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Energy-Efficient Speed Profile Optimization for High-Speed Railway Considering Neutral Sections

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ABSTRACT A neutral zone is a section of a railway line without traction current during high-speed train operations and is very common in high-speed railway lines. However, previous studies on energy-efficient speed profile (ESP) optimization did not consider the electric neutral section constraints. Thus, a risk of being unable to drive out of the neutral section because of a low initial speed and a deviation in the energy consumption effect occur. This paper studies the characteristics of the neutral section to optimize the operating curves of high-speed trains. It can aid in trains passing through the neutral section smoothly. The proposed method is feasible and close to reality. By analyzing the actual coasting distance of a train in the neutral section and building its mathematical model, we provide special double-speed limits and stability constraints. Subsequently, by simulating vehicle performance, operating conditions, and the location of the neutral section, a train ESP optimization model is constructed. Moreover, an improved dynamic programming algorithm is proposed, which uses the vehicle maximum traction and braking force to simplify the computation of state transitions. Finally, the numerical results of simulations demonstrate the efficiency of the proposed model and solution methodology. Compared with the solution without neutral section constraints, an energy-efficient effect of 1.43% is achieved. Further, the proposed method performs better with a stability and high-quality solution than the genetic algorithm.

INDEX TERMS Neutral section constraint, high-speed railway, energy-efficient speed profile, optimization, dynamic programming.

I. INTRODUCTION

Because of its outstanding advantages of capacity, rapidity, safety, and punctuality, the high-speed railway (HSR) is gradually becoming the core of China's railway transportation system. With the rapid construction of the HSR network, its operating mileage reached 35000 km by the end of 2019, of which a neutral section exists every other 15–20 km. And it is a basic and necessary process for the train to pass through the neutral section. Generally, the electrical multiple unit (EMU) of a train adopts forced coasting without traction current in neutral sections by auto-passing the neutral section device of an automatic train protection (ATP). Hence, the driver or automatic train operation (ATO) system must understand the auto-passing speed, particularly on trunk lines with higher permitting speeds and operation densities.

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Otherwise, when the train stops in a neutral section because of an untimely operating, the transportation order of the entire line, even to the railway network, will be affected.

From the mature application experience of the ATO in the urban rail transit (URT), it is a successful experiment to equip trains with an ATO system based on 2/3 level of the Chinese Train Control System (CTCS-2/3). Consider the CTCS-3 + ATO system as an example: an ATO unit, GPRS radio, and related supporting equipment are newly included in the on-board equipment, and a new function of platform door protection is included in the ATP; the ground equipment adds new functions to the train control center (TCC), temporary speed restriction server (TSRS), and centralized traffic control (CTC); precise positioning balises are mounted on the track in stations (Fig. 1). The ATO unit for the HSR has a classic double-layer control structure: an optimization layer and a control layer. The optimization layer calculates an optimal recommended speed curve for a specific offline or

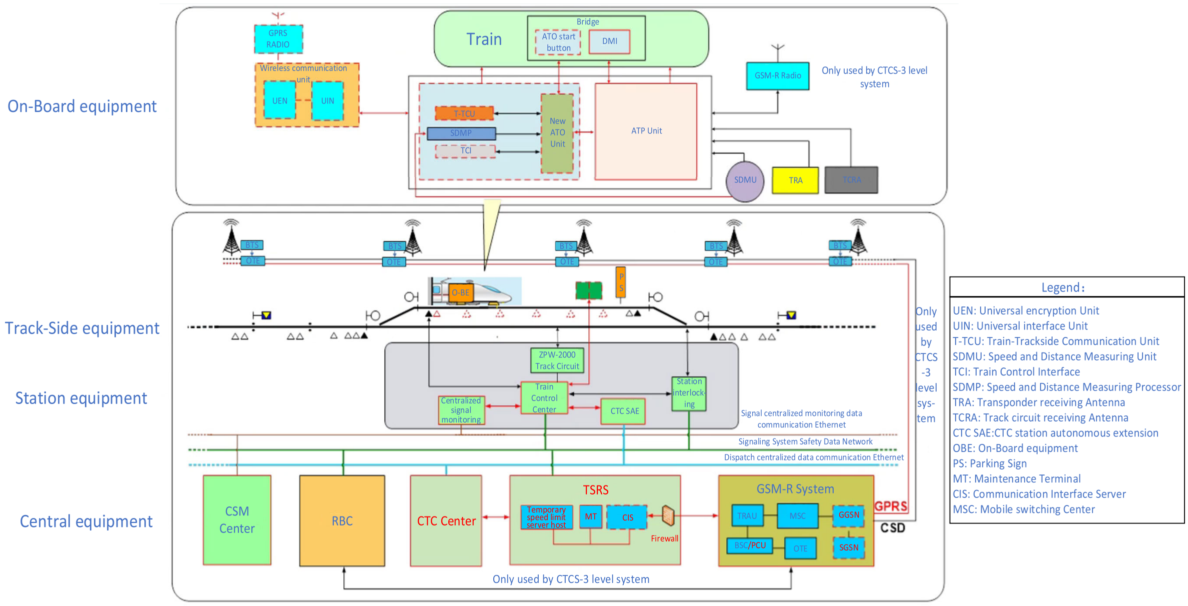

FIGURE 1. Architecture of CTCS-3 + ATO system.

online target according to line data, vehicle performance, and operating conditions, while the control layer outputs a control variable to drive the train automatically according to the guide profile. The control layer has many advanced control methods to help us realize the tracking of the train operation well [1]–[4], but this paper we emphasize the offline optimization of recommended speed profile based on neutral section constraints, and the controller design is not the focus of this paper.

Neutral zone is very common in railway lines and passing through it is a basic feature of high-speed train operations. According to our investigation data of Beijing-Shanghai HSR, it happens from time to time that the train stops into the neutral section, causing transportation disorder and increasing the work pressure of dispatchers. However, there is almost no literature in theory to consider the influence of neutral section from the perspective of optimizing train operation curve, thus studying energy-efficient speed profile (ESP) optimization with consideration to the neutral section constraints is a new idea and it effectively avoids stopping into neutral section of the train which will improve operational efficiency.

Many advantages such as energy efficiency, more comfortable and improving punctuality are shown in the existing different types of literature on the optimization of train speed. On this basis, we propose a method of ESP optimization by considering neutral section constraints. The main objective is to guarantee that a high-speed train passes smoothly through a neutral section meanwhile to improve energy efficiency. The main contributions of the paper are as follows:

1) Double-speed limits model and stability constraints are provided by analyzing the actual coasting distance of the train in the neutral section; these can be directly used by the ESP optimization.
2) An improved dynamic programming (DP) algorithm is proposed to solve an ESP optimization problem that considers neutral section constraints. It uses the vehicle maximum traction and braking force to reduce the calculation times of state transition. In comparison to previous works our team have done, the improved DP Effectively improve the solving efficiency.
3) The problem studied in this paper is not only a supplement to the train operation curve optimization, but also provides a prerequisite guarantee for the train to pass through neutral section smoothly and improve the operation efficiency. Since there is no traction energy consumption in neutral section, the energy consumption calculated in this paper is more suitable for the actual situation compared with the other existing literature.

