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Dissipative Control for Fuzzy Singular Markov Jump Systems With State-Dependent Noise and Asynchronous Modes

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ABSTRACT In this paper, the issue of dissipative control for a class of discrete-time T-S fuzzy singular Markov jump systems with state-dependent noise and asynchronous modes is investigated. The hidden Markov model (HMM) is introduced to observe the asynchronous phenomenon between the original system modes and the controller modes. Sufficient conditions are established in the form of strict LMIs to guarantee that the closed-loop system is stochastically admissible and strictly $(Q, S, R) - \beta$ dissipative. An asynchronous output feedback controller is successfully proposed, which is more general than synchronous and mode-independent ones. Finally, the effectiveness and validity of the obtained results are illustrated by a numerical example.

INDEX TERMS Singular Markov jump systems, asynchronous dissipative control, hidden Markov model, linear matrix inequalities (LMIs), Takagi-Sugeno (T-S) fuzzy model.

I. INTRODUCTION

Singular systems, also known as descriptor systems, implicit systems and differential-algebraic systems, have been paid close attention by researchers in the past few decades [1]–[6]. This is because those of systems are often used to simulate a large number of dynamical systems, such as power grid systems, electric and electronic engineering, Leontief models and chemical engineering etc [7]–[11]. It should be emphasized that singular systems are more complex compared with the state-space systems, since the regularity and causality need to be considered to ensure the existence and uniqueness of impulse-free solutions to system equations [3]–[6]. Up to now, singular systems have been widely investigated and a variety of results have been proposed, for instance robust stability and stabilization [12], [13], H_∞ control [6], dissipative control [14]–[17], sliding mode control [18], [19].

In many practical circumstances, the structure and parameter of systems can suffer abrupt changes and variations that caused by component failure, repair and disconnection etc [20]–[24]. Under this condition, Markov jump parameters are introduced into the system model and call such systems as Markov jump systems (MJSs). To date, a lot of works about MJSs have been done by the researchers from the different control fields and control issues. For instance, the sliding-mode control for slow-sampling singularly perturbed MJSs was discussed in [25]. The persistent dwell-time switching rule was introduced to describe changes in transition probabilities, and some valuable results were obtained for complex dynamic networks [26], singularly perturbed switched systems [27] and fuzzy Markov jump chaotic systems [28], respectively. As far as singular Markov jump systems (SMJSs) are concerned, a large number of research results have been obtained, we refer the reader to [18], [29], [30] and references therein. Nevertheless, most of existing results on SMJSs based on the same assumption that the information of system modes is completely available to controller modes. In this case, synchronous or independent controller was designed [31], [32]. However, in the network communication link, packet loss and time delay inevitably appear in the system, which often lead to the information loss. In view of this, the hidden Markov model in [33] was presented to describe the asynchronization between the system modes and control modes. Subsequently, some results

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concerning the asynchronous control of state-space Markov jump systems are reported one after another. For example, by a stochastic Lyapunov function approach, [34] designed a dissipativity-based asynchronous controller for discrete-time Markov jump systems with mixed time delays. When the controller and quantizer are both asynchronous with the controlled systems, H_∞ control was discussed for Markov jump time-delay systems in [35]. The asynchronous static output feedback control was addressed for MJSs with extended dissipative performance including both continuous- and discrete-time cases [36]. Nevertheless, it can be found that little attention is paid on the control of SMJSs based on HMM due to the complexity of system itself. Considering the time delay in switching signal and state, an asynchronous state feedback controller in [37] was designed for guaranteeing the admissibility of SMJSs. The reference [38] investigated the asynchronous H_∞ filter design for discrete-time SMJSs with packet losses. Although the mentioned literatures made some contributions to the asynchronous control of SMJSs, the sotchastic noise was neglected in the system. It was shown from [39]–[43] that any system can be disturbed by environment noise, stochastic noise played an important role in the system model. Also, as pointed out by [40]–[42], there are some differences in methodology between singular systems and singular stochastic systems. The asynchronous dissipative control for discrete-time SMJSs with multiplicative noises was investigated in [44], in which finite piecewise homogeneous state feedback controller was designed. More recently, the authors in [45] explored stochastic H_∞ control for a class of uncertain SMJSs with multiplicative noise based on HMM. But it was known that H_∞ performance just is a special form of the dissipative performance [34], [46]. And the dissipativity has wide applications in some real systems such as power systems and electrical networks. Consequently, researching the dissipative control for the discrete-time SMJSs subject to state-dependent noise and asynchronous modes is one of the motivations of the current work.

On the other hand, T-S fuzzy model has been very popular among researchers in the past few years. The reason concentrating on such topic is that the Takagi-Sugeno (T-S) fuzzy method can approximate the nonlinear system as several linear subsystems by making use of the fuzzy membership function [47]. Compared with piecewise linear function and single linear function, T-S fuzzy model has a more compact mathematical form, which is convenient for further processing [48]. It was shown that lots of results for fuzzy MJSs had been obtained. For example, to save the scarce communication bandwidth, an event-triggered reliable controller for fuzzy MJSs with the mixed H_∞ and passive performance was proposed in [49]. When actuator faults and time-varying delays appeared in the fuzzy MJSs simultaneously, the reliable controller was designed in [50] such that the resultant closed-loop system was stochastically stable and strictly dissipative. As for the asynchronous control of fuzzy linear MJSs, some encouraging results have been proposed as well in recent years. [46] studied the dissipativity, in which the given results can be regarded as the corresponding generalization of [33]. The same problem was further discussed in [51] when intermittent measurements and quantization were concerned.

So far, according to our knowledge, little attention has been paid on the asynchronous control for T-S fuzzy SMJSs with state-dependent noise and the dissipativity, which encourages our current research. In this paper, we will discuss the asynchronous $(Q, S, R) - \beta$ dissipative control for discrete-time fuzzy SMJSs with state-dependent noise based on HMM. Firstly, we provide sufficient conditions to ensure the closed-loop system to be stochastically admissible and $(Q, S, R) - \beta$ dissipative. Then, by utilizing the augmentation technique and slack variable matrices, the asynchronous output feedback controller gains are derived by solving a set of strict linear matrix inequalities. Finally, the validity of the design method is verified by a numerical example. The main contributions of this paper are highlighted as follows.

(i) The asynchronous dissipative control is the first time discussed for SMJSs with state-dependent noise, which is formulated by T-S fuzzy model.

(ii) The Lyapunov function adopted in this paper not only depends on the fuzzy rules but also the system modes, which can reduce the conservativeness of obtained results. Besides, the auxiliary variable matrices and augmentation technique are employed in order to obtain the tractable LMIs conditions.

(iii) It is worth noting that singular matrix, noise and fuzzy rules occur concurrently in the system, which is more general. Also, what we presented cover mode-independent and synchronous results as special cases and are also applicable to H_∞ control [45] and passive control [33].

The rest of this article is given as follows. The second section describes some preliminary knowledge. In the third section, we propose the conditions that ensure the stochastic admissibility and strict $(Q, S, R) - \beta$ dissipativity of the system. In the fourth section, an example is given to illustrate the authenticity and validity of the results. Finally, the conclusion of this paper is given in the fifth section.

Notations: The symbols used in this paper are all standard. $\mathbb{N}$ denotes the positive integer set; $\mathbb{R}^n$ stands for n-dimensional Euclidean space; $\mathbb{R}^{m\times n}$ is a set of spaces composed of $m \times n$ real matrices; $A > 0$ means a real symmetric positive definite matrix; $sym(P)$ is defined as the sum of P and P^T; $\mathcal{E}(\cdot)$ and $\|\cdot\|$ refer to the expectation operator and Euclidean vector norm. In the symmetric block matrix, the symbol $*$ represents the ellipsis of the term introduced for symmetry. For a matrix $E \in \mathbb{R}^{n\times n}$ with $rank(E) = \iota$, let $E^\perp \in \mathbb{R}^{(n-\iota)\times n}$ be any matrix with full row rank satisfying $E^\perp E = 0$ and $E^\perp(E^\perp)^\top > 0$. Finally, it is assumed that the matrices in this paper are compatible with the algebraic operation.

II. PRELIMINARIES

Consider the following discrete-time T-S fuzzy SMJSs with state-dependent noise. Plant Rule i: IF θ_{1k} is η_{i1}, θ_{2k}

is $\eta_{i2}, \ldots, \theta_{vk}$ is η_{iv}, THEN:

$$\begin{cases} Ex(k+1) = [A_i(\delta_k) + A_{wi}(\delta_k)w(k)]x(k) + B_i(\delta_k)u(k) \\ \qquad\qquad +C_i(\delta_k)v(k), \\ z(k) = D_i(\delta_k)x(k) + F_i(\delta_k)u(k) + H_i(\delta_k)v(k), \\ y(k) = G_i(\delta_k)x(k), \end{cases} \tag{1}$$

where $E \in \mathbb{R}^{n\times n}$ is a singular matrix with $0 < rank(E) = \iota \le n$, $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control input, $v(k) \in l_2[0, +\infty)$ is disturbance signal, $z(k) \in \mathbb{R}^p$ is the controlled output and $y(k) \in \mathbb{R}^l$ is measured output; $w(k)$ is a sequence of random variables taking values on the given space $(\Omega, \mathcal{F}, P)$, which satisfies $\mathcal{E}[w(k)] = 0$, $\mathcal{E}[w^2(k)] = 1$, $\mathcal{E}[w(k)w(s)] = 0 (k \neq s)$. Moreover, $\theta_k = [\theta_{1k}, \theta_{2k}, \cdots, \theta_{vk}]$ is the premise variable vector. And $\eta_{i\mu} (i \in \mathcal{I} = \{1, 2, \ldots, r\}, \mu \in \mathcal{V} = \{1, 2, \ldots, \nu\})$ is the fuzzy set, where the variable i means the ith fuzzy rule and r is the total number of fuzzy rules. The parameter δ_k, taking values in $\mathcal{M} = \{1, 2, \cdots, M\}$, denotes a discrete-time Markov chain with transition probability matrix $\Xi = [\alpha_{st}]$, where

$$Pr\{\delta_{k+1} = t | \delta_k = s\} = \alpha_{st}, \quad s, t \in \mathcal{M} \tag{2}$$

with $\alpha_{st} \in [0, 1]$ and $\sum_{t=1}^{M} \alpha_{st} = 1$. For $\delta_k = s$, we denote $A_i(\delta_k) = A_{is}, A_{wi}(\delta_k) = A_{wis}, B_i(\delta_k) = B_{is}, C_i(\delta_k) = C_{is}, D_i(\delta_k) = D_{is}, F_i(\delta_k) = F_{is}, H_i(\delta_k) = H_{is}, G_i(\delta_k) = G_{is}$. $A_{is}, A_{wis}, B_{is}, C_{is}, D_{is}, F_{is}, H_{is}, G_{is}$ are real constant matrices with appropriate dimensions.

