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Limited Stop Services Design Considering Variable Dwell Time and Operating Capacity Constraints

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ABSTRACT This article proposes an optimization model to set frequencies, vehicle capacities, required fleet and the stops serving each route along a transit corridor which minimize the total user and operating costs. The optimization problem is solved by applying the ''Black Hole'' algorithm, which imitates the movement of stars (solutions), towards a black hole (Best solution). The main contributions of the model are based on incorporating variable dwell times depending on bus stop demand not only to the passenger perceived journey times but also to the bus cycle times and on considering capacity constraints in both vehicles and bus tops. This led to a more accurate and realistic operating times and user perceived journey times. The application of the model to two case studies and the sensitivity analysis carried out demonstrate that for low levels of demand, constant dwell times can be assumed but being these times different between the different stops of the corridor, considering their demand. However, with high level of demand the difference found in operating costs and travel times strongly recommend incorporating variable dwell times in the model in order to achieve a more realistic design of transit corridor strategies.

INDEX TERMS Limited stop services, bus rapid transit, dwell time, express services, transit corridor, black hole algorithm.

I. INTRODUCTION

Large public transport corridors are being increasingly used in cities as a more economical and/or complementary alternative to mass transit systems. Specifically, bus services based corridors like the successful cases of the RIT in Curitiba (Brazil) and TransMilenio in Bogotá (Colombia) have been growing and have spread around the world [1], providing very high transport capacities and high commercial speeds. These systems are known as Bus Rapid Transit (BRT) and are characterized by having dedicated and segregated infrastructure (bus lanes, platform stations design based at bus stops), operating facilities (pre-payment at stations, signal priority control, fully implemented ITS) and high operating frequencies, are integrated within the existing transport systems in order to optimize the service provided to users. One of the advantages of this type of system is its flexible operation compared with track-based modes (metro and light railway) along with a much lower infrastructure cost. However, different systems have failed due to badly designed operating strategies which resulted in longer journey times for passengers or were too expensive to run because of the high number of vehicles required. Once the corridor has been defined and all the stops located along it, its correct functioning depends on the design of the services running along it, in terms of operating strategies, the sequence of the stops being served (full stop, skip-stop, deadheading, etc.), frequencies and type/size of vehicle [2]. It has been demonstrated that the probability of choosing skip-stop service increases with the increase in travel distance and the decrease in in-vehicle time [3].

The optimal design of these corridors is a subject that has been amply addressed in the bibliography. The first

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studies to propose methodologies for optimizing more than one operating variable along transport corridors date from the end of the 1970s and start of the 1980s, with research to optimize frequencies and limited stop pattern [4]–[7], however, with the arrival of BRT systems and their large scale introduction, the number of studies has increased over recent years. Most of analysed work has continued to concentrate on the same two variables (sequencing of stops and limited stop pattern). Thus, Sun *et al.* [8] applied an optimization model for three services: all-stop service, express service and zonal service, minimizing an overall cost function for users and operation, although they did not consider constraints on vehicle capacity. Later, Leiva *et al.* [9] proposed a model which also minimized an overall cost function (user and operation), with different variants depending on whether vehicle capacity or transfer constraints were being considered. They made an interesting sensitivity analysis on the influence of penalizing transfers, strategy type (dead-heading, express, super express, etc.), vehicles being used and the demand profile along the corridor in the final solution. This model formed the basis for Larrain *et al.*, [10] who proposed an optimization model (limited stop services and frequencies) for zonal services which serviced all the stops of a determined zone at the start and the end of the corridor, and skipped all the stops in between. In all cases limited stop services tended to improve the performance of the corridor. In this sense, García Albarracín and Jaramillo-Ramírez [11] demonstrated by applying an optimization model that the required capacity can be significantly reduced that splitting an isolated high frequency all-stop service into one limited stop service and an all-stop service with the same fleet size.

A different approach, not applied to a corridor, was proposed by Szeto and Jiang [12], suggesting a bi-level optimization model for route and frequency design on a network. Their model included the minimization of transfers in the upper level objective function, and an optimal strategy assignment model on the lower level, solved using a ‘‘hybrid artificial bee colony’’ algorithm. This research highlighted the importance of considering capacity constraints in the passenger assignment model rather than as a constraint on the objective function (as proposed in [8, 13, 14]), in spite of significantly complicating the solution of the assignment problem and reducing the efficiency of the solution algorithms. Modelling congestion as a constraint can result in unrealistic assignments as the user does not consider the extra waiting time in their choice of route or optimal strategy [11] or even distort the solution towards a system optimum rather than a user equilibrium [14].

An important advance in the problem of design strategies for corridors was the work of Soto *et al.* [15]. In this research the authors propose a complete methodology based on various sub-models to solve the problem of designing service stops and frequencies proposing assignment models with and without transfers based on deterministic and stochastic equilibrium. Furthermore, it solves the problem of generating initial solutions for the pre-design of the service to be optimized using a heuristic approach. In this methodology, the model finds the required optimum from a group of possible services after the authors rejected those services with a frequency of fewer than 1.5 buses/hour. Although the approach was very thorough, the authors recommend that a review must be performed on the model's solution in order to reorganize very similar services or reject others. This work served as a reference for Torabi and Salari [16], who also proposed a bi-level approach. They used a metaheuristic analysis to compare several of the existing algorithms (genetic, tabu search, etc.) to avoid localised optimals when generating express services. A heuristic analysis was applied to optimize frequencies and an assignment model on the lower level based on the concept of hyperpaths [17] without capacity constraint served as a constraint on the objective function of the upper level. According to the authors, the bi-level optimization method is a frequently used approach to the problem because of its hard-NLP nature, as direct optimization (solver-based) assumes simplifications which may limit the model's applicability to real situations. An example of the solver-based approach has recently been reported in the work of Wang *et al.* [18], who solve the problem of stop sequencing and frequencies by considering the variable/constraint of the available fleet size (given a fixed vehicle type). They incorporated the effect of capacity (as a constraint on the objective function) and transfer constraints. This approach was different in that it proposes a linearization of the objective function and thereby addresses the problem by obtaining an global optimum. However, given the complexity of the problem, the model is only applied to a very simplified network.

As can be seen, most of these recent articles present methodologies with a similar focus. They may propose: (i) a bi-level optimization model with an objective function on the upper level (usually based on the minimization of a user and operator cost function) and they use a passenger assignment algorithm for the public transport service (deterministic or stochastic) on the lower level (with or without the consideration of transfers and capacity constraints) or (ii) the optimization of a unique cost function incorporating the capacity constraint as a problem constraint. Alternative approaches has also proposed simulation based optimization techniques [19]. They are applied to real or quasi-real corridors after their application to a test corridor and they may or may not consider the presence of transfers in their models. However, none of these works appear to consider the dwell-time variable either at the stops or in the overall perceived journey time as part of the route choice during the assignment process, nor as part of the journey time to calculate the lines' turnaround time.

The omission of these items leads to an inexact estimation of both the user costs (perceived total travel time) and the operating costs. The dwell time variable and its importance, as well as the effect of congestion not only on the passengers but also on the vehicle journey times has already been highlighted as a necessary contribution to the state of the art [10], [15], [16], [18], [20] and their importance in the estimation of passenger assignment and required fleet size has already been reported by Alonso *et al.* [21] and Lam *et al.* [22] respectively. In this sense the proposed model in this article fills these gaps highlighted in the literature.