The remainder of this paper is organized as follows. In section II, we review the literature on solving the ESP optimization problem. In Section III, by analyzing the characteristics of the neutral section, we develop a coasting distance mathematical model and provide the double-speed limits constraints. In Section IV, an optimization model that considers neutral section constraints is constructed; subsequently, we present the solution process. Section V presents an experimental study based on data from the Beijing-Shanghai HSR in China. Finally, conclusions and future considerations are presented.

II. LITERATURE REVIEW

The ESP optimization problem has been popular since Ishikawa first studied the optimal train control model using the maximum principle [5]. Over the decades, scholars in China and other countries have conducted in-depth analyses on it from different perspectives. The methods in the literature can be crudely grouped into two types: analytical and numerical algorithms. Analytical algorithms can obtain precise optimal solutions while some simplified measures must be conducted to ensure the good properties of the objective function. Numerical algorithms do not require objective functions; however, there is a trade-off between efficiency and accuracy.

A. ANALYTICAL ALGORITHMS

In 1980, Moliry formulated the energy-efficient driving problem of trains as a continuous optimal control model, which established the basis for modern train optimal control theory. Later, it was developed by the Scheduling and Control Group of the University of South Australia. Based on the Pontryagin's Maximum Principle (PMP), further researches have been done by them ranging from the optimization model (continuous model-discrete model-continuous model) to line conditions (flat track to steep track, constant speed limit to changed speed limit) [6]–[14]. They even proved that any continuous reference curve can be approximated by a discrete optimal curve [15], which means both the continuous and discrete control models can be applied to vehicles with continuous or discrete traction and braking forces [16]. For continuous control problems, Khmelnitsky solved the optimal running curve using the PMP within a given scheduled time with consideration to speed limits and changed slopes, and proved that the calculation complexity is determined by slope and speed-limit complexity [17]. Liu *et al.* obtained an analytical solution and optimal control sequence of a train operation by analyzing the energy-efficient optimization calculation process, and they proposed a complete solving scheme of optimal speed curve [18]. However, because of the complex nonlinear characteristics of vehicles, solving them using analytical algorithms is difficult in some instances [19], while numerical algorithms provide better solutions.

B. NUMERICAL ALGORITHMS

Since Asnis theoretically proved that the ESP of trains should include four driving phases, i.e., ''maximum traction, cruise, coasting, and maximum braking'' [20], reasonably increasing the coasting time of driving strategies is generally considered to achieve energy saving [21]. The genetic algorithm (GA) has demonstrated promising performance in solving this problem [22]. In [23], a GA was used to solve the coasting control problem. Subsequently, for a single coasting point scenario, they studied some of the classic search algorithms such as the golden search method as a supplement to the GA study [24]. Based on this, the GA was improved by dynamically allocating coasting points into the chromosomes This enhanced its practical application [25]. In [26], a coasting control strategy in the automatic operation mode was designed based on the GA with consideration to the changing gradient, line curve, and speed limits, etc. The solving efficiency is well-known to determine the practicality of a numerical algorithm. Ke *et al.* proposed a new method of optimizing the ESP using the MAX-MIN ant colony algorithm. The computation time was lowered to a reasonable level, thereby enabling online optimization based on numerical algorithms [27]. In addition, DP performs well in optimizing the running profiles of trains. Franke transformed optimization of the velocity curve into discrete optimal control problem to implement the real-time optimal control that combines Discrete Dynamic Programming (DDP) and Nonlinear Model Predictive Control (NMPC) [28]. Ko formulated train operation process as a multi-stage decision process, with DP applied to search for the optimal control strategy directly [29]. Miyatake adopted DP, Gradient Descent and Sequential Quadratic Programming (SQP) to generate profiles which DP was contributed to process state variable constraints [30]. Lu proposed the graphic model of train trajectory searching, upon which DP, ant colony optimization (ACO) and GA were applied to search for the optimum speed at each preset position directly [19]. Ghaviha designed the recommended speed curve for Driver Advisory System (DAS) in which dynamic losses are taken into account in energy calculation [31]. Haahr proposed a speed curve graph with heuristic rules to solve the energy-efficient speed curve problem [32]. Zhou presented space-time-speed grid to complete the joint train timetable and speed profile optimization by using the Lagrangian relaxation [33]. In recent, some scholars adopted Approximate Dynamic Programming (ADP) to address the classical ''curse of dimensionality''. Huang proposed an energy-saving model with on-board energy storage devices and a heuristic dynamic programming (HDP) was designed for optimal speed curve [34]. Wang combined off-line ADP and on-line search process to reduce the probability of delayed arrival and energy saving with the uncertainty of train traction and resistance [35]. Xun designed an algorithm for maximizing the utilization of regenerative energy (MURE) by using ADP to adjust the speed curve of the accelerating train. Then, three approximation methods such as rollout method, interpolation method, and neural network were applied to verify the effectiveness of MURE [36]. Tang studied the energy-efficient driving problem of fixed planning time. For each section, a compound trapezoidal formula was used to calculate the operating distance and time; subsequently, DP was used to search for the optimal solution [37]. The above analysis indicates that in-depth Researches on energy-efficient driving exist. However, the effect of the neutral section requires further exploration.

III. PROBLEM OF ESP OPTIMIZATION WITH CONSIDERATION TO THE NEUTRAL SECTION

The following are definitions of the notations and hypotheses in this paper.

TABLE 1. Notation definition.

Symbol	Description
N	Number of stages
T	Scheduled trip time (s)
S	Total trip distance (m)
t	The actual running time
E	The total energy consumption
i	The speed index of the current stage
j	The speed index of next stage
k	Stage k index
x_k	The start location node of stage k
$s_k(v_k, x_k)$	The state of stage k
$v_{\lim it}$	The speed limit
$v_{neutral}$	The lower speed limit in the electrical neutral section
Δx	The length of an interval
v	The speed of the train
TC_k	A sparse matrix to store the time consumption of all nodes from stage k to $k+1$
EC_k	A sparse matrix to store the energy consumption of all nodes from stage k to $k+1$
x_{start}	The entrance position of the electrical neutral section
x_{end}	The exit position of the electrical neutral section
x'_{start}	The entrance of the transition zone
x'_{end}	The exit of the neutral transition zone
L_{poweroff}	Distance from the "power off" mark to the entrance of the electrical neutral section
$L_{poweron}$	Distance from the "power on" mark to the exit of the electrical neutral section
$L_{neutralzone}$	The length of the electrical neutral section
u	Control variable
$g(x)$	The gradient resistance per unit mass
$r(v)$	The basic resistance per unit mass
$F(v)$	The traction
$u_{tra}(v)$	The accelerate by traction
$u_{bra}(v)$	The deceleration by the braking force
$a_{\max\_tra}$	The maximum total accelerate of the tractive process
$a_{\max\_bra}$	The maximum total decelerate of the tractive process
H	The operation condition
B_{air}	The braking condition by air
C	The coasting condition

HYPOTHESES

1) Vehicles can obtain the theoretical maximum traction and braking force under any conditions.
2) The braking force and traction in each stage is a constant value.
3) Attempting the coasting condition first in the neutral section, the air brake can be used when the train will overspeed. The delay in establishing the air brake is assumed to be negligible.

