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Optimal Search Facilities Selection Model for Joint Aeronautical and Maritime Search

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ABSTRACT Joint aeronautical and maritime search and rescue is the most effective way of performing rescues at sea. The value and effectiveness of a search and rescue (SAR) are far greater when using a coordinated air-maritime search than when using only vessels or aircraft. However, the harmonization of aeronautical and maritime SAR is complex and potentially life-threatening. When the location of the target in distress is unknown, the search process must be carried out. As the sole way to locate and rescue survivors, the search process is the most costly, hazardous, and complicated part of the whole SAR operation. This article focuses on the key problem of the optimal selection of search facilities, that is often encountered in large-area maritime search practice and urgently needs to be solved in joint aeronautical and maritime search operations. The problem may be abstracted into an optimization model with vessel and aircraft quantitative constraints that fully considers the area of the sea region to be searched, maximum speeds, search capabilities, initial distances of vessels and aircraft from the search area, and maximum endurance of aircraft. By introducing 0-1 decision variables, the search facility selection can be judged and optimized directly and effectively. By analyzing the results with different vessel and aircraft quantities, and taking the relationship between search coverage time and the number of search facilities (cost) into account, the optimal (most economic and feasible) search facility selection scheme can be produced.

INDEX TERMS Joint aeronautical and maritime search, marine safety, search facility, optimal model.

I. INTRODUCTION

Joint aeronautical and maritime search and rescue (SAR) is an activity in which surface forces (vessel facilities) and air forces (aircraft facilities) are coordinated, which has proven to be the most effective way to perform SAR at sea [1]. However, joint aeronautical and maritime SAR is a very complex and life-threatening activity. In the Malaysia Airlines Flight 370 (MH370) accident, the approximate maximum flight radius of the airplane was 5250 kilometers, and the theoretical search area exceeded 86 million square kilometers, making it the largest search operation at sea in history. ''Science'' magazine published an editorial saying that the search for MH370 was the largest and most difficult search task in history. At least 160 ships and aircraft (including 65 aircraft and 95 ships) from 26 countries participated in this unprecedented search operation [2]–[5]. In such a case, when search facilities from different countries are employed, a problem that search commanders urgently need to solve is how to choose the optimal search facilities to participate in the search operation and efficiently complete the search coverage. In forming an effective mechanism for search information support and scientific decision-making, there should be at least two elements: one is a way to grasp the information of all available forces in a timely and accurate manner, and the other is an optimal maritime search model. The former is a prerequisite for planning the search while the latter is a scientific and rational approach by a mathematical program. Maritime SAR methods have been studied for many years, and the International Aeronautical and Maritime Search and Rescue (IAMSAR) Manual has become a programmatic document for taking search action [6], [7]. In addition, some countries have introduced their own maritime SAR manuals based on their conditions [8].

In recent years, with the development of artificial intelligence technology, some unmanned equipment such as

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unmanned aerial vehicles (UAVs) and unmanned surface vehicles (USVs), has gradually begun to be used in maritime SAR. Some studies [9]–[11] examined path planning and communication link issues in the process of maritime SAR using UAVs and USVs. Others [12]–[14] studied target location and recognition methods when using UAVs to detect marine targets. Another [15] studied the target location and recognition method when using UAV to detect marine targets.

Search and rescue decision-making support methods have always been a research hotspot in the maritime industry and research on this topic has been extensive [16]–[27]. A three-stage decision support method to optimize the type and number of resources used when developing SAR schemes to formulate an emergency response more efficiently and effectively has been developed [28]. Some studies [29]–[31] provided useful models and algorithms for increasing the probability of detection (POD) and the probability of success (POS). A feasibility study on geographic information system (GIS)-based cost distance modeling to support strategic maritime SAR planning has been provided [32]. Another study [33] introduced a simulation process of sea-air search trends at sea using 3D GIS technology, which helps search commanders judge search trends including search facility dynamics and the degree of area coverage. A case study using agent-based maritime search-operation simulation demonstrated a model verification and validation (V&V) technique called test-driven simulation modelling (TDSM) [34]. The optimal selection of vessels for participating in maritime search has been studied and a corresponding model was established in [35]–[37] In particular, [35] provides an important reference value for solving the optimal search facility selection problem. The main contributions can be summarized as follows:

(1) The maritime search facility selection problem that urgently needs to be solved in search operation is abstracted as an optimizations problem with vessel and aircraft quantitative constraint conditions, and an optimal model is established.
(2) The model's solution complexity is studied. We found that there is a combinatorial explosion in the solution space of the model that cannot be solved effectively by the traditional exhaustive method An effective solution algorithm is provided
(3) By considering the relationship between the search coverage time and selected search facilities (cost), the optimal (most economic and feasible) search scheme can be produced.

This article is organized as follows: Section 2 introduces the search facility selection problem, outlines the model establishment process and analyzes the model solving complexity; Section 3 gives the model solution algorithm; Section 4 provides an example of a maritime search case; and Section 5 concludes this article and proposes the future focus of additional work.

II. MODEL ESTABLISHMENT

A. PROBLEM DESCRIPTION

To master the information of overall search facilities in the vicinity of the region to be searched in a timely, accurate, and comprehensive manner is the prerequisite for search planning. However, the first practical problem that the search and rescue coordinator (SC) must solve is how to select the available search facilities from all the available search facilities in order to develop the most reasonable plan for completing the search operation in the shortest possible time. To solve this problem the SC must go through two processing steps:

Step 1: Preliminary screening of the search facilities to eliminate those that do not meet the search conditions and

Step 2: Selection of the optimal search facilities that meet the conditions for participating in the search operation.

The first step requires comprehensive consideration of the environmental conditions of the sea area to be searched (wind, waves, currents, air temperature, water temperature, etc.) and the actual situation of the person in distress (the nature of the distress, the time of the distress, etc.) combined with the conditions of the search facilities (initial positions, speeds, types of ship or aircraft, tonnage, maneuverability, wind resistance, shipping cargo, etc.) as well as an expert knowledge base. We focus on the second step, which is realizing the optimization of the available search facilities by establishing a corresponding mathematical model.