By using T-S fuzzy rules, the following fuzzy systems can be obtained for $\delta_k = s$

$$\begin{cases} Ex(k+1) = [A_{hs} + A_{whs}w(k)]x(k) + B_{hs}u(k) \\ \qquad\qquad +C_{hs}v(k), \\ z(k) = D_{hs}x(k) + F_{hs}u(k) + H_{hs}v(k), \\ y(k) = G_{hs}x(k), \end{cases} \tag{3}$$

where

$$A_{hs} = \sum_{i=1}^{r} h_i(\theta_k)A_{is}, A_{whs} = \sum_{i=1}^{r} h_i(\theta_k)A_{wis},$$
$$B_{hs} = \sum_{i=1}^{r} h_i(\theta_k)B_{is}, C_{hs} = \sum_{i=1}^{r} h_i(\theta_k)C_{is},$$
$$D_{hs} = \sum_{i=1}^{r} h_i(\theta_k)D_{is}, F_{hs} = \sum_{i=1}^{r} h_i(\theta_k)F_{is},$$
$$H_{hs} = \sum_{i=1}^{r} h_i(\theta_k)H_{is}, G_{hs} = \sum_{i=1}^{r} h_i(\theta_k)G_{is},$$
$$h_i = h_i(\theta_k) = \frac{\prod_{\mu=1}^{\nu} \eta_{i\mu}(\theta_{\mu k})}{\sum_{i=1}^{r} \prod_{\mu=1}^{\nu} \eta_{i\mu}(\theta_{\mu k})},$$

$h_i(\theta_k)$ is expressed as the standardized membership function, and $\prod_{\mu=1}^{\nu} \eta_{i\mu}(\theta_{\mu k}) \ge 0$. So it implies that $h_i(\theta_k) \ge 0, \sum_{i=1}^{r} h_i(\theta_k) = 1$.

The purpose of this note is to design an asynchronous fuzzy output feedback controller for the fuzzy SMJSs (3). The premise variable of controller is assumed to the same as that of the plant.

Controller Rule i: IF θ_{1k} is η_{i1}, θ_{2k} is $\eta_{i2}, \ldots, \theta_{vk}$ is η_{iv}, THEN:

$$u(k) = K_i(\psi_k)y(k), \tag{4}$$

where $K_i(\psi_k)$ is the controller gain matrix to be obtained under the rule i. The variable ψ_k is employed to observe the system modes, which belongs to the positive integer set $\mathcal{N} = \{1, 2, \ldots, N\}$ and satisfies the conditional transition probability matrix $\Psi = [\varphi_{sl}]$, where

$$Pr\{\psi_k = l | \delta_k = s\} = \varphi_{sl}, \quad s \in \mathcal{M}, l \in \mathcal{N} \tag{5}$$

with $\varphi_{sl} \in [0, 1]$ and $\sum_{l=1}^{N} \varphi_{sl} = 1$.

Remark 1: Remarkably, the asynchronous controller to be designed in this paper includes the following two special cases: (i) When $\mathcal{M} = \mathcal{N}$ and $\varphi_{ss} = 1$, the controller (4) becomes a mode-dependent (synchronous) one. (ii) When $\mathcal{N} = \{1\}$, the controller (4) degenerates to a mode-independent controller.

Remark 2: In reality, the abnormal transmission of the system will lead to the loss of system state information. So the system modes are sometimes hidden from the controller, which leads to the asynchronous phenomenon between the controller modes and the system modes. In view of this, as in [33], the random variable ψ_k here is introduced to represent the modes of controller, which depends on the conditional transition probability matrix $\Psi = [\varphi_{sl}]$ to establish a connection with the system modes. In this case, a HMM $(\delta_k, \psi_k, \Xi, \Psi)$ is constructed.

Let $\psi_k = l$, then (4) yields

$$u(k) = K_{hl}y(k) \tag{6}$$

with $K_{hl} = \sum_{i=1}^{r} h_i K_{il}$. Combining system (3) and controller (6), we obtain the following closed-loop system:

$$\begin{cases} Ex(k+1) = \bar{A}_{hsl}x(k) + A_{whs}x(k)w(k) + C_{hs}v(k), \\ z(k) = \bar{D}_{hsl}x(k) + H_{hs}v(k), \end{cases} \tag{7}$$

where

$$\bar{A}_{hsl} = A_{hs} + B_{hs}K_{hl}G_{hs} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \bar{A}_{ijsl},$$
$$\bar{D}_{hsl} = D_{hs} + F_{hs}K_{hl}G_{hs} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \bar{D}_{ijsl},$$
$$\bar{A}_{ijsl} = A_{is} + B_{is}K_{jl}G_{is}, \bar{D}_{ijsl} = D_{is} + F_{is}K_{jl}G_{is}.$$

Before proceeding, we present an important assumption and several basic definitions which will be used to develop our results in the sequel.

Assumption 1: $rank(E) = rank(E, A_{whs})$ for each $s \in \mathcal{M}$.

Remark 3: The above Assumption is proposed to guarantee the existence and uniqueness of the solution of the system (7). Under Assumption 1, the number of the independent variables of system (7) is $rank(E) = \iota$, that is, the diffusion term

$A_{whs}x(k)w(k)$ doesn't change the system structure in essence [41], [42].

Definition 1 [45]: (i) The system (7) with $v(k) = 0$ is said to be stochastically stable if for any initial condition $x_0 \in \mathbb{R}^n$ and $\delta_0 \in \mathcal{M}$, there exists a scalar $\digamma(x_0, \delta_0) > 0$ such that

$$\lim_{L\to\infty} \mathcal{E}\left\{ \sum_{k=0}^{L} \|x(k)\|^2 | x_0, \delta_0 \right\} < \digamma(x_0, \delta_0). \tag{8}$$

(ii) The system (7) is said to be stochastically admissible if it is stochastically stable and has an impulse-free solution.

Definition 2 [34]: The system (7) is said to be strictly $(Q, S, R) - \beta$ dissipative under the zero initial condition and $v(k) \neq 0$, if there exists a scalar $\beta > 0$ such that

$$\sum_{k=0}^{n} \mathcal{E}\{r(v(k), z(k))\} > \beta \sum_{k=0}^{n} v^{\mathrm{T}}(k)v(k), \quad \forall n > 0, \tag{9}$$

where $r(v(k), z(k)) = z^{\mathrm{T}}(k)Qz(k) + 2z^{\mathrm{T}}(k)Sv(k) + v^{\mathrm{T}}(k)Rv(k)$ is called as the energy supply function, Q, S, R are real rational matrices, Q and R are symmetrical. Assume that $(Q_{-}^{\frac{1}{2}}) = (-Q)^{\frac{1}{2}} \geq 0$ satisfies $-Q = (Q_{-}^{\frac{1}{2}})^2$, that is to say $Q \leq 0$.

Remark 4: As observed in [34], [46], [51], H_∞ control and passive control are integrated in the dissipative control. In other words, by selecting appropriate dissipativity parameters Q, S, R, the above two performance indexes can be respectively obtained. (i) When $Q = -I, S = 0, R = (\varsigma^2 + \beta)I$ (ς is a positive scalar), the dissipativity corresponds to H_∞ performance requirement. (ii) When $Q = 0, S = I, R = 2\beta I$, the dissipativity coincides with the passivity.

III. MAIN RESULTS

In this section, we propose sufficient conditions such that the considered system (7) is stochastically admissible and $(Q, S, R) - \beta$ dissipative. And then, the output feedback controller is given by solving the strict LMIs.

Theorem 1: Given a scalar $\beta > 0$, the real matrices Q, S and R, where Q is negative semidefinite matrix and Q, R are symmetric matrices. System (7) is stochastically admissible and strictly $(Q, S, R) - \beta$ dissipative, if there exist matrices $P_{is} > 0, P_{ct} > 0, W_{isl} > 0$, and $U = U^T$ such that the following conditions hold for any $s, t \in \mathcal{M}$, $l \in \mathcal{N}$ and $c, i, j \in \mathcal{I}$

$$\sum_{l=1}^{N} \varphi_{sl} W_{isl} < P_{is}, \tag{10}$$

$$\Gamma_{ciisl} < 0, \tag{11}$$

$$\Gamma_{cijsl} + \Gamma_{cjisl} < 0, \quad i < j, \tag{12}$$

where

$$\Gamma_{cijsl} = \begin{bmatrix} \Gamma_{cijsl11} & \Gamma_{cijsl12} & \bar{D}_{ijsl}^T (Q_{-}^{\frac{1}{2}})^T \\ * & \Gamma_{cijsl22} & H_{is}^T (Q_{-}^{\frac{1}{2}})^T \\ * & * & -I \end{bmatrix},$$

$$\Gamma_{cijsl11} = -E^T W_{isl} E + \bar{A}_{ijsl}^T \bar{X}_{cs} \bar{A}_{ijsl} + A_{wis}^T \bar{X}_{cs} A_{wis},$$

$$\Gamma_{cijsl12} = \bar{A}_{ijsl}^T \bar{X}_{cs} C_{is} - \bar{D}_{ijsl}^T S,$$

$$\Gamma_{cijsl22} = C_{is}^T \bar{X}_{cs} C_{is} - sym(H_{is}^T S) - R + \beta I,$$

$$\bar{X}_{cs} = \sum_{t=1}^{M} \alpha_{st} X_{ct}, X_{ct} = P_{ct} - (E^{\perp})^T U E^{\perp}.$$

