$U_j \le U_i - u_i X_{ij} + u\left(1 - X_{ij}\right) \quad \forall ij \in E \tag{A11}$$

$$A_0 = 0,\ U_0 = u,\ Y_0 = y \tag{A12}$$

$$Y_j \le Y_i - (h \cdot d_{ij}) X_{ij} + y\left(1 - X_{ij}\right) \quad \forall ij \in E_v \tag{A13}$$

$$Y_j \le Y_i + Z_i - (h \cdot d_{ij}) X_{ij} + y\left(1 - X_{ij}\right) \quad \forall ij \in E_f \tag{A14}$$

$$D_i \ge A_i + g \cdot Z_i \quad \forall i \in F' \tag{A15}$$

$$Y_i + Z_i \le y \quad \forall i \in F' \tag{A16}$$

$$\sum_{t \in T} S_{it} = 1 \quad \forall i \in F' \tag{A17}$$

$$\sum_{t \in T} t \cdot S_{it} = A_i \quad \forall i \in F' \tag{A18}$$

$$\sum_{t \in T} F_{it} = 1 \quad \forall i \in F' \tag{A19}$$

$$\sum_{t \in T} t \cdot F_{it} = D_i \quad \forall i \in F' \tag{A20}$$

$$L_{it} = \sum_{r=0}^{t} \left(S_{ir} - F_{ir}\right) \quad \forall i \in F',\ t \in T \tag{A21}$$

Objective (A1) is the weighted sum of the number of used EVs, total travel distance, and total energy cost, each of which is computed in constraints (A2)-(A4), respectively. Constraints (A5)-(A16) denote constraints for

TABLE 2. Parameters and decision variables of MIP.

Parameters	
$0, N+1$	depot nodes
V	set of customers $V = \{1, \dots, N\}$
F	set of CSs
F'	set of dummy vertices generated to permit several visits to each vertex in F
V'	$V' = V \cup F'$
$V_0, V_{0,N+1}$	$V_0 = V \cup \{0\}$ and $V_{0,N+1} = V \cup \{0, N+1\}$
V'_0, V'_{N+1}	$V'_0 = V' \cup \{0\}$ and $V'_{N+1} = V' \cup \{N+1\}$
$V'_{0,N+1}$	$V'_{0,N+1} = V' \cup \{0, N+1\}$
s_i	service time at customer $i \in V'_0$
e_i	the earliest start
l_i	the latest arrival time
u_i	load demand of customer $i \in V'_0$
u	cargo loading capacity of EV
g	recharging rate, i.e., the recharging time for the unit battery level
h	battery charge consumption rate
y	battery capacity of EV
d_{ij}	distance from i to j
t_{ij}	travel time from i to j
τ	set of tuples $\{i, j, t_{ij}\}$
E	set of arcs which can be contained in a feasible route $E = \{ij \in V'_0 \times V'_{N+1} \mid i \neq j, u_i + u_j \leq u, h \cdot d_{ij} \leq y, e_i + s_i + t_{ij} \leq l_j\}$
E_v, E_f	$E_v = \{ij \in E \mid i \in V_0\}$ and $E_f = \{ij \in E \mid i \in F'\}$
s	end of time horizon
T	set of discrete times $T = \{0, \dots, s-1\}$
c_t	price of electricity during interval $[t, t+1), t \in T$
M_1 and M_2	big numbers ($M_1 = 1000000$; $M_2 = 10$)

Decision variables	
X_{ij}	1 if an arc $ij \in E$ is used by an EV and 0 otherwise
U_i	remaining cargo level on arrival at vertex $i \in V'_{0,N+1}$
Y_i	remaining battery level on arrival at vertex $i \in V'_{0,N+1}$
Z_i	amount of battery charging on vertex $i \in F'$
A_i	arrival time on vertex $i \in V'_{0,N+1}$
D_i	departure time from vertex $i \in F'$
S_{it}	1 if an EV arrives on vertex $i \in F'$ at time $t \in T$ and 0 otherwise
F_{it}	1 if an EV departs on vertex $i \in F'$ at time t and 0 otherwise
L_{it}	1 if an EV is charging on vertex $i \in F'$ at $[t, t+1)$ and 0 otherwise
TV	total number (units) of used EVs
TD	total travel distance (Km) of EVs
TC	total energy cost ($) for recharging