The first article to propose an assignment model taking these factors into account is that of Larrain and Muñoz [23]. Later, Yu *et al.* [24] included a penalty function to model dwell times depending on the on-board congestion, however, these times did not depend on the real demand at each stop and the authors did assume an average value to be penalized. Only the research carried out by Alonso *et al.* [21] incorporates demand dependent dwell times in the transit assignment problem, also including capacity constraints both in the vehicles and at the stops, due to bus bunching effects. This approach of variable dwell times is similar than the one followed later by Tang *et al.* [25] in an optimization model to find fares and operational strategies in a transit network without considering congestion effects nor transfers.

The proposed model contributes to the state of the art forward by designing a single solution containing all the main operating variables that affect public transport corridors such as: frequency, vehicle capacity, required fleet size and the stops to service on each line. This will complement existing research and add the following contributions: (i) the first, and in our view, the most important contribution, is based on the effect of boarding and alighting times depending on demand, not only on user journey times, but also on the turnaround (cycle) times of the lines which affect the route choice process and the required fleet, respectively; (ii) the proposed design also incorporates the effect of congestion and capacity constraints on the passenger assignment model, modelling the capacity constraint with an increasing function of waiting time [26], the discomfort perceived by the user due to the high vehicle occupancy [27] and the congestion found at the stops themselves due to vehicle saturation [21], all of which are incorporated into the perceived journey times and the cycle times; (iii) the proposal of optimization in a single model addressing stop sequencing along the lines, frequencies, fleet size and vehicle typology and (iv) a demonstration of the method's applicability to a real corridor with real demand (many-to-many).

A metaheuristic algorithm is used to solve the problem, which, unlike in the work of Wang *et al.* [18] does not guarantee the optimal solution to the problem. However, tests of the algorithm are provided which do guarantee a good performance in this type of NLP-hard problems.

The article is structured in the following way: after analysing the state of the art and justifying the contribution being made, section II explains the model being implemented in this work; section III provides details of the metaheuristic algorithm being used and the results are presented in section IV. A sensitivity analysis is then performed and discussed in section V and the article finishes with the main conclusions drawn and suggestions for future work (section VI).

II. PROPOSED MODEL

A. INTRODUCTION AND PREVIOUS HYPOTHESES

The problem of designing an operational strategy along an urban public transport corridor can be conceived as a cooperative game on two levels or a Stackelberg Game [15]. The following players take part in this game: the planner, who determines the operational characteristics of the corridor and its users, who tend to minimize their generalized journey costs, producing a flow pattern for the corridor. Included in the generalized journey cost are all the elements that an individual considers important when making their journey.

On the first level, the planner defines the operation of the services, establishing the type of service (stops to be served), frequencies and vehicle types being used in the provision of each service (physical design problem). The users of the system are found on the second level, they react to the structure of services being provided and generate a flow profile along the proposed public transport services.

The objective of the problem to be solved is the minimization of the total cost involved in the transport system, which will include the costs of providing the services (perceived by the public transport operators) and the costs of making the journey (perceived by the users).

For the solution to the proposed problem we assume that a public transport corridor is served by a group L of bus lines, and that the stops ($p = 1,\ldots,P$) are able to be served by each line l. The model assumes homogeneous fleet for each line: each line l can only be operated by one type of bus k. A fixed and known demand is also considered which is defined by a trip matrix between stops. The network and route choice model proposed by De Cea and Fernández [26] will be used.

The approach followed is based on a number of lines defined by the planner; however the proposed methodology can be integrated into other research which solves this particular problem [10], [16], [20]. Finally, the notations are summarized in Table 1.

B. UPPER LEVEL: COST FUNCTION

The upper level objective function is based on a cost structure which distinguishes the costs of the public transport system (Co) (equation 2) operating the corridor and the public transport user cost (Cu) (Equation 3). This provides the cost function that needs to be minimized (equation 1):

$$\min z = Co + Cu \tag{1}$$

With:

$$Co = (CK + CF + CP) \cdot CI \tag{2}$$

$$Cu = TWT \cdot \Phi_w + TIVT \cdot \Phi_v + TTT \cdot \Phi_T \tag{3}$$

Running costs (CK) are modelled as follows:

$$CK = \sum_{l} \sum_{k} \left(LC_l \cdot f_l \cdot 60 \cdot Ck_k \cdot \delta_{k,l} \right) \tag{4}$$

Personnel costs (CP) and Fixed costs will directly depend on the required fleet size and the type of vehicles used. Note

TABLE 1. List of Notation.

p	bus stop
P	Number of total bus stops in the corridor
CK	Running costs, depending on the km covered by each type of bus k (€/h)
CF	Fixed costs associated to the costs of acquiring and maintaining the vehicles, according to type (€/h)
CP	Driver costs (€/h)
CI	Indirect running costs, assumed to be a percentage of the sum of the above costs
TWT	Total user waiting time in the system (h), obtained from the passenger assignment on the lower level
$TIVT$	Total passenger in-vehicle journey time (h), obtained from the passenger assignment on the lower level
TTT	Total transfer time (h), , obtained from the passenger assignment on the lower level
Φ_w, Φ_v, Φ_T	Weighting of waiting times, journey times and transfer times, respectively (€/h)
$\delta_{s,w}$	Binary variable taking a value of 1 if route section s belongs to the shortest path of the origin destination pair w and 0 in other cases.
LC_l	Length of route l turnaround (cycle), in this case equal to the total length of the corridor LC (km)
f_l	Frequency of the line l (b/min)
Nb_l	Number of buses of the line l
CK_k	Unit cost per km of a type k bus *(€/km).*
$\delta_{k,l}$	Dummy variable taking a value of 1 if bus type k is assigned route l and 0 in other cases
tc_l	Cycle time of line l (min).
C_p	Hourly unit cost of personnel per bus (€/h·b)
Cf_k	Fixed cost of bus type k (€/h·bus).
$\delta_{l,p}$	Binary variable taking a value of 1 if line l serves stop p and 0 in other cases.
td	Total stop time at terminals (hours)
$t_{l,p}$	Time spent by a bus on line l at stop p for passenger alighting and boarding and complementary operations
t_b	Average passenger boarding time (min/pax)
t_a	Average passenger alighting time (min/pax)
$B_{l,p}$	Passengers on line l with origin (boarding) at stop p
$A_{l,p}$	Passengers on line l with destination (alighting) at stop p
$t_{o,c}$	Time spent opening and closing doors and rejoining traffic (min)
d_p	Average delay of buses in line waiting to service stop p
s	Route section
f_l^*	Effective frequency of line l
$v_{l,p}$	Passengers on line l at stop p (see Alonso et al. [21])
C_k	Capacity of bus type k
α, β, ζ	Parameters to be calibrated
$v_{l,a}$	Passengers on line l as it passes along link a
$dt_{l,p}$	stoppage time of line l at stop p
w	Origin – destination of a trip (bus stop origin – bus stop destination)
Tw	Number of trips of the origin-destination pair w.

that because interlining for the vehicles is not assumed in this approach, fleet size must be an integer number and it will be rounded up in order to satisfy the required frequency:

$$CP = C_p \cdot \sum_l Nb_l \rightarrow C_p \cdot \sum_l round.up\left(tc_l \cdot f_l\right) \quad (5)$$

$$CF = \sum_l \sum_k Nb_l \cdot Cf_k \cdot \delta_{k,l} \quad (6)$$

In equations (5) and (6) the turnaround time or cycle time is calculated as the running time plus the time spent at each stop on each line. The running time between the stops, on a link or stretch of road a along the corridor, is defined exogenously, and will be a function of the distance covered, assuming an average given speed:

$$tc_l = \sum_a tr_{l,a} + \sum_p t_{l,p} \cdot \delta_{l,p} + td \quad (7)$$

The time spent by a bus on line l at stop p for passenger alighting and boarding and complementary operations ($t_{l,p}$), can be expressed as:

$$t_{l,p} = \left(max\left(B_{l,p} \cdot t_b; A_{l,p} \cdot t_a\right)\right) / \left(f_l \cdot 60\right) + t_{o,c} + d_p \quad (8)$$

The average delay of buses in line waiting to service stop p, which, according to Alonso *et al.* [28] can be modelled by a flow delay function depending on the saturation at the stop:

$$d_p = a \cdot exp\left(b \cdot \sum_l \left(f_l \cdot 60 \cdot \delta_{l,p}\right)/BSC_p\right) \quad (9)$$

where a and b are parameters to be calibrated and BSCp is the capacity of stop p according to the methodology suggested by the Transit Capacity and Quality of Service Manual (TCQSM) [29].

