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Impact of MPC Embedded Performance Index on Control Quality

PAWEŁ D. DOMAŃSKI AND MACIEJ ŁAWRYŃCZUK
Faculty of Electronics and Information Technology, Institute of Control and Computation Engineering, Warsaw University of Technology, 00-665 Warsaw, Poland
Corresponding author: Maciej Ławryńczuk (m.lawrynczuk@ia.pw.edu.pl)

ABSTRACT Model Predictive Control (MPC) is a well-established advanced process control technology. There are many successful implementations of different predictive strategies in process industry. There may be found various modifications of the MPC, however, one aspect remains fixed. MPC performance index is in quadratic form. Nonetheless, statistical analysis frequently points out that the quadratic regression formulation has some drawbacks. It is sensitive against the outliers. This work analyzes alternative and robust formulations of the MPC embedded performance index. It is shown that the quadratic formulation is not an optimal one, while the linear ℓ_1 weight improves control. Classical ℓ_2 norm together with robust Cauchy and Dynamic Covariance Scaling gives worse results.

INDEX TERMS MPC, performance index, robust regression, ℓ_1 norm, dynamic covariance scaling.

I. INTRODUCTION

Model Predictive Control (MPC) constitutes a major component of the Advanced Process Control (APC). Its story starts with the first formulation of the linear quadratic regulator (LQR) by Kalman. Within the next ten years the ongoing research enabled successful industrial applications of MPC in industry. Model Predictive Heuristic Control (known as Model Algorithmic Control (MAC)) has been presented followed by Dynamic Matrix Control (DMC) [1]. Next, Generalized Predictive Control (GPC) using minimum variance control ideas [2] has been proposed. The backbone of the MPC framework is established.

Further research investigated in details baseline formulation extending its basic idea towards various scenarios and configurations. The researchers have investigated multivariate, nonlinear constrained MPC configurations [3], [4]. As a result, MPC algorithms have been applied to numerous processes, ranging from relatively slow process control plants such as chemical reactors [5], distillation columns [6], NOx control [7] and coal mills [8] to very systems such as fast robots [9], micro grids [10], electric drives [11], spark-ignition gasoline engines [12] and autonomous vehicles [13]. Recently, Williams *et al.* [14] proposed a sampling-based and derivative-free MPC algorithm, known as Model Predictive Path Integral (MPPI) control framework, that can be easily utilized without requiring the first- or second-order approximation of the system dynamics and quadratic approximation of the objective functions. The MPPI control framework has been successfully applied to a variety of robotic systems for tasks such as aggressive autonomous driving and autonomous flying through 2D/3D cluttered environments [15].

Though the approach may vary, one single element remains unchanged in the majority of variants. It is a quadratic formulation of the embedded performance index. Exceptions from this rule are very rare. In the beginning, authors improved classical raw quadratic error adding weighting. Davidson [16] has proposed an alternate "cheap control" performance index transferred from discrete time servo control context. The quadratic formulation is named as a ℓ_2-MPC. There are also alternate formulations of MPC performance index in terms of mean and variance [17], however, they are equivalent. Interesting discussion about applicability and deficiency of the MPC quadratic cost function may be found in recent work of [18]. Authors show that typical choice of ℓ_2-MPC is not always suitable, in particular, a sufficiently long horizon satisfying MPC asymptotically stable control. Limitations and drawbacks of the quadratic performance index might also be observed in other than MPC contexts, as for instance statistics and regression.

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The research focuses on outliers, which pose a serious challenge for any control. The classical quadratic performance index is highly sensitive to any kind of the outliers [19], as it is characterized by the 0% breakdown point. Similar breakdown value appears for normal standard deviation. Integral of absolute error is only a little bit better, but still holds 0% breakdown point. Robust statistics [20] proposes M-estimators that exhibit 50% breakdown.

An outlier is a strange data observation [21], [22]. A single outlier may originate from an erroneous observation (exogenous) or can be an intrinsic symptom of some unknown underlying mechanism [23] (endogenous). Outliers impede analysis as they increase the variance and reduce the power of statistical tests [24], destroy Gaussianity, introduce tails [25], and bias data regression [19]. An α-stable distribution is a common approach [26] to model them.

Following the above indications, statistics suggest to use mean absolute error, which is formulated as a ℓ_1 norm. Statistically, this index is only slightly better, as it is robust to an outlier in y-direction, but is sensitive to outliers in x-direction (leverage points) and still holds a 0% breakdown [19]. Formulation and an analysis of the ℓ_1-MPC can be found in [27]. It was also quite natural that the researchers also considered other norms as for instance ℓ_∞-MPC [28]. Some other approach to improve MPC quadratic performance index has been proposed [29]. Classical least squares approach has been regularized with ℓ_1 component.

Investigation of ℓ_1 and ℓ_∞ MPCs is mostly driven by two factors: simplification of the MPC evaluation due to the computationally less complex problem linear programming formulation. On the other hand, there have been observed some issues with the performance and suitability of ℓ_1 and ℓ_∞ criteria. The main observed consequence was that they may yield either dead-beat or idle control performance [30]. It must be noted that the outlier robustness aspect is not as such addressed. On the other hand, robust indexes are successfully used in the MPC control performance assessment [31]–[33], which is independent of the controller operation, however, should reflect predictive operation.