Assume that there are many professional SAR ships (distributed mainly around the SAR bases or fixed standby points) and some professional SAR aircraft (located mainly at air bases) available to participate in the search action. In addition, some passing vessels may also be used to search. All these ships or vessels and aircraft constitute the overall joint aeronautical and maritime search facilities. Different ships may have different initial distances, maximum speeds, and search capabilities. The maximum speed, search capability, and maximum endurance may also differ for various professional aircraft. How to choose the available vessels and aircraft and make them work together, to complete the full coverage in the shortest time is a question often encountered in maritime SAR practice, as shown in Fig.1.

B. MATHEMATICAL MODEL BUILDING

The above problem is an optimization problem in pursuit of an efficient and economical search solution; therefore a mathematical model can be established as follows:

Set

① Sea area to be searched is S $nmile^2$;
② There are M vessels (each one is denoted as Ves_i, $i = 1, \cdots, M$) and N aircraft (each denoted as Air_j, $j = 1, \cdots, N$) available for this search operation;
③ The initial distance of Ves_i is D_i^v $nmile$, $i = 1, \cdots, M$ and the initial distance of Air_j is D_j^a $nmile$, $j = 1, \cdots, N$;

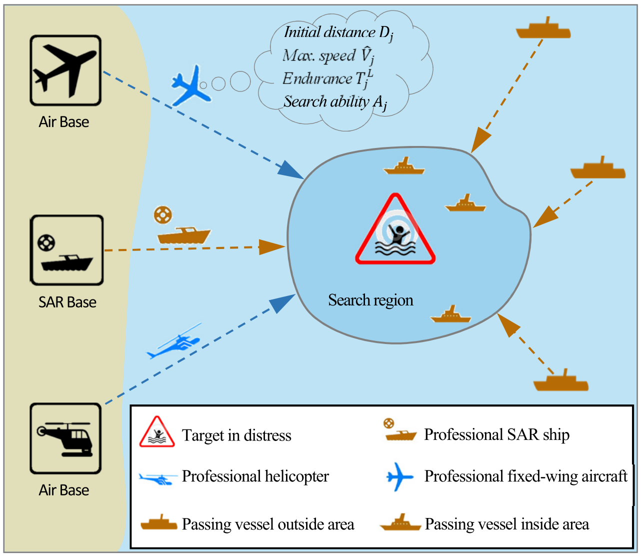

FIGURE 1. **Diagram of joint aeronautical and maritime search.**

④ The maximum speed of Ves_i is $\hat{V}_i^v, i = 1, \cdots, M$, and the maximum speed of Air_j is $\hat{V}_j^a, j = 1, \cdots, N$;

⑤ The search capability (area covered per hour) of Ves_i is A_i^v $nmile^2/h$ and the searching capability of Air_j is A_j^a $nmile^2/h$;

⑥ The maximum endurance of Air_j is T_j^L $h, j = 1, \cdots, N$;

⑦ The number of sorties performed by Air_j is L_j, $j = 1, \cdots, N$;

⑧ The quantitative restrictions of vessel and aircraft are Q^v and Q^a respectively;

⑨ The search operation takes T hours to achieve full coverage of the area.

Then

① Vessel Ves_i takes $\hat{T}_i^v = \frac{D_i^v}{\hat{V}_i^v}, i = 1, \cdots, M$ hours to arrive at the search region at full (maximum) speed;

② Vessel Ves_i takes $\bar{T}_i^v = T - \hat{T}_i^v, i = 1, \cdots, M$ hours to carry out search operations inside the search region;

③ Aircraft Air_j takes $\hat{T}_j^a = \frac{2D_j^a}{\hat{V}_j^a}, j = 1, \cdots, N$ hours for a round trip between the search region and its air base;

④ Aircraft Air_j takes $\hat{T}_j^a = T - \hat{T}_j^a, j = 1, \cdots, N$ hours to carry out search operations within the search region;

⑤ The number of sorties for Air_j is $L_j = \frac{T}{T_j^L}$, $j = 1, \cdots, N$.

The goal is to choose the optimal search facilities (vessels and aircraft) to perform search operations so that the time consumption T used to complete full coverage of the area is minimized. Therefore, we introduce the following decision variables.

Let

$$x_i = \begin{cases} 1 & \textit{if } Ves_i \textit{ joins operation} \\ 0 & \textit{if } Ves_i \textit{ does not joins operation} \end{cases} \quad (i = 1, \cdots, M) \tag{1}$$

and

$$y_j = \begin{cases} 1 & \textit{if } Air_j \textit{ joins operation} \\ 0 & \textit{if } Air_j \textit{ does not joins operation} \end{cases} \quad (j = 1, \cdots, N) \tag{2}$$

To implement fast and efficient search coverage over the search region in the shortest time, it is necessary to analyze the composition of the time spent by vessels and aircraft during the entire search operation.

As shown in Fig.2, suppose the start time of the search operation is t_s and the end time is t_e; then, the entire search operation time is

$$T = t_e - t_s \tag{3}$$

First, let us analyze the time for vessels to participate in the action: considering that the maximum speed and initial distance to the region to be searched are different for each vessel, the moment of arrival at the search region is also different. Fig.2 shows that Ves_1 and Ves_2 can reach the search site before the search operation is over, but Ves_4 will arrive at the search site after the search operation is over. Therefore, not all vessels have the opportunity to participate in the search operation. For each Ves_i that has the opportunity to participate in the search operation, the time it takes (denoted as T_i^v) is equal to the time (denoted as T) used in the entire search operation. T_i^v consists of two parts: one part is the time (denoted as $\overrightarrow{T}_i^v$) it takes for Ves_i to rush to the region to be searched, and the other part is the time (denoted as $\bar{T}_i^v$) it takes for Ves_i to carry out search operations in the search region. Therefore,

$$T = \overrightarrow{T}_i^v + \bar{T}_i^v \tag{4}$$

In addition, for passing vessels that are already in the sea region to be searched at the beginning of the search operation (such as Ves_3 in Fig.2), since they do not need to consume time to rush to the area (meaning $\overrightarrow{T}_3^v = 0$), the time ($\bar{T}_3^v$) for their search operations in the sea area is equal to the entire search time (T) spent in action.