A. PROBLEM DESCRIPTION

As Fig. 2 shows, curve a is the optimal speed profile based on an energy-efficient strategy. It does not consider the constraints of the neutral section. In practice, the train's trajectory may result as curves b or c depending on the differences in the lines or the entrance speed into the neutral section, which may not guarantee a smooth passage of the train. Hence, the original solution may not be feasible when a neutral section exists. In the following, we analyze the train operation process in the neutral section in detail.

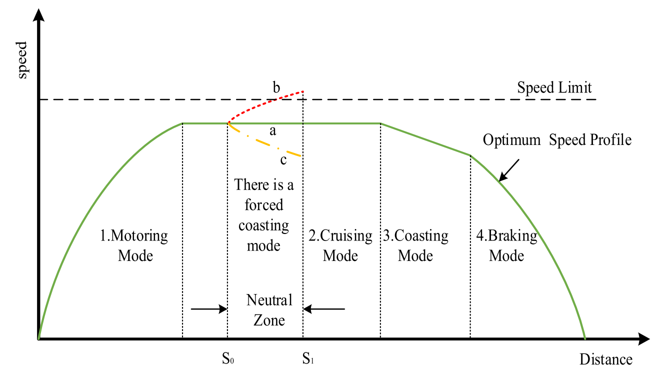

FIGURE 2. Possible speed profiles on the condition of neutral zone constraints.

B. NEUTRAL SECTION ANALYSIS

1) COASTING DISTANCE MODELING

This requires that the handle is returned to the zero level in advance and the main circuit switch is disconnected, that is, the actual coasting distance of the train in the neutral section is greater than its physical length. In this paper, the actual coasting distance is called the logical neutral section. Fig. 3 shows the detailed process of auto-passing the neutral section by the ATP.

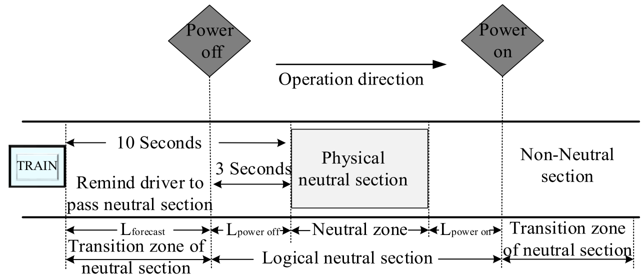

FIGURE 3. Length of a logical neutral zone based on the ATP auto-passing neutral section.

1) 10 seconds before the entrance of the neutral section, the on-board ATP continuously reminds the driver that the train is about to pass; hence, the driver disconnects the main circuit switch.
2) 3 seconds from the entrance of the neutral section, the automatic device disconnects the main circuit switch, causing the train to coast passively.

3) After passing the "power on" sign, the train is energized and operates normally.

Therefore, we can formulate the actual coasting distance as follows:

$$L_{\log ical} = L_{\text{poweroff}} + L_{neutralzone} + L_{poweron} \tag{1}$$

2) DOUBLE-SPEED LIMIT CONSTRAINT

The train's acceleration in the neutral section is determined by the resistance it encounters. According to the hypothesis(3), we simplify the problem and only analyze the decelerated condition in the neutral section. As Fig. 4 shows, if the train enters the neutral zone at different speeds, a corresponding unique exit speed will be obtained. However, when the entrance speed is lower than v_p, and the exit speed will be lower than v'_{q}. Here, the train is likely to stop in the neutral section because of insufficient power. In this paper, the speed at point P is called the lower speed limit, and the original limited speed is considered the upper speed limit; thus, they form the double-speed limit constraint in the neutral section. The velocity at point P is determined by the resistance acceleration. Frequently, it will be set to 5 km/h.

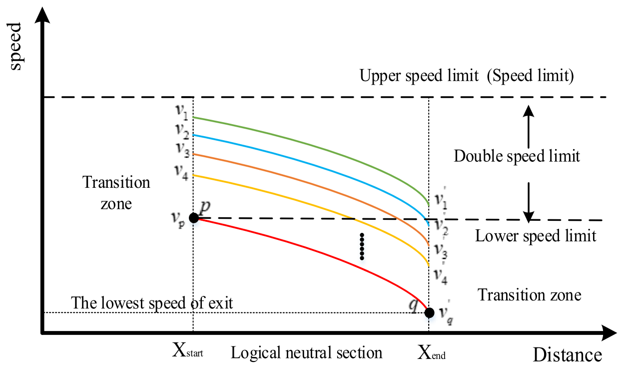

FIGURE 4. Diagram of the double speed limit.

v_{limit} and v_{neutral} are the upper and lower speed limits, respectively. Thus, the constraints of the neutral section is formulated as follows:

$$\begin{cases} F(v) = 0, & x_{start} \leq x \leq x_{end} \\ v_{neutral} \leq v \leq v_{\lim it}, & x_{start} \leq x \leq x_{end} \\ H \in \{B_{air}, C\}, & x_{start} \leq x \leq x_{end} \end{cases} \tag{2}$$

To avoid the shock caused by the sudden disconnection of the strong traction current, which would be extremely uncomfortable, and to satisfy the comfort requirements of the high-speed railway ATO standard, this paper expresses the stability constraint of the neutral section as follows:

$$\begin{cases} |a_{k+1}| \leq |a_k|, & x_k \in (x'_{start}, x_{start}) \\ |a_{k+1}| \geq |a_k|, & x_k \in (x_{end}, x'_{end}) \\ |a_{k+1} - a_k| \leq 0.15 \end{cases} \tag{3}$$

C. DYNAMICS MODEL

The operation of high-speed trains is restricted by many aspects such as line conditions, temporary speed limits, and vehicle conditions. The additional resistance (gradient, curve, tunnel, etc.) is frequently related to the train's location. For a more convenient description of the additional resistance, the single-mass point train motion equation is described as a position-based model:

$$\begin{cases} (1+\gamma) \cdot \dfrac{dv}{dx} = \dfrac{u_{tra}(v) - u_{bra}(v) - (r(v) + g(x)) \cdot g}{v} \\ \dfrac{dt}{dx} = \dfrac{1}{v} \\ r(v) = \text{a} + bv + cv^2 \end{cases} \tag{4}$$

where γ is the rotating coefficient, which is generally 0.6. We use the parameters of CR400AF EMU as the basic vehicle simulation data. Its traction, braking force, and basic resistance are shown in Fig. 5.