In contrary, robust regression estimators are successfully used in other engineering contexts, as for instance camera-based localization. Robust function in Iteratively Reweighted Least-Squares (IRLS) problem allows to deal with frequent correspondence outliers present in visual navigation [34]. These results have stimulated the presented work. Quadratic ℓ_2 estimator inside of MPC performance index formulation is compared with three robust estimators: ℓ_1, M-estimator with Cauchy function [35] and Dynamic Covariance Scaling (DCS) [36], which is not present in the literature as a robust cost function, but can be expressed as such.

Concluding, the paper addresses the following subjects:

- The most popular formulations of the MPC control incorporate quadratic internal performance index norm ℓ_2. However quadratic index is sensitive to the outliers as 0% breakdown point.
- Robust regression research delivers other estimators with a larger breakdown, like linear ℓ_1, robust Cauchy ℓ_{Cauchy} and Dynamic Covariance Scaling ℓ_{DCS}.
- The paper investigates and compares the impact of various robust norm formulations on an overall nonlinear MPC control performance.

In the following sections, we will introduce robust regression estimators and the way how they are incorporated into MPC performance index. Then, the simulations of MPC controller using four selected index formulations are performed using nonlinear control problem in the form of the pH neutralization reactor. The simulations are summarized with the results and their discussion. Advantages and possible areas for further research are highlighted in conclusions.

II. NONLINEAR MODEL PREDICTIVE CONTROL

Description of the robust MPC formulations consists of two following sections. First, the general layout of applied MPC is presented, which is followed by the specific performance index formulations addressed in the simulation analysis.

A. MPC ALGORITHM

The process input, i.e. Manipulated Variable (MV) is denoted by u and the output, i.e. Controlled Variable (CV), is denoted by y. The vector of decision variables determined on-line at each discrete sampling instant ($k = 0, 1, 2, \ldots$) by MPC algorithm [3] is

$$\triangle\boldsymbol{u}(k) = [\triangle u(k|k)\ \triangle u(k+1|k) \ldots \triangle u(k+N_{\text{u}}-1|k)]^{\text{T}}, \tag{1}$$

where N_{u} is the control horizon, i.e. the number of calculated future control increments defined as backward differences, i.e. $\triangle u(k|k) = u(k|k) - u(k-1)$ and $\triangle u(k+p|k) = u(k+p|k) - u(k+p-1|k)$ for $p = 1, \ldots, N_{\text{u}}-1$. For $p \geq N_{\text{u}}$ it is assumed that the manipulated variable is constant, i.e. $u(k+p|k) = u(k+N_{\text{u}}-1|k)$. The decision variables of MPC (1) are calculated from an optimization problem. It's typical form is

$$\begin{aligned}
&\min_{\triangle\boldsymbol{u}(k)} \left\{J_{\text{y}}(k) + J_{\text{u}}(k)\right\},\\
&\text{subject to}\\
&u^{\min} \leq u(k+p|k) \leq u^{\max},\ p = 0, \ldots, N_{\text{u}}-1,\\
&-\triangle u^{\max} \leq \triangle u(k+p|k) \leq \triangle u^{\max},\ p = 0, \ldots, N_{\text{u}}-1,\\
&y^{\min} \leq \hat{y}(k+p|k) \leq y^{\max},\ p = 1, \ldots, N,
\end{aligned} \tag{2}$$

where two components of the minimized cost-function are

$$J_{\text{y}}(k) = \sum_{p=1}^{N} \ell(e(k+p|k)), \tag{3}$$

$$J_{\text{u}}(k) = \lambda \sum_{p=0}^{N_{\text{u}}-1} (\triangle u(k+p|k))^2. \tag{4}$$

The role of the first part of MPC cost-function is to minimize predicted control errors over prediction horizon N

$$e(k+p|k) = \mathrm{sc}(y^{\mathrm{sp}}(k+p|k) - \hat{y}(k+p|k)). \tag{5}$$

Setpoint and predicted values of the process output for future sampling instant $k+p$ known/calculated for current instant k are denoted by $y^{\mathrm{sp}}(k+p|k)$ and $\hat{y}(k+p|k)$, respectively, sc is a scaling factor. Predicted values of the process output are calculated on-line using a mathematical model of the controlled process. The role of the second part of the cost-function is to eliminate excessive changes of the manipulated variable, $\lambda > 0$. In general, the constraints may be imposed on: (i) future values of the manipulated variable (over the control horizon), the minimal and maximal allowed values are $u^{\min}$ and $u^{\max}$, respectively, (ii) future changes of that variable, the maximal value is $\triangle u^{\max}$, (iii) predicted values of the controlled variable (over the prediction horizon), the minimal and maximal values are $y^{\min}$ and $y^{\max}$, respectively. Although at each sampling instant whole sequence of decision variable (1), is calculated, only its first entry is applied to the process. At the next sampling instant, $k+1$, measurement of the process output is updated and the procedure is repeated. In this work, a nonlinear dynamical model is used for prediction calculation. It means that the MPC optimization problem (2) is a nonlinear task. It is solved by the Sequential Quadratic Programming algorithm.

B. PERFORMANCE INDEX FORMULATION

Four performance index formulations used in the analysis are described in the following paragraphs. Although different functions ℓ are used to measure the influence of predicted control errors reflected in the first part of the cost-function (3), the classical quadratic second part of the cost-function (4) is used.

1) ℓ_2-MPC

The classical approach is to use a sum of squared predicted control errors [3]

$$J_{\mathrm{y}}(k) = \sum_{p=1}^{N} (e(k+p|k))^2. \tag{6}$$

When a linear model is used for prediction, the both parts of the optimized MPC cost-function are quadratic in terms of the calculated decision variables (1) and the whole MPC optimization problem (2) becomes a quadratic optimization problem (a quadratic cost-function and linear constraints). Such a problem may be very efficiently solved on-line.