original E-VRPTW while constraints (A17)-(A21) represent the TOU pricing. In details, constraints (A5)-(A7) establish the bound of number of visits and flow conservation of each vertex. Constraints (A8)-(A9) guarantee relation between departure and arrival times of two consecutive vertices visited by the same EV. Constraint (A10) enforces the time window constraint. Constraint (A11) computes load level of the EV when it arrives at any vertex. Constraint (A12) guarantees initial conditions of the depot. Constraints (A13)-(A14) guarantee relation of battery levels of two consecutive vertices visited by the same EV. Constraint (A15) enforces that departure time is no less than arrival time plus recharging time. Constraint (A16) enforces battery capacity of EV. Constraints (A17)-(A18) and constraints (A19)-(A20) discretize the arrival and departure times at the recharging station, respectively. In details, constraints (A17)-(A18) force $S_{it} = 1$ if an EV arrives at vertex i at time t(that is, $A_i = t$) and $S_{it} = 0$ otherwise and constraints (A19)-(A20) force $F_{it} = 1$ if an EV departs from vertex i at time t(that is, $D_i = t$) and $F_{it} = 0$ otherwise. Finally, constraint (A21) captures the recharging time interval, that is, $L_{it} = 1$ if $A_i \leq t \leq D_i - 1$ and $L_{it} = 0$ otherwise. We term this proposed model as MIP-1.

This MIP-1 can be improved by adding valid inequality. Let W be the set of triples which cannot be contained in a feasible route as follows:

$$W = \{ijk \in V'_0 \times V' \times V'_{N+1} \wedge (W_1 \vee W_2 \vee W_3)\,,$$

where

$$W_1 = \left\{ijk \mid u_i + u_j + u_k > u\right\},$$
$$W_2 = \left\{ijk \mid h\left(d_{ij} + d_{jk}\right) > y\right\}, \text{ and}$$
$$W_3 = \left\{ijk \mid e_i + s_i + t_{ij} + s_j + t_{jk} > l_k\right\}.$$

Then, for a triple $ijk \in W$, two arcs ij and jk cannot be taken simultaneously and the following constraint is valid:

$$X_{ij} + X_{jk} \leq 1 \quad \forall ijk \in W \tag{A22}$$

We term the model in which the constraint (A22) is added as MIP-2.

B. CONSTRAINT PROGRAMMING MODEL

E-VRPTW can be formulated in a CP by utilizing a *cumulative* function that tracks a resource usage (battery-level in our application) over time. A vehicle is forced to visit a CS for recharging to keep the battery-level from below zero. On the other hand, the TOU pricing can be captured by a *step* function that tracks the efficiency of a resource over time. In particular, the relation between the size and the length of an interval variable (the size is the sum of the intensity step function between the start and end of the interval variable) can be used to represent the total cost at the time of recharging.

A CP formulation looks very different with an MIP. There is a standard notation in MIP formulation, however there is no standard in CP formulation until IBM team has very recently proposed a standard typesetting. We make our best effort to follow the standard. Table 3 summarizes parameters and decision variables of CP model.

The E-VRPTW under TOU pricing is formulated as a CP as follows:

$$\text{Minimize } staticLex\,(TV, TD, TC) \tag{B1}$$

$$\text{s.t. } TV = \sum_{k \in K} Z_k \tag{B2}$$

$$TD = \sum_{k \in K} \sum_{i \in V'_{0,N+1}} d_{typeOfPrev(Seq_k, X_{i,k}), i} \tag{B3}$$

TABLE 3. Parameters and decision variables of CP.