Proof: Using the fuzzy rules, we give the following expressions.

$$P_{hs} = \sum_{i=1}^{r} h_i P_{is} > 0, W_{hsl} = \sum_{i=1}^{r} h_i W_{isl} > 0,$$

$$P_{h't} = \sum_{c=1}^{r} h'_c P_{ct} > 0, X_{h't} = \sum_{c=1}^{r} h'_c X_{ct},$$

$$\bar{X}_{h's} = \sum_{c=1}^{r} h'_c \bar{X}_{cs}, \tag{13}$$

where $h' = h_{k+1}$ represents the membership function at $k+1$. From (10) and (13), we can get

$$\sum_{l=1}^{N} \varphi_{sl} W_{hsl} < P_{hs}. \tag{14}$$

At first, we show that the system (7) ($v(k) = 0$) has an impulse-free solution. By combining (11) - (13), it is obtained that

$$\begin{aligned}\Gamma_{h'hsl} &= \sum_{c=1}^{r}\sum_{i=1}^{r}\sum_{j=1}^{r} h'_c(\theta_k) h_i(\theta_k) h_j(\theta_k) \Gamma_{cijsl} \\ &= \sum_{c=1}^{r} h'_c \left[\sum_{i=1}^{r} h_i^2 \Gamma_{ciisl} + \sum_{i=1}^{r-1} \sum_{j=i+1}^{r} h_i h_j (\Gamma_{cijsl} + \Gamma_{cjisl}) \right] \\ &< 0,\end{aligned} \tag{15}$$

where

$$\Gamma_{h'hsl} = \begin{bmatrix} \Gamma_{h'hsl11} & \Gamma_{h'hsl12} & \bar{D}_{hsl}^T (Q_{-}^{\frac{1}{2}})^T \\ * & \Gamma_{h'hsl22} & H_{hs}^T (Q_{-}^{\frac{1}{2}})^T \\ * & * & -I \end{bmatrix},$$

$$\Gamma_{h'hsl11} = -E^T W_{hsl} E + \bar{A}_{hsl}^T \bar{X}_{h's} \bar{A}_{hsl} + A_{whs}^T \bar{X}_{h's} A_{whs},$$

$$\Gamma_{h'hsl12} = \bar{A}_{hsl}^T \bar{X}_{h's} C_{hs} - \bar{D}_{hsl}^T S,$$

$$\Gamma_{h'hsl22} = C_{hs}^T \bar{X}_{h's} C_{hs} - sym(H_{hs}^T S) - R + \beta I.$$

Under Assumption 1, there exists a pair of nonsingular matrices M and N such that

$$MEN = \begin{bmatrix} I_\iota & 0 \\ 0 & 0 \end{bmatrix}, MA_{whs}N = \begin{bmatrix} A_{whs11} & A_{whs12} \\ 0 & 0 \end{bmatrix}. \tag{16}$$

Set

$$M\bar{A}_{hsl}N = \begin{bmatrix} \bar{A}_{hsl11} & \bar{A}_{hsl12} \\ \bar{A}_{hsl21} & \bar{A}_{hsl22} \end{bmatrix} \tag{17}$$

and

$$M^{-T} X_{h't} M^{-1} = \begin{bmatrix} X_{h't11} & X_{h't12} \\ X_{h't21} & X_{h't22} \end{bmatrix},$$

$$M^{-T} W_{hsl} M^{-1} = \begin{bmatrix} W_{hsl11} & W_{hsl12} \\ W_{hsl21} & W_{hsl22} \end{bmatrix}, \tag{18}$$

where $X_{h't12} = X^T_{h't21}$, $W_{hsl12} = W^T_{hsl21}$. It follows from (15) that

$$-E^T W_{hsl} E + \bar{A}^T_{hsl} \bar{X}_{h's} \bar{A}_{hsl} + A^T_{whs} \bar{X}_{h's} A_{whs} < 0. \quad (19)$$

In view of $W_{hsl} > 0$, $P_{h't} > 0$ and $E^{\perp}E = 0$, we obtain

$$\begin{aligned} E^T W_{hsl} E &\geq 0, \\ E^T X_{h't} E &= E^T(P_{h't} - (E^{\perp})^T U E^{\perp})E \\ &= E^T P_{h't} E - (E^{\perp}E)^T U E^{\perp} E \\ &= E^T P_{h't} E \\ &\geq 0. \end{aligned} \quad (20)$$

By (16), (17) and (18), we have

$$\begin{aligned} &E^T X_{h't} E \\ &= N^{-T}(MEN)^T M^{-T} M^T (M^{-T} X_{h't} M^{-1}) M \\ &\quad M^{-1}(MEN)N^{-1} \\ &= N^{-T} \begin{bmatrix} X_{h't11} & 0 \\ 0 & 0 \end{bmatrix} N^{-1}, \end{aligned} \quad (21)$$

$$\begin{aligned} &E^T W_{hsl} E \\ &= N^{-T}(MEN)^T M^{-T} M^T (M^{-T} W_{hsl} M^{-1}) M \\ &\quad M^{-1}(MEN)N^{-1} \\ &= N^{-T} \begin{bmatrix} W_{hsl11} & 0 \\ 0 & 0 \end{bmatrix} N^{-1}, \end{aligned} \quad (22)$$

$$\begin{aligned} &\bar{A}^T_{hsl} X_{h't} \bar{A}_{hsl} \\ &= N^{-T}(M\bar{A}_{hsl}N)^T M^{-T} M^T (M^{-T} X_{h't} M^{-1}) M \\ &\quad M^{-1}(M\bar{A}_{hsl}N)N^{-1} \\ &= N^{-T} \begin{bmatrix} \Pi_{h'hstl11} & \Pi_{h'hstl12} \\ * & \Pi_{h'hstl22} \end{bmatrix} N^{-1}, \end{aligned} \quad (23)$$

$$\begin{aligned} &A^T_{whs} X_{h't} A_{whs} \\ &= N^{-T}(MA_{whs}N)^T M^{-T} M^T (M^{-T} X_{h't} M^{-1}) M \\ &\quad M^{-1}(MA_{whs}N)N^{-1} \\ &= N^{-T} \begin{bmatrix} \Lambda_{wh'hst11} & \Lambda_{wh'hst12} \\ * & \Lambda_{wh'hst22} \end{bmatrix} N^{-1}, \end{aligned} \quad (24)$$

where

$$\begin{aligned} \Pi_{h'hstl11} &= \bar{A}^T_{hsl11} X_{h't11} \bar{A}_{hsl11} + \bar{A}^T_{hsl21} X_{h't21} \bar{A}_{hsl11} \\ &\quad + \bar{A}^T_{hsl11} X_{h't12} \bar{A}_{hsl21} + \bar{A}^T_{hsl21} X_{h't22} \bar{A}_{hsl21}, \\ \Pi_{h'hstl12} &= \bar{A}^T_{hsl11} X_{h't11} \bar{A}_{hsl12} + \bar{A}^T_{hsl21} X_{h't21} \bar{A}_{hsl12} \\ &\quad + \bar{A}^T_{hsl11} X_{h't12} \bar{A}_{hsl22} + \bar{A}^T_{hsl21} X_{h't22} \bar{A}_{hsl22}, \\ \Pi_{h'hstl22} &= \bar{A}^T_{hsl12} X_{h't11} \bar{A}_{hsl12} + \bar{A}^T_{hsl22} X_{h't21} \bar{A}_{hsl12} \\ &\quad + \bar{A}^T_{hsl12} X_{h't12} \bar{A}_{hsl22} + \bar{A}^T_{hsl22} X_{h't22} \bar{A}_{hsl22}, \\ \Lambda_{wh'hst11} &= A^T_{whs11} X_{h't11} A_{whs11}, \\ \Lambda_{wh'hst12} &= A^T_{whs11} X_{h't11} A_{whs12}, \\ \Lambda_{wh'hst22} &= A^T_{whs12} X_{h't11} A_{whs12}. \end{aligned}$$

Applying (22)-(24) to (19), we derive

$$\begin{aligned} -E^T W_{hsl} E &+ \bar{A}^T_{hsl} \bar{X}_{h's} \bar{A}_{hsl} + A^T_{whs} \bar{X}_{h's} A_{whs} \\ &= N^{-T} \begin{bmatrix} \nabla_1 & \nabla_2 \\ * & \nabla_3 \end{bmatrix} N^{-1} < 0, \end{aligned} \quad (25)$$

where

$$\begin{aligned} \nabla_3 &= \bar{A}^T_{hsl12} \bar{X}_{h's11} \bar{A}_{hsl12} + \bar{A}^T_{hsl22} \bar{X}_{h's21} \bar{A}_{hsl12} \\ &\quad + \bar{A}^T_{hsl12} \bar{X}_{h's12} \bar{A}_{hsl22} + \bar{A}^T_{hsl22} \bar{X}_{h's22} \bar{A}_{hsl22} \\ &\quad + \bar{A}^T_{whs12} \bar{X}_{h's11} \bar{A}_{whs12}, \\ \bar{X}_{h's11} &= \sum_{t=1}^{M} \alpha_{st} X_{h't11}, \bar{X}_{h's12} = \sum_{t=1}^{M} \alpha_{st} X_{h't12}, \\ \bar{X}_{h's21} &= \sum_{t=1}^{M} \alpha_{st} X_{h't21}, \bar{X}_{h's22} = \sum_{t=1}^{M} \alpha_{st} X_{h't22}. \end{aligned}$$