2) ℓ_1-MPC

When the sum of absolute control errors is minimized, the first part of the MPC cost-function is [27]

$$J_{\mathrm{y}}(k) = \sum_{p=1}^{N} |e(k+p|k)|. \tag{7}$$

3) ℓ_{Cauchy}-MPC

In this approach [35],

$$J_{\mathrm{y}}(k) = \sum_{p=1}^{N} 0.5c^2 \log\left(1 + \frac{(e(k+p|k))^2}{c}\right). \tag{8}$$

In all simulations, $c = 1$.

4) ℓ_{DCS}-MPC

In this approach [36],

$$J_{\mathrm{y}}(k) = \begin{cases} 0.5(e(k+p|k))^2 & \text{if } e(k+p|k) \leq \phi \\ \dfrac{2\phi(e(k+p|k))^2}{\phi+(e(k+p|k))^2} - 0.5e(k+p|k) & \text{if } e(k+p|k) \geq \phi. \end{cases} \tag{9}$$

The parameter ϕ is a threshold. Its value must be chosen taking into account properties of a particular application. In all simulations discussed in this work, $\phi = 1$.

III. SIMULATION

Simulation layout has been prepared for the analysis. The nonlinear process has been selected in the form of the pH neutralization reactor [37]. It has one controlled input and one considered as the unmeasured disturbance. Simulation layout is sketched in Fig. 1.

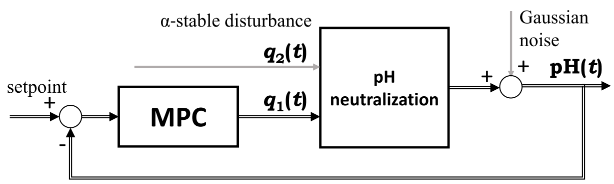

FIGURE 1. Simulation environment of pH neutralization reactor.

Two different simulation scenarios have been tested. The main set of results has been obtained using the undisturbed reactor, i.e. unmeasured disturbance buffer inlet $q_2(t)$ remains unchanged. In the second part $q_2(t)$ is simulated as a non-Gaussian process with α-stable distribution and stability factor $\alpha = 1.70$. Such a selection enables to analyze industrial-like aspects with sufficient process excitement.

Another aspect that requires testing is the range of applied cost function denoted as sc. The following values have been tested to reflect possible shapes of the M-estimator costs function: sc = {1, 2, 5, 10, 20, 50}.

We use control performance measures, called KPIs (Key Performance Indicators) to evaluate the quality of a control system. In general, in Control Performance Assessment (CPA), there are two classes of methods: model-driven and data-driven. Model-driven techniques need some assumed process knowledge that is used to identify the process model further used to calculate the KPI. Data-driven methods require only raw process data without any further assumptions. Classical KPIs are not always sufficient and industry searches for robust measures that would be suitable in the broader sense. This analysis uses measures that use control error ($\epsilon(k) = y^{\mathrm{sp}}(k) - y(k)$) time series of length N_{p} as the basis for index calculation [32].

1) MSE: mean square error – commonly used, but not robust against outliers, exhibits 0% breakdown

$$\mathrm{MSE} = \frac{1}{N_{\mathrm{p}}} \sum_{k=1}^{N_{\mathrm{p}}} \epsilon^2(k), \tag{10}$$

2) IAE: integral absolute error – not robust against outliers, exhibits 0% breakdown

$$\mathrm{IAE} = \frac{1}{N_{\mathrm{p}}} \sum_{k=1}^{N_{\mathrm{p}}} |\epsilon(k)|, \tag{11}$$

3) LMS: least median square – robust against outliers and exhibits 50% breakdown [19]

$$\mathrm{LMS} = \operatorname*{med}_{k} \epsilon(k)^2, \tag{12}$$

4) σ_{G}: Gaussian standard deviation calculated as

$$\sigma_{\mathrm{G}} = \sqrt{\frac{\sum_{k=1}^{N\mathrm{p}} (\epsilon(k) - \epsilon_0)^2}{N\mathrm{p} - 1}}, \tag{13}$$

where ϵ_0 is mean value of the control error,

5) γ: stable scaling factor evaluated for the fitted α-stable PDF

$$F^{\mathrm{stab}}_{\alpha,\beta,\delta,\gamma}(\epsilon) = \exp\left\{ i\delta\epsilon - |\gamma\epsilon|^{\alpha} \left(1 - i\beta l(\epsilon)\right)\right\}, \tag{14}$$

where

$$l(\epsilon) = \begin{cases} \operatorname{sgn}(\epsilon)\tan\left(\frac{\pi\alpha}{2}\right) & \text{for } \alpha \neq 1 \\ \operatorname{sgn}(\epsilon)\frac{2}{\pi}\ln|\epsilon| & \text{for } \alpha = 1. \end{cases} \tag{15}$$

The coefficient $0 < \alpha \leq 2$ is called a stability index or a characteristic exponent, $|\beta| \leq 1$ is a skewness coefficient, $\delta \in \mathbb{R}$ is distribution location and $\gamma > 0$ is called a scale or dispersion [38].