Second, let us analyze the time for aircraft to participate in the operation. Similar to the situation of vessels, not all aircraft have the opportunity to participate in the operation. The only aircraft that can reach the search area before the end of the search operation can participate. Fig.2 shows that Air_1 can participate in the operation, and Air_2 cannot participate in the operation. Each Air_j, due to its limited endurance, needs to perform searches in multiple sorties.

The time of each sortie is equal to the maximum endurance of the aircraft, which is composed of the following three parts:

① The time (denoted as $\overrightarrow{T}_j^a$) it takes for Air_j to rush to the search region;

② The time (denoted as $\bar{T}_j^a$) it takes for Air_j to carry out search operations within the search region;

③ The time (denoted as $\overleftarrow{T}_j^a$) it takes for Air_j to return to the air base.

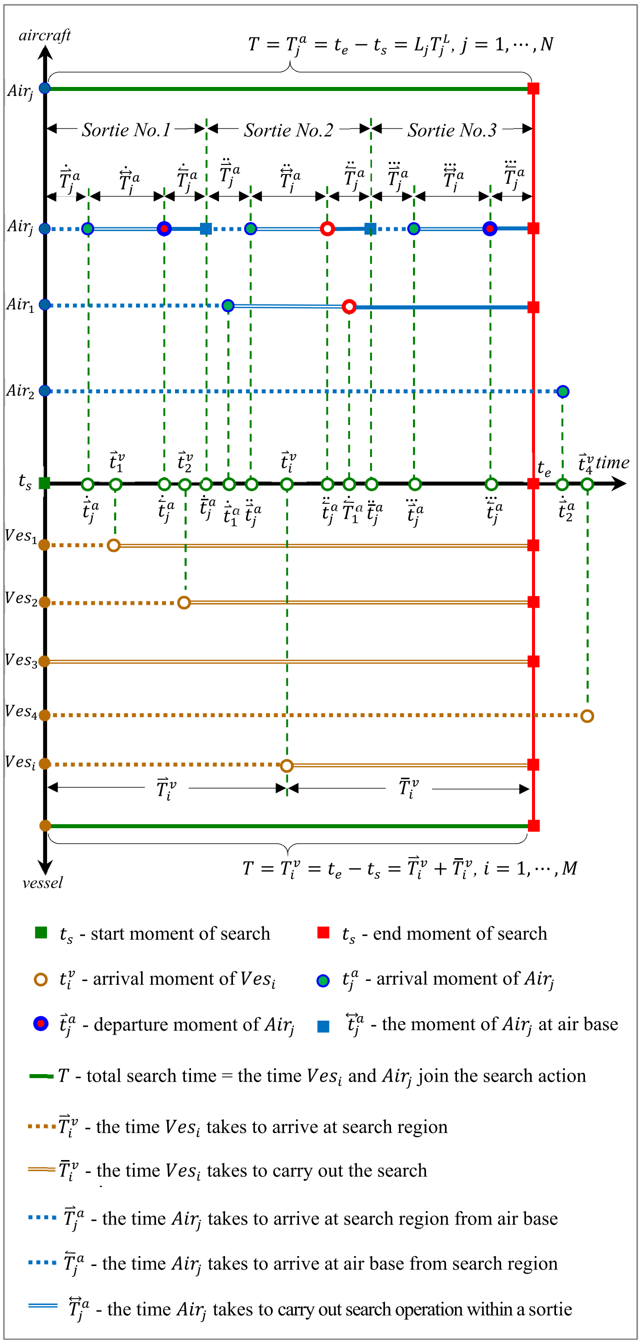

FIGURE 2. **Time taken by search facilities during search activity.**

For processing convenience, we make the following assumption: each vessel and aircraft are moving at the maximum speed when they rush to the search region and the search sub-area of each facility is non-overlapping; the time taken by the aircraft to travel between the search region and the air base is equal, and the time taken by the aircraft to refuel at the air base is not considered. In this way, the total time (denoted as $\bar{T}_j^a$) for Air_j to carry out search operations during the entire search operation is equal to the sum of the search operations for all sorties, that is,

$$\bar{T}_j^a = L_j \overleftrightarrow{T}_j^a \tag{5}$$

The search operation time ($\overleftrightarrow{T}_j^a$) for each sortie is equal to the aircraft's maximum endurance time (T_j^L) minus the round-trip time ($\hat{T}_j^a$) to and from the sea area to be searched, that is,

$$\bar{T}_j^a = L_j \left(T_j^L - \hat{T}_j^a\right) = L_j (T_j^L - 2D_j^a / \hat{V}_j^a) \tag{6}$$

The above analysis indicates that to achieve complete coverage of the search region, the following condition must be met:

$$\sum_{i=1}^{M} \bar{T}_i^v A_i^v x_i + \sum_{j=1}^{N} \bar{T}_j^a A_j^a y_j = S \tag{7}$$

That is

$$\sum_{i=1}^{M} \left(T - \hat{T}_i^v\right) A_i^v x_i + \sum_{j=1}^{N} \left(T - \frac{T}{T_j^L} \hat{T}_j^a\right) A_j^a y_j = S \tag{8}$$

Thus, after solving T, the model can be expressed as follows:

$$P\text{-}I \begin{cases} min\ T = \dfrac{\sum_{i=1}^{M} \hat{T}_i^v A_i^v x_i + S}{\sum_{i=1}^{M} A_i^v x_i + \sum_{j=1}^{N} \left(1 - \frac{\hat{T}_j^a}{T_j^L}\right) A_j^a y_j} & (9) \\ s.t. \begin{array}{l} \sum_{i=1}^{M} Q^v, Q^v \in \{0, 1, \cdots M\} \quad (10) \\ \sum_{j=1}^{N} Q^a, Q^a \in \{0, 1, \cdots N\} \quad (11) \end{array} \end{cases}$$

The target of the above model (denoted as *P-I*) is to seek the minimum time T throughout the search. Note that *P-I* has two constraints (called quantitative constraints), namely, the vessel's quantitative constraint (10) and the aircraft's quantitative constraint (11). Quantitative constraints are introduced for two reasons. First, objectively, due to the size of the search region, vessel tonnage and maneuverability and aircraft type (helicopters, fixed-wing airplanes), manipulable performance, etc., should not be assigned to every facility to be involved in the search action. Second, subjectively, by adding the quantitative constraint, the minimum time consumption can be obtained for different numbers of search facilities, so that the SC can balance the time cost and search facilities cost and then develop an optimal scheme with less time consumption while using the fewer possible search facilities.