IV. SOLUTION METHOD BASED ON DYNAMIC PROGRAMMING

A. OBJECTIVE FUNCTION

There are many complicated nonlinear constraints in the optimal control problem of train. Certainly, some simplification measures need to be done in the laboratory environment. And choosing a suitable algorithm is critical. DP can directly deal with difficult constraints of an optimal control problem [29]. And it is frequently used to solve discrete problems. Therefore, we constructed the train energy-efficient optimization model as a discrete model. Meanwhile, the boundary conditions and safe operation constraints are met.

$$\min E$$

$$E = \begin{cases} \sum_{\text{k}} = 1^{\text{N}} \dfrac{1}{2}(u + |u|) \cdot \Delta x_k, \; x_k \in ((0, x_{start}) \cup (x_{end}, S)) \\ 0, \; \Delta x_i \in (x_{start}, x_{end}) \end{cases}$$

$$\text{s.t} \begin{cases} v(0) = 0, & x(0) = 0 \\ v(T) = 0, & x(T) = S \\ v_k < v_{k,\lim it}, & x_k \\ !! \in ((0, x_{start}) \cup (x_{end}, S)) \\ v_{k,neutral} < v_k < v_{k,\lim it}, & x_k \in (x_{start}, x_{end}) \\ \left| \sum_{i=1}^{N} t_i - T \right| \leq \delta \end{cases} \tag{5}$$

B. SOLUTION PROCESS

The basic concept of DP is to divide the problem to be solved into several interrelated sub-problems. When the sub-problems are solved, the solution to the original problem would be determined. The solution process is generally divided into three steps: discrete state space, state transition calculation, and search for the optimal solution.

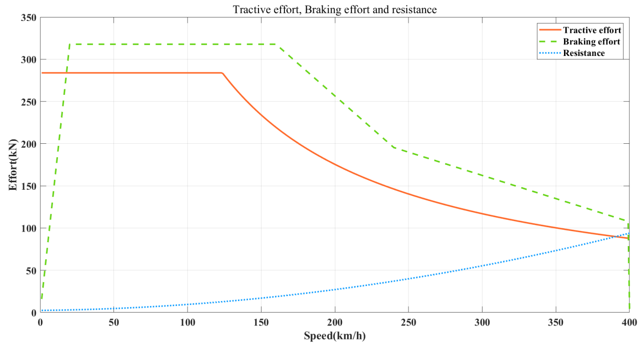

FIGURE 5. Tractive effort, braking effort, and resistance of the CR400AF EMU.

1) STATE DISCRETE

The entire interval is divided into N stages with equivalent distances of 10 m, and the starting point of each stage is the preset position of the train in that state. We assume that the slope and speed limit values in each stage are constant to ensure the control variable in each stage is constant. Thus, if conditions 1, 2, or 3 in Fig. 6 were encountered, the following processes would be conducted respectively.

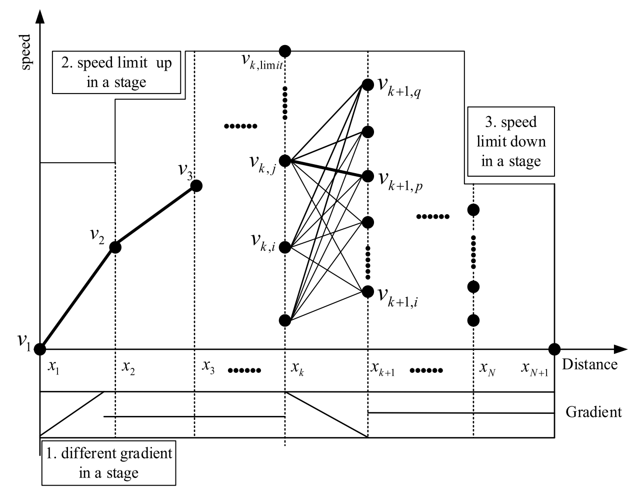

FIGURE 6. Schematic of stage divisions.

Scenario 1: the slopes have different values at a certain stage. In this scenario, the slope at the starting position of a stage is used as the slope of this stage.

Scenario 2: the speed limit increases at a certain stage. The lower speed limit should be considered the maximum speed limit at this stage to ensure safety.

Scenario 3: the speed limit decreases at a certain stage. Similarly, the lower speed limit must be used as the max speed limit at this stage.

Subsequently, the effective speed range $[v_{\min}, v_{\max}]$ would be easily obtained using the fastest and slowest running curves [38]. Here, we discrete it using $\Delta v = 0.5$km/h to obtain the speed set as $\{v_{\min}, v_{\min} + \Delta v, v_{\min} + 2\Delta v, v_{\min} + 3\Delta v \ldots\ldots v_{\lim it}\}$ for each stage. Thus, we obtain the entire state space Ω of the problem and state $s_k(v_k, x_k) \subseteq \Omega$ for $k = 1, 2, 3 \ldots\ldots N + 1$. The initial and final states can be expressed as $s_1(0, 0)$ and $s_{N+1}(0, x_{N+1})$, respectively.

2) STATE TRANSITION CACULATION

Restricted by the maximum traction and braking force when the train is out of the neutral section, a certain range in which the train transfers from one stage to another exists, which is defined by (6):

$$\begin{cases} v_a = \sqrt{v_k^2 + 2a_{\max\_tra} \cdot \Delta s} \\ v_b = \sqrt{v_k^2 + 2a_{\max\_bra} \cdot \Delta s} \end{cases} \tag{6}$$

Moreover, the state transition of stage k is

$$s_k(v_{k,i}, x_k) \xrightarrow{u} s_{k+1}(v_{k+1,j}, x_{k+1}), v_{k+1,j} \in [v_a, v_b] \tag{7}$$

In the neutral section, a range of the state transition does not exist, and the variables exhibit a one-to-one correspondence (8).

$$\begin{cases} v_{k+1} = \sqrt{v_k^2 + 2a_{\text{neutral}} \cdot \Delta x_k}, & x_k \in [x_{start}, x_{end}] \\ a_{\text{neutral}} = (r(v) + g(x)) \cdot g & \\ v_{k+1} = v_{k+1}, & v_{k+1} \leq v_{k+1,\lim it} \\ v_{k+1} = v_{k+1,\lim it}, & v_{k+1} > v_{k+1,\lim it} \end{cases} \tag{8}$$

Equation (9) can be used to calculate the energy and time cost of each state transition, and two sparse matrices, EC_k and TC_k, are used to store them in stage k. After calculating all the stages, the energy and time consumption costs of the entire state space are stored in two three-dimensional matrices.

$$\begin{cases} \begin{cases} E_k(k, i, j) = \dfrac{1}{2}(u(k, i, j) + |u(k, i, j)|) \cdot \Delta x_k \\ x_k \in ((0, x_{start}) \cup (x_{end}, S)) \end{cases} \\ E_k(k, i, j) = 0, x_k \in (x_{start}, x_{end}) \\ T_k(k, i, j) = \dfrac{2\Delta x_k}{v_{k,i} + v_{k+1,j}} \end{cases} \tag{9}$$

3) SEARCHING FOR AN OPTIMAL SOLUTION

We introduce the utility function in (10) to evaluate the decision-making in each stage, and we use indicator function $J(v)$ to assess the advantages and disadvantages of a policy; $J^*(v)$ is the value of the optimal strategy.