6) σ_{H}: robust scale M-estimator with logistic ψ function [39],

7) H_{diff}: differential entropy [40]

$$H_{\mathrm{diff}} = -\int_{-\infty}^{\infty} \gamma(\epsilon) \ln \gamma(\epsilon)\, d\epsilon, \tag{16}$$

8) H_{rat}: rational entropy [41]

$$H_{\mathrm{rat}} = -\int_{-\infty}^{\infty} \gamma(\epsilon) \log\left(\frac{\gamma(\epsilon)}{1+\gamma(\epsilon)}\right) d\epsilon. \tag{17}$$

A. PROCESS DESCRIPTION

In the considered neutralization reactor a base (NaOH) stream q_1, a buffer ($NaHCO_3$) stream q_2 and an acid (HNO_3) stream q_3 are mixed in a constant volume tank. The process has one input (manipulated) variable which is the base flow rate q_1 [ml/s] and one output (controlled) variable which is the value of pH (Fig. 2). Continuous-time fundamental model of the process comprises of two ordinary differential equations

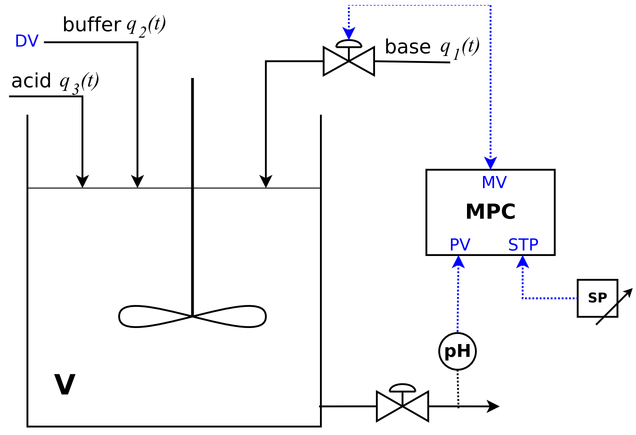

FIGURE 2. Schematic diagram of the pH neutralization reactor.

TABLE 1. Parameters of pH neutralization model.

$W_{\mathrm{a}_1} = -3.05 \times 10^{-3}$ mol	$W_{\mathrm{b}_1} = 5 \times 10^{-5}$ mol
$W_{\mathrm{a}_2} = -3 \times 10^{-2}$ mol	$W_{\mathrm{b}_2} = 3 \times 10^{-2}$ mol
$W_{\mathrm{a}_3} = 3 \times 10^{-3}$ mol	$W_{\mathrm{b}_3} = 0$ mol
$K_1 = 6.35$	$V = 2900$ ml
$K_2 = 10.25$	

TABLE 2. Nominal operating point of pH reactor.

$\bar{q}_1 = 15.55$ ml/s	$\overline{W}_{\mathrm{a}} = -4.32 \times 10^{-4}$ mol
$\bar{q}_2 = 0.55$ ml/s	$\overline{W}_{\mathrm{b}} = 5.28 \times 10^{-4}$ mol
$\bar{q}_3 = 16.60$ ml/s	$\overline{\mathrm{pH}} = 7$

$$\begin{aligned} \frac{\mathrm{d}W_{\mathrm{a}}(t)}{\mathrm{d}t} &= \frac{q_1(t)(W_{\mathrm{a}_1} - W_{\mathrm{a}}(t))}{V} + \frac{q_2(W_{\mathrm{a}_2} - W_{\mathrm{a}}(t))}{V} \\ &\quad + \frac{q_3(W_{\mathrm{a}_3} - W_{\mathrm{a}}(t))}{V} \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{\mathrm{d}W_{\mathrm{b}}(t)}{\mathrm{d}t} &= \frac{q_1(t)(W_{\mathrm{b}_1} - W_{\mathrm{b}}(t))}{V} + \frac{q_2(W_{\mathrm{b}_2} - W_{\mathrm{b}}(t))}{V} \\ &\quad + \frac{q_3(W_{\mathrm{b}_3} - W_{\mathrm{b}}(t))}{V} \end{aligned} \tag{19}$$

and one algebraic output equation

$$\begin{aligned} &W_{\mathrm{a}}(t) + 10^{\mathrm{pH}(t)-14} - 10^{-\mathrm{pH}(t)} \\ &\quad + W_{\mathrm{b}}(t)\frac{1 + 2 \times 10^{\mathrm{pH}(t)-K_2}}{1 + 10^{K_1-\mathrm{pH}(t)} + 10^{\mathrm{pH}(t)-K_2}} = 0. \end{aligned} \tag{20}$$

State variables W_{a} and W_{b} are reaction invariants. Parameters of the first-principle model are given in Table 1 and the values of process variables for the nominal operating point in Table 2. Buffer inflow $q_2(t)$ is the disturbance, while the acid stream $q_3(t)$ is constant.

B. SIMULATION RESULTS

The parameters of all compared MPC variants are the same: the prediction and control horizons are $N = 10$ and $N_{\mathrm{u}} = 3$, respectively, the weighting coefficient is $\lambda = 0.5$, the

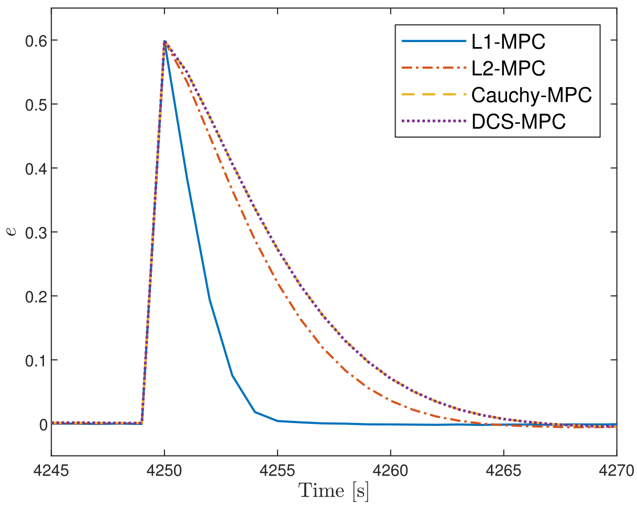

FIGURE 3. Control error time trends for MPC controllers with coefficient sc = 1 simulated without disturbances.