Parameters	
K	set of EVs
$fe(t)$	we pre-compute a piecewise linear function that gives the electricity price of at time t
Decision variables	
N_i	interval representing each node $i \in V'_{0,N+1}$, optional $(i \in F')$
$X_{i,k}$	optional interval representing node $i \in V'_{0,N+1}$ by EV k $(k \in K)$
I_i	optional interval representing node $i \in F'$ *intensity* $fe(t)$. The *intensity* is a function that applies a measure of usage or utility over an interval length; it is a stepwise function with integer values.
$Seq_k \leftarrow [X_{i,k}]$	sequenced collection of intervals assigned to EV k
Y_k	cumulative function representing the energy level of EV k $(k \in K)$ throughout the tour
Z_k	1 if EV k $(k \in K)$ is used and 0 otherwise

$$TC = \sum_{i \in F'} sizeOf\ (I_i) \tag{B4}$$

$$Z_k \geq \sum_{i \in V} presenceOf\ \left(X_{i,k}\right) \quad \forall k \in K \tag{B5}$$

$$startOf\ (I_i) = startOf\ (N_i) \cdot 100 \quad \forall i \in F' \tag{B6}$$

$$endOf\ (I_i) = endOf\ (N_i) \cdot 100 \quad \forall i \in F' \tag{B7}$$

$$noOverlap\left(Seq_k, \tau\right) \quad \forall k \in K \tag{B8}$$

$$e_i \leq startOf\ (N_i) \leq l_i \quad \forall i \in V \tag{B9}$$

$$sizeOf\ (N_i) = s_i \quad \forall i \in V \tag{B10}$$

$$first\left(Seq_k, X_{0,k}\right) \quad \forall k \in K \tag{B11}$$

$$last\left(Seq_k, X_{N+1,k}\right) \quad \forall k \in K \tag{B12}$$

$$\sum_{i \in V} presenceOf\ \left(X_{i,k}\right) \cdot u_i \leq u \quad \forall k \in K \tag{B13}$$

$$Y_k = step\ (0,y) - \sum_{i \in V'_{0,N+1}} stepAtStart\left(X_{i,k}, 0,y\right) + \sum_{i \in F'} stepAtEnd(X_{i,k}, 0,y) \quad \forall k \in K \tag{B14}$$

$$height\ At\ Start\left(X_{i,k}, Y_k, 0\right) = -h \cdot d_{typeOfPrev(Seq_k, X_{i,k}),i} \quad \forall k \in K, i \in V'_{0,N+1} \tag{B15}$$

$$heightAtEnd\left(X_{i,k}, Y_k, 0\right) = g \cdot sizeOf(X_{i,k}) \quad \forall k \in K, i \in F' \tag{B16}$$

Objective (B1) sequentially minimizes the number of used EVs, total travel distance, and total energy cost, each of which is computed in constraints (B2)-(B4), respectively. Constraint (B5) determines whether each EV is used or not. Constraints (B6)-(B7) synchronize two interval variables to calculate the total energy price during the charging by using intensity function. The size of an interval variable must be smaller than its length, so we rescale the interval variable by multiplying a hundred. Constraint (B8) prevents intervals in a sequence from overlapping on each EV. Constraints (B9)-(B10) enforce the time window constraint and service time, respectively. Constraints (B11)-(B12) force the vehicle to start from a depot and return to the same depot at the end of the tour. Constraint (B13) ensures the cargo capacity restriction. Constraints (B14)-(B16) ensure vehicle battery-level stays within limits. The constraints collectively set the initial battery-level at time 0, consume the battery as each vehicle operates, and charge the battery at CS, throughout the tour, by using a cumulative function with negative impact for travel and a positive impact for recharging.

C. COMBINED MODEL OF MIP AND CP (MCP)

The preliminary results showed that the two-index MIP model for E-VRPTW turned out to be inefficient for this variant with TOU, due to the large number of binary variables to compute the energy-cost at the time of charging. The preliminary results also showed that the CP model that explicitly describes each EV was not efficient either. Here we propose a combined model to use the complementary strengths of MIP and CP for solving E-VRPTW under TOU that was otherwise intractable using either of these two methods alone.

The combined model (MCP) inherits most of the efficient MIP equations and replaces TOU related variables/equations with CP. The proposed MCP model does not use binary variables S_{it}, F_{it}, L_{it} and equations (A4), (A17)-(A21) which were essential to calculate TOU pricing in a pure MIP model. Instead, the intensity function I_i and its associated constraints are borrowed from CP and some constraints are appropriately modified.

Decision variables:

X_{ij} : interval variable representing each arc $ij \in E$, 0 otherwise (note there is no vehicle index).