$$\begin{cases} U_k(v_{k,i}, v_{k+1,j}) = c \cdot (1 - \alpha)EC_k(i, j) \\ \qquad +\alpha \cdot \left| TC_k(i, j) - \dfrac{T}{N} \right| \\ k \in [1, N + 1], i, j \in [1, M] \end{cases}, \tag{10}$$

where c is the coefficient that balances the energy and time consumption, and $c = 1e - 7$ in this paper;α is the weight coefficient, which indicates the importance of punctuality. We assume that the indicator function value of the final state $s_{N+1}(0, x_{N+1})$ is $J_{N+1}(0, x_{N+1}) = 0$, the indicator function of each state in the $(N - 1)$th stage is

$$J_N(v_{N,i}) = U_N(v_{N,i}, 0) + J_{N+1}(0, x_{N+1}) \tag{11}$$

The minimum indicator function value of all state in the $(N-1)$th stage is the optimal indicator function. Therefore, the optimal indicator function of the kth stage is

$$J_k^*(v_{k,i}) = \begin{cases} \begin{cases} \min\{U_k(v_{k,i}, v_{k+1,j}) + J_{k+1}^*(v_{k+1,j})\}, \\ k \in [1, N), i, j \in [1, M] \end{cases} \\ 0, k = N+1 \end{cases} \tag{12}$$

where M is the maximum number of state variables in a stage. Using $v_k^* = \arg J_k^*(v_k)$, we can obtain the optimal speed sequence.

V. RESULTS AND DISCUSSION

To test and verify the efficiency of the proposed model and solution method, we present a study case based on the data from the Beijing-Shanghai HSR in China. Considering the flexibility of evolutionary algorithm, genetic algorithm is one of the most widely used evolutionary algorithms in solving the train curve optimization problem. In this paper, the flow chart of genetic algorithm presented in reference [39] is taken as an example to compare with the results of dynamic programming algorithm. In addition, the slope data were rounded to safe values which is more secure for train operation. The processed slope and speed limit are shown in Fig. 7. The parameters of simulation EMU and the control parameters of the algorithm are listed in Tables 2 and 3, respectively. We performed the simulation on a PC (2.2 GHz processor speed and 4 GB RAM) with the Windows 10 platform, and the simulation software was MATLAB R2018b.

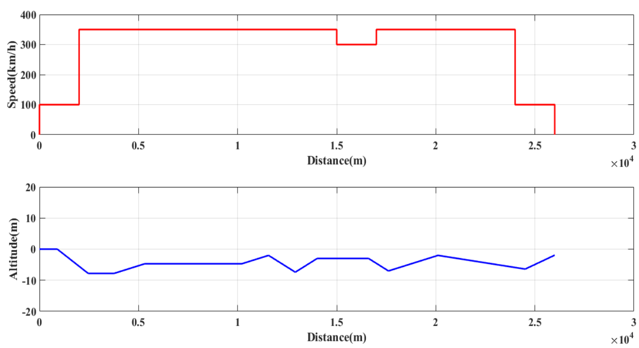

FIGURE 7. Journey altitude and speed limit profile.

According to the regulations of the Chinese railway transportation department on setting neutral section signs, we processed the original neutral zone data as shown in Table 4.

According to the neutral section model established in section III, the solution process in section IV, the simulation parameters and line data. The specific solving steps of DP algorithm are as follows:

Step 1: dividing the simulation section into 2600 stages with equal interval of 10m, with total 2601 position nodes, each position node $x_k \in \{0, 10, 20, \ldots\ldots 2600\}$.

Step 2: discrete speed limit uses $\Delta v = 0.5$km/h. If $x_{start} < x < x_{end}$, the speed set at each stage is

TABLE 2. Numerical parameters of the CR400AF electrical multiple unit.

Symbol	Value
M	472.3 t
$v_{operation}$	350 km/h
$F_{m(traction)}$	283.87 kN
$F_{m(braking)}$	317 kN
P_{wheel}	9750 kW
P_{motor}	650 kW
Start acceleration	1.908 km/h/s (0.53 m/s^2)
$r(v)$	$0.53+0.00388v+0.000114v^2$

TABLE 3. Table iii. Numerical parameters of the DP algorithm.

Symbol	Value
α	0.28
Distance	26 km
Scheduled time	720 s
Lower speed limit (in the neutral section)	40 km/h

TABLE 4. Neutral section information of Wuxi east to north of Suzhou.

name	Start mileage	Ending mileage	Length (m)
Actual neutral section	K1213+209	K1213+715	506
Logical neutral section	K1213+120	K1214+120	1000
Forward transition	K1212+950	K1213+120	170
Backward transition	K1214+120	K1214+290	170

$\{v_{neutral}, v_{neutral} + \Delta v \ldots\ldots v_{\lim it}\}$, else the speed set at each stage is $\{0, \Delta v, 2\Delta v, 3\Delta v \ldots\ldots v_{\lim it}\}$.

Step 3: calculating the fastest running curve $v_{\max}(x)$ of train.

Step 4: according *step 3*, compressing the speed set at every stage, if $x_{start} < x < x_{end}$, the speed set at each stage is $\{v_{neutral}, v_{neutral} + \Delta v \ldots\ldots v_{\max}(x_k)\}$, else the speed set at each stage is $\{0, \Delta v, 2\Delta v, 3\Delta v \ldots\ldots v_{\max}(x_k)\}$. thus, we can use a V matrix of $m * n$ dimension to store all discrete speed and obtain all states, each state $s_k = (x_k, v_{k,i})$. Where n is number of position nodes and m is Maximum number of state nodes in a stage.

Step 5: if $x_{start} < x < x_{end}$ (in neutral section), calculation the time cost and energy consumption of each state transition process with state in stage k transferring to a state in stage $k+1$ by (8), (9), else calculation the time cost and energy consumption of each state transition process with state in stage k transferring to a state in stage $k+1$ by (6), (7), (9). And storing them in sparse matrix $TC_{\mathrm{k}}(i,j)$ and $EC_k(i,j)$ respectively.

Step 6: for each stage $k(1 \leq k \leq 2600)$, execute step5. The energy and time cost of state transition in the whole state space are stored by two three-dimensional matrices $EC(i, j, k)$ and $TC(i, j, k)$ respectively.

Step 7: two $m * n$ dimensional matrix J and matrix I are established to store the optimal indicator and velocity index.

Step 8: for stage N, it is end state, $v_{N+1,1} = 0$ and the optimal indicator $J^*(v_{N+1,1}, N+1) = 0$, so the value of $J(1, N+1)$ in matrix J is $J(1, N+1) = 0$ and the value of $I(1, N+1)$ in matrix I is $I(1, N+1) = 1$. For stage $N-1$, for each state, the optimal indicator $J^*(v_{N,i}, N)$ is get by (10), (12), the value of $J(\mathrm{i}, N)$ in matrix J is $J^*(v_{N,i}, N)$ and the value of $I(i, N)$ in matrix I is $I(i, N) = j$, where j is subscript of $v_{N+1,j}$, $v_{N+1,j} = \arg(J^*(v_{N,i}, N))$. For stage $k(1 \leq k \leq N-2)$, execute the iterative calculation described as above.