The objective function may be subjected to the operational and budgetary constraints which need to be considered in each case:

$$\delta_{k,l} \in \{0, 1\} \quad (10)$$

$$\delta_{l,p} \in \{0, 1\} \quad (11)$$

$$\sum_k \delta_{k,l} = 1, \forall l \quad (12)$$

$$C_{k,} \in [30, 60, 90, 120, 150, 180] \quad (13)$$

$$\sum_l \left(Nb_l \cdot \delta_{k,l}\right) \leq Nb_k, \quad \forall k \quad (14)$$

$$\sum_l \sum_k \left(C_k \cdot \delta_{k,l}\right) \cdot (60/h_l) \geq \max v_a \quad (15)$$

$$f_l \geq 0, \quad \forall l \quad (16)$$

$$\sum\nolimits_l \delta_{l,p} \geq 1, \quad \forall p \quad (17)$$

$$\delta_{l,p} = 1, \quad \forall l, p = 1, p = P \quad (18)$$

$$\sum\nolimits_l f_l \cdot \delta_{l,p} \leq BSC_p, \quad \forall p \quad (19)$$

$$Co \leq C \quad (20)$$

Constraints (10) and (11) define the binary nature of the assignment variables for vehicle type and stops to lines, respectively, already defined in Table 1; (12) restricts a homogenous fleet assignment per line; (13) defines the capacities of the vehicles being used in the model; (14) is applied where there is a constraint on the number of vehicles of each type; (15) guarantees the sufficient supply of the system for the given solution to satisfy the demand, although this is already implicitly considered in the lower level assignment model; (16) establishes the lower limit of the frequency values; (17) makes sure that all the stops are serviced by at least one line; (18) obliges all the lines to run the entire length of the corridor; (19) guarantees the capacity constraint of the vehicles at the stops and (20) represents the operator budgetary constraints. This research does not consider constraints (14) and (20).

C. LOWER LEVEL: CONGESTED TRANSIT ASSIGNMENT MODEL INCORPORATING DWELL TIMES

The lower level is based on an equilibrium assignment model for public transport corridors. The journey times are influenced by the time the buses spend at the stops due to passengers boarding and alighting and the effects of congestion in terms of discomfort and longer waiting times at stops, proposed by Alonso *et al.* [21], which takes its basis from the approach of De Cea and Fernández [26] and Larraín and Muñoz [23].

The model assumes that the users choose the sub-group that minimizes their total journey time (in the bus and waiting) from all the possible lines joining a given pair of stops. This sub-group of routes is grouped into what is known as a route sections, defined as a fictitious link with the origin stop as the origin node and the destination stop as the destination node of the link and the sub-group of lines (attractive lines) that service it ($l \in S$) [26]. Once the passenger boards the vehicle, the journey time to destination ETT^w will be equal to the sum of the expected travel time of each route section belonging the bet origin-destination path ETT_s^w which corresponds to the running time in the vehicle $TIVT_s^w$ plus the waiting time TWT_s^w:

$$ETT^w = \sum_s ETT_s^w \cdot \delta_{s,w} \rightarrow \sum_s \left(TWT_s^w + TIVT_s^w\right) \cdot \delta_{s,w} \quad (21)$$

The waiting time can be assumed to be equal to the inverse of the effective frequency along route section s:

$$TWT_s^w = 1/f_s^* \rightarrow 1/\sum_{l \in S} f_l^* \quad (22)$$

The effective frequency of line l (f_l^*) at each stop p, can be defined as:

$$f_{l,p}^* = f_l / \left(1 + v_{l,p} / \left(\sum_k \left(C_k \cdot \delta_{k,l}\right) \cdot f_l\right)^{\zeta}\right), \quad \forall p \quad (23)$$

The in-vehicle journey time can be split into running time between origin stop and destination stop w along route section s plus the time spent at the stops in between. Therefore, the expected in-vehicle journey time of the passengers between a pair of stops (origin-destination) w along route section s will be:

$$TIVT_s^w = \left(\begin{array}{l} \sum_{l \in s} \left(tv_i^w \cdot f_l^*\right) / f_s^* + \\ + \sum_{l \in s} \left(\sum_p dt_{l,p} \cdot \delta_{l,p}\right) \cdot f_l^* / f_s^* \end{array}\right) \quad (24)$$

In equation 24, tv_l^w is the perceived journey time, which includes bus waiting time queuing to service a stop as well as a "discomfort" term [21], [27], [28] depending on vehicle occupancy:

$$tv_l^w = \sum_a \left(tr_{l,a} + \sum_p d_p \cdot \delta_{l,p}\right) \cdot \left(1 + \alpha \left(v_{l,a} / \left(\sum_k \left(C_k \cdot \delta_{k,l}\right) \cdot f_l\right)\right)^{\beta}\right) \quad (25)$$

The stoppage time of line l at stop p ($dt_{l,p}$) follows a similar expression to equation 7:

$$dt_{l,p} = \left(max\left(B_{l,p} \cdot t_b; A_{l,p} \cdot t_a\right)\right) / f_l + t_{o,c} \quad (26)$$

The next step is to find the variational inequality which will provide the equilibrium flows in the route sections and, therefore, on each line:

$$E\bar{T}T\left(\bar{Y}^*\right) \cdot \left(\bar{Y}^* - \bar{Y}\right) \leq 0, \quad \forall \bar{Y} \in \Omega \quad (27)$$

where $\bar{Y}$ is any feasible flow vector in route sections (Y_s^w) for each origin destination pair w, $\bar{Y}^*$ represents the solution of equilibrium flows and Ω is the group of feasible solutions of the flow vector (feasible transit assignment flows). In the assignment process, we assume the same approach followed by Tang *et al.* [25] in which passengers would not transfer between vehicles operated by different strategies because of high transferring cost considered. In case of allowing transfers, the total transfer time can be computed as:

$$TTT = \left(\sum_s \sum_w Y_s^w - \sum_w T_w\right) \quad (28)$$

Access time is not considered as user cost since a fixed demand at each bus stop is considered, so this component would be a constant in the proposed formulation.

The problem is solved applying a sequential transit assignment algorithm [21] and the widely used method of successive averages (MSA).

Note that this assignment model not only considers the boarding and alighting times at stops, but also the capacity constraints on the vehicles and their effect on increasing waiting times, the effect of on-board discomfort due to congestion inside the vehicle and the delays caused by bus saturation at the stops themselves. All of which are very relevant factors from both the point of view of the users and the operators and never tackled together when addressing the problem of designing the operation of public transport corridors [15].