C. MODEL SOLVING COMPLEXITY ANALYSIS

After the model is established, the most urgent problem is how to solve it. Because the values of the decision variables in *P-I* model can only be 0 or 1, there is only a limited variety of decision-making options (feasible solutions), which ensures that the optimal solution must exist Theoretically it is possible to use the exhaustive method (list all feasible solutions one by one and then compare the target function value of each feasible solution) to find the optimal solution. Actually, it is not feasible to use the exhaustive method to find the optimal solution of *P-I*, for the following reason.

Suppose the total quantity of available search facilities is n (called the problem scale). In *P-I* model, if we do not consider quantitative constraints, the total number of all solutions is

$$S = 2^n. \tag{12}$$

Assume that we can complete the calculation of a scheme in 1 *ns* (10^{-9} seconds). Table 1 gives the time consumption for the exhaustive method used to calculate the solution for all schemes with different scale of the question.

TABLE 1. Time consumption for the exhaustive method to identify the optimal solution without quantitative constraints.

Scale of the question (n)	Number of schemes (S)	Time consumption (t)
30	2^{30}=33554432	approximately 1.07 seconds
50	2^{50} = 1125899906842624	approximately 13.03 days
100	2^{100} = 1267650600228229401496703205376	approximately 4.02 × 10^{13} years

After the quantitative constraint conditions are introduced, the number of vessel selection schemes is the combination of the number of available vessels M and the corresponding quantitative constraint Q^v that is $C_M^{Q^v}$. The number of air facilities selection is the combination of the number of available aircraft N and the corresponding quantitative constraint Q^a, that is $C_N^{Q^a}$ Thus the total number of search schemes S is

$$S = C_M^{Q^v} \cdot C_N^{Q^a}. \tag{13}$$

Let us take 15 search facilities as an example (10 vessels and 5 aircraft) Table 2 gives the time consumption for the exhaustive method with scales of 30, 50, and 100.

TABLE 2. Time consumption for the exhaustive method to identify the optimal solution with quantitative constraints.

Scale of the question (n)	Number of schemes (S)	Time consumption (t)
20 vessels 10 aircraft	$C_{20}^{10} \cdot C_{10}^{5}$ = 46558512	approximately 0.05 seconds
40 vessels 10 aircraft	$C_{40}^{10} \cdot C_{10}^{5}$ = 213610453056	approximately 3.56 days
90 vessels 10 aircraft	$C_{90}^{10} \cdot C_{10}^{5}$ = 1441602661439556	Approximately 400.45 hours

Tables 1 and 2 show that the exhaustive method cannot obtain the optimal solution within a reasonable time when the scale of the question is large. In the practice of maritime search, search planners can often master a large amount of available search facility information through various technical means. Therefore the exhaustive method is not feasible in the actual solution process. It is valuable to study the effective solving algorithm of the *P-I* model

III. MODEL SOLUTION ALGORITHM

A. KNOWLEDGE OF OPTIMIZATION

Optimization, also known as mathematical programming, is a process of selecting the most reasonable scheme from many possible schemes to reach the optimal goal. All mathematical problems that pursue optimal goals are optimization problems.

The general form of the mathematical model of the optimization problem can be expressed as follows:

$$P\begin{cases} target\ min\, f(\boldsymbol{x}) & (14)\\ s.t.\ \ g_i(\boldsymbol{x}) \geq 0,\ i = 1, \cdots, m & (15)\\ \quad\ \ h_j(\boldsymbol{x}) = 0,\ j = 1, \cdots, n & (16)\end{cases}$$

The variable $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)^T \in \boldsymbol{R}^n$ is an n-dimensional vector called the solution vector. Each variable $x_1, x_2, \cdots, x_n$ is the decision variable; $f(\boldsymbol{x})$ is the target function; and $g_i(\boldsymbol{x}), i = 1, \cdots, m$ and $h_j(\boldsymbol{x}), j = 1, \cdots, n$ are the constraint conditions. A solution that satisfies constraints (15) and (16) is a feasible solution (or feasible point). The set of all feasible solutions is the feasible set, denoted by $\boldsymbol{S}$, namely,

$$\boldsymbol{S} = \{\boldsymbol{x} | g_i(x) \geq 0, i = 1, \cdots, m; h_j(x) = 0, j = 1, \cdots, n\}. \tag{17}$$

If $\exists \boldsymbol{x}^* \in \boldsymbol{S}$ and, for $\forall \boldsymbol{x} \in \boldsymbol{S}$, satisfies $f(\boldsymbol{x}^*) \leq f(\boldsymbol{x})$, then $\boldsymbol{x}^*$ is the optimal solution (or minimum point) of model P. The set of all optimal solutions $\boldsymbol{x}^*$ of P is called the optimal solution set, which is denoted by $\boldsymbol{S}^*$.

B. MODELSOLVING METHOD ANALYSIS

The *P-I* model (optimal search facilities selection model) established here has a special target function (a fractional target function for which the decision variables in the numerator and denominator are only 0 or 1) and two linear constraints, similar to the integer programming knapsack problem [38], [39]. This type of special fractional programming problem is the so-called fractional knapsack problem (FKP) and can be solved by the Dinkelbach algorithm in polynomial time [40].

To clearly explain the method of solving *P-I*, the target function (9) can be transformed as follows:

Set

$$① \quad p_0 = \sum\nolimits_{j=1}^{N} \left(1 - \frac{\hat{T}_j^a}{T_j^L}\right) A_j^a y_j; \tag{18}$$

$$② \quad p_i = A_i^v, \quad i = 1, \cdots, M; \tag{19}$$

$$③ \quad q_0 = S; \tag{20}$$

$$④ \quad q_i = \hat{T}_i^v A_i^v, \quad i = 1, \cdots, M. \tag{21}$$

From the known conditions, it is easy to know that

$$S > 0, \quad \hat{T}_i^v A_i^v \geq 0,\ A_i^v > 0,\ i = 1, \cdots, M.$$

Thus,

$$q_0 > 0, \quad q_i \geq 0,\ p_i > 0,\ i = 1, \cdots, M.$$

Then, *P-I* can be equivalently transformed into the following model (denoted as *P-II*):

$$P\text{-}II\begin{cases} max\ T = \dfrac{\sum_{i=1}^{M} p_i x_i + p_0}{\sum_{i=1}^{M} q_i x_i + q_0} & (22) \\ s.t.\ \sum_{i=1}^{M} x_i = Q^v, \quad 1 \le Q^v \le M & (23) \end{cases}$$

The target function of *P-II* is a fractional expression. The coefficient p_i of decision variable x_i in the numerator is a positive number, the coefficient q_i of the decision variable x_i in the denominator is a nonnegative number, and the constant q_0 is a positive number.