Since ∇_1 and ∇_2 are independent of the results discussed below, their concrete expressions are omitted. According to (19)-(25), it is easy to see that $\nabla_3 < 0$ and $X_{h't11} \geq 0$. Then we have

$$\begin{aligned} &\bar{A}^T_{hsl22} \bar{X}_{h's21} \bar{A}_{hsl12} + \bar{A}^T_{hsl12} \bar{X}_{h's12} \bar{A}_{hsl22} \\ &\qquad + \bar{A}^T_{hsl22} \bar{X}_{h's22} \bar{A}_{hsl22} < 0. \end{aligned} \quad (26)$$

(26) implies that $\bar{A}_{hsl22}$ is nonsingular. Let $\check{x}(k) = N^{-1}x(k) = \left[x_1^T(k)\ x_2^T(k)\right]^T$ for $k \in \mathbb{N}$, and denote

$$\bar{M}_{hsl} = \begin{bmatrix} I & -\bar{A}_{hsl12}\bar{A}^{-1}_{hsl22} \\ 0 & I \end{bmatrix} M. \quad (27)$$

By (16), (17) and (27), we immediately have

$$\begin{aligned} \bar{M}_{hsl} EN &= \begin{bmatrix} I_{\iota} & 0 \\ 0 & 0 \end{bmatrix}, \\ \bar{M}_{hsl} A_{whs} N &= \begin{bmatrix} A_{whs11} & A_{whs12} \\ 0 & 0 \end{bmatrix}, \\ \bar{M}_{hsl} \bar{A}_{hsl} N &= \begin{bmatrix} \bar{A}_{hsl11} - \bar{A}_{hsl12}\bar{A}^{-1}_{hsl22}\bar{A}_{hsl21} & 0 \\ \bar{A}_{hsl21} & \bar{A}_{hsl22} \end{bmatrix}. \end{aligned}$$

Then, the system (7) with $v(k) = 0$ is equivalent to

$$\begin{cases} x_1(k+1) = (\bar{A}_{hsl11} - \bar{A}_{hsl12}\bar{A}^{-1}_{hsl22}\bar{A}_{hsl21})x_1(k) \\ +[A_{whs11}x_1(k) + A_{whs12}x_2(k)]w(k), \\ 0 = \bar{A}_{hsl21}x_1(k) + \bar{A}_{hsl22}x_2(k). \end{cases} \quad (28)$$

Since $\bar{A}_{hsl22}$ is a nonsingular matrix, this implies

$$x_2(k) = -\bar{A}^{-1}_{hsl22}\bar{A}_{hsl21}x_1(k). \quad (29)$$

Substituting (29) into (28), it means that (28) is a normal discrete-time stochastic system. Thus, the system (7) with $v(k) = 0$ has an impulse-free solution.

In the following, we will prove that the system (7) ($v(k) = 0$) is stochastically stable. Consider the following Lyapunov function

$$V(x(k), \delta_k) = x^T(k) E^T P_{h\delta_k} Ex(k), \quad (30)$$

where $P_{h\delta_k} = \sum_{i=1}^{r} h_i P_{i\delta_k} > 0$. In the case of $v(k) = 0$, by making use of (7) and the difference of $V(x(k), \delta_k)$, we have

$$\begin{aligned} &\mathcal{E}\{\Delta V_k\} \\ &= \mathcal{E}\{V(x(k+1), \delta_{k+1})\} - \mathcal{E}\{V(x(k), \delta_k)\} \\ &= \mathcal{E}\{\mathcal{E}[V(x(k+1), \delta_{k+1}=t)|x(k), \delta_k = s]\} - \mathcal{E}\{V(x(k), \delta_k \end{aligned}$$

$$= s)\} - \mathcal{E}\{x^T(k+1)E^T(E^{\perp})^T UE^{\perp}Ex(k+1)\}$$
$$= \mathcal{E}\{\sum_{l=1}^{N}\varphi_{sl}[(\bar{A}_{hsl}x(k) + A_{whs}x(k)w(k))^T(\sum_{t=1}^{M}\alpha_{st}(P_{h't}$$
$$-(E^{\perp})^T UE^{\perp}))(\bar{A}_{hsl}x(k) + A_{whs}x(k)w(k))]\}$$
$$-\mathcal{E}\{x^T(k)E^T P_{hs}Ex(k)\}$$
$$= \mathcal{E}\{\sum_{l=1}^{N}\varphi_{sl}[(\bar{A}_{hsl}x(k) + A_{whs}x(k)w(k))^T\bar{X}_{h's}(\bar{A}_{hsl}x(k)$$
$$+A_{whs}x(k)w(k))]\} - \mathcal{E}\{x^T(k)E^T P_{hs}Ex(k)\}$$
$$= \mathcal{E}\{x^T(k)\sum_{l=1}^{N}\varphi_{sl}(\bar{A}_{hsl}^T\bar{X}_{h's}\bar{A}_{hsl} + A_{whs}^T\bar{X}_{h's}A_{whs}$$
$$-E^T P_{hs}E)x(k)\}. \quad (31)$$

The derivation process here is similar to that in [45]. Let $\Lambda_{h'hsl} = \sum_{l=1}^{N}\varphi_{sl}(\bar{A}_{hsl}^T\bar{X}_{h's}\bar{A}_{hsl} + A_{whs}^T\bar{X}_{h's}A_{whs} - E^T P_{hs}E)$, (14), (19) and (31) deduce

$$\mathcal{E}\{\Delta V_k\} = \mathcal{E}\{x^T(k)\Lambda_{h'hsl}x(k)\} < 0. \quad (32)$$

For (32), there exists a scalar $\gamma = \lambda_{min}(-\Lambda_{h'hsl})$ such that

$$\mathcal{E}\{\Delta V_k\} < -\gamma\mathcal{E}\{\|x(k)\|^2\}. \quad (33)$$

Summing up k both sides of (33) from 0 to τ, we get

$$\mathcal{E}\{V(x(0), \delta_0)\} - \mathcal{E}\{V(x(\tau+1), \delta_{\tau+1})\} > \gamma\mathcal{E}\left\{\sum_{k=0}^{\tau}\|x(k)\|^2|x(0), \delta_0\right\}. \quad (34)$$

Taking the limit on (34), we obtain

$$\lim_{\tau\to\infty}\mathcal{E}\left\{\sum_{k=0}^{\tau}\|x(k)\|^2|x(0), \delta_0\right\} < \mathcal{F}(x(0), \delta_0), \quad (35)$$

where $\mathcal{F}(x(0), \delta_0) = \frac{\mathcal{E}\{V(x(0),\delta_0)\}}{\gamma}$. Therefore, according to Theorem 1, the system (7) with $v(k) = 0$ is stochastically admissible. At last, we verify that the system (7) is strictly $(Q, S, R) - \beta$ dissipative. By the energy supply function and a series of computation, we obtain

$$\mathcal{E}\{\Delta V_k - r(z(k), v(k)) + \beta v^T(k)v(k)\}$$
$$= \mathcal{E}\{V(x(k+1), \delta_{k+1}) - V(x(k), \delta_k) - (z^T(k)Qz(k)$$
$$+2z^T(k)Sv(k) + v^T(k)Rv(k)) + \beta v^T(k)v(k)\}$$
$$= \mathcal{E}\{\sum_{l=1}^{N}\varphi_{sl}[(\bar{A}_{hsl}x(k) + A_{whs}x(k)w(k) + C_{hs}v(k))^T$$
$$\bar{X}_{h's}(\bar{A}_{hsl}x(k) + A_{whs}x(k)w(k) + C_{hs}v(k))$$
$$-x^T(k)E^T P_{hs}Ex(k) - (\bar{D}_{hsl}x(k) + H_{hs}v(k))^T Q$$
$$(\bar{D}_{hsl}x(k) + H_{hs}v(k)) - 2(\bar{D}_{hsl}x(k) + H_{hs}v(k))^T Sv(k)$$
$$-v^T(k)Rv(k) + \beta v^T(k)v(k)]\}$$
$$= \mathcal{E}\{\zeta_k^T\Xi_{h'hsl}\zeta_k\}, \quad (36)$$

where $\zeta_k = \begin{bmatrix} x^T(k) & v^T(k)\end{bmatrix}^T$ and

$$\Xi_{h'hsl} = \sum_{l=1}^{N}\varphi_{sl}\begin{bmatrix}\Xi_{h'hsl11} & \Xi_{h'hsl12}\\ * & \Xi_{h'hsl22}\end{bmatrix},$$

$$\Xi_{h'hsl11} = \bar{A}_{hsl}^T\bar{X}_{h's}\bar{A}_{hsl} + A_{whs}^T\bar{X}_{h's}A_{whs} - E^T P_{hs}E$$
$$-\bar{D}_{hsl}^T Q\bar{D}_{hsl},$$
$$\Xi_{h'hsl12} = \bar{A}_{hsl}^T\bar{X}_{h's}C_{hs} - \bar{D}_{hsl}^T QH_{hs} - \bar{D}_{hsl}^T S,$$
$$\Xi_{h'hsl22} = C_{hs}^T\bar{X}_{h's}C_{hs} - H_{hs}^T QH_{hs} - sym(H_{hs}^T S)$$
$$-R + \beta I.$$

Employing Schur complement lemma to (15) and considering $\varphi_{sl} \geq 0$, we derive

$$\mathcal{E}\{\Delta V_k - r(v(k), z(k)) + \beta v^T(k)v(k)\} < 0. \quad (37)$$

Summing up (37) from $k = 0$ to n, we have

$$\mathcal{E}\{V(x(n+1), \delta_{n+1})\} - \mathcal{E}\{V(x(0), \delta_0)\}$$
$$-\sum_{k=0}^{n}\mathcal{E}\{r(v(k), z(k))\} + \beta\mathcal{E}\left\{\sum_{k=0}^{n}v^{\mathrm{T}}(k)v(k)\right\} < 0. \quad (38)$$

Because of $\mathcal{E}\{V(x(0), \delta_0)\} = 0$ and $\mathcal{E}\{V(x(n+1), \delta_{n+1})\} > 0$, (38) results in

$$\sum_{k=0}^{n}\mathcal{E}\{r(v(k), z(k))\} > \beta\sum_{k=0}^{n}v^{\mathrm{T}}(k)v(k), \forall n \geq 0.$$

Consequently, the system (7) is strictly $(Q, S, R) - \beta$ dissipative. The proof is completed.