III. SOLUTION ALGORITHM

Given the combinatorial nature of the problem and considering the large search area, we propose the application of a "Black Hole" metaheuristic technique, proposed by Hatamlou [30]. This algorithm has been found as a very effective algorithm considering the few parameters needed and its simple structure [31], [32] and has been validated in various problems of Machine Learning [33], as well as highly complex problems such as "Set Covering" [34], "Solving Manufacturing Cell Design" [35], "Travelling Salesman" [36], industry applied problems such as "Critical Slip Surface of Soil Slope" [37] and reinforcing bridges using tied arches [38].

One of the advantages of using this algorithm is that only a parametric search must be performed to find a feasible population to define the searching space. For instance, a Genetic Algorithm could also be applied, but in that case more calibration parameters should be defined such crossing and mutation probabilities and type of crossover.

The Black-Hole algorithm is based on the phenomenon of black holes in space, which form when a large star collapses and any object which approaches the event horizon will be absorbed by the black hole and will disappear forever.

Its main characteristics are:

- It is based on a population of candidates (stars) which are randomly generated and distributed in the search area.
- The star which demonstrates the best performance is chosen as the black hole.
- At each iteration, if a star presents a better performance than the black hole then they exchange places.

While the stars move towards the black hole, a star could reach a position where it performs better than the black hole, in which case, they exchange places. The algorithm then continues with the black hole in its new position and the stars start to move towards this new location. At each iteration the stars join with the black hole, using Equation 29.

$$x_i^d(t+1) = x_i^d(t) + rand \cdot \left(x_{BH}^d - x_i^d(t)\right); \quad i = 1, 2, \ldots, N; d = 1, 2, \ldots, D \tag{29}$$

where $x_i^d(t)$ y $x_i^d(t+1)$ is the position of dimension d of star i at iteration t and $t+1$, respectively. x_{BH}^d is the position of the dimension d of the black hole in the search area, while *rand* is a random number at the interval [0,1], and N is the number of stars (candidates for the solution).

Algorithm 1 Black Hole

Objective Function: min or max $f(x)$, $x = (x_1, x_2, \ldots, x_N)$
Initialize population of N starts with random position in the searching space
$t = 0$
while $(t < MaxGeneration)$ or (stop criterion) **do**
  Evaluate each $(x_1, x_2, \ldots, x_N)$ *in the Objective Function*
  $BH = Best(x_1, x_2, \ldots, x_N)$
  for $i = 1 : N$ **do**
    $x_i = x_i + rand \cdot (x_{BH} - x_i)$
  end for
  $R = \frac{of_{BH}}{\sum_{i=1}^{N} of_i}$
  for $i = 1 : N$ **do**
    if $f(x_i) > f(BH)$ **then**
      swap(x_i, BH)
    end if
    if $(x_i - BH) \leq R$ **then**
      Move x_i *to a random position in the searching space*
    end if
  end for
  $t = t + 1$
end while=0

FIGURE 1. Black Hole Algorithm.

The stars can cross over the event horizon during their movement towards the black hole. Each star which reaches this radius will be absorbed by the black hole. Each time a star disappears in this way, another solution candidate (star) appears at a new random location in the search area and a new search begins. This process keeps the number of solution candidates constant, as well as opening up the exploration of the search area. The radius of the event horizon in the BHA is calculated by using Equation 30:

$$R = of_{bh} / \sum_{i=1}^{N} of_i \tag{30}$$

where R is the radius of the event horizon, of_{bh} is the value of the aptitude of the black hole, of_i is the value of the aptitude of star i and N is the number of stars. When the distance between a candidate and the black hole is lower than R, the candidate collapses, and a new candidate is created and randomly distributed in the search area. The functioning of this technique is summarised, according to the author, in Figure 1.

Therefore, the search algorithm itself has the task of introducing feasible solutions to the problem, thereby solving the problem of generating express services, given the number of lines fixed by the planner. This procedure is described in Figure 2.

Firstly, feasible solutions (stars) of frequencies (f), bus types (K) and stop pattern (p) per line are generated according a preliminary parametric tuning to define the initial population. These feasible solutions are introduced in the model and the passenger assignment problem is solved. The resulting values based on perceived travel times will be the input for the user costs calculation. For its part, running times will be used for estimating the turnaround (cycle) times for each line. These times are needed to estimate both the fleet size and the number of drivers which address fixed costs and personnel costs respectively to estimate operating costs. Once the user

Algorithm 2 Solution Algorithm

```
while d ≤ N do
    Initial feasible solutions generation (f, K, p).
    Run Transit assignment
    Travel Times estimation
    Fleet Size estimation
    Evaluate the Objective Function
end while
Get the Best Solution
while (t ≤ MaxGeneration) or (stop criterion) do
    while d ≤ N do
        Generate movement
        while solution != Feasible do
            Generate new movement
        end while
    end while
    Evaluate the Objective Function of new solution
end while
Return (Best Solution)
```

FIGURE 2. **Solution Algorithm.**

and operating costs are known, the value of the required objective function is estimated for each star. The algorithm performs a search to find the best solution (black hole) and based on the values obtained modifies the decision variables (*f, K, p*) vector of the stars. In this step, radius R explained in (30) allows the model to skip local optimum values and therefore controlling the exploration-exploitation balance. Once this step has been performed, the algorithm continues with the iterative process of evaluating the objective function of the population, finding the best solution and performing new movements while the defined stop criterion are not reached. At each iteration the star with the best value of the objective function is converted into a new black hole, being the previous one converted into a new star. The overall process can be seen in Figure 3.

IV. RESULTS

The model is initially applied to a smaller sized corridor, as presented in the work of Wang. et. al. [18], a corridor running only in one direction with 10 stops and with passenger flows only to nodes 6 and 10. The combinations of possible resultsfor both the assignment and the cost minimization are therefore low and drastically reduce the search area.

In order to validate the correct convergence of the model, it is then applied to a larger more complex case such as is that of Leiva et. al. [9], composed of 10 stops in each direction, with a bidirectional passenger flow between all the stops. The generated search area results in a long computing time, justifying the use of an incomplete technique in order to reach the optimal solution.

A. TEST CORRIDOR

The example proposed by Wang *et al.* [18] involves 10 stops and passenger flows in only one direction, where passengers only alight at stops 6 and 10. The demand for these stops is detailed in Figure 4.

The algorithm is initially tested by applying the simplest case which is to optimize a single line, stopping at all the stops (all-stop service), with a constant dwell time of 1 minute and a vehicle capacity fixed at 60 pax/bus, using the same user and operator unit costs (see Wang *et al.* [18]). Complete optimization models are known to solve problems with these characteristics in acceptable computing times with improved performance over metaheuristic methods.

Nevertheless, the implementation of Black Hole to this particular problem aims to evaluate the capacity of the metaheuristic technique in finding the overall optimal and its convergence.

The optimal result found by Wang *et al.* [18] was for a frequency of 9 buses/h, a required fleet size of 5 vehicles and a total cost of 3,318 \$. Once the cost function for the assignment of the optimal solution using our model was found to be identical, the model was applied without the fleet optimization component and, as it was an all-stop service, only the frequency was optimized. The algorithm was run 30 times and the overall optimal was reached on 19 occasions (63%) while the difference between the optimal solution and the one found previously was lower than 1.5% in 90% of the cases, finding the solution in less than 0.01 seconds. These results allowed us to check the algorithm performance in a simplified problem, which optimal value was known. With this initial validation process, the algorithm was then applied to a somewhat more complicated case with 3 lines, of which only one was all-stop.