Let us study the value of p_0 and introduce the method of determining the value of decision variable y_j. As shown in (18), the value of p_0 is determined by the following three parameters:

① A_j^a – the search capability of Air_j, and $A_j^a > 0$;

② $\hat{T}_j^a$ – the time required for Air_j to make a round trip between the search region and the air base, and $\hat{T}_j^a = \frac{2D_j^a}{V_j^a} > 0$;

③ T_j^L – the maximum endurance of Air_j, and $T_j^L > 0$;

and the value of decision variable y_j.

Note that for any aircraft, only it meets the following condition,

$$T_j^L > \hat{T}_j^a \tag{24}$$

can it be available for search operations; that is, the aircraft must be able to make a round trip within its maximum endurance (T_j^L). Hereinafter, (24) is referred to as a prerequisite. For any aircraft that meets this prerequisite, there is

$$\left(1 - \frac{\hat{T}_j^a}{T_j^L}\right) A_j^a > 0. \tag{25}$$

For the convenience of description, let $\bar{a}_j = \left(1 - \frac{\hat{T}_j^a}{T_j^L}\right) A_j^a$; then, $\bar{a}_j > 0$, and we can always exchange the order to satisfy the following relationship:

$$\bar{a}_1 \ge \bar{a}_2 \ge \cdots \ge \bar{a}_{\bar{N}} \tag{26}$$

where $\bar{N}$ is the number of aircraft that meet the above prerequisite. To maximize the target function value T in *P-II* model, it is obvious that the following value

$$p_0 = \sum_{j=1}^{\bar{N}} \bar{a}_j y_j \tag{27}$$

should be maximized.

Let $y_j = 1, j = 1, \cdots, \bar{N}$. Considering the quantitative constraint of aircraft, the decision variable y_j in *P-I* are

① for $T_j^L \le \hat{T}_j^a, j = 1, \cdots, N, y_j \equiv 0$;

② for $T_j^L > \hat{T}_j^a, j = 1, \cdots, \bar{N}$,

$$\begin{cases} if\ j \le Q_j^a, & then\ y_j = 1; \\ if\ j > Q_j^a, & then\ y_j = 0. \end{cases}$$

In summary, after the values of all decision variables $y_j, j = 1, \cdots, N$ are determined, p_0 is a constant, and $p_0 > 0$. The *P-II* model can be solved by the Dinkelbach algorithm, which constructs an auxiliary problem with parameters that have the same optimal solution as the original model and then solves it by an iterative method. The procedure of the algorithm is as follows:

Let

$$f_1(\boldsymbol{x}) = \sum_{i=1}^{M} p_i x_i + p_0 \tag{28}$$

$$f_2(\boldsymbol{x}) = \sum_{i=1}^{M} q_i x_i + q_0 \tag{29}$$

Then, *P-II* can be expressed in the following form (denoted as *P-III*):

$$P\text{-}III\begin{cases} max\ f(\boldsymbol{x}) = \dfrac{f_1(\boldsymbol{x})}{f_2(\boldsymbol{x})} & (30) \\ \boldsymbol{x} \in \boldsymbol{S} & (31) \end{cases}$$

where

$$\boldsymbol{S} = \{\boldsymbol{x} | \boldsymbol{x} \in \{0, 1\}^M, \sum_{i=1}^{M} x_i = Q^v, 1 \le Q^v \le M\}$$

is the feasible domain and satisfies

$$f_1(\boldsymbol{x}) > 0, \quad f_2(\boldsymbol{x}) > 0.$$

We construct an auxiliary model (denoted as *P-IV*) with the same optimal solution as *P-III* with the following parameters (set as λ):

$$P\text{-}IV\begin{cases} G(\lambda) = max\ g(\boldsymbol{x}) = f_1(\boldsymbol{x}) - \lambda f_2(\boldsymbol{x}) & (32) \\ \boldsymbol{x} \in \boldsymbol{S} & (33) \end{cases}$$

Fig.3 shows the procedure of the Dinkelbach algorithm.

The auxiliary model *P-IV* can be solved quickly by the greedy method as follows:

From (32), we can get

$$\begin{aligned} G(\lambda) &= max\ g(\boldsymbol{x}) = f_1(\boldsymbol{x}) - \lambda f_2(\boldsymbol{x}) \\ &= (p_0 - \lambda q_0) + \sum_{i=1}^{M} (p_i - \lambda q_i) x_i \end{aligned} \tag{34}$$

Let

$$\begin{aligned} w_0 &= p_0 - \lambda q_0, \\ w_i &= p_i - \lambda q_i, \quad i = 1, \cdots, M \end{aligned}$$

then *P-IV* can be expressed as

$$P\text{-}V\begin{cases} G(\lambda) = max\ g(\boldsymbol{x}) = w_0 + \sum_{i=1}^{M} w_i x_i & (35) \\ \boldsymbol{x} \in \boldsymbol{S} & (36) \end{cases}$$

For *P-V*, we can set

$$w_{i_1} \ge w_{i_2} \ge \cdots \ge w_{i_m} > 0, \quad 1 \le m \le Q^v$$

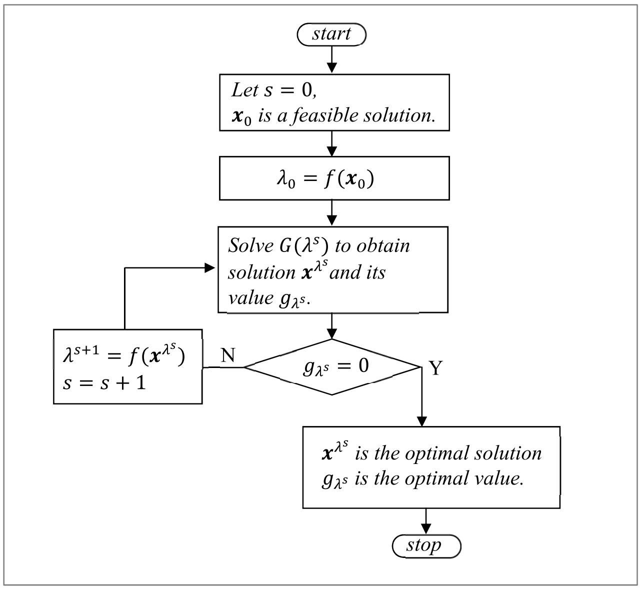

FIGURE 3. Dinkelbach algorithm flowchart.