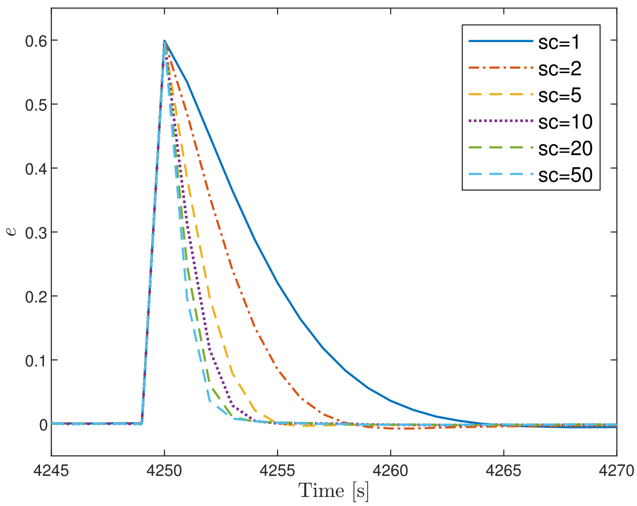

FIGURE 4. Control error time trends for various scaling using ℓ_2-MPC simulated without disturbances.

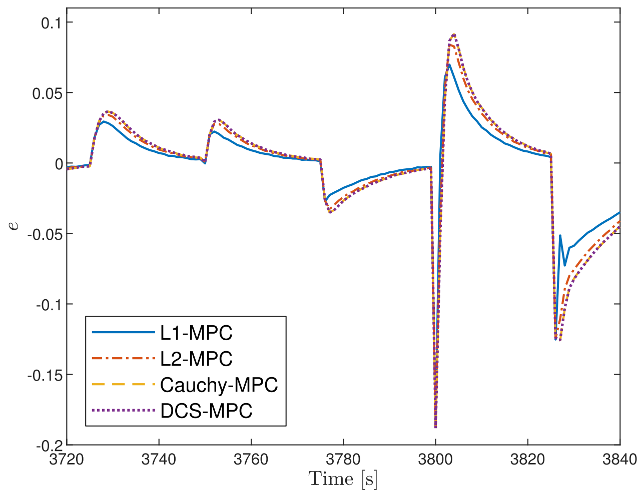

FIGURE 5. Control error time trends for MPC controllers simulated with disturbances.

TABLE 3. Impact of a controller on step response.

	ℓ_1-MPC	ℓ_2-MPC	ℓ_{Cauchy}-MPC	ℓ_{DCS}-MPC
κ	0.598%	0.850%	0.805%	0.785%
T_{set}	100	160	200	200

TABLE 4. Impact of scaling coefficient sc on step response performance indexes.

sc	1	2	5
κ	0.850%	0.795%	0.680%
T_{set}	160	120	100
sc	10	20	50
κ	0.660%	0.640%	0.638%
T_{set}	90	90	90

magnitude constraints imposed on the manipulated variable are defined by $q_1^{\min} = 0$ ml/s and $q_1^{\max} = 30$ ml/s, respectively.

At first no disturbances are taken into account during simulations. Fig. 3 shows the control error for one step set-point change for four MPC controllers in case of sc $= 1$. The ℓ_1-MPC algorithm is the fastest approach, the ℓ_2-MPC one is significantly slower, both ℓ_{Cauchy}-MPC and ℓ_{DCS}-MPC ones are the worst ones (they both give practically the same results). Next, Fig. 4 presents also undisturbed time trends showing an effect of scaling sc changes for ℓ_2-MPC. Of course, the higher the scaling factor, the faster the control. Finally, Fig. 5 presents disturbed time trends (all MPCs share the same disturbance realization). Similarly to Fig. 3, The ℓ_1-MPC algorithm gives the fastest response, the ℓ_2-MPC one is significantly slower, both ℓ_{Cauchy}-MPC and ℓ_{DCS}-MPC ones are the worst ones.

Numerical results of settling time T_{set} and overshoot κ are presented in a tabular way. Table 3 shows values of T_{set} and κ for MPCs having the same scaling sc $= 1$. It is well seen that ℓ_1-MPC has the shortest settling time. Though overshoot is small in all cases, it reaches the minimum for ℓ_1-MPC as well.

Table 4 presents an effect of performance scaling sc for classical ℓ_2-MPC, however the same effect appears for other MPCs. It is well seen that increasing the scaling improves control, both in the settling time and the overshoot.

Further comparison of the obtained results is presented in a set of the figures showing the relationship between scaling factor sc and CPA index simultaneously for four considered robust estimators of MPC performance index: ℓ_2-MPC, ℓ_1-MPC, ℓ_{Cauchy}-MPC and ℓ_{DCS}-MPC.