Remark 5: Note that in Theorem 1, HMM is implemented to detect the system modes and stochastically admissible conditions for dissipativity of fuzzy SMJSs are proposed. When system modes and control modes are synchronous or independent, Theorem 1 can not only be attributed to corresponding results of [3], [6], [14] but also be regarded as the discrete-time counterpart of [15]. Moreover, Lyapunov function selected in Theorem 1 relies on both fuzzy rules and system modes, although the amount of variables increases, the results are less conservative.

Remark 6: (i) The reference [46] studied the fuzzy MJSs, which was limited to the scope of normal linear systems. However, we discuss singular systems in this paper, which are more complex than normal systems. Because the regularity and non-impulsiveness must be considered to ensure that the system has an impulse-free solution. (ii) The results of [44] were different from those of this paper, which proposed the piecewise homogeneous Markov chain to reflect the asynchronization. Concretely speaking, the controller modes obeyed the finite piecewise homogeneous Markov chain, and the system modes followed the homogeneous Markov chain. But we have described the asynchronous phenomenon by a hidden Markov model, in which the controller modes and the system modes are connected through a certain transition probability matrix. On the other hand, [44] investigated state feedback control, and the method adopted was not suitable for dealing with output feedback control. While the system augmentation approach is used in Theorem 2 to design the output feedback controller, which is more extensive and effective for studying the problem of state feedback control.

It should be pointed out that the criterion obtained in Theorem 1 can not be directly applied to design the desired asynchronous output feedback controller. To solve this problem, some tractable conditions will be presented in the next theorem.

Before proceeding, we introduce the augmented dimension form of system (7)

$$\begin{cases}\tilde{E}\tilde{x}(k+1)=\tilde{A}_{hsl}\tilde{x}(k)+\tilde{A}_{whs}\tilde{x}(k)w(k)+\tilde{C}_{hs}v(k),\\ z(k)=\tilde{D}_{hs}\tilde{x}(k)+H_{hs}v(k),\end{cases} \tag{39}$$

where

$$\tilde{E}=\begin{bmatrix}E & 0\\ 0 & 0\end{bmatrix},\ \tilde{A}_{hsl}=\begin{bmatrix}A_{hs} & B_{hs}\\ K_{hl}G_{hs} & -I\end{bmatrix},$$
$$\tilde{A}_{whs}=\begin{bmatrix}A_{whs} & 0\\ 0 & 0\end{bmatrix},\ \tilde{C}_{hs}=\begin{bmatrix}C_{hs}\\ 0\end{bmatrix},$$
$$\tilde{D}_{hs}=\begin{bmatrix}D_{hs} & F_{hs}\end{bmatrix},\ \tilde{x}(k)=\begin{bmatrix}x^T(k) & u^T(k)\end{bmatrix}^T.$$

Theorem 2: Given a scalar $\beta > 0$, the real matrices Q, S and R, where Q is negative semidefinite matrix and Q, R are symmetric matrices. System (7) is stochastically admissible and strictly $(Q, S, R) - \beta$ dissipative if one of the following two conditions holds.

(i) There exist matrices $\mathscr{P}_{is} > 0$, $\mathscr{P}_{ct} > 0$, $\mathscr{W}_{isl} > 0$ and $\mathscr{U} = \mathscr{U}^T$ for any $s, t \in \mathcal{M}$, $l \in \mathcal{N}$ and $c, i, j \in \mathcal{I}$ such that

$$\sum_{l=1}^{N}\varphi_{sl}\mathscr{W}_{isl} < \mathscr{P}_{is}, \tag{40}$$
$$\Upsilon_{ciisl} < 0, \tag{41}$$
$$\Upsilon_{cijsl}+\Upsilon_{cjisl} < 0,\quad i<j, \tag{42}$$

where

$$\Upsilon_{cijsl}=\begin{bmatrix}\Upsilon_{cijsl11} & \Upsilon_{cijsl12} & \tilde{D}_{is}^T(Q_-^{\frac{1}{2}})^T\\ * & \Upsilon_{cijsl22} & H_{is}^T(Q_-^{\frac{1}{2}})^T\\ * & * & -I\end{bmatrix},$$
$$\Upsilon_{cijsl11}=-\tilde{E}^T\mathscr{W}_{isl}\tilde{E}+\tilde{A}_{ijsl}^T\tilde{X}_{cs}\tilde{A}_{ijsl}+\tilde{A}_{wis}^T\tilde{X}_{cs}\tilde{A}_{wis},$$
$$\Upsilon_{cijsl12}=\tilde{A}_{ijsl}^T\tilde{X}_{cs}\tilde{C}_{is}-\tilde{D}_{is}^TS,$$
$$\Upsilon_{cijsl22}=\tilde{C}_{is}^T\tilde{X}_{cs}\tilde{C}_{is}-sym(H_{is}^TS)-R+\beta I,$$
$$\tilde{X}_{cs}=\sum_{t=1}^{M}\alpha_{st}\tilde{X}_{ct},\quad \tilde{X}_{ct}=\mathscr{P}_{ct}-(\tilde{E}^{\perp})^T\tilde{\mathscr{U}}\tilde{E}^{\perp},$$
$$\tilde{E}^{\perp}=\begin{bmatrix}E^{\perp} & 0\\ 0 & I\end{bmatrix},\quad \tilde{\mathscr{U}}=\begin{bmatrix}\mathscr{U} & 0\\ 0 & 0\end{bmatrix},$$
$$\tilde{A}_{ijsl}=\begin{bmatrix}A_{is} & B_{is}\\ K_{jl}G_{is} & -I\end{bmatrix},\quad \tilde{A}_{wis}=\begin{bmatrix}A_{wis} & 0\\ 0 & 0\end{bmatrix},$$
$$\tilde{C}_{is}=\begin{bmatrix}C_{is}\\ 0\end{bmatrix},\quad \tilde{D}_{is}=\begin{bmatrix}D_{is} & F_{is}\end{bmatrix}.$$

(ii) There exist matrices $\mathscr{P}_{is} > 0$, $\mathscr{P}_{ct} > 0$, $\mathscr{W}_{isl} > 0$, $\mathscr{V}_l$, $\mathscr{Y}_l$ and $\mathscr{U} = \mathscr{U}^T$ satisfying (40) and the following LMIs hold for any $s, t \in \mathcal{M}$, $l \in \mathcal{N}$ and $c, i, j \in \mathcal{I}$

$$\tilde{\Upsilon}_{ciisl} < 0, \tag{43}$$
$$\tilde{\Upsilon}_{cijsl}+\tilde{\Upsilon}_{cjisl} < 0,\quad i<j, \tag{44}$$

where

$$\tilde{\Upsilon}_{cijsl}=\begin{bmatrix}\tilde{\Upsilon}_{cijsl11} & -\mathscr{V}_l+\mathscr{A}_{ijsl}^T\mathscr{Y}_l^T\\ * & \mathscr{X}_{cs}-\mathscr{Y}_l-\mathscr{Y}_l^T\end{bmatrix},$$
$$\mathscr{X}_{cs}=\begin{bmatrix}\tilde{X}_{cs} & 0\\ 0 & I\end{bmatrix},$$
$$\tilde{\Upsilon}_{cijsl11}=\mathscr{K}_{isl}+\mathscr{A}_{wis}^T\mathscr{X}_{cs}\mathscr{A}_{wis}+sym(\mathscr{V}_l\mathscr{A}_{ijsl}),$$
$$\mathscr{K}_{isl}=\begin{bmatrix}-\tilde{E}^T\mathscr{W}_{isl}\tilde{E} & -\tilde{D}_{is}^TS\\ * & -sym(H_{is}^TS)-R+\beta I\end{bmatrix},$$
$$\mathscr{A}_{ijsl}=\begin{bmatrix}\tilde{A}_{ijsl} & \tilde{C}_{is}\\ Q_-^{\frac{1}{2}}\tilde{D}_{is} & Q_-^{\frac{1}{2}}H_{is}\end{bmatrix},\ \mathscr{A}_{wis}=\begin{bmatrix}\tilde{A}_{wis} & 0\\ 0 & 0\end{bmatrix}.$$

Proof: From (41)-(42), it follows that

$$\begin{aligned}\Upsilon_{h'hsl}&=\sum_{c=1}^{r}\sum_{i=1}^{r}\sum_{j=1}^{r}h_c'(\theta_k)h_i(\theta_k)h_j(\theta_k)\Upsilon_{cijsl}\\&=\sum_{c=1}^{r}h_c'\Big[\sum_{i=1}^{r}h_i^2\Upsilon_{ciisl}+\sum_{i=1}^{r-1}\sum_{j=i+1}^{r}h_ih_j\\&\quad\times(\Upsilon_{cijsl}+\Upsilon_{cjisl})\Big]<0.\end{aligned} \tag{45}$$