Obviously for those w_{i_j} that are less than or equal to 0, there must be $x^*_{i_j} = 0$, and the optimal solution of *P-V* is

$$\boldsymbol{x}^* = \{x_{i_j} = 1, j = 1, \cdots, m; x_{i_j} = 0, j = m+1, \cdots, M\} \tag{37}$$

It can be proven that the Dinkelbach algorithm requires $O(log(Mv))$ iterations in the worst case when solving *P-II*, where $v = \max\{max\,|p_i|\,, \max|q_i|\,, 1\}\,, i = 1, \cdots, M$. Therefore, this algorithm meets the actual calculation needs.

IV. MARITIME SEARCH CASE

A. MODEL EXAMPLE

Assume that the search region is 2000 $nmile^2$ and that there are 15 vessels (of which one is located in the search area and the other 14 vessels are located around the search area) and 5 aircraft available for a joint aeronautical and maritime search. Table 3 shows each vessel's initial distance, maximum speed, and search capability. Table 4 shows each aircraft's initial distance, maximum speed, maximum endurance, and search capability.

The search facility data are inserted into the *P-I* model, and the results are shown in Table 5.

The calculation results indicate that there are 37 feasible and optimal search actions, namely:

(1) 15 kinds of vessel selection schemes when no aircraft join the search action;
(2) 10 kinds of vessel selection schemes when only one aircraft joins the search action;
(3) 6 kinds of vessel selection schemes when two aircraft join the search action; and
(4) 6 kinds of vessel selection schemes when three aircraft join the search action.

TABLE 3. Vessels for the search.

Vessel no.	Initial distance $D_i^v(nmile)$	Max. speed $\hat{V}_i^v(kn)$	Search capability $A_i^v(nmile^2/h)$
1	0	8	9
2	21	10	12
3	22	33	50
4	25	12	24
5	26	31	56
6	69	12	21
7	75	21	42
8	77	17	25
9	87	16	21
10	88	13	24
11	92	15	19
12	93	16	27
13	95	21	47
14	97	22	58
15	99	23	62

TABLE 4. Aircraft for the search.

Aircraft no.	Initial distance $D_j^a(nmile)$	Max. speed $\hat{V}_j^a(kn)$	Endurance $T_j^L(h)$	Search capability $A_j^a(nmile^2/h)$
1	21	155	4.26	180
2	35	175	5.25	220
3	225	135	3.44	150
4	412	155	4.26	180
5	717	175	5.25	220

Among 37 kinds of schemes, the one that takes the minimum time to complete the search coverage is dispatching three aircraft (Nos. 1, 2, and 3) and six vessels (Nos. 1, 2, 3, 4, 5, and 7), with time consumption of approximately 4.05 h.

Every vessel that has an opportunity to take part in the action must arrive at the scene before the other search facilities (vessels or aircraft) finish the search task. Table 6 lists each vessel's time required to rush from its initial position to the search region. When no aircraft are involved in the operation, all vessels have the opportunity to participate in search activities. When an aircraft is dispatched in the operation, the 11th vessel (No. 6), the 12th vessel (No. 9), the 13th vessel (No. 10), the 14th vessel (No. 11) and the 15th vessel (No. 12) take 5.75 h, 5.44 h, 6.77 h, 6.13 h, and 5.81 h, respectively, to arrive at the region, and the entire search operation takes only 5.36 h; therefore, a maximum of ten vessels can participate in this search. Similarly, if two aircraft are dispatched, a maximum of six vessels can participate in

TABLE 5. Optimal search scheme.

Q^a	Q^v	T (h)	Join search action: Vessel no.	Join search action: Aircraft no.	Have opportunity to join search action: Vessel no.	Have opportunity to join search action: Aircraft no.
0	1	36.55	5	none	1,2,3,4,6,7,8,9,10,11,12,13,14,15	1,2,3
	2	19.61	5,15	none	1,2,3,4,6,7,8,9,10,11,12,13,14	1,2,3
	3	13.97	3,5,15	none	1,2,4,6,7,8,9,10,11,12,13,14	1,2,3
	4	11.52	3,5,14,15	none	1,2,4,6,7,8,9,10,11,12,13	1,2,3
	5	10.27	3,5,7,14,15	none	1,2,4,6,8,9,10,11,12,13	1,2,3
	6	9.41	3,5,7,13,14,15	none	1,2,4,6,8,9,10,11,12	1,2,3
	7	8.90	3,4,5,7,13,14,15	none	1,2,6,8,9,10,11,12	1,2,3
	8	8.60	3,4,5,7,8,13,14,15	none	1,2,6,9,10,11,12	1,2,3
	9	8.39	1,3,4,5,7,8,13,14,15	none	2,6,9,10,11,12	1,2,3
	10	8.19	1,2,3,4,5,7,8,13,14,15	none	6,9,10,11,12	1,2,3
	11	8.04	1,2,3,4,5,7,8,12,13,14,15	none	6,9,10,11	1,2,3
	12	7.91	1,2,3,4,5,7,8,9,12,13,14,15	none	6,10,11	1,2,3
	13	7.81	1,2,3,4,5,6,7,8,9,12,13,14,15	none	10,11	1,2,3
	14	7.74	1,2,3,4,5,6,7,8,9,11,12,13,14,15	none	10	1,2,3
	15	7.70	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15	none	none	1,2,3
1	1	7.90	5	2	1,2,3,4,6,7,8,9,10,11,12,13,14,15	1,3
	2	6.73	3,5	2	1,2,4,6,7,8,9,11,12,13,14,15	1,3
	3	6.32	3,5,15	2	1,2,4,6,7,8,9,11,12,13,14	1,3
	4	6.04	3,5,7,15	2	1,2,4,6,8,9,12,13,14	1,3
	5	5.83	3,4,5,7,15	2	1,2,6,8,9,12,13,14	1,3
	6	5.66	3,4,5,7,14,15	2	1,2,8,9,13	1,3
	7	5.56	1,3,4,5,7,14,15	2	2,8,9,13	1,3
	8	5.47	1,3,4,5,7,13,14,15	2	2,8,9	1,3
	9	5.40	1,2,3,4,5,7,13,14,15	2	8	1,3
	10	5.36	1,2,3,4,5,7,8,13,14,15	2	none	1,3
2	1	4.79	5	1,2	1,2,3,4,7,8,13,14,15	3
	2	4.35	3,5	1,2	1,2,4,7,15	3
	3	4.25	3,4,5	1,2	1,2,7	3
	4	4.17	1,3,4,5	1,2	2,7	3
	5	4.12	1,2,3,4,5	1,2	7	3
	6	4.08	1,2,3,4,5,7	1,2	none	3
3	1	4.73	5	1,2,3	1,2,3,4,7,8,13,14,15	none
	2	4.31	3,5	1,2,3	1,2,4,7,15	none
	3	4.21	3,4,5	1,2,3	1,2,7	none
	4	4.13	1,3,4,5	1,2,3	2,7	none
	5	4.09	1,2,3,4,5	1,2,3	7	none
	6	4.05	1,2,3,4,5,7	1,2,3	none	none