The comparison starts with Fig. 6, which shows the relationship reflected with the mean square error. It shows that ℓ_1-MPC controller reaches the best performance according to the MSE. It is also seen that three of the controllers saturate on the same performance ℓ_1-MPC, ℓ_2-MPC and ℓ_{Cauchy}-MPC, except ℓ_{DCS}-MPC as scaling factor reaches sc $= 20$. It is due to the fact that then the shapes of the performance index

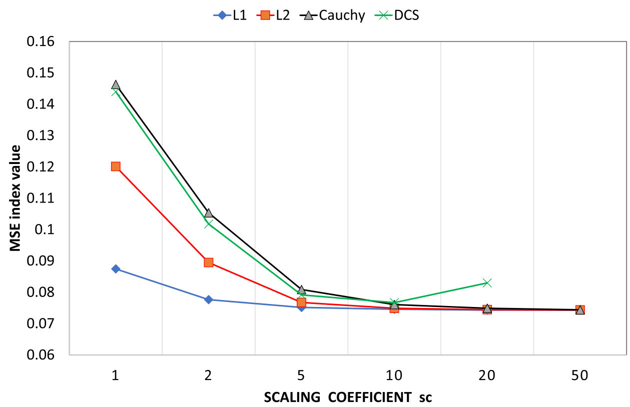

FIGURE 6. Performance index impact measured with MSE.

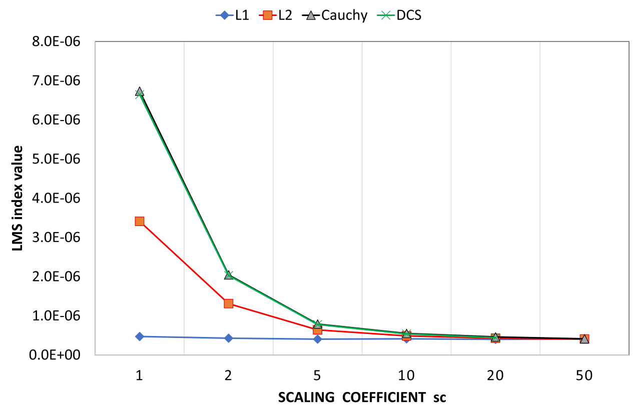

FIGURE 9. Performance index impact measured with LMS.

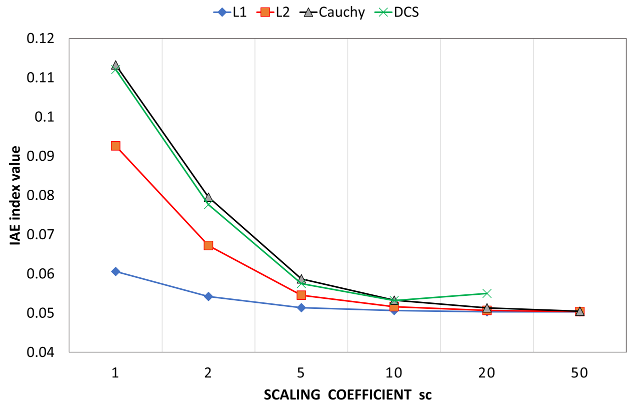

FIGURE 7. Performance index impact measured with IAE.

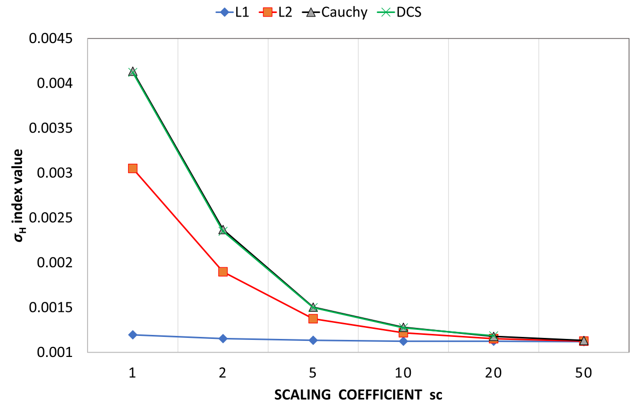

FIGURE 10. Performance index impact measured with σ_H.

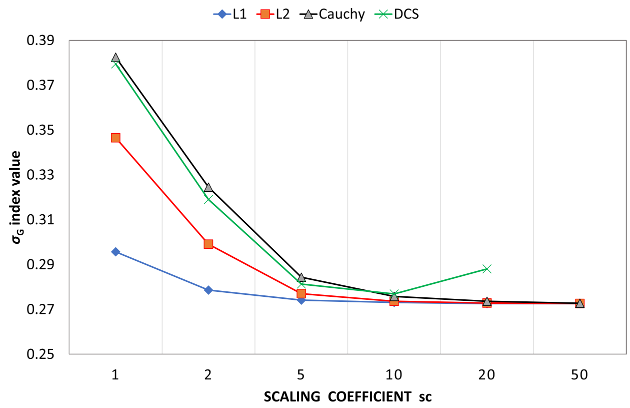

FIGURE 8. Performance index impact measured with σ_G.

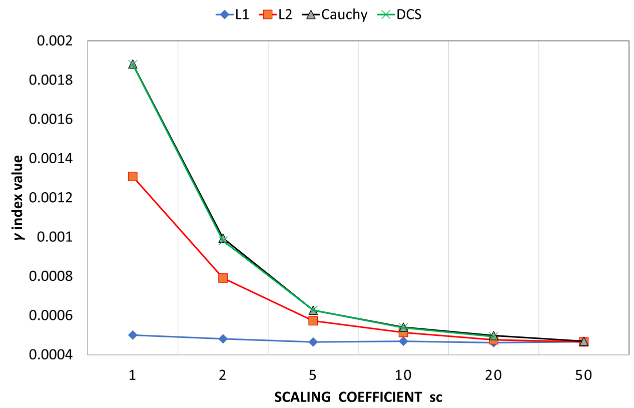

FIGURE 11. Performance index impact measured with γ.

function converge to the same function. It should be noticed that for large scaling sc = 20 the ℓ_{DCS}-MPC loses stability.