Step 9: according to the value of $I(1, 1)$ in matrix I, we can easily search the subscript of optimal velocity in next stage. after searching for all stages, a vector of optimal-velocity index is certain, then find the velocity value in matrix V by the vector. Thus, the optimal solution is obtained.

In this study, the fastest running curve, optimal speed profile with neutral section constraints, and optimal speed profile without neutral section constraints were simulated and analyzed. Fig. 8 indicates that the fastest operating strategy operated the train at the maximum speed in almost every stage, including maximum traction, cruise, and maximum braking conditions. The optimal speed profile with neutral section constraints had a passive coasting process in the a–b segment (in the neutral zone). Note that the neutral section affected the entire interval, particularly the segment of the speed profile before the neutral section area. Compared with the optimal speed profile without neutral section constraints, the solution in this study would accelerate in advance to reduce speed loss in the neutral section. This is also known as "speed grabbing" in engineering. The comparisons of the three profiles are shown in Table 5. The difference between the running and planned time of three curves was less than three minutes, which satisfied the punctuality requirements of HSRs, and the comfort level also satisfied the required value of 0.75 $\mathrm{m \cdot s^{-3}}$. In addition, compared with the solution without neutral section constraints, an energy-efficient effect of 1.43% was achieved.

Fig. 9 shows the speed curve, resultant acceleration, and slope in the neutral section. The recommended speeds at the entrance and exit of the neutral section were 238.5 and 235.7 km/h, respectively, which meant that a speed loss of 2.8 km/h occurred. In the neutral zone, the train continued decelerating and the deceleration change range was from -0.06 to -0.02. The deceleration was consistent with the change trend of the line slope, which indicated the rationality of the speed profile.

Fig. 10 shows the entire energy consumption in the interval, energy consumption in each stage, and change in the control variable. The figure indicates that energy consumption was concentrated in the traction section. Thus, energy consumption in the neutral section was zero. The entire change trend of the control variable was relatively stable with a few local violent changes with higher energy consumption.

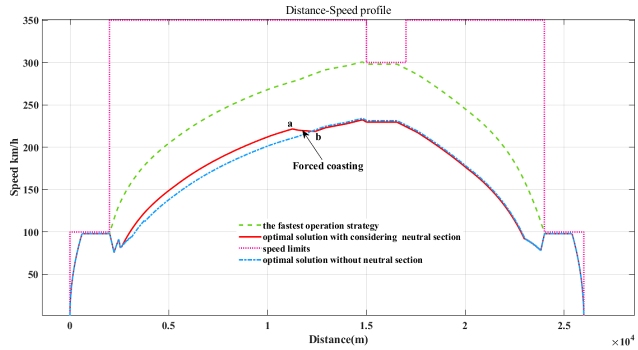

FIGURE 8. Three types of speed trajectories.

TABLE 5. Comparison of the results of three operation curves.

Result comparison	Operation time (s)	Traction energy consumption (kWh)	Average jerk $(\mathrm{m \cdot s^{-3}})$	Energy-saving ratio (%)
Optimal Solution A	678.7780	573.08	0.0567	—
Optimal solution B without neutral section	689.8535	581.41	0.0673	—
The fastest strategy	545.0067	819.89	0.0088	—
Compared on the fastest strategy	—	—	—	30.20
Compared on solution B	—	—	—	1.43

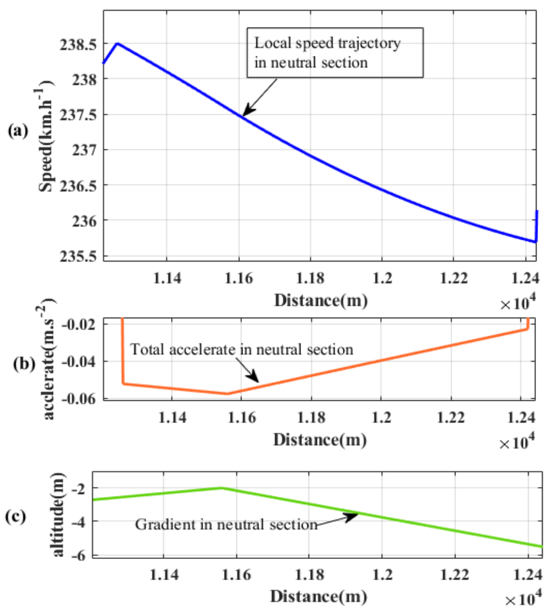

FIGURE 9. Details of the speed trajectory, total accelerate, and gradient in the neutral zone.

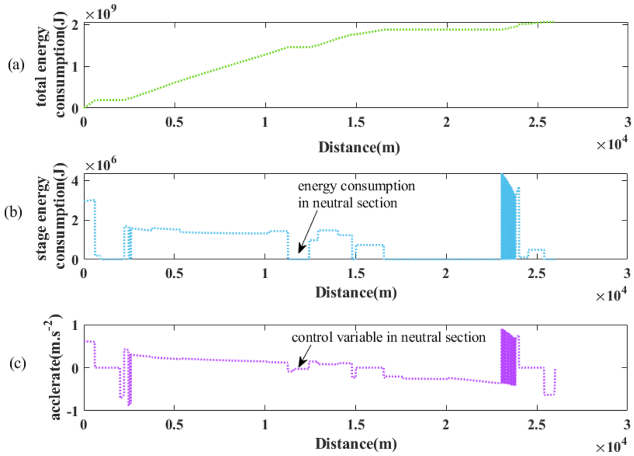

FIGURE 10. Total energy consumption, energy consumption in each stage, and total control variable.

According to the neutral section model established in section III, the solution process in section IV, the simulation parameters and line data, and reference [39], the specific solving steps of GA algorithm are as follows:

Step 1: First we set the number of genetic individuals (num) is 100. Individuals are randomly generated as the first generation $G_1 = (S_1^1, S_1^2, \ldots, S_1^{num})$ .The individual's genes represent the distance and acceleration, which are the variables of the solution $S_k^i = (x_1, a_1, x_2, a_2, \ldots, x_n, a_n)$.

Step 2: The solution of each individual is required an evaluation of the fitness function to determine whether the individual can be preserved. if $x_{start} < x < x_{end}$(in neutral section), calculation the time cost and energy consumption of each individual by (8), (9), else by (6), (7), (9). Then, the expression of the fitness function is

$$f_k^n = \frac{1}{c \cdot (1-\alpha) E_k^n + \alpha \cdot \left| T_k^n - T \right|} \tag{13}$$

where f_k^n is the fitness of the nth individual of the k generation; E_k^n and T_k^n are the traction energy consumption and the journey time of it.

Step 3: The individuals are arranged in descending order according to fitness function, and the sorted individuals are operated in a series of steps, including selection, crossover, mutation and replacement. In this study, the specific parameters of the algorithm are based on the reference [39].