TABLE 6. Time required by vessels rushing to sea region for search.

Vessel no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Rush time/h	0.00	2.10	0.67	2.08	0.84	5.75	3.57	4.53	5.44	6.77	6.13	5.81	4.52	4.41	4.30

TABLE 7. Aircraft endurance and round-trip time.

Aircraft no.	Endurance $T_j^L(h)$	Round-trip time $\hat{T}_j(h)$	Meets prerequisites
1	4.26	0.27	yes
2	5.25	0.40	yes
3	3.44	3.33	yes
4	4.26	5.32	no
5	5.25	8.19	no

the search, and if three aircraft are dispatched, a maximum of six vessels can be involved.

Each aircraft must meet the prerequisite that it has to be able to fly back and forth within its maximum endurance to participate in the search operation. As shown in Table 7, only aircraft Nos. 1, 2, and 3 can participate in this action; therefore, at most three aircraft can participate.

B. ANALYSIS RESULTS

The above 37 feasible and optimal search schemes can be obtained by solving the *P-I* model. Among these schemes, as the number of search facilities participating in the operation increases, the time required to complete the search coverage gradually decreases, and the extent of this reduction shrinks. The reason for this phenomenon is that as the search operation progresses, the remaining area to be searched accounts for the proportion of the total area of the search region continuously decreasing, resulting in the fact that facilities newly joining the search contribute increasingly less to the entire operation. By analyzing the changes in the time required to complete search coverage under the constraints of different quantitative search facilities, it is helpful for search decision makers to select an optimal plan that takes the least time and uses the fewest search facilities.

First, we analyze the change in the time required to complete the search coverage with different vessel quantities when the number of aircraft remains unchanged. When three aircraft are dispatched to participate in the operation, the search coverage can be completed in the shortest time. Then, there are six vessel selection schemes, that is, one to six vessels are selected to participate in the operation. The time required for these six schemes is shown in Fig.4.

Table 8 compares the time required for two adjacent vessel quantities. For two vessels, it takes approximately 6 *min* longer to participate in the action than for three vessels, approximately 11 *min* longer than for four vessels, approximately 13 *min* longer than for five vessels, and

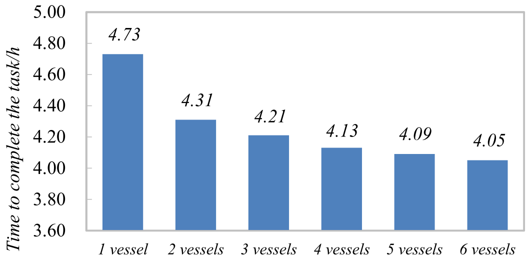

FIGURE 4. Action time required comparison among schemes with different vessel quantitative constraints.

TABLE 8. Time difference among different vessel schemes.

Vessel quantity	Time difference $\Delta T_{min}/min$
1~2	25
2~3	6
3~4	5
4~5	2
5~6	2

approximately 15 *min* longer than that for six vessels. Table 6 shows that it takes 2.08 *h* for the third vessel (No. 4) to rush to the search site, and the actual time it spends participating in the search operation is 1.97 *h*; it takes 2.10 *h* for the fifth vessel (No. 2) to rush to the search site, and the time it spends participating in the search operation is 1.95 *h*; and it takes 3.57 *h* for the sixth vessel (No. 7) to rush to the search site, and the actual time it spends participating in the search operation is 0.48 *h*. The time required for the above vessels to rush to the search site is longer than the actual search operation time in the search region. Fig.5 shows the workloads (covered areas) of different quantities of search facilities. Although the fourth vessel (No. 1) is already inside the search region when the search starts, its search area is approximately 37 $nmile^2$, accounting for only 1.8% of the total area. The third vessel (No. 4) can search approximately 47 $nmile^2$, accounting for only 2.4% of the total area. The fifth vessel (No. 2) can search approximately 23 $nmile^2$, accounting for only approximately 1.2% of the total area. The sixth vessel can search approximately 20 $nmile^2$, accounting for only approximately 1.0% of the total area. Since the vessels participating in the search operation need to rush to the search site at full speed and the speed is closely related to the vessel's fuel consumption, usually in a cubic relationship, passing vessels are often called into service, and their costs will be higher due to delayed shipping schedules. The economic cost of each additional vessel involved in the search operation is very high. By analyzing the relationship between the time required for each search action plan (benefit) and the total number of vessels participating in the operation (cost), we can exclude vessels that do not contribute much to the search operation when the search time requirements are not extremely urgent. Therefore, it is reasonable to increase the

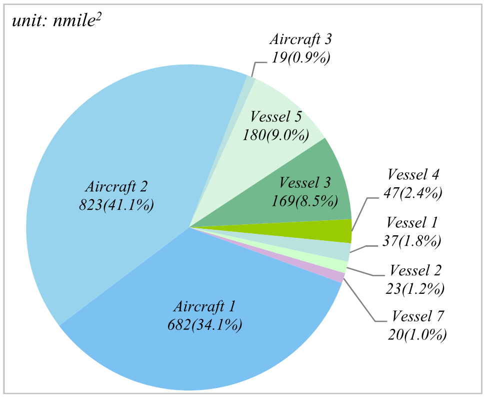

FIGURE 5. Search workload with different search facilities.

search operation time appropriately to save huge search costs. In this example, it is ideal to select two vessels to participate in the action, which takes only approximately 15 *min* longer than when six vessels participate in the action.