Similar behavior appears for IAE and σ_G CPA measures. Three robust measures: LMS (Fig. 9), σ_H (Fig. 10) and γ (Fig. 11) show slightly different pattern, however still ℓ_1-MPC exhibits the best values. It confirms outlier robustness properties observed in the CPA analysis [32].

Differential entropy H_{diff} sketched in Fig. 12 detects performance in a similar way to MSE, while rational entropy H_{rat} presented in Fig. 13 exhibits the highest scattering of the curves.

Table 5 presents CPA indexes for four undisturbed MPC controllers, while the next Table 6 shows similar comparison for the simulations disturbed with α-stable disturbance variable. Tabular results confirm earlier observations. In an undisturbed case the ℓ_1-MPC reaches the best performance according to all used CPA measures. The results for disturbed simulations show a different pattern. Outlier sensitive CPA

TABLE 5. CPA indexes in undisturbed simulations (best value highlighted).

MPC type	MSE	IAE	σ_{G}	σ_{H}	γ	H_{diff}	H_{rat}	LMS
ℓ_1-MPC	$\mathbf{7.457 \times 10^{-2}}$	$\mathbf{5.065 \times 10^{-2}}$	$\mathbf{2.731 \times 10^{-1}}$	$\mathbf{1.123 \times 10^{-3}}$	$\mathbf{4.681 \times 10^{-4}}$	$\mathbf{1.413 \times 10^{3}}$	$\mathbf{1.342}$	$\mathbf{4.114 \times 10^{-7}}$
ℓ_2-MPC	7.487×10^{-2}	5.163×10^{-2}	2.736×10^{-1}	1.218×10^{-3}	5.128×10^{-4}	1.406×10^{3}	1.354	4.864×10^{-7}
ℓ_{Cauchy}-MPC	7.606×10^{-2}	5.328×10^{-2}	2.758×10^{-1}	1.278×10^{-3}	5.399×10^{-4}	1.399×10^{3}	1.391	5.479×10^{-7}
ℓ_{DCS}-MPC	7.668×10^{-2}	5.319×10^{-2}	2.769×10^{-1}	1.273×10^{-3}	5.371×10^{-4}	1.400×10^{3}	1.373	5.377×10^{-7}

TABLE 6. CPA indexes in disturbed simulations (best value highlighted).

MPC type	MSE	IAE	σ_{G}	σ_{H}	γ	H_{diff}	H_{rat}	LMS
ℓ_1-MPC	7.975×10^{-2}	6.772×10^{-2}	2.824×10^{-1}	$\mathbf{1.016 \times 10^{-2}}$	$\mathbf{5.278 \times 10^{-3}}$	$\mathbf{1.272 \times 10^{3}}$	1.422	$\mathbf{3.959 \times 10^{-5}}$
ℓ_2-MPC	$\mathbf{7.797 \times 10^{-2}}$	$\mathbf{6.687 \times 10^{-2}}$	$\mathbf{2.792 \times 10^{-1}}$	1.123×10^{-2}	5.878×10^{-3}	1.244×10^{3}	$\mathbf{1.392}$	4.944×10^{-5}
ℓ_{Cauchy}-MPC	7.938×10^{-2}	6.858×10^{-2}	2.818×10^{-1}	1.178×10^{-2}	6.175×10^{-3}	1.238×10^{3}	1.400	5.393×10^{-5}
ℓ_{DCS}-MPC	7.957×10^{-2}	6.836×10^{-2}	2.821×10^{-1}	1.178×10^{-2}	6.153×10^{-3}	1.234×10^{3}	1.441	5.387×10^{-5}

TABLE 7. CPA index relative change versus the best one in undisturbed case (best value highlighted).

MPC type	MSE	IAE	σ_{G}	σ_{H}	γ	H_{diff}	H_{rat}	LMS
ℓ_1-MPC	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**
ℓ_2-MPC	0.40%	1.92%	0.20%	8.39%	9.55%	0.52%	0.91%	18.25%
ℓ_{Cauchy}-MPC	1.99%	5.19%	0.99%	13.78%	15.33%	0.96%	3.61%	33.19%
ℓ_{DCS}-MPC	2.82%	5.01%	1.40%	13.33%	14.74%	0.91%	2.33%	30.71%

TABLE 8. CPA index relative change versus the best one in disturbed case (best value highlighted).

MPC type	MSE	IAE	σ_{G}	σ_{H}	γ	H_{diff}	H_{rat}	LMS
ℓ_1-MPC	2.28%	1.27%	1.15%	**0.00%**	**0.00%**	**0.00%**	2.16%	**0.00%**
ℓ_2-MPC	**0.00%**	**0.00%**	**0.00%**	10.53%	11.37%	2.20%	**0.00%**	24.88%
ℓ_{Cauchy}-MPC	1.81%	2.56%	0.93%	15.94%	17.00%	2.67%	0.57%	36.22%
ℓ_{DCS}-MPC	2.05%	2.23%	1.04%	15.94%	16.58%	2.99%	3.52%	36.07%

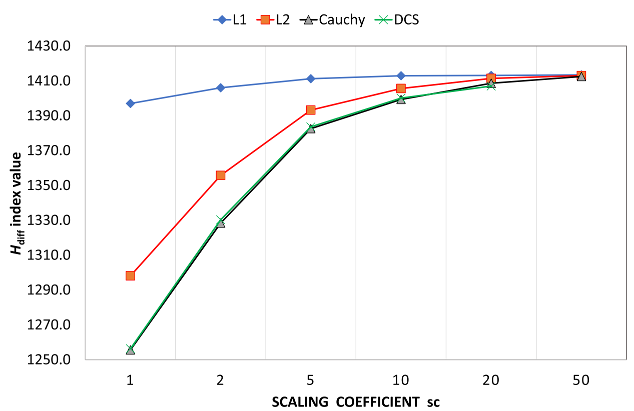

FIGURE 12. Performance index impact measured with H_{diff}.