Minimize (A1)

s.t. (A2) – (A3), (B4), (A5) – (A16)

$$replace\ X_{ij}\ with\ presence\ Of\ \left(X_{ij}\right)$$

$$startOf\ (I_i) = A_i \cdot 100 \quad \forall i \in F' \tag{C17}$$

$$lengh\ Of\ (I_i) = (D_i - A_i) \cdot 100 \quad \forall i \in F' \tag{C18}$$

Constraints (C17)-(C18) synchronize interval variable (I_i) of CP and MIP variables (A_i and D_i) to calculate the total energy price during the charging by using intensity function. The size of an interval variable must be smaller than its length, so we rescale the interval variable by multiplying a hundred. We term this combined model MCP-1. In addition, MCP-2 is a model with the following constraint (C19) added to MCP-1.

$$presenceOf\ \left(X_{ij}\right) + presence\ Of\ \left(X_{jk}\right) \leq 1 \quad \forall ijk \in W \tag{C19}$$

IV. COMPUTATIONAL EXPERIMENTS

All experiments including MIP, CP, and flow control models are implemented in OPL 12.10.0 on an Intel®Core i7-4770 CPU with 16 GB of RAM. The Cplex codes, test instances and results are available at the following link: https://github.com/hamcruise/eVRP_TOU

TABLE 4. Comparison of models in terms of costs based on benchmark instances proposed by Schneider *et al.* (2014).

Inst	TV	MIP-TOU				MIP-TOU2				CP-TOU			MCP-TOU			MCP-TOU2		
		TD	TC	Gap	Bin	TD	TC	Gap	Bin	TD	TC	Gap	TD	TC	Gap	TD	TC	Gap
C101-5	2	266	240.8	9.0%	22341	264	232.3	0.0%	22341	268	18.9	2.2%	267	233.4	9.1%	266	239.9	9.0%
C103-5	1	177	213.0	6.2%	14909	177	214.1	3.5%	14910	—	—	—	177	229.1	12.0%	177	225.9	11.8%
C206-5	1	—	—	—	81481	—	—	—	81481	—	—	—	251	436.0	15.1%	267	508.2	21.3%
C208-5	1	—	—	—	61118	—	—	—	61118	—	—	—	165	168.0	9.8%	164	154.2	8.6%
R104-5	2	136	4.3	**0.0%**	4240	136	4.3	**0.0%**	4244	136	17.3	1.3%	136	4.3	0.3%	136	4.3	0.3%
R105-5	2	156	9.5	**0.0%**	4232	156	9.5	**0.0%**	4240	—	—	—	156	9.5	**0.0%**	156	9.5	0.6%
R202-5	1	143	17.8	0.0%	18100	143	17.8	0.3%	18100	—	—	—	143	18.0	1.2%	147	19.1	4.0%
R203-5	1	205	45.9	11.7%	24125	197	34.9	7.7%	24125	—	—	—	199	31.6	8.5%	197	35.3	7.7%
RC105-5	2	242	14.7	**0.0%**	5878	242	14.7	**0.0%**	5878	—	—	—	242	15.3	0.6%	246	15.9	2.3%
RC108-5	2	252	17.0	0.0%	5882	252	17.0	0.0%	5883	—	—	—	260	18.3	3.8%	252	17.3	0.7%
RC204-5	1	185	18.6	1.0%	23166	185	18.6	1.0%	23166	185	132.5	6.7%	189	30.1	3.7%	189	30.1	3.7%
RC208-5	1	168	17.5	1.0%	17382	175	17.8	5.0%	17382	—	—	—	168	16.6	1.0%	175	17.0	4.9%
C101-10	5	—	—	—	37366	—	—	—	37379	—	—	—	682	614.5	16.1%	624	500.9	7.4%
C104-10	3	429	557.4	11.9%	29950	—	—	—	29950	427	28.0	0.7%	480	552.2	20.2%	471	488.0	17.9%
C202-10	3	—	—	—	101995	—	—	—	101995	—	—	—	529	697.7	20.0%	488	634.4	13.1%
C205-10	3	—	—	—	61227	—	—	—	61227	—	—	—	465	610.8	12.8%	480	671.1	16.1%
R102-10	5	444	30.3	0.7%	5769	444	30.6	0.7%	5784	—	—	—	449	32.9	1.8%	462	35.4	4.6%
R103-10	3	336	33.3	1.0%	4374	336	33.4	1.0%	4380	—	—	—	343	35.1	3.0%	340	35.3	2.2%
R201-10	3	335	43.5	3.9%	24252	328	38.3	1.8%	24252	—	—	—	351	47.0	8.3%	364	52.1	11.7%
R203-10	2	—	—	—	30327	—	—	—	30327	—	—	—	335	52.0	3.3%	418	81.7	22.8%
RC102-10	5	493	18.0	0.3%	6002	493	18.3	0.2%	6013	—	—	—	495	19.9	0.8%	664	49.4	26.3%
RC108-10	5	629	42.4	0.7%	6035	629	41.5	0.7%	6038	—	—	—	639	43.1	2.2%	636	43.6	1.8%
RC201-10	4	440	30.7	2.0%	23287	436	24.5	1.0%	23287	—	—	—	434	22.6	0.5%	443	44.0	3.0%
RC205-10	4	—	—	—	23300	—	—	—	23300	—	—	—	693	82.1	2.9%	684	71.7	1.5%
															6.5%			8.5%