*Step 4:*in the selection phase, the 10 top fitness function ranking individuals are retained to form the new generation.

Step 5: during the crossover phase, two individuals are selected randomly as parents, and some allele genes from the parents are exchanged, this step is repeated 35 times.

Step 6: in the mutation phase, individuals are randomly selected and some genes are replaced with random values, this step is repeated 10 times.

Step 7: during the replacement phase, new individuals are randomly generated to replace the last 10 individuals sorted by the fitness function.

*Step 8:*after the four stages of the GA, the total number of individuals in each generation remains unchanged and a new generation is born. *Step 2–7* are repeated until the number of generations reaches at 50 or the change of fitness function less than the expected set value.

The simulation results of the GA are shown in Fig. 11. GA fail to find a smooth trajectory with frequent traction and braking. However, speed profile based on DP performed better with a stable cruising phase and smooth trajectory according to the above results analysis.

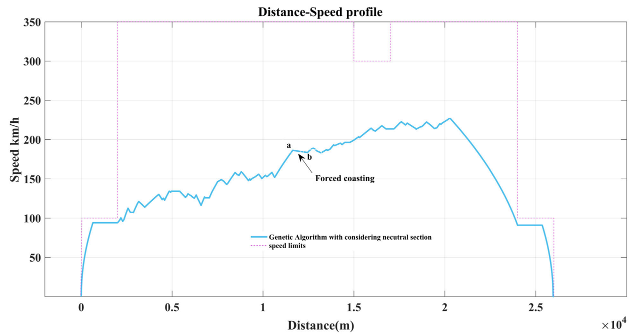

FIGURE 11. Speed trajectory using GA.

Fig. 12 shows how the fitness function evolves with the generation. The fitness function of each generation decreases gradually with the change of generation, which converges to the optimal solution quick.

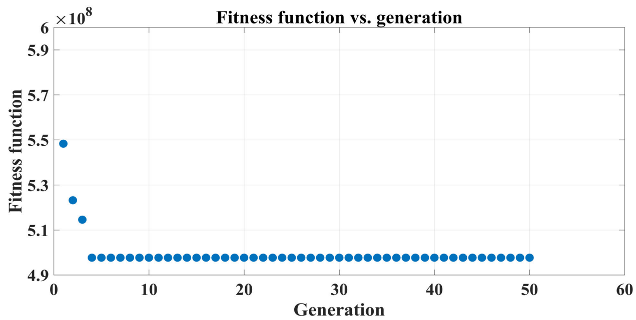

FIGURE 12. Fitness function at each generation for GA.

TABLE 6. Performance comparison between the two algorithms.

Algorithms	Operatio n time (s)	Traction energy consumption (kWh)
GA	735.301 4	709.81
DP	678.778 0	573.08

DP is able to obtain the optimal solution compared with the GA, since the different states in each stage correspond to a certain journey time, energy consumption and distance.

The optimized solution is found by traversing all solutions set, while the GA may fall into local optimum.

Table 6 shows a performance comparison between the two algorithms.

VI. CONCLUSION AND SUMMARY

The main contribution of this paper is to consider the constraints of the neutral section in the energy-efficient optimization problem of the HSR to determine a solution with more practical engineering application significance. In particular, by analyzing the operation characteristics of the train in the neutral section, a mathematical model of the actual coasting distance in the neutral section is constructed. The double speed limit and stability constraints are provided to ensure that the train passes through the neutral section smoothly. Additionally, based on these constraints, we build an energy-efficient optimization model of speed profiles. When solving the model, we improved the calculation method of the state transition of the DP algorithm according to the features of the neutral section. We conduct a case study and combine with the GA comparison to elucidate the efficiency of the proposed approach based on actual track data from the Beijing-Shanghai HSR. The results of the numerical experiment indicated that the neutral section affects the speed curve in the entire interval and not only in the neutral zone.

Note that the accuracy of the speed quantification frequently affects the accuracy of the solution. When the acceleration in the neutral section is sufficiently small, the actual speed reached in the next stage may be between two discrete speed values. Meanwhile, some approximate processing measures are required to ensure a complete solution can be searched. In addition, using DP to solve the energy-efficient optimization problem requires a large storage memory. The storage space increases non-linearly with the state space even if a sparse storage method is adopted. The solving efficiency is another key factor for scholars to use DP algorithms, particularly when the train headway is increasingly becoming smaller. Generally, when the state space is determined, the total number of calculation operations is almost certain. Using high-configuration computers to increase the calculation efficiency or reduce the number of calculations times are the two main methods of increasing solution efficiency. Therefore, the future consideration of this study is methods of reducing the invalid state transition calculation process.

REFERENCES

[1] R.-E. Precup, R.-C. David, E. M. Petriu, A.-I. Szedlak-Stinean, and C.-A. Bojan-Dragos, "Grey wolf optimizer-based approach to the tuning of Pi-fuzzy controllers with a reduced process parametric sensitivity," *IFAC-PapersOnLine*, vol. 49, no. 5, pp. 55–60, Jul. 2016.

[2] G. Rigatos, P. Siano, D. Selisteanu, and R. E. Precup, "Nonlinear optimal control of oxygen and carbon dioxide levels in blood," *Intell. Ind. Syst.*, vol. 3, no. 2, pp. 61–75, Jun. 2017.

[3] S. Preitl, R.-E. Precup, Z. Preitl, S. Vaivoda, S. Kilyeni, and J. K. Tar, "Iterative feedback and learning control. Servo systems applications," *IFAC Proc. Volumes*, vol. 40, no. 8, pp. 16–27, Apr. 2007.

[4] G. Wen, S. S. Ge, C. L. P. Chen, F. Tu, and S. Wang, "Adaptive tracking control of surface vessel using optimized backstepping technique," *IEEE Trans. Cybern.*, vol. 49, no. 9, pp. 3420–3431, Sep. 2019.

[5] K. Ichikawa, "Application of optimization theory for bounded state variable problems to the operation of train," *Bull. JSME*, vol. 11, no. 47, pp. 857–865, 1968.

[6] P. G. Howlett, "Existence of an optimal strategy for the control of a train," South Aust. Inst. Technol., School Math. Rep., Adelaide, SA, Australia, Tech. Rep. 3, 1987.

[7] P. G. Howlett, "Necessary conditions on an optimal strategy for the control of a train," South Aust. Inst. Technol., School Math. Rep., Adelaide, SA, Australia, Tech. Rep. 3, 1987.

[8] P. Howlett, "An optimal strategy for the control of a train," *J. Austral. Math. Soc. B, Appl. Math.*, vol. 31, no. 4, pp. 454–471, Apr. 1990.

[9] P. G. Howlett and A. Leizarowitz, "Optimal strategies for vehicle control problems with fifinite control sets," *Dyn. Continuous, Discr. Impuls. Syst. B, Appl. Algorithms*, vol. 8, no. 1, pp. 41–69, Jan. 2001.

[10] C. Jiaxin and P. Howlett, "Application of critical velocities to the minimisation of fuel consumption in the control of trains," *Automatica*, vol. 28, no. 1, pp. 165–169, Jan. 1992.