The complete optimization was now left to run freely with the stops, frequencies and fleet type, however, in order to not differentiate excessively from the comparison model, dwell time was kept constant at 20 seconds per stop. In this case a problem appeared with a higher combinatory, so a parametric calibration was performed to find which values would be more suitable for the population and number of iterations used by the algorithm.

Ten experiments were performed with different combinations of populations and iterations to reach an average of 5042 attempts at the objective function. These initial experiments allowed the authors to compare the behaviour when the initial population were changed. The results were similar to those found in the bibliography for most of the combinatorial problems: the greater the population, the better the performance of the algorithm, but, in this case, very similar solutions were found (Figure 5). This preliminary analysis was used to to test the convergence of the algorithm and to perform a sensitivity analysis to find the best initial population in order to run the algorithm in the considered application.

After performing the parametric analysis, the algorithm was run for the 3 lines situation. The calculation time was 107 seconds using an Intel Xeon E5 with 24 Gb of RAM. Table 2 shows the difference in the results. This case is resolved by Wang *et al.* [18] using a model which considers transfers, meaning the results are not directly comparable with those obtained by our model in terms of cost.

When transfers are not considered the model tends to reduce the number of express services at a cost of decreasing

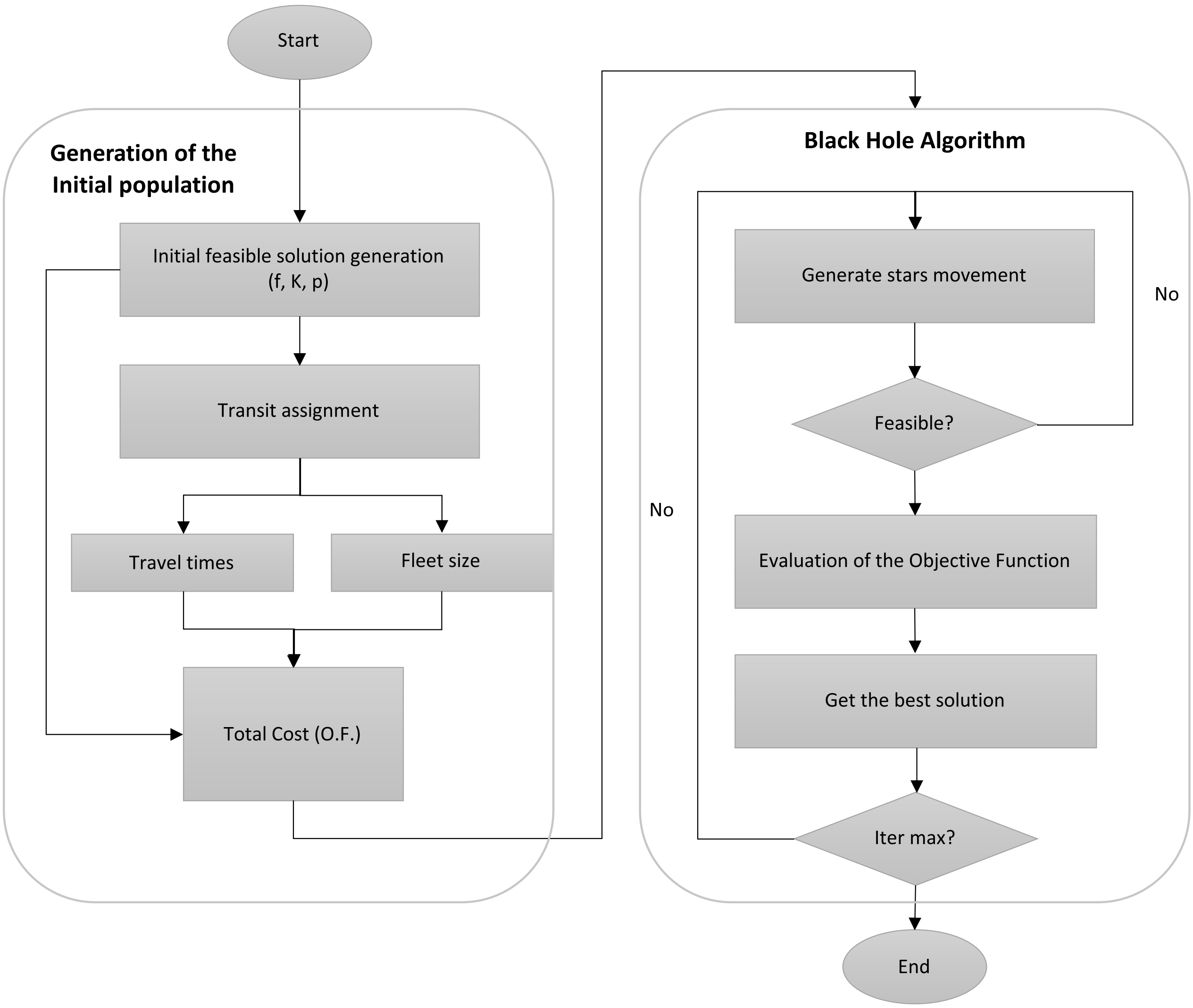


FIGURE 3. Solution Algorithm chart flow.

the frequencies, offering more effective frequencies at most of the stops along the corridor. Furthermore, the value of the objective function turned out to be lower than that found by Wang. et al [18]. A better solution was found partly because operating costs were reduced by considering a heterogenic fleet in the model.

B. AVENUE LOS PAJARITOS CORRIDOR (SANTIAGO DE CHILE)

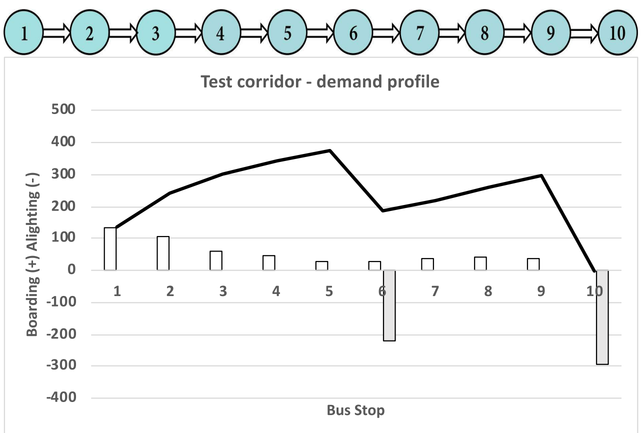
After checking the model and metaheuristic analysis were working correctly, they were applied to the Pajaritos corridor, with similar operating conditions to those proposed by Leiva *et al.* [9]. This is a transit corridor with 10 stops and two-way traffic (Figure 6) with a strong passenger imbalance between the 2 directions (3680 pax/h in direction 1 to 10 and 16870 pax/h in direction 10 to 1).

Although the model, as in the previous instance, considered transfers, a sensitivity analysis performed by the authors showed they had only a slight influence on the final result (lower than 1%, see Leiva *et al.* [9]). The capacity of the proposed model was tested by designing asymmetric operating strategies, using different stop sequences in each direction yet maintaining the same frequencies and, the same vehicle type in both directions on each line. The 4 lines set-up was maintained, where one of them was of the conventional ''all-stop'' type and the rest were express. Operating costs and values of time were kept the same (Table 3), the difference in this case being that the boarding/alighting time was incorporated using values of 1.75 s/pax for the boarding time and 1 s/pax for the alighting time, whereas Leiva *et al.* [9] assumed a constant time of 1 minute.