In the same way, (43)-(44) leads to

$$\begin{aligned}\tilde{\Upsilon}_{h'hsl}&=\sum_{c=1}^{r}\sum_{i=1}^{r}\sum_{j=1}^{r}h_c'(\theta_k)h_i(\theta_k)h_j(\theta_k)\tilde{\Upsilon}_{cijsl}\\&=\sum_{c=1}^{r}h_c'\Big[\sum_{i=1}^{r}h_i^2\tilde{\Upsilon}_{ciisl}+\sum_{i=1}^{r-1}\sum_{j=i+1}^{r}h_ih_j\\&\quad\times(\tilde{\Upsilon}_{cijsl}+\tilde{\Upsilon}_{cjisl})\Big]<0,\end{aligned} \tag{46}$$

where

$$\Upsilon_{h'hsl}=\begin{bmatrix}\Upsilon_{h'hsl11} & \Upsilon_{h'hsl12} & \tilde{D}_{hs}^T(Q_-^{\frac{1}{2}})^T\\ * & \Upsilon_{h'hsl22} & H_{hs}^T(Q_-^{\frac{1}{2}})^T\\ * & * & -I\end{bmatrix},$$
$$\tilde{\Upsilon}_{h'hsl}=\begin{bmatrix}\tilde{\Upsilon}_{h'hsl11} & -\mathscr{V}_l+\mathscr{A}_{hsl}^T\mathscr{Y}_l^T\\ * & \mathscr{X}_{h's}-\mathscr{Y}-\mathscr{Y}^T\end{bmatrix},$$
$$\Upsilon_{h'hsl11}=-\tilde{E}^T\mathscr{W}_{hsl}\tilde{E}+\tilde{A}_{hsl}^T\tilde{X}_{h's}\tilde{A}_{hsl}+\tilde{A}_{whs}^T\tilde{X}_{h's}\tilde{A}_{whs},$$
$$\Upsilon_{h'hsl12}=\tilde{A}_{hsl}^T\tilde{X}_{h's}\tilde{C}_{hs}-\tilde{D}_{hs}^TS,$$
$$\Upsilon_{h'hsl22}=\tilde{C}_{hs}^T\tilde{X}_{h's}\tilde{C}_{hs}-sym(H_{hs}^TS)-R+\beta I,$$
$$\tilde{\Upsilon}_{h'hsl11}=\mathscr{K}_{hsl}+\mathscr{A}_{whs}^T\mathscr{X}_{h's}\mathscr{A}_{whs}+sym(\mathscr{V}\mathscr{A}_{hsl}),$$
$$\mathscr{K}_{hsl}=\begin{bmatrix}-\tilde{E}^T\mathscr{W}_{hsl}\tilde{E} & -\tilde{D}_{hs}^TS\\ * & -sym(H_{hs}^TS)-R+\beta I\end{bmatrix},$$
$$\mathscr{A}_{whs}=\begin{bmatrix}\tilde{A}_{whs} & 0\\ 0 & 0\end{bmatrix},\ \mathscr{X}_{h's}=\begin{bmatrix}\tilde{X}_{h's} & 0\\ 0 & I\end{bmatrix},$$
$$\mathscr{A}_{hsl}=\begin{bmatrix}\tilde{A}_{hsl} & \tilde{C}_{hs}\\ Q_-^{\frac{1}{2}}\tilde{D}_{hs} & Q_-^{\frac{1}{2}}H_{hs}\end{bmatrix},$$
$$\tilde{X}_{h's}=\sum_{t=1}^{M}\alpha_{st}\tilde{X}_{h't},\tilde{X}_{h't}=\mathscr{P}_{h't}-(\tilde{E}^{\perp})^T\tilde{\mathscr{U}}\tilde{E}^{\perp}.$$

(i) By the matrix decomposition technique, matrices $\tilde{A}_{hsl}, \tilde{A}_{whs}, \tilde{D}_{hs}$ and $\tilde{E}$ are respectively expressed in the following forms,

$$\begin{aligned}
\tilde{A}_{hsl} &= \begin{bmatrix} I & -B_{hs} \\ 0 & I \end{bmatrix}\begin{bmatrix} \bar{A}_{hsl} & 0 \\ 0 & -I \end{bmatrix}\begin{bmatrix} I & 0 \\ -K_{hl}G_{hs} & I \end{bmatrix}, \\
\tilde{A}_{whs} &= \begin{bmatrix} A_{whs} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I & 0 \\ -K_{hl}G_{hs} & I \end{bmatrix}, \\
\tilde{D}_{hs} &= [D_{hs} \quad F_{hs}]\begin{bmatrix} I & 0 \\ K_{hl}G_{hs} & I \end{bmatrix}\begin{bmatrix} I & 0 \\ -K_{hl}G_{hs} & I \end{bmatrix}, \\
\tilde{E} &= \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I & 0 \\ -K_{hl}G_{hs} & I \end{bmatrix}.
\end{aligned} \tag{47}$$

Let

$$\begin{aligned}
\mathscr{P}_{hs} &= \begin{bmatrix} \mathscr{P}_{1hs} & \mathscr{P}_{2hs} \\ * & \mathscr{P}_{3hs} \end{bmatrix} > 0, \\
\mathscr{W}_{hsl} &= \begin{bmatrix} \mathscr{W}_{1hsl} & \mathscr{W}_{2hsl} \\ * & \mathscr{W}_{3hsl} \end{bmatrix} > 0,
\end{aligned} \tag{48}$$

where

$$\mathscr{P}_{hs} = \sum_{i=1}^{r} h_i \mathscr{P}_{is}, \quad \mathscr{W}_{hsl} = \sum_{i=1}^{r} h_i \mathscr{W}_{isl},$$
$$\mathscr{P}_{is} = \begin{bmatrix} \mathscr{P}_{1is} & \mathscr{P}_{2is} \\ * & \mathscr{P}_{3is} \end{bmatrix} > 0, \mathscr{W}_{isl} = \begin{bmatrix} \mathscr{W}_{1isl} & \mathscr{W}_{2isl} \\ * & \mathscr{W}_{3isl} \end{bmatrix} > 0.$$

Substituting (47) into (45) yields

$$\Theta_{hsl}^T \begin{bmatrix} \Theta_{1hsl} & \cdots & \Theta_{2hsl} \\ \cdots & \cdots & \cdots \\ \Theta_{2hsl}^T & \cdots & \Theta_{3hsl} \end{bmatrix} \Theta_{hsl} < 0, \tag{49}$$

where

$$\begin{aligned}
\Theta_{hsl} &= \begin{bmatrix} I & 0 & 0 \\ -K_{hl}G_{hs} & I & 0 \\ 0 & 0 & I \end{bmatrix}, \\
\Theta_{1hsl} &= \bar{A}_{hsl}^T \tilde{X}_{1h's} \bar{A}_{hsl} + A_{whs}^T \tilde{X}_{1h's} A_{whs} - E^T \mathscr{W}_{1hsl} E \\
&\quad - \bar{D}_{hsl}^T Q \bar{D}_{hsl}, \\
\Theta_{2hsl} &= \bar{A}_{hsl}^T \tilde{X}_{1h's} C_{hs} - \bar{D}_{hsl}^T Q H_{hs} - \bar{D}_{hsl}^T S, \\
\Theta_{3hsl} &= C_{hs}^T \tilde{X}_{1h's} C_{hs} - H_{hs}^T Q H_{hs} - sym(H_{hs}^T S) \\
&\quad - R + \beta I, \\
\tilde{X}_{1h's} &= \sum_{t=1}^{M} \alpha_{st} \tilde{X}_{1h't}, \tilde{X}_{1h't} = \mathscr{P}_{1h't} - (E^{\perp})^T \mathscr{U} E^{\perp}, \\
\mathscr{P}_{1h't} &= \sum_{c=1}^{r} h'_c \mathscr{P}_{1ct} > 0.
\end{aligned}$$

By (49), we have

$$\begin{bmatrix} \Theta_{1hsl} & \Theta_{2hsl} \\ * & \Theta_{3hsl} \end{bmatrix} < 0. \tag{50}$$

According to (40) and (48), it follows that

$$\sum_{l=1}^{N} \varphi_{sl} \mathscr{W}_{1hsl} < \mathscr{P}_{1hs}. \tag{51}$$

It is noted that (50) and (51) satisfy the condition of Theorem 1. As a result, system (7) is stochastically admissible and strictly $(Q, S, R) - \beta$ dissipative.

(ii) Pre- and post-multiplying (46) by $\begin{bmatrix} I & \mathscr{A}_{hsl}^T \end{bmatrix}$ and $\begin{bmatrix} I & \mathscr{A}_{hsl}^T \end{bmatrix}^T$, we obtain

$$\begin{bmatrix} \tilde{A}_{hsl}^T \tilde{X}_{h's} \tilde{A}_{hsl} + \tilde{A}_{whs}^T \tilde{X}_{h's} \tilde{A}_{whs} & \tilde{A}_{hsl}^T \tilde{X}_{h's} \tilde{C}_{hs} - \tilde{D}_{hs}^T S \\ -\tilde{E}^T \mathscr{W}_{hsl} \tilde{E} & \\ & \tilde{C}_{hs}^T \tilde{X}_{h's} \tilde{C}_{hs} - R \\ * & -sym(H_{hs}^T S) + \beta I \end{bmatrix}$$
$$- \begin{bmatrix} \tilde{D}_{hs}^T Q \tilde{D}_{hs} & \tilde{D}_{hs}^T Q H_{hs} \\ * & H_{hs}^T Q H_{hs} \end{bmatrix} < 0. \tag{52}$$

By Schur complement lemma, (52) is equivalent to (45). The proof is completed.

Remark 7: It is shown that, by the augmentation dimension scheme, system (7) is transformed into (39) and two novel conditions for stochastic admissibility and dissipativity of system (7) are obtained in Theorem 2. Actually, the condition (ii) can deduce the condition (i). Besides, the condition (ii) introduces more supplementary free variables $\mathscr{V}_l, \mathscr{Y}_l$, which makes the LMIs conditions (43) and (44) more tractable by Matlab software. Correspondingly, the desired output feedback controller can be design successfully. In spite of computational increase, which is a trade-off between conservatism and computation.