[11] C. Jiaxin and P. Howlett, "A note on the calculation of optimal strategies for the minimization of fuel consumption in the control of trains," *IEEE Trans. Autom. Control*, vol. 38, no. 11, pp. 1730–1734, Nov. 1993.

[12] P. J. Pudney and P. G. Howlett, "Optimal driving strategy for a train journey with speed limits," *J. Austral. Math. Soc. B. Appl. Math.*, vol. 36, no. 1, pp. 38–49, Jul. 1994.

[13] P. G. Howlett and J. Cheng, "Optimal driving strategies for a train on a track with continuously varying gradient," *J. Austral. Math. Soc. B, Appl. Math.*, vol. 38, no. 3, pp. 388–410, Jan. 1997.

[14] J. Cheng, "Analysis of optimal driving strategies for train control systems," Ph.D. dissertation, Dept. Math., Univ. South Aust., Adelaide, SA, Australia, 1997.

[15] P. G. Howlett, "The optimal control of a train," *Ann. Oper. Res.*, vol. 98, nos. 1–4, pp. 65–87, Dec. 2000.

[16] X. Yang, X. Li, B. Ning, and T. Tang, "A survey on energy-efficient train operation for urban rail transit," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 1, pp. 2–13, Jan. 2016.

[17] E. Khmelnitsky, "On an optimal control problem of train operation," *IEEE Trans. Autom. Control*, vol. 45, no. 7, pp. 1257–1266, Jul. 2000.

[18] R. Liu and I. M. Golovitcher, "Energy-efficient operation of rail vehicles," *Transp. Res. A, Policy Pract.*, vol. 37, no. 10, pp. 917–931, 2003.

[19] S. Lu, S. Hillmansen, T. K. Ho, and C. Roberts, "Single-train trajectory optimization," *IEEE Trans. Intell. Transp. Syst.*, vol. 14, no. 2, pp. 743–750, Jun. 2013.

[20] W. ShangGuan, X.-H. Yan, B.-G. Cai, and J. Wang, "Multiobjective optimization for train speed trajectory in CTCS high-speed railway with hybrid evolutionary algorithm," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 4, pp. 2215–2225, Aug. 2015.

[21] I. A. Asnis, A. V. Dmitruk, and N. P. Osmolovskii, "Solution of the problem of the energetically optimal control of the motion of a train by the maximum principle," *USSR Comput. Math. Math. Phys.*, vol. 25, no. 6, pp. 37–44, Jan. 1985.

[22] J. Xing, X. Yang, B. Ning, Y. H. Wang, and K. F. Li, "Survey on trajectory optimization for train operation," *J. China Railway Soc.*, vol. 36, no. 4, pp. 14–22, 2014.

[23] K. K. Wong and T. K. Ho, "Coast control of train movement with genetic algorithm," in *Proc. Congr. Evol. Comput. (CEC)*, vol. 2, Dec. 2003, pp. 1280–1287.

[24] K. Wong and T. Ho, "Coast control for mass rapid transit railways with searching methods," *Proc. Inst. Elect. Eng.-Elect. Power Appl.*, vol. 151, no. 3, pp. 365–376, May 2004.

[25] K. K. Wong and T. K. Ho, "Dynamic coast control of train movement with genetic algorithm," *Int. J. Syst. Sci.*, vol. 35, nos. 13–14, pp. 835–846, Oct. 2004.

[26] S. H. Han, Y. S. Byen, J. H. Baek, T. K. An, S. G. Lee, and H. J. Park, "An optimal automatic train operation (ATO) control using genetic algorithms (GA)," in *Proc. IEEE Region Conf. Multimedia Technol. Asia–Pacific Inf. Infrastruct. (TENCON)*, Cheju Island, South Korea, Sep. 1999, pp. 360–362.

[27] B. Ke, C. Lin, and C. Yang, "Optimisation of train energy-efficient operation for mass rapid transit systems," *IET Intell. Transp. Syst.*, vol. 6, no. 1, pp. 58–66, Mar. 2012.

[28] R. Franke, P. Terwiesch, and M. Meyer, "An algorithm for the optimal control of the driving of trains," in *Proc. 39th IEEE Conf. Decis. Control*, Sydney, NSW, Australia, 2000, pp. 2123–2128.

[29] H. Ko, T. Koseki, and M. Miyatake, "Application of dynamic programming to the optimization of the running profile of a train," *Adv. Transp.*, vol. 74, pp. 103–112, May 2004.
[30] M. Miyatake and H. Ko, "Optimization of train speed profile for minimum energy consumption," *IEEJ Trans. Electr. Electron. Eng.*, vol. 5, no. 3, pp. 263–269, May 2010.
[31] N. Ghaviha, M. Bohlin, C. Holmberg, E. Dahlquist, R. Skoglund, and D. Jonasson, "A driver advisory system with dynamic losses for passenger electric multiple units," *Transp. Res. C, Emerg. Technol.*, vol. 85, pp. 111–130, Dec. 2017.
[32] J. T. Haahr, D. Pisinger, and M. Sabbaghian, "A dynamic programming approach for optimizing train speed profiles with speed restrictions and passage points," *Transp. Res. B, Methodol.*, vol. 99, pp. 167–182, May 2017.
[33] L. Zhou, L. Tong, J. Chen, J. Tang, and X. Zhou, "Joint optimization of high-speed train timetables and speed profiles: A unified modeling approach using space-time-speed grid networks," *Transp. Res. B, Methodol.*, vol. 97, pp. 157–181, Mar. 2017.
[34] Y. Huang, L. Yang, T. Tang, Z. Gao, F. Cao, and K. Li, "Train speed profile optimization with on-board energy storage devices: A dynamic programming based approach," *Comput. Ind. Eng.*, vol. 126, pp. 149–164, Dec. 2018.
[35] P. Wang, A. Trivella, R. M. P. Goverde, and F. Corman, "Train trajectory optimization for improved on-time arrival under parametric uncertainty," *Transp. Res. C, Emerg. Technol.*, vol. 119, Oct. 2020, Art. no. 102680.
[36] J. Xun, T. Liu, B. Ning, and Y. Liu, "Using approximate dynamic programming to maximize regenerative energy utilization for metro," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 9, pp. 3650–3662, Sep. 2020.
[37] H. Tang and X. Feng, "Train running time allocation algorithm based on dynamic programming," in *Proc. 32nd Chin. Control Conf.*, 2013, pp. 8157–8160.
[38] H. Dong, L. Zhang, Y. Chen, B. Ning, and W. Bai, "Multi-objective train trajectory design based on dynamic programming," in *Proc. 33rd Chin. Control Conf.*, Jul. 2014, pp. 738–743.
[39] N. Zhao, Z. Tian, L. Chen, C. Roberts, and S. Hillmansen, "Driving strategy optimization and field test on an urban rail transit system," *IEEE Intell. Transp. Syst. Mag.*, early access, Feb. 20, 2020, doi: 10.1109/MITS.2019.2926369.

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