The travel time between 2 consecutive stops is 1.2 minutes. Finally, deadheading strategies were not allowed in this case, meaning that all the lines were obliged to reach the end of the corridor. This method was used because Leiva *et al.* [9] found that this strategy did not produce improvements any greater than 0.1% in the example being considered.

TABLE 2. Comparison between the applications for 3 lines.

	Proposed model	Wang et al. [18]
	Optimal frequency	
L1	3 buses/h	5 buses/h
L2	6 buses/h	6 buses/h
L3	2 buses/h	5 buses/h
	Optimal stop sequence	
L1	1-2-3-4-5-6-7-8-9-10	1-2-3-4-5-6-7-8-9-10
L2	1-2-3------6-7-8-9-10	1-------------7-8----10
L3	1-2-3----5-6-7-8---10	1-2-3---5------------10
	Optimal fleet size	
L1	60 pax/bus (2 buses)	60 pax/bus (3 buses)
L2	30 pax/bus (3 buses)	60 pax/bus (3 buses)
L3	60 pax/bus (1 bus)	60 pax/bus (3 buses)
Total cost	3.237$/h	3.431 $/h



Node	1	2	3	4	5	6	7	8	9	10
Demand to node 6	75	65	40	25	15	0	0	0	0	0
Demand to node 10	60	40	20	20	15	30	35	40	35	0

FIGURE 4. The Wang *et al.* Corridor [18] and Hourly demand.

TABLE 3. Unit costs used for the Pajaritos corridor example (600 CH$.

Variable	Input used
CK	bus 30pax: 125; bus 60pax: 175; bus 90pax: 225
(CH$/km)	bus 120 pax: 300; bus 150pax:358; bus 180 pax: 400
CF+CP	bus 30pax:3200; bus 60pax: 3500; bus 90pax: 3711
(CH$/h)	bus 120 pax: 4200; bus 150pax: 4666; bus 180 pax: 5000
Φ_v (CH$/h)	900
Φ_w (CH$/h)	1800

A parametric analysis (number of iterations and population) was once again performed to evaluate the behaviour of the technique and its convergence and choose the correct parameters for this particular problem. In this case the best result was found with an initial population of 50 feasible solutions.

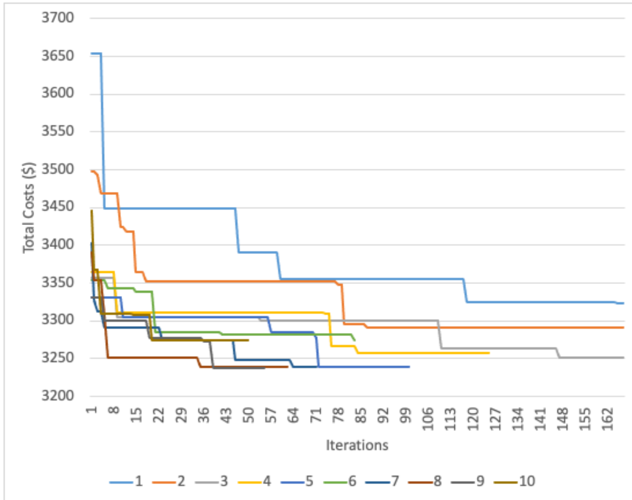


FIGURE 5. Parametric analysis and convergence analysis of the work of Wang *et al.* [18] for 3 lines.

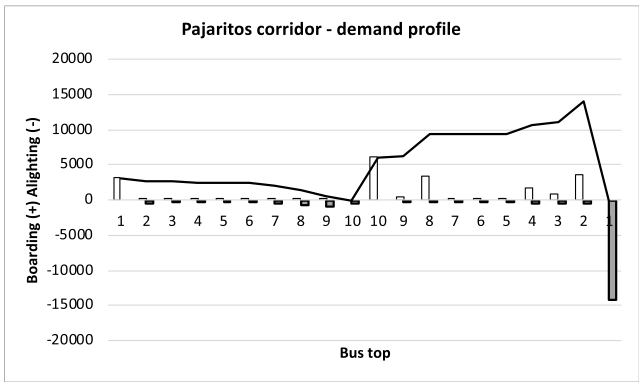


FIGURE 6. Demand along the Pajaritos corridor [9].

Once all the inputs had been defined, the model was applied to the example. The results can be seen in Table 4, where they are compared with those obtained by Leiva *et al.* [9] with the lines organised in such a way as to be comparable between both cases. As can be seen, the proposed model obtains slight variations in the optimal headways and reinforces the conventional service (''all-stop''), providing greater frequencies and higher vehicle capacities. The express service strategies are similar for lines L3 and L4 in terms of the number of stop skips that occur, and even the L3 strategy coincides in direction 10→1. Generally, the model provides greater headways using larger buses, reinforces the express services on higher demand O-D relationships and supports the remainder with the ''all-stop'' line.

Despite the differences, the required fleet size and the value of the objective function are consistent with the results that have already been reported. It must be noted that the required fleet size is lower in our model because cycle times are slightly different due to the consideration of variability in the dwell time and the obtained frequencies are lower. In this sense, significant differences up to 8% were found in cycle times which also affects the fleet needs.

TABLE 4. Results obtained and comparison with the solution of Leiva et. al. [9].

	Proposed model	Leiva et al. [9]
	Optimal headway	
L1	60 buses/h	33.5 buses/h
L2	4.5 buses/h	14.2 buses/h
L3	60 buses/h	97 buses/h
L4	6 buses/h	11.3 buses/h
	Optimal stop sequencing	
L1	1-2-3-4-5-6-7-8-9-10-9-8-7-6-5-4-3-2-1	1-2-3-4-5-6-7-8-9-10-9-8-7-6-5-4-3-2-1
L2	1------4--------8-9-10-9---------5-4----2-1	1---3---1
L3	1---3--------7----9-10----8---------4----2-1	1-2---4---------8---10----8--------4----2-1
L4	1---3-4-5-6---8----10-------7-6-5-4-3-2-1	1-2-3-4------7-8-9-10-9-8-7------4-3-2-1
	Optimal fleet size	
L1	120 pax/bus (38 buses)	90 pax/bus (22 buses)
L2	90 pax/bus (3 buses)	90 pax/bus (1 bus)
L3	150 pax/bus (32 buses)	90/150 pax/bus (17+32 buses=49 buses)
L4	120 pax/bus (4 buses)	90 pax/bus (7 buses)
	Total fleet: 77 buses	Total fleet: 79 buses
	Objective function (User cost+Operator cost)	
	70.637 CH$/min	68.153 CH$/min

Variable dwell times and congestion also affect the perceived user times, and this increases the value of the objective function, reporting higher values than those previously reported. However, if variables dwell times were considered in the solution proposed by Leiva *et al.* [9], the resulting fleet would increase up to 89 buses, while the objective function would become 83.341 CH$/min, representing more than 22% difference in costs and 12.6% in vehicle resources.

These differences demonstrate the importance of considering boarding and alighting times at stops within the perceived journey times, as well as in the turnaround times, as concluded by Alonso *et al.* [21]. It has also been confirmed that the proposed model provided a positive result for the case study.

V. DWELL TIME AND DEMAND LEVEL SENSITIVITY ANALYSIS

A sensitivity analysis was performed to check the effect of different dwell time values and demand levels on the operating costs and the value of the objective function.