A. PROBLEM INSTANCES

Schneider *et al.* [25] generated E-VRPTW benchmark instances based on the instances of VRPTW by Solomon [23]. These instances are divided into three classes, depending on the geographical distribution of the customer locations: random customer distribution (R), clustered customer distribution (C), and a mixture of both (RC). They located one recharging station at the depot. They limited the charging locations to be reachable from the depot using at most two different recharging stations. The battery capacity was set to the maximum of the following two values: (1) the charge needed to travel 60% of the average route length of the best-known solution to the corresponding VRPTW instance, and (2) twice the amount of battery charge required to travel the longest arc between a customer and a station. Finally, we have added the hourly day-ahead TOU pricing/mWh (c_t) that was retrieved from hourlypricing.comed.com and duplicated for the rest of the planning horizon.

B. RESULTS

1) PERFORMANCE OF PROPOSED MODELS

Table 4 compares the results based on the benchmark instances. Column 1 identifies the name of the instance and TV, TD, TC are total number (units) of used EVs, total travel distance (Km) of EVs, and total energy cost ($) for recharging of the solutions found within 600 seconds time limit, respectively, for each model.

When the proposed model could not find a feasible solution, the column shows the symbol –. The optimality gaps and the counts of binary variables are recorded in Gap and Bin columns, respectively. The bold font indicates the optimal solution. The big-M method in MIP and lexicographic objective in CP led to an incorrect optimality gap calculation so we set the number of used EVs as a hard constraint to obtain a meaningful optimality gap.

The results show the pure MIP and CP models were not efficient, failing finding feasible solutions in many instances. It is observed that MIP models could not produce any feasible solution when the number of binary integer variables exceeds around 24000. On the other hand, the combined models (MCP-1 and MCP-2) successfully found feasible solutions of all instances. In terms of solution quality, combined models outperformed MIP and CP models. We can concluded that combined models exploited the complementary strengths of MIP and CP for solving E-VRPTW under TOU. We think this was possible because the following two reasons: (1) adopting

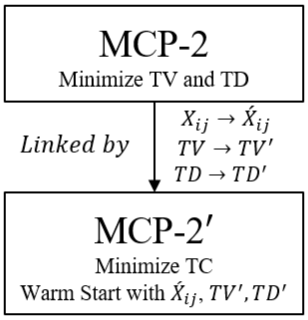

FIGURE 4. A proposed two-stage method to access the potential cost-saving.

TABLE 5. Comparison of conventional scheduling (minimizing number of used vehicles and total travel distance without considering energy-cost) and energy-aware scheduling (minimizing energy-cost as it maintains the same number of used vehicles and travel distance.