Theorem 3: Given a scalar $\beta > 0$, the real matrices Q, S and R, where Q is negative semidefinite matrix and Q, R are symmetric matrices. System (7) is stochastically admissible and strictly $(Q, S, R) - \beta$ dissipative, if for any $s, t \in \mathcal{M}$, $l \in \mathcal{N}$ and $c, i, j \in \mathcal{I}$, there exist matrices $\mathscr{P}_{ct} = \begin{bmatrix} \mathscr{P}_{1ct} & \mathscr{P}_{2ct} \\ * & \mathscr{P}_{3ct} \end{bmatrix} > 0$, $\mathscr{P}_{is} = \begin{bmatrix} \mathscr{P}_{1is} & \mathscr{P}_{2is} \\ * & \mathscr{P}_{3is} \end{bmatrix} > 0$, $\mathscr{W}_{isl} = \begin{bmatrix} \mathscr{W}_{1isl} & \mathscr{W}_{2isl} \\ * & \mathscr{W}_{3isl} \end{bmatrix} > 0$, $\mathscr{U} = \mathscr{U}^T$, $V_{11l}, V_{13l}, V_{21l}, V_{22l}, V_{3l}, Y_{11l}, Y_{13l}, Y_{21l}, Y_{22l}, Y_{3l}, T_{jl}$ and nonsingular matrix V_{12l}, such that the following conditions hold:

$$\sum_{l=1}^{N} \varphi_{sl} \mathscr{W}_{isl} < \mathscr{P}_{is}, \tag{53}$$
$$\Phi_{ciisl} < 0, \tag{54}$$
$$\Phi_{cijsl} + \Phi_{cjisl} < 0, \quad i < j, \tag{55}$$

where

$$\begin{aligned}
\Phi_{cijsl} &= \begin{bmatrix} \bar{\Phi}_{11cijsl} & \bar{\Phi}_{12ijsl} \\ * & \bar{\Phi}_{22csl} \end{bmatrix}, \\
\bar{\Phi}_{11cijsl} &= \begin{bmatrix} \Phi_{11cijsl} & \Phi_{12ijsl} & \Phi_{13ijsl} \\ * & \Phi_{22isl} & \Phi_{23isl} \\ * & * & \Phi_{33isl} \end{bmatrix}, \\
\bar{\Phi}_{12ijsl} &= \begin{bmatrix} \Phi_{14ijsl} & \Phi_{15ijsl} & \Phi_{16isl} \\ \Phi_{24isl} & \Phi_{25isl} & \Phi_{26isl} \\ \Phi_{34isl} & \Phi_{35isl} & \Phi_{36isl} \end{bmatrix}, \\
\bar{\Phi}_{22csl} &= \begin{bmatrix} \Phi_{44csl} & \Phi_{45csl} & \Phi_{46l} \\ * & \Phi_{55csl} & \Phi_{56l} \\ * & * & \Phi_{66l} \end{bmatrix},
\end{aligned}$$

$$
\begin{aligned}
\Phi_{11cijsl} &= -E^T W_{1isl} E + A_{wis}^T[\mathscr{P}_{1cs} - (E^{\perp})^T \mathscr{U} E^{\perp}]A_{wis} \\
&\quad + sym(V_{11l}A_{is} + \mathcal{Z}T_{jl}G_{is} + V_{21l}Q_{-}^{\frac{1}{2}}D_{is}),\\
\Phi_{12ijsl} &= V_{11l}B_{is} - \mathcal{Z}V_{12l} + V_{21l}Q_{-}^{\frac{1}{2}}F_{is} + A_{is}^T V_{13l}^T + G_{is}^T T_{jl}^T \\
&\quad + D_{is}^T (Q_{-}^{\frac{1}{2}})^T V_{22l}^T,\\
\Phi_{13ijsl} &= -D_{is}^T S + V_{11l}C_{is} + V_{21l}Q_{-}^{\frac{1}{2}}H_{is} + D_{is}^T(Q_{-}^{\frac{1}{2}})^T V_{3l}^T,\\
\Phi_{14ijsl} &= A_{is}^T Y_{11l}^T + G_{is}^T T_{jl}^T \mathcal{Z}^T + D_{is}^T (Q_{-}^{\frac{1}{2}})^T Y_{21l}^T - V_{11l},\\
\Phi_{15ijsl} &= A_{is}^T Y_{13l}^T + G_{is}^T T_{jl}^T + D_{is}^T (Q_{-}^{\frac{1}{2}})^T Y_{22l}^T - \mathcal{Z}V_{12l},\\
\Phi_{16isl} &= D_{is}^T (Q_{-}^{\frac{1}{2}})^T Y_{3l}^T - V_{21l},\\
\Phi_{22isl} &= sym(V_{13l}B_{is} - V_{12l} + V_{22l}Q_{-}^{\frac{1}{2}}F_{is}),\\
\Phi_{23isl} &= -F_{is}^T S + V_{13l}C_{is} + V_{22l}Q_{-}^{\frac{1}{2}}H_{is} + F_{is}^T (Q_{-}^{\frac{1}{2}})^T V_{3l}^T,\\
\Phi_{24isl} &= B_{is}^T Y_{11l}^T - V_{12l}^T \mathcal{Z}^T + F_{is}^T (Q_{-}^{\frac{1}{2}})^T Y_{21l}^T - V_{13l},\\
\Phi_{25isl} &= B_{is}^T Y_{13l}^T - V_{12l}^T + F_{is}^T (Q_{-}^{\frac{1}{2}})^T Y_{22l}^T - V_{12l},\\
\Phi_{26isl} &= F_{is}^T (Q_{-}^{\frac{1}{2}})^T Y_{3l}^T - V_{22l},\\
\Phi_{33isl} &= sym(-H_{is}^T S + V_{3l}Q_{-}^{\frac{1}{2}}H_{is}) - R + \beta I,\\
\Phi_{34isl} &= C_{is}^T Y_{11l}^T + H_{is}^T (Q_{-}^{\frac{1}{2}})^T Y_{21l}^T,\\
\Phi_{35isl} &= C_{is}^T Y_{13l}^T + H_{is}^T (Q_{-}^{\frac{1}{2}})^T Y_{22l}^T,\\
\Phi_{36isl} &= H_{is}^T (Q_{-}^{\frac{1}{2}})^T Y_{3l}^T - V_{3l},\\
\Phi_{44csl} &= \mathscr{P}_{1cs} - (E^{\perp})^T \mathscr{U} E^{\perp} - Y_{11l} - Y_{11l}^T,\\
\Phi_{45csl} &= \mathscr{P}_{2cs} - \mathcal{Z}V_{12l} - Y_{13l}^T,\\
\Phi_{46l} &= -Y_{21l},\ \Phi_{55csl} = \mathscr{P}_{3cs} - V_{12l} - V_{12l}^T,\\
\Phi_{56l} &= -Y_{22l},\ \Phi_{66l} = I - Y_{3l} - Y_{3l}^T,\\
\mathscr{P}_{1cs} &= \sum_{t=1}^{M} \alpha_{st}\mathscr{P}_{1ct},\ \mathscr{P}_{2cs} = \sum_{t=1}^{M}\alpha_{st}\mathscr{P}_{2ct},\\
\mathscr{P}_{3cs} &= \sum_{t=1}^{M}\alpha_{st}\mathscr{P}_{3ct},\ \mathcal{Z} = \begin{bmatrix} I_m & 0_{m\times(n-m)} \end{bmatrix}^T.
\end{aligned}
$$

Moreover, the asynchronous output feedback controller gains can be solved by $K_{jl} = V_{12l}^{-1}T_{jl}$.

Proof: The matrices $\mathscr{V}_l$ and $\mathscr{Y}_l$ are given as

$$
\mathscr{V}_l = \begin{bmatrix} V_{11l} & \mathcal{Z}V_{12l} & V_{21l} \\ V_{13l} & V_{12l} & V_{22l} \\ 0 & 0 & V_{3l} \end{bmatrix}, \tag{56}
$$

$$
\mathscr{Y}_l = \begin{bmatrix} Y_{11l} & \mathcal{Z}V_{12l} & Y_{21l} \\ Y_{13l} & V_{12l} & Y_{22l} \\ 0 & 0 & Y_{3l} \end{bmatrix}. \tag{57}
$$

By setting $T_{jl} = V_{12l}K_{jl}$ and substituting (56)-(57) into the condition (ii) of Theorem 2, Theorem 3 is derived.

IV. A NUMERICAL EXAMPLE

In this section, a numerical example is given to show the validity of the proposed methods. Consider the T-S fuzzy singular stochastic Markov jump systems (7) with two fuzzy rules and three modes, the associated parameters are presented as follows:

- Mode 1

$$
\begin{aligned}
A_{11} &= \begin{bmatrix} -1.01 & 0.02 \\ 0.05 & 1.12 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -1.05 & 0.09 \\ 0.05 & 1.03 \end{bmatrix},\\
A_{w11} &= \begin{bmatrix} 0.15 & 0.1 \\ 0 & 0 \end{bmatrix}, \quad A_{w21} = \begin{bmatrix} 0.07 & 0 \\ 0 & 0 \end{bmatrix},\\
B_{11} &= \begin{bmatrix} 0.11 \\ 0.01 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 0.1 \\ 0.02 \end{bmatrix},\\
C_{11} &= \begin{bmatrix} 0.11 \\ 0.01 \end{bmatrix}, \quad C_{21} = \begin{bmatrix} 0.24 \\ 0.08 \end{bmatrix},\\
G_{11} &= \begin{bmatrix} 0.05 & 0.13 \end{bmatrix}, \quad G_{21} = \begin{bmatrix} 0.31 & 0 \end{bmatrix},\\
D_{11} &= \begin{bmatrix} 0.91 & 0 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0.13 & 0.04 \end{bmatrix},\\
F_{11} &= 0.02, F_{21} = -0.11, H_{11} = 0.62, H_{21} = 0.81.
\end{aligned}
$$