Second, we analyze the change in the time required to complete the search coverage with different aircraft quantities when the number of vessels remains unchanged. When six vessels are dispatched to participate in the operation, there are four aircraft selection schemes, that is, dispatching zero to three aircraft to participate in the operation. The time required for these four options is shown in Fig.6(a). The time to dispatch one aircraft to participate in the operation is 3.75 *h* less than that of the scheme without using the aircraft. Therefore, the use of aircraft to participate in the search can greatly shorten the entire search operation. Similarly, as the quantity of participating aircraft increases, the magnitude of this reduction gradually decreases. It takes 1.58 *h* more to dispatch one aircraft than to dispatch two aircraft to participate in the operation; to dispatch two aircraft to participate in the operation takes only about 2 *min* longer than to dispatch three aircraft, and the workload (area covered) of the third aircraft (no. 3) accounts only for 1.0% of the total area in the entire operation. Considering that marine accidents often occur in extremely poor weather and rough sea conditions, the aircraft also face a huge safety threat. Therefore, when comprehensively considering meteorological and sea conditions, the aircraft's ability to withstand a harsh environment, and the aircraft's workload, it is reasonable to extend the search action time to reduce the number of aircraft used. In this example, it takes only 2 *min* more to dispatch two aircraft to participate in the action than it takes to dispatch three aircraft. Therefore, two aircraft should be selected to participate in the action when the search time requirements are not extremely tight.

By comparatively analyzing the time consumption in the case of different vessel and aircraft quantitative constraints, superior facilities (contributing a larger workload for the entire search operations) can be found and inferior facilities (contributing little workload to the entire search operation) can be excluded. In the above example, dispatching two aircraft (Nos. 1 and 2) and two vessels (Nos. 3 and 5) to take part

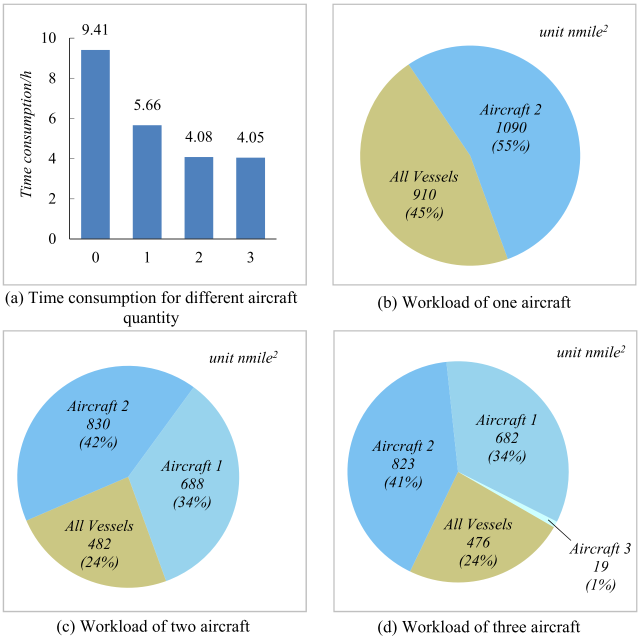

FIGURE 6. Analysis of schemes with different aircraft quantitative constraints.

in the search action is ideal, taking just 4.35 *h*, approximately 18 *min* more than the shortest scheme (4.05 *h* taken by three aircraft and six ships). The workload of each facility is shown in Fig.7.

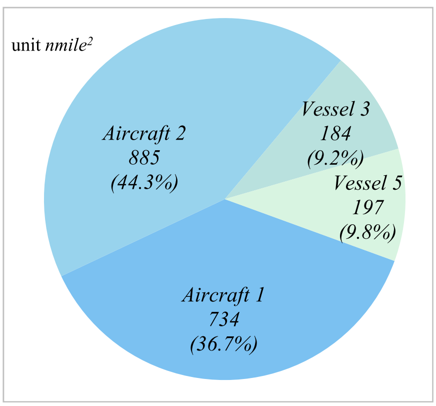

FIGURE 7. The workload of two vessels and two aircraft.

V. CONCLUSION AND FUTURE WORK

The facility selection problem of joint aeronautical and maritime search is analyzed and solved in this article. An optimal model for search effort selection is established. The model fully considers factors including the area of the sea region to be searched, the maximum speed of search vessels and aircraft, and the search capabilities and initial distances of the vessels and aircraft as well as each aircraft's endurance. By introducing 0-1 decision variables, the search facility selection can be judged and optimized directly and effectively. By analyzing the optimal results with different vessel and aircraft quantities, and considering the relationship between search coverage time and the number of search facilities (cost), an economic and feasible search scheme can be produced that provides commanding officers with mathematical model support for scientific decision-making in a joint aeronautical and maritime search at sea.

In future work, we can focus on solving how to select available search efforts by establishing a corresponding model. This question requires comprehensive consideration of the environmental conditions of the sea area to be searched (wind, waves, currents, air temperature, water temperature, etc.) and the actual situation of the person in distress (the nature of the distress, the time of the distress, etc.) combined with the conditions of the search facilities (initial positions, speeds, types of ship (or aircraft), tonnage, maneuverability, wind resistance, shipping cargo, etc.) as well as an expert knowledge base. However, choosing the best search facilities is only one step in the maritime search operation. The next step is to determine specific search subareas for these search facilities [41]. The region of maritime search operations is usually represented by polygons. A region partition algorithm suitable for maritime search should be studied so that these search facilities can coordinate operations in their respective search subareas

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