In a first analysis, the overall corridor was analysed using constant dwell time values of 20, 40 and 60 seconds and for variable dwell times with boarding and alighting times of: 2-1.2 sec/passenger, 1.75-1 sec/passenger and 1-0.75 sec/passenger respectively, to cover different boarding and alighting ranges [39]–[41] to cover different boarding/alighting disciplines (by using all the doors, only front or rear doors, etc.) and with different payment systems.

The weight of each operating stage on cycle times can be seen in Figure 7. Of course, running times of each line are the same in all the scenarios and are no dependent of dwell times. However, total queueing times at bus stops may strongly affect cycle times even in case where constant dwell times are similar to variable dwell times. Total queueing times up to 4.6 minutes were obtained in spite of assigning bus flows under the bus stop capacity. This amount of time may lead to extra vehicle needs for a transit line in order to satisfy the designed frequency and cannot be avoided.

Furthermore, total dwelling times per line have strong influence on cycle times. Ranges of 25% to 35% of cycle time can be spent in bus stops due to passengers' operations when variable dwell times are considered. This value strongly depends on the unit boarding/alighting time considered. Thus, differences from up to 5 minutes have been found. This fact highlights the well-known importance of designing an appropriate payment system in order to reduce additional delays. Furthermore, the influence of total dwelling times on cycle times varies in all cases with variable dwell times but strongly affects the cycle times of the express lines. As can be seen in Figure 7, significant differences have been found for lines 2 and 3 in all the scenarios. The results show that using constant dwell times tend to underestimate cycle times and therefore user travel times, affecting the final fleet needed to satisfy the designed frequencies and the users' choices in their trips, leading to significant differences in the passenger loads along the corridor.

This strong variations in the passenger's assignment results can be better explained if a disaggregated analysis of dwell times is performed at each bus stop along the corridor. Figure 8 represents the dwell time for line 1 (all-stop) and line 3 (express) in both directions along the corridor. As can be seen, constant dwell times (for example, 60s) introduce an additional delay of almost 1.5 minutes for the 3-6 pair, representing a difference of 30% compared to the real journey time.

The same happens in the opposite direction but because of the lower demand the difference is even greater for the all-stop line, representing almost a 30% overestimation of

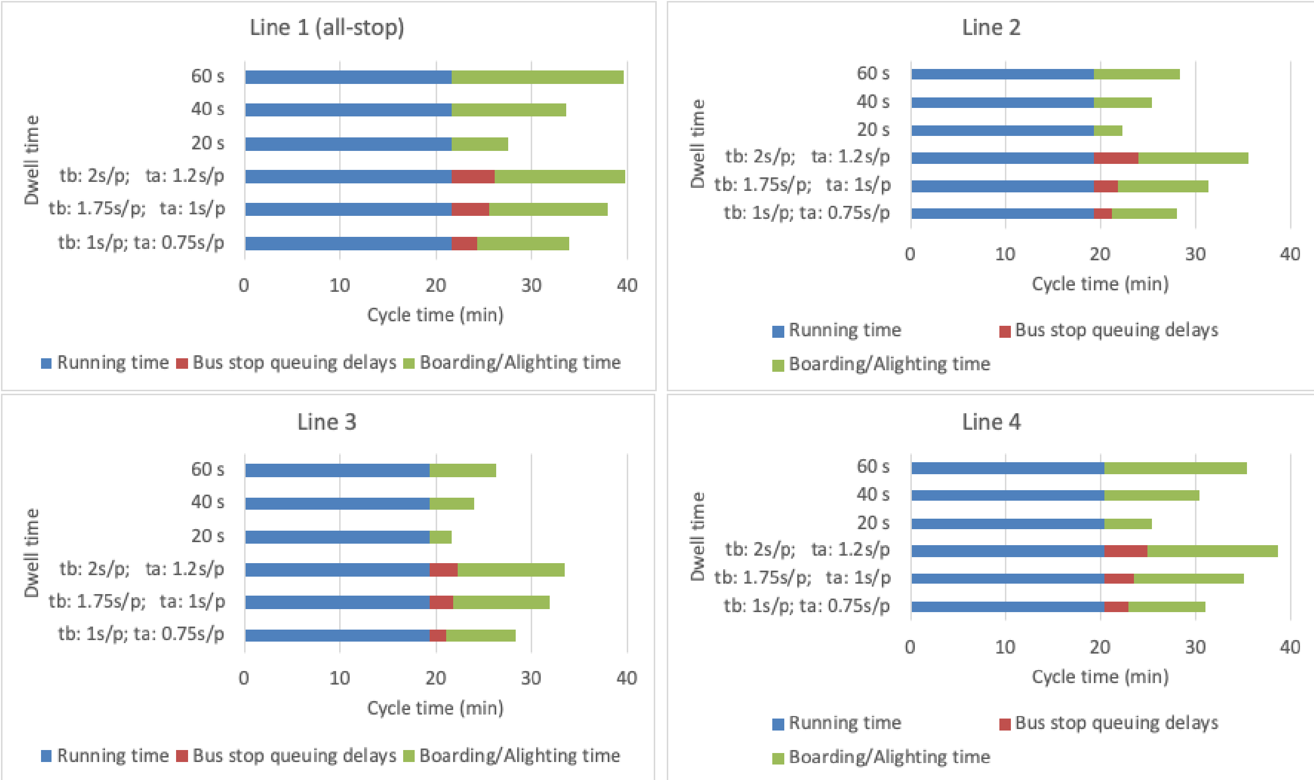


FIGURE 7. Cycle time estimation and operating stage disaggregation per transit line for different dwell times.