Inst	Conventional			Energy-Aware			
	TV	TD	TC	TV	TD	TC	$TC\downarrow$
C101-5	2	264	239.47	2	264	233.47	2.5%
C103-5	1	176	279.30	1	176	254.95	8.7%
C206-5	1	250	478.73	1	250	478.73	0.0%
C208-5	1	164	154.22	1	164	154.22	0.0%
R104-5	2	136	4.33	2	136	4.33	0.0%
R105-5	2	156	9.53	2	156	9.53	0.0%
R202-5	1	143	17.98	1	143	17.77	1.2%
R203-5	1	185	49.35	1	185	42.87	13.1%
RC105-5	2	242	15.33	2	242	14.88	2.9%
RC108-5	2	252	17.23	2	252	17.23	0.0%
RC204-5	1	185	18.82	1	185	18.77	0.3%
RC208-5	1	168	16.60	1	168	16.60	0.0%
C101-10	5	624	602.83	5	624	584.65	3.0%
C104-10	3	427	418.63	3	427	418.63	0.0%
C202-10	3	479	656.92	3	479	574.45	12.6%
C205-10	3	459	635.17	3	459	589.23	7.2%
R102-10	5	444	31.17	5	444	30.88	0.9%
R103-10	3	336	33.98	3	336	33.85	0.4%
R201-10	3	326	51.18	3	326	48.33	5.6%
R203-10	2	329	44.97	2	329	44.97	0.0%
RC102-10	5	493	22.90	5	493	22.90	0.0%
RC108-10	5	629	42.32	5	629	42.32	0.0%
RC201-10	4	434	42.00	4	434	35.47	15.6%
RC205-10	4	681	83.10	4	681	82.57	0.6%
							3.1%

efficient two-index formulation from MIP model that does not explicitly model each vehicle and (2) borrowing efficient energy-cost calculation at the time use from CP model that does not use a binary variable. When comparing the results of MCP-1 and MCP-2, we could not find a significant difference.

2) EFFECT OF AWARENESS OF ENERGY-COST

We were interested in quantifying the potential amount of cost-saving from this proposed model. A two-stage optimization approach was adopted to demonstrate the proposed method can reduce the energy-cost without compromising other conventional objectives (Fig. 4). At the first stage, the proposed MCP-2 generates a solution that minimizes the number of used EVs and the total travel distance without being aware of energy-cost. The solution is passed to the second stage where MCP-2 resumes a search for a solution that minimizes energy-cost within a solution space that is limited by the calculated conventional objective values at the first stage ($\acute{X}_{ij}, TV', TD'$). Namely, when MCP-2 starts a search at the second stage, it refers to the solution from the first stage (warm-start).

Table 5 contrasts the conventional scheduling vs. the energy-aware scheduling. The conventional scheduling minimizes the number of used EVs and the total travel distance without being aware of energy-cost, whereas the energy-aware scheduling minimizes the energy-cost as it maintains the same number of used EVs and the total travel distance. The results show a 3.1% saving on average. This mild savings can be largely explained by the time-window restriction that forces a vehicle to visit a customer at the prescribed time, mitigating an opportunity of cost-saving. However, 15.6% and 13.1% savings on RC201-10 and R203-5 instances, respectively, demonstrated a significant potential.

Note we can still use the proposed MCP models to handle a dynamic order. The proposed models will generate an initial schedule for a static order. Then the same models can reschedule for a dynamic order in real-time based on warm-start method to reduce a run-time and a service disruption.

V. CONCLUSION

This paper directs an electrical vehicle routing problem with time window (E-VRPTW) toward the energy-aware scheduling by considering time-of-use (TOU) pricing, aiming to minimize the electricity costs as well as the number of used vehicles and total travel distance, for the emerging robotaxi service providers. The proposed method exploits the hour-by-hour varying electric prices and shifts battery charging to off-peak periods and adjusts the charging duration in order to reduce the cost. First, the problem is carefully carved in a mixed integer programming model. Second, a constraint programming model is built. Third, combined models are constructed to exploit the strengths of both models. The computational study demonstrates that combined models outperformed a mixed integer programming model and a constraint programming model. The proposed method reduces the electricity cost by 3.1% on average. It also shows a significant potential for energy-saving.

In future research, we will study the home energy management system to efficiently schedule energy consumptions by EVs, heating and air conditioning, and home appliances under the dynamic TOU pricing.

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