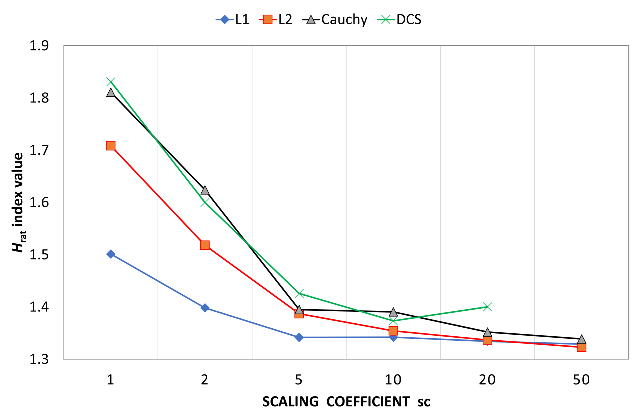

FIGURE 13. Performance index impact measured with H_{rat}.

indexes (MSE, IAE, σ_{G} and H_{rat}) select ℓ_2-MPC, while the robust ones (LMS, σ_{H}, γ and H_{diff}) point out the ℓ_1-MPC. It is due to the fact that the applied disturbance is fat-tailed. In such situations, the robust indexes are more appropriate. In all cases two other controllers, i.e. ℓ_{Cauchy} and ℓ_{DCS}-MPC exhibit worse performance.

Moreover, the use of the ℓ_1-cost function seems to be less sensitive to scaling sc in view of obtained results. That seems to be a key advantage of the ℓ_1-MPC as it significantly facilitates its application.

Tables 7 and 8 present how the MPC control performance is assessed by various performance measures. Tables show relative improvement measured versus the best case. Such data presentation informs about the scale of the improvement exhibited by the best measure.

Above observations are consistent with the ones noticed in the context of the controller tuning according to the mean square error and absolute error cost function. Research shows that tuning minimizing the MSE punishes large deviations and causes aggressive control [42]. The absolute error is less conservative and it is often used for an on-line controller tuning. The IAE has the closest relation with economic aspects [43]. It penalizes continued cycling.

Finally, let us compare the calculation time of all four considered MPC algorithms. It turns out that the ℓ_1-MPC method is almost twice more computationally demanding than the other MPC schemes. Let the scaled computational time for the ℓ_1-MPC algorithm equals to 100%. The obtained times are: 55.37%, 54.72% and 53.31% for ℓ_2-MPC, ℓ_{Cauchy}-MPC and ℓ_{DCS}-MPC approaches.

IV. CONCLUSION AND FURTHER RESEARCH

Performed research focuses on the possibility to improve MPC control through the use of other, outlier robust, performance index formulation. Classical ℓ_2 cost function is compared against ℓ_1-MPC and two other formulations originating from other contexts, i.e. ℓ_{Cauchy}-MPC and ℓ_{DCS}-MPC.

The results show consistent observations. ℓ_1-MPC always exhibits the best performance, independently on the applied CPA measure. Furthermore, it is less sensitive to scaling sc. At the same time, it obtains the shortest step response settling time and the smallest overshoot. It shows that linear introduction of the cost function is more realistic than the quadratic one. Finally, obtained results show that two selected robust estimators using Cauchy function and Dynamic Covariance Scaling are not improving MPC operation. As positive properties of the ℓ_1 norm are well known, it is expected that similar results might be observed in other MPC applications as well.

It is worth to evaluate the performance of other possible robust regression functions. As in the present research, the control cost element in the MPC performance index is still quadratic, one might be interested in verifying whether its formulation has any impact. The other opportunity would be to exchange mean operator in MPC performance index with a median.

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PAWEŁ D. DOMAŃSKI was born in Warsaw, Poland, in 1967. He received the M.S., Ph.D., and D.Sc. degrees from the Faculty of Electronics and Information Technology, Warsaw University of Technology, in 1991, 1996, and 2018, respectively, all in control engineering.

Since 1991, he has been working with the Institute of Control and Computational Engineering, Warsaw University of Technology. Apart from scientific research, he participated in dozens of industrial implementations of APC and optimization in power and chemical industries. He is the author of one book and more than 100 publications. His main research interests include industrial APC applications, control performance quality assessment, and optimization.

MACIEJ ŁAWRYŃCZUK was born in Warsaw, Poland, in 1972. He received the M.Sc., Ph.D., and D.Sc. degrees from the Faculty of Electronics and Information Technology, Warsaw University of Technology, in 1998, 2003, and 2013, respectively, all in automatic control.

He has been with the Institute of Control and Computation Engineering, Warsaw University of Technology, since 1998, where he is currently an Associate Professor. He is the author or coauthor of six books and more than 100 other publications, including more than 40 journal articles. His research interests include advanced control algorithms, in particular, Model Predictive Control (MPC) algorithms, set-point optimization algorithms, soft computing methods, in particular neural networks, modeling, and simulation.