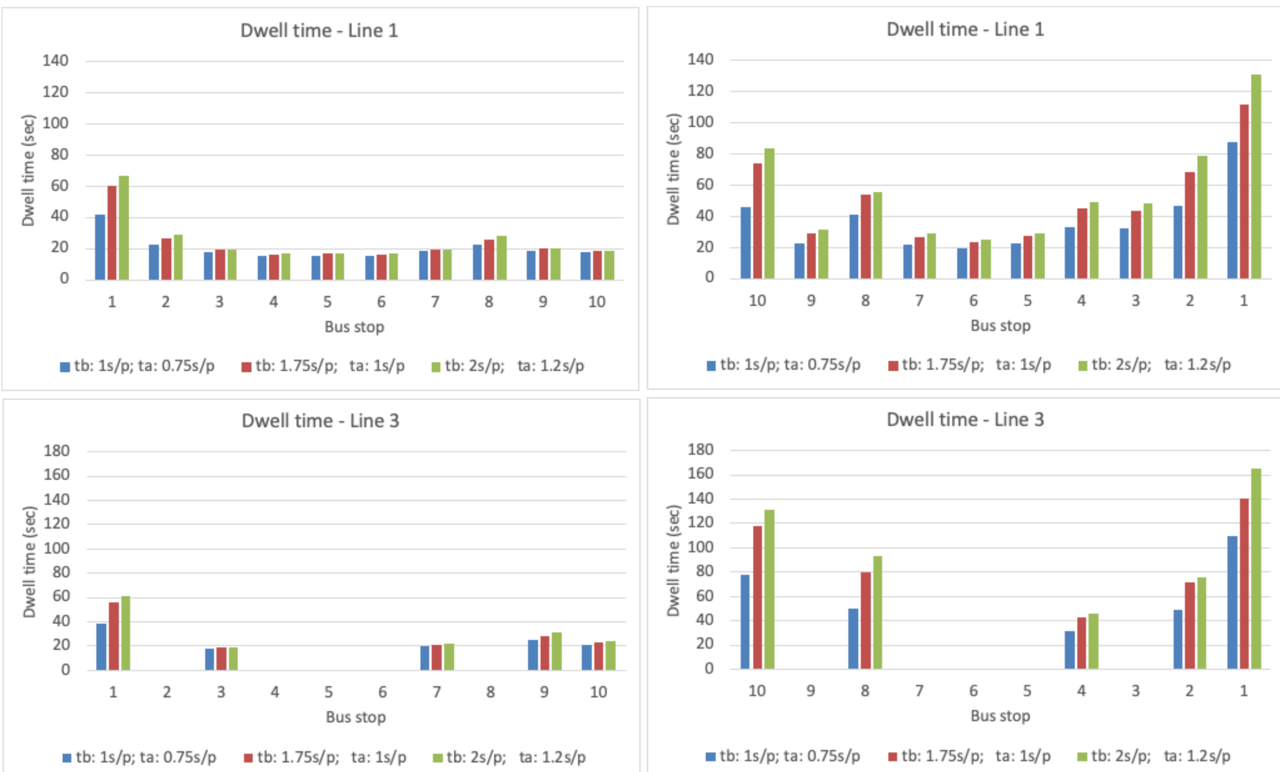


FIGURE 8. Dwell time for different boarding and alighting times in both directions along the corridor.

the journey time. In the case of the express line L3 for the 10-1 pair the difference is 2.5 minutes, or 21% of the journey time. However, as can be seen in the right side of Figure 8, for low levels of demand, constant dwell times can be assumed

FIGURE 9. **Comparative analysis of operational costs and fleet sizes for different dwell times at high (left) and low (right) demand levels.**

but being these times different between the different stops of the corridor, considering their demand.

Thus, the observed difference between the variable and constant dwell times significantly changes the journey times between the different O-D pairs. It leads to high differences in the volume of passengers of each line which can also be observed in each load profile along the corridor. If the assignment results of the base scenario is analysed (60 sec for constant dwell time and 1,75-1 sec for variable dwell time) Line 1 reported differences up to 35% in passenger load and a 30% of average passenger load along the corridor. The same differences were observed for express line 3, where the maximum occupancy grew from 82% to 96%. The most extreme differences were found for Line 4. The occupancy rates of this line moved from 23% to 59% with variable dwell time. In fact, with the preliminary designed value, the operator could assign a smaller bus to this line, leading to an under-sizing mistake.

In order to analyse the combined effect of variable dwell times and the level of demand, a total of 12 scenarios have been simulated, combining 6 different cases of dwell time with high and low demands. Given that the demands varied greatly depending on the direction run along the Los Pajaritos corridor there was a strong imbalance (16.870 pax/h vs. 3668 pax/h), both directions were analysed independently.

The obtained results were used to perform an aggregated and comparative analysis between both directions summarised in Figure 9. As expected, the main differences were found for high demands, in which the different dwell-time scenarios underestimated the cycle time and therefore, the required fleet size, between 12% and 30% more vehicles than those obtained assuming constant dwell times.

Using 60 seconds as a constant value at each stop and only low values for boarding and alighting (1s-0.75s.) provides similar fleet values, while with the other scenarios the difference is +7.3% and +12.2%, representing 3 and 5 extra buses, respectively. This will have repercussions on the operating costs, which, although cushioned by the constant of the rolling costs, show increases of 4.5% and 7.3% respectively. In the case of low demand we find differences of between 9.6% and 13% (+3 and +4 buses) in the variable dwell-time cases compared with constant times of 20 seconds, and −5.4% to −8.1% (−2 and −3 buses) with respect to times of 40 seconds (the 60 second option was not considered as it was not realistic at these levels of demand). However, this effect has fewer repercussions for the operating costs, given that the rolling costs have more weight in this case, as the line frequencies are the same for both levels of demand.

The large differences previously seen for various O-D relationships on the different lines have repercussions on the assignment results, already partly reported by Alonso *et al.* [21], implying significant variations in the user cost function (Figure 9). The high demand scenarios with constant times of 60 s show variations of −10% for the lowest boarding and alighting times (1s.-0.75s.) to + 7% for the highest values (2s-1.2s.). These differences reach up to

FIGURE 10. **Comparative analysis of user costs and objective function for different dwell time values at high demand (left) and low demand (right).**

16% comparing constant times of 40 s. For the low demand scenarios, the user cost values are lower in the cases of variable dwell times compared with the constant 40s, providing differences of between −9.2% and −13.8% for the highest and lowest boarding and alighting values, respectively. In the case of 20 s, the values are similar in all cases with variable dwell time, except for the very lowest values (1s-0.75) which show a reduction of 4.2%.

An analysis of the value of the objective function (overall costs) for the high demand scenarios provides variations of up to 7% with constant dwell times of 60 s and up to 16% at 40 s (Figure 10). Significant differences are also found with low demand values, with variations of up to −8.5%. The variations in the boarding and alighting times are seen to have a lesser impact during the low demand scenarios, being very similar at dwell time values of 20 s. However, when demand is high, the boarding and alighting times become more important for the value of the objective function, between −10% and +5% with respect to the base scenario reported in section 4.2. For these cases, the variations with constant times of 20s reach over 20%, and 10%−13% for times of 40s and between 5% and 10% for times of 60s.

VI. CONCLUSION

This research has proposed an optimization model for the operational design of bus lines along public transport corridors. Frequencies, vehicle type per line and the group of stops to be served for each line along the corridor are estimated in order to minimize the total cost of the corridor, comprising by operating costs and users perceived costs. An important contribution of this research is to incorporate variable dwell times, depending on the boarding and alighting times at each stop and operating capacity constraints, not only as part of the travel time perceived by the users but also as part of the cycle time which affect the fleet size required as well.

The proposed optimization problem was solved by applying the ‘‘Black Hole’’ algorithm which imitates the movement of stars (solutions found) towards a black hole (the best solution found so far) with good results in both instances. The results provide a complete design of each transit line by setting its skip-stop pattern, frequency, vehicle type and fleet needed. The fact of considering variable dwell times and congestion effects report big differences with respect constant dwell time. Thus, in the case of the Pajaritos Avenue (Chile), we found differences of more than 22% in overall costs and 12% for the required fleet size to provide the service along the corridor.

The sensitivity analysis performed highlighted the importance of considering variable dwell times at stops depending on the demand not only during the assignment process as user perceived journey time, but also when estimating the cycle time of the line and, therefore, the fleet required to provide a given frequency. This influence has been found to be higher in case of high levels of demand, where the model with constant dwell times tends to underestimate the required fleet (12-30%), and, therefore, the operational costs. User travel times also reported a wide variation range in several origin-destination trips. Differences up to 30% of travel time were found. This lead to a different assignment results in terms of total trips and load profiles per line, even when the User Cost function (and/or the value of the objective function) gives similar absolute values. The results found demonstrate that for low levels of demand, constant dwell times can be assumed but being these times different between the different stops of the corridor, considering their demand. However, with high level of demand the difference obtained strongly recommend incorporating variable dwell times in the assignment process.

Therefore, this model provides an integrated tool to fully design the operating strategies and fleet needs with a more accurate estimation of travel times and operating times, which are critical parameters for a successful implementation of bus rapid transit corridors.

Future research should extend this study by incorporating transfers in the transit assignment model and by combining the proposed model with methodologies solving the Limited Stop Service Generation, in order to provide an integrated solution. Finally, as we have considered fixed demand, an adapted model considering elastic demand would reflect possible variations of the results reported in the state of the art.

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