- Mode 2

$$
\begin{aligned}
A_{12} &= \begin{bmatrix} 0.73 & 0.43 \\ 0.01 & 0.02 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -0.3 & 0.03 \\ 0 & 1.05 \end{bmatrix},\\
A_{w12} &= \begin{bmatrix} 0.11 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{w22} = \begin{bmatrix} 0.04 & 0.01 \\ 0 & 0 \end{bmatrix},\\
B_{12} &= \begin{bmatrix} 0.13 \\ 0.02 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0.07 \\ 0.01 \end{bmatrix},\\
C_{12} &= \begin{bmatrix} -0.1 \\ -0.03 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 0.1 \\ 0.02 \end{bmatrix},\\
G_{12} &= \begin{bmatrix} 0.1 & 0.04 \end{bmatrix}, \quad G_{22} = \begin{bmatrix} 0.1 & 0.06 \end{bmatrix},\\
D_{12} &= \begin{bmatrix} 0.4 & 0.01 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0.08 & 0 \end{bmatrix},\\
F_{12} &= 0.09, \quad F_{22} = 0.21, H_{12} = 0.79, H_{22} = 0.7.
\end{aligned}
$$

- Mode 3

$$
\begin{aligned}
A_{13} &= \begin{bmatrix} 0.15 & 0.2 \\ 0.02 & 1.07 \end{bmatrix}, \quad A_{23} = \begin{bmatrix} 0.68 & 1 \\ -0.2 & 1.42 \end{bmatrix},\\
A_{w13} &= \begin{bmatrix} 0.1 & 0.02 \\ 0 & 0 \end{bmatrix}, \quad A_{w23} = \begin{bmatrix} 0.1 & 0.05 \\ 0 & 0 \end{bmatrix},\\
B_{13} &= \begin{bmatrix} 0.09 \\ 0.01 \end{bmatrix}, \quad B_{23} = \begin{bmatrix} 0.1 \\ 0.01 \end{bmatrix}, C_{13} = \begin{bmatrix} 0.31 \\ 0.1 \end{bmatrix},\\
C_{23} &= \begin{bmatrix} 0.09 \\ 0.01 \end{bmatrix}, \quad G_{13} = \begin{bmatrix} -0.1 & 0.2 \end{bmatrix},\\
G_{23} &= \begin{bmatrix} 0.12 & 0 \end{bmatrix},\\
D_{13} &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_{23} = \begin{bmatrix} 0.1 & 0.03 \end{bmatrix},\\
F_{13} &= 0.03, \quad F_{23} = 0.18, \ H_{13} = 0.9, \ H_{23} = 0.69.
\end{aligned}
$$

The transition probability matrix Ω is given as

$$
\Omega = [\alpha_{st}] = \begin{bmatrix} 0.4 & 0.25 & 0.35 \\ 0.1 & 0.4 & 0.5 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}.
$$

On the other hand, we choose $E = \begin{bmatrix} 1 & 0.4 \\ 0 & 0 \end{bmatrix}, E^{\perp} = \begin{bmatrix} 0 & 4 \end{bmatrix}$. And the fuzzy membership functions are given as

$$
\begin{aligned}
h_1(x_1(k)) &= \begin{cases} \dfrac{\sin x_1(k) - \varpi x_1(k)}{(1-\varpi)x_1(k)}, & x_1(k) \neq 0, \\ 1, & x_1(k) = 0, \end{cases}\\
h_2(x_1(k)) &= 1 - h_1(x_1(k)),
\end{aligned}
$$

where $\varpi = \frac{0.01}{\pi}$.

TABLE 1. The values of Q,S,R for different control criteria.

	Q	S	R
passivity	0	1	2β
H_∞	-1	0	$\varsigma^2+\beta$
$(Q,S,R)-\beta$ dissipativity	-0.4	0.5	1

TABLE 2. Designed passive controller based on different asynchronous levels.

	β	K_{11}	K_{12}	K_{13}	K_{21}	K_{22}	K_{23}
case 1	0.65	3.9483	-11.1404	-1.4948	2.1532	0.4607	-7.5760
case 2	0.58	3.3561	-9.5564	1.5208	2.2054	1.8423	-3.0327
case 3	0.3	-2.9977	-5.6111	-2.7888	2.7769	2.3857	-2.6954
case 4	0.05	-1.6021	-4.6662	-2.6741	1.2901	0.8003	0.3680

TABLE 3. Designed H_∞ controller under H_∞ performance level $\varsigma = 2.5$ and different asynchronous levels.

	β	K_{11}	K_{12}	K_{13}	K_{21}	K_{22}	K_{23}
case 1	0.65	3.8285	-7.0270	1.9948	1.8928	-0.0325	-5.5502
case 2	0.58	3.1855	-6.5900	2.9725	1.9014	1.5422	-3.0588
case 3	0.3	-2.1277	-2.1014	1.2902	2.3292	1.9043	-2.2932
case 4	0.05	0.2969	-0.5017	0.1104	0.3771	-0.0298	-0.4338

TABLE 4. Designed $(Q,S,R)-\beta$ dissipative controller based on different asynchronous levels.

	β	K_{11}	K_{12}	K_{13}	K_{21}	K_{22}	K_{23}
case 1	0.65	4.5277	-9.4069	0.9884	2.2967	0.1673	-6.7685
case 2	0.58	4.7560	-8.2662	3.4496	2.6780	2.0577	-2.0857
case 3	0.3	-3.7798	-4.9954	-0.8990	2.8348	2.4200	-2.4171
case 4	0.05	-1.1786	-2.5271	-1.5054	0.5657	0.1112	-0.3008

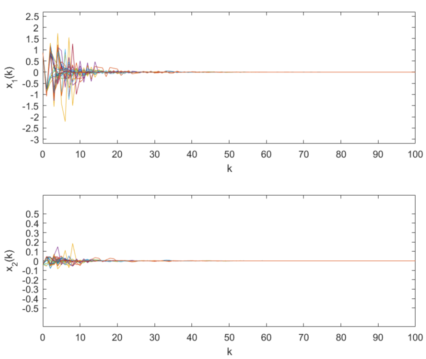

FIGURE 1. State trajectories of the closed-loop system (7).

The conditional probability matrices $\Psi = [\varphi_{sl}]$ based on different asynchronous level are given below, respectively:

synchronous (case 1): $\Psi_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

weakly asynchronous (case 2): $\Psi_2 = \begin{bmatrix} 0.5 & 0.15 & 0.35 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

strongly asynchronous (case 3): $\Psi_3 = \begin{bmatrix} 0.5 & 0.15 & 0.35 \\ 0.4 & 0.2 & 0.4 \\ 0 & 0 & 1 \end{bmatrix}$,

completely asynchronous (case 4): $\Psi_4 = \begin{bmatrix} 0.5 & 0.15 & 0.35 \\ 0.4 & 0.2 & 0.4 \\ 0.4 & 0.15 & 0.45 \end{bmatrix}$.

We give different values of Q, S and R in Table 1 to represent the passive, H_∞ and dissipative performance of

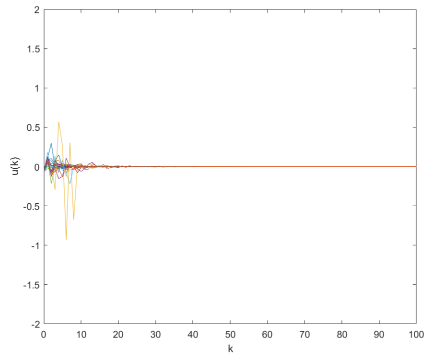

FIGURE 2. Trajectories of control input (6).

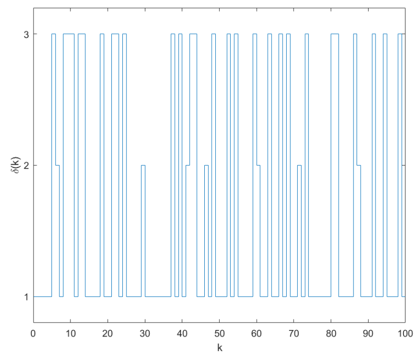

FIGURE 3. The curve of $\delta(k)$.

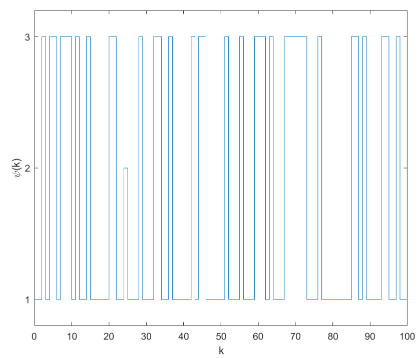

FIGURE 4. The curve of $\psi(k)$.

the system, respectively. By solving LMIs of Theorem 3, the output feedback gains for three different control issues are obtained in Tables 2-4, respectively. This means the dissipative control considered in this paper covers the passivity and H_∞ control as special cases. In the simulation, the initial state is $x(0) = \begin{bmatrix} 0.8426 & 0.0186 \end{bmatrix}^T$, the time k takes values from 1 to 102, and $w(k)$ takes 101 random numbers obeying the normal distribution. Due to the affection of $w(k)$, the state of system from k to $k+1$ is stochastic. We draw the state spline curves for closed-loop system (7) under the completely asynchronous controller given in the case 4 of Table 4. Figures 1-2 show 30 state trajectories and control input trajectories, respectively. It can be clearly seen from them that the curves approximate to zero as k increases. The obtained results demonstrate that our developed scheme is correct and effective. Furthermore, it is observed from Figures 3-4 that the modes of the system and controller are obviously asynchronous.

V. CONCLUSION

This note has studied the dissipative control for discrete-time T-S fuzzy SMJSs with state-dependent noise and asynchronous modes. The HMM has been applied to describe the asynchronous phenomenon appearing between the system modes and the controller modes, which makes the closed-loop system become the singular stochastic hidden Markov jump system. Firstly, based on Lyapunov function dependent on modes and fuzzy set, sufficient conditions, which guarantee the considered system to be stochastically admissible and strictly $(Q, S, R) - \beta$ dissipative, have been proposed. Secondly, two alternative conditions for facilitating the analysis of controller have been presented by employing the auxiliary variable matrices and augmentation approach. Thirdly, an asynchronous output feedback controller has been successfully designed via solving strict LMIs. Finally, an example has been presented to demonstrate the effectiveness of the design method. In the future, by making using of the obtained results of this paper, we will investigate the asynchronous dissipative filtering for singular stochastic Markov jump systems with time-varying delay.

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